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Voting for Legislators

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Abstract

In this paper we propose a model with uncertainty in which strategic voters vote, under poportional rule, for a Parliament and parties bargain to form a government. We prove that only consensus government form and only extreme parties take votes.

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1 Introduction

In this paper we introduce an extension of De Sinopoli and Iannantuoni (2007). In the former the authors study strategic voting under proportional rule in a complete information setting. The main result shows that essentially only a two-party equilibrium exists, in which rational voters vote only for the two extremist parties. This extension builds on the consideration that most of the parliamentary democracies are characterized by a legislative body, elected by proportional rule, and by an executive body, which derives its mandate from the legislature. Clearly, national policies reflect the described institutional complexity. We aim to explore in an incomplete information model, the strategic behavior of policy-motivated citizens who face such institutional design. Through such analysis we aim to better understand parliamentary democracies, and the manner in which political institutions shape national policy (see Persson, 2002).

We present a simple game theoretical model where strategic voters elect, by proportional rule, politicians (with preferences over policy and office-holding benefit) to form a legislature. Legislators sharing the same interests over policy constitute parties. Parties represented in Parliament bargain to form an executive, which implements the policy.

This paper relates to two strands of the political economy literature. As already mentioned we build on De Sinopoli and Iannantuoni (2007) for the strategic voting analysis in proportional representation systems. For the post-electoral legislative bargaining the closest paper is Baron and Diermeier (2001): the government results from an efficient proto-coalition bargaining process. The formateur, which is chosen with a probability equal to his votes share[1], selects a proto-coalition, i.e. a potential government coalition (it has the majority in Parliament by definition), which bargains over both the policy and the allocation of transfers. The role of transfers is precisely to allow parties, which have preferences over both dimensions, to bargain efficiently. The main advantage of this approach is that to any proto-coalition one can associate the policy it would implement if in power. The pioneering paper joining the two approaches is the one by Austen-Smith and Banks (1988), who analyze a multistage game of three party competition under proportional representation, leading to a plethora of different equilibria.

This paper differs from the previous literature in various respects, the most important one being the analysis of multicandidate elections in an incomplete information environment. Moreover, we do not impose any restriction on voters' distribution.

We accomplish the following results. The implemented policy is the weighted average of parties' positions, where the weights are given by the electoral outcomes. Within the proto-coalition the formateur, who has all the bargaining power in his hands, extracts all the rent associated to the implemented policy. Moreover, the proto-coalition selection always leads to consensus government. The intuition of the result is very simple: it follows from the assumption that the legislator's preferences includes a strong benefit term (alternatively, one can read this assumption as a very high cost associated to the status quo). Finally, strategic voters vote only for the extreme, i.e. for politicians belonging to the extreme parties.[2] This is indeed the most efficient manner in which a voter moves the final policy outcome towards his preferred policy.

The paper is organized as follows. Section 2 presents the model, we solve the proto-coalition bargaining in Section 3, the proto-coalition selection in Section 4, and the electoral stage in Section 5. Section 6 introduces an example, and Section 7 concludes the paper.

[1] See Diermeier and Merlo (2004) for an empirical support of this assumption.

[2] The "polarized" voting behavior can be studied in a multidimensional setting. Assuming that there are parties at the extreme of the policy space (for example, if the space is the unit square this would mean that parties are located at the four corners), and that voters' preferences are single peaked in each dimension with the peak independent from the other dimension then strategic voters vote only for those extreme parties (see De Sinopoli and Iannantuoni, 2008).

2 The Model

We consider a multicandidate model, with uncertainty about voters' preferences, in a one-dimensional policy space generating results on policy choice, on government formation, and on election outcomes. Let us introduce the elements of the model.

The policy space. The policy space $\mathbb{X}$ is a closed interval of the real line, and without loss of generality we assume $\mathbb{X} = [0, 1]$.

Parties. There is a given finite set of parties P. Each party $k \in P = \{1, ...k, ..., p\}$ may be represented by its bliss policy $\zeta_k \in [0, 1]$

