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Abstract

We reconsider the issue of equilibrium determinacy under the limited asset market participation hypothesis in a medium-scale model which accounts for external consumption habits. This allows to characterize concern for relative consumption in the preferences of agents which are heterogeneous in their wealth holdings. We find that external habits and consumption inequality have mutually reinforcing adverse effects on determinacy. We therefore uncover a causality link between long-run inequality and macroeconomic volatility in a New-Keynesian DSGE model. In our framework, redistributive polices targeting consumption inequality have beneficial implications for macroeconomic stability.

JEL codes: E52, E63
Keywords: Limited Asset Market Participation, DSGE, Determinacy, Consumption Habits

1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are based on the assumption that intertemporal optimization drives consumption-saving decisions. Due to its apparent ability to replicate some business cycle facts, a consensus has emerged around a specific model structure, the New Keynesian model, that incorporates a number of nominal and real frictions in goods, labor and financial markets. (Christiano et al. 2005, Smets and Wouters, 2003, 2005, 2007).

Following Mankiw (2000), a rapidly expanding literature has introduced the notion of consumers' heterogeneity, drawing a distinction between agents that have full access to financial markets (Ricardian agents henceforth) and agents that do not exploit financial markets to smooth consumption (RT consumers henceforth). (Galì et al 2004, 2007; Erceg, Guerrieri and Gust, 2006; De Graeve et al., 2010). Empirical research provides support for the RT consumers hypothesis. Estimates of the RT consumers share, obtained using a variety of econometric methods, ranging from 26 to 40% (Campbell and Mankiw, 1989; Jacoviello, 2004; Coenen and Straub, 2005; Forni et al., 2009) obtain values around 35%. Indirect support for the RT consumers hypothesis is also found in recent studies that document households responses to temporary tax-reductions and public transfers increases (Johnson et al., 2006; Shapiro and Slemrod, 2009; Parker et al. 2011) Indeed the fiscal stimuli implemented in response to the 2007-08 financial crisis (Oh and Reis, 2011) were largely based on increased public transfers, apparently meant to support consumption of liquidity-constrained households.

Standard DSGE models typically deliver the prescription that a monetary policy based on the Taylor principle is sufficient to ensure equilibrium determinacy. This result has been challenged in Bilbiie (2008, Bilbiie henceforth), who showed that satisfying the Taylor principle cannot ensure model determinacy in a very simple model where price stickiness and limited asset market participation (LAMP henceforth) are the only frictions. This obtains because imperfect price adjustment to wage increases causes profit losses which are entirely borne by Ricardian agents. As a consequence, a real interest rate increase may be associated to a surge in aggregate demand and production even if it induces a fall in the consumption of Ricardian agents. Natvik (2010) shows that in a unionized labor market, where wages are flexible, the equilibrium becomes indeterminate for a lower share of rule-of-thumb households. In contrast , Colciago (2011) and Ascari et al. (2011) show that a modest amount of nominal wage rigidity is sufficient to limit profit volatility and to restore the standard Taylor Principle even for a very large share of RT consumers.

In this paper we reconsider the issue of equilibrium determinacy under the LAMP hypothesis in a medium-scale model (see Smets and Wouters 2005) which accounts for a wide range of frictions beyond price and nominal wage rigidity (consumption habits, investment adjustment costs, variable capacity utilization, a working capital channel, inflation indexation). right from the outset, it is worth noting that this model is characterized by the assumption of external consumption habits. The alternative specification of internal habits is sometimes adopted in similar models (Christiano et al. 2005). Dennis (2009) shows that the two specifications bear almost identical implications in empirical New Keynesian models based of full asset market participation. Here we prefer the external habit specification because it allows to characterize concern for relative consumption in the preferences of agents which are heterogeneous in their wealth holdings. This feature of the model is new and allows to uncover certain characteristics of the model dynamics which have so far neglected even in contributions that allow for limited asset market participation but constrain external habits to be group specific (Coenen and Straub, 2005; Forni et al., 2009).

Numerical simulations of our model show that determinacy obtains only if the share of RT consumers does not exceed 27%, suggesting that other frictions may weaken the role of nominal wage stickiness and restore the original Bilbiie result. To get further insights on the causes of indeterminacy, we focus on a simpler version of the model, where capital is fixed, nominal wages are flexible and there is no cash-in-advance constraint on firms. This allows to obtain analytical results highlighting the role of consumption habits. Then, following a piecemeal approach, we add physical capital to the model and consider the role of each remaining friction in turn.

In a nutshell, our key conclusion is that habits increase the sensitivity of nominal wages (and profits) to business cycle conditions, thereby lowering the threshold share of RT consumers that is associated to equilibrium indeterminacy. In this regard, we find that external habits and consumption inequality have mutually reinforcing adverse effects on determinacy. This implies that equilibrium indeterminacy obtains at a lower threshold share of RT consumers for all the model features which raise consumption inequality between the two groups, i.e. physical capital and the working capital cash in advance constraint on firms.

In this framework, we consider redistributive fiscal polices targeting consumption inequality between the two consumer groups, finding that they have beneficial implications for macroeconomic stability.

The rest of the paper is organized as follows. In the next section we describe in detail the model structure, then in section 3 we present the results concerning the model stability. Section 4 proposes alternative ways to regain stability of the model. Section 5 concludes.

2 The Model

We extend a standard medium-sized New Keynesian model (Christiano et al. 2005, Smets and Wouters, 2003, 2005, 2007), to account for both Ricardian and RT households. Ricardian households participate in financial markets, base their choices on intertemporal optimization and react to real interest rate changes. By contrast, RT consumers do not hold any wealth. Due to the cash-in-advance constraint on firms, RT consumers receive their labor income at the beginning of the period and entirely use it to finance their current consumption.

2.1 Households preferences

Households are indexed by i, where $i \in [0,1]$. RT (rt) and Ricardian (o) consumers are defined over the intervals $[0,\theta]$ and $(\theta,1]$ respectively. The common preferences are characterized by the following utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln \left(c_t^i - b c_{t-1} \right) - \frac{\psi_l}{1+\phi_l} (h_t^i)^{1+\phi_t} + \frac{\psi_q}{1-\sigma_q} (q_t^i)^{1-\sigma_q} \right\} \tag{1}$$

where $q_t^i = \frac{Q_t}{P_t}$ represents households real money balances, $c^i = \left[\int_0^1 \left(c^i(z) \right)^{\frac{\eta-1}{\eta}} dz \right]^{\frac{\eta}{\eta-1}}$ represents individual consumption of a basket of differentiated goods, b denotes external habits as in Smets and Wouters (2003, 2005, 2007), $c = \int_0^1 c^i di$ is aggregate consumption and $h_t^i = \left(\int_0^1 \left(h_t^i(j) \right)^{\frac{\alpha_w - 1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_w - 1}}$ denotes individual labour supply of a differentiated labour bundle.

