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Distribution and Development**

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Occupational Choice, Wealth Distribution and Development *

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Abstract

This paper develops a model of occupational choice and income distribution in which both the wage rate and the interest rate are determined endogenously. We show the existence of multiple equilibria that depend on the initial distribution. In particular there can be a "development trap" characterized by many poor workers earning low wages and few rich entrepreneurs that exploit the low wage level and the high interest rate.

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1 Introduction

Under many points of view the transition process that is characterizing Eastern Europe, provides an interesting example to test the relevance of different economic theory and the effects of different policies. What makes this example interesting is the fact that these economies started the transition from similar initial conditions¹, (they had the same industrial structure, geographical location, trading partners and they had similar levels and distributions of income per capita); yet after only seven years since the beginning of reforms some economies seem to follow a development path that looks very much different from the one followed by others. The question of particular relevance is whether this is just a temporary phenomenon determined by the massive shock of the fall of the planned system, and therefore sometimes in the future they will all converge to some similar economic conditions, or in fact they are effectively taking different development path that will lead them very much apart from each other.

In other words it is important to understand if what we observe is just the transitional dynamics of a system that displays a unique steady state, or they are approaches to different equilibria displayed by the same system. Apart from the technical point of view this is crucial in terms of policy analysis; in the former case in fact policy does not matter very much: the best it can be done is to speed up or slow down the speed of transition to the steady state. In the latter case policy is extremely important: one shot policies in fact can have permanent effects and put the economy on a different development path.

The emphasis put in this paper on income distribution is an aspect of particular relevance for transitional economies. One of the results (perhaps the only one) that was achieved by the socialist system was the realization of a very low degree of inequality in the distribution of income; yet few years after the beginning of transition income inequality has considerably increased. It seems therefore interesting and appropriate, while analyzing the long run development of those economies, to address also the issue of income distribution.

The focus on income distribution, moreover, allows to tie the literature on transitional economies with the more general literature on development economics that has recently seen a resurgence of interest on themes related to distributional aspects. The present paper is related in particular with Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira

¹At least considering different groups of economies. i.e. baltic states, central Europe (Hungary, Poland and Czech Republic), NIS etc.

(1993) and Piketty (1997); in all those papers it is stressed the importance of initial conditions (in terms of income distribution) for the long run development of an economy, extending in this way one of the central ideas of the new growth theory (Lucas (1988), Romer (1986) and Murphy, Shleifer and Vishny (1989)).

In our model agents can choose between entrepreneurship and working as employees; in order to become entrepreneurs they need to borrow, but the existence of financial market imperfections implies that their investment decisions are constrained by the amount of wealth (collateral) they can put up front. This implies that the occupational choice and therefore the institutional structure of the economy is determined by the evolution of wealth. Moreover as the economy is closed, with the occupational choice the distribution of wealth determines also the equilibrium in the market for labor and capital. It turns out that in equilibrium there are many configurations of wage rate and interest rate that can support an equilibrium wealth distribution. In particular there can be a "development trap" in which the economy is characterized by many poor workers whose number depresses the wage rate and few rich entrepreneurs enjoying big rents deriving from low labor cost. This configuration is preserved by the fact that the low wage paid to workers depresses the supply of funds which in turn requires a high equilibrium interest rate to clear the capital market making more difficult for workers to borrow in order to become entrepreneurs.

From the technical point of view our model differs from other related papers (Aghion and Bolton (1997), Banerjee and Newman (1993, 94), Galor and Zeira (1993) and Piketty (1997)) in that we determine the evolution of wealth in a setting in which both the wage rate and the interest rate are determined endogenously. We show the existence, both analytically and numerically, of some classes of equilibria and provide a description of the evolution of the economy under those different configurations.

The remainder of the paper is organized as follows: sections (2) to (4) spell out the formal model and its dynamics implications, section (5) addresses the issue of income inequality; section (6) finally concludes.

2 Model Economy

The economy is closed and populated by a continuum of agents of mass 1. Each agent lives for one period in which he works, consumes and invests; the

remaining is left as bequest to his offsprings. The population is stationary, that is each agent has one child to take care of.

2.1 Preferences

Agents are assumed to be risk neutral and to have preferences over consumption and bequest.

$$U(c_t, b_t) = c_t^{1-s} b_t^s \quad (1)$$

Where c_t b_t denote respectively consumption and bequest. In every period agents maximize (1) with respect to c and b subject to the relevant budget constraint.

Denote by y_t the level of wealth (income) that each agent has at time t : the indirect utility function looks like

$$U(y) = Ay_t \quad (2)$$

where $A = s^s(1-s)^{(1-s)}$. This specification implies that consumption and bequest are a constant fraction of income: $b_t = sy_t$ and $c_t = (1-s)y_t$. Because of the bequest motive at each point of time the evolution of the economy can be represented by the distribution of wealth.

We assume that initial wealth is distributed over the support $[0, \bar{b}]$ with a distribution function $G_t(b_t)$. We also assume that $\bar{b} > \tilde{b}$ with \tilde{b} to be determined below; this last assumption ensures that whatever the dynamic evolution of the economy is, the equilibrium distribution of wealth will always be bounded.

2.2 Occupation

Each agent is endowed with one unit of labor. He can employ the labour endowment in four types of occupation:

- Work in a backyard activity: this is a safe activity that requires no investment and that yields a return of n .
- Work as an employee and enjoy the market wage w_t
- Set up a firm and become an employer.

- Set up a firm for self employment

The difference between self employment and entrepreneurship is given by the technology adopted.

2.3 Technology

There are two technologies available in the economy. One could invest in a labor intensive technology represented by the following specification:

$$F(k, l) = \begin{cases} R_1 \hat{k} & \text{with probability } p \\ R_0 \hat{k} & \text{with probability } (1 - p) \end{cases} \quad \text{if } k \geq \hat{k} \text{ and } l \geq 1 \quad (3)$$

$$F(k, l) = 0 \quad \text{otherwise}$$

As equation (3) shows the technology is characterized by non convexities: there is a minimum efficient scale that requires an investment of $\hat{k} > \bar{b}$ units of capital² that have to be combined with 1 unit of labor³ (in addition to the one provided by the entrepreneur). The combination of \hat{k} units of capital with 1 worker yields a return of $R_1 \hat{k}$ with probability p and $R_0 \hat{k}$ with probability $(1 - p)$. Denote by \bar{R} the expected value of R .

