

DEPARTMENT OF ECONOMICS, MANAGEMENT AND STATISTICS UNIVERSITY OF MILAN – BICOCCA

# DEMS WORKING PAPER SERIES

# **Constitutional Rules and Efficient Policies**

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No. 270 – March 2014

Dipartimento di Economia, Metodi Quantitativi e Strategie di Impresa Università degli Studi di Milano - Bicocca <u>http://dems.unimib.it/</u>

# Constitutional Rules and Efficient Policies.\*

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March 4, 2014

#### Abstract

This paper compares the ability to select the efficient policy of a parliamentary and a presidential constitutional setup. In order to do it we build a dynamic theoretical model with asymmetric information that succeeds in addressing both the politicians' accountability and the competence dimensions. The main difference between the two institutional frameworks is the presence of the confidence vote in the parliamentary system that may cause elections before the natural end of the legislature.

The equilibrium predictions suggest that, exactly because of the different incentives created by the confidence vote, the parliamentary system has a higher probability of selecting the efficient policy the higher is the quality of politicians that are member of the legislative body.

Keywords: presidential system, parliamentary system, comparison confidence vote, hierarchical accountability

JEL Classification: C72, D72

<sup>\*</sup>The authors gratefully acknowledge financial support from PRIN (2012-2015) " New approaches to political economics : positive political theories, empirical evidence and experiments in laboratory".

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# 1 Introduction

It has been well acknowledged, since the seminal works by Persson and Tabellini [2002, 2005], that institutional setups have a relevant impact on the shape of economic policies. However, in spite of the substantial amount of literature, both in the fields of political science and economics, there is still no received knowledge on which constitutional design may be more desirable. We contribute to this debate by comparing the performance of a presidential and a parliamentary system in selecting the efficient policy and we show that the parliamentary framework has a higher chance of being better the higher is the quality of the members of the legislative body.

We focus on the comparison between presidential and parliamentary systems under informational asymmetry; such asymmetry generates both a moral hazard problem (accountability) and an adverse selection one (politician's competence). We show that the two institutional frameworks respond differently to these two dimensions; the specific incentive schemes generated have a dramatic impact on the efficiency of the policies chosen by governments.

Specifically, we compare presidential versus parliamentary systems through the following two-period setup. The government is defined by an executive body, represented by a single player, and by a legislative body, represented by an assembly composed by three members. At the beginning of the game each player observes his type (i.e. congruent or not). In the first period the executive observes the true state of the world while the members of the assembly receive only an informative signal about it. At this point the executive proposes a policy that has to be approved by majority in the assembly. At the end of the first period each player observes the true state of the world, updates his beliefs and then period two occurs analogously. The difference between the two systems is the presence of the confidence vote as a key constitutional ingredient of the parliamentary system. The main implication of the confidence requirement is that if the policy proposed by the executive is rejected, new elections are called for both government bodies. This allows the parliamentary system to get rid of very bad politicians even before the natural conclusion of the legislature; in turn though, it makes the system also very sensitive to the incentives of the members of the assembly who may have private agendas themselves and not act in the interest of voters.

In spite of this big modelling difference the two systems behave alike in many regions of our parametric space, and this is due to the interplay of the different forces that drive the politicians' behavior. We can say though that, in general, the parliamentary system performs better than the presidential one the higher is the quality of the assembly because it will exercise more often its right of causing a government crisis and new elections to change a bad executive in power.

In our model voters, directly and through the assembly, are able to exercise a form of control over the executive branch of the government by using policy proposals and assembly votes as signals about the congruency of the executive.

Our work is related, as mentioned above, to the literature on the relation between constitutional design and economic policy that began with Persson and Tabellini [2002, 2005] and to the literature on incentives in political economy (see for example Besley [2007]).

More precisely, the idea that a good way to judge a political system is its ability to select the efficient policy comes from Besley and Coate [1998], where, in a different setup, they identify a political failure as the inability to undertake a potentially Pareto improving public investment with the available policy instrument.

In our model we show that politicians with a different tenure or time horizon have different incentives in choosing policies irrespective of their utility function as Maskin and Tirole [2004]. We also model in a simlar way the legacy motive present in congruent politicians (both in the executive and in the legislative body) and the value of being in office which characterizes all members of the political class. We do however modify the approach to politicians' accountability and the benefit of having elections which correct (or at least mitigate) inefficiencies due to both moral hazard (acting in the public interest) and adverse selection (weeding out the bad politicians), in order to take into account the hierarchical structure that comes from the presence of multiple levels of control, i.e. voters and assembly. We also observe some form of pandering in equilibrium, a perverse effect of politicians trying to be reelected, that is choosing to implement what is thought to be the popular policy to please the electorate.

Another related work is Diermeier and Vlaicu [2011] who study how constitutional features influence political behavior in terms of legislative success (passing of bills proposed by the executive) and they show that the confidence vote (that may send everybody home) is the critical feature that may explain the different performance of a parliamentary and presidential system in terms of legislative success.

Our hierarchical agency structure is related to the one in Vlaicu [2008] and Vlaicu and Whalley [2013] where they study accountability in government under different hierarchical controls without comparing different constitutions.

The structure of the papers is as follows: Section 2 describes the elements of the model, Sections 3 presents some benchmarks result and the one period version of the model, Section 4 contains the details of the presidential system, Section 5 those of the

parliamentary one, Sections 6 compares the two institutional setups and Section 7 briefly concludes. All proofs are in the Appendix.

# 2 The model

We analyze a two-period political system characterized by the presence of three (sets of) agents: the executive, the assembly and the voters.

**Policy environment.** Each period t = 1, 2 is characterized by a state of the world  $s_t \in \{H, L\}$ ; each state is equally likely,  $\mathbb{P}[s_t = k] = \frac{1}{2}$  for k = H, L, and states are independently distributed across periods.

In every period t a public good, A or B has to be produced. We indicate with  $g_t \in \{A, B\}$  the implemented policy at time t, i.e. the choice of the public good produced in period t. The production cost of the public good A is  $c_A > 0$  while, w.l.o.g., we normalized to 0 the cost of production of the public good B.

The selection process works as follows: the executive proposes a policy  $g_t^e \in \{A, B\}$ and the assembly votes on this proposal. If the assembly rejects the proposal a statusquo policy  $g^0$  is implemented. We analyze both  $g^0 = A$  and  $g^0 = B$ . If the executive wishes to implement the status-quo policy its proposal gets through with no vote.

The amount of resources in the country is  $\overline{Y}$ , and it can be used to provide the public good g or it can be privately consumed (through perks) by the executive.

**Voters.** The electorate is made of N homogeneous voters with a per period utility  $u(g, s_t)$ . We assume that the utility of the public good A is state-dependent; in particular we let u(A, H) > u(A, L). We assume instead that  $u(B, s_t)$  is constant across states, therefore we write it as u(B).

We assume that in state H it is efficient to produce good A  $(u(A, H) - c_A > u(B))$ , while in state L it is efficient to produce good B  $(u(B) > u(A, L) - c_A)$ .

We let  $g^{*}(s_{t})$  denote the efficient policy, where:

$$g^{*}(s_{t}) = \begin{cases} A, & \text{if } s_{t} = H \\ B, & \text{if } s_{t} = L \end{cases}$$

**Executive.** The executive body is made of a single member whose privately observed type is  $\theta^e \in \{0,1\}$  where  $\theta^e = 0$  indicates a non congruent executive and  $\theta^e = 1$  a congruent one. The executive is congruent with probability  $\mathbb{P}[\theta^e = 1] = \gamma$ .

The executive's utility function is:

$$V^{e} = R^{e}(g_{1}) + \theta^{e}u(g_{1}, s_{1}) + \pi \left(R^{e}(g_{2}) + \theta^{e}u(g_{2}, s_{2}) + \varepsilon\hat{\theta}^{e}\right)$$

where  $R^e(g_t) = \overline{Y} - c(g_t)$  indicates the amount of resources the executive consumes in each period in the form of rent;  $u(g_t, s_t)$  is the period t utility of voters when the state is  $s_t$  and the implemented policy is  $g_t$  and  $\pi$  is the probability of being in power in period two ( $\pi = 1$  for the presidential system and  $\pi \leq 1$  for the parliamentary). The last term  $\varepsilon \hat{\theta}^e$  represents the executive's concerns for reputation.  $\hat{\theta}^e$  is the ex-post voters' belief on the probability that the executive is congruent while  $\varepsilon$  is a positive real number which is small enough to satisfy:

$$0 \le \varepsilon \le u(B) - (u(A, L) - c_A)$$

This condition ensures that the reputation concerns cannot overcome a congruent executive's incentives to implement the correct policy in the last period.<sup>1</sup> To put it simply a non congruent executive cares only about his rent while a congruent one has a legacy motive that depends on the utility of the electorate. Both types will choose policy proposals in order to maximize their utility over the two periods taking into account the behavior of the assembly and the beliefs of the voters.

Assembly. The assembly is the legislative body which has to approve or reject the executive's policy proposal in each period. It is composed of three members (legislators), l = 1, 2, 3; each member has private information about his type  $\theta^{al} \in \{0, 1\}$  where  $\theta^{al} = 0$  is non congruent and  $\theta^{al} = 1$  is congruent. The probability that each member's type is congruent is  $\mathbb{P}[\theta^{al} = 1] = \gamma$ , and types are independent across members. We are therefore assuming that both executive and legislative posts are filled with politicians drawn from the same pool.

The utility function of the legislators is:

$$V^{al} = R^{a} + \theta^{al} \left( u(g_1, s_1) - c(g_1) \right) + \pi \left( R^{a} + \theta^{al} \left( u(g_2, s_2) - c(g_2) \right) + \varepsilon \hat{\theta}^{a} \right)$$

where  $R^a$  is the fixed rent from being in parliament,  $u(g_t, s_t) - c(g_t)$  is the surplus generated in period t when the state is  $s_t$  and the implemented policy is  $g_t$ . Moreover  $\hat{\theta}^a$  is the relevant reputation across electoral systems, that is the ex-post voters' belief

<sup>&</sup>lt;sup>1</sup>In particular the condition ensures that a congruent executive will not have incentive to do A in state L just to have a higher reputation.

on the probability that the majority of the assembly is congruent.

We assume  $R^a \in [\varepsilon, \max[u(A, H) - u(B) - c_A, u(B) - u(A, L) + c_A]]$  only to ensure that rent-seeking motives do not overshadow the legacy motives also for the congruent members of the assembly.

**Information structure.** As previously mentioned, members of the assembly and the executive have private information about their type.