Goods markets are monopolistically competitive, and good z is produced with the following technology:

$$y_t(z) = (k_t(z))^{\alpha} (h_t(z))^{1-\alpha}$$

where $k_t(z)$ defines the physical capital services obtained from households and $h_t(z)$ is the composite labor input used by each firm z. Firms z demand for labor type j is

$$h_t^j(z) = \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d(z) \tag{2}$$

where $W_t = \left(\int_0^1 \left(W_t^j \right)^{1-\alpha_w} dj \right)^{1/(1-\alpha_w)}$ defines the wage index.

At the beginning of period t firms borrow the wage bill $W_t h_t$ at the gross interest rate R_t.
Firm *z's* nominal total production cost is therefore given by

$$TC_t(z) = \left[R_t W_t h_t(z) + (1 + R_t^k) k_t(z) \right] (1-\rho) \tag{3}$$

where ρ defines a production subsidy which is financed is financed by lump-sum taxes, levied on firms. The real marginal costs are:

$$mc_t = \left[\left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{w_t R_t}{(1-\alpha)} \right)^{1-\alpha} \right] (1-\rho) \tag{4}$$

where $w_t = \frac{W_t}{P_t}$ and $r_t^k = \frac{R_t^k}{P_t}$ respectively define thee real wage rate and the real return on physical capital.

2.1.1 Sticky Prices

Price stickiness is based on the Calvo mechanism. In each period a fraction λ_p of firms cannot reoptimize and adjust their price to previous period inflation. For these firms the price-setting condition therefore is:

$$p_t(z) = (1+\pi_{t-1})^{\gamma_p} p_{t-1}(z) \tag{5}$$

where $(1+\pi_{t-1}) = \frac{P_{t-1}}{P_{t-2}}$ and $\gamma_p \in [0,1]$ represents the degree of price indexation.

The remaining $(1-\lambda_p)$ firms reoptimize and set the price $\widetilde{P}_t$ that maximizes the discounted sum of expected future profits:[1]

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s}\right) y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}}\right)^{-\eta} \tag{6}$$

where y_t^d is aggregate demand and λ_t is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} y_{t+s}^d \left[\begin{array}{c} (1-\eta)\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{1-\eta} \widetilde{P}_t^{-\eta} (P_{t+s})^{\eta} + \\ +\eta \widetilde{P}_t^{-\eta-1} P_{t+s}^{\eta+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{-\eta} \end{array}\right] = 0 \tag{7}$$

2.2 Ricardian Households

Ricardian households maximize 1 subject to the following period budget constraint.

$$M_{t+1} + P_t [i_t + c_t] \quad = \quad R_t [M_t - P_t q_t] + A_{j,t} + P_t q_t + P_t \left[r_t^k u_t - a(u_t)\right] \bar{k}_t + \tag{8}$$

$$+P_t d_t + h_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} dj \tag{9}$$

Where M_t defines nominal money balances, $R_t [M_t - Q_t]$ defines firms interest payments on the borrowed funds, $A_{j,t}$ and d_t respectively are the cash flow from participating in state-contingent securities and real dividends. Variable i_t denotes investment. Optimizing households own the physical stock of capital $\bar{k}_t$, and choose the degree of its utilization, u_t. The term $a(u_t)$ defines the real cost of using the capital stock with intensity u_t. Firms rent capital services at the real rate r_t^k. The household's stock of physical capital evolves as:

$$\bar{k}_{t+1} = (1-\vartheta) \bar{k}_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{10}$$

$$k_t = u_t \bar{k}_t \tag{11}$$

where ϑ and S respectively denote the physical rate of depreciation and investment adjustment costs.

The Euler equation is

$$\lambda_t^o = \beta E_t \lambda_{t+1}^o \frac{R_{t+1}}{\pi_{t+1}} \tag{12}$$

where

$$\lambda_t^o = \frac{1}{c_t^o - bc_{t-1}} \tag{13}$$

[1] These firms face symmetrical marginal costs.

defines the marginal utility of consumption. Ricardian households money demand depends positively on current consumption and negatively on the current interest rate.

$$\psi_q (q_t)^{-\sigma_q} = (R_t - 1)\,\lambda_t^o \tag{14}$$

The following first order conditions describe demand functions for capital[2] and investment and the optimal degree of capital utilization.

$$P_{k',t} = \beta E_t \left\{ \lambda_{t+1}^o \frac{r_{t+1}^k u_{t+1} - a\left(u_{t+1}\right) + (1-\vartheta)\, P_{k',t+1}}{\lambda_t^o} \right\} \tag{15}$$

$$\lambda_t^o = E_t \left\{ \begin{array}{c} \lambda_t^o P_{k',t}\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] + \\ +\beta \lambda_{t+1}^o P_{k',t+1}\left[S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right] \end{array} \right\} \tag{16}$$

$$r_t^k = a'\left(u_t\right) \tag{17}$$

The investment adjustment cost function and the capital utilization function are: [3]

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

$$a\left(u_t\right) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2$$

2.3 Money market Clearing

Money market clearing requires that

$$W_t L_t = M_t - Q_t \tag{18}$$

2.4 Rule-of-Thumb Households

At the beginning of each period RT consumers receive a monetary payment equal to their labor income. This is used for purchasing consumption goods by the end of the period.

$$c_t^{rt} = w_t h_t^{rt} \tag{19}$$

2.5 Labor market

For each labor input there is a union j which monopolistically sets the nominal wage, W_t^j, subject to (2). Each household i supplies all labour types at the given wage rate, and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{20}$$

Ricardian and non-Ricardian households work for the same amount of time because we assume that the two households groups are uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Individual labor income therefore is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{21}$$

[2] $P_{k',t}$ is the shadow relative price of one unit of capital with respect to one unit of consumption.

[3] Function $S(\cdot)$ satisfies the following properties. $S(1) = S'(1) = 0$ and $S''(1) > 0$. These restrictions imply the absence of adjustment costs up to a first order approximation around the deterministic steady state.The function $a(\cdot)$, instead, is assumed to satisfy $a(1) = 0$ and $a'(1), a''(1) > 0$. Moreover the parameters γ_1 and γ_2 are fixed given that $a'(u) = r^k$ at steady state.

