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The Expectations Hypothesis of Term Structure of Interest Rates Revisited

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Abstract

This paper investigates the validity of the Expectations Hypothesis of the Term Structure (EHTS) employing standard forward-spot regressions which accommodate for the presence of time-varying term premia. The novelty of this paper is that the analysis is conducted by taking advantage of the following two properties of the Kalman Filter: first, it makes possible to model time-varying term premia as unobservables, and second it delivers recursive estimations of forward-spot regressions as more data become available. In fact, previous studies have modelled term premia by means of macroeconomic variables. To the extent that term premia are influenced by political and social climates which are difficult to observe, it might be preferable to model them as unobservables, rather than by means of observed variables. Moreover, especially when tested over long periods of data, the EHTS might hold for certain periods while it might not for others. These periodic departures from and reversions to the EHTS cannot be detected by constant parameters models, which therefore can provide only broad brush evidence. This paper shows that the recursive nature of the Kalman filter can be employed to construct a test for the EHTS which gives more refined evidence. The analysis is carried out focusing on the short-end of the US term structure spectrum.

JEL classification: C32, E43
Keywords: Term structure of interest rates; Monetary regimes; Kalman filter

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1 Introduction

The basic idea underlying the Expectations Hypothesis of the Term Structure of interest rates (EHTS) is that, with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. A second version of the EHTS states that, with the exception of a term premium, forward rates are unbiased predictors of future spot rates. If forward rates are unbiased predictors of future spot rates, excess returns should not be predictable on the basis of predetermined information. Thus, a straightforward way to test the EHTS is to employ linear regression to determine whether excess returns are predictable. Predictability of excess returns have been tested by including in linear regressions explanatory variables like forward-spot rate differentials or term spreads.[1]
Throughout this paper analysis will be carried out that focuses on the second version of the EHTS and makes use of forward-spot regressions. Tests based on single regression models with fixed parameters consistently reject the null that the EHTS holds, especially when applied to the US term structure. Such rejections, in turn, can be motivated by either departures from rational expectations or time-varying term premium, or both (see, for instance, Fama (1984) and Fama and Bliss (1987)).
However, it is now well-established the importance of modelling term premia, and of identifying their linkages with monetary regimes and, more generally, with levels of riskiness of the economics environment. Dotsey and Otrok (1995), for instance, explore the linkage between the Fed's reaction function and time-varying term premia. Lee (1995) proposes a model which captures uncertainty on both the real and monetary sides of the economy, allowing term premia to depend on the conditional variance-covariance matrix of output and money supply (see also Engle et al. (1987) and Lauterbach (1989)). In turn, the acknowledgment of the presence of time-varying term premia resulted into single regression models to be supplemented in order to accommodate for their time variability. In these studies time-varying term premia are modelled by means of observables such as macroeconomic variables or riskiness indicators like conditional standard deviations, while the rational expectations hypothesis is tested by checking the statistical significance of the parameters attached to forward-spot differentials or term spreads (see, among others, Jones and Roley (1983), Simon (1989), Tzavalis and Wickens (1997), Boero and Torricelli (2002), Bams and Wolff (2003) and Harris (2004)). The EHTS is, in fact, a joint hypothesis of rational expectations and constant term premium. Given that the EHTS is soundly rejected when tested by making use of standard regression models with fixed parameters, these studies assume that the EHTS is rejected because of time-varying term premium and test for its second leg, i.e. for rational expectations. In other words, they accommodate for time varying term premium in the hope of obtaining evidence more in favor of rational expectations.

[1]The term "term spreads" is used in the literature to define the difference between a long-term and a short-term interest rate while the term forward-spot differentials indicates the difference between forward and actual spot interest rates of identical maturity.

One shortcoming of this strand of literature is that tests of the EHTS were carried out by making use of regressions with fixed parameters. Because of that, they can only provide broad brush conclusions on whether the expectations hypothesis holds. When, in fact, the expectation hypothesis is investigated over a given lapse of time, it might hold for some specific subperiods while it might be rejected for some others. This is especially true when long periods of data are considered, where changes in the economic and political environment might induce temporary departures from rational expectations. This, of course, cannot be captured by a constant parameters regression which, by definition, tend to average out periods of departure and periods of reversion to rational expectations. It is in this sense that tests for the expectations hypothesis based on constant parameters models can only provide broad brush conclusions. They cannot disentangle periods of departures from rational expectations from those where rational expectations hold. Equally important, they cannot measure how persistent are departures from rational expectations. A more refined method to test for rational expectations would be to recursively estimate the parameters of the single regression models. Recursive estimation, in fact, delivers at each stage of the recursion estimates of the various coefficients of the model as well as their standard errors. Thus, as more data is added to the estimating regression, the parameter attached to the forward-spot differentials (or term spreads) might assume values statistically significant for some specific time t and values not significant for some other time t in the sample period (therefore indicating temporary departures from and reversions to rational expectations).

The second shortcoming of the literature is related to the fact that the time-varying term premium must be treated like an unobservable (or latent) variable. For instance, the factors which affect the term premium are likely to be those influencing both the political and social climates (see Gravelle and Morley (2005)). Such factors, in turn, are difficult to observe. In order to model the time-varying term premium, a largely employed approach was to proxy for the term premium by means of observables. For instance, Boero and Torricelli (2002) model the term premium by making use of linear combinations of both yields and macroeconomic data.[2] Similarly, Dotsey and Otrok (1995) model the term premia by means of the Fed's reaction function while Tzavalis and Wickens (1997) use one version of the EHTS to work out the time-varying term premium as the difference between the expected holding return of holding an n-period bond for one period and the short rate. Yoo (2003), extending the econometric framework elaborated by Engle et al. (1987) in a Bayesian fashion, estimates the time-varying term premium by means of a GARCH-M model where the level of the term premium is a linear function of the regression's conditional variance. The common feature of all the previously mentioned studies is that, in order to recover the term premia, they adopt inherently *ad hoc* approaches.[3] In this regard, the least restricted methodology proposed by the

[2] Other studies which model time-varying term premia along these lines are, for instance, Jones and Roley (1983) and Simon (1989).

[3] For instance, in Yoo (2003) the time-varying term premium must necessarily depend on both past squared innovations and past conditional volatility.

literature is the imposition of a certain stochastic process, say an AR(1), on the unobservable time-varying term premium and detection of it by means of Kalman filtering techniques. This is actually the methodology followed by Wolff (1987), Iyer (1997) and Gravelle and Morley (2005).[4] All these authors pursue an alternative to the single regression approach to investigate the relevance of the EHTS. In fact, they construct the excess forward returns (i.e. forward rates minus future realized spot rates) as the sum of a (unobservable) time-varying term premium plus forecast errors. Then, assuming rational expectations, they employ the Kalman filter to extract the term premium from excess forward returns. Thus, without testing for the unbiasedness of forward rates, they can reject the EHTS if the term premium is found to be time-varying.

The novelty of this work is that the EHTS is investigated by means of single regression models with time-varying parameters, that are implemented by means of Kalman filter and maximum likelihood. It can be shown that this econometric framework is preferable to standard constant parameters models under many different respects. The first advantage is that, similarly to Wolff (1987), Iyer (1997) and Gravelle and Morley (2005), the time-varying term premium can be modelled as an unobservable variable. However, unlike these previous studies, this work incorporates the advantages of modelling the term premium by following the single regression approach. This, in turn, is equivalent to testing the EHTS without taking any *a priori* stance about the validity of the rational expectations hypothesis. Second, a desirable property of the Kalman filter is that it produces recursive estimations of the time-varying parameters together with their standard deviations at each time t included in the sample period. Exploiting this feature it becomes possible to disentangle the subperiods in which departures from the EHTS occur from those where the hypothesis holds, as well as to measure the persistency of these departures. This analysis is carried out by testing for the statistical significance of the parameter attached to forward-spot differentials. Third, as shown by Kim and Nelson (1999), the Kalman filter estimates of time-varying parameters models are preferable to standard recursive Least Squares estimates. More specifically, regardless of the hypothesis of Gaussian state space, the Kalman filter delivers recursive estimates of the parameters which are unbiased estimators of the true parameters of the regression, with minimum Mean Squared Error (MSE).[5] Fourth, as pointed out by the same authors, the Kalman filter gives insight into how a rational economic agent would revise his estimates of the coefficients in a Bayesian fashion when new information becomes available in a world of uncertainty, especially under changing policy regimes. In other words, it updates parameter estimates mimicking the behavior of a rational agent, as more data becomes available. Fifth, to the best of my knowledge, a test for the EHTS through a single regression model with time-varying parameters, that is implemented by means of Kalman

[4] Wolff (1987) applies the Kalman filtering technique to detect the time-varying term premium in the forward exchange rates market. His technique, however, can be straightforwardly applied to the term structure's context.

[5] In a Gaussian state space the disturbance terms of the measurement and transition equations as well as the initial state vector are normally distributed.

filter techniques, is new in the literature. More specifically, in Section 6 empirical estimations are carried out by employing a model which accounts for both time-varying parameters and moving average processes in the disturbance term which arises when data frequency is higher than the term of the investment. This last extension allows the testing of the EHTS on actual observed forward contracts with settlement dates of more than one period in the future, rather than just on the one-period-ahead case.
Since a large part of the literature supports the idea that term premia are time-varying because they are related to the monetary policy stances of central bankers, the EHTS is investigated across different monetary regimes. This is another original aspect of the analysis carried out in this study. Following Goodfriend (1998) and Bordo and Haubrich (2004), the period under analysis is divided into three sub-periods which correspond to different monetary regimes, and the EHTS is tested against each separate regime. The analysis is carried out employing three, six and twelve month spot interest rates for the United States. The period analyzed spans from January 1960 to May 2000.
The paper is organized as follows: Section 2 introduces the EHTS and the single regression model used in this study. Section 3 describes the data while Section 4 sets out the empirical estimates when term premia are assumed constant. Section 5 reports some tests for the stability of the parameters of single regression models employed while Section 6 sets out the models which accommodates for time-varying term premium, together with their empirical estimates. Section 7 presents an alternative approach to test the EHTS, based on the aforementioned property of the Kalman filtering technique which consists of delivering estimates of the state variables (the time-varying parameters) together with their standard deviations at each time t included in the sample period. Section 8 presents the conclusions.

2 The Expectations Hypothesis of Term Structure

The EHTS consists of the idea that, with the exception of a term premium, forward rates are unbiased predictors of future spot rates. Based on the no-arbitrage relation between forward and spot rates, the EHTS can be expressed as follows:

$$F^n_{n-m}(t) = E_t[R_m(t+n-m)] + TP^n_{n-m}(t) \tag{1}$$

where $R_m(t)$ is the m-period short rate and $F^n_{n-m}(t)$ the m-period forward rate, i.e. the rate at trade date t for a loan between periods $(t+n-m)$ and $(t+n)$, E_t is the Expectations Operator conditional on information available at time t and $TP^n_{n-m}(t)$ is the associated constant term premium. If we allow the term premium to be time-varying, then eq.(1) turns into an arbitrage relationship in which the only restriction imposed is that the term premium must be station-

ary.[6] This relationship, therefore, turns out to be more general than the EHTS which, in its pure form, requires a term premium equal to 0 and, in its weaker form (the Liquidity Preference Theory), requires a positive and constant term premium.
By isolating all the terms of eq.(1) on its right-hand side, by multiplying by -1 and then by adding $[R_m(t+n-m) - R_m(t)]$ to both sides, one can obtain:

$$[R_m(t+n-m) - R_m(t)] = [F^n_{n-m}(t) - R_m(t)] - TP^n_{n-m}(t) + \mu_{t+n-m}(t) \quad (2)$$

where $\mu_{t+n-m}(t) = [R_m(t+n-m) - E_t(R_m(t+n-m))]$ is a random forecast error with a conditional mean of zero.[7] According to eq.(2), the change in the short term interest rates between time t and t+n-m must be equal to the term spread adjusted for a term premium plus a random forecast error $\mu_{t+n-m}(t)$. From this relationship one can construct the first version of the test for the validity of the EHTS:

$$[R_m(t+n-m) - R_m(t)] = \delta_0 + \delta_1[F^n_{n-m}(t) - R_m(t)] + \mu_{t+n-m}(t). \quad (3)$$

If the EHTS holds, then $\delta_1 = 1$ and the term premium, here assumed to be constant, is equal to $-\delta_0$.
The second version of the test for the validity of the EHTS can be obtained from eq.(3) by subtracting $[F^n_{n-m}(t) - R_m(t)]$ and then multiplying by -1 each side with the following result:

$$[F^n_{n-m}(t) - R_m(t+n-m)] = \beta_0 + \beta_1[F^n_{n-m}(t) - R_m(t)] + \varepsilon_{t+n-m}(t) \quad (4)$$

where $\varepsilon_{t+n-m}(t) = -\mu_{t+n-m}(t)$. If the EHTS holds, then $\beta_1 = 0$ and the term premium, assumed to be constant, is equal to β_0.[8]
Eq.(4) is the baseline relationship which is considered in order to inspect the validity of the EHTS and it is known in the literature as forward-spot regression. For instance, focusing on three-month interest rates and a forecast horizon of three months, it is possible to simplify the notation setting $n = 6$ and $m = 3$ to obtain:

$$[F^6_3(t) - R_3(t+3)] = \beta_0 + \beta_1[F^6_3(t) - R_3(t)] + \varepsilon_{t+3}(t). \quad (5)$$

Throughout the paper estimations of different versions of eq.(4) are carried out. While in Section 4 the term premium is assumed to be constant, in Section 6 the above equation accommodates for time-varying term premia. However, before discussing the merits of estimations, it seems appropriate to provide some theoretical justifications for the employment of eq.(1).