Alternatively one can invest in a "technology intensive" technology that requires the same investment \hat{k} and yields the same expected return (the return in each state is R'_1 with probability q and R'_0 with probability $(1 - q)$). This technology does not require any labor input in addition to the one provided by the entrepreneur; however the entrepreneur has to incur in a cost $c > n$ to use the technology. The cost c can be thought as training cost. The expected return from becoming an entrepreneur is given by:

$$\bar{R}\hat{k} - (1 + r_t)\hat{k} - w_t \quad (4)$$

While if one becomes self employed gets

$$\bar{R}\hat{k} - (1 + r_t)\hat{k} - c \quad (5)$$

²We have assumed that $\hat{k} \geq \bar{b}$ so that even the richest individual will need to borrow in order to become entrepreneur. Minor modifications would be needed to allow for the fact that there can be agents with $b \geq \hat{k}$.

³The assumption that this technology requires only 1 unit of labor is purely for simplifying matters. We could have a more general formulation that allowed $m > 1$ units of labor without affecting any of the results.

The occupational choice will be for the

$$\text{Max} : \left\{ \bar{R}\hat{k} - (1 + r_t)\hat{k} - w_t, \bar{R}\hat{k} - (1 + r_t)\hat{k} - c, w_t \right\}$$

The existence of the backyard activity implies that there is a minimum wage $\underline{w} = n$; if the wage rate is below \underline{w} everybody will prefer to work at the backyard activity. We assume that, that is at the minimum possible wage (and at the interest rate associated with it) entrepreneurial production is more profitable than self employment which in turn is more profitable than working. Therefore

$$\bar{R}\hat{k} - (1 + r_t(\underline{w}))\hat{k} - \underline{w} > \bar{R}\hat{k} - (1 + r_t(\underline{w}))\hat{k} - c > \underline{w}$$

2.4 Financial Market Imperfections

Financial market are imperfect; there are several ways to model financial market imperfections; here we adopt a simplified version of Banerjee and Newman (1994): in particular we assume that each borrower can evade debt payment by moving to another place once he received the loan. This move leaves his investment opportunities intact. Lenders have however a positive probability of catching the renegeing borrower; let us denote this probability by π . If caught the borrowed obtains the maximum punishment, that is his income is held at zero⁴.

Because of these imperfections loan contracts need to satisfy the following incentive compatibility constraint for entrepreneurs:

$$\bar{R}\hat{k} - (1 + r_t)\hat{k} - w_t + (1 + r_t)b_t \geq (1 - \pi)\bar{R}\hat{k} - w_t \quad (6)$$

and for self employed

$$\bar{R}\hat{k} - (1 + r_t)\hat{k} - c + (1 + r_t)b_t \geq (1 - \pi)\bar{R}\hat{k} - c$$

That is the expected return from being an entrepreneur (or self employed) must be greater than the expected profit from renegeing on the loan. Both incentive compatibility constraint determine a unique threshold level of wealth

⁴Other forms of imperfections due to moral hazard like those adopted by Aghion and Bolton (1997) and Piketty (1997) would yield similar results.

$$\hat{b} = \frac{1}{1+r_t} \left[(1+r_t)\hat{k} - \pi\bar{R}\hat{k} \right] \quad (7)$$

We assume that π is small enough such that the threshold level of wealth is positive. From (7) it is also clear that \hat{b} increases with the interest rate. It is \hat{b} which determines the occupational choice: anyone with wealth $b_t = \hat{b}$ will be indifferent between becoming an entrepreneur or working as employee. Everyone with $b_t < \hat{b}$ will be denied credit and therefore will work. Everyone with $b_t > \hat{b}$ will become entrepreneur, either employer or self employed. The distinction between these two status is determined by the equilibrium conditions in the labor market to which now we turn.

3 Equilibrium Conditions and Factor Prices

3.1 Labor Market Equilibrium

The non convex technology allows us to have quite a simple representation of the labor market. Let us define a wage level \bar{w} such that the expected return from being an entrepreneur equals the expected return from being self employed

$$\bar{R}\hat{k} - (1+r_t)\hat{k} - \bar{w} = \bar{R}\hat{k} - (1+r_t)\hat{k} - c$$

That implies

$$\bar{w} = c$$

We now turn at the determination of demand and supply of labor. For wage levels greater than \bar{w} the labor demand will be zero. For all wage level below \bar{w} the labor demand will be determined by the number of potential entrepreneur in the market that is $[1 - G(\hat{b})]$. At $w = \bar{w}$ the labor demand will be the interval $\{0, [1 - G(\hat{b})]\}$. The labor supply on the other side will be 0 for $w < \underline{w}$, the interval $\{0, G(\hat{b})\}$ at $w = \underline{w}$, and will be $G(\hat{b})$ for $w > \underline{w}$. Demand and supply of labor are illustrated in figure (1).

