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Giorgio Motta, Patrizio Tirelli

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Money Targeting, Heterogeneous Agents and Dynamic Instability*

Giorgio Motta*) Patrizio Tirelli**)

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Abstract

Following a seminal contribution by Bilbiie (2008), the Limited Asset Market Participation hypothesis has triggered a debate on DSGE models determinacy when the central bank implements a standard Taylor rule. We reconsider the issue here in the context of an exogenous money supply rule, documenting the role of nominal and real frictions in determining these results. A general conclusion is that frictions matter for stability insofar as they redistribute income between Ricardian and non-Ricardian households when shocks hit the economy. Finally, we extend the model to allow for the possibility that consumers who do not participate to the market for interest-bearing securities hold money. In this case endogenous monetary transfers between the two groups allow to smooth consumption differences and the model is determinate provided that the non-negativity constraint on individual money holdings is satisfied.

Jel Classification E52;E58 **Keywords:** Rule of Thumb Consumers, DSGE, Determinacy, Limited Asset Market Participation, Money Targeting

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1 Introduction

New Keynesian business cycle models are typically characterized by optimizing agents (Ricardian agents henceforth), and by a number of nominal and real frictions in goods, labor and capital markets (Christiano et al. 2005, CEE henceforth). Following a seminal contribution by Mankiw (2000), who introduced the notion of heterogeneous consumers (savers and spenders), a second strand of New Keynesian (NK henceforth) literature emphasizes the role of agents that do not exploit financial markets to smooth consumption, but fully consume their current income (RT consumers henceforth). Galì et al (2004, 2007) showed that RT consumers can substantially affect both the stability of NK business cycle models and their dynamic adjustment to government spending shocks. Empirical research cannot reject the RT consumers hypothesis. Estimated structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989). More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain values around 35%. Erceg, Guerrieri and Gust (2006) calibrate the share of RT consumers to 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Bilbiie, Meier and Mueller (2007) argue that changes in asset market participation help explaining the change in transmission of fiscal policy shocks in the U.S. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets. In fact, increasing regulation in the aftermath of the 2008 crisis is likely to increase the share of liquidity-constrained households (OECD 2009).

The Limited Asset Market Participation hypothesis (LAMP henceforth) has triggered a debate on the stability of DSGE models when the central bank implements a standard Taylor rule. Bilbiie (2008, Bilbiie henceforth) showed that an interest rate policy based on the Taylor principle cannot ensure model determinacy in a simple model where price stickiness and LAMP are the only frictions, and the share of RT consumers is of a realistic size. The reason for this is that imperfect price adjustment to wage increases has an adverse wealth effect on Ricardian agents who participate to financial markets and therefore bear profit losses. The strength of such a wealth effect is increasing in the share of RT consumers and may invert the standard Keynesian logic: increases in the real interest rate may in fact be associated to a surge in production even if consumption of Ricardian agents falls. Inversion of the IS curve is a necessary prerequisite for indeterminacy in the Bilbiie model. This result has been challenged by the theoretical contributions in Colciago (2011) and Ascari et al.(2011), who have shown that a mild degree of wage stickiness is sufficient to restore the standard IS slope and the validity of the Taylor principle even for a very large share of RT consumers. More recently, Motta and Tirelli (2012) have shown that consumption habits, which are associated to a more rigid labor supply schedule, are sufficient to restore the Bilbiie result in spite of nominal wage rigidity.

Some empirical contributions confirm the importance of the inverted IS curve at least for certain historical periods. Using a single-equation, reduced-form GMM estimation, Bilbiie and Straub (2008) document that the sign of the IS curve's slope was negative in the 70s and turned positive in the early 80s. More recently, Bilbiie and Straub (2012) estimate a NK model using Bayesian techniques. They find that LAMP plays an important role in explaining U.S. macroeconomic performance and monetary policy before and after the 1980s, and document that the increase in the share of Ricardian consumers was a key factor behind the shift in the output-inflation volatility frontier that lies at the root of the great moderation period.

In this paper we analyze the issue of stability/determinacy under LAMP when the central bank implements an exogenous money supply rule, taking into account a number of extensions of the basic NK model. In the first step of our analysis we use a model which differs from Bilbiie only in the assumption that monetary policy is characterized by an exogenous money growth path. In this setup we are able to obtain analytical solutions that are in line with the original Bilbiie results. When the Calvo-pricing formalism is extended to include price indexation to past inflation the model turns from indeterminate to unstable if the indexation coefficient is sufficiently large. This is in contrast with the results obtained under Taylor rules, when indexation does not matter for determinacy. The intuition behind our finding is that, unlike the Taylor rule, the exogenous money supply rule pins down the steady-state price level to a unique steady-state path. In this case a sufficiently strong indexation mechanism, combined with an inverted IS curve, makes it impossible to obtain mean reversion of the price level.

The next step in our analysis is to introduce other frictions that characterize NK models (see for example CEE). A general conclusion is that frictions matter for stability insofar as they redistribute income between the two consumers groups through the basic mechanism highlighted in Bilbiie, i.e. real wage variations that are absorbed by profit margins when prices are sticky. Of particular interest here are nominal wage rigidity and consumption habits. We find that under sticky wages the short-run labour supply is more elastic and the income redistribution between the two household groups is substantially reduced so that the effect of LAMP on determinacy apparently becomes a lesser problem. The opposite results are obtained under consumption habits, since they are associated with a more rigid labour supply schedule. When sticky wages and consumption habits are jointly considered numerical simulations show that the model is indeterminate for a plausible range of values in the share of RT consumers.

Finally, we extend the model to allow for the possibility that RT consumers hold money. Previous work has focused on cashless models and in this context the LAMP hypothesis implies that RT consumers neither react to interest rate changes nor smooth consumption. Our framework allows for a less restrictive definition of the LAMP hypothesis, where agents do not participate to the market for interest-bearing securities but can smooth consumption by adjusting their money holdings. To the best of our knowledge, this is the first contribution in this field, with the notable exception of Choi (2011) who focuses on a pure exchange economy. We find that endogenous monetary transfers between the two household groups allow to limit the correlation between current income and consumption choices, preventing the inversion of the IS and ensuring determinacy even for a very large share of RT consumers. The importance of this result is obviously limited by the requirement that the non-negativity constraint of individual money holdings be satisfied.

The rest of the paper is organized as follows: In the next section we describe in detail the model structure, we then present the results concerning the model stability in section 3. Section 4 proposes the two extensions of the benchmark model, i.e. wage rigidity and habits in consumption. Section 5 concludes.

2 The Model

In this section we lay out the structure of the basic model, which is equivalent to the one in Bilbiie. The key distinction between Ricardian and RT consumers concerns intertemporal optimization. Ricardian consumers' choices take into account future utility when choosing consumption and portfolio composition. In contrast, RT consumers spend their whole income every period, thus they do not hold any wealth.

2.1 Households' preferences

We assume a continuum of households indexed by i, where $i \in [0,1]$. RT (rt) and Ricardian (o) consumers are defined over the intervals $[0,\theta]$ and $(\theta,1]$ respectively. All households share the same utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(c_t^i\right) - \frac{\psi_l}{1+\phi_l}(h_t^i)^{1+\phi_l} + \frac{\psi_q}{1-\sigma}(q_t^i)^{1-\sigma} \right\} \tag{1}$$

where $q_t^i = \frac{Q_t^i}{P_t}$ represents households real money balances, c_t^i represents total individual consumption, and h_t^i denotes individual labour supply.

