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between Taxation and Long Term Equilibrium
in a Model of Endogenous Lifetime and
Economic Growth**

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The Non-monotonic Relationship between Taxation and Long Term Equilibrium in a Model of Endogenous Lifetime and Economic Growth

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Abstract

In this paper, we shed light on the relationship between taxation and steady states by analyzing the OGL model of Chakraborty [2004]. We show that there is (i) a non-monotonic relationship between taxation and growth; (ii) a threshold value of taxation in such a way that there is no effect on the steady state; and finally, (iii) the effect on the two non-trivial steady states is not equal: an increase in tax rate will initially reduce (increase) the unstable (stable) steady state and will then increase (reduce) it.

JEL: H51, I12, O1, O41

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1. Introduction

Different authors have recently analyzed the relationship between life expectancy and economic growth. Longevity has been treated as exogenous in earlier works (cfr. Enhrlich and Lui,1991; de la Croix and Licandro, 1999) while recently it has been endogenized through either (or both) public or/and private health expenditure (i.e. Kalemly-Ozcam,2002 and Chakraborty, 2004). Both models reveal a positive relationship between length of life and growth, confirming the empirical evidence. Moreover endogenous models are even able to explain the persistent low level of per capita income, known as the poverty trap.

Chakraborty (2004) introduces the probability of surviving in a two-period overlapping generation model. The probability depends strictly on public investment in health, which is financed through a proportional taxation. The poverty trap can arise, depending strictly on high output elasticity and a second condition already discussed in Bunzel and Qiao (2005). His main outcome is that deviations in the growth pattern can be explained by looking at differences in health and/or physical capital.

In this paper, analyzing the Chakraborty model, we shed (analytically) light on the relationship between the tax rate and steady states¹. Focusing on the case of the existence of the poverty trap (when there are three steady states) and using a specific function for the surviving probability, suggested by Chakraborty, we find that (i) there is a non-monotonic relationship between taxation and growth; (ii) there is a threshold value of taxation in such a way that there is no effect on the steady state; and finally, (iii) the effect on the two non-trivial steady states is not equal: an increase in tax rate will initially reduce the unstable steady state (reducing the area of the poverty trap) and will then increase it. On the contrary, the positive stable steady state initially has a positive relationship and then a negative one with the tax rate. In the second paragraph we briefly explain the Chakraborty model, while in the third one a proposition on the relation between taxation and steady states is stated and demonstrated, reporting some numerical simulations. The final part examines policy considerations and draws some conclusions.

2.The Model

¹ In a recent paper Ponthiere (2006) has underlined that in the Chakraborty model there is an unknown effect of the tax rate on steady states, but his statement has been not demonstrated.

The exact model employed by Chakraborty (2004) is used. There is an economy of a two-period lived overlapping generation and an infinitive-lived-government. In each period young agents of measure one are born with a time endowment of one unit, which they inelastically supply, earning a wage ω . The probability of young agents born at t , surviving to the next period depends on their health capital, h_t , which is a non-decreasing concave function $\phi_t \equiv \phi(h_t)$, that satisfies the following assumptions: $\phi(0) = 0$, $\lim_{h \rightarrow \infty} \phi(h) = \beta \leq 1$ and $\lim_{h \rightarrow 0} \phi'(h) = \gamma < \infty$.

We employ the specific function of $\phi(h)$ suggested by Chakraborty in Note 4 (on page four):

$$\phi(h) = \frac{\beta h}{(1+h)} \quad \text{with } \gamma = \beta$$

[1]

The health capital is a function of public health expenditure, financed through proportional taxation $\tau \in (0,1)$ on labour income.

$$h_{t+1} = g(\tau w_t) = \tau w_t$$

[2]