The executive observes the state of the world in every period while each member of the assembly receives a common signal  $\sigma_t$  on the state of the world; the signal has precision  $\rho$  and is observed in each period before voting. Formally the signal is as follows:

$$\sigma_t = \begin{cases} H \text{ with probability } \rho \\ L \text{ with probability } 1 - \rho \end{cases} \text{ if } s_t = H;$$
  
$$\sigma_t = \begin{cases} L \text{ with probability } \rho \\ H \text{ with probability } 1 - \rho \end{cases} \text{ if } s_t = L;$$

Voters and assembly will perfectly observe  $s_1$  before the beginning of period 2, so that the update on the executive's probability of being congruent is based on the true realization of the state of the world.

#### Timing.



Both systems are analyzed over two periods. In the presidential system at t = 0 each player observes his private type, at t = 1 the executive observes the state of the world  $s_1$  while the assembly receives an informative signal  $\sigma_1$  about  $s_1$ . Then the executive makes a policy proposal and, if it is different from the status quo policy, the assembly votes to accept of reject. At the very end of period 1 both assembly and voters observe the state of the world of the period that just ended and update their beliefs on the type of the executive. In period two things happen exactly like in period one until the very end when there are new elections for both the assembly and the executive.

In the parliamentary setup the information structure and the game are very similar to the presidential system with the following exceptions: the policy proposal made by the executive is observed also by the voters and every vote on policy is like a confidence vote so that when a policy is rejected there are new elections. If the assembly rejects the policy a new executive  $(\tilde{E})$  and a new assembly  $(\tilde{A})$  will be in place at the beginning of period two.

# **3** Preliminary analysis

#### 3.1 Assembly

We begin our analysis from the comparison of the different incentives that members of the assembly face in the two institutional setups.

**Presidential system.** In the presidential system there is always an equilibrium in which members of the assembly of every type vote according to what they believe is optimal in that period. That is they vote yes to a policy if they believe that the probability that the proposed policy is the optimal one is larger than  $\frac{1}{2}$ . This is always an equilibrium given that by doing so, their expost reputation is the same as the ex-ante one; moreover, for the congruent members this is strictly better than the other possible strategies because in this way they maximize the component of their utility function that depends on  $u(\cdot)$  while in such an equilibrium non-congruent members are actually indifferent between voting in favor or not. This is because in equilibrium both actions are observed, therefore they cannot signal a higher reputation in any way.

**Parliamentary system** In the parliamentary system, instead, the first period choice of the policy changes the continuation payoff in a relevant way, given the presence of the confidence vote. In this case, given that the reputation concerns are not too high, noncongruent types always approve the executive's proposal, so to stay in power and keep their rent  $R^a$  also for the second period. The behavior of the congruent members depends instead on the ex post probability that the proposed policy is the correct one, but in any case, given that they are pivotal with positive probability, they will vote sincerely, for the alternative that maximizes their utility. All this is true in the first period of the parliamentary system. In the second period the assembly of the parliamentary system behaves exactly as the presidential one, given that new elections are called at the end of the period regardless of the assembly behavior.

#### 3.2 One period model

In order to understand some of the mechanisms underlying the two different systems, we start from the analysis of a one period version of our model. In this simpler setup, all the differences between the parliamentary system and the presidential one disappear, except for the following : in the presidential system voters can only observe the implemented policy, and not the proposed one, while in the parliamentary system they can observe both the proposed policy and the implemented one (and therefore they know also the vote of the assembly). Importantly, this allows us to verify how the two systems behave in absence of the threat of the confidence vote for the parliamentary system.

As discussed above for the last period of the two-perdiod model, in the one period model there is an equilibrium in which the assembly votes according to the probability that the proposed policy is the optimal one (i.e. they approve the policy iff the probability that the policy is optimal  $\geq \frac{1}{2}$ ) in both electoral systems. We focus on this equilibrium behavior for the assembly.

In the one period model, for both systems and both status-quo policies ( $g^0 = A$  and  $g^0 = B$ ), given the behavior of the assembly and our parametric assumptions, there is a unique pure strategy equilibrium in which the congruent executive proposes the correct policy, and the non-congruent executive always proposes B. Proposition 1 derives the behavior of the assembly given these executive's strategies, while Propositions 2 and 3 fully characterize the executive's equilibrium behavior in the two systems.

**Proposition 1** In the one period version of both systems, when  $g^0 = A$  and the equilibrium strategies are  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$ , the assembly votes always yes after B if  $\rho < \frac{1}{2-\gamma}$ , while it votes according to its signal if  $\rho \ge \frac{1}{2-\gamma}$ . Moreover, when  $g^0 = B$  and the equilibrium strategies are  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$  the assembly always vote yes after A.

The assembly, given  $g^0$  and  $\rho$ , uses the policy proposal made by the executive as a signal. Given the equilibrium strategies described, policy A is proposed only by a congruent type in s = H and therefore it is always passed. Policy B, instead, is proposed when it is efficient (i.e. s = L) but also by a non-congruent executive in state H. If the information of the assembly is sufficiently precise then they will choose according to their signal and may vote against.

The following proposition characterizes the pure strategy equilibrium of the presidential one-period system.

**Proposition 2** In the one period version of the presidential system, there is only one pure-strategy equilibrium, in which the executive's proposal  $g^e(s, \theta^e)$  is as follows  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$ . When  $g^0 = B$  or  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$  such equilibrium exists iff  $c_A \ge \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ ; when  $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$  such equilibrium exists iff  $c_A \ge \varepsilon \left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} - \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)$ .

In the one-period presidential system a congruent executive always proposes the efficient policy and sometimes he is voted against when proposing B, while the non congruent type always chooses the "cheaper" alternative B. This equilibrium exists if  $c_A$  is sufficiently large so that the non-congruent type never chooses policy A in exchange for a better end of period reputation that he could obtain by pooling with a congruent type across states of the world.

The equilibrium in the parliamentary system is qualitatively not very different, as the following proposition explain.

**Proposition 3** In the one period version of the parliamentary system, there is only one pure-strategy equilibrium, in which the executive's proposal  $g^e(s, \theta^e)$  is as follows  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$ . When  $g^0 = B$  or  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$  such equilibrium exists iff  $c_A \ge \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ ; when  $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$  such equilibrium exists iff  $c_A \ge \varepsilon \left(1 - \rho\gamma \left(1 - \rho\right) \left(\frac{1}{(1-\rho)\gamma + (1-\gamma)} + \frac{1}{1-\gamma+\gamma\rho}\right)\right)$ .

The equilibrium behavior in the one period parliamentary system is identical to the one of the presidential one. The thresholds for the existence of the pure strategy equilibrium differ because end of period reputations are different. This is due to the fact that in the parliamentary system voters observe not only the implemented policy but also the proposed one. In this way they have more information that they can use to update their prior on the executive's type and form their posterior belief. Voters are therefore able to exploit the superior information of the assembly when updating their beliefs on the probability of the executive being congruent. It is important to stress, however, that the structure of the equilibrium in the two systems is exactly the same. This will no longer be the case when we analyze the twoperiod version of the problem, and introduce the confidence vote in the parliamentary system.

# 4 The presidential system

We can now focus on the two-period presidential model. Periods in this system have the same structure; moreover politicians that are in power in the first period are sure to be present also in the second period. The only difference between the two periods rests in the executive's reputation: by the second period both the assembly and the voters have received additional information (from the executive's behavior and from the state of the world) that allowed them to have updated beliefs on the executive,  $\hat{\gamma}^a$  and  $\hat{\gamma}^v$ , that may differ from the initial one,  $\gamma$ .

Notice moreover that, in general, members of the assembly and voters will hold different beliefs on the executive ( $\hat{\gamma}^a \neq \hat{\gamma}^v$ ): the members of the assembly update their beliefs after observing  $g_1^e$  and  $s_1$ , while the voters update on the basis of  $g_1$  and  $s_1$ . In this setting voters do not have the possibility of observing the proposed policy, and they cannot generally infer it from the implemented one.

We argued in Section 3 that in the presidential system there is always an equilibrium in which the behavior of the assembly does not depend on the type of the members of the assembly but only on the signal they observe. We focus on this equilibrium behavior of the assembly, that is, we assume that the assembly votes according to the probability that the proposed policy is the efficient one.

If this is the case, Proposition 1 describes the first period equilibrium behavior. Moreover, the second period behavior of the assembly differs from Proposition 1 as follows: when  $g^0 = A$  and the equilibrium strategies are  $g_2^e(s_2, 1) = g^*(s_2)$  and  $g_2^e(s_2, 0) = B$ , the assembly votes always yes after B if  $\rho < \frac{1}{2-\hat{\gamma}^a(g_1^e,s_1)}$ , while it votes according to its signal if  $\rho \geq \frac{1}{2-\hat{\gamma}^a(g_1^e,s_1)}$ , where  $\hat{\gamma}^a(g_1^e,s_1) = \Pr[\theta^e = 1|g_1^e,s_1]$ . In other words, the assembly behaves similarly in both periods, the difference being in the treshold that determines what is a good enough signal as this threshold depends on the executive's reputation at the beginning of each period.

We are now ready to characterize the pure-strategy equilibria of this system.

**Proposition 4** In the two-period presidential system we can have the following purestrategy equilibria: (E1)  $g_t^e(s_t, 1) = g^*(s_t), g_t^e(s_t, 0) = B$ ; this is an equilibrium under the following conditions:

- if 
$$c_A > 2\varepsilon$$
 when  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ ;

1. - if 
$$c_A > \varepsilon \gamma \left( \frac{1+\gamma-\rho\gamma}{2\gamma+\rho-2\rho\gamma} \right)$$
 when  $g^0 = A$  and  $\rho > \frac{1}{2-\gamma}$ ;  
- if  $c_A > \varepsilon$  when  $g^0 = B$ .

(E2)  $g_t^e(s_t, 1) = g^*(s_t), g_1^e(s_1, 0) = g^*(s_1), g_2^e(s_2, 0) = B$ ; this is an equilibrium under the following conditions:

$$1. \quad \text{if } c_A \in \left(\varepsilon \frac{2-2\gamma}{2-\gamma}, 2\varepsilon \frac{\gamma}{2-\gamma}\right) \text{ when } g^0 = A \text{ and } \rho < \frac{1}{2-\gamma};$$
  
-  $\text{if } c_A \in \left(\varepsilon \left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} - \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right), \frac{1}{2}\varepsilon \left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\right) \text{ when } g^0 = A \text{ and } \rho \ge \frac{1}{2-\gamma};$   
-  $\text{if } c_A \in \left(\varepsilon \frac{2-2\gamma}{2-\gamma}, \varepsilon \frac{\gamma}{2-\gamma}\right) \text{ when } g^0 = B$ 

The above proposition describes the two possible pure-strategy equilibria of the presidential system.

The first equilibrium (E1) arises when the production of the public good A is costly enough. In this equilibrium both types of the executive replicate twice the behavior of the one period version of the model: the congruent executive always proposes the efficient policy, while the non-congruent one always proposes B. Notice that the threshold on  $c_A$  that determines the existence of E1 is higher than the corrisponding threshold of the one-period version (see Proposition 2). The reason is that the reputation carried over from the first period influences the probability of having the second period proposal accepted, therefore a higher threshold on  $c_A$  is needed to prevent deviations of a non congruent executive that could trade off some of the period 1 payoff for a higher expected second period one.

