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A model of fashion: endogenous preferences in social interaction

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Abstract

The aim of this paper is to investigate the dynamics of the fashion cycle as originally described by Simmel (1904). The theoretical models used in the more recent economic literature (Stigler & Becker 1977; Karni & Schmeidler 1990; Matsuyama 1992; Coelho & McCure 1993; Corneo & Jeanne 1997) have the undeniable advantage of making the cycle widely applicable, and consequently appropriate for the analysis of the most varied fields of consumption activity. However, in the process, the originality of Simmel's thought has been lost. Since they are built on the principles of standard economics, the above-mentioned models generally assume that preferences are exogenous and overlook the fact that individual tastes change in time, partly in line with choices previously made by the social group. This paper proposes a model of the fashion cycle in which conspicuous consumption 'snob' and 'bandwagon' preferences (Leibenstein 1950) are determined endogenously and depend on previous consumption experience, both personal and that of other consumers. Thus the particular contribution of this work in comparison to the preceding economic literature is dual in nature. By assuming preferences to be endogenous, it reflects more accurately the dynamics causing perpetual motion in the fashion cycle. By assuming preferences to be shaped through the social interaction of a heterogeneous community of individuals, the model manages to identify more closely the psycho-sociological nuances that, according to Simmel, give rise to the cycle.

Keywords: fashion cycles, endogenous preferences, snob and bandwagon effects, bifurcation, complex dynamics

JEL classification: C61; D11

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Introduction

Fashion is, of course, an important social phenomenon, as has been documented by numerous studies in sociology and economics. In sociology, the treatment of subject was made famous by Simmel (1904), who held that the fashion cycle is driven by the masses' imitation of the élite. According to Simmel, the upper classes seek to set themselves apart from the masses by adopting a new style, which the masses then imitate. The tendency to imitate expresses a primary need for social approval, while the tendency to stand out expresses the exact opposite, the fundamental need to affirm one's personality.

In early economics, the fashion cycle gained the attention of researchers such as Veblen (1899) who studied sudden shifts in consumer behavior and wrote of patterns of conspicuous consumption in which individuals motivate their purchases by making comparisons with their slightly better-off similars, in order to express their buying power. Half a century later, Leibenstein, who was primarily interested in welfare analysis, (1950) classed aggregate consumption phenomena as a 'non-functional demand' comprising 'bandwagon effects', deriving from a desire to join the others, and 'snob effects, originating in a desire to stand apart. Drawing on the field of motivational psychology, Scitovsky (1976) then explained the fashion cycle as the result of balancing two opposed human drives, namely, the pursuit of novelty and the need to conform to collective standards. A different explanation of fashion demand can be found in Stigler & Becker (1977), who claim that tastes are stable over time and need not be part of a theory of demand, as long as they are held in common by consumers. Karni & Schmeidler (1990) have instead pointed out that equilibrium selection in a dynamic game of complete information with overlapping generations of players divided into different classes can also lead to cyclical demand variations.

The main focus of recent literature on fashion is on inter-temporal mechanisms that, coupled with externalities, may give rise to fashion dynamics. In an evolutionary framework, Matsuyama (1992) identifies the social environments that give rise to limit cycles, in which nonconformists act as fashion leaders and conformists as fashion followers. Coelho and McClure (1993) stress the potential role of consumer expectations in generating the fashion cycle. Corneo & Jeanne (1997) develop a model in which the signaling value of conspicuous consumption depends on the number of consumers, with consumer behavior being characterized by either snobbism or conformism. The models of the fashion cycle developed from Becker onwards have the undeniable advantage of generalizing the fashion cycle and therefore making it appropriate for the analysis of the most varied fields of consumption activity.

However, the originality of Simmel's thought does not emerge from the above papers, given that they are constructed on the principles of standard economics and generally assume that fashion cycle models are exogenous, overlooking the fact that tastes change over time, and vary also in reaction to choices previously made by the social group.