Legislators. A legislator $l \in M$, with $m := |M|$, is characterized by his bliss policy $\zeta_l \in [0, 1]$, such that $\zeta_l = \zeta_k$ whenever legislator l belongs to party k. Each legislator l has preferences over both policies and office-holding benefits $y_l \in \Re$, where $\sum_{l=1}^{m} y_l \leq Y$, with $y_l = 0$ if legislator l is not in the government. Legislator l's preferences are represented by a quasi linear utility function $U_l(x, y)$ given by

$$U_l(x, y) = B + u_l(x) + y_l,\ B > 1,\ l = 1, ..., m.$$

Clearly $u_l(x)$ is the legislator l's policy motivation. Let us assume $u_l(x)$ to be quadratic (as, for example, in Baron and Diermeier 2001):

$$u_l(x) = -(x - \zeta_l)^2$$

Voters. There is a finite set of voters $N = \{1, ..., i, ..., n\}$. Each voter $i \in N$ is characterized by a bliss point θ_i that is his private information. Voters' bliss points are independently distributed according to the commonly known distribution $F(\cdot)$ that has support $[0, 1] \subset \mathbb{R}$. Voters have quadratic utility functions over government policies. Voter i's preferences are represented by a utility function $u_i(x; \theta)$,

$$u_i(x; \theta) = -(x - \theta_i)^2.$$

The pure strategy space of player i, given his bliss point θ_i, is $S_i(\theta_i) = \{1, ..., k, ..., p\}$ where each $k \in S_i(\theta_i)$ is a vector of p components with all zeros except for a one in position k, which represents the vote for party k. A mixed strategy of player i is a vector $\sigma_i(\theta_i) = (\sigma_i^1(\theta_i), ..., \sigma_i^k(\theta_i), ..., \sigma_i^p(\theta_i))$, with $\sum_{k=1}^{p} \sigma_i^k(\theta_i) = 1$, where each $\sigma_i^k(\theta_i)$ represents the probability that player i votes for party k when his bliss point his θ_i.

The electoral rule. Given $s(\theta) = (s_1(\theta_1), ..., s_i(\theta_i), ..., s_n(\theta_n))$, we derive the share of votes each party gets under that strategy combination $s(\cdot)$, i.e. $v_k(s(\cdot)) = \frac{1}{n} \sum_{i=1}^{n} s_i^k(\cdot)$. We assume that such share of votes is equal to the share of seats each party (composed by members with the same bliss policy) has in parliament.

*Status quo***.** If a proto-coalition doesn't agree on the policy, a status quo policy, $q \in \mathbb{R}$ is implemented. Notice that there are no transfers associated to the implementation of the status quo policy.[3]

The game is a three stage game.

In the first stage policy motivated voters determine the composition of the parliament by voting under proportional rule.

The second stage consists of a government formation process. Every party has a probability that one of its members is selected as the formateur (we think about him as the selected prime minister) equal to the party vote share. The formateur, f, selects a proto-coalition G such

[3] Let us notice that we could have alternatively written the setup of the model assuming $B = 0$ and $u_l(q) = -1$.

that the formateur belongs to it, $f \in G$, and such that is gets the support of the majority in parliament. The formateur makes a take-it-or-leave-it offer on both policy and transfers dimensions. In case the proto-coalition does not get the majority in parliament, the status quo will be implemented, with no transfer associated.

In the third stage legislative bargaining occurs. As already mentioned in the introduction, we solve this stage by relying on the efficient proto-coalition bargaining (Baron and Diermeier, 2001). The formateur proposes to the proto-coalition G a policy, x^G, and office-holding benefits y^G to maximize the sum of the utilities of the parties in G. Hence, the policy outcome is an average of government parties' bliss points, while, given the nature of the bargaining stage (i.e. the take-it-or-leave-it offer) the formateur will get all the transfers.

We solve the game backward, and start by solving the proto-coalition bargaining.

3 The proto-Coalition Bargaining

Given a Parliament composition (i.e. given $(v_1(s), v_2(s), ..., v_p(s))$), let G be a proto-coalition with the majority in Parliament. An efficient policy x^G for the proto-coalition G is given by:

$$\begin{aligned} \left(x^G, y^G\right) &\in \arg\max_{x,y} \sum_{l \in G} \left[B + u_l(x) + y_l\right] \\ & \text{such that} \\ B + u_l(x) + y_l &\geq u_l(q), \qquad \forall l \in G \\ \sum_{l=1}^{m} y_l &\leq Y. \end{aligned}$$

For any formateur f in the proto-coalition G the optimal policy x^G is the efficient policy:

$$x^G = \frac{1}{|G|} \sum_{l \in G} \zeta_l.$$

When the formateur f makes a take-it-or-leave-it offer to obtain the support of a legislator l belonging to the proto-coalition, the transfer, y_l^G, necessary to implement x^G are given by

$$\begin{aligned} y_l^G &= u_l(q) - B - u_l\left(x^G\right), \quad l \in G/f \\ y_l^G &= 0, \qquad\qquad\qquad\qquad l \notin G. \end{aligned}$$

The utility of each member of the proto-coalition other than the formateur is given by

$$W_l\left(x^G\right) \equiv U_l\left(x^G, y^G\right) = u_l(q).$$

The utility of the formateur f is given by

$$W_f\left(x^G\right) \equiv U_f\left(x^G, y^G\right) = |G| B + Y - \sum_{l \in G/f} u_l(q) + \sum_{l \in G} u_l\left(x^G\right).^4$$

The bargaining power is clearly in the hand of the formateur. For this reason, in equilibrium, the partners of the formateur, belonging to the proto-coalition, reach an agreement on a policy by paying the formateur, who extracts all the rent associated to x^G.