The second equilibrium (E2) arises for lower values of  $c_A$ . In this case all types of the executive pool on offering the efficient policy in the first period. By doing so they induce the assembly to approve every policy offer in period 1; moreover they enter the second period with the initial reputation  $\gamma$ , as nothing can be learned from their behavior in period 1. The second period coincides exactly with the one-period version of Proposition 2. This equilibrium therefore exists if  $c_A$  is high enough to preserve the second period behavior of the non-congruent type but not too high so that even a noncongruent type may be willing to choose A in the right state of the world because of the

	$g^0 = A$ and $\rho \ge \frac{1}{2-\gamma}$	$g^0 = A$ and $\rho < \frac{1}{2-\gamma}$	$g^0 = B$
E1	$\frac{5}{4}\gamma - \frac{5}{4}\gamma\rho + 2\rho$	$\frac{3}{4} + \frac{5}{4}\gamma + \frac{1}{2}\rho\left(1 - \gamma\right)$	$1 + \gamma$
E2	$1 + \frac{\gamma}{2} \left(1 - \rho\right) + \rho$	$rac{3}{2}+rac{\gamma}{2}$	$\frac{3}{2} + \frac{\gamma}{2}$

Table 1: Welfare in the presidential system

gain in reputation which will grant him a greater probability of policy approval in the second period.

In both equilibria the behavior of the congruent executive is driven by legacy motives; as for the non congruent type, he may always propose B (as in the one-period version of the model) or he may choose the efficient policy just because of the gain in reputation that he obtains by making the same offer as the congruent type. The second period reputation may therefore have a disciplining effect that is at work in E2. This disciplining effect and the learning that is happening across the periods distinguish the two-period model from a repetition of the one-period version.

#### 4.1 Welfare Analysis

We can now try to compare the equilibria discussed above by looking at what they achieve in terms of welfare. For this purpose we assume that the gain from implementing the right policy in each state is equal across states. In analytical terms this amounts to imposing the following symmetry:  $u(A, H) - c_A - u(B) = u(B) - u(A, L) + c_A$ . As a consequence the welfare will be higher the higher is the probability of doing the right thing.

The following table summarizes the total probability, over the two periods, of choosing the efficient policy for each equilibrium:

We can first of all notice that the welfare is always increasing in  $\gamma$ . This is very intuitive, since in this system a higher quality of the executive implies directly that the efficient policy is proposed more often. The behavior of welfare when  $\rho$  varies, instead, depends on the status quo. When  $g^0 = A$  the welfare is increasing in  $\rho$ : the legislators make an effective use of their information, therefore better information translates in higher welfare. When  $g^0 = B$  instead, the total probability of implementing the efficient policy does not depend on  $\rho$ , given that the assembly does not have the chance to make good use of their information.

Overall we can say that when  $g^0 = A$  the two-period model presents a higher probability of choosing the efficient policy than the hypothetical case in which we would have two independent one period models and the reason is twofold: first in the two period model there is learning (*signalling effect*), secondarily the presence of a second period where an executive enters with a reputation carried over from the previous one makes non-congruent types behave better (*disciplining effect*).

It's worth noting that E1 performs better when the assembly exploits the good quality information they have  $(g^0 = A \text{ and } \rho \ge \frac{1}{2-\gamma})$  while E2 is better when the information is relatively poor  $(g^0 = A \text{ and } \rho < \frac{1}{2-\gamma})$  or when there is no chance of using it  $(g^0 = B)$ . This is due to the fact that in the second equilibrium the non congruent executive behaves as a congruent one in the first period. In fact the main driving force in E1 is the signalling effect so the welfare is higher when the signal is good; when the signal is poor we see that E2, whose driving force is the disciplining effect, achieves a higher welfare.

Notice moreover that the total probability of choosing the efficient policy in E1 is lower when  $g^0 = B$  than when  $g^0 = A$  because of the absence of learning. Finally when  $g^0 = B$  the assembly alone could do better than the executive-assembly pair.

### 5 The parliamentary system

We now analyze the equilibrium of a different institutional setting, the parliamentary system. The main difference with the presidential system is that in this case any assembly vote is a confidence vote; therefore if the executive's proposal is rejected there are new elections. This modifies the incentives of the executive and of members of the assembly. In particular the voting incentives of non-congruent legislators in the first period change, as their main concern is remaining in office. As a consequence the behavior of the assembly depends on the type of its majority.

**Remark 1** The probability that the majority of the legislators is congruent is  $\Gamma = \gamma^3 + 3\gamma^2 (1-\gamma) = 3\gamma^2 - 2\gamma^3$ . Notice that  $\Gamma > \gamma$  if  $\gamma < \frac{1}{2}$ , while  $\Gamma < \gamma$  if  $\gamma > \frac{1}{2}$ .

We start the analysis of this system by considering the equilibrium behavior of legislators in relation with their type, period and executive's behavior<sup>2</sup>.

**Proposition 5** Non-congruent legislators approve every proposal in the first period and behave as congruent in the second period.

Congruent legislators, in the second period, behave according to Proposition 1.

In the first period, when  $g^0 = A$ , congruent legislators behave as follows, given  $g_2^e(s_2, 1) = g^*(s_2)$  and  $g_2^e(s_2, 0) = B$ :

 $<sup>^{2}</sup>$ In this proposition we describe only the equilibrium behavior of the assembly as a response to possible equilibrium behaviors of the executive.

- if  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = B$  they approve B always when  $\rho < \frac{1}{2-\gamma}$  and follow the signal when  $\rho \ge \frac{1}{2-\gamma}$ ;
- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = g^*(s_1)$  they always approve B;
- if  $g_1^e(s_1, 1) = A$  and  $g_1^e(s_1, 0) = B$  they follow the signal if  $\rho \ge \frac{1}{2} + \frac{\gamma}{6}$  and vote against B otherwise;
- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = A$  they follow the signal.

When  $g^0 = B$ , congruent legislators behave as follows:

- if  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = B$  they always approve A;
- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = g^*(s_1)$  they always approve A.

Proposition 5 shows one of the crucial differences between the two institutional systems: non congruent legislators always approve any policy proposals in the first period because the want to stay in power as long as possible. In the second period they behave like in the one-period version of the model, therefore doing what they believe to be the efficient thing in order to maximize end of period reputation. Congruent legislators instead want to maximize the total probability of doing the right thing over the two period stretch.

Given the behavior of the assembly described above, we can now characterize the equilibria of the two-period parliamentary system.

**Proposition 6** In the two-period parliamentary system, when  $g^0 = A$ , in every pure strategy equilibrium the second period behavior is  $g_2^e(s_2, 1) = g^*(s_2)$  and  $g_2^e(s_2, 0) = B$ . In the first period we can have the following equilibrium behavior:

- **(E1)**  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = B$ ;
- **(E2)**  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = g^*(s_1);$
- **(E3)**  $g_1^e(s_1, 1) = A \text{ and } g_1^e(s_1, 0) = B;$
- **(E4)**  $g_1^e(s_1,1) = g_1^e(s_1,0) = A.$

The existence conditions of these equilibria are provided in Table 2.

$E1\\ \rho \ge \frac{1}{2-\gamma}$	$c_A \ge \max \left\{ \begin{array}{c} \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}, \\ \left\{ \begin{array}{c} \frac{1}{(1-\rho)\Gamma}u\left(A,L\right)\left(\frac{1}{2}-\frac{\rho(1-\rho)\Gamma}{4}\right) \\ +\frac{1}{(1-\rho)\Gamma}u\left(A,H\right)\left(-\frac{1}{2}+(1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right) \\ +\overline{Y}+\frac{1}{2}\varepsilon\left(1+\rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}+(1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{array} \right\} \end{array}\right\}$
$\begin{array}{c c} E1\\ \rho < \frac{1}{2-\gamma} \end{array}$	$c_A \geq 2\varepsilon$
$E2\\ \rho \ge \frac{1}{2-\gamma}$	$c_A \in \left[ \varepsilon \left( 1 - \rho \gamma \left( 1 - \rho \right) \left( \frac{1}{1 - \rho \gamma} + \frac{1}{1 - \gamma + \gamma \rho} \right) \right), \frac{\varepsilon}{2} \left( \frac{\rho \gamma}{1 - \gamma + \gamma \rho} + \frac{(1 - \rho) \gamma}{1 - \rho \gamma} \right) \right]$
$\begin{array}{c c} E2\\ \rho < \frac{1}{2-\gamma} \end{array}$	$c_A \in \left[\varepsilon\left(\frac{2-2\gamma}{2-\gamma}\right), 2\varepsilon\frac{\gamma}{2-\gamma}\right]$
E3	$c_A \in \left[ \frac{\rho \geq \frac{1}{2} + \frac{\gamma}{6}}{\frac{1}{(1-\rho)\Gamma} u(A,L) \left(\frac{1+\rho}{4} - \frac{\rho(1-\rho)\Gamma}{4}\right)}{+\frac{1}{(1-\rho)\Gamma} u(A,H) \left(-\frac{1+\rho}{4} + (1-\rho)\Gamma \left(1+\frac{\rho}{4}\right)\right)} + \overline{Y} + \frac{\varepsilon}{(1-\rho)\Gamma} \right) \right\}$
$E4\\\rho \ge \frac{1}{2-\gamma}$	$c_{A} \in \begin{bmatrix} \varepsilon \left(1 - \rho \gamma \left(1 - \rho\right) \left(\frac{1}{(1 - \rho)\gamma + (1 - \gamma)} + \frac{1}{1 - \gamma + \gamma \rho}\right)\right), \\ \frac{2(1 - \rho)\Gamma}{2 - (1 - \rho)\Gamma} \left(\overline{Y} + \varepsilon \frac{1}{2} \left(\frac{\rho \gamma}{1 - \gamma + \rho \gamma} + \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)}\right)\right) \\ \left\{ \begin{array}{c} \frac{1}{(1 - \rho)\Gamma} u \left(A, L\right) \left(\frac{1}{2} - \frac{\rho(1 - \rho)\Gamma}{4}\right) \\ + \frac{1}{(1 - \rho)\Gamma} u \left(A, H\right) \left(-\frac{1}{2} + (1 - \rho)\Gamma \left(1 + \frac{\rho}{4}\right)\right) \\ + \overline{Y} + \frac{1}{2}\varepsilon \left(1 + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} + (1 - \rho) \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)}\right) \end{array} \right\} \end{bmatrix}$
$\begin{array}{c} \text{E4} \\ \rho \geq \frac{1}{2-\gamma} \end{array}$	$(u(A,H) - u(A,L)) \left(\frac{3}{4}(1-\rho)\Gamma - \frac{1}{2}\right) + (1-\rho)\Gamma\left(\overline{Y} + \frac{1}{2}\varepsilon\left(1+\frac{\gamma}{2-\gamma}\right)\right) > 0$ $c_A \in \left[\varepsilon\frac{2-2\gamma}{2-\gamma}, \frac{(1-\rho)\Gamma}{1-(1-\rho)\Gamma}\left(\overline{Y} + \varepsilon\frac{\gamma}{2-\gamma}\right)\right]$

Table 2: Existence conditions for Proposition 6

The first two equilibria, E1 and E2, correspond to the equilibria of the presidential system. As for the presidential system, in E1 described the congruent executive proposes the efficient policy in both periods and the non congruent one always chooses B. It exists if  $c_A$  large enough, so that a non congruent as no incentive to deviate and implement the efficient policy in some state of the world. In E2 both types choose the efficient policy in every state of the world, and separate in the second period where the equilibrium behavior of the one-period model is preserved. In E2 the disciplining effect of the second period is strong enough to make a non congruent executive behave alike a congruent one.