The goal of this paper is to offer an explanation for collective behavior that presents the characteristics of fashion as originally described by Simmel. The study models fashion as a dynamic phenomenon in the presence of conspicuous consumption 'bandwagon' and 'snob' effects (Leibenstein 1950). In particular, it proposes a model in which agents' consumption preferences are determined endogenously and depend on the experience of previous consumption by oneself and others. Originally proposed by Benhabib & Day (1981), the model has been utilized to study local and global interdependent preferences (Naimzada & Tramontana 2009) and public goods dynamics (Di Giovinazzo & Naimzada 2012). As in fact will be shown, the endogenous nature of preferences and social interaction are necessary preconditions for setting off a fashion cycle. The model demonstrates that a fashion cycle emerges when both types of agents – snobs and 'bandwagoners' – co-exist in a meaningful way. As far as we know, no-one has yet tried formalizing the fashion cycle with endogenous preferences in social interaction.

The remainder of the paper is structured as follows. Section 1 presents the basic model and provides conditions for the formation of a fashion cycle. Section 2 discusses some examples, while the last section offers our concluding remarks.

1. The model

In this study, we experiment with modeling the fashion cycle on the basis of the work of Benhabib & Day. The latter model assumes agents' preferences to be endogenous, in particular with preferences in any given period depending on past consumption experiences. It does not contemplate, however, the possibility of consumption being determined by social interaction, even though the latter is likely fundamental in the fashion world, where social influence is critical. The model proposed here is the result of continuous interaction between the two types of agent, known as the 'snob' and the 'bandwagoner', via an endogenous transformation of the parameter representing the preference for conspicuous consumption. For the 'bandwagoners', it is assumed that preference for a particular kind of consumption increases with the rise in the social group's average consumption of the previous period. For the snobs, preference for a certain purchase is believed to drop with the rise of the average collective consumption of the preceding period.

1.1 Assumptions

We consider a discrete-time economy populated by *N* agents, where preferences are defined by Cobb-Douglas utility functions:

$$U^i(u,y;\alpha^i) = x^{\alpha^i} y^{(1-\alpha^i)} \quad , 0 < \alpha < 1, \qquad 1)$$

where u_i represents the utility of the *i*-th agent and is defined by: an observable good *x*, referred to as the conspicuous good, and a numéraire good *y* (Corneo 1997), where α^i represents the preference of *i*-th agent for the good *x*. The budget constraint is indicated by:

$$m^i = px + qy ,$$

where m^i is the income of the *i*-nth agent, *p* is the price of the good *x* and *q* is the price of the good *y*. The economy is populated by two types of individual: the snob and the bandwagoner. We define bandwagoners *as* agents whose preference for a conspicuous purchase increases with the rise in social group's past average consumption of the good *x.* We then define as snobs the agents whose preference for a good falls with the social group's past increase in consumption. The population is composed by a share ω of individuals of the bandwagon type and (1-ω) individuals of the snob type, with $0 < \omega < 1$; ωN, therefore, is the number of bandwagon agents and $(1-\omega)N$ the number of snob agents. It is hypothesized that, inside each group, agents are homogeneous both for income, m^i, and preferences, α^i. For the bandwagoners we have $m^i = m^B$ and $\alpha_t^i = \alpha_t^B$ per $i \in (1,2,\ldots,\omega N)$ .For the snobs we have $m^i = m^S$ e $\alpha_t^i = \alpha_t^S$ per $i \in (\omega N + 1, 2, \ldots, N)$. In general, it is presumed that $m^B \leq m^S$. The optimality condition calls for the first order conditions to be satisfied, as follows:

$$\begin{cases} \dfrac{\alpha^i x^{\alpha^i - 1} y\left(1-\alpha^i\right)}{\left(1-a^i\right) x^{\alpha^i} y^{-\alpha^i}} = \dfrac{p}{q} \\ m^i - px^i - qy^i = 0. \end{cases} \qquad 2)$$

$i = 1, \ldots, N$.

Solving the optimization problem we obtain the demand functions of the two goods:

$$x^i = \frac{\alpha^i m^i}{p}$$

and

$$y^i = \frac{\left(1-\alpha^i\right)m^i}{q}.$$

The dependence of these demand functions upon experience is obtained by supposing that the parameter of the utility function representing preferences depends endogenously on the social group's past choices concerning the conspicuous good. It is hypothesized that the parameter α_t^i depends on the social group's past consumption choices of the conspicuous good: $\alpha_{t+1}^i = f(x_t^1, x_t^2, \ldots, x_t^N)$.