[4] Notice that $U_f\left(x^G, y^G\right) \geq U_f(q, 0)$.

4 The Proto-Coalition Selection

Let us now analyze the government formation stage in which elected legislators bargain to form a government. Recall that legislators sharing the same bliss policy constitute a party, and that each party has a probability that one of its members is selected as the formateur equal to the party vote share. The formateur, f, selects a proto-coalition G such that the formateur belongs to it, $f \in G$, and such that it gets the support of the majority in parliament. The formateur makes a take-it-or-leave-it offer on both policy and transfers dimensions. If no agreement is reached, the status quo will be implemented. We assume that no transfer are associated to the status quo.

We prove that the optimal size of the proto-coalition leads always to consensus governments. This result builds on the existence of high office-holding benefits, B.

Proposition 1 *Given the result of the proto-coalition bargaining stage, the outcome of a government formation process is always $G = M$.*

Proof. Suppose that there is a legislator $j \notin G$, we prove that

$$W_f\left(x^{G\cup\{j\}}, y^{G\cup\{j\}}\right) > W_f\left(x^G, y^G\right).$$

The above expression can be written as

$$|G+1|\,B + Y + \sum_{l\in G\cup\{j\}} u_l\left(x^{G\cup\{j\}}\right) - \sum_{l\in G\cup\{j\}/f} u_l(q) - Y - |G|\,B - \sum_{l\in G} u_l\left(x^G\right) + \sum_{l\in G/f} u_l(q) > 0,$$

that is

$$B - u_j(q) + \sum_{l\in G\cup\{j\}} u_l\left(x^{G\cup\{j\}}\right) - \sum_{l\in G} u_l\left(x^G\right) > 0.$$

Let's now focus on the term $\sum_{l\in G\cup\{j\}} u_l\left(x^{G\cup\{j\}}\right) - \sum_{l\in G} u_l\left(x^G\right)$. Given that $x^{G\cup\{j\}}$ is the efficient policy when the proto-coalition is $G \cup \{j\}$, the following must hold:

$$\sum_{l\in G\cup\{j\}} u_l\left(x^{G\cup\{j\}}\right) - \sum_{l\in G} u_l\left(x^G\right) \geq \sum_{l\in G\cup\{j\}} u_l\left(x^G\right) - \sum_{l\in G} u_l\left(x^G\right) = u_j\left(x^G\right) \geq -1,$$

where the last inequality holds because of our assumptions on the utility function.

We can therefore conclude that

$$B - u_j(q) + \sum_{l\in G\cup\{j\}} u_l\left(x^{G\cup\{j\}}\right) - \sum_{l\in G} u_l\left(x^G\right) \geq B - u_j(q) - 1 > 0$$

given $B > 1$ and $u_j(q) \leq 0$. ■

5 Strategic Voting

Let now focus on the electoral stage in which voters, who are characterized by preferences over the final policy outcome, cast their ballot for one of the candidates. Note that each voter's preferences are his private information. We show that strategic voters vote only for the extreme candidates (i.e. candidates belonging to the extreme parties). Such a behavior is rational because it allows voters to influence the final outcome in the strongest possible way.

Let's define L as the leftmost legislator/s and R as the rightmost/s (i.e., $L = \arg\min_{l\in M} \zeta_l$, $R = \arg\max_{l\in M} \zeta_l$).

Let $\sigma(\theta) = (\sigma_1(\theta_1), ..., \sigma_n(\theta_n))$, $\sigma = E_\theta(\sigma_1(\cdot), ..., \sigma_n(\cdot))$, $\bar{\mu}^{\sigma,\theta} = \sum_{i\in N} \frac{\sigma_i(\theta_i)}{n}$ and $\bar{\mu}^{\sigma} = \sum_{i\in N} \frac{\sigma_i}{n}$. With abuse of notation, let $X(\bar{\mu}^{\sigma,\theta}) = \sum_{k=1}^{m} \zeta_k \bar{\mu}_k^{\sigma,\theta}$ and $X(\bar{\mu}^{\sigma}) = \sum_{k=1}^{m} \zeta_k \bar{\mu}_k^{\sigma}$.