2.5.1 Sticky wages

In each period a fraction λ_w of unions cannot reoptimize and index their wages to lagged inflation:

$$W_t^j = W_{t-1}^j \left(1 + \pi_{t-1}\right)^{\gamma_w}$$

where γ_w stands for the degree of wage indexation. Following Colciago (2011), the remaining $(1 - \lambda_w)$ unions set $\widetilde{W}_t$ to maximize a weighted average of the two household types utility functions, conditional to the probability that the wage cannot be reoptimized in the future.

$$L^u = E_t \sum_{s=0}^{\infty} \left(\beta\lambda_w\right)^s \left\{\left[\left(1-\theta\right) U^o(c_{t+s}^o) + \theta U^{rt}(c_{t+s}^{rt})\right] - U(h_{t+s})\right\} \tag{22}$$

The relevant constraints are (20), (8), (19). In addition, the wage setting decision takes into account that in the future it might not be able to reoptimize. In this case, the real wage at the generic period $t+s$ will read as

$$w_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \tag{23}$$

The union's first-order condition is:

$$E_t \sum_{s=0}^{\infty} \left(\beta\lambda_w\right)^s \left[\left(1-\theta\right)\lambda_{t+s}^o + \theta\lambda_{t+s}^{rt}\right] h_{t+s}^d \left(w_{t+s}\right)^{\alpha_w} \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}}\right)^{-\alpha_w} \cdot \tag{24}$$

$$\cdot \left[\widetilde{w}_t \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}}\right) - \mu^w \frac{\psi_l h_{t+s}^{\phi_l}}{\left[\left(1-\theta\right)\lambda_{t+s}^o + \theta\lambda_{t+s}^{rt}\right]}\right] \quad = \quad 0$$

where $\lambda_t^{rt} = \frac{1}{c_t^{rt} - bc_{t-1}}$ and $\mu^w = \frac{\alpha_w}{(\alpha_w - 1)}$.

2.6 Aggregation

Aggregation yields:

$$y_t = \left(\left(u_t\right) k_t\right)^{\alpha} \left(h_t\right)^{1-\alpha} = c_t + i_t + a\left(u_t\right) k_t$$

where

$$c_t = \theta c_t^{rt} + (1-\theta) c_t^o \tag{25}$$

$$i_t = (1-\theta) \int_{\theta}^{1} i_t^o(j)\ dj \tag{26}$$

$$k_t = (1-\theta) \int_{\theta}^{1} k_t^o(j)\ dj \tag{27}$$

$$h_t = \int_{\theta}^{1} h_t^i\ di \tag{28}$$

2.7 Monetary Policy

Monetary policy follows a standard Taylor rule

$$\frac{R_t}{R} = \left(\pi_t\right)^{\phi_\pi} \tag{29}$$

This obviously implies that the money supply is endogenous and ensures money market clearing at the central bank desired level of the interest rate.

3 Stability Analysis

The complexity of the model requires that stability analysis be carried out through numerical methods.[4] The parameter governing the degree of habit persistence, b, and the labor utility parameter, ϕ_l are respectively set at 0.65 and 3, well in the ranges of the estimates obtained by Smets and Wouters (2005). We calibrate the parameters γ_1 and γ_2 in order to have $\frac{a''}{a'} = 2.01$ as in Altig, et al. (2005) and the parameter ψ_l in order to have steady state level of worked hours close to 0.3. We set $\beta = (1.03)^{-0.25}$ which implies a steady-state annualized real interest rate of about 3%. Values for price and nominal wage stickiness and inflation indexation are also taken from Christiano et al. ($\lambda_p = 0.6$, $\lambda_w = 0.64$, $\gamma_p = \gamma_w = 1$), who find that prices and nominal wages are optimized every 2.5 and 2.8 quarters respectively. [5]

The baseline version of the model is indetermined for $\theta > 0.27$. In simpler models, Colciago (2011) and Ascari er al. (2011) have shown that a minimal degree of nominal wage stickiness is sufficient to achieve determinacy in the Bilbiie model. Our result suggests that other frictions may restore the original Bilbiie result. To get further insights on the causes of indeterminacy, we focus on a version of the model where capital is fixed, nominal wages are flexible and there is no cash-in-advance constraint on firms. This allows to obtain analytical results highlighting the role of consumption habits. Then, following a piecemeal approach, we shall consider the effects of each remaining friction in turn.

3.1 Model 1

In this version of the model we remove price and wage indexation ($\gamma_p = \gamma_w = 0$), the cash in advance constraint on firms and the capital accumulation channel ($\alpha = 0$). In addition, we set $\lambda_w = 0$, implying that nominal wages are flexible. The wage setting condition therefore is

$$w_t = \frac{\alpha_w}{\alpha_w - 1}\psi_l \frac{(h_t)^{\phi_l}}{\left[(1-\theta)\lambda^o_{t+s} + \theta\lambda^{rt}_{t+s}\right]} \tag{30}$$

3.1.1 The model in log-linear form

We take a log-linear approximation around the steady state[6]

Supply side

$$\hat{y}_t = \hat{h}_t \tag{31}$$

$$\hat{w}_t = \phi_l \hat{h}_t - \left[(1-\theta)\hat{\lambda}^o_t + \theta\hat{\lambda}^{rt}_t\right] \tag{32}$$

$$\hat{\lambda}^o_t = -\frac{1}{\left(\frac{c^o}{c}\right) - b}\left[\left(\frac{c^o}{c}\right)\hat{c}^o_t - b\hat{c}_{t-1}\right] \tag{33}$$

$$\hat{\lambda}^{rt}_t = -\frac{\left(\frac{c^{rt}}{c}\right)}{\left(\frac{c^{rt}}{c}\right) - b}\left[\left(\frac{c^o}{c}\right)\hat{c}^{rt}_t - b\hat{c}_{t-1}\right] \tag{34}$$

$$\widehat{mc}_t = \hat{w}_t \tag{35}$$

$$\hat{\pi}_t = \kappa\left(\frac{1}{1-b} + \phi_l\right)\hat{y}_t - \kappa\left(\frac{b}{1-b}\right)\hat{y}_{t-1} + \beta\hat{\pi}_{t+1}; \quad \kappa = \frac{(1-\lambda_p)(1-\beta\lambda_p)}{\lambda_p} \tag{36}$$

[4] In Appendix A we present the loglinear version of the model.

[5] We do not report numerical values for parameters that appear in the money market equilibrium conditions (14) and (18) because in this model they are irrelevant for determinacy analysis under an interest rate rule.

[6] Hatted variables denote the log-deviation of a variable from its zero-inflation, deterministic steady-state value.

Demand side

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t \tag{37}$$

$$\hat{\lambda}_t^o = E_t \hat{\lambda}_{t+1}^o + \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{38}$$

$$\hat{y}_t = \hat{c}_t = (1-\theta)\frac{c^o}{c}\hat{c}_t^o + \theta \frac{c^{rt}}{c}\hat{c}_t^{rt} \tag{39}$$

Monetary policy rule

$$\hat{R}_t = \phi_\pi \hat{\pi}_t. \tag{40}$$

3.1.2 Stability analysis

To limit the analytical complexity of the model, we posit that the production subsidy $\rho = \rho^*$ brings production at the competitive level. Bearing in mind that the subsidy is entirely financed by lump-sum taxes levied on firms, as in Ascari et al. (2011), this implies that in steady state firms profits are nil, and that both consumption and the marginal rate of substitution are identical for the two consumer groups $\left(\frac{c^o}{c} = \frac{c^{rt}}{c} = 1\right)$. In this case, from (31), (32), (33), (34) we get

$$\hat{w}_t = \phi_l \hat{h}_t + \frac{1}{1-b}\hat{y}_t - \frac{b}{1-b}\hat{y}_{t-1} \tag{41}$$