2.1.1 Consumption Bundles

c_t^i is a standard consumption bundle

$$c_t^i = \left[\int_0^1 c\,(z)_t^{\frac{\eta-1}{\eta}}\, dj \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where η represents the price elasticity of demand for the individual goods. The aggregate consumption price index is

$$P_t = \left(\int_0^1 p\,(z)_t^{1-\eta}\, dj \right)^{\frac{1}{1-\eta}}.$$

2.2 Firms

Goods markets are monopolistically competitive, and good z is produced with the following technology:

$$y_t\,(z) = h_t\,(z)$$

The real marginal costs are:

$$mc_t = w_t\,(1-\rho) \tag{3}$$

where $w_t = \frac{W_t}{P_t}$ defines the real wage rate and ρ is a fiscal subsidy, entirely financed by lump-sum taxes on firms profits, as in Ascari et al. (2011).

2.2.1 Sticky Prices

Price stickiness is based on the Calvo mechanism. In each period firm z faces a probability $1-\lambda_p$ of being able to re-optimize its price. The re-optimized price, $\widetilde{P}_t$, is chosen to maximize the discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda^*_{t+s} \left(\widetilde{P}_t - P_{t+s} mc_{t+s}\right) y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = y^d_{t+s} \left(\frac{\widetilde{P}_t}{P_{t+s}}\right)^{-\eta} \tag{4}$$

where Y^d_{t+s} is aggregate demand and λ^*_{t+s} is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda^*_{t+s} y^d_{t+s} \left[\begin{array}{c} (1-\eta)\widetilde{P}_t^{-\eta}(P_{t+s})^{\eta} + \\ +\eta \widetilde{P}_t^{-\eta-1} P_{t+s}^{\eta+1} mc_{t+s} \end{array}\right] = 0 \tag{5}$$

2.3 Ricardian Households

Ricardian households maximize (1) subject to the following period budget constraint:

$$P_t c^o_t + Q^o_t + B_t = A_{j,t} + Q^o_{t-1} + R_{t-1}B_{t-1} + \tag{6}$$

$$+P_t d_t + h^o_t W_t \tag{7}$$

Where Q^o_t represents nominal cash balances, $A_{j,t}$ defines the net cash flow from participating in state-contingent securities. Optimizing households own firms and receive real dividends, d_t. Finally, B_t denotes a nominally riskless bond.

The Euler equation is

$$\lambda^o_t = \beta E_t \left(\lambda^o_{t+1} \frac{R_t}{\pi_{t+1}}\right) \tag{8}$$

where

$$\lambda_t^o = \frac{1}{c_t^o} \tag{9}$$

Ricardian households' money demand depends positively on current consumption and negatively on the current interest rate.

$$\psi_q (q_t^o)^{-\sigma} = \left(\frac{R_t - 1}{R_t} \right) \lambda_t^o \tag{10}$$

In the competitive labour market, the individual labour supply is

$$w_t = \psi_l (h_t^o)^{\phi_l} c_t^o \tag{11}$$

2.4 Rule-of-Thumb Households

RT households always consume their current income

$$c_t^{rt} = w_t h_t^{rt} \tag{12}$$

Given (1) and (12), they choose a constant labour supply

$$h_t^{rt} = (\psi_l)^{\frac{1}{1+\phi_l}} \tag{13}$$

2.5 Monetary Policy

The monetary rule is

$$M_t = M_{t-1} \exp \mu_t \tag{14}$$

with

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t$$

where and ε_t is an i.i.d. exogenous shock with zero mean and standard deviation σ_ε

2.6 Aggregation

The aggregate production function is

$$y_t = \int_0^1 h_t(z)\, dz = h_t$$

where

$$y_t = c_t$$

and

$$c_t = \int_0^1 c_t^i(j)\ dj = \int_0^\theta c_t^{rt}(j)\ dj + \int_\theta^1 c_t^o(j)\ dj = \theta c_t^{rt} + (1-\theta) c_t^o$$

In the money market

$$\frac{M_t}{P_t} = (1-\theta)\, q_t^o \tag{15}$$

2.7 The model in log-linear form

The deterministic, zero-inflation steady state is easily obtained. In order to limit the analytical complexity of the model, we posit that the production subsidy $\rho = \rho^*$ brings production at the competitive level, where $w = \left[\left(\frac{\eta}{\eta-1}\right)(1-\rho^*)\right]^{-1} = 1$. In the steady-state firms profits are nil because the subsidy is entirely financed by lump-sum taxes levied on firms. Both consumption and the marginal rate of substitution are therefore identical for the two consumer groups $\left(\frac{c^o}{c} = \frac{c^{rt}}{c} = \frac{h^o}{h} = \frac{h^{rt}}{h} = 1\right)$. The real interest rate is $R = \frac{1}{\beta}$. We choose parameter

ψ_l to obtain $c = h = 1$. We take a log-linear approximation around the steady state.[1]

2.7.1 Supply side

$$\hat{y}_t = \hat{h}_t \tag{16}$$

$$\hat{h}_t = (1-\theta)\hat{h}_t^o \tag{17}$$

$$\hat{w}_t = \phi_l \hat{h}_t^o + \hat{c}_t^o \tag{18}$$

$$\widehat{mc}_t = \hat{w}_t \tag{19}$$

$$\begin{aligned} \hat{\pi}_t &= \delta \hat{y}_t + \beta \hat{\pi}_{t+1}; && (20) \\ where \;\; \delta &= k\left(1+\phi_l\right) && (21) \\ k &= \frac{\left(1-\lambda_p\right)\left(1-\beta\lambda_p\right)}{\lambda_p} && (22) \end{aligned}$$

2.7.2 Demand side

$$\hat{c}_t^{rt} = \hat{w}_t \tag{23}$$

$$\hat{\lambda}_t^o = \hat{\lambda}_{t+1}^o + E_t\left(\hat{R}_t - \hat{\pi}_{t+1}\right) \tag{24}$$

$$\hat{\lambda}_t^o = -\hat{c}_t^o \tag{25}$$

$$\hat{y}_t = \hat{c}_t = (1-\theta)\hat{c}_t^o + \theta \hat{c}_t^{rt} \tag{26}$$

[1] Hatted variables denote the log-deviation of a variable from its zero-inflation, deterministic steady-state value.