Agents' preferences are represented by the following utility function:

$$U_t = \ln c_t^t + \phi_t \ln c_{t+1}^t$$

[3]

where ϕ_t , the survival probability of a young person born at t , is the positive discount factor. Utility is maximized subject to the budget constraint:

$$c_t^t \leq (1 - \tau_t)w_t - z_t \quad \text{and} \quad c_{t+1}^t \leq \hat{R}_{t+1} z_t$$

where savings, z_t , are invested in capital with a gross return of \hat{R}_{t+1} . In each period, young agents save a part of their disposal income $z_t = (1 - \tau_t)\sigma_t w_t$ where σ_t symbolizes the savings propensity and is a function of the increasing surviving probability

$$\sigma_t \equiv \frac{\phi_t}{1 + \phi_t} = \frac{\beta h}{1 + h + \beta h}$$

[4]

A canonical Cobb-Douglas with a constant return to scale technology represents the production function:

$F_t(K, L) = A_t K_t^\alpha L_t^{1-\alpha}$. Perfect competition guarantees that marginal products equalize prices of both labour and capital, hence:

$$w_t = (1 - \alpha)A_t k_t^\alpha$$

$$R_t = 1 + \alpha A_t k_t^{\alpha-1} - \delta$$

where k_t is the capital-labour ratio and δ the depreciation rate of capital. Equalizing savings and capital,

$z_t = k_{t+1}$, the general equilibrium follows:

$$k_{t+1} = (1 - \tau)(1 - \alpha)\sigma(k_t)A k_t^\alpha$$

[5]

Substituting a (1) for the (5), the following first order difference equation is obtained:

$$k_{t+1} = \frac{(1 - \tau)(1 - \alpha)^2 \beta \tau A^2 k_t^{2\alpha}}{1 + \tau(1 - \alpha)(1 + \beta)A k_t^\alpha}$$

[6]

According to Bunzel and Qiao (2005) we can define $J(k)$ as the right hand side of six (6); steady states are

identified from $J(k) = \bar{k}$. Hence by rewriting six and simplifying, the following relation can be obtained:

$$(1 + \beta) = (1 - \tau)\beta A(1 - \alpha)k^{\alpha-1} - \frac{k^{-\alpha}}{A(1 - \alpha)\tau} = G(k)$$

[7]

The left hand side of seven is a constant, while the right hand side is an unimodal curve in k initially increasing and then decreasing. The intersections between the two curves determine the number of fixed points.

Substituting the value of k that maximizes the right hand side in $G(k)$, it is easy to find the value of parameters that guarantees the existence of steady states. Hence:

$$k^{\max} = \left(\frac{\alpha}{A^2 (1 - \alpha)^3 \tau (1 - \tau) \beta} \right)^{\frac{1}{2\alpha-1}}$$

[8]

Specifically, a bifurcation fold leads to the existence of three steady states, the trivial stable solution ($k_0 = 0$)

and two positive steady states ($\bar{k}_1 < \bar{k}_2$) if:

$$G(k^{\max}) > (1 + \beta)$$

that is, as already demonstrated by Bunzel and Qiao (2005) when

$$\alpha > 1/2 \text{ and } (A(1-\alpha))^{\frac{1}{2\alpha-1}} \left(\frac{(1-\alpha)}{\alpha} (1-\tau)\beta\tau \right)^{\frac{\alpha}{2\alpha-1}} \left(\frac{2\alpha-1}{(1-\alpha)\tau} \right) > (1 + \beta).$$

There is a poverty trap if the initial capital condition lies in the interval $[0, \bar{k}_1)$, given that zero is stable.

Otherwise the capital pro-capita converges to \bar{k}_2 ; if it lies in the interval (\bar{k}_1, ∞) . \bar{k}_1 is always unstable.

3. Taxation Analysis

Taxation has a double effect in the model. Tax rate has the classic negative effect on a steady state through the reduction of savings. However, in this model financing public health expenditure, taxation has an indirect effect, increasing the probability of surviving and therefore saving.