The average consumption of good *x* at time *t* is therefore:

$$\bar{x}_t = \frac{\sum_{i=1}^{n} x_t^i}{N} = \frac{\omega N x_t^B + (1-\omega N) x_t^S}{N}. \qquad (3)$$

The dependence of the parameters that determine the preference for the conspicuous good of the two groups of agents on the past average social group's consumption is then considered.

For the bandwagoners the dependence is given by $\alpha_t^B = f^B(\bar{x}_{t-1})$; with the following characteristics:

1a) $f^B : \Re_+ \to [0,1]$;

2a) f^B continuous;

3a) f^B increasing.

For the group of the snobs the function $\alpha_t^S = f^S(\bar{x}_{t-1})$; has the following characteristics:

1b) $f^S : \Re_+ \to [0,1]$;

2b) f^S continuous;

3b) f^S decreasing.

The mass of the population is normalized to unity, *N*=1. Introducing the dependence of the preferences with respect to the average consumption of the previous period, we obtain the following equation:

$$\bar{x}_t = \frac{\omega m^B}{p} f^B(\bar{x}_{t-1}) + \frac{(1-\omega)m^S}{p} f^S(\bar{x}_{t-1}). \qquad (4)$$

The above equation defines a discrete dynamic system of the first order in the variable $\bar{x}$.

2. Dynamic analysis

To develop the dynamic analysis two specific functional forms will be used, which reflect the assumptions previously made concerning α_t^B e α_t^S.

For the snob group, the dependence of the parameter representing their preferences with respect to the average past consumption of the population is determined by the following linear function:

$$\alpha_{t+1}^S = f^S(\bar{x}_t) = 1 - b\bar{x}_t, \text{ with } b > 0, \ \ \bar{x} < 1/b. \qquad (5)$$

The condition $\bar{x} < 1/b$ assures the positivity of α_{t+1}^S.

Fig. 1

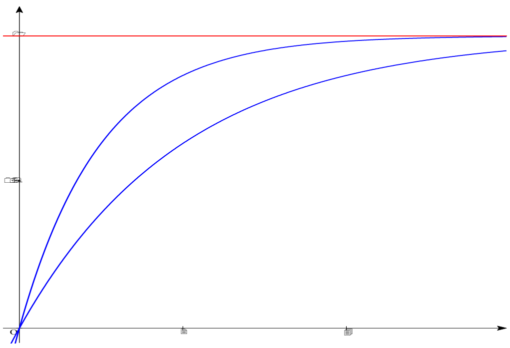


Figure 1. Two different functions describing how α_t^S is determined with different *b* values: $b' = 0{,}2$ and $b'' = 0{,}4$.

The parameter *b* defines the slope of the f^S; it represents the reactivity of the preferences of snob agents with respect to the average consumption of the conspicuous good in the previous period, $\bar{x}_{t-1}$.

For the bandwagoners the dependence is determined by the following non-linear function:

$$\alpha_{t+1}^B = f^B(\bar{x}_t) = \left[1 - \exp(-a\,\bar{x}_t)\right], \text{ with } a>0. \qquad (6)$$

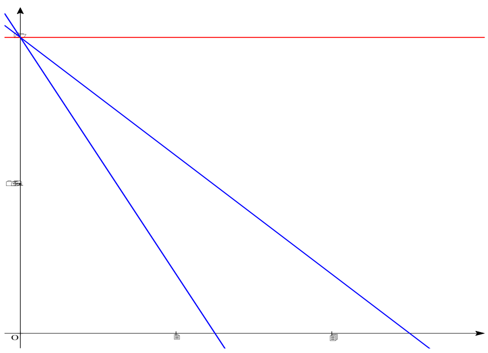


Fig. 2. Two different functions describing how α_t^B is determined with different *a* values: $a' = 0{,}5$ and $a'' = 1$.

The parameter *a* determines the slope of f^B; it represents the reactivity of the preferences of bandwagon agents to the average consumption of the conspicuous good in the previous period, $\bar{x}_{t-1}$.

We then consider the system dynamics with homogeneous agents, and hypothesize a group of agents formed exclusively by bandwagoners or, conversely, by snobs. In the case of a population formed entirely by snobs or $\omega = 0$, the map representing the system dynamics becomes:

$$\bar{x}_{t+1} = \frac{m^S}{p}(1 - b\bar{x}_t). \qquad (7)$$

Proposition 1. *The system representing the evolution of the group with only snob agents,* $\omega = 0$, *has a unique, positive steady state:* $x^S = \frac{m^S}{p + bm^S}$. *The steady state,* x^S, *is globally stable when b is sufficiently small:* $b < \frac{p}{m^S}$.