2.7.3 Money market

$$\sigma \hat{q}_t^o = \hat{c}_t^o - \frac{1}{R-1}\hat{R}_t \tag{27}$$

$$\hat{m}_t = \hat{q}_t^o \tag{28}$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + \mu_t \tag{29}$$

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t \tag{30}$$

3 Stability analysis

Note that, due to price stickiness, firms profits are the inverse of real wage deviations from steady state.[2]

$$\hat{d}_t = -\widehat{mc}_t = -\hat{w}_t = -\left(1+\phi_l\right)\hat{y}_t \tag{31}$$

Using (16), (17), (18), (23) and (26) it is easy to see that in equilibrium each optimizing household must consume

$$\hat{c}_t^o = \hat{w}_t + \hat{h}_t^o + \frac{\hat{d}_t}{1-\theta} = \frac{\hat{y}_t}{1-\theta} - \frac{\theta\hat{w}_t}{1-\theta} = \chi^{FW}\hat{y}_t \tag{32}$$

where $\chi^{FW} = \left(1 - \frac{\theta\phi_l}{(1-\theta)}\right)$. When $\theta = 0$ (and $\chi^{FW} = 1$), an increase in output is associated with a fall in profits and with a real wage increase that exactly offset each other. In this case $\hat{c}_t^o = \hat{y}_t$. In contrast, when $\theta > 0$ the real wage increase is associated with a reduction in Ricardian households income. This happens because, due to the presence of RT consumers, asset holders bear individual profit losses which are larger than the increase in their wage income. The net size of this effect, $\left(-\frac{\theta\hat{w}_t}{1-\theta}\right)$ is determined by the share of RT consumers and by the

[2] Due to the efficient steady state assumption, profits are defined here as a fraction of steady state output.

inverse of the Frish elasticity, ϕ_l. For "large" values of θ, profit losses exceed the positive labor income variation determined by the increase in output. In this case $\chi^{FW} < 0$ and $\hat{c}_t^o$ is inversely related to $\hat{y}_t$.

This part of the model closely resembles Bilbiie (2008, Bilbiie henceforth), who showed that LAMP may invert the standard relation between output and the nominal interest rates. In fact, by substituting (32) into (24) we get the New Keynesian IS curve

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\chi^{FW}} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) \tag{33}$$

When the share of asset holders is not too large ($0 < \chi^{FW} < 1$), an increase in θ strengthens the aggregate demand elasticity to interest rate changes. In contrast, when θ exceeds a critical threshold $\theta^* = (1 + \phi_l)^{-1}$, the value of χ^{FW} turns negative. Hence, for a relatively large share of RT consumers the relationship between consumption of optimizing agents and total output is negative and the IS curve is inverted, i.e. a real interest rate increase *raises* aggregate demand.

Under the assumption of no inflation indexation, Bilbiie shows that this model is indeterminate when $\theta > \theta^*$ and the central bank implements a forward-looking Taylor rule $\left(\hat{R}_t = \phi^\pi \hat{\pi}_{t+1} \right)$. Under a contemporaneous Taylor rule $\left(\hat{R}_t = \phi^\pi \hat{\pi}_t \right)$ the critical value of θ that causes indeterminacy is larger than θ^*.

Substituting (26), (27), (29) and (33), into (28) we get the *implicit* interest rate rule that obtains under an exogenous money supply rule.

$$R_t = \left(\frac{R-1}{R} \right) E_t \left\{ \hat{\pi}_{t+1} + \chi^{FW} \hat{y}_{t+1} - \sigma \left[\hat{m}_{t-1} + \mu_t - \hat{\pi}_t \right] \right\} \tag{34}$$

The small size of coefficient $\frac{R-1}{R}$ implies that the interest rate response to current inflation and to next-period expected inflation and output gap is very weak if compared with standard Taylor rules. However, in contrast with Taylor rules, under the exogenous money growth rule future interest rates are constrained to react to inherited deviations of real money holdings from the steady state. In fact P in steady state is independent from past real shocks and only reacts to changes in M.[3] This is the key factor that pins down inflation expectations and ensures determinacy under full asset market participation, i.e. when $\theta = 0$, $\chi^{FW} = 1$.

Proposition 1 *The model is stable and uniquely determined if θ does not exceed a threshold θ^{**} such that*

$$\frac{\chi^{FW}}{\delta} > -\frac{R\left(\sigma\left(R-1\right)+2\right)}{\left(1+R^2\right)+2\left(1+\frac{1}{R}\right)}. \tag{35}$$

Below this threshold the model is indeterminate.

Proof. See Appendix B ■

Our determinacy analysis implies that only for a certain range of θ values can Ricardian consumers choose a initial response to the shock which is consistent with a uniquely determined convergence path. Parameter χ^{FW} identifies the constant wedge between $\hat{c}_t^o$ and $\hat{y}_t$, which falls in θ and is independent from the consumption decision of Ricardian consumers. Given that $R > 1$, inversion of (33) is necessary but not sufficient to violate condition (35).

The economic meaning of the ratio $\frac{\chi^{FW}}{\delta}$ is easily interpretable: in fact from (20), and (33) we know that $-\frac{\delta}{\chi^{FW}}$ defines the current inflation response to a change in current output which, in turn, is determined by a real interest rate variation. Condition (35) shows that dynamic stability cannot obtain if an increase in the interest rate raises output (IS inversion), and if the ensuing positive inflation response is too weak.

Under the benchmark calibration the model is indetermined beyond the threshold identified in (35). Numerical simulations[4] show that the critical value for the share of RT consumers is $\theta > 0.31$, larger than the

[3] By contrast, under a Taylor rule temporary real shocks do affect the steady state price level.

[4] Parameter values are reported in table 1

value of θ which causes the inversion of the IS curve, $\theta^* = 0.25$.

4 Extensions

4.1 The role of price indexation

Following CEE, firms that cannot re-optimize adjust their price to previous-period inflation:

$$p_t(z) = (1 + \pi_{t-1})^{\gamma_p} p_{t-1}(z) \tag{36}$$

where $\gamma_p \in [0, 1]$ represents the degree of price indexation. With price indexation equation (20) becomes

$$\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) = \delta \hat{y}_t + \beta \left(\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t\right) \tag{37}$$

Proposition 2 *Under an exogenous money growth rule the model is stable and uniquely determined if θ does not exceed a threshold θ^{**} such that*

$$\frac{\chi^{FW}}{\delta} > -\frac{R\left(\sigma\left(R-1\right)+2\right)}{2\left(1+R^2\right)\left(1+\gamma_p\right)}, \tag{38}$$

Proof. See appendix B ■

Numerical simulations show that if $\frac{\chi^{FW}}{\delta} < -\frac{R(\sigma(R-1)+2)}{2(1+R^2)(1+\gamma_p)}$ the model is indeterminate when γ_p is relatively small, whereas the model becomes unstable for larger values of γ_p (see figure 1) In appendix B we provide analytical results about instability when $\gamma_p = 1$.

To understand this result one should bear in mind that: 1) unlike inflation targeting regimes, the exogenous money supply rule pins down the steady state price level to a unique value, independent of past non monetary shocks. This implies that any accumulated output (and profit) gaps must be subsequently reversed. 2) A necessary condition for violation of (38) is that $\chi^{FW} < 0$ and the IS curve is inverted, i.e. a nominal interest rate increase is associated to a positive output gap. In the no-indexation case, when (35) is violated a self-fulfilling conjecture of a positive output and inflation gap in t is possible and is always consistent with the rational expectation of future output gaps and inflation values that ensure convergence to steady state. Observe that without indexation, the state of the economy in $t+s$ is independent from past realizations. With indexation, by contrast, a positive output and inflation gap in t impacts on inflation in $t+1$ through the indexation mechanism. This, in turn, has a positive effects on $\hat{R}_{t+1}$ and, because of $\chi^{FW} < 0$, also on $\hat{y}_{t+1}$. If the indexation coefficient is sufficiently large the model turns from indeterminate to unstable. Note that the instability result under indexation is specific to the money growth rule and would not apply to the case of a standard Taylor rule.[5]

4.2 Sticky wages and consumption habits

Here we modify our benchmark model to account for consumption habits and for wage stickiness.[6]

Condition (1) now becomes

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(c_t^i - bc_{t-1}^i\right) - \frac{\psi_l}{1+\phi_l}(h_t^i)^{1+\phi_l} + \frac{\psi_q}{1-\sigma}(q_t^i)^{1-\sigma} \right\} \tag{39}$$

where b defines internal habits, and h_t^i represents a labor bundle. For each household type, the marginal utility of consumption is defined as

$$\lambda_t^i = \frac{1}{c_t^i - bc_{t-1}^i} - \frac{\beta b}{c_{t+1}^i - bc_t^i}$$

[5] Proof available upon request. The key mechanism at work in our model is that temporary gaps in real variables cannot have permanent effects on the price level.