Lemma 1 *There are two possible equilibrium wage rates: either \underline{w} or \bar{w} .*

Proof. Not considering the non generic case in which $G(\hat{b}) = [1 - G(\hat{b})]$, we note that the labor market allows two possible configurations:

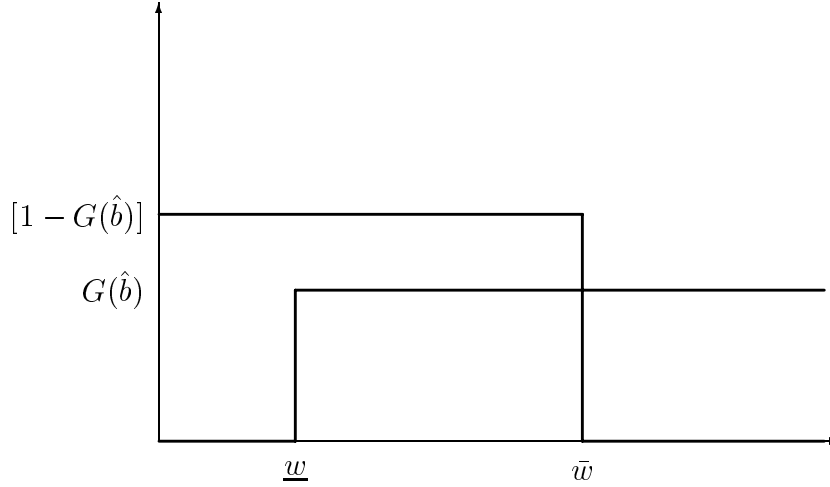


Figure 1: Labor Market Equilibrium

1. $G(\hat{b}) > [1 - G(\hat{b})]$; the prevailing wage is \underline{w} . There is excess supply of labor and a fraction of the potential workers works at the backyard activity (at \underline{w} agents are indifferent between working as employees and at the backyard activity). The probability of working as employee is $\rho = \frac{(1-G(\cdot))}{G(\cdot)}$ while the probability of working in the backyard activity is $(1 - \rho)$.
2. $G(\hat{b}) < [1 - G(\hat{b})]$; the prevailing wage is \bar{w} . There is excess demand of labor: all potential entrepreneurs who satisfy the IC constraint obtain the loan. Of those potential entrepreneurs a fraction G find a match with a worker; this happens with probability $\rho' = \frac{G(\cdot)}{1-G(\cdot)}$ while the others who do not find a match with the worker (with probability $(1 - \rho')$) become self employed.

The existence of self-employed and of workers employed in the backyard activity, provides the "buffer" necessary for the two wages \underline{w} and \bar{w} to clear the market in both the two configurations. ■

Note that there will be agents working at the backyard activity only when the wage rate is \underline{w} , (excess supply) because at \bar{w} they are always better off working as employees rather than at the backyard activity; on the other side there will be self-entrepreneurs only when the market wage is \bar{w} (excess

demand) because at \underline{w} everybody with $b_t \geq \hat{b}$ can become employer and at that wage he will be better off rather than being self employed.

Since there are self entrepreneurs only in presence of excess demand of labor and since $\bar{w} = c$, even if ex ante employers and self-entrepreneurs can have different expected returns, ex post the expected returns are always equal.

3.2 Capital Market Equilibrium

The supply of funds is determined by all agents who are working

$$S^k(r_t) = \int_0^{\hat{b}(r_t)} b dG_t(b_t) \quad (8)$$

while capital demand will be determined by employers and self-entrepreneurs. Because both need the same initial investment the demand of capital is simply:

$$D^k(r_t) = \int_{\hat{b}(r_t)}^{\bar{b}} (\hat{k} - b) dG_t(b_t) \quad (9)$$

The demand of capital is decreasing in r while the supply is increasing in r . S^k and D^k uniquely determine the equilibrium interest rate. This can be established noting that given G_t at time t all variables are determined by the past; for any given level of b_t and for any given distribution G_t an increase in the interest rate, increasing the threshold \hat{b} , increase the supply and decreases the demand of credit.

4 Market Equilibrium Dynamics

There are four types of individual transition functions corresponding to the four classes of agents that characterize the economy. Those transition functions are represented below:

For those agents with $b_t < \hat{b}$ and work as employees

$$b_{t+1} = s[(1 + r_t)b_t + w_t] \quad (10)$$

For those with $b_t < \hat{b}$ and work at the backyard activity

$$b_{t+1} = s[(1 + r_t)b_t + n] \quad (11)$$

For those with $b_t > \hat{b}$ and become employers

$$b_{t+1} = \begin{cases} s \left[R_1 \hat{k} + (1 + r_t)b_t - (1 + r_t)\hat{k} - w_t \right] & \text{with probability } p \\ s \left[R_0 \hat{k} + (1 + r_t)b_t - (1 + r_t)\hat{k} - w_t \right] & \text{with probability } (1 - p) \end{cases} \quad (12)$$

For those with $b_t > \hat{b}$ and become self employed

$$b_{t+1} = \begin{cases} s \left[R'_1 \hat{k} + (1 + r_t)b_t - (1 + r_t)\hat{k} - c \right] & \text{with probability } q \\ s \left[R'_0 \hat{k} + (1 + r_t)b_t - (1 + r_t)\hat{k} - c \right] & \text{with probability } (1 - q) \end{cases} \quad (13)$$

Transition functions like those in equation (10) and (12) are represented in figures (2) and (3).

We place the restriction that $s(1 + r) < 1$ so that wealth does not grow without bounds; hence the recurrent distribution is bounded between 0 and \tilde{b} , where

$$\tilde{b} = \frac{s}{(1 - s(1 + r))} [R'_1 \hat{k} - (1 + r_t)\hat{k} - w_t] \quad (14)$$

The two bounds mean that nobody can receive a transfer less than zero and that whoever receives a transfer greater than \tilde{b} , even if it becomes a successful entrepreneur, will leave to his descendants a transfer smaller than the one he has received. This implies that, even if the support for the initial distribution is the interval $[0, \bar{b}]$, in steady state the relevant support for the distribution is $[0, \tilde{b}]$.

To describe the dynamics of the system we can use the transition functions (10) (12) and figures (2) and (3). We note however that those transition functions and figures give only a partial description of the dynamic evolution of the economy: they are snapshots taken in a given moment of time for a given distribution of wealth.