Note that due to price stickiness, firms profits are the inverse of marginal cost deviations from steady state.[7]

$$\hat{d}_t = -\widehat{mc}_t = -\left(1 + \phi_l + \frac{1}{1-b}\right)\hat{y}_t + \frac{b}{1-b}\hat{y}_{t-1} \tag{42}$$

Using (31), (39), (41), and (42) it is easy to see that in equilibrium each optimizing household must consume

$$\hat{c}_t^o = \hat{w}_t + \hat{h}_t + \frac{\hat{d}_t}{1-\theta} = \chi \hat{y}_t + \frac{\theta}{(1-\theta)}\frac{b}{1-b}\hat{y}_{t-1} \tag{43}$$

where $\chi = \left[1 - \frac{\theta}{(1-\theta)}\left(\phi_l + \frac{1}{1-b}\right)\right]$. When $\theta = 0$ (and $\chi = 1$), an increase in current output is associated with a real wage increase and with a profits reduction that exactly offset each other. By contrast, when $\theta > 0$ the increase in output entails a redistribution of income from asset holders to RT consumers. For "large" values of θ, profit losses exceed the positive labor income variation determined by the increase in output. In this case $\chi < 0$ and $\hat{c}_t^o$ is inversely related to $\hat{y}_t$. Note that habits increase the sensitivity of wages to output and therefore unambiguously strengthen the income redistribution that takes place when output changes.

In Bilbiie, under a forward-looking Taylor rule $\left(\hat{R}_t = \phi_\pi \hat{\pi}_{t+1}\right)$, indeterminacy obtains as soon as θ exceeds the critical value θ^* that causes a negative relation between $\hat{c}_t^o$ and $\hat{y}_t$ and the inversion of the standard relation between output and the nominal interest rates.[8] Under a contemporaneous Taylor rule $\left(\hat{R}_t = \phi_\pi \hat{\pi}_t\right)$ the critical value of θ that causes indeterminacy is larger than θ^*.

By substituting (43) into (38) we get the New Keynesian IS curve

$$\hat{y}_t = \frac{A}{(A+B)}\hat{y}_{t-1} + \frac{B}{(A+B)}\hat{y}_{t+1} - \frac{\left(\hat{R}_t - \hat{\pi}_{t+1}\right)}{(A+B)} \tag{44}$$

where $A+B = \frac{1}{1-b}\left[\left(b\left(1 - \frac{1}{1-b}\frac{\theta}{1-\theta}\right)\right) + \chi\right] = \frac{1}{1-b}\left[(1+b)\left(1 - \frac{1}{1-b}\frac{\theta}{1-\theta}\right) - \frac{\theta\phi_l}{(1-\theta)}\right]$. Note that a real interest rate increase has a positive impact on current output, $A+B<0$, if $\theta > \theta^* \simeq 0.18$.

[7] Due to the efficient steady state assumption, profits are defined here as a fraction of steady state output.

[8] This version of the model differs from Bilbiie because we assume monopolistic competition in the labor market and consumption habits.

Proposition 1 *Under a Taylor rule that controls contemporaneous inflation the model is stable and uniquely determined if θ does not exceed a threshold θ^{**} such that $(A+B) > -\frac{\kappa\left(\left(\phi_l+\frac{1}{1-b}\right)+\frac{b}{1-b}\right)(\phi_\pi+1)}{2(\beta+1)}$, that is*

$$\theta^{**} = \frac{\left(1+b+\frac{(1-b)\kappa\left(\left(\phi_l+\frac{1}{1-b}\right)+\frac{b}{1-b}\right)(\phi_\pi+1)}{2(\beta+1)}\right)}{\left\{\left(\frac{1+b}{1-b}+\phi_l\right)+\left(1+b+\frac{(1-b)\kappa\left(\left(\phi_l+\frac{1}{1-b}\right)+\frac{b}{1-b}\right)(\phi_\pi+1)}{2(\beta+1)}\right)\right\}} \tag{45}$$

Proof. See Appendix B ■

Under our benchmark calibration we obtain $\theta^{**} \simeq 0.21$, slightly larger than the value of θ that causes an inversion of (44).[9] Consumption habits unambiguously lower the threshold value θ^{**}. In fact we obtain $\frac{\partial\theta^{**}}{\partial b} < 0$ for $0.2 < \phi_l < 100$, $0 \leq b < 1$, $1 < \phi_\pi < 100$. This happens because habits strengthen the sensitivity of $\hat{\lambda}_t^i$ to $\hat{c}_t^i$ ($\frac{\partial\hat{\lambda}_t^i}{\partial\hat{c}_t^i} = -\frac{1}{1-b}$) and therefore increase the sensitivity of the real wage to output variations ($\frac{\partial\hat{w}_t}{\partial\hat{y}_t} = \phi_l + \frac{1}{1-b}$), the key factor that causes indeterminacy in this model.

If we relax the efficient steady state assumption ($\rho = 0$) the threshold value θ^{**} falls ($\theta^{**} \simeq 0.13$). To grasp intuition, note that: *i)* in steady state $\lambda^o < \lambda^{rt}$ because $c^o = \frac{1}{\mu}\left(1+\frac{\mu-1}{1-\theta}\right)h > c^{rt} = \frac{1}{\mu}h$; *ii)* the representative union objective function (22) is a weighted average of the two household groups utility functions, and the union is therefore concerned with limiting consumption gaps between the two groups. As a consequence, the sensitivity of $\hat{w}_t$ to $\hat{y}_t$ is now stronger than in the efficient steady state case (Figure 1, left panel). Note that for $\hat{\theta} > 15\%$ the wage elasticity would become negative. This paradoxical result may be explained as follows. In a standard model the representative wage setter would react to an increase in labour demand by raising the real wage. This outcome may be observed in equilibrium only if the corresponding change in consumption is associated to an increase in the marginal rate of substitution that matches the wage increase (see 30). The presence of RT consumers does not affect this result when the steady state is efficient. By contrast, when the steady is inefficient and θ is relatively large, a positive wage response to a labour demand increase generates a very strong fall in the marginal utility of RT consumption that would not only outweigh the corresponding increase in the marginal utility of Ricardian agents consumption, but would also lead to a more than proportional increase in the average marginal rate of substitution. As a result, the representative union first order condition (30) can be satisfied only if the wage response to an increase in labor demand is reversed. This new finding is of limited relevance in the very simple model considered here because $\hat{\theta} > \theta^{**}$, but the issue will be reconsidered in the context of the full model. To conclude our discussion, the right panel of Figure 1 shows that without habits the positive response of the income wage elasticity to theta would grow at a much slower rate. All in all, these results suggest that external habits and consumption inequality have mutually reinforcing adverse effects on determinacy.