[6] On the working paper version of this article (http://ideas.repec.org/p/mib/wpaper/193.html) we study the determinacy properties of a model which, as in CEE, incorporates endogenous capital accumulation, investment adjustment costs, variable capacity utilization and a cash in advance constraint on firms. These additional features do not play a major role in ensuring the dynamic determinacy of the model with LAMP.

The composite labor input used by each firm is

$$h_t(z) = \left(\int_0^1 \left(h_t^j(z) \right)^{\frac{\alpha_w - 1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_{w-1}}} \tag{40}$$

where $\alpha_w > 1$ is the intratemporal elasticity of substitution between labor inputs. For each labor input there is a monopolistic wage-setting union. Each household i supplies all labour types at the given wage rate As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time. The full characterization of wage-setting behavior is presented in the Appendix. In addition to equations (16), (19), (20), (24), (26), (27), (28), (29), (30) the model in log-linear form includes the condition for real wage dynamics

$$\left[\begin{array}{c} \hat{w}_t - \frac{\beta\lambda_w}{\left(1+\beta\lambda_w^2+\lambda_w k\right)} \hat{w}_{t+1} + \\ - \frac{\lambda_w}{\left(1+\beta\lambda_w^2+\lambda_w k\right)} \hat{w}_{t-1} \end{array} \right] = (1-\lambda_w) \left[\begin{array}{c} \frac{(1-\beta\lambda_w)}{\left(1+\beta\lambda_w^2+\lambda_w k\right)} \left(\phi_l + \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} \right) \hat{h}_t + \\ - \frac{(1-\beta\lambda_w)}{\left(1+\beta\lambda_w^2+\lambda_w k\right)} \left(\frac{\beta b \hat{h}_{t+1}}{(1-\beta b)(1-b)} + \frac{b \hat{h}_{t-1}}{(1-\beta b)(1-b)} \right) \end{array} \right],$$

the identical labour supply condition for the two groups

$$\hat{h}_t^o = \hat{h}_t^{rt} = \hat{h}_t, \tag{41}$$

the new period budget constraint for RT consumers

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t, \tag{42}$$

and the marginal utility of consumption under the habits assumption

$$\hat{\lambda}_t^i = \frac{\beta b}{(1-\beta b)(1-b)} E_t \hat{c}_{t+1}^i - \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} \hat{c}_t^i + \frac{b}{(1-\beta b)(1-b)} \hat{c}_{t-1}^i.$$

Condition (32) now reads as follows,

$$\hat{c}_t^o = \chi^{SW} \hat{y}_t - \frac{\theta}{(1-\theta)\left(1+\beta\lambda_w^2+\lambda_w k\right)} \left(\beta\lambda_w \hat{w}_{t+1} + \lambda_w \hat{w}_{t-1} - \frac{b(1-\beta\lambda_w)}{(1-\beta b)(1-b)} (1-\lambda_w) \left(\beta \hat{h}_{t+1} + \hat{h}_{t-1}\right)\right)$$

where $\chi^{SW} = 1 - \frac{\theta}{1-\theta} \frac{(1-\lambda_w)(1-\beta\lambda_w)}{\left(1+\beta\lambda_w^2+\lambda_w k\right)} \left(\phi_l + \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}\right)$.

Note that under flexible wages and no habits, i.e. $\lambda_w = b = 0$, $\chi^{SW} = \chi^{FW}$ and we replicate the results obtained in section (3). Under sticky wages and no habits, $b = 0$, the relationship between $\hat{c}_t^o$ and $\hat{y}_t$ is equivalent to Ascari et al. (2011). in this case $\chi^{SW} < \chi^{FW}$ for any given value of θ. Even in case of a limited degree of wage stickiness[7] ($\lambda_w = 0.5$, that is, the wage contract is renegotiated every two quarters) numerical simulations show that the model is now stable and uniquely determined for $\theta < 0.72$. The intuition behind this result is very simple: wage stickiness dampens the income redistribution effect associated with output variations and limits the possibility that RT consumption choices cause the inversion of the IS curve. To the contrary, the inclusion of habits unambiguously increases the real wage sensitivity to changes in output, potentially restoring the inverted IS curve. Numerical simulations confirm this result. Imposing a conservative value for the degree of habit formation, i.e. $b = 0.7$, the model is uniquely determinate for $\theta < 0.22$ if wages are reset every two quarters and $\theta < 44$ if wages are reset every three quarters.

The Bilbiie original intuition is therefore confirmed: the strength of the wealth redistribution between consumer groups is the key to understand the stability problems that arise in DSGE models under LAMP.

[7] Christiano et al. (2005) and Smets and Wouters (2007) respectively report λ_w estimates at 0.64 and 0.7.

4.3 RT consumers hold money

The period budget constraint (12) now allows for the RT consumers' choice of holding money. An additional constraint is determined by the impossibility that money holdings of RT consumers become negative. We therefore restrict our analysis to the case where, for a given amount of RT money holdings in steady state, the size of shocks does not require that $Q_t^{rt} < 0$.

$$P_t c_t^{rt} + Q_t^{rt} = W_t h_t^{rt} + Q_{t-1}^{rt}$$

In log-linear form the labor supply of RT consumers satisfies:

$$\hat{w}_t = \phi_l \hat{h}_t^{rt} - \hat{\lambda}_t^{rt} \tag{43}$$

where

$$\hat{\lambda}_t^{rt} = -\hat{c}_t^{rt} \tag{44}$$

defines the marginal utility of consumption. Observe that RT labor supply now is endogenous to business cycle conditions. As a result the total labor supply is

$$\hat{h}_t = (1-\theta)\hat{h}_t^o + \theta \hat{h}_t^{rt} \tag{45}$$

The first order condition for RT consumers' money holdings is: $\psi_q (q_t^{rt})^{-\sigma} = \frac{1}{c_t^{rt}} - \beta E_t \left(\frac{1}{c_{t+1}^{rt} \pi_{t+1}} \right)$. It is interesting to note that RT consumers now adjust their desired real money holdings in order to smooth consumption across periods. For instance they react to an increase in expected consumption by lowering their current money holdings in order to raise their current consumption. Similarly, an increase in expected future inflation is met with a reduction in q_t^{rt} because future inflation reduces next-period consumption value of current money holdings. In log-linear form the RT money demand function is:

$$\sigma_m \hat{q}_t^{rt} = \frac{R}{R-1}\left(\hat{c}_t^{rt} - \beta E_t\left(\hat{c}_{t+1}^{rt} + \hat{\pi}_{t+1}\right)\right) \tag{46}$$

As for the definition of RT consumption, condition (23) becomes[8]

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t^{rt} - q\left(\hat{q}_t^{rt} - \hat{q}_{t-1}^{rt}\right) \tag{47}$$

The money market equilibrium condition (28) changes into

$$\hat{m}_t = (1-\theta)\hat{q}_t^{o} + \theta\hat{q}_t^{rt} \tag{48}$$

The loglinearized model now is defined by (16), (18), (19), (20), (24), (25), (26), (27), (29), (30), (43), (44), (45), (46), (47), (48).