At the beginning of each period, given the distribution of wealth G_t , b_t is given, that is it is the result of equilibrium conditions of the previous period. Once received the bequest b_t , agents make their occupational choice;

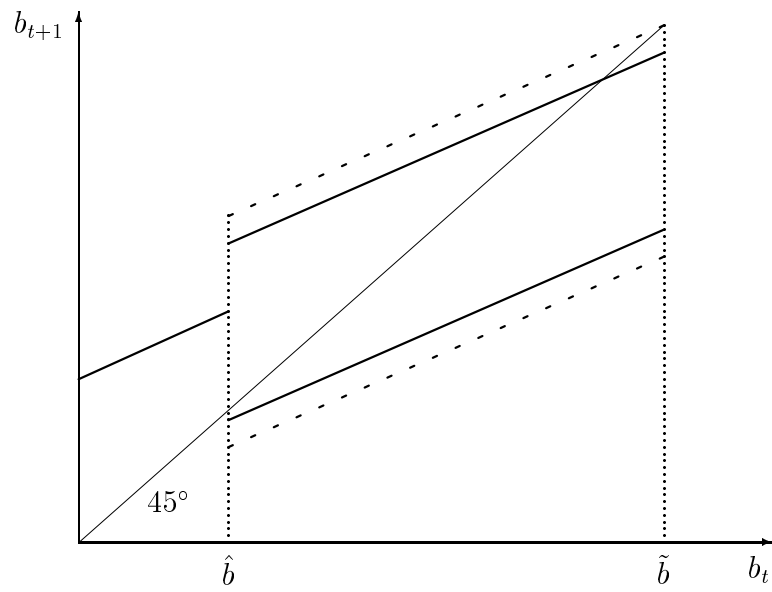


Figure 2: Individual Transitions: mobility in both directions; high w and low r

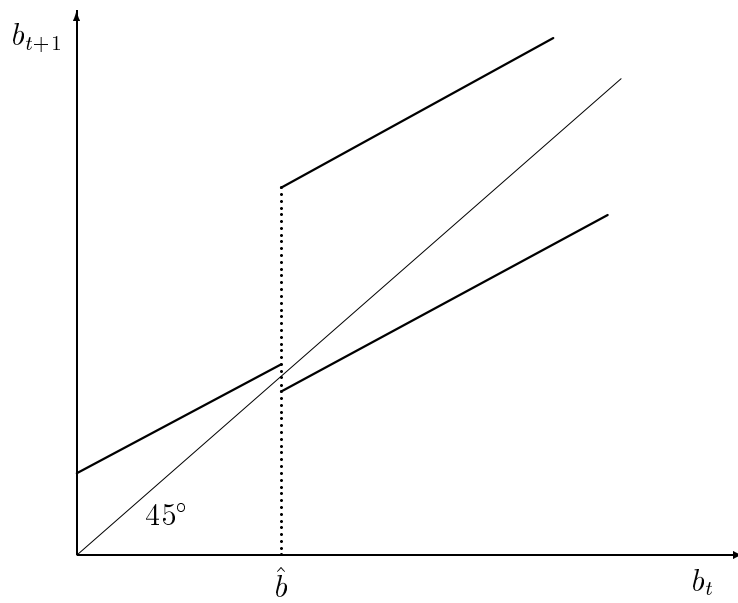


Figure 3: Individual Transitions: mobility in both directions: low w and high r

this choice determines the demand and supply of labor and funds and therefore determines the distribution of wealth and the equilibrium levels of the wage and the interest rate. Finally, given the equilibrium wage and interest rate, each agent bequeath a fraction s of his income to his descendants, determining b_{t+1} .

A formal analysis of the dynamics described above is however quite difficult. The difficulties come from two sources:

1. The state space is the set of wealth distributions over $[0, \bar{b}]$ and not only the wealth interval itself.
2. The recursive map on the space of wealth distributions is non linear

In other words wealth follows a non stationary Markov process because the interest rate and the wage rate affect the distribution itself and as the distribution evolves so do w and r . There are very few mathematical results that allow us to deal with Markov processes that are non stationary and we are thus constrained in the dynamical analysis that we can carry out. In particular we cannot determine any description of the transitional dynamics and we have to restrict the analysis to the steady state. More precisely, following the classification by Owen and Weil (1998) we distinguish between *conditional* and *unconditional* steady states.

- A *conditional steady state* is defined as a fixed point of the recursive map that describes the dynamic evolution of the distribution of wealth, holding the wage rate and the interest rate constant.
- An *unconditional steady state* is a fixed point of that map such that itself generates the equilibrium wage and interest rate.

4.1 Conditional Steady States

To formally define a conditional steady state, we begin by considering the dynamic process followed by the distribution of wealth, keeping the wage rate and the interest rate fixed. This process can be represented by the following equation:

$$G_{t+1}(b) = H(G_t(b), w, r) \tag{15}$$

For $G_0(b)$ given. A conditional steady state is a fixed point of the map defined in (15).

$$G_{ss}^c(b) = H(G_{ss}^c(b), w, r) \quad (16)$$

Where the subscript ss denotes steady state values.

The properties of the distribution of wealth that characterize a conditional steady state depend on the degree of mobility that there exists between classes. A sufficient condition for the existence of upward mobility is determined by:

$$\frac{s}{1 - s(1 + r)}\bar{w} > \hat{b} \quad (17)$$

In the case of high wage and by:

$$\frac{s}{1 - s(1 + r)}w > \hat{b} \quad (18)$$

In the case of low wage. That is parents that are working as employees or in the backyard activity will eventually bequeath to their children an amount of wealth sufficient to enable them to become entrepreneurs.

On the other side a sufficient condition for downward mobility is determined by:

$$\frac{s}{1 - s(1 + r)} \left[R_0 \hat{k} - (1 + r)\hat{k} - \bar{w} \right] < \hat{b} \quad (19)$$

In the case of high wage, and by

$$\frac{s}{1 - s(1 + r)} \left[R_0 \hat{k} - (1 + r)\hat{k} - \underline{w} \right] < \hat{b} \quad (20)$$

In the case of the low wage. That is, after a sufficient number of bad draws entrepreneurs (or self employed) will eventually give to their descendants a transfer small enough such that they will not be able to pass the threshold \hat{b} . There can be many configurations in which there is mobility in both directions, only upwards and only downwards, or no mobility at all.

We will skip the analysis of cases in which there is mobility only in one direction (we will analyze those cases when considering unconditional steady states) and we will concentrate on the two cases of no-mobility and of mobility in both directions.