Proof. See Appendix.

When we analyze the dynamic equation (6) and apply the parameters b=0,4, m^S=2 and p=1, the following map of the first iterate is obtained (Fig. 3). As can at once be noted from the analysis of the graph, the average consumption of the conspicuous good converges to x^S irrespective of the initial condition.

Fig. 3a

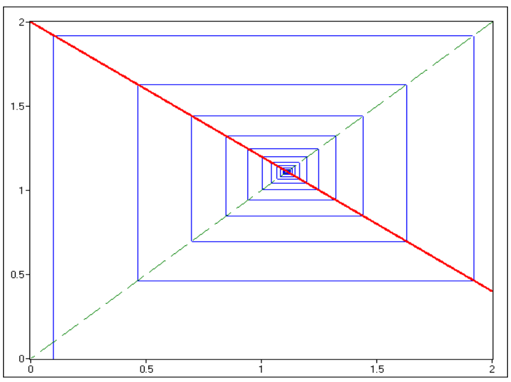


Fig. 3b

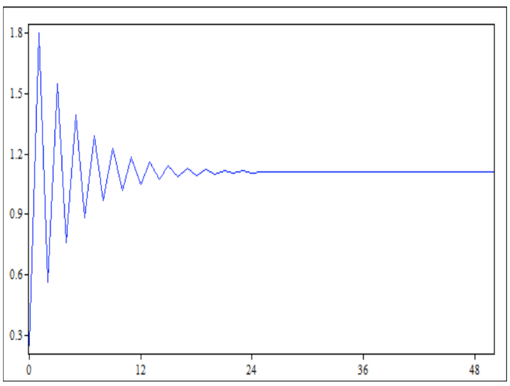


Fig. 3. First iterate with snob agents alone, with a globally stable steady state: a) first iterate; b) $\bar{x}$ versus time.

If the reactivity of the snobs is relatively high, they will react by reducing their demand for the conspicuous good. In this way, a dynamic is produced with damping oscillations converging towards a steady state, as shown in Fig. 3.

In the case of the population being entirely formed of bandwagoners, or $\omega = 1$, the map representing the system dynamics becomes:

$$\bar{x}_{t+1} = \frac{m^B}{p}\left[1 - \exp(-a\,\bar{x}_t)\right], \tag{8}$$

Proposition 2. *The system representing the evolution of the group with only bandwagon agents,* $\omega = 1$, *has a unique positive globally stable steady state given by the origin,* $x_1^B = 0$ *when a is sufficiently small:* $a < \frac{p}{m^B}$. *When a is sufficiently large,* $a > \frac{p}{m^B}$, *the system has two steady states,* $x_1^B = 0$ *and* $x_2^B > 0$; x_1^B *is an unstable steady state, while* x_2^B *is stable.*

Proof. See Appendix.

When we analyze the dynamic equation (8) and apply the parameters a=2, m^B=2 and p=1, the following map of the first iterate is obtained (Fig. 4). Two steady states exist, one always unstable.

Fig. 4a

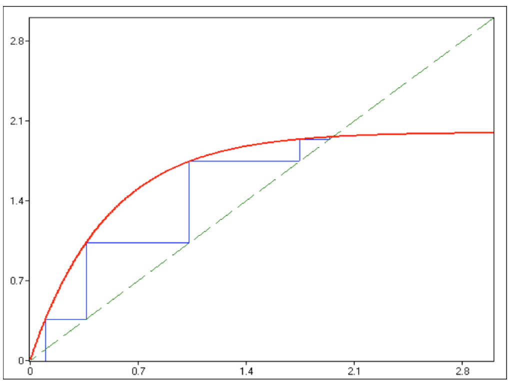


Fig. 4b

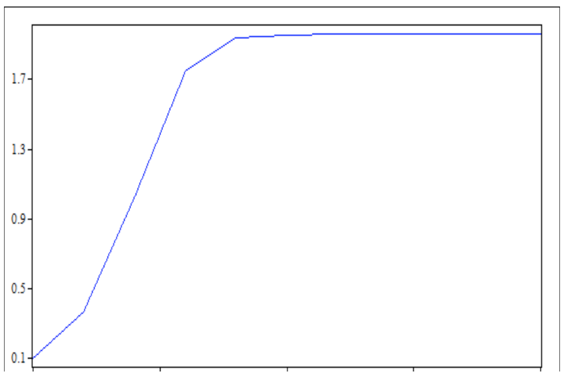


Fig. 4. First iterate with only bandwagon agents with an unstable steady state at the origin and a positive globally stable steady state; a) first iterate; b) $\bar{x}$ versus time.