Proposition. Assuming that there are three steady states, the trivial stable solution ($k_0 = 0$) and two positive steady states ($\bar{k}_1 < \bar{k}_2$):

(i) $\bar{\tau}$ is a critical value, as such there are no variations of \bar{k}_i . Particularly, $\frac{\partial k_i}{\partial \tau} \leq 0$ if $\tau < \bar{\tau}$, $\frac{\partial k}{\partial \tau} = 0$ if

$$\tau = \bar{\tau} \text{ and } \frac{\partial k}{\partial \tau} \geq 0 \text{ if } \tau > \bar{\tau};$$

(ii) let us define \bar{k}^* as a fixed point and τ^* the corresponding threshold value of the tax rate. When all other is equal, for a lower value of $\bar{k} \leq \bar{k}^*$ there is a lower threshold value $\bar{\tau} \leq \tau^*$;

(iii) $\bar{\tau}$ is a minimum value for \bar{k}_1 and a maximum value for \bar{k}_2 ; hence $\frac{\partial k_1}{\partial \tau} < 0$ if $\tau < \bar{\tau}$ and $\frac{\partial k_1}{\partial \tau} > 0$

when $\tau > \bar{\tau}$, vice versa for \bar{k}_2 , $\frac{\partial k_2}{\partial \tau} > 0$ for $\tau < \bar{\tau}$ and $\frac{\partial k_2}{\partial \tau} < 0$ if $\tau > \bar{\tau}$.

Proof.

Equation (7) is a relation between k and τ . It can be rewritten as:

$$g(k, \tau) = (1 + \beta) - (1 - \tau)\beta A(1 - \alpha)k^{\alpha-1} + \frac{k^{-\alpha}}{A(1 - \alpha)\tau}$$

[9]

We use the implicit function theorem to work out the derivate of k compared to τ . Hence:

$$k'(\tau) = -\frac{\frac{\partial g}{\partial \tau}}{\frac{\partial g}{\partial k}} = -\left[\frac{[\beta A^2(1 - \alpha)^2 \tau^2 k^{2\alpha-1} - 1]k}{\beta A^2(1 - \alpha)^3 \tau(1 - \tau)k^{2\alpha-1} - 1} \right]$$

[10]

$k'(\tau) = 0$ when the numerator equals zero, the values at which there is no taxation effects on the steady states can be detected. That is, trivially for $k = 0$ and for $\tau = 0$, or for $\beta A^2(1 - \alpha)^2 \tau^2 k^{(2\alpha-1)} - 1 = 0$

hence for

$$\bar{\tau} = \sqrt{\frac{1}{\beta A^2(1 - \alpha)^2 k^{(2\alpha-1)}}$$

[11]

Furthermore for a particular configuration of the parameters it is true that $0 < \bar{\tau} \leq 1$.

The statement (ii) can be achieved directly by exploring equation (11). For $\alpha > 1/2$, the lower the value of the steady state is, the higher the threshold value $\bar{\tau}$. If there is shock present that changes the other parameters of our model, and these changes trigger a decrease of the steady state capital, the corresponding threshold value ($\bar{\tau}$) is higher.

However the effect of τ on k depends strictly on the denominator. Therefore to demonstrate the proposition (iii), we have to study the signs of both numerator and denominator.

The numerator is positive for:

$$k < k_a = \left(\frac{1}{\beta A^2 (1-\alpha)^2 \tau^2} \right)^{\frac{1}{2\alpha-1}}$$

[12]