[6] Let us suppose, for example, that the time-varying term premium from positive turns out to be negative. What happens is that the forward rate, which at the beginning was higher than the expected future spot rate, becomes lower. As a result traders, who at the beginning are locked in a lending position with a forward contract and have borrowed at the lower expected spot rate, reverse their positions.

[7] It is important to recall that the EHTS can be viewed as a joint hypothesis where the first component is given by eq.(1) while the second component, the so called "rational expectations leg of the joint hypothesis", requires $\mu_{t+n-m}(t)$ to be orthogonal, or uncorrelated, to the information set available at time t.

[8] Notice that, since the regressor in eqs.(3) and (4) is the same, and the sum of the regressands equals the regressor, the two equations are entirely complementary. This, in turn, implies that $\beta_0 = -\delta_0$ and $\beta_1 = 1 - \delta_1$.

2.1 A Simple Model for the Forward-Spot Regressions

In this paragraph an asset pricing model with time-varying term premium is employed to provide theoretical justification for the use of forward-spot regressions based on condition (1). To do so I make use of a model that relies on discrete-time, inter-temporal asset pricing models such as those of Lucas (1978, 1982) and assumes that investors maximize expected discounted utility function defined on the future stream of consumption, subject to sequential budget constraints. In particular, setting the investment horizon to n, the optimal consumption path in equilibrium is given by the following Euler condition:

$$U^{'}(C_t) = \beta^n E_t[U^{'}(C_{t+n})R_n(t)] \tag{6}$$

where $U^{'}(C_t)$ is the marginal utility to consumption, β is the rate of patience and $R_n(t)$ is the rate of return from a n-period investment. Being the marginal utility to consumption at time t known, the Euler condition can be rewritten as follows:

$$1 = \beta^n E_t[\frac{U^{'}(C_{t+n})}{U'(C_t)} R_n(t)]. \tag{7}$$

Empirically, this class of models has been vastly employed to inspect the pricing of both common stocks and Treasury Bills as well as the term premium's determinants in foreign exchange markets (see, for instance, Grossman and Shiller (1981) and Mark (1985)). Defining $R_n(t)$ and $R_m(t)$ respectively the long and short interest rate, investors can choose to invest either into a n-period bond or roll-over m-period bonds. Setting n=6 and m=3 to simplify the notation, in the first case the condition (7) becomes:

$$1 = \beta^6 E_t\{\frac{U^{'}(C_{t+6})}{U'(C_t)} R_6(t)\}. \tag{8}$$

When, on the other hand, investors roll-over three period bonds the above condition becomes:

$$1 = \beta^6 E_t\{\frac{U^{'}(C_{t+6})}{U'(C_t)} [R_3(t) + R_3(t+3)]\}. \tag{9}$$

Taking the difference between conditions (8) and (9) and dividing by the rate of patience β^6 one obtains the following condition:

$$E_t\{\frac{U^{'}(C_{t+6})}{U'(C_t)} [R_6(t) - R_3(t) - R_3(t+3)]\} = 0 \tag{10}$$

which states that the conditional first cross-moment between the inter-temporal marginal rate of substitution to consumption and speculative excess return in bond markets, must be zero. Exploiting the conditional variance decomposition, it is then possible to write:

$$\begin{aligned} E_t\{R_6(t) - R_3(t) - R_t(t+3)]\} = \\ -\frac{Cov_t\{\frac{U^{'}(C_{t+6})}{U'(C_t)} [R_6(t) - R_3(t) - R_t(t+3)]\}}{E_t\{\frac{U^{'}(C_{t+6})}{U'(C_t)}\}}. \end{aligned} \tag{11}$$

Assuming absence of arbitrage between securities in the spot and forward markets, the following condition must hold:

$$R_6(t) = R_3(t) + F_3^6(t) = 0. \tag{12}$$

Substituting eq.(12) into eq.(11) one obtains:

$$E_t\{F_3^6(t) - R_3(t+3)\} = -\frac{Cov_t\{\frac{U'(C_{t+6})}{U'(C_t)}[F_3^6(t) - R_3(t+3)]\}}{E_t\{\frac{U'(C_{t+6})}{U'(C_t)}\}}. \tag{13}$$

The above condition shows that the term premium, defined as the difference between forward and future spot interest rates, must be proportional to the conditional covariance between the inter-temporal marginal rate of substitution to consumption and returns from interest rates speculation. Since the marginal rate of substitution is always positive, the sign of the term premium is determined by the above covariance which can depend on sample information and be time-varying. As such, also the term premium can fluctuate over time assuming both positive and negative values (see, for instance, Modigliani and Sutch (1966)).
Eq.(13) is the baseline model which has been used in the literature to identify the determinants of the time-varying term premium under rational expectations, and it provides a theoretical justification for the employment of eq.(1) in the analysis undertaken in this paper. Although the determinants of the term premium in eq.(13) are left in implicit form, it is possible to sophisticate the above framework in order to make them explicit. For instance, Lee (1995) employing a similar model shows the conditional (time-varying) variance-covariances of output and money supply to be among the determinants of the term premium (see also Engle et al. (1987), Lauterbach (1989) and Castillo and Fillion (2002)).

3 Data

The dataset employed consists of three, six and twelve months US Treasury Bill rates (average auctions). These data are taken from the FRED database at the Federal Reserve Bank of St. Louis. The forward interest rates $F_3^6(t)$ and $F_6^{12}(t)$ are the rates implicit in the yield curve extracted using the three, six and twelve month rates. The implicit forward interest rates are worked out assuming efficient financial markets. Under this assumption, there should be no arbitrage opportunities at the equilibrium between the rate of return of a long-term (twelve and six-month) security on the spot market, and the rate of return obtained by investing on short-term (six and three-month) securities on the spot and the forward markets over the same period.
The observed yield on each bill has been derived from the price of that bill on a given day (last trading day of the month) so that the data relate to bills which are identical in all respects other than term. While data for the three and six-month Treasury Bills are available for the period from January 1960 to May

2000, twelve-month Treasury Bill data begin only in January 1964 and extends in May 2000.
Following Goodfriend (1998) and Bordo and Haubrich (2004) the dataset is divided into three sub-periods according to the characteristics of the monetary regimes that were at work at that time. More specifically, the dataset can be split into the following sub-samples: Period I from 1960:01 to 1965:12, period II from 1966:01 to 1985:12 and period III from 1986:01 to 2000:05. According to the above authors, the expected inflation rate was high and volatile in subperiod II while it was low and stable in subperiods I and III. Once one believes the level and volatility of inflation to be a key factor for the credibility of a monetary regime and the term premium to depend on both a liquidity and a risk premium, then the term premium should mirror the particular level of riskiness, or credibility, which characterizes the monetary regime at any time t. Moreover, the subperiod II encompasses three different monetary institutional arrangements which are distinguished by the degree of interest rate targeting undertaken by the Federal Reserve. The first, covering the period up to September 1979, corresponds to a period during which the Federal Reserve targeted interest rates. The period from October 1979 to September 1982 covers the Federal Reserve's "new operating procedures", when it ceased targeting interest rates in favor of monetary aggregates. The third, from October 1982 onward, corresponds to the abandonment of the "new operating procedures" and the resumption of partial interest rates targeting.

4 The Constant Term Premium Model

In this Section estimations of eq.(4) are carried out employing a number of different methodologies. Two different sets of estimations are carried out. The first set, which considers three month spot and forward interest rates (i.e. $R_3(t)$ and $F_3^6(t)$), is reported in Tables 1 to 3, while the second set, which employs six month spot and forward rates (i.e. $R_6(t)$ and $F_6^{12}(t)$), is set out in Tables 4 to 6. First of all, because monthly data are employed and the investment horizons are respectively three and six months, the data frequency is higher than the terms of the investments. This implies that the estimation errors follow respectively MA(2) and MA(5) stochastic processes.[9] As a result, Least Squares (LS) estimations of eq.(4) produce unbiased but inefficient estimates and therefore inference is invalid. Secondly, as it will be shown later, the disturbance term $\mu_{t+n-m}(t)$ follow a GARCH stochastic process.[10] This is something that must be taken into account if one wants to improve the efficiency of the estimation.

[9]For instance, considering eq.(4) it can be shown that $Cov(\varepsilon_{t+3}(t), \varepsilon_{t+3}(t-1)) = 2\sigma_\epsilon^2$, $Cov(\varepsilon_{t+3}(t), \varepsilon_{t+3}(t-2)) = \sigma_\epsilon^2$ and $Cov(\varepsilon_{t+3}(t), \varepsilon_{t+3}(t-j)) = 0$ for $j \geq 3$. This shows that the disturbance term evolves according to a MA(2) stochastic process. More generally, if the investment term is k months, then the overlapping nature of the monthly observations implies that a MA(k-1) process is induced into the disturbance term.

[10]In fact, the Lagrange Multiplier ARCH tests reject the null hypothesis of homoscedasticity for most of the sub-samples under analysis.

In order to obtain reliable estimates of eq.(4) the first problem can be solved in two different ways. In Tables 1 and 4 eq.(4) is estimated by making use of the Heteroscedasticity and Autocorrelation Consistent (HAC) covariance matrix proposed by Newey and West (1987), while in Table 2 and 5 the same equation is estimated by means of maximum likelihood which accounts for the MA processes followed by the disturbance terms.[11] In Tables 3 and 6 estimates of eq.(4) are worked out taking into account both the moving average processes followed by the disturbance terms and the GARCH processes followed by their second moments.

While the estimation of the HAC covariance matrix is common in the literature, it seems opportune to specify in detail the econometric models employed to accommodate for the moving average and GARCH processes in the disturbance terms. When the model accommodates for moving average processes, then eq.(4) is jointly estimated with the following stochastic process for the disturbance term:

$$\varepsilon_{t+n-m}(t) = e(t) + \vartheta_1 e(t-1) + ... + \vartheta_{n-m-1} e(t-n+m+1). \qquad (14)$$

When the conditional heteroscedasticity in the disturbance terms together with the moving average process are accounted for, eq.(4) is jointly estimated with eq.(14) and the GARCH process followed by the conditional variance of $\varepsilon_{t+n-m}(t)$:

$$\sigma^2_{\varepsilon_{t+n-m}}(t) = \alpha_0 + \alpha_1 \varepsilon^2_{t+n-m}(t-1) + \alpha_2 \sigma^2_{\varepsilon_{t+n-m}}(t-1). \qquad (15)$$

4.1 Estimates for the Three-Month Term Spread Regression

The estimates reported in Table 1 show that the three month term spread significantly helps predict the excess returns indicating, therefore, strong departures from the EHTS throughout the entire sample and the different subperiods. The only subperiod in which the estimate of β_1 is in favor of the EHTS is 1986-2000. This period turns out to be quite insightful for the interpretation of the results. In fact, one might notice that when the estimate of β_1 are more favorable to the EHTS, like in the subperiod 1986-2000, the estimate of β_0, the constant term premium, is statistically different from 0. In turn, this could also imply that whenever the estimates of β_1 are significantly different from 0, the term premium varies over time so that the estimates of β_0 in the constant term premium model are not statistically significant.[12] This interpretation is also fostered by the fact that β_0 assumes values statistically different from 0 only in the subperiods 1960-1965 and 1986-2000 (which, according to Goodfriend (1998) and Bordo and Haubrich (2004), are characterized by low and stable inflation)

[11] The HAC covariance matrix is a procedure which computes consistent estimates of the parameters with robust standard errors in the presence of serial correlation and heteroscedasticity. Since it is only asymptotically justified, the reported t-ratios are not exactly t distributed in a finite sample, but only asymptotically normal.

[12] This is actually what happens for the entire period 1960-2000 and for the sub-period 1966-1985.

while in the subperiod 1966-1985 (characterized by high and volatile inflation) it takes not significant values. Once one expects the term premium to be related to the degree of uncertainty present in the different subperiods, one should also expect the term premium to be highly volatile during the period 1966-1985. Such volatility is then captured, in the constant term premium model, by an estimate of β_0 which is not statistically significant. Although estimates of β_0 not statistically significant do not necessarily imply time-varying term premium, it will be shown later that a model which accommodates for time-varying term premium delivers results more in favor of the EHTS.
In Table 2 the estimates are refined taking into account that, with overlapping contracts, it is expected *a priori* that the disturbance term will follow a MA(2) stochastic process. The results turn out to be consistent with the estimates reported in Table 1. There is overwhelming empirical evidence which indicates strong departures from the EHTS. With regard to the parameters β_0 and β_1, the only difference with respect to the previous estimates is in the subperiod 1986-2000 where the evidence, perhaps very weak, in favor of the EHTS disappears. The combination of statistically significant β_1 and not statistically significant β_0 is in line with the conjecture according to which whenever the estimates of β_1 is statistically significant, the term premium should vary over time so that the estimate of β_0, in the constant term premium model, turns out to be not significantly different from zero. The moving average parameters ϑ_1 and ϑ_2 are both highly significant while the goodness of fit has remarkably improved.
Both Tables 1 and 2 report the Box-Ljung Q-statistic and the Breusch-Godfrey (B-G) test for serial correlation, the Breusch-Pagan (B-P) and the Lagrange Multiplier (LM) ARCH test for heteroscedasticity as well as the Jarque-Bera (J-B) test for normality in the residuals.[13] More specifically, testing for autoregressive conditional heteroscedasticity (ARCH) in the residuals is motivated by the observation that in many financial time series, the magnitude of the conditional variance appears to be related to the magnitude of past residuals. While ARCH in itself does not invalidate standard maximum likelihood inference, ignoring its presence may result in loss of efficiency. According to the Box-Ljung Q-statistics and the B-G tests, both the specifications with and without the inclusion of the MA(2) process for the disturbance term show the residuals to be strongly serially correlated. The presence of heteroscedasticity in the forms detected by both the B-P and the LM tests, however, turns out to be significantly reduced in the specification which accounts for the MA(2) process in the disturbance term for the subperiods 1960-1965 and 1986-2000.[14] Finally, the residuals of both the specifications are far from being normally distributed, as highlighted by the J-B tests.