In what follows we apply some results recently shown by Hopenhayn and Prescott (1992) (henceforth HP) that rely on the property of monotonicity of Markov processes⁵.

Lemma 2 *Let P be the transition function that describes the Markov process followed by wealth. P is monotonically increasing.*

Proof. See the Appendix.

Monotonicity alone is enough to ensure the *existence* of an invariant distribution of wealth. This result can be established by applying Corollary 4 by HP that shows the existence of fixed points for monotone maps defined over compact sets; these maps need only to be monotone, and not necessarily linear. The formal statement of the corollary as of other results obtained by HP is reported in the appendix.

Monotonicity allows us to establish only the existence of a limiting wealth distribution. In fact there can be *many* of those distribution; fortunately in addition to monotonicity we can establish some other properties of the transition functions (10) and (12) that allow us to apply Theorem 2 by HP and state the following proposition:

Proposition 1 *For any steady state equilibrium pair (r_{ss}, w_{ss}) the associated limiting wealth distribution is unique if there is mobility between classes; there is a large set of associated limiting wealth distribution if there is no mobility between classes.*

Proof. (Follows from Theorem 2 by HP) See the appendix.

Even if the limiting wealth distribution is unique there is nothing that guarantees that the equilibrium pair (r_{ss}, w_{ss}) is unique. Most likely there will be many equilibrium pairs that can sustain a limiting distribution; we will try to characterize how the different equilibrium configurations can look like in section (4.3), for the moment we note that because of the particular configuration of the labor market we can divide all possible equilibria in two classes: one characterized by low wages and the other by high wages.

⁵As clear from the figures, the map from b_t to b_{t+1} is discontinuous; therefore we cannot use results based on continuity of Markov operators as in Futia (1982).

4.2 Unconditional Steady States

So far we have limited the analysis to conditional steady states; we now turn at the analysis of unconditional steady states. They can be defined in the following way:

i) Define H^* as the function that maps a pair $\{w, r\}$ into a conditional steady state distribution of wealth; that is

$$H^*(\{w, r\}) = \{G_{ss}^c \mid G_{ss}^c = H(G_{ss}^c, \{w, r\})\} \quad (21)$$

ii) Consider now the following function: for any value of $G(\cdot)$ the function maps the set of wages and interest rates generated by such a value of $G(\cdot)$ and the set of conditional steady state distribution generated by each pair $\{w, r\}$ into the set of values of $G(\cdot)$. Call this function $\Phi(G)$. Then

$$\Phi(G) = \psi(H^*(w_{ss}, r_{ss})) \quad (22)$$

An unconditional steady state is a fixed point of

$$G_{ss}^u = \Phi(G_{ss}^u) \quad (23)$$

The wage rate and the interest rate are uniquely determined by the equilibrium condition on the labor market and on the market of capital; in case of both upward and downward mobility, for any given wage and interest rate there is a unique conditional steady state distribution of wealth (that is $H^*(\cdot)$ is a singleton), however there can be more than one fixed point of (23) as there can be more than one equilibrium interest rate and wage level. Still there is a one to one correspondence between each equilibrium pair $\{r_{ss}, w_{ss}\}$ and the limiting distribution $G_{ss}^u(b)$. In case of no-mobility then H^* is not anymore unique and the set of limiting distribution will generally be large.

Proposition 2 *In case of partial mobility*

- a) *There cannot be an unconditional steady state with only upward mobility.*
- b) *There exist an unconditional steady state with only downward mobility.*

Proof. Part a) If there is only upward mobility, the number of workers will progressively decline, and so will the supply of funds. Eventually the resulting interest rate will be so high that the implied threshold will shut down mobility.

Part b) If there is only downward mobility the economy will eventually collapse to a "development trap" in which nobody can afford to become entrepreneur and everybody has to work in the backyard activity. ■

Proposition 3 *For some configuration of parameters there exist an unconditional steady state in which there is no mobility in any direction.*

Proof. See the Appendix.

Proposition 4 *Numerical analysis shows that for some configuration of parameters there exist an unconditional steady state in which there is mobility in both directions.*

The model was simulated as follows: given the initial distribution and the initial number of agents total wealth is determined. The total wealth, given the project size \hat{k} determines in turn how many project can be financed; sorting the agents by wealth this in turn determines the threshold \hat{b}_t . Once determined the threshold, given the other parameter values we can determine the equilibrium interest rate. The wage, interest and the realization of the project return determine a new wealth distribution and the process is repeated until convergence. The simulations assumed 2000 agents distributed according to a uniform distribution over the interval $[10, 300]$ ⁶. The model was run for 400 periods; the wage rate was set at 100, \hat{k} was set at 1200; the other parameters were as follows: $R_g = 1.3$ $R_b = 1.2$, $p = 0.8$, $\pi = 0.8$, $s = 0.6$ ⁷

Figure (4, a) show that with a wage of 100 and the other parameters as described above the resulting equilibrium interest rate is 9.97% and the implied threshold \hat{b} is 82.6. The associated stationary distribution is represented in figure (4, b); the values of the parameters and the particular convex technology give to entrepreneurs a big rent with respect to workers (entrepreneurial (average) income net of wages and interest rate is 208.3, more than the double of workers' income) this determines a skewed wealth distribution with few very rich entrepreneurs and many "poor" workers.

⁶Almost identical results were obtained with a normal distribution.

⁷The saving rate is assumed to take such high values because it denotes saving out of total wealth and not out of income.