The reduced-form dynamic system is too complex to obtain analytical results about determinacy. According to our simulations the system is stable and uniquely determined even for $\theta = 0.99$. To support intuition, note that from equations (46), (47) we obtain

$$\hat{c}_t^{rt} = \frac{\sigma\left(R-1\right)\left(\hat{w}_t + \hat{h}_t^{rt} + q\hat{q}_{t-1}^{rt}\right) + q\left(\hat{c}_{t+1}^{rt} + \hat{\pi}_{t+1}\right)}{\sigma\left(R-1\right) + qR} \tag{49}$$

[8] Straightforward manipulations also show that in the efficient steady state case $q^{rt} = q^{o} = q = \left(\frac{\psi_q}{1-\beta}\right)^{\frac{1}{\sigma_q}}$ is the index of money velocity.

RT consumption is less sensitive to current income and reacts to expectations about the future. Further, bear in mind that in equilibrium even the consumption of Ricardian households must satisfy the period budget constraint

$$\hat{c}_t^o = -\frac{\theta \hat{w}_t}{1-\theta} + \hat{h}_t^o - q\left(\hat{q}_t^o - \hat{q}_{t-1}^o\right)$$

Now, consider what happens if the real wage increases. In this case there is an income redistribution between the two groups, but there is an incentive for both groups to trade money in order to stabilize consumption: money holdings of Ricardians will fall and vice versa. In figure 2 we plot IRFS to a money supply shock when RT consumers hold (do not) money and $\theta = 0.26$, a value that would cause an inversion of the IS in model discussed in section 3. It is easy to see that redistribution of financial wealth allows both groups to smooth consumption. In fact Ricardian consumers now increase their consumption and decumulate their money holdings. RT consumers instead limit the increase in their current consumption and accumulate money holdings that will finance a consumption increase in the subsequent periods.

5 Conclusion

We have investigated the role of limited asset market participation in a New-Keynesian model under an exogenous money supply rule. The basic version of the model confirms the results obtained in Bilbiie under a Taylor rule, i.e. a sufficiently large share of RT consumers causes inversion of the IS curve and indeterminacy. A non-trivial difference arises when we consider inflation indexation in the price-setting mechanism. In this case the persistence of inflation generates instability. A key contribution of the paper is that RT consumers matter when a combination of shocks and frictions acts to redistribute wealth between the two consumer groups. In this regard, wage stickiness and consumption habits generate opposing effects.

We have also found that the model is always determinate if RT consumers can exploit money holdings as a consumption-smoothing device. A key requirement to obtain this result is that optimal decisions of RT consumers do not violate the non-negativity constraint on money holdings. Future research should investigate the role of monetary policy in facilitating consumption smoothing of RT consumers.

Notes

[9]Note that $|X| \gtrless 1 \iff x \gtrless 0$

[10]See Samuelson (1941) and more recently, Felippa and Park (2004)- section 4 page 18, Ascari et al. (2011) and Rossi (2011).

[11]Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

[12]It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

References

[1] Ascari Guido, Andrea Colciago and Lorenza Rossi, 2011 "Limited Asset Market Participation: Does it Really Matter for Monetary Policy" Bank of Finland Research Discussion Papers 15 - 2011.

[2] Bilbiie, Florin O., 2008. "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, Elsevier, vol. 140(1), pages 162-196, May.

[3] Bilbiie, Florin O, André Meier and Gernot J. Müller, 2008. "What Accounts for the Changes in U.S. Fiscal Policy Transmission?," *Journal of Money, Credit and Banking*, Blackwell Publishing, vol. 40(7), pages 1439-1470, October.

[4] Bilbiie, Florin O, and Roland Straub, 2013. "Asset market participation, monetary policy rules and the great inflation," *The Review of Economics and Statistics*, May 2013, Vol. 95, No. 2, Pages 377-392

[5] Bils, Mark and Peter J. Klenow, 2004. "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, University of Chicago Press, vol. 112(5), pages 947-985, October.

[6] Campbell, John Y. and Gregory N. Mankiw, 1989. "Consumption, Income, and Interest Rates: Reinterpreting the Time-Series Evidence," NBER Macroeconomics Annual.

[7] Choi, Hyung S. 2011. "Monetary Policy And Endowment Risk In A Limited Participation Model," *Economic Inquiry*, vol. 49(1), pages 89-93, 01

[8] Christiano, Lawrence J, Martin Eichenbaum and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, University of Chicago Press, vol. 113(1), pages 1-45, February.

[9] Coenen, Günter & Roland Straub, 2005. "Does Government Spending Crowd in Private Consumption?

Theory and Empirical Evidence for the Euro Area," *International Finance*, Blackwell Publishing, vol. 8(3), pages 435-470, December.

[10] Colciago, Andrea, 2011. "Rule of Thumb Consumers Meet Sticky Wages," *Journal of Money, Credit and Banking* Vol.43 (2), 325-353

[11] Erceg, Christopher J., Luca Guerrieri and Christopher Gust, 2006. "SIGMA: A New Open Economy Model for Policy Analysis," *International Journal of Central Banking*, International Journal of Central Banking, vol. 2(1), March.

[12] Felippa, Carlos. A. and K.C. Perk, 2004. "Synthesis Tools for Structural Dynamics and Partitioned Analysis of Coupled Systems," Aerospace Engineering Sciences and Center for Aerospace Structures.

[13] Forni, Lorenzo, Libero Monteforte and Luca Sessa, 2009. "The general equilibrium effects of fiscal policy: Estimates for the Euro area," *Journal of Public Economics*, Elsevier, vol. 93(3-4), pages 559-585, April.

[14] Galí, Jordi, J. David López-Salido and Javier Vallés, 2004. "Rule-of-Thumb Consumers and the Design of Interest Rate Rules," NBER Working Papers 10392, National Bureau of Economic Research, Inc.

[15] Galí, Jordi, J. David López-Salido and Javier Vallés, 2007. "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, MIT Press, vol. 5(1), pages 227-270, 03.

[16] Jacoviello, Matteo, 2004. "Consumption, house prices, and collateral constraints: a structural econometric analysis," *Journal of Housing Economics*, Elsevier, vol. 13(4), pages 304-320, December.

[17] Mankiw, N. Gregory, 2000. "The Savers-Spenders Theory of Fiscal Policy," *American Economic Review*, American Economic Association, vol. 90(2), pages 120-125, May

[18] Motta, Giorgio and Patrizio Tirelli, 2012 "Optimal Simple Monetary and Fiscal Rules under Limited Asset

Market Participation". *Journal of Money, Credit and Banking*, Vol. 44, No. 7

[19] OECD 2009 "The Financial Crisis. Reform and Exit Strategies"
http://www.oecd.org/document/20/0,3343,en_2649_34813_43726868_1_1_1_37467,00.html

[20] Rossi, Raffaele, 2011. "Designing Monetary and Fiscal Policy Rules in a New Keynesian Model with Rule-of-Thumb Consumers". *Macroeconomic Dynamics*, available on CJO2012. doi:10.1017/S1365100512000430.