[13] The Q-statistics reported are the values computed at lag 4 and 8, the B-G test is constructed under the null hypothesis that there is no serial correlation in the residuals up to lag 4. The B-P test is computed using both the term spread and its squared values while the LM ARCH test is constructed under the null that there is no ARCH effect up to order 1, 2, 4 and 8 in the residuals.

[14] With regard to the sub-periods 1960-1965 and 1986-2000, such evidence is also fostered by the non statistically significance of the Q-stats obtained from the correlograms of the squared residuals. For better readability, these results are not presented in detail.

Table 3 presents, for the subperiods 1966-1985 and 1960-2000, estimates of the constant term premium model in which the first and second moment of the disturbance term follow, respectively, a MA(2) and a GARCH(1,1) stochastic process.[15] In line with the previous estimations, also in this case the results indicate that the term spread significantly helps predict excess returns. As with Table 2, the moving average coefficients are highly significant while the sum of the ARCH and GARCH coefficients ($\alpha_0+\alpha_1$) is very close to one indicating that the volatility shocks have very high persistence.[16] Overall, when the analysis is conducted for three and six month spot interest rates, the constant term premium model provides overwhelming empirical evidence against the validity of the EHTS. Even when the estimations are refined by including the MA(2) process for the disturbance term and the GARCH(1,1) process for its conditional variance, the empirical evidence delivers the same conclusions.

4.2 Estimates for the Six-Month Term Spread Regression

While the previous paragraph has investigated the validity of the EHTS for three and six month spot interest rates, this paragraph replicates the same analysis considering six and twelve month rates.

The estimates reported in Table 4 show that the six month term spread helps predict excess returns suggesting, therefore, departures from the EHTS for the entire sample and across the different subperiods. Unlike previous estimations, the parameter β_0 is never statistically significant while β_1 is always significant. This, in turn, would suggest that whenever the estimates of β_1 are statistically different from zero, the term premium varies over time so that the estimate of β_0, in the constant term premium model, turns out to be not significantly different from zero. As previously explained, although estimates of β_0 not statistically significant do not necessarily imply time-varying term premium, it will be shown later that a model which accommodates for time-varying term premium delivers results more in favor of the EHTS.

In Table 5 the estimates are refined by accounting for the fact that the disturbance term follows now a MA(5) stochastic process. The results turn out to be consistent with the estimates reported in Table 4, showing strong departures from the EHTS. The moving average parameters turn out to be highly statistically significant, and the goodness of fit remarkably improved. Tests for serial correlation and heteroscedasticity are reported in the bottom part of both the tables. Similarly to the evidence obtained for the three month term spread, both the Box-Ljung Q-statistics and the B-G test show the disturbance terms to be strongly serially correlated. The presence of heteroscedasticity turns out to be significantly reduced in the specification which accounts for the presence

[15]The inclusion of the GARCH(1,1) process is motivated by the fact that the LM ARCH tests reject the null hypothesis of no ARCH effect up to order 1, 2, 4 and 8 in the residuals. Although the LM tests would suggest an ARCH(8) process, in order to reduce the number of parameters estimated, I choose to implement a GARCH(1,1).

[16]In other words, there is evidence that the conditional variances follow an Integrated GARCH processes.

of moving average process in the disturbance term for the subperiod 1986-2000. Moreover, unlike previous estimations, the null of residuals normally distributed cannot be rejected for the same subperiod.
Table 6 reports estimates of the constant term premium model which accounts for the presence of moving average and GARCH processes in the disturbance term.[17] In line with the evidence previously obtained, the estimates indicate that the term spread significantly helps predict excess returns. The moving average coefficients turn out to be highly significant as well as the coefficients related to the GARCH process.
Summing up, when tests for the EHTS are conducted for the short end of the term structure (i.e. for three, six and twelve month spot interest rates) by means of the constant term premium model, empirical results show that such hypothesis is soundly rejected. When the empirical estimates are refined by modeling the disturbance terms as moving average and GARCH processes, the empirical evidence delivers the same results. This conclusion, in turn, is consistent with findings of previous studies (see, for instance, Fama (1984), Fama and Bliss (1987) and Boero and Torricelli (2002)). However, in the next Sections it will be shown that such conclusion can be partially reversed once single regression models which accommodate for time-varying term premia are employed in the analysis. In fact, the diagnostic tests suggest that there might be an omitted variable problem since the residuals display serial correlation and conditional heteroscedasticity even after the presence of moving average processes in the disturbance terms is accounted for. In order to inspect the possibility that the omitted variable problem is due to a time-varying term premium, and to have some indications about the importance of its variations, in Section 5 the lower bound for the standard deviations of the term premia are carried out when the correlation between term premium and term spread attains its maximum value of unity.

5 Testing the Stability of the Constant Term Premium Model

This Section provides both empirical and theoretical evidence for the parameters of the constant term premium model to be time-varying. Paragraph 5.1 sets out the Mankiw and Miron's (1986) argument that whenever the estimation of β_1 in eq.(4) departs from zero, the term premium must be time-varying. Paragraph 5.2 carries out some stability tests which provides extensive empirical evidence that the parameters are not constant over time. Although from different perspectives, this Section provides convincing empirical and theoretical evidence for the case of a time-varying parameters model which accommodates

[17]Estimations which accommodate for moving average and GARCH processes in the disturbance term are worked out also for the subperiod 1986-2000 even though in Table 5 it is shown that the presence of GARCH process for this subperiod is weak.

for a time-varying term premium. Such evidence holds for both three and six month, and for six and twelve month spot interest rates.

5.1 The Mankiw and Miron argument

A strong argument which supports the case for a time-varying term premium comes from Mankiw and Miron (1986). They have been among the first to highlight the fact that the term premium must be time-varying for $plim\hat{\beta}_1 \neq 0$ (i.e. the predicted value of β_1 to be something other than zero).[18] To show this, I report the probability limit of $\hat{\beta}_1$ in eq.(4) for the three-month spread (note that the same intuition carries over to m=6 and n=12 as well):

$$plim\hat{\beta}_1 = \frac{\sigma^2[TP_3^6(t)] + \rho\sigma[R_3^e(t+3) - R_3(t)]\sigma[TP_3^6(t)]}{\sigma^2[R_3^e(t+3) - R_3(t)] + \sigma^2[TP_3^6(t)] + \rho\sigma[R_3^e(t+3) - R_3(t)]\sigma[TP_3^6(t)]} \tag{16}$$

where $\sigma^2[R_3^e(t+3) - R_3(t)]$ is the variance of the expected change in the three months (short term) interest rate, ρ is the correlation index between expected changes in the three months interest rate and the term premium, while $\sigma^2[TP_3^6(t)]$ is the variance of the time-varying term premium.[19]
Notice that for a non stochastic term premium, the term $\sigma^2[TP_3^6(t)]$ collapses to zero as does $plim\hat{\beta}_1$. Hence a stochastic term premium is required for the probability limit to be significantly different from zero. Also observe that the greater the volatility of the term premium, the more $plim\hat{\beta}_1$ will depart from zero, while the higher the variance of the expected changes in the three months interest rate, the more $plim\hat{\beta}_1$ approaches zero. More generally, the departure of $\hat{\beta}_1$ from zero will depend on the relative importance of the variance of the term premium with respect to the variance of the expected changes in the short term interest rate.[20]

5.1.1 A lower bound for the volatility of the term premium

Some idea of the importance of variations in the term premium can be obtained by constructing a lower bound for its standard deviation when the correlation index between the term premium and the term spread attains its maximum value of unity. Following Fama (1984) and Wickens and Thomas (1991), the probability limit of $\hat{\beta}_1$ defined in eq.(16) can be rewritten as follows (note that

[18] See also Fama (1984).

[19] Notice that Mankiw and Miron (1986) derive the probability limit $plim\hat{\delta}_1 \neq 0$ referring to eq.(3). However, eq.(16) can be worked out recalling that, because eqs.(3) and (4) are entirely complementary, $\beta_1 = 1 - \delta_1$.

[20] Notice that the presence of the variance of the expected change in the short term interest rate in eq.(16) highlights the existing linkage between monetary policy and EHTS. Mankiw and Miron (1986) show that such variance became much smaller after the creation of the Federal Reserve System and attribute this finding to the Fed's concern for interest rate smoothing. That is why the same authors found, prior to the creation of the Fed, evidence in favor of the EHTS.

the same intuition carries over to m=6 and n=12 as well):

$$plim\hat{\beta}_1 = \rho \frac{\sigma[TP_3^6(t)]}{\sigma[F_3^6(t) - R_3(t)]} \tag{17}$$

where $\sigma[F_3^6(t) - R_3(t)]$ is the standard deviation of the term spread.[21] The lower bound standard deviation for the term premium, $\sigma_L[TP_3^6(t)]$, can be easily derived from eq.(17) by taking the asymptotic limit of the OLS estimator of β_1, $\hat{\beta}_1$, and assuming the correlation index ρ equal to 1.

The value of the lower bound of the standard deviation of the term premium for the three-month spread (i.e. when n=6 and m=3), $\sigma_L[TP_3^6(t)]$, computed considering the estimate of β_1 reported in Table 1, turns out to be 0.068. According to this calculations, the estimate of $\sigma_L[TP_3^6(t)]$ indicates that the term premium variability must account for at least 70.3% of the standard deviation of the term spread.[22] Similarly, the lower bound of the standard deviation of the term premium for the six-month spread (i.e. when n=12 and m=6), can be worked out considering the estimate of β_1 reported in Table 4. In this case the lower bound of $\sigma_L[TP_6^{12}(t)]$ turns out to be 0.156. As a result, the term premium variability must account for at least 89.7% of the standard deviation of the term spread.[23] This, in turn, suggests that the time-varying term premium can be very important to explain the failure of the EHTS for the short-end of the term structure over the period 1960-2000.

5.2 Stability Tests

The battery of stability tests reported in this paragraph consists of the CUSUM of Squared Residuals tests, as well as the tests for structural changes proposed by Andrews (1993), and Andrews and Ploberger (1994). These tests are carried out for both m=3 and n=6, and m=6 and n=12. An important shortcoming of the standard tests used to detect breakpoints in the parameters is that the breakdate has to be known a priori.[24] As the true datebreak can be different from the datebreak picked by the researcher (based on some known features of the data), these tests can be potentially misleading. The solution to this problem has been to treat the breakdates as unknown. Among the different

[21]Notice that eq.(17) can be seen as an expression for the asymptotic bias in the estimate of $\hat{\beta}_1$. The magnitude of the asymptotic bias depends not only on the value of ρ but also on the ratio between the standard deviation of the term premium and the term spread. If the former is approximately zero, then $\hat{\beta}_1$ will converge to 0, for any value of ρ.

[22]When the lower bound of the standard deviation of the term premium is computed considering the estimate of β_1 reported in Tables 2 and 3, it assumes the values 0.069 and 0.089. In this case, the term premium variability must account, respectively, for at least 91.7 and 96.1% of the standard deviation of the term spread. The standard deviation of the term spread employed in the calculations, $\sigma[F_3^6(t) - R_3(t)]$, is 0.098.

[23]When the lower bound of the standard deviation of the term premium is computed considering the estimate of β_1 reported in Tables 5 and 6, it assumes the values 0.175 and 0.179. In this case, the term premium variability must account, respectively, for at least 100% and 102.5% of the standard deviation of the term spread. The standard deviation of the term spread employed in the calculations, $\sigma[F_6^{12}(t) - R_6(t)]$, is 0.175.

[24]This is true, for instance, for the Chow Breakpoint test.