As can be noted from the analysis of the graph, the average consumption of the conspicuous good converges to x_2^B, irrespective of the positive initial condition. In the case where the population is formed only by bandwagoners, starting from a low level of average consumption, in the following period bandwagoner consumption rises, leading to an increase in average consumption. In this manner, the process leads to successive decreasing (successively smaller?) increases in bandwagon consumption, and therefore also in average consumption. This process of monotonic increase in average consumption converges asymptotically towards a positive steady state.

The case is then considered of heterogeneous agents, $0 < \omega < 1$; here, the dynamic evolution of average consumption is represented by the following equation:

$$\bar{x}_{t+1} = \frac{\omega m^B}{p}\left[1 - \exp(-a\bar{x}_t)\right] + \frac{(1-\omega)m^S}{p}\left(1 - b\bar{x}_t\right). \tag{9}$$

With the more general case, the existence is shown of a single positive stationary state.

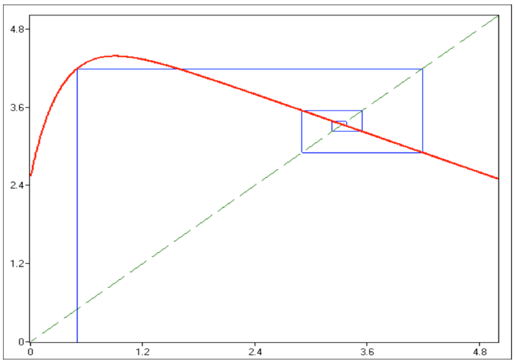
Proposition 3. *The map* (8) *has a unique positive steady state.*

Proof. See Appendix.

Given the non-linearity, the map does not allow us to determine analytically the value of the stationary state, and therefore it is not possible to study analytically its stability. This is why the heterogeneous case, $0 < \omega < 1$, will be analyzed via numerical simulations. Three possible scenarios will be treated. In the first, the presence of snob agents is dominant and therefore ω is close to 0. In the second, the situation of bandwagon agent dominance is considered: ω close to 1. Finally, in the third scenario, a situation is considered in which there is a balanced presence of the two groups: ω close to 0,5.

Considering the first two scenarios, we fix the parameters of the system in the following way: $p = 0,2; m^B = 1; a = 3, m^S = 1; b = 0,2$. In Fig. 5a) there is a predominance of snob agents while in Fig. 5b) the bandwagon agents dominate.

Fig. 5a **Fig. 5b**



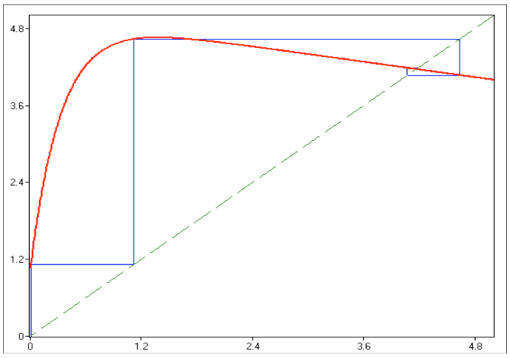
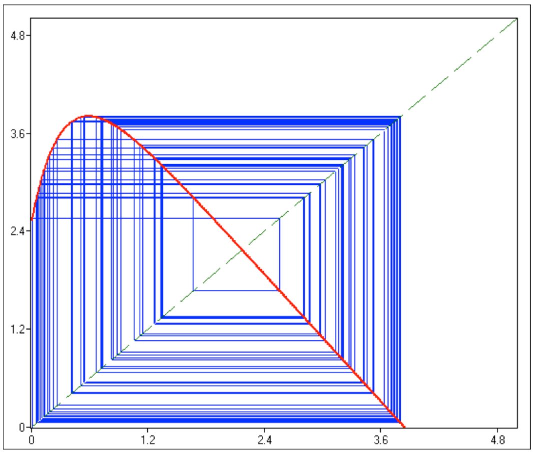