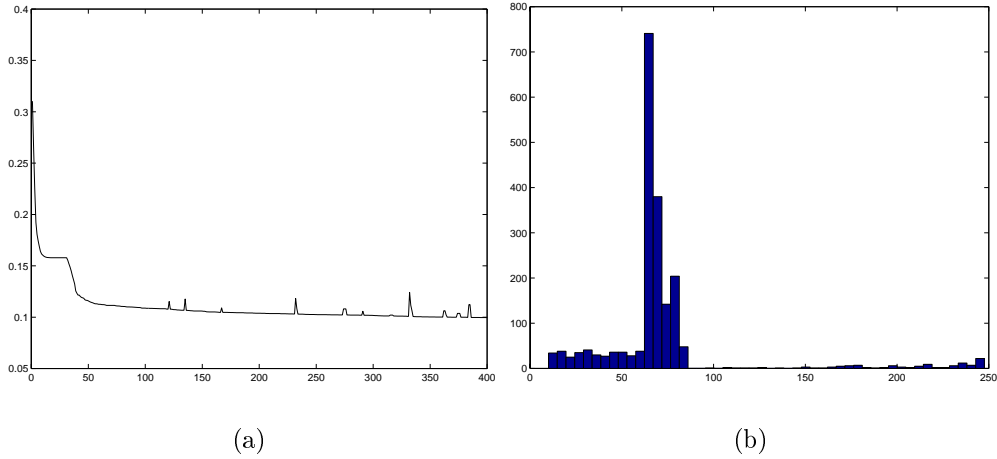


Figure 4: Equilibrium interest rate and stationary distribution

4.3 A qualitative analysis of the steady state

So far we have established the existence of conditional and unconditional steady states and we have stressed the fact that there will be a multiplicity of such steady states. However we have not analyzed what are the characteristics and qualitative features of those steady states.

We have already noticed that we can divide all the possible equilibria in two classes: one characterized by low wages and the other by high wages. Unfortunately we cannot say much about the behavior of the interest rate within those two classes of equilibria unless we make some assumptions on the distribution of wealth.

The behavior of the interest rate is represented by figure (5) as a function of $G(\cdot)$. For $0.5 > G > 0$ the high wage prevails; as G approaches 0.5 the interest rate declines monotonically. The value \tilde{G} in figure (5) represents the value of G such that the associated interest rate equals $(1 - s)/s$; as we have stressed above in equilibrium it has to be the case that $r < (1 - s)/s$ to guarantee that wealth does not grow without bounds; therefore the relevant region for steady state analysis is $G(\cdot) > \tilde{G}$.

At $G(\cdot) = 0.5$ there is a jump in the interest rate as for virtually the same number of workers their wage drops to w and so does the supply of funds. The interest rate subsequently declines monotonically as $G(\cdot)$ approaches 1.

Some particular configurations of w and r are worth to be analyzed.

Proposition 5 *There can be an equilibrium pair in which the low wage level*

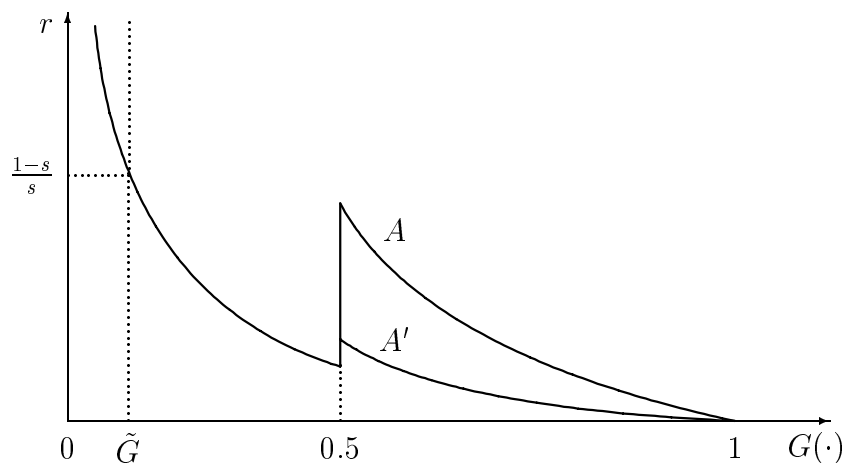


Figure 5: Capital market equilibrium

is associated with a high interest rate while the with high wage is associated with a low interest rate.

The intuition for this result is the following: suppose that the economy is settled in a high wage low interest rate equilibrium. A positive shock to the interest rate determines two effects:

1. The threshold level of wealth (\hat{b}) increases determining a reduction in the number of entrepreneurs: there will be fewer agents that can afford to become entrepreneurs and more agents that will have no option rather than work as employees. This effect can lead the system to the dynamics associated with the low wage.
2. As workers are net lenders an increase in the interest rate will make them better off. In addition there will be more of them. However they will experience a reduction of the wage from \bar{w} to \underline{w} . In equilibrium if the wage effect is strong enough (this in general will happen if there is a consistent difference between \bar{w} and \underline{w}) on aggregate there will be a decrease in the supply of funds which will make the high interest rate self sustaining.

Using the same parameters value described above we increased the wage to 120. The equilibrium interest rate decreased to 8.89% and the resulting

	w=100	w=120
r (%)	9.93	8.89
\hat{b}	82.60	71.50
Entr. Income	208.30	188.20

Table 1:

threshold of wealth was reduced to 71.5. Entrepreneurial income decreased as well (to 188.2) following higher wages that have to be paid to workers. Table 1 summarizes the results for the two wage levels.

The likelihood of the result previously underlined clearly depends on the magnitude of the wage effect. If the relative difference between \bar{w} and \underline{w} is limited, then the jump in the interest rate at $G = 0.5$ will be small (path A' in figure (5)) and it will be unlikely that the high wage equilibria will have a lower interest rate than the low wage equilibria

Conversely if the relative difference between \bar{w} and \underline{w} is consistent, then r will have a big jump (path A) and it will be likely that it will be higher in the low wage equilibrium than in the high wage equilibrium.

For example suppose the economy is settled in an equilibrium in which $(1 - G) - G = \varepsilon$ where ε is small; the interest rate associated with \bar{w} is very low. A small positive shock to the interest rate, if it makes $(1 - G)$ lower than G will have a big impact on the wage rate; therefore the negative effect on the supply of funds will be quite strong while the positive effect given by the higher interest rate is likely to be small. In this way the high interest rate can be self sustaining.