The fact that the destiny of the executive is linked to the vote of the assembly in our parliamentary system, where any vote over policy proposal is a confidence vote, introduces the possibility of pandering. In E3 the congruent executive proposes A in both states; this holds in fact when the information of the assembly is particularly good ( $\rho$  high but not as high as our usual treshold, in fact  $\frac{1}{2} + \frac{\gamma}{6} \leq \frac{1}{2-\gamma}$ ) so that legislators tend to follow their signal.

In E4 both types of executive propose policy A regardless of efficiency issues, just because that option is somehow the more popular one. This equilibrium involves no learning at all but also no disciplining effect as the main force driving the executive behavior is staying in power.

It's worth noting that E1 and E3 can coexist<sup>3</sup>, while E4 does not coexist with E1 and E3 when  $\rho \geq \frac{1}{2-\gamma}$ .<sup>4</sup>

The following proposition characterizes the pure-strategy equilibria when  $g^0 = B$ .

**Proposition 7** In the two-period parliamentary system, when  $g^0 = B$ , in every pure strategy equilibrium the second period behavior is  $g_2^e(s_2, 1) = g^*(s_2)$  and  $g_2^e(s_2, 0) = B$ . In the first period we can have the following equilibrium behavior:

(E1)  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = B$ ; this is an equilibrium if  $c_A > \varepsilon$ ;

(E2)  $g_1^e(s_1,1) = g_1^e(s_1,0) = g^*(s_1)$ ; this is an equilibrium if  $c_A \in \left(\varepsilon \frac{2-2\gamma}{2-\gamma}, \varepsilon \frac{\gamma}{2-\gamma}\right)$ .

$$\frac{{}^{3}\text{In fact}}{\left\{\begin{array}{c} \frac{1}{(1-\rho)\Gamma}u\left(A,L\right)\left(\frac{1+\rho}{4}-\frac{\rho(1-\rho)\Gamma}{4}\right)+\frac{1}{(1-\rho)\Gamma}u\left(A,H\right)\left(-\frac{1+\rho}{4}+(1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right)\\ +\overline{Y}+\frac{\varepsilon}{(1-\rho)\Gamma}\right\} \\ < \left\{\begin{array}{c} \frac{1}{(1-\rho)\Gamma}u\left(A,L\right)\left(\frac{1}{2}-\frac{\rho(1-\rho)\Gamma}{4}\right)+\frac{1}{(1-\rho)\Gamma}u\left(A,H\right)\left(-\frac{1}{2}+(1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right)\\ +\overline{Y}+\frac{1}{2}\varepsilon\left(1+\rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}+(1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right)\end{array}\right\}$$

<sup>4</sup>In fact  $\frac{2(1-\rho)\Gamma}{2-(1-\rho)\Gamma} \left(\overline{Y} + \varepsilon \frac{1}{2} \left( \frac{\rho\gamma}{1-\gamma+\rho\gamma} + \frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)} \right) \right) < \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}$ 

	$g^0 = A$ and $\rho \ge \frac{1}{2-\gamma}$	$g^0 = A$ and $\rho < \frac{1}{2-\gamma}$	$g^0 = B$
Eq. 1	$\frac{\frac{1-\Gamma}{2}+\rho\left(1+\Gamma\right)+\frac{\gamma}{2}\left(1-\rho\Gamma\right)}{+\frac{3}{4}\gamma\left(1-\rho\right)+\frac{(1-\gamma)\gamma}{4}\rho\Gamma\left(1-\rho\right)}$	$\frac{3}{4} + \frac{5}{4}\gamma + \frac{1}{2}\rho\left(1 - \gamma\right)$	$1 + \gamma$
Eq. 2	$1 + \frac{\gamma}{2} \left(1 - \rho\right) + \rho$	$\frac{3}{2} + \frac{\gamma}{2}$	$\frac{3}{2} + \frac{\gamma}{2}$
Eq. 3	$ \begin{array}{c} \gamma + \left( 1 - \gamma \right) \rho + \frac{1}{2} \\ + \frac{\Gamma}{4} \left( 1 - \gamma \right) \left( 2 \left( 2\rho - 1 \right) + \gamma \left( 1 - \rho \right) \right) \end{array} $	$ \begin{array}{c} \gamma + (1 - \gamma)  \rho + \frac{1}{2} \\ + \frac{\Gamma}{4} \left( 1 - \gamma \right) \left( 2 \left( 2\rho - 1 \right) + \gamma \left( 1 - \rho \right) \right) \\ (\text{only if } \rho \geq \frac{1}{2} + \frac{\gamma}{6} ) \end{array} $	x
Eq. 4	$\frac{1}{2} + \frac{\gamma}{2} \left(1 - \rho\right) + \rho$	$1+rac{\gamma}{2}$	x

Table 3: Welfare in the parliamentary system

When  $g^0 = B$  we only have E1 and E2, exactly as in the presidential system. In this case pandering is never optimal because the fact that policy B does not require approval limits the incentives of a non congruent executive to pool with a congruent one.

#### 5.1 Welfare in the parliamentary system

We proceed now to check the behavior of the pure-strategy equilibria of the parliamentary system in terms of welfare. Given the assumption that the gain from implementing the right policy in each state is equal across states, the welfare will be higher the higher is the probability of doing the right thing and choosing the efficient policy.

The following table summarizes the total probability, over the two periods, of choosing the efficient policy for each equilibrium:

Table 3 shows that there are several aspects of the parliamentary system that are similar to the presidential one. First of all, the behavior when  $g^0 = B$  is identical across systems: only two equilibria exist also in the parliamentary one (E1 and E2), and they deliver the same welfare as in the presidential system.

Second, the welfare is still (weakly) increasing in  $\rho$ : better informed legislators take better decisions also in the parliamentary system.

Let us now analyze the effects that are specific to the parliamentary system. We can start by noticing that in this system there are two new types of pure-strategy equilibria, E3 and E4. These new equilibria where politicians pander on A to some degree, however, perform worse than the first two types of equilibria. In particular E4 is always worse than E2, as the probability of implementing the correct policy in E4 is  $\frac{1}{2}$  less than in E2. This is due to a different behavior in the first period, while in the second period the two equilibria deliver the same behavior. Notice also that the two equilibria are defined over overlapping regions. Moreover E1 is better than E3, implying that it is better to have a congruent executive that offers the efficient policy in each state and runs the risk of being voted against by an ill informed but congruent assembly (that cannot fully separate a congruent from a non congruent executive) than having a congruent executive that always offers the safe option to avoid being sent home. E1 and E3 are also defined over overlapping regions.

Another difference from the presidential system is the behavior of the welfare with respect to  $\gamma$ . Better politicians increase the welfare in E2 and E4 as they did in the presidential system. However there are cases, as for example E3, in which the probability of selecting the correct policy is not increasing in  $\gamma$ , at least in some regions. In E3 this is particularly intuitive: a congruent executive always proposes A in the first period, and proposes the efficient policy in the second period; in a sense, the congruent executive misbehave in period 1, and behave optimally in period 2. An increase in the quality of the executive may increase (if  $\gamma$  is low) or decrease (if  $\gamma$  is high) the total probability of selecting the correct policy, depending on which effect prevails.

Finally, some of the equilibria depend on  $\gamma$  through the quality of the assembly, that is through  $\Gamma$ . The effect of  $\Gamma$ , whenever it affects the welfare, is unambigously positive: better legislators induce better policy outcomes. This is particularly interesting because  $\Gamma$  may also be affected by the size of the assembly; if the quality of the legislators ( $\gamma$ ) is sufficiently high,  $\Gamma$ , and in turn welfare, increases with the size of the assembly. Parliamentary systems therefore seem to perform better with large assemblies than with small ones.

#### 6 Comparison between the two systems

We can now compare the welfare properties of the two institutional setup and verify which one allows to implement the efficient policy more often and under which parametric conditions.

First of all notice, as mentioned before, that the two systems behave alike when  $g^0 = B$ : under this specification of the status-quo they display the same equilibria and induce the same welfare for any level of  $c_A$ .

When  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$  the behavior of the two systems is similar in E1 and E2; in these equilibria the two systems generate the same welfare. This is due to the fact that the confidence vote never bites in these cases. However in this parametric region the parliamentary system may display other two equilibria, E3 and E4, that are dominated by E1 and E2 respectively in terms of welfare. This would suggest that in this parametric region the presidential system is better as the best welfare induced by both

systems is the same, but the parliamentary system is characterized by a multiplicity of equilibria that may arise and lower the welfare.

Finally when  $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$  we have the interplay of several effects. First of all notice that the two systems behave in the same way in E2. Moreover, the parliamentary system displays two equilibria, E3 and E4, that are dominated in terms of welfare by E1 and E2 respectively. For both systems, however, E1 is the equilibrium that induces the highest welfare in this region. If we want to compare the behavior of the two systems in E1 things are not so clear cut. The parliamentary system performs better for  $\Gamma$ large enough: as the quality of the assembly improves, the parliamentary system, that relies more heavily on the work of the assembly, outperforms the presidential one. The confidence vote that allows to change the "wrong" politicians has a positive effect on welfare only if it is exercised, and this happens the larger is the share of congruent members of the assembly. Notice that the welfare in E1 of the parliamentary system is increasing in  $\Gamma$  and  $\Gamma$  is increasing with n (the assembly size) for  $\gamma$  large enough; so our result therefore can be somehow thought as a lower bound to the parliamentary performance in this equilibrium.

# 7 Concluding remarks.

We have shown how two different constitutional systems perform in selecting the efficient policy when in presence of asymmetric information and possibly non congruent politicians. Not surprisingly the parliamentary system that assigns to the assembly a particular role through the confidence vote has a better performance than the presidential one when the legislative body has a higher chance of having a congruent majority.