Fig. 5. First iterate; a) with a predominance of snob agents, $\omega = 0,2; p = 0,2; m^B = 1; a = 3, m^S = 1; b = 0,2;$ b) with a predominance of bandwagon agents, $\omega = 0,8; p = 0,2; m^B = 1; a = 3, m^S = 1; b = 0,2.$

As can be observed from the simulations given in Fig. 5, in the case of heterogeneous groups with the predominance of one type of agent, there is a simple dynamic, or a convergence oscillating towards the single steady state. It is also possible to observe the inverse relation between the steady state and the quota of snob agents. With the increase in the quota of snob agents, there is a decrease in the value of the steady state. When, however, the weight of the two groups is sufficiently balanced, the dynamic system can become unstable and can create dynamics with irregular oscillations around the steady state, i.e. chaotic dynamics. In order to consider this scenario, we fix in the following way the parameters $\omega = 0,5; p = 0,2; m^B = 1; a = 3; m^S = 1; b = 0,52$. With a more uniform distribution of the population and with a greater reactivity of snob agents (b=0,52), the first iterate is obtained and the behavior of $\bar{x}$ in the time is represented in the following figure:

Fig. 6a **Fig. 6b**



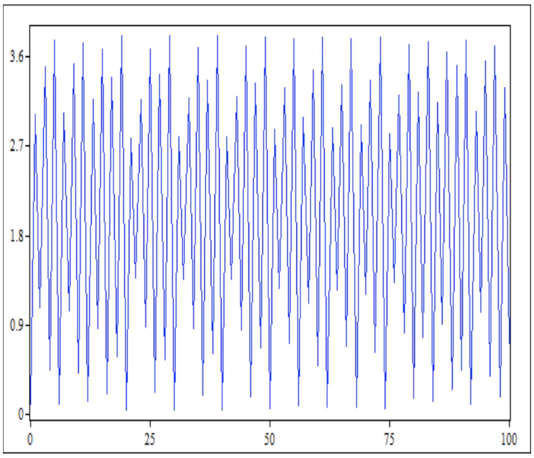


Fig. 6. Chaotic motion with $\omega = 0,5; p = 0,2; m^B = 1; a = 3; m^S = 1; b = 0,52;$ a) first iterate; b) $\bar{x}$ versus time.

We hypothesize starting from a situation of relatively low average consumption $\bar{x}$; this implies that in the following period snob consumption undergoes an increase together with that of the bandwagoners, later giving rise to relatively high average consumption of the conspicuous good. In this kind of situation, there is a sharp reduction in the snobs' consumption of the conspicuous good and a marginal increase of that of the bandwagoners. On the whole, the net outcome of this contrast, given the reactivity of the snob agents, is to reduce average consumption. Irregular dynamic cycles are therefore foreseen.

In order to have a synthesis of the dynamical behaviors of the system, we propose in Fig. 7 a bifurcation diagram with respect to the parameter *ω*, when we set other parameters in the following way: $p = 0,2; m^B = 1; a = 3; m^S = 1; b = 0,52.$ The bifurcation diagram represents the set of limit points as a function of parameter *ω:* for each *ω* we have the stable attractor, be it a single point (corresponding to a stable steady state), two points (corresponding to a stable 2-cycles), or whatever. We observe that, for intermediate values, the unique limit set is given by an infinite set of limiting points, i.e. a chaotic region. Increasing *ω* further, we have a sequence of period-halving bifurcations, corresponding to cycles of all periods; beyond a certain value of *ω*, there is the stable region, in which the limiting set is given by a stable steady state.