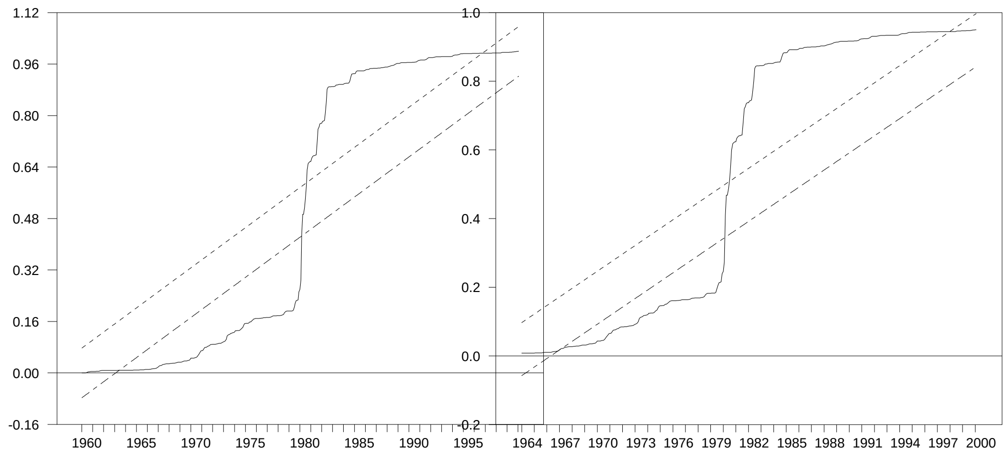

Figure 1: Cumulative Sum of Squared Residuals (solid line) with upper and lower bound of the confidence interval (dotted lines) for m=3 and n=6 (left panel), and for m=6 and n=12 (right panel).

tests proposed by the literature, the tests employed in the analysis are those proposed by Andrews (1993), and Andrews and Ploberger (1994). These tests are based on the idea that an appropriate method to estimate the parameters, including the breakdates, is the Least Squares. Operationally, the sample is split at each possible breakdate, the other parameters estimated by Least Squares and the sum of squared errors calculated and stored. The Least Squares breakdate estimate is the date that minimizes the full-sample residual variance. These tests allow for testing instability in any single parameter of regressions, as well as joint instability in a group of parameters (see, for instance, Hansen (2001), and McConnell and Perez-Quiros (2000)).

Table 7 reports the Andrews and Andrews-Ploberger tests for the null hypotheses of individual stability and joint stability in the parameters β_0^*, β_1^* and the variance of the disturbance term σ_ε^{*2}. While the upper panel sets out the tests when m=3 and n=6, the bottom panel reports the tests for m=6 and n=12. The two tests provide similar results. When three and six month interest rates are taken into consideration they show strong evidence of structural breaks in the parameter β_1^*, while for the parameters β_0^* and σ_ε^* the two tests highlight, respectively, weak evidence and no evidence of structural breaks. The datebreaks identified by the two tests fall within the interval January 1980 - August 1982, when the Federal Reserve ceased targeting interest rates. Focusing on the twelve and six month rates, the two tests show that structural changes occur in both the parameter β_0^* and β_1^* while there is weak evidence of structural change in the variance of the disturbance term. Once again, the datebreaks are located in the early 80s, in correspondence with the Federal Reserve's "new operating procedure". On the other hand, both the Andrews and the Andrews-Ploberger tests soundly reject the null of no structural break when tested against the alternative of the presence of structural breaks in all the parameters. Summing up, both the Andrews and the Andrews-Ploberger provide evidence in favor of the non-constancy of the parameters of eq.(4). This, in turn, is in line with the parameter estimates reported in Tables 1-5 which indicate substantial shifts. Applying the test for multi structural changes recently proposed by Bai and Perron (2003), the results are, all in all, similar. In fact, structural changes which occur in the early 80s are shown to be present in all the parameters but σ_ε^*.[25]

Figure 1 reports the Cumulative Sum of Squared residuals (CUSUM Squared) computed for n=6 and m=3 and for n=12 and m=6. The two diagrams are quite similar, in fact both of them fall outside the two standard error bands for the most part of the sample, strongly suggesting at least one break in the parameters.

6 The Time-Varying Parameters Model

Given the strong evidence in favor of the non constancy in the parameters of eq.(4), this section sets out a simple time-varying parameters model which accommodate for the presence of time-varying term premium. The first step, in order to construct the model is to cast it in State Space form. State Space forms are essentially a notational convenience developed to make tractable what would otherwise be notationally intractable estimation problems (see, for instance, Harvey (1992) and Hamilton (1994)). These models consist of two equations, called respectively "measurement" and "transition" equation, and they serve as the basis for virtually all linear estimation methods such as the Kalman filtering technique. In the present context, casting a model in state space form allows for the possibility of estimating both β_0^* and β_1^* as time-varying parameters.

The model is constructed casting in state space form eqs.(4) and (14). Because such model takes into account the presence of MA stochastic processes in the disturbance term, it can provide potentially more refined estimations of the parameters involved as well as of the time-varying term premium. Moreover, as already mentioned, accommodating for the presence of the moving average process permits the examination of the stochastic properties of the term premium on actual observed forward contracts with settlement dates of more than one period in the future, rather than just the one period ahead case. The measurement equation consists of eqs.(4) and (14) in which the parameters β_0^* and β_1^* are made time-varying and it assumes the following form:

$$(F_{n-m}^n(t) - R_m(t+n-m)) = \qquad (18)$$

[25] Results for the Bai and Perron (2003) test are not reported since they provide evidence identical to both the Andrews and Andrews-Ploberger tests.

$$= \begin{bmatrix} 1 & (F^n_{n-m}(t) - R_m(t)) & 1 & \theta_1 & \theta_2 & ... & \theta_{n-m-1} \end{bmatrix} \cdot \begin{bmatrix} \beta^*_{0,t} \\ \beta^*_{1,t} \\ e_t \\ e_{t-1} \\ e_{t-2} \\ \vdots \\ e_{t-n+m+1} \end{bmatrix}.$$

The disturbance term e_t is a scalar assumed to be normally distributed with zero mean and variance σ^2_e which is unknown but assumed finite. The transition matrix F^* is a square matrix whose dimension, in the case of n=6 and m=3, is (5×5) while $\overline{v}^*_t$ is a random disturbance vector assumed to be normally distributed with zero mean and covariance matrix Q^*. The elements in the matrix F^* as well as in the covariance matrix Q^* are unknown and have to be estimated. To simplify the estimation procedure, independence among $v_{0,t}$, $v_{1,t}$ and e_t is assumed in the covariance matrix Q^* which is also of dimension (5×5). With regard to the transition equation (19), it is a first order stochastic difference equation with constant coefficients. As a result, they require starting values. Thus, the initial values of the state vectors as well as the initial values of the covariance matrice Q^* are assumed to be known.[26]

$$\begin{bmatrix} \beta^*_{0,t} \\ \beta^*_{1,t} \\ e_t \\ e_{t-1} \\ e_{t-2} \\ \vdots \\ e_{t-n+m+1} \end{bmatrix} = \begin{bmatrix} \phi^*_0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & \phi^*_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \beta^*_{0,t-1} \\ \beta^*_{1,t-1} \\ e_{t-1} \\ e_{t-2} \\ e_{t-3} \\ \vdots \\ e_{t-n+m} \end{bmatrix} + \begin{bmatrix} v^*_{0,t} \\ v^*_{1,t} \\ e_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (19)$$

$$\overline{\beta}^*_t = F^* \overline{\beta}^*_{t-1} + \overline{v}^*_t \quad (20)$$

$$Q^* = E_t[\overline{v}^*_t \overline{v}^{*'}_t] = \begin{bmatrix} \sigma^2_{v1*} & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma^2_{v2*} & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma^2_e & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (21)$$

The model has been set out in the most comprehensive form possible. It encompasses the "stationary", the "non stationary", the "pure random" and the "constant" specification of the term premium. This is a desirable feature for a model which accommodates for the presence of time-varying term premium

[26] This means that in the estimation process initial guesses for both the state vectors and the covariance matrices must be provided.

because it does not force the researcher to take any a priori stance about the stochastic process followed by the term premium itself. The stationary specification requires $|\phi_0^*| < 1$, $|\phi_1^*| < 1$, $\sigma_{v0}^{*2} > 0$ and $\sigma_{v1}^{*2} > 0$. In this case, both $\beta_{0,t}^*$ and $\beta_{1,t}^*$ are assumed to follow a stationary first order autoregressive process. The non stationary specification, on the other hand, implies $\phi_0^* = 1$, $\phi_1^* = 1$, $\sigma_{v0}^{*2} > 0$ and $\sigma_{v1}^{*2} > 0$. According to this specification, both the parameters are assumed to follow a driftless random walk. If, on the other hand, $\sigma_{v0}^{*2} = 0$ and $\sigma_{v1}^{*2} = 0$, then we fall in the constant specification while the pure random specification requires $\phi*_0 = 0$, $\phi*_1 = 0$, $\sigma_{v0}^{*2} > 0$ and $\sigma_{v1}^{*2} > 0$. The non-stationary specification is taken into consideration to accommodate for the possibility that permanent shocks dominate the term premium.[27] On the other hand, the stationary specification must be considered to accommodate for the possibility of a mean-reverting term premium.
The configurations consistent with the EHTS, i.e. consistent with β_0 constant and $\beta_1 = 0$, are the pure random and, as long as the estimate of $\beta_{1,t}^*$ turns out to be not statistically significant, the constant specification.

6.1 The Time-Varying Term Premium Model

The state space model given by eqs.(18)-(21) encompasses a class of models largely employed in the literature to detect time-varying term premia under rational expectations. These models can be constructed by making use of condition (1). Subtracting the future realized spot rate $R_m(t+n-m)$ from both the left and right-hand-side, one can write the *(n-m)*-period excess forward return as follows:

$$F_{n-m,n}(t) - R_m(t+n-m) = TP_{n-m}^n + u_{t+n-m}(t) \tag{22}$$

where $u_{t+n-m}(t) = E_t[R_m(t+n-m)] - R_m(t+n-m)$ is a *(n-m)*-period market forecast error. Setting n=6 and m=3 to simplify the notation, the equation above can be rewritten as follows:

$$F_{3,6}(t) - R_3(t+3) = TP_3^6 + u_{t+3}(t). \tag{23}$$

Under rational expectations - i.e. under the assumption that the market forecast errors are uncorrelated with prior information and $E_t[u_{t+n-m}(t)] = 0$ - the EHTS implies a constant term premium TP_{n-m}^n. Thus, the Kalman filter can be used to detect the unobserved time-varying term premium from the *(n-m)*-period excess forward returns under the assumption of rational expectations. The only requirements of the model is the specification of the stochastic processes for the unobserved components TP_{n-m}^n in eq.(22). Once such specification is chosen, the model can be put in state space form and the term premium estimated. If the term premium is found to fluctuate over time, then the null of EHTS is rejected because of time-varying term premium.

[27] The findings in Evans and Lewis (1994) and Gravelle et al. (1999) on cointegration between spot and forward interest rates support this specification. Time-varying term premia which evolve as random walks have been highlighted also in Iyer (1997).

It can be shown that the above model originates from the general models given by eqs.(18)-(21) when the restriction $\phi_1^* = \sigma_{v1}^{*2} = 0$ is imposed. Thus, assuming that the above restrictions hold, the model can be used to detect the term premium by means of the time-varying parameter $\beta_{0,t}^*$. It follows that testing the null hypothesis $\phi_1^* = \sigma_{v1}^{*2} = 0$ is equivalent to testing for the hypothesis that departures from rational expectations do not play any role in the invalidation of the EHTS. These tests will be carried out in Section 6.6.

Models similar to the restricted versions of eqs.(18)-(21) have been employed, for instance, by Wolff (1987), Iyer (1997) as well as Gravelle and Morley (2005) to estimate time-varying term premia under rational expectations.[28]

Having set out the model that will be employed to inspect the validity of the EHTS as well as its restricted version, the next step is to address the problem of estimating the unknown parameters. These parameters are called "hyperparameters". For the (unrestricted) model with n=6 and m=3, for instance, they consist of ϕ_0^*, ϕ_1^*, θ_1, θ_2, σ_{v1}^{*2}, σ_{v2}^{*2} and σ_e^2.

6.2 Methodology

Once the model has been put in state space form, the hyperparameters can be estimated by means of Kalman filter and maximum likelihood, which is a recursive procedure for calculating the optimal estimator of the state (or latent) variables given all the information which is currently available. This paragraph sets out the main characteristics of the methodology.[29]

Defining $\widehat{\beta}_{t|t-1}$ the estimate of the state at time t conditional on the information available at time t-1, it is possible to write eq.(19) in the following form:

$$\widehat{\beta}_{t|t} = F\widehat{\beta}_{t|t-1} + \overline{v}_t \tag{24}$$

while its covariance matrix can be obtained applying the Variance Operator to eq.(24), namely:

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q. \tag{25}$$

When, at time t, a new observation of the dependent and independent variables become available, it is possible to compute the "one step ahead prediction error" η_t and its variance f_t as follows:

$$\eta_t = (F_{n-m}^n(t) - R_m(t+n-m)) - (F_{n-m}^n(t) - R_m(t))\widehat{\beta}_{t|t-1} \tag{26}$$

$$f_t = (F_{n-m}^n(t) - R_m(t))P_{t|t-1}(F_{n-m}^n(t) - R_m(t)) + \sigma_e^2. \tag{27}$$

Once the one step ahead prediction error as well as its variance have been worked out for each time t in the sample period, it becomes also possible to construct the likelihood function in term of the above components. Defining T as the

[28] Wolff (1987) applies this econometric model to estimate the time-varying term premium in forward exchange markets.