3.2 Model 2: Capital accumulation

We now add physical capital and the "working capital", cash-in-advance constraint on firms.[10] The log-linear version of model 2 is obtained by setting $\lambda_w = 0$ (flexible wages) in equation (72) of the full model (see Appendix A). Given the previous discussion about the complementarity between external habits and income inequality, we expect that capital holdings in the hands of Ricardian consumers, which increase income inequality, should lower the determinacy threshold. In fact under our benchmark calibration we obtain a very low determinacy threshold, $\theta^{**} \simeq 0.02$, which increases to $\theta^{**} \simeq 0.14$ when steady state firms profits are constrained to be zero.

3.3 Model 3: Sticky wages

Adding wage stickiness ($\lambda_w = 0.64$) we return to the benchmark model described in section 2. As discussed in section 2.5, the wage-setting decision maximizes a weighted average of the two household types utility

[9] Just like Bilbiie we find that in principle a strong antinflation response can ensure determinacy under a Taylor rule for any value of θ, but this would require implausibly large values for ϕ_π. For instance, in our case $\theta^* = 0.54$ if we set $\phi_\pi = 40$ and $\theta^* = 0.72$ if we set $\phi_\pi = 100$.

[10] Relative to Model 1, in our simulations we also account for price and wage indexation. This is irrelevant for the determination of θ^{**}. Results available upon request.

functions, conditional to the probability that the wage cannot be reoptimized in the future. Then all households supply labour on demand at the given wage rate. Wage dynamics is now described by equations (72) and (73) in Appendix A.

Wage stickiness dampens movements of the real wage bill and limits the multiplier effect produced by RT consumers, and should prevent the reversal of the slope of the IS curve that we observed under full wage flexibility. Due to the inclusion of habits this effect is quantitatively limited, as the determinacy threshold now is $\theta^{**} \simeq 0.28$. [11]

3.4 Sensitivity analysis

For values of b in the range $[0.60, 0.67]$, θ^{**} varies between 0.53 and 0.08.[12] Changing the value of the inverse of Frish elasticity, ϕ_l in the range $[0.2, 5]$, the threshold value θ^{**} lies in the interval $[0.21, 0.29]$. Wage stickiness is another key parameter: raising the frequency of wage adjustment to 4 quarters ($\lambda_w = 0.8$) the threshold value θ^{**} reaches 0.51. The opposite result obtains considering the same frequency of price stickiness ($\lambda_p = 0.8$): in this case determinacy arises for $\theta = 0.19$.

Our results are robust to alternative values of $\kappa \in [0.5, 5]$, $\sigma_m \in [1, 100]$. Given the calibration on the other parameters, changing the values for the size of investment adjustment costs does not significantly affect the threshold θ^{**} that causes indeterminacy. Finally wage and price indexation to past inflation plays no role in determining the value of θ^{**} for this model.[13]

4 Dynamic performance

In Figure 2 we compare the impulse response functions to a 1% interest rate shock when asset market participation is full ($\theta = 0$) and when the number of RT consumers is very limited ($\theta = 0.05$). Under full asset market participation[14] the interest rate shock has an adverse effect on the wage bill and therefore on marginal costs. This generates a stagflationary outcome: inflation increases whereas output, worked hours and the real wage fall. Even a minimal amount of RT consumers is sufficient to reverse outcomes observed under full asset market participation. In particular, we see that the relation between output and real wages is apparently inverted. The real wage fall redistributes income between the two households groups, benefiting Ricardian agents whose consumption now increases. It is interesting to note that under full asset market participation Ricardian consumers decumulate capital in order to smooth consumption, whereas this is no longer necessary under LAMP.

4.1 Can we rescue the model?

So far our results emphasize the role of consumption inequality as a source of indeterminacy. In the real world governments do implement redistributive policies, and our model suggests that such policies might have beneficial effects on the stability of the economy by reducing the consumption gap between the two households groups. To assess the potential effects of such policies in our framework we now introduce a fiscal transfer to RT consumers, financed by a lump-sum tax paid by Ricardian agents as in Galì et al.. Such transfer is proportional to the steady state income gap between the two households types.

$$T = \gamma_T \left(c^o - c^{rt} \right)$$

In Table 2 we report the thresholds θ^{**} associated with different values of γ_T. We find that $\frac{c^o}{c^{rt}}$ falls from 1.65 to 1.19 as γ_T grows from 0 to 0.5. The corresponding θ^{**} values would range between 0.27 and 0.71. Figure 3 plots the impulse response functions when $\gamma_T = 0.2$ and $\theta = 0.2$. Dynamics of aggregate variables are now very close what is observed under full asset market participation, with the exception of aggregate consumption. In addition, monetary policy cannot control the asymmetric pattern of consumption dynamics for the two consumer groups.

[11] Simulating model 3 without habits we obtain $\theta^{**} \simeq 0.97$ replicating the results obtained in Colciago (2011) and Ascari et al. (2011).

[12] $b = 0.68$ is the maximum value we can set for the habit persistence in order to avoid a negative steady state marginal utility of consumption for RT consumers.

[13] Results available upon request.

[14] In this case our model broadly replicates Christiano et al. (2005) under a Taylor rule.

5 Conclusion

We embodied limited asset market participation in a well known medium scale New Keynesian DSGE model. We showed that external habits and consumption inequality have mutually reinforcing adverse effects on determinacy, uncovering a causality link between long-run inequality and macroeconomic volatility. We have also shown that redistributive polices targeting consumption inequality have beneficial implications for macroeconomic stability.

Both common wisdom and recent empirical work (Immervoll and Richardson, 2011) suggest that governments implement a substantial amount of income redistribution through their tax/benefits systems. We provide an additional theoretical argument supporting such policies. Further research should investigate how the causal link between consumption inequality and macroeconomic instability affects the design of transfer schemes.

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A The full model: steady-state solution and log-linear form

A.1 Steady state

The presence of RT consumers influences the steady state uniquely for what concerns households individual consumption levels. From equations (14) and (15), and assuming zero inflation in steady state, it holds true that

$$R = \frac{1}{\beta} \tag{46}$$

$$r^k = \frac{1}{\beta} - 1 + \vartheta \tag{47}$$

$$mc = \left(\frac{\eta}{\eta - 1}\right)^{-1}$$

From the cost minimization problem we obtain:

$$\left(\frac{k}{h}\right) = \left[\left(\frac{\eta}{\eta-1}\right)\left(\frac{1}{\beta} - 1 + \vartheta\right)\frac{1}{\alpha}\right]^{\frac{1}{\alpha-1}} \tag{48}$$

$$w = \frac{\left(\frac{\eta}{\eta-1}\right)^{-1}(1-\alpha)\left(\frac{k}{h}\right)^{\alpha}}{R} \tag{49}$$