Fig. 7

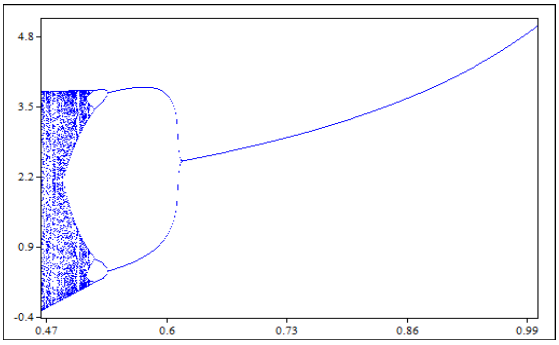


Fig. 7. The graph of the bifurcation diagram for the parameter ω, given the other parameters $p = 0,2; m^B = 1; a = 3; m^S = 1; b = 0,52$.

Conclusions

Simmel stated, "Just as soon as the lower classes begin to copy their style, thereby crossing the line of demarcation, the upper classes turn away from this style and adopt a new one, which in its own turn differentiates them from the masses; and thus the game goes merrily on." (1957 [1904], p. 545). There have been many attempts to formalize the fashion cycle on the basis of this assumption. The analysis in this paper provides a simple formal framework for studying the fashion cycle, adding a double contribution to the extant economic literature. Hypothesizing the endogeneity of preferences, it manages to reflect more faithfully the dynamics that create perpetual motion in the fashion cycle. Working from the hypothesis that preferences are molded by the social interaction of a heterogeneous community of individuals, the model manages to identify more fully the intimate psycho-sociological nuances, which, according to Simmel, give rise to the fashion cycle. The simulations conducted on the basis of such hypotheses offer an explanation of not only why people conform but also why the coexistence of the two groups allows the emergence and perpetuation of fashion cycles. They confirm that only as almost uniform distribution of the two typologies and a sufficiently high reactivity of snob agents set off the dynamic behaviors which assure the fashion cycle phenomenon. An important future for the model could be to extend its use to the production sphere, in order to strengthen its predictive capacity.

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Appendix

Proof Proposition 1. The map (7) is a simple linear first-order difference equation with negative slope. Local stability, in this case, is given when $-b\frac{m^S}{p} > -1$, that is, when $b < \frac{p}{m^S}$. Given the linearity of the map, local stability is equivalent to global stability.

Proof Proposition 2. Imposing the steady state condition, $\bar{x}_{t+1} = \bar{x}_t = x^*$ to (on?) map (8), we obtain:

$$1 - \frac{p}{m}x^* = e^{-ax^*}. \qquad 10)$$

It is clear that the origin is $x_1^B = 0$. The left and the right side of equation (10) are continuously decreasing maps and they have an intersection point at the origin. The left side has an intersection with the *x* axis at $x^* = \frac{m^B}{p}$, while the *x* axis is a horizontal asymptote for the right side; furthermore, the right side is a convex function. When, at the origin, the slope of the left side is smaller than the slope of the right side, that is, $a < \frac{p}{m^B}$, there are no more intersections between the map representing the two sides of equation (10). Otherwise, when the slope of the right side is larger than the slope on the left side, that is $a > \frac{p}{m^B}$, there is another positive intersection, $\bar{x}_2^B$, between the two sides. When $a < \frac{p}{m^B}$, $\bar{x} > 0$, the map (8) is increasing concave and lies under the identity map. Given any initial condition, $\bar{x}_0$, we have a sequence of iterates converging towards the origin. When $a > \frac{p}{m^B}$, the slope of map (8) in the origin is positive and greater than 1, the steady state at the origin becomes locally unstable; furthermore, there is also in this situation a positive steady state obtained via an intersection from above of map (8) with the identity map. Initial conditions $\bar{x}_0$ belonging to the interval (0,

$\bar{x}_2^B$) determine an increasing sequence converging on $\bar{x}_2^B$, while initial conditions $\bar{x}_0 > \bar{x}_2^B$ determine a decreasing sequence converging on $\bar{x}_2^B$.

Proof Proposition 3. Imposing the steady state condition, $\bar{x}_{t+1} = \bar{x}_t = x^*$, and arranging the equation (9), the following equation is obtained:

$$\omega\frac{m^B}{p} + \frac{(1-\omega)m^S}{p}(1-bx^*) - x^* = \frac{m^B}{p}e^{-ax^*} \qquad (10)$$

The left side of (10) is a linear decreasing map with positive intercepts on both axes, while the right side is a decreasing function with a vertical intercept smaller than the vertical intercept on the right side. In addition, the right side has the horizontal axis as its asymptote. Given the continuity of the two parts, the intersection is positive and gives the only positive steady state.