[21] Smets, Frank and Raf Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, MIT Press, vol. 1(5), pages 1123-1175, 09.

A Determinacy Analysis

A.1 Proof of Proposition 1:

The reduced form dynamic system

$$\begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \\ \hat{m}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\beta} & -\frac{1}{\beta}\delta & 0 \\ \frac{R}{\chi}\left(\sigma\left(\frac{R-1}{R}\right)-1\right) & R\left(1+\frac{\delta}{\chi^{FW}}\right) & -\frac{\sigma}{\chi}\left(R-1\right) \\ -1 & 0 & 1 \end{bmatrix}}_{T} \begin{bmatrix} \hat{\pi}_t \\ \hat{y}_t \\ \hat{m}_{t-1} \end{bmatrix}$$

now has two jump variables ($\hat{\pi}_t$ and $\hat{y}_t$) and one state variable ($\hat{m}_{t-1}$). The Characteristic polynomial is

$$P_T(X) = X^3 + a_1 X^2 + a_2 X + a_3 = 0$$

The stability properties of the system depend on the location of the roots inside the unit circle in the complex plane, i.e. $|X_i| < 1$, which may be very difficult to identify. By adopting the conformal involuntary transformation

$$X = \left(\frac{1+x}{1-x}\right),$$

it is in general possible to turn $P_T(X)$ into a Hurwitz polynomial[9] $P_H(X)$, whose stability properties depend on the location of the roots in the left hand plane $\mathcal{R}(X) < 0$:[10]

$$P_H(x) = \left(\frac{1+x}{1-x}\right)^3 + a_2\left(\frac{1+x}{1-x}\right)^2 + a_1\left(\frac{1+x}{1-x}\right) + a_0$$

[9] Note that $|X| \gtrless 1 \iff x \gtrless 0$

[10] See Samuelson (1941) and more recently, Felippa and Park (2004)- section 4 page 18, Ascari et al. (2011) and Rossi (2011).

which can be rewritten as

$$\begin{aligned} P_H(X) &= x^3 + \underbrace{\left(\frac{(3a_0 - a_1 - a_2 + 3)}{(a_1 - a_0 - a_2 + 1)}\right)}_{d_2} x^2 + \\ &\quad + \underbrace{\left(\frac{(a_2 - a_1 - 3a_0 + 3)}{(a_1 - a_0 - a_2 + 1)}\right)}_{d_1} x + \\ &\quad + \underbrace{\frac{(a_0 + a_1 + a_2 + 1)}{(a_1 - a_0 - a_2 + 1)}}_{d_0} = 0 \end{aligned}$$

where

$$a_0 = -\det(T) = -R^2$$

$$a_1 = sum\ of\ leading\ minors\ (T) = \left(\frac{R\,(\sigma\,(R-1)+1)\,\delta}{\chi^{FW}} + (R+2)\,R\right).$$

$$a_2 = -Tr(T) = -\left(R + R\left(\frac{\delta}{\chi^{FW}} + 1\right) + 1\right)$$

Simplifying and rearranging $P_H(X)$ we obtain

$$\begin{aligned} d_0 &= \frac{(\sigma\,(R-1))}{\frac{1}{R}\frac{\chi^{FW}}{\delta}\left(2\,(1+R) + \left(1+\frac{2}{R}\right) + R^2\right) + (\sigma\,(R-1)+2)} = \\ &= -x_1 x_2 x_3 \\ d_1 &= \frac{2\,(1-R)^2\,\frac{1}{R}\frac{\chi^{FW}}{\delta} - (\sigma\,(R-1)+2)}{\left(\frac{1}{R}\right)\frac{\chi^{FW}}{\delta}\left(2\,(1+R) + \left(1+\frac{2}{R}\right) + R^2\right) + (\sigma\,(R-1)+2)} = \\ &= x_1 x_2 + x_1 x_3 + x_2 x_3 \\ d_2 &= \frac{4\left(1-R^2\right)\left(\frac{1}{R}\right)\frac{\chi^{FW}}{\delta} - (\sigma\,(R-1))}{\left(\frac{1}{R}\right)\frac{\chi^{FW}}{\delta}\left(2\,(1+R) + \left(1+\frac{2}{R}\right) + R^2\right) + (\sigma\,(R-1)+2)\,\frac{\chi^{FW}}{\delta}} = \\ &= -\left(x_1 + x_2 + x_3\right) \end{aligned}$$

where x_i, $i = 1, 3$ are the roots of $P_H(x)$.

The necessary condition for model's stability is:

$$d_0 > 0 \;\Leftrightarrow\; \frac{\chi^{FW}}{\delta} > -\frac{R\left(\sigma\left(R-1\right)+2\right)}{\left(1+R^2\right)+2\left(1+\frac{1}{R}\right)} \tag{50}$$

Given (50), stability obtains if either d_1 or d_2 or both are negative.

Since the numerator of d_1 is positive if

$$\frac{\chi^{FW}}{\delta} > \frac{\left(\sigma\left(R-1\right)+2\right)R}{2\left(R-1\right)^2}$$

it follows that $d_1 < 0$ when $d_0 > 0$ and $\frac{\chi^{FW}}{\delta} < \frac{(\sigma(1-\beta)+2\beta)}{2(1-\beta)^2}$. Determinacy obtains as long as

$$-\frac{\left(\sigma\left(R-1\right)+2\right)R}{\left(2\left(1+R\right)+\left(1+\frac{2}{R}\right)+R^2\right)} < \frac{\chi^{FW}}{\delta} < \frac{\left(\sigma\left(R-1\right)+2\right)R}{2\left(R-1\right)^2}$$

But since $\chi^{FW} = \left(1 - \frac{\theta\phi_l}{(1-\theta)(1-\alpha)}\right)$ satisfies $\frac{\chi^{FW}}{\delta} < \frac{(\sigma(R-1)+2)R}{2(R-1)^2}$ $\nabla\ \theta \in [0,1]$, then

$$\frac{\chi^{FW}}{\delta} > -\frac{R\left(\sigma\left(R-1\right)+2\right)}{\left(1+R^2\right)+2\left(1+\frac{1}{R}\right)}$$

is the necessary and sufficient condition for determinacy.

Consider now the case where the model is not determinate, i.e. $d_0 < 0$ because

$$\frac{\chi^{FW}}{\delta} < -\frac{\left(\sigma\left(R-1\right)+2\right)R}{\left(2\left(1+R\right)+\left(1+\frac{2}{R}\right)+R^2\right)}.$$

We can rule out 3 unstable roots if either $d_1 < 0$ or $d_2 > 0$. Term d_1 is positive if

$$\frac{\chi^{FW}}{\delta} > \frac{(\sigma(R-1)+2)R}{2(R-1)^2}.$$

This is ruled out because $d_0 < 0$

The term d_2 is positive for

$$\frac{\chi^{FW}}{\delta} < -\frac{\sigma}{4\left(1+\frac{1}{R}\right)}.$$

Studying the space

$$\frac{(\sigma(R-1)+2)R}{\left(2(1+R)+\left(1+\frac{2}{R}\right)+R^2\right)} - \frac{\sigma}{4\left(1+\frac{1}{R}\right)} = 0$$

as a function of σ we find that it is positive for $\sigma > 2$ implying that when $d_0 < 0$ indeterminacy obtains for a large range of parameters.