[29] A friendly introduction to the topic is offered by Welch and Bishop (2001) while more detailed treatments are provided in Hamilton (1994) and Kim and Nelson (1999).

number of observations and j as the number of parameters being estimated, the likelihood function L takes the following form

$$L = -0.5(T-k)(ln\ 2\pi + ln\ \sigma_e^2) - 0.5 \sum_{t=k+1}^{T} f_t - 0.5\sigma_e^{-2} \sum_{t=k+1}^{T} \frac{\eta_t^2}{f_t}. \qquad (28)$$

Maximum likelihood estimates of the hyperparameters, as well as of their covariance matrix, are then obtained applying numerical optimization techniques to the above function.[30] Once maximum likelihood estimates of the hyperparameters have been obtained, the Kalman filter can be re-run to compute recursively the mean of the state variables, as well as their covariance matrix for each time t in the sample period. The Kalman filter is applied, in this context, to make inferences on the changing regression coefficients. As already mentioned, a desirable feature of the Kalman filter is that, according to eqs.(24)-(25), it delivers estimates of $\beta^*_{0,t}$ and $\beta^*_{1,t}$ together with their standard deviations at each time t included in the sample period. In Section 7 it will be shown how to exploit such a characteristic to construct an alternative test for the EHTS.

6.3 Hyperparameters Instability and Monetary Regimes

Before proceeding to comment on the estimates obtained, some theoretical and empirical arguments are introduced in order to justify the partition of the analysis into the subperiods 1960-1965, 1966-1985 and 1986-2000.
While Section 5 has certified the instability of the parameters of the constant term premium model, there is a second set of parameters that must be considered. These are the hyperparameters of the model defined by eqs.(18)-(21). These may well vary over the sample period and the question of their stability must also be considered. It can be shown, for instance, that the usual tests for heteroscedasticity carried out in Tables 1 to 5 return positive values for models with constant parameters but with shifts in the hyperparameters. According to Wells (1995), a test which is particularly indicated in detecting the latter shifts is the CUSUM of squared residuals. Thus, in this context, Figure 1 has a twofold valence. Because the statistic falls outside the upper and lower bands, it provides evidence of parameters instability. However, the abrupt change in the slope of the statistic which occurs in correspondence to 1965 and 1985 indicates also that shifts in the hyperparameters might have occurred in both the dates. This, in turn, fosters Goodfriend (1998) and Bordo and Haubrich's (2004) arguments, according to which the period 1960-2000 can be split into three sub-periods, each of them characterized by a different monetary regime. This introduces the second argument used to justify the partition of the analysis into sub-periods. According to these authors, in fact, the long bond rate is particularly well suited to help a central bank assess the degree to which it

[30]For all models in this paper, Maximum Likelihood estimations have been conducted using the BFGS (Broyden, Fletcher, Goldfarb, Shanno) and the SIMPLEX algorithm in RATS. While the former requires twice-continuously differentiable functions, the latter is a derivative free algorithm which requires only continuity.

has achieved credibility of low inflation. Thus, inspecting the behavior of the yield on long-term nominal bonds they are able to distinguish among periods of high and low credibility. The period 1960-1965 is depicted by the authors as a period of high credibility in which inflation averaged around 1 to 2 percent and the long-term inflation expectations were no more than that.[31] In sharp contrast, the period 1966-1985 is defined as a period of low credibility characterized by rising inflation where long-term bond returns became high and increasingly variable, signalling high and volatile inflation and inflation expectations.[32] The subperiod 1966-1985 encompasses the abrupt change in the Federal Reserve's monetary policy which occurred between October 1979 and September 1982. With regard to the subperiod 1986-2000, the fact that the FED succeeded in bringing inflation down below 3 percent, and the decline in long bond rates to below 5 percent indicates the return to a regime of full credibility.[33]

6.4 Parameter Estimates of the Three-Month Term Spread Regression

Tables 8-9 set out the parameter estimates for the model identified by eqs.(18)-(21). All the maximum likelihood estimations have been conducted using both the SIMPLEX and the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm in RATS. Because the former is less sensitive to the choice of initial values than the latter, the SIMPLEX algorithm is used to refine initial estimates before using the BFGS. This mitigates the risk of finding a local rather than a global maximum.[34] Once the final estimates have been obtained, a sensitive analysis to check the stability of the estimates has been conducted. Such analysis consists in feeding the SIMPLEX and the BFGS algorithms with the final estimates obtained in the first stage, and to check that they deliver estimates consistent with those previously obtained. The estimation is conducted for the entire sample period as well as for the subperiods which correspond to the monetary regimes identified by Goodfriend (1998) and Bordo and Haubrich (2004). Focusing on the period 1960-2000, both the autoregressive coefficients ϕ_0^* and ϕ_1^* as well as the variances σ_{v0}^{*2} and σ_{v1}^{*2} turn out to be statistically significant. This means that both $\beta_{0,t}^*$ and $\beta_{1,t}^*$ are time-varying. Since $\phi_0^* = 0.747$, the parameter $\beta_{0,t}^*$ follows a mean reverting first order autoregressive stochastic process with half life of 4 months and a half. With regard to $\beta_{1,t}^*$, it evolves according to a mean reverting process with a half life of 2 and a half months.[35] The meaning

[31] During this period, in fact, the long bond rate averaged around 3 to 4 percent.

[32] For instance, when interest rates peaked in 1981, inflation was well above 10 percent per year.

[33] During this period the Chairman of the FED had been Alan Greenspan who emphasized the importance of price stability such that "the expected rate of change of the general level of prices ceases to be a factor in individual and business decision-making".

[34] The drawback of this strategy is that when the SIMPLEX algorithm feeds the BFGS with initial values estimated with a fairly high level of precision, the BFGS takes too few iterations to achieve the convergence. If this happens, the BFGS might produce poorly estimated standard errors.

[35] The half-life is commonly employed as a measure of the speed at which a stochastic process reverts back to its mean. In other words, it is the time t^* required for a divergence from the

of a time-varying $\beta^*_{1,t}$ is that the importance attached to the term spread, as explanatory variable of the excess forward returns (the forward rate minus the future realized spot rate), varies over time. More specifically, the statistical significance of both ϕ^*_1 and σ^{2*}_{v1} suggests that the importance of the term spread in explaining the excess forward returns is statistically significant. As a result, the EHTS must be rejected also when the hypothesis is tested making use of a time-varying parameters model which accommodates for time-varying term premium. Moreover, because the above estimates suggest a $\beta^*_{1,t}$ statistically significant over time, the time-varying parameter $\beta^*_{0,t}$ cannot be interpreted as an estimation of the time-varying term premium.
The model fares pretty well in fitting actual data, in fact it presents a higher $R^2_{t|t-1}$ with respect to the one set out in Table 1 for the constant term premium model. Overall, the statistical significance of the hyperparameters related to both $\beta^*_{0,t}$ and $\beta^*_{1,t}$ suggests that the EHTS does not hold because of both time-varying term premium and departures from rational expectations. The departures from rational expectations are highlighted by the fact that $\beta^*_{1,t}$ is likely to assume, over time, values different from zero which, in turn, means that the term spread contains information at time t which has remained unexploited. Unlike in the constant term premium model, the moving average coefficients turn out to be non statistically significant. This might be explained by the fact that when both the parameters β_0 and β_1 become time-varying, they absorb part of the explanatory power of the moving average process.
Table 9 reports the Box-Ljung Q-statistic and the Breusch-Godfrey (B-G) test for autocorrelation, the Breusch-Pagan (B-P) and the Lagrange Multiplier (LM) ARCH test for heteroscedasticity, as well as the Jarque-Bera (J-B) test for normality in the residuals. Despite the model accommodates for the time-varying term premium, the diagnostic tests suggest that there may still exist an omitted variable problem, since the residuals are serially correlated and conditional heteroscedastic. This, in turn, suggests that there is information available to agents at time t not included in the term spread which has remained unexploited. As a result, a further role for the presence of irrationality arises in the rejection of the EHTS. These results are consistent with the findings of previous studies (see, for instance, Jones and Roley (1983), Fama (1984), Fama and Bliss (1987), Mishkin (1988) and Harris (2004)).

6.4.1 Parameter estimates across different monetary regimes

In line with the arguments set out in Paragraph 6.3, this paragraph shows that the hyperparameter estimates of eqs.(18)-(21) are regime dependent. This is especially true for the parameter $\beta^*_{0,t}$ which accommodate for the time-varying term premium. In fact, consistently with the economic characteristics of the different monetary regimes, the parameter turns out to be driven by a deterministic process in the period 1960-1965 and by stochastic processes respectively with

average to dissipate by one half. Since the processes governing both $\beta^*_{0,t}$ and $\beta^*_{1,t}$ are AR(1), the half life it is computed by $t^* = ln(0.5)/ln(\phi^*)$.

high and low variance during the periods 1966-1985 and 1986-2000.[36]
Because the period 1960-1965 is a credible monetary regime, one would expect the term premium not to play a significant role. This is confirmed by the fact that the variance σ_0^{*2} is very small and not statistically significant and $|\phi_0^*| < 1$. In other words, the parameter $\beta_{0,t}^*$, which accommodates for time-varying term premium, is not stochastic and converges quickly toward zero. With regard to $\beta_{1,t}^*$, since both the autoregressive coefficient and the variance turn out to be statistically significant, it evolves according to a mean reverting first order autoregressive process. Because of the constancy of $\beta_{0,t}^*$ and therefore the absence of time-varying term premium, one would also expect the constant parameters model to be a reliable model. This is confirmed by the fact that the R^2 statistics for both the constant and the time-varying parameter models assume similar values. Thus, the evidence of non statistically significant $\beta_{0,t}^*$ and highly significant ϕ_1^* and σ_{v1}^{*2} suggests that the EHTS does not hold because $\beta_{1,t}^*$ is likely to assume, over time, values different from zero. In other words, the EHTS is invalidated because the term spread contains information at time t which has remained unexploited. Table 9 report diagnostic statistics for the residuals originated by the model. Despite the model accommodates for the time-varying term premium, the diagnostic tests show that the residuals remain serially correlated and conditional heteroscedastic, suggesting the existence of an omitted variable problem. This, in turn, suggests that there is information available to agents at time t not included in the term spread which has remained unexploited. Thus, limitedly to the period 1960-1965, a further role for departures from rational expectations arises in the rejection of the EHTS. Finally, unlike in the constant term premium models, the residuals originated by the model turn out to be normally distributed.
Conversely, in a monetary regime characterized by low credibility like the subperiod 1966-1985, one would expect the term premium to be relevant and the time-varying parameter model to be more reliable than the constant term premium model for the assessment of the EHTS. The empirical estimates of Table 8 support this view. In fact, the variance of the stochastic process governing $\beta_{1,t}^*$ turns out to be non-statistically significant, suggesting a non-stochastic parameter which converges quickly toward zero. On the other hand, the relevant role reserved to the term premium is suggested by the statistical significance of the parameters which govern the stochastic process of $\beta_{0,t}^*$, as well as by its high variability - as indicated by the large values taken by $SD(\beta_{0,t}^*)$.[37] The higher reliability can be deduced from the higher $R^2_{t|t-1}$ statistic of the time-varying parameters model with respect to the statistic reported in Table 1 for the constant term premium model. Thus, limitedly to the period 1966-1985, the hyperparameter estimates suggest that the EHTS does not hold because of time-varying term premium, while a limited role is reserved to departures from rational expectations. The analysis of residuals reported in Table 9 highlights

[36]Notice, in fact, that the parameter σ_{v0}^{*2} is statistically significant for the subperiods 1966-1985 and 1986-2000, while it turns out not significant for the subperiod 1960-1965.
[37]$SD(\beta_{0,t}^*)$ is the standard deviation of the time-varying parameter $\beta_{0,t}^*$.

the presence of serial correlation as well as conditional heteroscedasticity. Moreover, similarly to the constant term premium model, the residuals turn out to be not normally distributed.
The sub-period 1986-2000, which coincides in large part with the Greenspan's regime, is characterized by low turbulence and high credibility so that, once again, one would expect a reduced relevance of the time-varying term premium as well as the constant term premium to be a reliable model to investigate the EHTS. Also for this sub-period, σ_{v1}^{*2} turns out to be not significant, highlighting the marginal role played by departures from rational expectations. Thus, the above result might suggest that also for the subperiod 1986-2000 the parameter $\beta_{0,t}^{*}$ is an estimation of the term premium. Although remarkably reduced with respect to the previous subperiod, its variance σ_{v0}^{*2} remains statistically significant as well as the autoregressive parameter ϕ_0^*. Unlike the period 1960-1965, therefore, the term premium is driven by a stochastic process. However, its degree of variability turns out to be greatly reduced with respect to the period 1966-1985.[38] Because of the desirable characteristics of this monetary regime, one would also expect a higher degree of mean reversion with respect to the previous period while this is not the case.[39] Overall, the above estimates suggest that the EHTS does not hold because of time-varying term premium. In such a context, it is not surprising that the time-varying term premium model outperforms the constant term premium model.[40] Also for this subperiod the diagnostic statistics reported in Table 9 highlight residuals serially correlated and conditionally heteroscedastic as well as not normally distributed.
Summing up, the evidence so far obtained for both the subperiods 1966-1985 and 1986-2000 suggests that the EHTS is rejected solely because of time-varying term premium. Such hypothesis, however, can be formally tested imposing the restrictions $\phi_1^* = \sigma_{v1}^{2*} = 0$ on the models given by eqs.(18)-(21). These test will be carried out in the paragraph 6.6.