Proposition 2 *(α) if $\theta_i \leq X(\bar{\mu}^\sigma) - \frac{1}{n}$ then $\sigma_i = L$*
(β) if $\theta_i \geq X(\bar{\mu}^\sigma) + \frac{1}{n}$ then $\sigma_i = R$.

Proof. (α) Given a mixed strategy σ_i, and his type θ_i, player i's vote is a random vector $\tilde{s}_i(\theta_i)$ with $Pr(\tilde{s}_i = k) = \sigma_i^k(\theta_i)$; given the mixed strategy σ_i, player i's expected probability of voting for party k is $Pr(\tilde{s}_i = k) = \sigma_i^k$. Given $\sigma_{-i}(\theta_{-i}) = (\sigma_1(\theta_1), ...\sigma_{i-1}(\theta_{i-1}), \sigma_{i+1}(\theta_{i+1}), ...\sigma_n(\theta_n))$, let $\bar{\mu}^{\sigma_{-i}} = \frac{1}{n-1} \sum_{j\in N/i} \sigma_j$; given $\sigma_{-i} = (\sigma_1, ...\sigma_{i-1}, \sigma_{i+1}, ...\sigma_n)$, let $\bar{\mu}^{\sigma_{-i}} = \frac{1}{n-1} \sum_{j\in N/i} \sigma_j$. We show that if $\theta_i \leq X(\bar{\mu}^\sigma) - \frac{1}{n}$

$$E_{\theta_{-i}} \left[u\left(X\left(\sigma_{-i}(\theta_{-i}), L\right), \theta_i\right) - u\left(X\left(\sigma_{-i}(\theta_{-i}), c\right), \theta_i\right) \right] > 0 \qquad \forall c \in M/L.$$

Recalling the quadratic form of the utility function, we can write the previous inequality as:

$$E_{\theta_{-i}} \left[u\left(X\left(\sigma_{-i}(\theta_{-i}), L\right), \theta_i\right) - u\left(X\left(\sigma_{-i}(\theta_{-i}), c\right), \theta_i\right) \right] =$$

$$E_{\theta_{-i}} \left[\begin{array}{c} -\left[X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, L\right) - \theta_i\right]^2 - VAR\left(X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, L\right)\right) \\ +\left[X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, c\right) - \theta_i\right]^2 + VAR\left(X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, c\right)\right) \end{array} \right] =$$

$$E_{\theta_{-i}} \left[-\left[X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, L\right) - \theta_i\right]^2 + \left[X\left(\bar{\mu}^{\sigma_{-i},\theta_{-i}}, c\right) - \theta_i\right]^2 \right] =$$

$$-\left[X\left(\bar{\mu}^{\sigma_{-i}}, L\right) - \theta_i\right]^2 - VAR\left(X\left(\bar{\mu}^{\sigma_{-i}}, L\right)\right) + \left[X\left(\bar{\mu}^{\sigma_{-i}}, c\right) - \theta_i\right]^2 + VAR\left(X\left(\bar{\mu}^{\sigma_{-i}}, c\right)\right) =$$

$$\left[X\left(\bar{\mu}^{\sigma_{-i}}, c\right) - X\left(\bar{\mu}^{\sigma_{-i}}, L\right)\right]\left[X\left(\bar{\mu}^{\sigma_{-i}}, c\right) + X\left(\bar{\mu}^{\sigma_{-i}}, L\right) - 2\theta_i\right] > 0.$$

Obviously, the first part of the above inequality is positive, i.e. $X(\bar{\mu}^{\sigma_{-i}}, c) - X(\bar{\mu}^{\sigma_{-i}}, L) > 0$. In order to prove our statement it will be enough to prove that $X(\bar{\mu}^{\sigma_{-i}}, L) > \theta_i$.

$$\begin{aligned} X\left(\bar{\mu}^{\sigma_{-i}}, L\right) &= \frac{n-1}{n} \sum_k \bar{\mu}_k^{\sigma_{-i}} \zeta_k + \frac{1}{n}\zeta_L = \\ &= X(\bar{\mu}^\sigma) - \frac{1}{n} \sum_k \sigma_i^k \zeta_k + \frac{1}{n}\zeta_L \geq X(\bar{\mu}^\sigma) - \frac{1}{n}(\zeta_R - \zeta_L) \geq X(\bar{\mu}^\sigma) - \frac{1}{n} \geq \theta_i. \end{aligned}$$

(β) A similar argument applies. ■

6 Example

We now introduce an example that illustrates the main results of the paper an that clarifies the driving forces operating in this setup. Consider an election where 7 voters have to elect a parliament that is composed by $m < 7$ members. Voters are characterized by a bliss point θ_i. Bliss points are voters' private information and they are independently drawn from the following distribution:

$$\Pr(\theta_i = 0) = \Pr\left(\theta_i = \frac{1}{2}\right) = \Pr(\theta_i = 1) = \frac{1}{3}.$$

There are three parties, L, with $\zeta_L = 0$, C with $\zeta_C = \frac{1}{2}$ and R with $\zeta_R = \frac{3}{5}$.