A.2 Proof of proposition 2

We consider the reduced form of the model

$$\begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \\ \hat{m}_t \\ \hat{\pi}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\left(\beta\gamma_p+1\right)}{\beta} & -\frac{\delta}{\beta} & 0 & -\frac{\gamma_p}{\beta} \\ \frac{\left(\sigma(1-\beta)-\beta\gamma_p-1\right)}{\beta\chi} & \frac{\left(1+\frac{\delta}{\chi^{FW}}\right)}{\beta} & -\frac{\sigma(1-\beta)}{\beta\chi} & \frac{\gamma_p}{\beta\chi} \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{T} \begin{bmatrix} \hat{\pi}_t \\ \hat{y}_t \\ \hat{m}_{t-1} \\ \hat{\pi}_{t-1} \end{bmatrix}$$

The system is characterized by two jump variables ($\hat{\pi}_t$ and $\hat{y}_t$) and two state variables ($\hat{\pi}_t$ and $\hat{m}_{t-1}$).

The Characteristic polynomial is

$$
\begin{aligned}
P_T(X) &= X^4 + a_1 X^3 + a_2 X^2 + a_3 X + a_4 = \\
&= X^4 + \underbrace{\left(-\frac{\left(\frac{\delta}{\chi^{FW}} + 1\right)}{\beta} - \frac{\left(\beta\gamma_p + 1\right)}{\beta} - 1 \right)}_{a_1\text{=-Trace of T}} X^3 + \\
&\quad + \underbrace{\left(\frac{\left(1 + (\sigma + \beta - \sigma\beta)\frac{\delta}{\chi^{FW}} + 2\beta + \beta^2\gamma_p + 2\beta\gamma_p\right)}{\beta^2} \right)}_{a_2\text{= sum of the principal second-order minors of T}} X^2 + \\
&\quad + \underbrace{\left(\frac{-\left(\gamma_p + 2\beta\gamma_p + 1\right)}{\beta^2} \right)}_{a_3\text{= }-\text{(sum of the principal third-order minors) of T}} X + \\
&\quad + \underbrace{\left(\frac{\gamma_p}{\beta^2} \right)}_{a_4\text{= Determinant of T}} .
\end{aligned}
$$

We transform $P_T(X)$ into a Hurwitz polynomial:

$$
P_H(x) = \left(\frac{1+x}{1-x}\right)^4 + a_1\left(\frac{1+x}{1-x}\right)^3 + a_2\left(\frac{1+x}{1-x}\right)^2 + a_3\left(\frac{1+x}{1-x}\right) + a_4
$$

which can be rewritten as

$$
\begin{aligned}
P_H(X) &= x^4 + \underbrace{\left(\frac{2\left(a_3 - a_1 - 2a_4 + 2\right)}{\left(a_2 - a_1 - a_3 + a_4 + 1\right)}\right)}_{b_1} x^3 + \\
&\quad + \underbrace{\left(\frac{2\left(3a_4 - a_2 + 3\right)}{\left(a_2 - a_1 - a_3 + a_4 + 1\right)}\right)}_{b_2} x^2 + \\
&\quad + \underbrace{\left(\frac{2\left(a_1 - a_3 - 2a_4 + 2\right)}{\left(a_2 - a_1 - a_3 + a_4 + 1\right)}\right)}_{b3} x + \\
+ &\quad \underbrace{\left(\frac{a_1 + a_2 + a_3 + a_4 + 1}{\left(a_2 - a_1 - a_3 + a_4 + 1\right)}\right)}_{b_4} = 0
\end{aligned}
$$

Simplifying and taking into account that $\frac{1}{\beta} = R$:

$$
\begin{aligned}
b_1 &= \frac{2\left(3 + R\frac{\delta}{\chi^{FW}} + 2R + \gamma_p - R^2 - 2R\gamma_p - 3R^2\gamma_p\right)}{\frac{\delta}{\chi^{FW}}\left(2R + \frac{\chi^{FW}}{\delta}\left(4R + 2\gamma_p + 4R\gamma_p + 2R^2\gamma_p + 2R^2 + 2\right) - R\sigma + R^2\sigma\right)} = \\
&= -(x_1 + x_2 + x_3 + x_4)
\end{aligned}
$$

$$
\begin{aligned}
b_2 &= \frac{-2\left(\left(1 + \sigma\frac{\delta}{\chi^{FW}} - 3\gamma\right)R^2 + \left(\frac{\delta}{\chi^{FW}} + 2 - \sigma\frac{\delta}{\chi^{FW}} + 2\gamma\right)R - 3 + \gamma\right)}{\frac{\delta}{\chi^{FW}}\left(2R + \frac{\chi^{FW}}{\delta}\left(4R + 2\gamma_p + 4R\gamma_p + 2R^2\gamma_p + 2R^2 + 2\right) - R\sigma + R^2\sigma\right)} = \\
&= x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4
\end{aligned}
$$

$$
\begin{aligned}
b_3 &= \frac{2\left(\left(1 - \gamma\right)R^2 + \left(2\gamma - 2 - \frac{\delta}{\chi^{FW}}\right)R + 1 - \gamma\right)}{\frac{\delta}{\chi^{FW}}\left(2R + \frac{\chi^{FW}}{\delta}\left(4R + 2\gamma_p + 4R\gamma_p + 2R^2\gamma_p + 2R^2 + 2\right) - R\sigma + R^2\sigma\right)} = \\
&= -\left(x_1x_2x_3 + x_1x_2x_4 + x_2x_3x_4 + x_1x_3x_4\right)
\end{aligned}
$$

$$
\begin{aligned}
b_4 &= \frac{R\sigma(R-1)}{\left(2R+\frac{\chi^{FW}}{\delta}\left(4R+2\gamma_p+4R\gamma_p+2R^2\gamma_p+2R^2+2\right)+R\sigma(R-1)\right)} = \\
&= x_1x_2x_3x_4
\end{aligned}
$$

where x_i, $i=1,4$ are the roots of $P_H(x)$

Focus on b_4,. We want it to be positive since we need two positive and two negative roots according to the Blanchard-Kahn conditions (Blanchard, Kahn, 1980). This requires

$$
\frac{\chi^{FW}}{\delta} > \frac{-R(2+\sigma(R-1))}{\left(4R+2\gamma_p+4R\gamma_p+2R^2\gamma_p+2R^2+2\right)}
$$

When $\gamma_p = 1$, it turns out that $b_4 > 0$ if condition (**??**) in the text holds, and it is straightforward to demonstrate that when $b_4 > 0$ and $\frac{\chi^{FW}}{\delta} < 0$ then $b_1 < 0$ $b_2 < 0$ and $b_3 < 0$. In this case the characteristic polynomial (**??**) has two changes of sign and (by Descartes rule) two positive roots.