6.5 Parameter Estimates of the Six-Month Term Spread Regression

Table 10 reports the parameter estimates for the model defined by eqs.(18)-(21) when n=12 and m=6. Focusing on the period 1964-2000, both the autoregressive coefficients ϕ_0^* and ϕ_1^* as well as the variances σ_{v0}^{*2} and σ_{v1}^{*2} turn out to be statistically significant. This means that both $\beta_{0,t}^{*}$ and $\beta_{1,t}^{*}$ are time-varying parameters. Since $\phi_0^* = 0.870$, the parameter $\beta_{0,t}^{*}$ follows a mean reverting first order autoregressive stochastic process with half life of nearly five months. With regard to $\beta_{1,t}^{*}$, it evolves according to a mean reverting process with a half life of one and a half months. Also in this case, similarly to the results obtained for the three-month term spread, the statistical significance of both ϕ_1^* and σ_{v1}^{*2} suggests

[38] The reduced volatility is also highlighted by $SD(\beta_{0,t}^{*})$.

[39] The half life t^* for the period 1986-2000 is 4 and a half months, while for the previous period it is slightly more than two months.

[40] Notice, in fact, that the R^2 statistics for the time-varying term premium model are much higher than the R^2 for the constant term premium model.

that the importance of the term spread in explaining the excess forward returns is statistically significant. As a result, the EHTS must be rejected also when the hypothesis is tested making use of a time-varying parameters model which accommodates for time-varying term premium. Moreover, because the above estimates suggest a $\beta_{1,t}^{*}$ statistically significant over time, the time-varying parameter $\beta_{0,t}^{*}$ cannot be interpreted as an estimation of the time-varying term premium. The model fares pretty well in fitting actual data, in fact it presents a higher $R_{t|t-1}^{2}$ with respect to the one set out in Table 4 for the constant term premium model.

Overall, the statistical significance of the hyperparameters which govern both $\beta_{0,t}^{*}$ and $\beta_{1,t}^{*}$ suggests that the EHTS does not hold because of both time-varying term premium and departures from rational expectations. The departures from rational expectations are highlighted by the fact that $\beta_{1,t}^{*}$ is likely to assume, over time, values different from zero which, in turn, means that the term spread contains information at time t which has remained unexploited. This result parallels the empirical evidence obtained for the three-month term spread. Unlike in the constant term premium model, the moving average coefficients turn out to be non statistically significant. This might be explained by the fact that when both the parameters β_{0}^{*} and β_{1}^{*} become time-varying, they absorb part of the explanatory power of the moving average process.

Table 11 reports the Box-Ljung Q-statistic and the Breusch-Godfrey (B-G) test for autocorrelation, the Breusch-Pagan (B-P) and the Lagrange Multiplier (LM) ARCH test for heteroscedasticity, as well as the Jarque-Bera (J-B) test for normality in the residuals. Similarly to the three-month term spread regressions, the diagnostic tests suggest that there may still exist an omitted variable problem, since the residuals are serially correlated. This, in turn, suggests that a further role for the presence of irrationality, other than the role played by the time-varying term premium, arises in the rejection of the EHTS. The Breusch-Pagan (B-P) and the Lagrange Multiplier (LM) ARCH test highlight the presence of conditional heteroscedasticity.

6.5.1 Parameter estimates across different monetary regimes

In line with the previous analysis, this paragraph works out the hyperparameter estimates of eqs.(18)-(21) for the subperiods 1964-1985 and 1986-2000. The empirical estimates reported in Table 10 turn out to be regime dependent. This is especially true for the parameter $\beta_{0,t}^{*}$ which accommodates for time-varying term premium. In fact, consistently with the economic characteristics of the different monetary regimes, it turns out to be driven by stochastic processes with, respectively, high and low variance during the subperiods 1964-1985 and 1986-2000. This supports the view that in a monetary regime characterized by low credibility like the subperiod 1966-1985, one should expect the term premium to play a relevant role and the time-varying parameter models to be more reliable than the constant term premium model for the assessment of the EHTS. The variance σ_{v1}^{2*} turns out to be not statistically significant. In other words, $\beta_{1,t}^{*}$ is a deterministic process which converges quickly towards zero. Thus, limitedly to the subperiod 1964-1985, the hyperparameter estimates suggests that the time-varying term premium is the only cause of rejection of the EHTS, while a limited role is reserved to departures from rational expectations. The higher reliability of the time-varying parameters model can be deduced from the higher $R^2_{t|t-1}$ statistic with respect to the statistic reported in Table 4 for the constant term premium model. The analysis of residuals reported in Table 11 highlights the presence of serial correlation as well as conditional heteroscedasticity. Moreover, similarly to the constant term premium model, the residuals turn out to be not normally distributed.

As already explained the subperiod 1986-2000 was characterized by low turbulence and high credibility so that one would expect a reduced relevance of the time-varying term premium as well as the constant term premium to be a reliable model to investigate the EHTS. Similarly to the results obtained for the three-month term spread regression, the variance σ^{*2}_{v1} turns out to be not statistically significant, highlighting the marginal role played by departures from rational expectations. Thus, the above result suggest that also for the subperiod 1986-2000 the parameter $\beta^*_{0,t}$ could be an estimation of the term premium. Although remarkably reduced with respect to the previous subperiod, its variance σ^{*2}_{v0} remains statistically significant as well as the autoregressive parameter ϕ^*_0.[41] Because of the desirable characteristics of this monetary regime, one would also expect a higher degree of mean reversion with respect to the previous period, however this is not the case.[42] Overall, the above estimates suggest that the EHTS does not hold because of time-varying term premium. In such a context, it is not surprising that the time-varying term premium model outperforms the constant term premium model.[43] Also for this subperiod the diagnostic tests reported in Table 11 highlight residuals serially correlated and conditionally heteroscedastic. However, the null of normality is rejected only at 10% significance level.

Summing up, the evidence so far obtained for both the subperiods 1966-1985 and 1986-2000 suggests that the EHTS is rejected solely because of time-varying term premium. As already mentioned, such hypothesis can be formally tested imposing the restrictions $\phi^*_1 = \sigma^{2*}_{v1} = 0$ on the model given by eqs.(18)-(21). These tests will be reported in the next paragraph.

6.6 Testing for Departures from Rational Expectations

In this paper tests for the EHTS have been carried out making use of the forward-spot spread regression given by eq.(4). This equation, in fact, in both its constant and time-varying parameters versions, must be regarded as a testing equation which can be employed to test for the EHTS (see, for instance, Fama

[41]The reduced volatility is also highlighted by $SD(\beta^*_{0,t})$.

[42]The half life t^* for the period 1986-2000 is seven and a half months, while for the previous period it is slightly more than four months.

[43]Notice, in fact, that limitedly to the subperiod 1986-2000 the R^2 statistics for the time-varying term premium model are much higher than the R^2 for the constant term premium model.

(1984) and Tzavalis and Wickens (1997)). As highlighted in paragraph 6.1, however, if the parameter $\beta^*_{1,t}$ is equal to zero then eq.(4), in its time-varying parameters version, becomes a tool which can be employed to extract the term premium from excess forward returns under the hypothesis of rational expectations. In fact, as long as the already mentioned restriction holds, the above model becomes equivalent to the econometric framework employed by Wolff (1987), Iyer (1997) and Gravelle and Morley (2005). Moreover, from a theoretical perspectives it can be seen that if the restriction $\beta^*_{1,t} = 0$ holds then eq.(4) falls into the class of models of the time-varying term premium set out in par.2.1. In this case, the unobservable term premium is modelled by means of the time-varying parameter $\beta^*_{0,t}$.
Given the econometric framework of eqs.(18)-(21), the restrictions $\beta^*_{1,t} = 0$ can be evaluated by testing the null hypothesis $\phi^*_1 = \sigma^{*2}_{v1} = 0$. Testing for the above restrictions has a twofold motivation. Firstly, such tests are essentially tests for rational expectations. As long as the null cannot be rejected, then the role of departures from rational expectations in the invalidation of the EHTS can be excluded, and the restricted version of eq.(4) can be employed to detect the time-varying term premium. Secondly, there is strong indication that for the three and six-month term spread regressions the empirical estimates of σ^{*2}_{v1} are not statistically significant for the subperiods 1966-1985 and 1986-2000, showing evidence in favor of the above restrictions. Thus, standard Likelihood Ratio (LR) tests are employed to test for the null $\phi^*_1 = \sigma^{*2}_{v1} = 0$ for the subperiod 1966-2000.[44]

6.6.1 Restrictions for the Three-Month Term Spread Regression

Table 12 reports the estimates for the unrestricted and restricted model given by eqs.(18)-(21) for the subperiod 1966-2000 when n=6 and m=3. While the parameter estimates of the unrestricted and restricted models turn out to be quite similar, the LR test is 21.67. Being the degrees of freedom equal to 2, the null of rational expectations is rejected at standard significance levels. Thus, although the non statistical significance of σ^{*2}_{v1} reported in Table 8 provides evidence in favor of rational expectations, when this hypothesis is formally tested it is soundly rejected. Moreover, identical conclusions can be drawn when the null hypothesis is tested separately for the subperiods 1966-1985 and 1986-2000.[45]
As a result, limitedly to the period 1966-2000, the theoretical framework set out in paragraph 6.1 cannot be employed to estimate the time-varying term premium under rational expectations. Moreover, recalling that the EHTS is a joint hypothesis of rationality and risk neutrality, the above result shows that a large part of the literature which has inspected the hypothesis assuming that

[44] The likelihood ratio tests are based on the result that, under the null, the statistic $LR = -2log(L_R/L_{UR})$ is asymptotically distributed as χ^2_m where m is the number of restrictions and where L_R and L_{UR} are the values of the log likelihood functions under the restricted and unrestricted case.

[45] To save space the empirical estimates of the restricted and unrestricted models as well as the likelihood ratio for these subperiods are not reported.

the rational expectation leg of the theory holds and testing for its second leg - i.e. the presence of time-varying term premium - might be flawed when applied to three and six month spot interest rates.

6.6.2 Restrictions for the Six-Month Term Spread Regression

When the above analysis is carried out for the six-month term spread regressions (i.e. when n=12 and m=6), the conclusions turn out to be slightly different. In line with the outcomes obtained for the three-month term spread regression, the restriction for model (18)-(21) are soundly rejected when tested over the period 1966-2000 and the subperiod 1966-1985. To save space the estimates of the restricted and unrestricted models as well as the likelihood ratio for these periods are not reported. When, however, the same restrictions are tested over the subperiod 1986-2000 there is partial evidence in favor of rational expectations. The empirical estimates of the unrestricted and restricted models given by eqs.(18)-(21) for the subperiod 1986-2000 are reported in Table 12. Being the LR test equal to 1.960, the null of rational expectations cannot be rejected at conventional significance levels. This, in turn, suggests that the theoretical framework set out in paragraph 6.1 is a valid instrument to detect the time-varying term premium under rational expectations, when applied to twelve and six month spot interest rates.

7 Testing the EHTS from a Different Perspective

All the estimates carried out in the previous sections provide empirical evidence that the EHTS is invalidated because of either or both time-varying term premium and departures from rational expectations. With regard to the constant term premium models set out in Section 4, they provide overwhelming empirical evidence against the validity of the EHTS. Once one believes the term premium to be time-varying, however, such models become flawed. In fact, the Kalman filter estimates of Section 6 suggest that there might be a pre-eminent role for the time-varying term premium in the invalidation of the EHTS. This, in turn, could explain why the EHTS is so soundly rejected when tested using the constant parameters model. As argued in Section 4, in presence of time-varying term premium, the time-varying coefficients models become preferable because they can accommodate for the presence of time-varying term premium (see, for instance, Simon (1989), Jones and Roley (1983), Tzavalis and Wickens (1997) and Boero and Torricelli (2002)). Because of that, such models might provide results more favorable to the EHTS. However, when estimations of these models are carried out, the evidence provided is far from being reversed. Because both the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$ are found to be driven by autoregressive stochastic processes, the EHTS remains invalidated, as it is shown by the empirical estimates of Tables 8 and 10.

As already mentioned, a desirable feature of the Kalman filter is that it delivers estimates of the state variables $\beta^*_{0,t}$ and $\beta^*_{1,t}$ together with their standard deviations at each time t included in the sample period. This section shows that the voluminous evidence against the EHTS can be partially reversed when the hypothesis is tested exploiting this particular characteristic. Applied to this context, such feature becomes very desirable because it permits the checking over time of the statistical significance of both the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$. In fact, although in Section 5 both the parameters have been found to be time-varying, they might assume, for some specific time t in the sample period, values not statistically significant, thus providing evidence for the EHTS to hold. More specifically, at each time t the EHTS is validated as long as both the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$ turn out to be not statistically significant. On the other hand, the EHTS can be rejected because of time-varying term premium. This occurs when the parameter $\beta^*_{0,t}$ turns out to be statistically significant while the parameter $\beta^*_{1,t}$ is not. The same hypothesis can also be rejected because of both time-varying term premium and departures from rational expectations. In this case both the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$ must be jointly statistically significant.