The two systems differ significantly when the confidence vote has bite, in fact the equilibria are strategically equivalent when the status-quo is B, and nobody votes against A, and in the one period model where there can never be early elections.

Through the interplay of asymmetric information and the institutional characteristics our equilibria show the presence of two different positive effect that induce the executive into proposing the efficient policy more often: a signalling effect, that builds on the learning done by the assembly, and a disciplining effect, that is caused by the presence of the second period into which an executive carries the reputation built in the first one. We also show how important is the status-quo choice, in fact when  $g^0 = B$  the assembly has very little role in the play of the game and the signalling and disciplining effect do nothing to incentivate the executive.

We also show that the equilibria in which some form of pandering is present are never

welfare maximising. This demonstrates that in a framework where the assembly plays a bigger role through the possibility of early elections some of the incentives that drive the executive policy proposal may have a negative impact.

This paper contributes to the literature at least in three directions: it develops a dynamic theoretical model with informational asymmetries to compare different government constitutions which generates intuitive equilibrium predictions; it sheds light on the logic of governments by explaining how the choice of the efficient policy is function of the specific incentive scheme generated by the constitutional features; it generates testable hypothesis providing ideas for further empirical research.

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## 8 Appendix

#### 8.1 Proofs from Section 3 (One Period Model)

**Proof of Proposition 1.** A congruent legislator maximizes his utility by voting for what he believes to be the efficient policy given the executive's proposal and equilibrium strategy. A non-congruent one maximizes his utility by behaving as a congruent in order to maximize his end of period reputation.

Consider the case in which  $g^0 = A$ . If the assembly observes  $g^e = B$  and  $\sigma = L$  it approves B because the signal that the legislators receive is compatible with the policy that is proposed by the executive. If the assembly observes  $g^e = B$  and  $\sigma = H$ , instead, it computes  $\Pr[s = L|g^e = B, \sigma = H]$  in order to decide on its vote. Such probability is

$$\Pr[s=L|g^e=B,\sigma=H] = \frac{\Pr[g^e=B,\sigma=H|s=L]\cdot\Pr[s=L]}{\Pr[g^e=B,\sigma=H]} = \frac{1-\rho}{1-\gamma\rho};$$

the assembly approves B after  $\sigma = H$  iff  $\Pr[s = L|g^e = B, \sigma = H] > \frac{1}{2}$ , which happens when  $\rho < \frac{1}{2-\gamma}$ .

Consider the case in which  $g^0 = B$ . In this case the assembly only has to vote when A is proposed. Given the equilibrium strategies, whenever A is proposed it is also efficient, therefore the assembly always approves A.

**Proof of Proposition 2.** We start from the case in which either  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ , or  $g^0 = B$ . As shown in Proposition 1 in both these cases the assembly approves every policy offer, regardless of the signal  $\sigma$  received. Therefore the voters know that  $g^e = g$ . The ex-post reputation after offering A therefore is 1 (only the congruent executive offers A) and the ex-post reputation after offering B is

$$\begin{aligned} \Pr[\theta^e &= 1|g=B] = \Pr[\theta^e = 1|g^e = B] \\ &= \frac{\Pr[g^e = B|\theta^e = 1]\Pr[\theta^e = 1]}{\Pr[B]} = \frac{\gamma}{2-\gamma}. \end{aligned}$$

The strategies  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$  constitute a pure strategy NE if no type of executive has incentive to deviate. When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g^e(H, 0) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right) \ge \overline{Y} - c_A + \varepsilon (1)$$

which is satisfied if  $c_A \geq \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ . The same condition prevents the deviation to

 $g^e(L,0) = A.$ 

A type  $\theta^e = 1$  could deviate and choose  $g^e(H, 1) = B$ . For this not to be a profitable deviation it must be:

$$\overline{Y} - c_A + u(A, H) + \varepsilon(1) \ge \overline{Y} + u(B) + \varepsilon\left(\frac{\gamma}{2 - \gamma}\right)$$

which is trivially satisfied because of assumption  $u(A, H) - c_A > u(B)$ .

A type  $\theta^e = 1$  could deviate and choose  $g^e(L, 1) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + u(B) + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right) \ge \overline{Y} - c_A + u(A, L) + \varepsilon (1)$$

which is also always satisfied because of assumption  $u(B) > u(A, L) - c_A + \varepsilon$ .

Now let's consider the case in which  $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$ . In this case the assembly, after observing *B* votes according to its signal. The ex-post reputation after *A* is no longer equal to 1, because there are cases in which the executive proposes *B* and *B* is not approved; in that case the executive may also be non-congruent, therefore the voters' belief on the executive being congruent after observing *A* is less than one. More precisely we have:

$$\Pr[\theta_e = 1 | g = A] = \frac{\Pr[g = A | \theta_e = 1] \Pr[\theta_e = 1]}{\Pr[A]} = \frac{(2 - \rho) \gamma}{1 + \gamma - \rho \gamma}$$

and the ex-post reputation after B is

$$\Pr[\theta_e = 1 | g = B] = \frac{\Pr[g = B | \theta_e = 1] \Pr[\theta_e = 1]}{\Pr[B]} = \frac{\rho \gamma}{1 - \gamma + \rho \gamma},$$

where  $\Pr[\theta_e = 1 | g = A] > \gamma > \Pr[\theta_e = 1 | g = B].$ 

The strategies  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$  constitute a pure strategy NE if no type of executive has incentive to deviate. When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g^e(H, 0) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} - \rho c_A + \varepsilon \left( \rho \frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + (1-\rho) \frac{\rho\gamma}{1-\gamma+\rho\gamma} \right) \ge \overline{Y} - c_A + \varepsilon \left( \frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} \right)$$

which is satisfied if  $c_A \geq \varepsilon \left( \frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} - \frac{\rho\gamma}{1-\gamma+\rho\gamma} \right)$ . The same condition also prevents the deviation to  $g^e(L,0) = A$ .

A type  $\theta^e = 1$  could deviate and choose  $g^e(H, 1) = B$  or choose  $g^e(L, 1) = A$  and

these are not profitable deviations for the very same reasons as for the case in which  $\rho < \frac{1}{2-\gamma}$ .

We have to prove that this is the only NE in pure strategies. The strategies available to an executive, whatever his type, are:  $g^e(s, \theta^e) = g^*(s), g^e(s, \theta^e) = A, g^e(s, \theta^e) = B, g^e(s, \theta^e) \neq g^*(s).$ 

Under our assumptions  $g^e(s,1) = A$  cannot be an equilibrium strategy because when s = L a congruent executive will prefer to play B irrespective of voter's beliefs. Under our assumptions  $g^e(s,1) = B$  cannot be an equilibrium strategy because when s = H a congruent executive will prefer to play A irrespective of voter's beliefs. Then  $g^e(s,1) = g^*(s)$  is the only possible candidate for an equilibrium strategy for a type  $\theta_e = 1$ . For analogous reasons it cannot be an equilibrium strategy  $g^e(s,1) \neq g^*(s)$ (that is  $g^e(H,1) = B$  and  $g^e(L,1) = A$ ).

We could have  $g^e(s, \theta^e) = g^*(s)$  for  $\theta^e = 0, 1$ . In that case a policy offer will not signal anything and reputation will remain unchanged through the legislative process. A type  $\theta^e = 0$  will always have an incentive to deviate because doing A will reduce his rent extraction and not increase his ex-post reputation.

Moreover  $g^e(s,1) = g^*(s)$  and  $g^e(s,0) \neq g^*(s)$  cannot be an equilibrium. This is so because one of the following two cases applies: it can be that the assembly every proposal, because the signal that the assembly receives is not precise enough; in this case the non-congruent executive has the incentive to deviate to  $g^e(s,0) = B$ . Otherwise, it can be that the assembly votes according to its signal after having  $g^e = B$ , when  $g^0 = A$ ; in this case  $g^e(s,0) \neq g^*(s)$  is dominated by  $g^e(s,0) = g^*(s)$  that induces a higher reputation and allows the non-congruent executive to implement B more often. Finally it can be that the assembly votes according to its signal after observing  $g^e = A$ , when  $g^0 = B$ ; in this case the non-congruent executive has either an incentive to deviate to  $g^e(H,0) = A$ , or to  $g^e(L,0) = B$ .

A fortiori  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = A$  cannot be an equilibrium because in this case  $\Pr[\theta^e = 1 | g = B] = 1$  (since only the congruent type offers B) therefore doing A brings a reduction in rents and a reduction in reputation.

**Proof of Proposition 3.** We start from the case in which either  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ , or  $g^0 = B$ . As shown in Proposition 1 in both these cases the assembly approves every policy offer, regardless of the received signal  $\sigma$ . Therefore the ex-post reputation after offering A is 1 and the ex-post reputation after offering B is

$$\Pr[\theta^e = 1 | g = B] = \frac{\Pr[g = B | \theta^e = 1] \Pr[\theta^e = 1]}{\Pr[B]} = \frac{\gamma}{2 - \gamma}$$

The strategies  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$  constitute a pure strategy NE if no type of executive has incentive to deviate. When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g^e(H, 0) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right) \ge \overline{Y} - c_A + \varepsilon (1)$$

which is satisfied if  $c_A \geq \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ . This condition also prevents the deviation to  $g^e(L,0) = A$ .

A type  $\theta^e = 1$  could deviate and choose  $g^e(H, 1) = B$ . For this not to be a profitable deviation it must be:

$$\overline{Y} - c_A + u(A, H) + \varepsilon(1) \ge \overline{Y} + u(B) + \varepsilon\left(\frac{\gamma}{2 - \gamma}\right)$$

which is trivially satisfied because of assumption  $u(A, H) - c_A > u(B)$ .

A type  $\theta^e = 1$  could deviate and choose  $g^e(L, 1) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + u(B) + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right) \ge \overline{Y} - c_A + u(A, L) + \varepsilon (1)$$

which is also always satisfied because of assumption  $u(B) > u(A, L) - c_A + \varepsilon$ .