When $b_4 > 0$ and $\frac{\chi^{FW}}{\delta} > 0$ stability obtains if $b_1 < 0$.This in turns requires $\frac{\delta}{\chi^{FW}} < 4(\frac{(R-1)(1+R)}{R})$. Note that with our calibration $4(\frac{(1-R)(1+R)}{R}) = 0.079$. The maximum value of χ^{FW} is 1. Therefore obtaining $\frac{\delta}{\chi^{FW}} < 4(\frac{(R-1)(1+R)}{R})$ would require $\lambda_p > 0.873$ if $\phi = 3$, and $\lambda_p > 0.778$ if $\phi = 0.2$.These appear rather large degrees of price stickiness (See CEE and Bils and Klenov, 2004) . If we can rule out $\frac{\delta}{\chi^{FW}} < 4(\frac{(R-1)(1+R)}{R})$ when $\chi^{FW} > 0$, then by Descartes rule, the characteristic polynomial is characterized by only two changes of sign, which are associated to two positive roots.

Note that when $b_4 < 0$ (i.e. $\chi^{FW} < 0$) the model is unstable. In this case $b_3 > 0$, $b_2 > 0$,$b_1 < 0$. and by Descartes rule we can have three changes of sign, that is, three positive roots and model's instability.

B Sticky wages

For any given level of firm labor demand $h_t(z)$, the optimal allocation across labor inputs implies

$$h_t^j(z) = \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d(z) \tag{51}$$

where $W_t = \left(\int_0^1 \left(W_t^j\right)^{1-\alpha_w} dj\right)^{1/(1-\alpha_w)}$ is the standard nominal wage index. For each labor input there is a union j which monopolistically supplies the labor input j in the labor market j.

Each union sets the nominal wage, W_t^j, subject to (51). Each household i supplies all labour types at the given wage rate[11] and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{52}$$

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time, h_t. Individual nominal labor income is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{53}$$

In each period a union faces a constant probability $1 - \lambda_w$ of being able to reoptimize the nominal wage. Following Colciago(2008), the representative union objective function is defined as

[11] Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

$$L^u = \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left\{ \left[(1-\theta) \, U^o(C^o_{t+s}) + \theta U^{rt}(C^{rt}_{t+s}) \right] - U(h_{t+s}) \right\} \tag{54}$$

Where U^o_s, U^{rt}_s are defined as in (39). Thus the wage-setting decision about $\widetilde{W}_t$ maximizes a weighted average of the two household types utility functions, conditional to the probability that the wage cannot be reoptimized in the future.[12] The relevant constraints are (52), (6) and (12). The union's first-order condition is:

$$\begin{aligned} \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left[(1-\theta) \, \lambda^o_{t+s} + \theta \lambda^{rt}_{t+s} \right] h^d_{t+s} \left(W_t \right)^{\alpha_w} \cdot & \\ \cdot \left[\widetilde{W}_t - \mu^w \frac{\psi_l h^{\phi_l}_{t+s}}{\left[(1-\theta) \, \lambda^o_{t+s} + \theta \lambda^{rt}_{t+s} \right]} \right] &= 0 \end{aligned} \tag{55}$$

[12] It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

C Tables and Figures

Table 1: Calibration

Parameter	Value	Description
β	0.99	subjective discount factor
η	6	price-elasticity of demand for a differentiated good
λ_p	0.6	degree of price stickiness
σ	10.62	money demand elasticity
ϕ_m	0.5393	preference parameter
ψ_l	1	preference parameter
γ_p	1	indexation on prices
ϕ_l	3	Inverse of Frish elasticity
ρ_μ	0.5	money growth rate autocorrelation

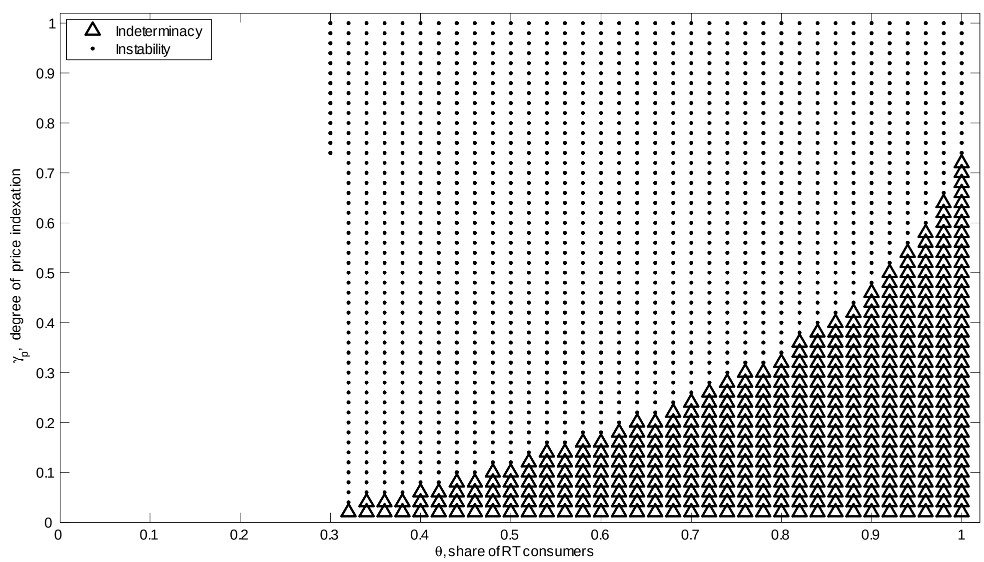

Figure 1: Indeterminacy and instability regions

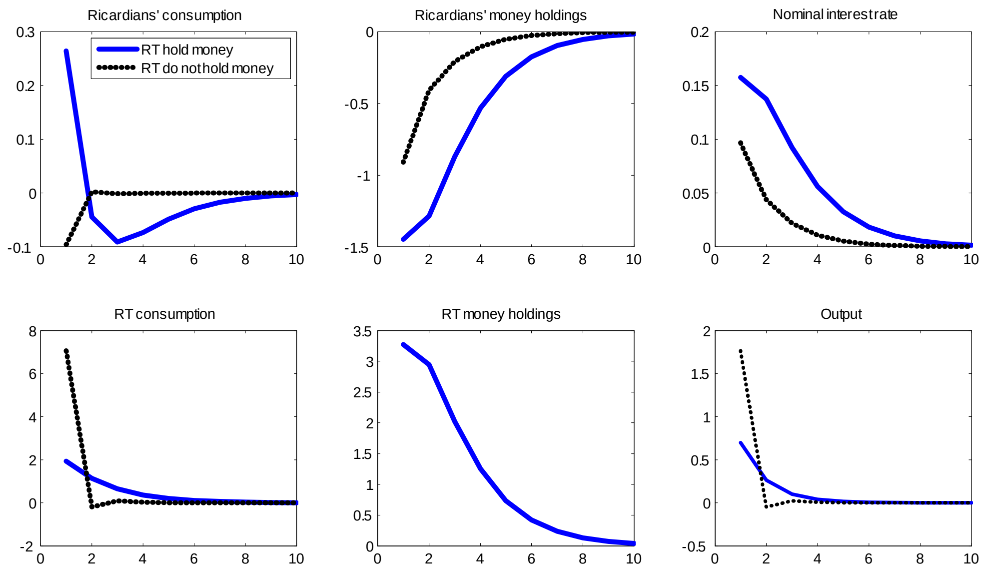

Figure 2: IRF's to a 1% positive monetary shock