From the production function we get

$$\frac{y}{h} = \left(\frac{k}{h}\right)^{\alpha} \tag{50}$$

Since

$$\frac{i}{y} = \vartheta\frac{k}{y}$$

the aggregate resource constraint reads as:

$$y = c + i \tag{51}$$

$$1 = \frac{c}{y} + \frac{i}{y} \tag{52}$$

The aggregate consumption-output ratio is

$$\frac{c}{y} = 1 - \vartheta\frac{k}{h}\left(\frac{y}{h}\right)^{-1} \tag{53}$$

The equation for the optimal wage allows us to derive the solution for worked hours

$$h = \left[\frac{\alpha_w - 1}{\alpha_w}\left(\frac{(1-\theta)}{\frac{c^o}{c} - b} + \frac{\theta}{\frac{c^{rt}}{c} - b}\right)\frac{c^{rt}}{c}\right]^{\frac{1}{(\phi_l+1)}}$$

so that

$$k = \frac{k}{h}h \tag{54}$$

RT individual consumption is

$$c^{rt} = w\ h$$

therefore

$$\frac{c^{rt}}{c} = \left(\frac{c}{y}\right)^{-1} w\left(\frac{y}{h}\right)^{-1} \tag{55}$$

From the total consumption identity $c = (1-\theta)\,c^o + \theta c^{rt}$ we get

$$\frac{c^o}{c} = \frac{1}{1-\theta} - \frac{\theta}{1-\theta}\frac{c^{rt}}{c} \tag{56}$$

Optimizing households consumption at steady state is given by the sum of labour income, firms profits return of capital and returns of money rents to firms:

$$c^o = wh + \frac{1}{1-\theta}\left(d + r^k K + (R-1)\,wh\right) \tag{57}$$

where $d = (1-mc)y = (1-\frac{1}{\mu})y$ and $\mu = \frac{\eta}{\eta-1}$ denotes firms markup. Thus optimizing agents are richer the higher the share of RT consumers. Aggregate consumption can be finally rewritten as

$$c = (1-\theta)\,c^o + \theta c^{rt} = wh + \Pi + r^k K + (R-1)\,wh \tag{58}$$

A.2 Model in log-linear form

Aggregate consumption is defined by:

$$\hat{c}_t = (1-\theta)\frac{c^o}{c}\hat{c}^o_t + \theta\frac{c^{rt}}{c}\hat{w}_t + \theta\frac{c^{rt}}{c}\hat{h}_t \tag{59}$$

Marginal costs are given by

$$\widehat{mc}_t = (1-\alpha)\left(\hat{w}_t + \hat{R}_t\right) + \alpha\hat{r}^k_t \tag{60}$$

The following equation combines firms' F.O.C. with respect to production factors

$$\hat{h}_t + \hat{w}_t + \hat{R}_t = \hat{k}_{t-1} + \left(1 + \frac{\gamma_1}{\gamma_2}\right)\hat{r}^k_t \tag{61}$$

Production function is given by

$$\hat{y}_t = \alpha\hat{k}_{t-1} + \alpha\frac{\gamma_1}{\gamma_2}\hat{r}^k_t + (1-\alpha)\,\hat{h}_t \tag{62}$$

Aggregate resource constraint

$$\hat{y} = \frac{i}{y}\hat{\imath}_t + \frac{c}{y}\hat{c}_t + \gamma_1\frac{\gamma_1}{\gamma_2}\frac{k}{y}\hat{r}^k_t \tag{63}$$

RT consumption

$$\hat{c}^{rt}_t = \hat{w}_t + \hat{h}_t \tag{64}$$

Euler equation

$$\hat{\lambda}^o_t = E_t\hat{\lambda}^o_{t+1} + E_t\left(\hat{R}_{t+1} - \hat{\pi}_{t+1}\right) \tag{65}$$

Households marginal utility of consumption

$$\hat{\lambda}^o_t = -\frac{1}{\left(\frac{c^o}{c}\right) - b}\left[\left(\frac{c^o}{c}\right)\hat{c}^o_t - b\hat{c}_{t-1}\right] \tag{66}$$

$$\hat{\lambda}^{rt}_t = -\frac{1}{\left(\frac{c^{rt}}{c}\right) - b}\left[\left(\frac{c^{rt}}{c}\right)\hat{c}^o_t - b\hat{c}_{t-1}\right] \tag{67}$$

Investment decisions

$$\hat{\imath}_t - \frac{1}{k\,(1+\beta)}\hat{P}_{k',t} - \frac{1}{(1+\beta)}\hat{\imath}_{t-1} - \frac{\beta}{(1+\beta)}E_t\hat{\imath}_{t+1} = 0 \tag{68}$$

$$E_t\hat{\pi}_{t+1} + \beta\,(1-\vartheta)\,E_t\hat{P}_{k',t+1} - \hat{P}_{k',t} = E_t\hat{R}_{t+1} - \beta r^k E_t\hat{r}^k_{t+1} \tag{69}$$

Capital accumulation

$$\hat{k}_t = (1-\vartheta)\,\hat{k}_{t-1} + \vartheta\hat{\imath}_t \tag{70}$$

Phillips Curve

$$\frac{\lambda_p}{1-\lambda_p}\left(\hat{\pi}_t-\gamma_p\hat{\pi}_{t-1}\right)=\left(1-\beta\lambda_p\right)\widehat{mc}_t+\beta\lambda_p\left(E_t\hat{\pi}_{t+1}-\gamma_p\hat{\pi}_t\right)+\beta\frac{\lambda_p^2}{1-\lambda_p}\left(E_t\hat{\pi}_{t+1}-\gamma_p\hat{\pi}_t\right) \tag{71}$$

Monetary Rule

$$\hat{R}_t=\phi_\pi\hat{\pi}_t+\varepsilon_t$$

Wage dynamics

$$\left[\begin{array}{c}\left(\frac{1}{1-\lambda_w}+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)\hat{w}_t-\beta\frac{\lambda_w}{1-\lambda_w}E_t\hat{w}_{t+1}+\\-\left(\beta\lambda_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)E_t\hat{\pi}_{t+1}+\\+\left(\beta\lambda_w\gamma_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\gamma_w+\frac{\lambda_w}{1-\lambda_w}\right)\hat{\pi}_t+\\-\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t-1}-\frac{\lambda_w}{1-\lambda_w}\gamma_w\hat{\pi}_{t-1}\end{array}\right]=\left(1-\beta\lambda_w\right)\left(\varphi\hat{h}_t-\hat{\psi}_t\right) \tag{72}$$

where

$$\hat{\psi}_t=\frac{\frac{(1-\theta)}{\frac{c^o}{c}-b}}{\frac{(1-\theta)}{\frac{c^o}{c}-b}+\frac{\theta}{\frac{c^{rt}}{c}-b}}\hat{\lambda}_t^o+\frac{\frac{\theta}{\frac{c^{rt}}{c}-b}}{\frac{(1-\theta)}{\frac{c^o}{c}-b}+\frac{\theta}{\frac{c^{rt}}{c}-b}}\hat{\lambda}_t^{rt} \tag{73}$$

B Determinacy Analysis

B.1 Model 1 : proof of Proposition 1:

Consider the reduced form of the model

$$\begin{array}{c}\hat{\pi}_{t+1}\\ \hat{y}_{t+1}\\ \hat{y}_t\end{array}=\begin{array}{ccc}\frac{1}{\beta} & -\frac{\kappa}{\beta}\left(\phi_l-\frac{1}{b-1}\right) & -b\frac{\kappa}{\beta(b-1)}\\ \frac{1}{B}\phi_\pi-\frac{1}{B\beta} & \frac{1}{B}\left(A+B\right)+\frac{1}{B}\frac{\kappa}{\beta}\left(\phi_l-\frac{1}{b-1}\right) & \frac{1}{B}b\frac{\kappa}{\beta(b-1)}-\frac{A}{B}\\ 0 & 1 & 0\end{array}\begin{array}{c}\hat{\pi}_t\\ \hat{y}_t\\ \hat{y}_{t-1}\end{array}$$

where

$$\begin{aligned}A&=\frac{b}{1-b}\left(1-\frac{1}{1-b}\frac{\theta}{1-\theta}\right)\\ B&=\frac{\left(1-\theta-\theta\left(\phi_l+\frac{1}{1-b}\right)\right)}{\left(1-b\right)\left(1-\theta\right)}\\ k&=\frac{\left(1-\lambda_p\right)\left(1-\beta\lambda_p\right)}{\lambda_p}\end{aligned}$$

The system is characterized by two jump variables ($\hat{\pi}_t$ and $\hat{y}_t$) and one state variable ($\hat{y}_{t-1}$).
The Characteristic polynomial is

$$\begin{aligned}&P_T\left(X\right)=X^3+a_2X^2+a_1X+a_0=\\ =\;&X^3+\underbrace{\left(-\frac{1}{\beta}-\frac{(A+B)}{B}-\frac{\kappa}{B\beta}\left(\phi_l-\frac{1}{b-1}\right)\right)}_{a_2=-\text{Trace}}X^2+\\ &+\underbrace{\left(\begin{array}{c}\frac{1}{\beta}\left(\frac{(A+B)}{B}+\frac{\kappa}{B\beta}\left(\phi_l-\frac{1}{b-1}\right)\right)+\frac{A}{B}+\\-\frac{\kappa}{\beta}\left(\phi_l-\frac{1}{b-1}\right)\left(\frac{1}{B\beta}-\frac{\phi_\pi}{B}\right)-\frac{\kappa}{B\beta}\frac{b}{(b-1)}\end{array}\right)}_{a_1=\text{Sum of leading minors}}X+\\ &+\underbrace{\left(\frac{(A-Ab+b\kappa\phi_\pi)}{B\beta\left(b-1\right)}\right)}_{a_0=-\text{Determinant}}\end{aligned}$$

The stability properties of the system depend on the location of the roots inside the unit circle in the complex plane, i.e. $|X_i| < 1$. By adopting the conformal involuntary transformation

$$X = \left(\frac{1+x}{1-x}\right),$$

it is in general possible to turn $P_T(X)$ into a Hurwitz polynomial[15] $P_H(x)$, whose stability properties depend on the location of the roots in the left hand plane $\mathcal{R}(X) < 0$:[16]

$$P_H(x) = \left(\frac{1+x}{1-x}\right)^3 + a_2\left(\frac{1+x}{1-x}\right)^2 + a_1\left(\frac{1+x}{1-x}\right) + a_0$$

which can be rewritten as

$$\begin{aligned} P_H(X) &= x^3 + \underbrace{\left(\frac{(3a_0 - a_1 - a_2 + 3)}{(a_1 - a_0 - a_2 + 1)}\right)}_{d_2} x^2 + \\ &+ \underbrace{\left(\frac{(a_2 - a_1 - 3a_0 + 3)}{(a_1 - a_0 - a_2 + 1)}\right)}_{d_1} x + \\ &+ \underbrace{\frac{(a_0 + a_1 + a_2 + 1)}{(a_1 - a_0 - a_2 + 1)}}_{d_0} = 0 \end{aligned}$$

Therefore:

$$\begin{aligned} d_0 &= \frac{1 + a_2 + a_1 + a_0}{1 - a_2 + a_1 - a_0} = -x_1 x_2 x_3 \\ d_1 &= \frac{3 + a_2 - a_1 - 3a_0}{1 - a_2 + a_1 - a_0} = x_1 x_2 + x_1 x_3 + x_2 x_3 \\ d_2 &= \frac{3 - a_2 - a_1 + 3a_0}{1 - a_2 + a_1 - a_0} = -(x_1 + x_2 + x_3) \end{aligned}$$

where x_i, $i = 1, 3$ are the roots of $P_H(x)$.
The necessary condition for model's stability is:

$$d_0 = \frac{\kappa(\phi_l + 1)(\phi_\pi - 1)}{\left(\kappa\left(\phi_l + \frac{1}{1-b}\right) + b\frac{\kappa}{1-b}\right)(\phi_\pi + 1) + (2\beta + 2)(A + B)} > 0 \tag{74}$$

Under the Taylor principle $\phi_\pi - 1 > 0$, therefore

$$d_0 > 0 \;\Leftrightarrow (A + B) > -\frac{\kappa\left(\left(\phi_l + \frac{1}{1-b}\right) + \frac{b}{1-b}\right)(\phi_\pi + 1)}{2(\beta + 1)} \tag{75}$$

that is

$$\theta < \frac{\left(1 + b + \frac{(1-b)\kappa\left(\left(\phi_l + \frac{1}{1-b}\right) + \frac{b}{1-b}\right)(\phi_\pi + 1)}{2(\beta+1)}\right)}{\frac{1+b}{1-b} + \phi_l + \left(1 + b + \frac{(1-b)\kappa\left(\left(\phi_l + \frac{1}{1-b}\right) + \frac{b}{1-b}\right)(\phi_\pi + 1)}{2(\beta+1)}\right)}$$

When (74) holds, by Descartes rule stability obtains if either d_1 or d_2 or both are negative.

[15] Note that $|X| \gtrless 1 \iff x \gtrless 0$

[16] See Samuelson (1941) and more recently, Felippa and Park (2004)- section 4 page 18, Ascari et al. (2011) and Rossi (2011).

Since $d_1 < 0$ if

$$\theta < \frac{\left(2\left(1-\beta\right)\left(1-b\right)+\kappa\left(1+b+\phi_l\left(1-b\right)\right)+\kappa\phi_\pi\left(\left(1+\phi_l\right)\left(1-b\right)-2b\right)\right)}{\left(2\left(1-\beta\right)\left(\phi_l-b+2\right)+\kappa\left(1+b+\phi_l\left(1-b\right)\right)+\kappa\phi_\pi\left(\left(1+\phi_l\right)\left(1-b\right)-2b\right)\right)}$$

which is always true when condition (74) is satisfied, condition (74) is the necessary and sufficient condition for determinacy under the Taylor principle.