Now let's consider the case in which  $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$ . In this case the assembly, after observing *B* votes according to its signal. In this case the ex-post reputation after offering *A* is

$$\Pr[\theta^e = 1 | g^e = A] = 1;$$

if the executive proposes B its proposal can be either accepted or rejected by the assembly. The ex-post reputations are as follows:

$$\Pr[\theta^{e} = 1 | g^{e} = B, g = B] = \frac{\Pr[g^{e} = B, g = B | \theta_{e} = 1] \Pr[\theta^{e} = 1]}{\Pr[g^{e} = B, g = B]} = \frac{\rho\gamma}{1 - \gamma + \gamma\rho}.$$
$$\Pr[\theta^{e} = 1 | g^{e} = B, g = A] = \frac{\Pr[g^{e} = B, g = A | \theta_{e} = 1] \Pr[\theta^{e} = 1]}{\Pr[g^{e} = B, g = A]} = \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)}.$$

The strategies  $g^e(s, 1) = g^*(s)$  and  $g^e(s, 0) = B$  constitute a pure strategy NE if no type of executive has incentive to deviate. When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g^e(H,0) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + \varepsilon \left( \rho \frac{(1-\rho)\gamma}{(1-\rho)\gamma + (1-\gamma)} + (1-\rho) \frac{\rho\gamma}{1-\gamma+\gamma\rho} \right) \ge \overline{Y} - c_A + \varepsilon (1)$$

which is satisfied if  $c_A \ge \varepsilon \left(1 - \rho \gamma \left(1 - \rho\right) \left(\frac{1}{(1-\rho)\gamma + (1-\gamma)} + \frac{1}{1-\gamma+\gamma\rho}\right)\right)$ . A type  $\theta^e = 0$  could deviate and choose  $g^e(L, 0) = A$ . For this not to be a profitable

A type  $\theta^e = 0$  could deviate and choose  $g^e(L, 0) = A$ . For this not to be a profitable deviation it must be:

$$\overline{Y} + \varepsilon \left( (1-\rho) \frac{(1-\rho)\gamma}{(1-\rho)\gamma + (1-\gamma)} + \rho \frac{\rho\gamma}{1-\gamma+\gamma\rho} \right) \ge \overline{Y} - c_A + \varepsilon (1)$$

which is satisfied if  $c_A \geq \varepsilon \left(1 - \frac{(1-\rho)^2 \gamma}{(1-\rho)\gamma + (1-\gamma)} - \frac{\rho^2 \gamma}{1-\gamma+\gamma\rho}\right)$ . Notice that this condition is implied by the previous one.

A type  $\theta^e = 1$  could deviate and choose  $g^e(H, 1) = B$  or choose  $g^e(L, 1) = A$  and these are not profitable deviations for the very same reasons as for the case in which  $\rho < \frac{1}{2-\gamma}$ .

This equilibrium is unique for the same reasons explained in the proof for the presidential setup.  $\blacksquare$ 

# 8.2 Proof from Section 4 (Presidential System)

**Proof of Proposition 4.** We call  $\hat{\gamma}^a$  the updated belief that the legislators have on the congruence of the executive at the beginning of period two, and  $\hat{\gamma}^v$  the updated belief that the voters have on the congruence of the executive at the beginning of period 2. Notice that  $\hat{\gamma}^a$  is relevant to determine the voting behavior of the legislators in period 2, while  $\hat{\gamma}^v$  is relevant to determine the executive's reputation incentives. Moreover in the presidential system the two beliefs may differ, given that  $\hat{\gamma}^a$  is an update of  $\gamma$  based on  $g_1^e$  and  $s_1$ , while  $\hat{\gamma}^v$  is an update of  $\gamma$  based on  $g_1$  and  $s_1$ , and in general  $g_1$  may differ from  $g_1^{e}$ .

**Equilibrium 1.** The second stage behaves as the one-period model where  $\hat{\gamma}^v$  is the relevant parameter that determines the existence conditions for the equilibria. We now proceed by considering three regions, that differ in terms of legislators' voting behavior:  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ ;  $g^0 = A$  and  $\rho > \frac{1}{2-\gamma}$ ;  $g^0 = B$ .

-  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ : Based on the equilibrium strategies the legislators' beliefs at the

beginning period 2 are:

$$\begin{aligned} \widehat{\gamma}^{a} (A, H) &= \Pr \left( \theta^{e} = 1 | g_{1}^{e} = A, s_{1} = H \right) = 1, \\ \widehat{\gamma}^{a} (A, L) &= \Pr \left( \theta_{e} = 1 | g_{1}^{e} = A, s_{1} = L \right) = \gamma, \\ \widehat{\gamma}^{a} (B, H) &= \Pr \left( \theta^{e} = 1 | g_{1}^{e} = B, s_{1} = H \right) = 0 \\ \widehat{\gamma}^{a} (B, L) &= \Pr \left( \theta^{e} = 1 | g_{1}^{e} = B, s_{1} = L \right) = \gamma \end{aligned}$$

Notice that in this case the beliefs held by the voters on the congruence of the executive at the end of period 1 coincide with the beliefs held by the legislators, that is  $\hat{\gamma}^v(g_1, s_1) = \hat{\gamma}^a(g_1^e, s_1)$ . This is due to the fact that in this parametric region the legislators always approve B when it is offered in the first period, therefore  $g_1 = g_1^e$  always.

- All the beliefs above are computed using Bayes' rule on the equilibrium path except for  $\widehat{\gamma}(A, L)$ . Given that neither type of executive has a predominant incentive to deviate to (A, L) we assume beliefs are passive and set  $\widehat{\gamma}(A, L) = \gamma$ .
- When  $g^0 = A$  a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u(A, H) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \varepsilon$$

$$\geq \overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + \rho u(B) + (1 - \rho)\left(u(A, L) - c_A\right)\right)\right)$$

which is always satisfied by assumption  $u(A, H) - c_A > u(B) > u(A, L) - c_A$ .

When  $g^0 = A$  a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . In this case for  $g_1^e(1, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} - c_A + u(A, L) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \varepsilon\frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)$$

which is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g_1^e(1, H) = A$ . For  $g_1^e(1, H) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right) \ge \overline{Y} - c_A + \overline{Y} + \varepsilon$$

which is satisfied if  $c_A > 2\varepsilon$ .

When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon \ge \overline{Y} - c_A + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon$$

which is satisfied if  $c_A > 0$ .

- The second period equilibrium behavior is unaffected and is described by Proposition 2. The more stringent condition is  $c_A > 2\varepsilon$ .
- $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$ . Reputation  $\widehat{\gamma}^a$  at the end of period 1 will be an update on the prior  $\gamma$  based on observed policy and  $s_1$ , the state of the world and equal to the previous case wehre  $\rho < \frac{1}{2-\gamma}$ .
  - In this case however, the voters' beliefs are different from the legislators' beliefs. Based on the equilibrium strategies, and on the voting behavior of the assembly the voters' beliefs at the beginning of period 2 are:

$$\begin{split} \widehat{\gamma}^{v}\left(A,H\right) &= \Pr\left(\theta^{e} = 1 | g_{1} = A, s_{1} = H\right) = \frac{\gamma}{\gamma + (1 - \gamma)\rho} > \gamma, \\ \widehat{\gamma}^{v}\left(A,L\right) &= \Pr\left(\theta_{e} = 1 | g_{1} = A, s_{1} = L\right) = \gamma, \\ \widehat{\gamma}^{v}\left(B,H\right) &= \Pr\left(\theta^{e} = 1 | g_{1} = B, s_{1} = H\right) = 0 \\ \widehat{\gamma}^{v}\left(B,L\right) &= \Pr\left(\theta^{e} = 1 | g_{1} = B, s_{1} = L\right) = \gamma \end{split}$$

When  $g^0 = A$  a type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u\left(A, H\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right) \\ + \frac{1}{2}\left(\frac{\left(2 - \rho\right)\widehat{\gamma}^v}{1 + \widehat{\gamma}^v - \rho\widehat{\gamma}^v} + \frac{\rho\widehat{\gamma}^v}{1 - \widehat{\gamma}^v + \rho\widehat{\gamma}^v}\right)\varepsilon \\ \ge \overline{Y} + u\left(B\right) + \left(\overline{Y} - \frac{1}{2}\left(-c_A + u\left(A, H\right) + \rho u\left(B\right) + (1 - \rho)\left(-c_A + u\left(A, L\right)\right)\right)\right)$$

where  $\widehat{\gamma}^{v} = \widehat{\gamma}^{v}(A, H)$ . The above condition is always satisfied by assumption since  $u(A, H) - c_{A} > u(B)$ .

When  $g^0 = A$  a type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . For  $g_1^e(1, L) = A$ 

not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + \rho u(B) + (1 - \rho) (-c_A + u(A, L)) \\ + \left( \overline{Y} + \frac{1}{2} (-c_A + u(A, H) + \rho u(B) + (1 - \rho) (-c_A + u(A, L))) \right) \\ + \frac{1}{2} \left( (2 - \rho) \frac{(2 - \rho) \gamma}{1 + \gamma - \rho \gamma} + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} \right) \varepsilon \\ \ge \quad \overline{Y} - c_A + u(A, L) + \left( \overline{Y} + \frac{1}{2} (-c_A + u(A, H) + \rho u(B) + (1 - \rho) (-c_A + u(A, L))) \right) \\ + \frac{1}{2} \left( (2 - \rho) \frac{(2 - \rho) \gamma}{1 + \gamma - \rho \gamma} + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} \right) \varepsilon \end{aligned}$$

which is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = A$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(1, H) = A$ . For  $g_1^e(1, H) = A$  not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + \rho\left(-c_A\right) + \left(\overline{Y} - \frac{1}{2}c_A\right) + \rho \frac{1}{2} \left(\frac{\left(2 - \rho\right)\widehat{\gamma}^v}{1 + \widehat{\gamma}^v - \rho\widehat{\gamma}^v} + \frac{\rho\widehat{\gamma}^v}{1 - \widehat{\gamma}^v + \rho\widehat{\gamma}^v}\right)\varepsilon\\ \geq \quad \overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2} \left(\frac{\left(2 - \rho\right)\widehat{\gamma}^v}{1 + \widehat{\gamma}^v - \rho\widehat{\gamma}^v} + \frac{\rho\widehat{\gamma}^v}{1 - \widehat{\gamma}^v + \rho\widehat{\gamma}^v}\right)\varepsilon\end{aligned}$$

where  $\widehat{\gamma}^{v} = \widehat{\gamma}^{v}(A, H)$ . The condition is satisfied if  $c_{A} > \frac{1}{2} \left( \frac{(2-\rho)\widehat{\gamma}^{v}}{1+\widehat{\gamma}^{v}-\rho\widehat{\gamma}^{v}} + \frac{\rho\widehat{\gamma}^{v}}{1-\widehat{\gamma}^{v}+\rho\widehat{\gamma}^{v}} \right) \varepsilon = \varepsilon\gamma \left( \frac{1+\gamma-\rho\gamma}{2\gamma+\rho-2\rho\gamma} \right)$ .

When  $g^0 = A$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + (1-\rho)(-c_A) + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2}\left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon$$

$$\geq \overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2}\left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon$$

which is satisfied if  $c_A > 0$ . The second period equilibrium behavior is unaffected and is described by proposition 2. The more stringent condition is  $c_A > \epsilon \gamma \left(\frac{1+\gamma-\rho\gamma}{2\gamma+\rho-2\rho\gamma}\right)$ .