Appendix B provides a more formal example of two possible equilibria configurations of the type envisaged above

This result allow us to combine some results previously obtained by the literature. In particular it reconciliates the results obtained by Banerjee and Newman (1993) and Piketty (1997) with the difference that while the former consider only the determination of the wage rate and the latter considers only the determination of the interest rate, here w and r are both endogenously determined. This example shows that the same intuitions goes through in a more general setting; it seems therefore that the existence of a "development trap" characterized by credit rationing and low wealth for individuals who are rationed, is a quite robust result of these types of models.

5 Development and Inequality

The wealth distributions associated with the two classes of equilibria can entail quite different degrees of inequality.

Proposition 6 *The set of low wage equilibria is generally characterized by a more unequal distribution of wealth than the set of high wage equilibria.*

To understand the last proposition it is useful to divide the analysis in two parts; we know that in equilibrium there will be two sets of equilibria one characterized by wage \underline{w} and one characterized by wage \bar{w} . We keep the notation used previously identifying variables and distributions associated with \underline{w} by subscript 1 and those associated with \bar{w} by subscript 2. Two cases are possible: either $r_1 > r_2$ or $r_1 < r_2$.

We measure inequality in a very simple way referring to the fixed points of individual transitions.

Consider the transition functions that characterize entrepreneurs with low wages. These transition functions will on average converge to the fixed point

$$b_1 = \frac{s}{1 - s(1 + r_1)} [\bar{R}\hat{k} - (1 + r_1)\hat{k} - \underline{w}]$$

On the other side transition functions of entrepreneurs with high wages will converge on average to

$$b_2 = \frac{s}{1 - s(1 + r_2)} [\bar{R}\hat{k} - (1 + r_2)\hat{k} - \bar{w}]$$

Consider the case where $r_1 < r_2$: as entrepreneurs are net borrowers the lower interest rate compounds the advantage given by the lower wage, therefore the transition functions of entrepreneurs with low wages will always stay above the transition functions of entrepreneurs with high wages. For the opposite reasons transition functions of workers with low wages will always stay below transition functions of workers with high wages. In this case to the low wage equilibrium is associated a more unequal distribution of income.

Consider now the case where $r_1 > r_2$: if the difference in wage rates is sufficiently high, and in particular if $(\bar{w} - \underline{w}) > (r_1 - r_2)\hat{k}$, then transition functions of entrepreneurs with low wages will converge on average to a higher fixed point than transition functions of entrepreneurs with high wages. Also in this case, if the condition above is satisfied, to the low wage equilibrium is associated a more unequal distribution of income.

6 Conclusions

In this paper we have characterized the dynamic evolution of an economy in which the distribution of wealth, the equilibrium conditions in the labor and the capital market are endogenously determined in presence of financial market imperfections.

Those imperfections prove to be not only important for the short run development but they affect also the long run evolution of an economy and the degree of inequality present in steady state.

In our model imperfections in financial markets are crucial in giving persistence to initial conditions. Without removing those imperfections (for instance with a clear definition of property rights, with a sound regulatory framework and with fair but severe bankruptcy laws) it will be very difficult for an economy to get out from a “development trap” in which few rich entrepreneurs are getting advantage of low wages paid to workers who are credit constrained.

Finally this paper makes scope for redistributive policies. Given the fact that there are financial market imperfections that can be eased only with difficulty, a government may want to engage in redistributive policies that reduce the degree of inequality. Such one shot policies, in our case, can be welfare improving having permanent effects.

Appendix A

The proofs contained in this appendix rely on some results obtained by Hopenhayn and Prescott (1992), henceforth (HP).

In what follows we take the state space to be a Borel set of an Euclidean space, $B \subseteq \mathbf{R}^l$ with Borel subset \mathcal{B} .

Proof of Lemma 2

Let $P : B \times \mathcal{B} \rightarrow [0, 1]$ be the transition function that corresponds to the Markov process followed by wealth. The interpretation is that $P(a, A) = \Pr\{b_{t+1} \in A \mid b_t = a\}$, that is the number $P(a, A)$ is the probability that the random variable b next period lies in the set A given that the current value is a .

P is monotone if it is increasing in its first arguments in the stochastic order sense: $b, b' \in B$ and $b \geq b'$ implies $P(b, \cdot) \geq P(b', \cdot)$. This property can be established immediately observing the individual transition functions and noting that b_{t+1} is an increasing function of b_t . ■

Let B be a compact metric space and let P be a transition function as defined above. P induces a mapping $T^* : \mathcal{P} \times (B) \rightarrow \mathcal{P} \times (B)$ defined by

$$(T^*\mu)(A) = \int P(b, A)\mu(db)$$

T^* is called the *adjoint* of the Markov operator T , μ is a probability measure and A is a Borel subset of B .

The interpretation is that if $\mu(A)$ is the probability that the current period the state b is in the set A , then $(T^*\mu)(A)$ is the probability that b lies in A next period.

Corollary 4, HP pp.1392: If B is a compact metric space with a minimum element and $P : B \times \mathcal{B} \rightarrow [0, 1]$ is an increasing monotone function, then the Markov process corresponding to P has a stationary distribution; i.e., there exists a fixed point for the mapping T^* induced by the process.

Theorem 2, HP pp.1397:

Suppose P is increasing, B contains a lower bound l and an upper bound u and the following condition is satisfied:

Monotone Mixing Condition⁸: there exists a point $b^* \in B$ and an integer m such that $P^m(u, [l, b^*]) > 0$ and $P^m(l, [b^*, u]) > 0$ ⁹.

⁸See Stokey, Lucas, Jr. and Prescott (1989), pp.381.

⁹Where $P^m(l, [b^*, u])$ denotes the probability of reaching the set $[b^*, u]$ starting from l after m iterations of the Markov Process

Then there is a unique stationary distribution λ^* for the process P and for any initial measure μ , $T^{*n}\mu = \int P^n(b, \cdot)\mu(db)$ converges to λ^* .

Proof of Proposition 1

We have already established the property of monotonicity; we need to establish that the transition functions a) operate on a bounded set, and b) satisfy the Monotone Mixing Condition.