Section 3 and 4 showed that in such an electoral competition the efficient policy is given by the average bliss policy of the members of the proto-coalition, and that a consensus government arises.

Voting stage. We can start the analysis of voters' behavior from voters with extreme types. When a voter has type $\theta_i = 0$, he votes for party L for sure, because for any given composition of the parliament he prefers a parliament with a higher share of legislators from party L. Similarly, voters with type $\theta_i = 1$ always vote for party R, regardless of their beliefs on the other voters.

The optimal behavior of a voter with type $\theta_i = \frac{1}{2}$ depends instead on the likelihood of other voters' types, and on his beliefs on their strategies. We show that in this case it is optimal for a voter with type $\theta_i = \frac{1}{2}$ to vote for party R given any possible strategy of the other voters when their type is $\frac{1}{2}$.

To show this, we compute the optimal policy for a voter with $\theta_i = \frac{1}{2}$, when he believes that other voters will vote for L if their type is 0, will vote for R if their type is 1 and will randomize with probabilities r, c and $1-c-r$ on R, C and L respectively, when their type is $\frac{1}{2}$. Let therefore $\widehat{\sigma}_{-i} = (\widehat{\sigma}_1, ..., \widehat{\sigma}_{i-1}, \widehat{\sigma}_{i+1}, ..., \widehat{\sigma}_n)$ where for any $j \neq i$

$$\widehat{\sigma}_j = \begin{cases} L & \text{if } \theta_i = 0, \\ (L, 1-c-r; C, c; R, r) & \text{if } \theta_i = \frac{1}{2}, \\ R & \text{if } \theta_i = 1. \end{cases}$$

The expected implemented policy, given $\widehat{\sigma}_{-i}$, and the probability distribution on other voters' types, when voter i votes for party R is

$$E\left(x \mid \theta_i = \frac{1}{2}, s_i\left(\frac{1}{2}\right) = R, \widehat{\sigma}_{-i}\right) = \left(\frac{1}{3}\right)^6 \left\{\frac{729}{7}c + \frac{4374}{35}r + \frac{1254}{7}\right\}.$$

where $c + r \leq 1$. Clearly $E\left(x \mid \theta_i = \frac{1}{2}, s_i\left(\frac{1}{2}\right) = R, \widehat{\sigma}_{-i}\right)$ is maximum when the strategy of the other voters prescribes $r = 1$ when their type is $\frac{1}{2}$. Let $\sigma^*_{-i} = \left(\sigma^*_1, ..., \sigma^*_{i-1}, \sigma^*_{i+1}, ..., \sigma^*_n\right)$ where for any $j \neq i$

$$\sigma^*_j = \begin{cases} L & \text{if } \theta_i = 0 \\ R & \text{otherwise.} \end{cases}$$

The expected implemented policy, given σ^*_{-i}, when i votes for party R is

$$E\left(x \mid \theta_i = \frac{1}{2}, s_i\left(\frac{1}{2}\right) = R, \sigma^*_{-i}\right) = \left(\frac{1}{3}\right)^6 \frac{10644}{35} = \frac{10644}{25515} < \frac{1}{2}.$$

Therefore it is optimal for voter i to vote for R when his type is $\theta_i = \frac{1}{2}$ when the strategy of the other voters is σ^*_j (and also for every possible strategy of the other voters). Hence the equilibrium strategy is $\sigma^* = (\sigma^*_1, ..., \sigma^*_n)$ where for any $i = 1, ..., n$.

$$\sigma^*_i = \begin{cases} L & \text{if } \theta_i = 0 \\ R & \text{otherwise.} \end{cases}$$

7 Conclusion

In this paper we have solved a game theoretical model of strategic voting and government formation in parliamentary democracies. We have shown that, if the bargaining stage is efficient, strategic voters vote only for the extreme parties. A first extension would be to write a multidimensional game in order to check if one could obtain a sort of "polarized" result. Another interesting extension would be to consider different bargaining processes. Our results could be easily extended as long as the bargaining processes guarantee that each vote casted by a citizen affects the outcome.

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