-  $g^0 = B$ . Let's now move to the case in which  $g^0 = B$ . In this case, both for the

assembly and for the voters, the beliefs at the beginning of period 2 are:

$$\begin{split} \widehat{\gamma}^{a}\left(A,H\right) &= \widehat{\gamma}^{v}\left(A,H\right) = \Pr\left(\theta^{e} = 1 | g_{1}^{e} = A, s_{1} = H\right) = 1, \\ \widehat{\gamma}^{a}\left(A,L\right) &= \widehat{\gamma}^{v}\left(A,L\right) = \Pr\left(\theta_{e} = 1 | g_{1}^{e} = A, s_{1} = L\right) = \gamma, \\ \widehat{\gamma}^{a}\left(B,H\right) &= \widehat{\gamma}^{v}\left(B,H\right) = \Pr\left(\theta^{e} = 1 | g_{1}^{e} = B, s_{1} = H\right) = 0 \\ \widehat{\gamma}^{a}\left(B,L\right) &= \widehat{\gamma}^{v}\left(B,L\right) = \Pr\left(\theta^{e} = 1 | g_{1}^{e} = B, s_{1} = L\right) = \gamma \end{split}$$

When  $g^0 = B$  a type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u\left(A, H\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right) + \varepsilon$$

$$\geq \overline{Y} + u\left(B\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + \frac{1}{2}u\left(B\right)\right)\right)$$

which is always satisfied by assumption  $u(A, H) - c_A > u(B)$ .

When  $g^0 = B$  a type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . In this case the assembly vote yes after A in the second period, because only the congruent executive proposes A. Therefore the condition for  $g_1^e(1, L) = A$  not to be a profitable deviation is:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} - c_A + u(A, L) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + \frac{1}{2}u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

The condition is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = B$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(1, H) = A$ . For  $g_1^e(1, H) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \overline{Y} \ge \overline{Y} - c_A + \overline{Y} + \varepsilon$$

which is satisfied if  $c_A > \varepsilon$ .

When  $g^0 = B$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon \ge \overline{Y} - c_A + \overline{Y} + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right)$$

The condition is satisfied if  $c_A > 0$ .

The second period equilibrium behavior is unaffected and is described by Proposition 2. The more stringent condition for existence is  $c_A > \varepsilon$ .

Equilibrium 2. First of all notice that in the first period in equilibrium both types of executive propose the efficient policy. As a consequence, the assembly always approves the policy proposed by the executive in the first period. Therefore  $\hat{\gamma}^a(g_1^e, s_1) = \hat{\gamma}^v(g_1, s_1)$ in all parametric regions, given that  $g_1^e = g_1$  for any  $\rho$  and any  $g_0$ . In this case, both for the assembly and for the voters, the beliefs at the beginning of period 2 are:

$$\begin{aligned} \widehat{\gamma}^{a}\left(A,H\right) &= \widehat{\gamma}^{v}\left(A,H\right) = \Pr\left(\theta^{e} = 1|g_{1}^{e} = A, s_{1} = H\right) = \gamma, \\ \widehat{\gamma}^{a}\left(A,L\right) &= \widehat{\gamma}^{v}\left(A,L\right) = \Pr\left(\theta_{e} = 1|g_{1}^{e} = A, s_{1} = L\right) = \text{undetermined}, \\ \widehat{\gamma}^{a}\left(B,H\right) &= \widehat{\gamma}^{v}\left(B,H\right) = \Pr\left(\theta^{e} = 1|g_{1}^{e} = B, s_{1} = H\right) = 0 \\ \widehat{\gamma}^{a}\left(B,L\right) &= \widehat{\gamma}^{v}\left(B,L\right) = \Pr\left(\theta^{e} = 1|g_{1}^{e} = B, s_{1} = L\right) = \gamma \end{aligned}$$

All the beliefs above are computed using Bayes' rule on the equilibrium path except for  $\hat{\gamma}^a(A, L)$  and  $\hat{\gamma}^a(B, H)$ . Given that neither type of executive has a predominant incentive to deviate to (A, L) we assume that  $\hat{\gamma}^a(A, L) = \gamma$ .(passive beliefs)

We assume that  $\widehat{\gamma}^{a}(B,H) = \widehat{\gamma}^{v}(B,H) = \Pr(\theta^{e} = 1|g_{1}^{e} = B, s_{1} = H) < \gamma$ , since, net of the reputation concerns, (B,H) generates a higher utility than (A,H) for a noncongruent executive and a lower utility for a congruent one. This is enough to prove that the congruent executive has no incentive to deviate. However, in order to simplify the analysis of the non-congruent executive, we assumed directly  $\widehat{\gamma}^{a}(B,H) = \widehat{\gamma}^{v}(B,H) =$  $\Pr(\theta^{e} = 1|g_{1}^{e} = B, s_{1} = H) = 0.$ 

-  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$ . When  $g^0 = A$  and  $\rho < \frac{1}{2-\gamma}$  a type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u\left(A, H\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} + u\left(B\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right) + \frac{1}{2}\left(1\right)\varepsilon$$

which is always satisfied by assumption  $u(A, H) - c_A > u(B)$ .

A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . In this case for  $g_1^e(1, L) = A$  not

to be a profitable deviation the following must hold:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} - c_A + u(A, L) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \varepsilon\frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)$$

which is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon \ge \overline{Y} + \overline{Y} - \frac{1}{2} c_A$$

which is satisfied if  $c_A < 2\varepsilon \frac{\gamma}{2-\gamma}$ .

When  $g^0 = A$  a type  $\theta^e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon \ge \overline{Y} - c_A + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon$$

which is satisfied if  $c_A > 0$ .

- The condition for the existence of the equilibrium in the second period is  $c_A > \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ . Since  $\frac{2-2\gamma}{2-\gamma} > 0 \ \forall \gamma$  the equilibrium exists in this region iff  $c_A \in \left(\varepsilon \frac{2-2\gamma}{2-\gamma}, 2\varepsilon \frac{\gamma}{2-\gamma}\right)$ , provided that the interval is well-defined. Notice that such equilibrium never exist if  $\gamma \leq \frac{1}{2}$ .
- $g^0 = A$  and  $\rho \ge \frac{1}{2-\gamma}$ . A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u\left(A, H\right) + \left(\overline{Y} - \frac{1}{2}\left(-c_A + u\left(A, H\right) + \rho u\left(B\right) + (1 - \rho)\left(-c_A + u\left(A, L\right)\right)\right)\right)$$
$$+ \frac{1}{2}\left(\left(2 - \rho\right)\frac{\left(2 - \rho\right)\gamma}{1 + \gamma - \rho\gamma} + \rho\frac{\rho\gamma}{1 - \gamma + \rho\gamma}\right)\varepsilon$$
$$\geq \overline{Y} + u\left(B\right) + \left(\overline{Y} - \frac{1}{2}\left(-c_A + u\left(A, H\right) + \rho u\left(B\right) + (1 - \rho)\left(-c_A + u\left(A, L\right)\right)\right)\right)$$

which is always satisfied by assumption  $u(A, H) - c_A > u(B)$ .

A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . For  $g_1^e(1, L) = A$  not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + u\left(B\right) \\ &+ \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + \rho u\left(B\right) + (1-\rho)\left(-c_A + u\left(A, L\right)\right)\right)\right) \\ &+ \frac{1}{2}\left(\left(2-\rho\right)\frac{\left(2-\rho\right)\gamma}{1+\gamma-\rho\gamma} + \rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon \\ \geq & \overline{Y} - c_A + u\left(A, L\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + \rho u\left(B\right) + (1-\rho)\left(-c_A + u\left(A, L\right)\right)\right)\right) \\ &+ \frac{1}{2}\left(\left(2-\rho\right)\frac{\left(2-\rho\right)\gamma}{1+\gamma-\rho\gamma} + \rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon \end{aligned}$$

which is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = A$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2}\left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon$$

$$\geq \overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right)$$

The condition is satisfied if  $c_A < \frac{1}{2} \left( \frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma} \right) \varepsilon$ .

When  $g^0 = A$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2}\left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon$$

$$\geq \overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A\right) + \frac{1}{2}\left(\frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} + \frac{\rho\gamma}{1-\gamma+\rho\gamma}\right)\varepsilon$$

which is satisfied if  $c_A > 0$ .

The condition for the existence of the equilibrium in the second period is  $c_A > \varepsilon \left( \frac{(2-\rho)\gamma}{1+\gamma-\rho\gamma} - \frac{\rho\gamma}{1-\gamma+\rho\gamma} \right)$ .

-  $g^0 = B$ . Let's now move to the case in which  $g^0 = B$ . A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + u\left(A, H\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} + u\left(B\right) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u\left(A, H\right) + u\left(B\right)\right)\right)$$

which is always satisfied by assumption  $u(A, H) - c_A > u(B)$ .

A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . In this case the assembly vote yes after A in the second period, because only the congruent executive proposes A. Therefore the condition for  $g_1^e(1, L) = A$  not to be a profitable deviation is:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

$$\geq \overline{Y} - c_A + u(A, L) + \left(\overline{Y} + \frac{1}{2}\left(-c_A + u(A, H) + u(B)\right)\right) + \frac{1}{2}\left(\frac{\gamma}{2 - \gamma} + 1\right)\varepsilon$$

The condition is satisfied by assumption since  $u(B) > u(A, L) - c_A + \varepsilon$ .

When  $g^0 = B$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(1, H) = B$ . For  $g_1^e(1, H) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + \overline{Y} + \left(\frac{\gamma}{2 - \gamma}\right)\varepsilon \ge \overline{Y} + \overline{Y}$$

which is satisfied if  $c_A < \left(\frac{\gamma}{2-\gamma}\right)\varepsilon$ .

When  $g^0 = B$  a type  $\theta_e = 0$  could deviate and choose  $g_1^e(0, L) = A$ . For  $g_1^e(0, L) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} + \overline{Y} + \frac{\gamma}{2 - \gamma} \varepsilon \ge \overline{Y} - c_A + \overline{Y} + \varepsilon \left(\frac{\gamma}{2 - \gamma}\right)$$

The condition is satisfied if  $c_A > 0$ .

The condition for the existence of the equilibrium in the second period is  $c_A > \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ . Since  $\frac{2-2\gamma}{2-\gamma} > 0 \quad \forall \gamma$  the equilibrium exists in this region iff  $c_A \in \left(\varepsilon \frac{2-2\gamma}{2-\gamma}, 2\varepsilon \frac{\gamma}{2-\gamma}\right)$ , provided that the interval is well-defined. Notice that such equilibrium never exist if  $\gamma \leq \frac{2}{3}$ .

#### 8.3 Proofs from Section 5 (Parliamentary System)

**Proof of Proposition 5.** A non-congruent legislator always approves any policy proposal in the first period given  $R^a > \varepsilon$ ; in the second period he behaves as in the one period model and mimics the congruent legislator to maximize his final reputation.