Part a) The set is bounded between 0 and $\max\{\tilde{b}, \check{b}\}$ where \check{b} is the largest endowment (inheritance) any individual starts up with at $t = 0$.

Part b) The mixing condition is indeed satisfied whenever there is both upward and downward mobility between classes.

We can therefore apply Theorem 2 by HP and establish the existence of a unique invariant distribution.

On the other side whenever there is no mobility between classes, the space $[0, \bar{b}]$ will be divided into two ergodic sets; within those two ergodic sets wealth converges to a unique stationary distribution (in this case there is no need of monotonicity as this can be established using standard arguments based on continuity (Futia (1982))), but total wealth will converge to a convex combination of the two stationary distributions, and there are many convex combinations. ■

Proof of Proposition 3

From equations (18) and (20) the conditions for no mobility (in case of low wages) are the following:

$$\underline{w} < \left[R_0 - (1 + r)\hat{k} - \underline{w} \right] \quad (\text{A.1})$$

Consider the case in which $G(\cdot) \rightarrow 1$. As $G(\cdot) \rightarrow 1$, then $r \rightarrow 0$. If parameters value are such that $[R_0 - \hat{k}] > 2\underline{w}$, choosing G in an ε -neighborhood (from below) of 1, condition (A.1) will still be satisfied, G will remain fixed over time (because of no-mobility) the wage will be fixed at \underline{w} and the interest rate will clear the capital market.

Finally one has to make sure that at $r = 0$ the threshold level of wealth is still positive, one can choose π to be sufficiently low such that this condition is satisfied.

More generally, still considering the low wage case, we have previously noticed that for $G > 0.5$ the interest rate is decreasing in r ; if, for a suitable choice of

parameters there is a G , let us call it G^* , such that equation (A.1) is satisfied with equality, then for any $G^* < G < 1$ inequality (A.1) will still be satisfied and one can construct an equilibrium like the one above in which there is no mobility between classes. ■

Appendix B

We now provide a formal example of a configuration of w and r like the one described in the text.

Let us identify with subscript 1 the interest rate, the threshold and the distribution associated with the wage \underline{w} and with subscript 2 those associated with the wage \bar{w} . Equilibrium in the capital market requires (from equation (8 and (9))

$$\int_0^{\hat{b}} bdG_1(\hat{b}_1) = [1 - G_1(\hat{b}_1)]\hat{k}$$

and

$$\int_0^{\hat{b}} bdG_2(\hat{b}_2) = [1 - G_2(\hat{b}_2)]\hat{k}$$

Subtracting the two equations we get

$$\int_0^{\hat{b}} bdG_2(\hat{b}_2) - \int_0^{\hat{b}} bdG_1(\hat{b}_1) = [G_1(\hat{b}_1) - G_2(\hat{b}_2)]\hat{k} \quad (\text{B.1})$$

Suppose that there is second order stochastic dominance between the two distributions, that is $G_1(\cdot)$ is a mean preserving spread of $G_2(\cdot)$ with the single crossing property. Let us denote by b^* the crossing point of the two distributions and let us assume that $\hat{b}_1, \hat{b}_2 \geq b^*$. Because of the assumption of mean preserving spread the right hand side of equation (B.1) is equal to 0; thus the only way for equation (B.1) to hold is that $G_1(\hat{b}_1) = G_2(\hat{b}_2)$. Above b^* , $G_2(\cdot) > G_1(\cdot)$, therefore it must be the case that $\hat{b}_1 > \hat{b}_2$ that, from equation (7) implies $r_1 > r_2$ ¹⁰

¹⁰Note that the likelihood of this particular case will be higher the closer is b^* to 0. This in turn would imply that inequality is concentrated among workers, that is there is a large difference between \underline{w} and \bar{w} as the intuition reported above suggested.

References

- Aghion, Philippe and Patrick Bolton**, “A Theory of Trickle-Down Growth and Development with Debt-Overhang,” *Review of Economic Studies*, 1997, *64*, 151–172.
- Banerjee, Abhijit and Andrew F. Newman**, “Occupational Choice and the Process of Development,” *Journal of Political Economy*, 1993, *101*, 274–298.
- and —, “Poverty, Incentives, and Development,” *American Economic Review Papers and Proceedings*, 1994, *84*, 211–215.
- and —, “Information, the Dual Economy, and Development,” *Review of Economic Studies*, 1998, *65* (4), 631–654.
- Bhattacharya, Jodeep**, “Credit Market Imperfections, Income Distribution and Capital Accumulation,” *Economic Theory*, 1998, *11* (1), 171–200.
- Futia, Carl A.**, “Invariant Distributions and the Limiting Behavior of Markovian Economic Models,” *Econometrica*, 1982, *50* (2), 377–408.
- Galor, Oded and Joseph Zeira**, “Income Distribution and Macroeconomics,” *Review of Economic Studies*, 1993, *60*, 35–52.
- Hopenhayn, Hugo A. and Edward C. Prescott**, “Stochastic Monotonicity and Stationary Distributions for Dynamic Economies,” *Econometrica*, 1992, *60* (6), 1387–1406.
- Lucas, Robert E.**, “On the Mechanics of Economic Development,” *Journal of Monetary Economics*, 1988, *22*, 3–42.
- Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny**, “Industrialization and the Big Push,” *Journal of Political Economy*, 1989, *97*, 1003–1026.
- Owen, Ann L. and David N. Weil**, “Intergenerational Earnings Mobility, Inequality and Growth,” *Journal of Monetary Economics*, 1998, *41*, 71–104.
- Piketty, Thomas**, “The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing,” *Review of Economic Studies*, 1997, *64*, 173–189.

Romer, Paul, “Increasing Returns and Long Run Growth,” *Journal of Political Economy*, 1986, *94*, 1002–1037.

Stokey, Nancy, Robert E. Lucas, Jr., and Edward C. Prescott, *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

World Bank, *World Development Report: Financial Systems and Development*, Oxford University Press, 1989.

—, *World Development Report: From Plan to Market*, Oxford University Press, 1996.