Tables and Figures

Table 1: Calibration

Parameter	Value	Description
b	0.65	degree of habit persistence
β	$(1.03)^{-0.25}$	subjective discount factor
α	0.36	share of capital
ϑ	0.025	depreciation rate
η	6	price-elasticity of demand for a differentiated good
α_w	21	intratemporal elasticity of substitution between labor inputs
κ	2.48	parameter governing investment adjustment costs
λ_w	0.64	degree of wage stickiness
λ_p	0.6	degree of price stickiness
γ_1	0.0324	parameter governing capacity adjustment costs
γ_2	0.0652	parameter governing capacity adjustment costs
ψ_l	1	preference parameter
γ_p	1	indexation on prices
γ_w	1	indexation on wages
ϕ_l	3	Inverse of Frish elasticity
ρ	0	production subsidy

Table 2: Fiscal transfer, redistribution and model stability

	$\gamma_T = 0$	$\gamma_T = 0.05$	$\gamma_T = 0.1$	$\gamma_T = 0.15$	$\gamma_T = 0.2$	$\gamma_T = 0.25$	$\gamma_T = 0.5$
θ^{**}	0.27	0.45	0.52	0.57	0.61	0.63	0.71
$\frac{c^o}{c^{rt}}$ when $\theta = 0.3$	1.65	1.52	1.43	1.37	1.33	1.29	1.19

Values represent percentage standard deviations

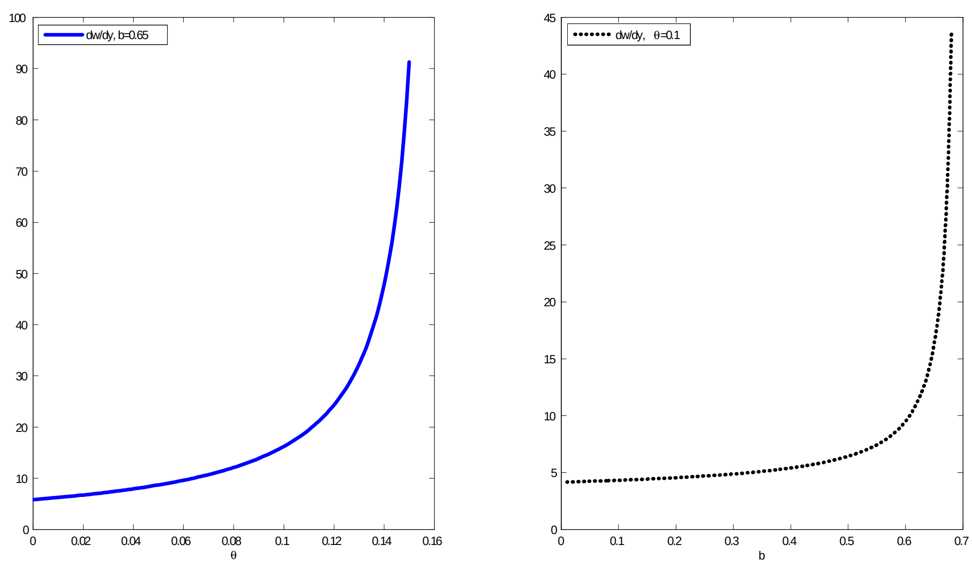

Figure 1: Elasticity of real wage to current output $\frac{\partial w_t}{\partial y_t}$

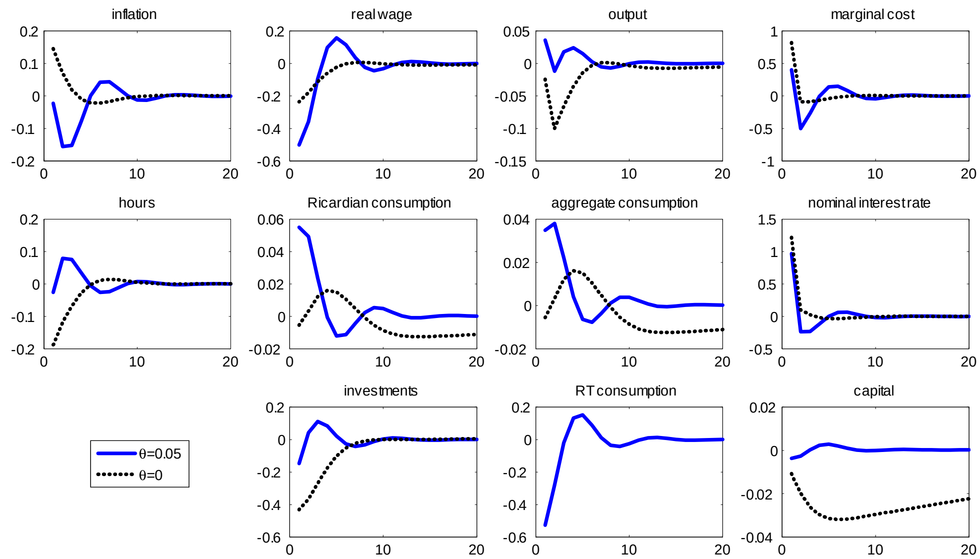

Figure 2: Impulse response functions to a 1% monetary shock

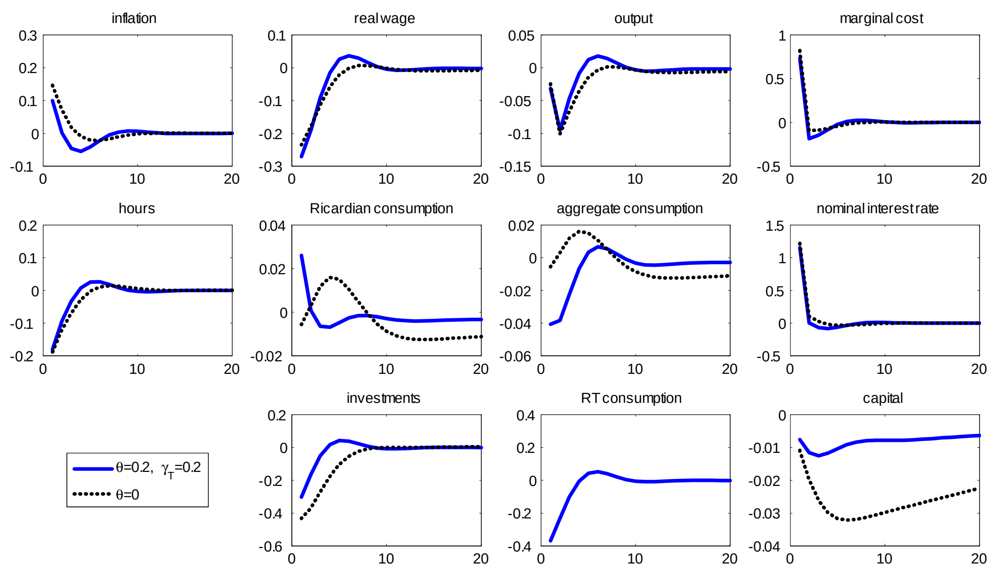

Figure 3: Impulse response functions to a 1% monetary shock