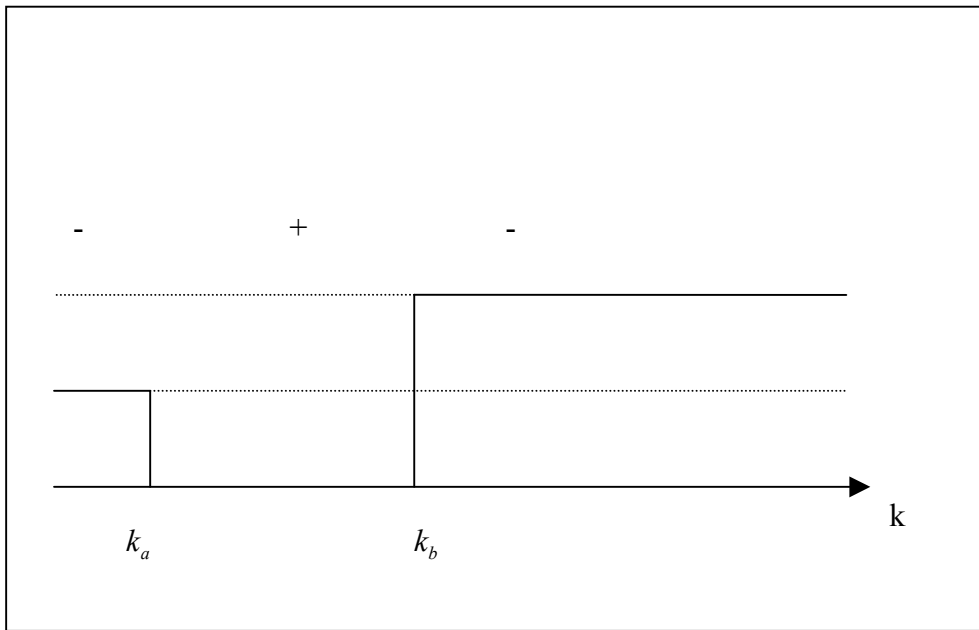
while the denominator is positive for

$$k > k_b = \left(\frac{1}{\beta A^2 (1-\alpha)^3 \tau (1-\tau)} \right)^{\frac{1}{2\alpha-1}}$$

[13]

Using straightforward algebra, and given our assumptions, it is always true that $k_a < k_b$, therefore the sign of the equation (10) follows the pattern reported in Figure 1.

Figure 1. Signs of denominator and numerator of equation (11)



To identify the different effects of τ on \bar{k}_1 and \bar{k}_2 we use the k^{\max} .

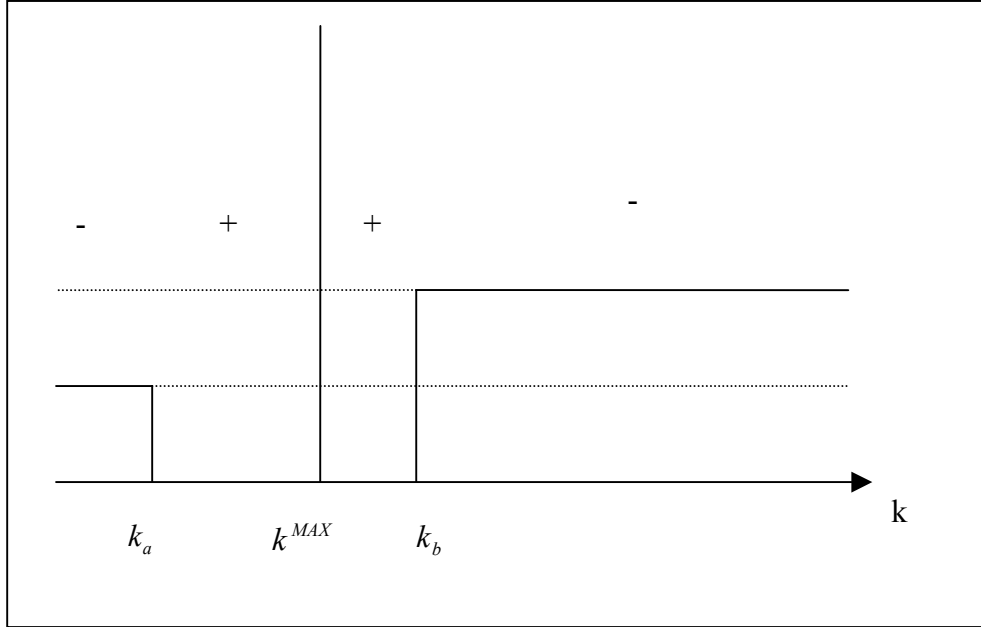
Given the bifurcation fold we already know that:

$$k_1 < k^{\max} = \left(\frac{\alpha}{A^2 (1-\alpha)^3 \tau (1-\tau) \beta} \right)^{\frac{1}{2\alpha-1}} < k_2$$

Therefore detecting k^{\max} in Figure 1, we are able to explore the relationship between τ and both \bar{k}_1 and \bar{k}_2 .

Particularly, it is easy to show that: $k_a < k^{MAX} < k_b$. In Figure 2, we show this relationship.

Figure 2. k^{\max} in the equation (11)



Therefore for values lower than k^{MAX} (i.e. \bar{k}_1), an increase in the tax rate initially reduces the steady state and then increases it. Otherwise, for values of capital higher than k^{MAX} (i.e. \bar{k}_2), an increase in the tax rate first leads to an increase in the capital pro-capita and then reduces it.

In Table 1, values of \bar{k}_2 for different values of τ and β are reported, fixing $\alpha = 0.55$ and $A = 1000$. For all values of β , an increase of the tax rate initially increases and then decreases the steady state. Furthermore, as stated in (ii), an increase of \bar{k}_2 , determined by an increase in β , affects the threshold value $\bar{\tau}$; particularly for $\beta = 0.1$ the threshold lies between $\tau = 0.005$ and $\tau = 0.005$, while for $\beta = 0.5$ it lies between $\tau = 0.001$ and $\tau = 0.005$.

Table 1: Values of the \bar{k}_2 calculated for different values of the parameters

	$\tau = 0.0005$	$\tau = 0.001$	$\tau = 0.005$	$\tau = 0.05$
$\beta = 0.1$	3452.7232	3627.4760	3739.0995	3403.0923
$\beta = 0.5$	67449.925	67865.828	67653.540	61120.738
$\beta = 0.9$	148318.18	148701.06	147819.14	133461.09

Other Parameters: $\alpha = 0.55$; $A = 1000$

4. Policy Considerations and Conclusions

In the last decade various authors have focused their work on the effects of life expectancy on economic growth. In a recent paper, Chakraborty (2004) endogenizing the probability of surviving in a standard overlapping generation model, revealed a positive relationship between length of life and growth and detected the possibility of a poverty trap. Specifically, he assumes that public health expenditure, financed by proportional taxation, can increase the probability of surviving and therefore the savings and capital of each worker.

Focusing on the case of the existence of a poverty trap and given the function of probability of surviving suggested by Chakraborty, we have studied the relationship between taxation and steady states. We find that this relation is non-monotonic, existing as a threshold value of taxation in such a way that there is no effect on the steady state. Taxation mainly affects the two non-trivial steady states in different ways: while an increase in the tax rate will initially reduce and then increase the unstable steady state, it has the opposite effect on the positive stable steady state.

These results arise from two opposite effects that taxation has on savings (and therefore on growth). While the classic negative effect on a steady state by means of a reduction of savings works, it has a positive effect, since financing public health expenditure, enables an increase in the length of life that is positively related to savings. It is worth analyzing the consequences in terms of political economy. A developing country, stylized by an initial condition that lies in the basin of the poverty trap basin, has an incentive to increase taxation as the positive

effect on surviving is greater than the negative effect on savings, which then leads to a reduction of the poverty trap.

The same effect arises for a developed country. The positive effect of a health programme is initially greater than the negative effect on savings. However the higher per worker capital is, the lower the tax rate threshold will be, which signifies that the more developed the country is, the more negative the effect of taxation on savings, and therefore on growth, will be.

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