7.1 Testing the EHTS using the Three-Month Term Spread Regression

In this paragraph the validity of the EHTS is tested by simultaneously evaluating the statistical significance of the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$ when three and six month spot interest rates are considered. Table 14 reports the relative frequency with which the parameters assume values statistically significant at 10% level. These results show that, during the period 1960-2000, the EHTS cannot be rejected for long periods reversing, therefore, the conclusions drawn in the previous sections. The time-varying term premium is the cause which invalidates the EHTS for 17.7% of the entire period (85 months out of 481) while the joint effect of time-varying term premium and departures from rational expectations becomes statistically relevant for 3.74% (18 months). It can be shown that the period in which departures for rational expectations play a role in the rejection of the EHTS is mainly concentrated within the interval from 1979 to 1982. This period, in turn, coincides with the abrupt change in the Federal Reserve's monetary policy which occurred between October 1979 and September 1982. Finally, the EHTS holds for the most part of the period 1960-2000 (i.e. 78.6% of the times, equivalent to 378 months out of 481).
The figures reported in Table 14 show that also the conclusions drawn in Section 6 regarding the different monetary regimes must be partially reviewed. Unlike the empirical evidence of Section 6, in fact, the period 1960-1965 is now fully characterized by the EHTS which holds 100% of the times. Although this result perfectly matches the descriptions given by Goodfriend (1998) and Bordo and Haubrich (2004), it must be taken with caution in view of the reduced number of data available for the period 1960-1965.[46] While the monetary regime

[46] In fact, once maximum likelihood estimates of the hyperparameters are obtained, the Kalman filter is re-run to compute recursively the values of $\beta^*_{0,t}$ and $\beta^*_{1,t}$, as well as their

at work during the subperiod 1966-1985 was found fully characterized by the predominant role of the time-varying term premium as well as by departures from rationality, there is now room for the EHTS to hold. More specifically, the EHTS remains valid for 158 months out of 240 (equivalent to 66.1% of the entire subperiod). The time-varying term premium is the major cause of rejection of the EHTS (27.2% of the times) while the joint effect of departures from rational expectations and time-varying term premium accounts for 7.5% of the times. With regard to the period 1986-2000, quite unexpectedly, in the previous sections, it was found characterized by stochastic properties similar to those of the subperiod 1966-1985, despite its characteristics of credible regime. The results of Table 14 assign now a pre-eminent role to the EHTS which holds 87.6% of the times. In this case, the only cause of rejection of the EHTS is the presence of time-varying term premium, while no role is reserved to departures from rational expectations. Such result reconciles the empirical evidence with the aforementioned characteristics of credible monetary regime highlighted by Goodfriend (1998) and Bordo and Haubrich (2004).

7.2 Testing the EHTS using the Six-Month Term Spread Regression

This paragraph tests for the validity of the EHTS when six and twelve month spot interest rates are considered. The relative frequency with which the parameters $\beta^*_{0,t}$ and $\beta^*_{1,t}$ assume values statistically significant at 10% level is reported in Table 15. Overall, the empirical evidence is similar to that obtained for six and three month spot interest rates.

During the period 1964-2000, the EHTS cannot be rejected for long periods reversing, therefore, the conclusions drawn in the previous sections. The time-varying term premium is the cause which invalidates the EHTS for 47.2% of the entire period (204 months out of 432). Thus, the presence of time-varying term premium plays a more pre-eminent role in the rejection of the EHTS when twelve and six month spot interest rates are considered. The joint effect of time-varying term premium and departures from rational expectations becomes statistically relevant for 1.2% of the times (5 months out of 432). Finally, the EHTS holds for the most part of the period (51.8% equivalent to 224 months). While the analysis carried out in paragraph 6.5 has highlighted that the subperiod 1966-1985 is fully characterized by the predominant role of the time-varying term premium as well as by departures from rationality, there is now room for the EHTS to hold. In particular, the EHTS remains valid for 97 months out of

covariance matrix. For this computation to be carried out, initial values for the two parameters as well as for their covariance matrix must be provided. However, unlike the starting values for β^*_0 and β^*_1, their covariance matrix is assumed to be unknown. This, in turn, is equivalent to set the initial values of the covariance matrix to infinite. As more data points are added one period at a time until the end is reached, the recursive estimates of the covariance matrix converge toward their true (natural) values. The convergency rate, however, could be relatively slow. When this occurs, the estimates of the covariance matrix turn out to be inflated for the initial part of the sample period considered. This might explain why, limitedly to the subperiod 1960-1965, both the parameters β^*_0 and β^*_1 result not statistically significant.

240 (equivalent to 40.4% of the entire subperiod). The presence of time-varying term premium represents the major cause of rejection of the EHTS (57.5% of the times equivalent to 138 months out of 240) while the time-varying term premium and departures from rational expectations becomes jointly relevant for 2.1% of the times. Finally, with regard to the regime at work during the period 1986-2000, the results of Table 15 still assign a quite relevant role to the time-varying term premium while no role whatsoever is reserved to departures from rational expectations. More specifically, the EHTS is rejected because of time-varying term premium for 37.9% of the subperiod, while it holds for the remaining 62.1%. This result, in turn, supports the empirical evidence of paragraph 6.6, where the null of rational expectations, limitedly to the subperiod 1986-2000, was not rejected at standard significance levels. Such results reconcile the empirical evidence with the aforementioned characteristics of credible monetary regime.

8 Conclusion

This paper inspects the validity of the EHTS making use of single regression models that accommodates for the presence of time-varying term premia. This is done by employing time-varying parameters models estimated by means of Kalman filter and maximum likelihood. It is this approach that informs the originality of this study. While standard single regression models employed in the literature require identification of the factors affecting term premium using observables, the Kalman filter approach does not require any *a priori* specification of such factors. This is the main advantage of this approach, since it is highly plausible that term premia are affected by factors difficult to observe. Moreover, the time-varying parameters models employed in this paper have some important features which make them preferable to standard constant parameters models employed in the literature. First, their econometric framework encompasses the models commonly employed in the literature to detect time-varying term premia under rational expectations. As such, their econometric framework can be employed to construct a formal test for the null of rational expectations. Second, the time-varying parameters models can be employed to test for the validity of the EHTS for each time t of the sample period.

Throughout this paper, the EHTS is investigated employing three different approaches. First, standard fixed parameters regressions are considered. This basic approach is then sophisticated by implementing time-varying parameters models by means of Kalman filter and maximum likelihood. Finally, the EHTS is investigated exploiting the fact that the Kalman filter delivers iterative estimations of the latent (unobservable) variables together with their standard deviations at each time t included in the sample period.

When constant parameters regressions are used to inspect the validity of the EHTS, they provide strong evidence against this hypothesis. However, in presence of time-varying term premia, the constant parameters models become flawed. Unlike these models, the time-varying parameter models can accom-

modate for time-varying term premia and, therefore, might deliver results more favorable to the EHTS. When, however, such models are employed, the results suggest that the EHTS is still rejected when tested on the short-end of the term structure. This evidence holds for both three and six month, and six and twelve month interest rates.
When the EHTS is inspected across different monetary regimes at work during the period 1960-2000, the evidence results quite different depending on the dataset considered. When the hypothesis is tested on three and six month interest rates, the Kalman filter estimates highlight a reduced role for departures from rational expectations for the period 1966-2000. However, when the null of rational expectations is formally tested, such hypothesis is soundly rejected. This shows that, although the role of departures from rational expectations as cause of rejection of the EHTS turns out to be reduced, it is still relevant along with the role played by the time-varying term premium. The empirical evidence obtained when six and twelve month rates are employed turns out to be different depending on the subperiod considered. When the analysis focusses on the subperiod 1966-1985, the Kalman filter estimates highlight a reduced but still relevant role for departures from rational expectations. However, when the subperiod 1986-2000 is taken into consideration, the empirical evidence shows that departures from rational expectations do not play any role whatsoever in the rejection of the EHTS. As a result, limitedly to this period, the EHTS turns out to be rejected exclusively because of the presence of time-varying term premium.
Finally, the EHTS is tested exploiting the fact that the Kalman filter delivers iterative estimations of the parameters of the models employed to test for the EHTS, together with their standard deviations at each time t included in the sample period. Unlike the evidence provided in previous sections, this analysis shows much stronger evidence in favor of the EHTS. In fact, the EHTS is shown to holds for large part of the sample periods under analysis, while the time-varying term premium is by far the most frequent cause for its rejection. Departures from rational expectations are clustered around the abrupt change in the Federal Reserve's monetary policy which occurred between October 1979 and September 1982. However, apart from this short period, they are rarely statistically relevant for the invalidation of the EHTS. This evidence holds for both three and six month, and six and twelve month interest rate. Given the extensive empirical literature regarding the US term structure which shows the non-validity of the EHTS when tested on the short-end of the term structure spectrum, this represents a quite surprising result. This finding, in turn, reconciles the US evidence with the evidence found for many European countries where the EHTS was found to be valid.

Table 1: HAC estimation of the constant term premium model for n=6 and m=3.

$F_3^6(t) - R_3(t+3) = \beta_0 + \beta_1[F_3^6(t) - R_3(t)] - \varepsilon_{t+3}(t)$				
	1960-1965	1966-1985	1986-2000	1960-2000
β_0	-0.069*	0.026	0.052*	0.028
	(0.030)	(0.059)	(0.019)	(0.030)
β_1	1.667*	0.737*	0.375	0.703*
	(0.417)	(0.314)	(0.213)	(0.229)
R^2	0.730	0.145	0.392	0.177
$Q(4)^a$	53.08*	141.6*	153.6*	289.3*
$Q(8)$	94.91*	166.1*	160.9*	323.9*
$B-G^b$	42.6*	155.2*	115.6*	314.9*
$B-P^c$	19.613*	3.872	4.932**	3.069
$LM(1)^d$	15.355*	73.561*	57.878*	158.36*
$LM(2)$	23.324*	76.832*	60.428*	163.97*
$LM(4)$	28.467*	93.026*	61.716*	197.86*
$LM(8)$	33.960*	106.23*	70.350*	226.54*
$J-B^e$	30.14*	740.6*	5.133**	5562*
Obs.	72	240	173	485

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 2: Estimation of the constant term premium model for n=6 and m=3 with MA(2) stochastic process in the disturbance term.

$F_3^6(t) - R_3(t+3) = \beta_0 + \beta_1[F_3^6(t) - R_3(t)] + \varepsilon_{t+3}(t)$ $\varepsilon_{t+3}(t) = e(t) + \vartheta_1 e(t-1) + \vartheta_2 e(t-2)$				
	1960-1965	1966-1985	1986-2000	1960-2000
β_0	0.003	-0.013	0.027	0.011
	(0.004)	(0.012)	(0.017)	(0.016)
β_1	0.930*	0.995*	0.878*	0.917*
	(0.037)	(0.021)	(0.071)	(0.034)
ϑ_1	0.898*	1.004*	1.045*	0.998*
	(0.054)	(0.004)	(0.033)	(0.004)
ϑ_2	0.906*	1.002*	0.851*	0.984*
	(0.031)	(0.004)	(0.030)	(0.008)
σ^2	0.0013*	0.034*	0.003*	0.019*
	(0.0002)	(0.001)	(0.0002)	(0.0004)
R^2	0.976	0.874	0.911	0.751
$Q(4)^a$	5.323	24.13*	35.25*	44.14*
$Q(8)$	12.10	48.02*	49.37*	81.67*
$B-G^b$	4.86**	25.6*	19.40*	50.04*
$B-P^c$	2.478	6.588*	4.114	5.939**
$LM(1)^d$	0.002	13.80*	0.173	31.09*
$LM(2)$	0.167	25.54*	0.942	59.65*
$LM(4)$	11.08*	25.72*	9.646*	59.63*
$LM(8)$	6.367	41.577*	12.280	93.09*
$J-B^e$	42.21*	358.4*	2577*	11445*

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 3: Estimation of the constant term premium model for n=6 and m=3 with MA(2) and conditional heteroscedastic disturbance term.

$F_3^6(t) - R_3(t+3) = \beta_0 + \beta_1[F_3^6(t) - R_3(t)] + \varepsilon_{t+3}(t)$ $\varepsilon_{t+3}(t) = e(t) + \vartheta_1 e(t-1) + \vartheta_2 e(t-2)$ $\sigma^2_{\varepsilon_{t+3}}(t) = \alpha_0 + \alpha_1 \varepsilon^2_{t+3}(t-1) + \alpha_2 \sigma^2_{\varepsilon_{t+3}}(t-1)$				
	1960-1965	1966-1985	1986-2000	1960-2000
β_0	-	-0.023*	-	-0.010
	(-)	(0.009)	(-)	(0.007)
β_1	-	0.997*	-	0.961*
	(-)	(0.034)	(-)	(0.031)
ϑ_1	-	0.950*	-	0.949*
	(-)	(0.005)	(-)	(0.008)
ϑ_2	-	0.999*	-	0.941*
	(-)	(0.005)	(-)	(0.018)
α_0	-	0.0008*	-	0.0001*
	(-)	(0.0003)	(-)	(0.00006)
α_1	-	0.646*	-	0.677*
	(-)	(0.016)	(-)	(0.049)
α_2	-	0.390*	-	0.392*
	(-)	(0.030)	(-)	(0.080)
R^2	-	0.871	-	0.740

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.

Table 4: HAC estimation of the constant term premium model for n=12 and m=6.

$F_6^{12}(t) - R_6(t+6) = \beta_0 + \beta_1[F_6^{12}(t) - R_6(t)] + \varepsilon_{t+6}(t)$			
	1964-1985	1986-2000	1964-2000
β_0	-0.014	0.042	-0.001
	(0.053)	(0.031)	(0.038)
β_1	0.935*	0.525*	0.897*
	(0.238)	(0.263)	(0.210)
R^2	0.171	0.065	0.159
$Q(4)^a$	292.3*	391.8*	520.1*
$Q(8)$	313.6*	339.5*	535.4*
$B-G^b$	198.1*	137.2*	328.1*
$B-P^c$	11.82*	6.497*	27.67*
$LM(1)^d$	181.3*	98.74*	224.1*
$LM(2)$	198.1*	101.4*	223.5*
$LM(4)$	200.8*	102.8*	332.3*
$LM(8)$	217.5*	103.3*	356.6*
$J-B^e$	71.5*	2.373	305.5*
Obs.	264	165	429

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 5: Estimation of the constant term premium model for n=12 and m=6 with MA(5) stochastic process in the disturbance term.

$F_6^{12}(t) - R_6(t+6) = \beta_0 + \beta_1[F_6^{12}(t) - R_6(t)] + \varepsilon_{t+6}(t)$ $\varepsilon_{t+6}(t) = e(t) + \vartheta_1 e(t-1) + \vartheta_2 e(t-2) + ... + \vartheta_5 e(t-5)$			
	1964-1985	1986-2000	1964-2000
β_0	0.009 (0.014)	-0.001 (0.036)	0.0021 (0.008)
β_1	1.005* (0.028)	1.070* (0.035)	1.002* (0.011)
ϑ_1	1.402* (0.161)	1.097* (0.010)	1.361* (0.095)
ϑ_2	1.228* (0.222)	1.166* (0.021)	1.278* (0.128)
ϑ_3	0.596* (0.180)	1.152* (0.021)	0.780* (0.143)
ϑ_4	0.146 (0.132)	1.060* (0.027)	0.407* (0.106)
ϑ_5	0.465* (0.103)	0.944* (0.016)	0.556* (0.070)
σ^2	0.031* (0.001)	0.004* (0.0004)	0.020* (0.0005)
R^2	0.882	0.924	0.877
$Q(6)^a$	40.82*	26.79*	51.18*
$Q(8)$	57.36*	28.63*	86.71*
$B-G^b$	104.2*	15.52*	106.5*
$B-P^c$	33.93*	4.866**	49.86*
$LM(1)^d$	55.96*	5.042*	88.05*
$LM(2)$	55.88*	5.021**	88.39*
$LM(4)$	68.66*	5.413	97.19*
$LM(8)$	96.80*	14.57**	153.8*
$J-B^e$	285.9*	0.788	1705*

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 6: Estimation of the constant term premium model for n=12 and m=6 with MA(5) and conditional heteroscedastic disturbance term.

$F_6^{12}(t) - R_6(t+6) = \beta_0 + \beta_1[F_6^{12}(t) - R_6(t)] + \varepsilon_{t+6}(t)$ $\varepsilon_{t+6}(t) = e(t) + \vartheta_1 e(t-1) + \vartheta_2 e(t-2) + \ldots + \vartheta_5 e(t-5)$ $\sigma^2_{\varepsilon_{t+6}}(t) = \alpha_0 + \alpha_1 \varepsilon^2_{t+6}(t-1) + \alpha_2 \sigma^2_{\varepsilon_{t+6}}(t-1)$			
	1964-1985	1986-2000	1964-2000
β_0	-0.076*	0.035	-0.041*
	(0.029)	(0.019)	(0.021)
β_1	1.023*	1.028*	1.025*
	(0.011)	(0.034)	(0.009)
ϑ_1	0.996*	1.069*	0.987*
	(0.014)	(0.024)	(0.007)
ϑ_2	1.025*	1.055*	1.014*
	(0.012)	(0.024)	(0.011)
ϑ_3	0.998*	0.955*	1.000*
	(0.020)	(0.025)	(0.013)
ϑ_4	0.958*	0.832*	0.962*
	(0.014)	(0.024)	(0.011)
ϑ_5	0.954*	0.766*	0.971*
	(0.016)	(0.022)	(0.007)
α_0	0.0001*	0.0004*	0.0001*
	(0.0001)	(0.0001)	(0.00005)
α_1	0.776*	0.452*	0.776*
	(0.032)	(0.046)	(0.025)
α_2	0.280*	0.494*	0.251*
	(0.047)	(0.077)	(0.034)
R^2	0.871	0.881	0.878

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.

Table 7: Andrews and Andrews-Ploberger instability test for n=6 and m=3 (upper panel) and for n=12 and m=3 (lower panel).

$F_3^6(t) - R_3(t+3) = \beta_0 + \beta_1[F_3^6(t) - R_3(t)] + \varepsilon_{t+3}(t)$			
	Andrews	Andrews-Ploberger	Datebreak
β_0	7.150**	0.867	1981:06
β_1	12.161*	2.223*	1981:01
σ^2_μ	3.404	0.722	1982:08
Joint	12.473*	2.902**	1981:01
$F_6^{12}(t) - R_6(t+6) = \beta_0 + \beta_1[F_6^{12}(t) - R_6(t)] + \varepsilon_{t+6}(t)$			
	Andrews	Andrews-Ploberger	Datebreak
β_0	22.82*	6.874*	1981:04
β_1	12.61*	2.600*	1980:04
σ^2_μ	5.012	1.491**	1982:09
Joint	23.56*	8.252*	1980:06

Notes: * (**) Significant at 5% (10%).

Table 8: Parameter estimates of the three-month time-varying model (n=6 and m=3) defined by eqs.(18)-(21) for the period 1960-2000 and for the different monetary regimes.

	1960-1965	1966-1985	1986-2000	1960-2000
ϕ_0^*	0.738*	0.740*	0.899*	0.747*
	(0.272)	(0.046)	(0.039)	(0.034)
ϕ_1^*	0.904*	0.928*	0.999*	0.908*
	(0.047)	(0.042)	(0.006)	(0.040)
θ_0	0.070	0.012	-0.116	-0.003
	(1.875)	(3.170)	(2.965)	(1.360)
θ_1	-0.118	0.055	-0.026	-0.091
	(1.645)	(2.652)	(3.420)	(1.480)
σ_{v0}^{2*}	0.00017	0.058*	0.0036*	0.027*
	(0.0001)	(0.005)	(0.0004)	(0.002)
σ_{v1}^{2*}	0.150*	0.089	0.004	0.195*
	(0.040)	(0.055)	(0.004)	(0.076)
$\sigma_{\varepsilon}^{2*}$	$4.04 \cdot 10^{-13}$*	$2.50 \cdot 10^{-13}$	$2.36 \cdot 10^{-13}$	$2.25 \cdot 10^{-13}$
	$(6.73 \cdot 10^{-9})$	$(2.02 \cdot 10^{-8})$	$(6.80 \cdot 10^{-9})$	$(1.55 \cdot 10^{-14})$
t^*	3.195	2.323	4.116	2.376
$SD(\beta_{0,t}^*)$	0.064	0.303	0.094	0.223
$R_{t\|t-1}^2$	0.945	0.787	0.841	0.799

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.

Table 9: Diagnostic tests for the model defined by eqs.(18)-(21) for the period 1960-2000 and for the different monetary regimes.

	1960-1965	1966-1985	1986-2000	1960-2000
$Q(4)^a$	25.15*	112.4*	84.30*	218.3*
$Q(8)$	41.60*	125.8*	90.73*	236.4*
$B-G^b$	20.41*	128.1*	82.48*	242.2*
$B-P^c$	1.302*	4.306	3.521	4.68**
$LM(1)^d$	8.240*	47.06*	29.108*	90.85*
$LM(2)$	8.810*	46.85*	33.51*	90.88*
$LM(4)$	7.701**	63.03*	38.77*	121.4*
$LM(8)$	5.820	67.13*	40.11*	121.1*
$J-B^e$	1.038	1286*	6.102*	13946*

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 10: Parameter estimates of the three-month time-varying model (n=12 and m=6) defined by eqs.(18)-(21) for the period 1964-2000 and for the different monetary regimes.

	1964-1985	1986-2000	1964-2000
ϕ_0^*	0.856*	0.912*	0.870*
	(0.032)	(0.032)	(0.024)
ϕ_1^*	0.705*	0.780*	0.645*
	(0.164)	(0.142)	(0.115)
θ_0	-0.009	0.501	-0.277
	(1.398)	(2.241)	(4.194)
θ_1	0.426	0.449	0.152
	(1.585)	(4.194)	(2.421)
θ_2	0.270	0.208	0.380
	(1.531)	(4.194)	(2.652)
θ_3	0.259	0.537	0.288
	(1.482)	(3.424)	(2.695)
θ_4	-0.049	0.223	0.081
	(1.875)	(4.194)	(2.097)
σ_{v0}^{2*}	0.050*	0.006*	0.030*
	(0.005)	(0.001)	(0.003)
σ_{v1}^{2*}	0.088	0.032	0.126*
	(0.055)	(0.020)	(0.046)
$\sigma_{\varepsilon}^{2*}$	$3.36 \cdot 10^{-14}$*	$9.44 \cdot 10^{-14}$	$2.48 \cdot 10^{-13}$
	$(8.07 \cdot 10^{-9})$	$(3.52 \cdot 10^{-9})$	$(1.65 \cdot 10^{-8})$
t^*	4.457	7.524	4.976
$SD(\beta_{0,t}^*)$	??	??	??
$R_{t\|t-1}^2$	0.766	0.560	0.791

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two-tailed test.

Table 11: Diagnostic tests for the model defined by eqs.(18)-(21) (n=12 and m=6) for the period 1964-2000 and for the different monetary regimes.

	1964-1985	1986-2000	1964-2000
$Q(4)^a$	175.9*	326.9*	184.8*
$Q(8)$	277.9*	352.5*	316.3*
$B-G^b$	141.3*	131.4*	190.9*
$B-P^c$	12.45*	6.052*	12.76*
$LM(1)^d$	63.85*	83.42*	71.49*
$LM(2)$	70.29*	82.70*	71.70*
$LM(4)$	76.25*	86.04*	78.43*
$LM(8)$	87.32*	85.20*	116.2*
$J-B^e$	462.4*	5.039**	340.9*

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%).
[a] Ljung-Box statistic, [b] Breusch-Godfrey test, [c] Breusch-Pagan test, [d] Lagrange Multiplier ARCH test, [e] Jarque-Bera test.

Table 12: Parameter estimates of the unrestricted and restricted time-varying model (n=6 and m=3) defined by eqs.(18)-(21) for the period 1966-2000.

	1966-2000	1966-2000
ϕ_0^*	0.745*	0.765*
	(0.036)	(0.032)
ϕ_1^*	0.916*	-
	(0.044)	(-)
θ_0	0.005	-0.004*
	(2.652)	(0.392)
θ_1	-0.041	-0.021*
	(2.965)	(0.526)
σ_{v0}^{2*}	0.032*	0.039*
	(0.002)	(0.003)
σ_{v1}^{2*}	0.166*	-
	(0.082)	(-)
$\sigma_{\varepsilon}^{2*}$	$2.55 \cdot 10^{-13}$	$2.48 \cdot 10^{-13}$
	$(1.23 \cdot 10^{-8})$	$(1.26 \cdot 10^{-8})$
$LR^{\dagger}$	-	21.676*
		(0.000)

Notes: Standard Error in parenthesis.
* (**) Significant at 5% (10%) in a two -tailed test. † likelihood ratio test for the null $\phi_1^* = \sigma_{v1}^{2*} = 0$.

Table 13: Parameter estimates of the unrestricted and restricted time-varying model (n=12 and m=6) defined by eqs.(18)-(21) for the period 1986-2000.

	1986-2000	1986-2000
ϕ_0^*	0.912*	0.908*
	(0.032)	(0.032)
ϕ_1^*	0.780*	-
	(0.142)	(-)
θ_0	0.501	0.363
	(2.241)	(0.667)
θ_1	0.449	0.591
	(4.194)	(0.655)
θ_2	0.208	0.647
	(4.194)	(0.655)
θ_3	0.537	0.454
	(3.424)	(0.680)
θ_4	0.223	0.316
	(4.194)	(0.651)
σ_{v0}^{2*}	0.006*	0.007*
	(0.001)	(0.001)
σ_{v1}^{2*}	0.032	-
	(0.020)	(-)
$\sigma_{\varepsilon}^{2*}$	$9.44 \cdot 10^{-14}$	$2.75 \cdot 10^{-13}$
	$(3.52 \cdot 10^{-9})$	$(8.15 \cdot 10^{-10})$
$LR^{\dagger}$	-	1.960
		(0.375)

* (**) Significant at 5% (10%) in a two -tailed test. † likelihood ratio test for the null $\phi_1^* = \sigma_{v1}^{2*} = 0$.

Table 14: Relative frequency with which the factors that invalidate the EHTS become statistically relevant (n=6 and m=3).

	1960-1965	1966-1985	1986-2000	1960-2000
$TVTP^{\dagger}$	0%	27.3%	12.5%	17.7%
$IRR^{\ddagger} \cap TVTP$	0%	7.6%	0%	3.7%
$EHTS$	100%	65.1%	87.5%	78.6%

Notes: † Time-varying term premium, ‡ Departure from rational expectations.

Table 15: Relative frequency with which the factors that invalidate the EHTS become statistically relevant (n=12 and m=6).

	1966-1985	1986-2000	1966-2000
$TVTP^{\dagger}$	57.5%	37.9%	47.2%
$IRR^{\ddagger} \cap TVTP$	2.08%	0%	1.15%
$EHTS$	40.4%	62.1%	51.8%

Notes: † Time-varying term premium, ‡ Departure from rational expectations.

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