A congruent legislator given his utility function and

$$R^{a} \leq \max \left[ u(A, H) - u(B) - c_{A}, u(B) - u(A, L) + c_{A} \right]$$

always votes for what he believes to be the efficient policy in the second period; therefore Proposition 1 holds and the relevant threshold is  $\rho \geq \frac{1}{2-\gamma_2}$ . In the first period a congruent legislator votes maximizing the total probability of implementing the efficient policy over the two periods. Therefore, when  $g^0 = A$ :

- if  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = B$  a congruent legislator follows the signal when  $\rho \geq \frac{1}{2-\gamma}$  as shown in Proof of Proposition 1. When  $\rho < \frac{1}{2-\gamma}$  they always approve B. However the Proof of Proposition 1 does not apply in this case as the legislator may be induced to reject B after  $\sigma_1 = H$  to improve on the expected quality of the executive in the second period. This is never the case as, by approving B, the total probability of implementing the efficient policy over two periods is  $\frac{1-\rho}{1-\gamma\rho}\left(\frac{3+\gamma}{2}-\rho\right)+\rho$  which is larger than  $\frac{3+\gamma}{2}-\frac{1-\rho}{1-\gamma\rho}$ , the total probability when voting against and having a new executive in the second period.
- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = g^*(s_t)$  they always approve *B* because *B* is proposed by any type of executive only when it is the efficient choice;
- if  $g_1^e(s_1, 1) = A$  and  $g_1^e(s_1, 0) = B$  the executive proposal reveal the type of the executive. In the event of  $g_1^e(s_1) = B$  and  $\sigma_1 = H$ , a congruent legislator knows that the executive is non-congruent and believes that A is more likely to be the efficient policy and therefore votes against B. In the event of  $g_1^e(s_1) = B$  and  $\sigma_1 = L$  a congruent legislator expects, by approving B, the correct policy to be implemented with probability  $\rho$  in the first period. If he approves B he is sure of the efficient policy with probability  $\rho$  (since he follows the signal in the second period). The total probability is then  $2\rho$ . If he votes against B instead, the probability is  $(1 \rho)$  in the first period; in the second period however the executive is congruent with probability  $\gamma$  and the probability of the efficient policy is  $\frac{3+\gamma}{2} \rho$ . The total probability is maximized by following the signal if  $\rho \geq \frac{1}{2} + \frac{\gamma}{6}$  and always voting

against otherwise.

- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = A$  they follow the signal because the executive proposal is uninformative as it is not state dependent.

When  $g^0 = B$ :

- if  $g_1^e(s_1, 1) = g^*(s_t)$  and  $g_1^e(s_1, 0) = B$  they always approve A because A is proposed only by the congruent executive when it is the efficient choice
- if  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = g^*(s_t)$  they always approve A because A is proposed by any type of executive only when it is the efficient choice.

**Proof of Proposition 6.** The second stage behaves as the one-period model where we call  $\hat{\gamma}$  the updated reputation of the executive at the beginning of period two. Notice that in the parliamentary system the belief held by legislators on the congruence of the executive,  $\hat{\gamma}^a$ , is the same as the belief held by the voters on the congruence of the executive,  $\hat{\gamma}^v$ , because both legislators and voters have observed, at the beginning of period two,  $g_1^e$  and  $s_1$ . Therefore we call such updated reputation  $\hat{\gamma}$ .

Equilibrium 1. Given the equilibrium behavior of the executive we have that:

$$\begin{split} \widehat{\gamma}\left(A,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=A,s_{1}=H\right)=1,\\ \widehat{\gamma}\left(A,L\right) &= \Pr\left(\theta_{e}=1|g_{1}^{e}=A,s_{1}=L\right)=\gamma,\\ \widehat{\gamma}\left(B,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=H\right)=0,\\ \widehat{\gamma}\left(B,L\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=L\right)=\gamma. \end{split}$$

All the beliefs above are computed using Bayes' rule on the equilibrium path except for  $\widehat{\gamma}(A, L)$ . Given that neither type of executive has a predominant incentive to deviate to (A, L) we assume that  $\widehat{\gamma}(A, L) = \gamma$ .(passive beliefs)

- $\rho < \frac{1}{2-\gamma}$ . In this case the legislators do not follow their signal after B in the first period.
  - A type  $\theta_e = 0$  could deviate and choose  $g_1^e(0, L) = A$  or  $g_1^e(0, H) = A$ , because since  $g^0 = A$  this would ensure being in power in period 2. He has the greatest incentive to deviate when s = H because, whatever  $\widehat{\gamma}(A, L)$  is, the additional gain in reputation is larger after (A; H). For  $g_1^e(0, H) = A$  not to be a profitable

deviation the following must hold:

$$\overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right) \ge \overline{Y} - c_A + \overline{Y} + \varepsilon$$

that is  $c_A \geq 2\varepsilon$ .

A type  $\theta_e = 1$  could deviate and choose  $g_1^e(1,L) = A$ . Let  $\widehat{\gamma}(A,L) > \frac{2\rho-1}{\rho}$ . For  $g_1^e(1,L) = A$  not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + u\left(B\right) + \\ &+ \left(\overline{Y} + \frac{1}{2}\left(u\left(A, H\right) - c_A\right) + \frac{1}{2}u\left(B\right)\right) + \varepsilon \frac{1}{2}\left(1 + \frac{\gamma}{2 - \gamma}\right) \\ \geq & \overline{Y} + \left(u\left(A, L\right) - c_A\right) + \left(\overline{Y} + \frac{1}{2}\left(u\left(A, H\right) - c_A\right) + \frac{1}{2}u\left(B\right)\right) + \varepsilon \frac{1}{2}\left(1 + \frac{\widehat{\gamma}}{2 - \widehat{\gamma}}\right) \end{aligned}$$

The condition becomes

$$c_A \ge u(A,L) - u(B) + \frac{\varepsilon}{2} \left( \frac{\widehat{\gamma}}{2 - \widehat{\gamma}} - \frac{\gamma}{2 - \gamma} \right).$$

which is always satisfied given that  $u(B) > u(A, L) - c_A + \varepsilon$ , and  $\frac{\widehat{\gamma}}{2-\widehat{\gamma}} - \frac{\gamma}{2-\gamma} < 1$ . If instead  $\widehat{\gamma}(A, L) < \frac{2\rho - 1}{\rho}$ , the r.h.s. of the inequality is even lower than the one above.

- A type  $\theta_e = 1$  could never deviate to  $g_1^e(1, H) = B$ , as it delivers lower first period utility, lower second period expected utility and lower final reputation.
- The executive enters the second stage with reputation either 1 or 0 or  $\gamma$ . In the first two cases the second period behavior is trivially an equilibrium one; in the last case the relevant condition is  $c_A \geq \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ , which is implied by the condition  $c_A \geq 2\varepsilon$ .
- $-\rho \geq \frac{1}{2-\gamma}$ . A type  $\theta^e = 0$  could deviate and choose  $g_1^e(0,L) = A$  or  $g_1^e(0,H) = A$ , because this would ensure being in power in period 2, given that  $g_0 = A$ .
- He has the greatest incentive to deviate when s = H because of the additional gain in reputation. For  $g_1^e(0, H) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} - \rho \Gamma c_A + (1 - \rho \Gamma) \left( \overline{Y} - \frac{1}{2} c_A \right) \ge \overline{Y} - c_A + \overline{Y} + \varepsilon$$

that is  $c_A \geq \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}$ . Notice that if a type  $\theta^e = 0$  has no incentive to deviate to  $g_1^e(0, H) = A$  he has even less incentives to deviate to  $g_1^e(0, L) = A$  as the condition that prevents such deviation has the same r.h.s, but a l.h.s that is higher both in terms of first period utility  $(\overline{Y} - (1-\rho)\Gamma c_A)$ , probability of being in power in the second period and final reputation.

When  $g^0 = A$  a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1, L) = A$ . Let  $\widehat{\gamma}(A, L) < \frac{2\rho - 1}{\rho}$ . For  $g_1^e(1, L) = A$  not to be a profitable deviation the following must hold:

$$\begin{split} \overline{Y} + (1 - (1 - \rho) \Gamma) u(B) + (1 - \rho) \Gamma (u(A, L) - c_A) + \\ + (1 - (1 - \rho) \Gamma) \left( \overline{Y} + \frac{1}{2} (u(A, H) - c_A) + \frac{1}{2} (\rho u(B) + (1 - \rho) (u(A, L) - c_A)) \right) \\ + (1 - (1 - \rho) \Gamma) \frac{1}{2} \varepsilon \left( 1 + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} + (1 - \rho) \frac{(1 - \rho) \gamma}{(1 - \rho) \gamma + (1 - \gamma)} \right) \\ \ge \overline{Y} + (u(A, L) - c_A) + \left( \overline{Y} + \frac{1}{2} (u(A, H) - c_A) + \frac{1}{2} (\rho u(B) + (1 - \rho) (u(A, L) - c_A)) \right) \\ + \frac{1}{2} \varepsilon \left( 1 + \rho \frac{\rho \widehat{\gamma}}{1 - \widehat{\gamma} + \rho \widehat{\gamma}} + (1 - \rho) \frac{(1 - \rho) \widehat{\gamma}}{(1 - \rho) \widehat{\gamma} + (1 - \widehat{\gamma})} \right) \end{split}$$

Given that  $\widehat{\gamma} = \gamma$  this condition becomes

$$c_{A} \geq \frac{2}{2-\rho(1-\rho)\Gamma} \begin{cases} (1-(1-\rho)\Gamma)(u(A,L)-u(B)) \\ +(1-\rho)\Gamma(\overline{Y}+\frac{1}{2}u(A,H)+\frac{1}{2}(\rho u(B)+(1-\rho)u(A,L))) \\ +(1-\rho)\Gamma\frac{1}{2}\varepsilon\left(1+\rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}+(1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{cases}$$
$$= \begin{cases} (u(A,L)-u(B))\left[1-(1-\rho)\Gamma-\frac{1}{2}\rho(1-\rho)\Gamma\right] \\ +\left(\frac{u(A,L)+u(A,H)}{2}+\overline{Y}\right)(1-\rho)\Gamma \\ +(1-\rho)\Gamma\frac{1}{2}\varepsilon\left(1+\rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}+(1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{cases}$$

Moreover we replace  $u(B) = \frac{u(A,H)+u(A,L)}{2} - c_A$ , therefore the above condition becomes:

$$c_A \ge \left\{ \begin{array}{c} \frac{1}{(1-\rho)\Gamma} u\left(A,L\right) \left(\frac{1}{2} - \frac{\rho(1-\rho)\Gamma}{4}\right) + \frac{1}{(1-\rho)\Gamma} u\left(A,H\right) \left(-\frac{1}{2} + (1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right) \\ +\overline{Y} + \frac{1}{2}\varepsilon \left(1 + \rho\frac{\rho\gamma}{1-\gamma+\rho\gamma} + (1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{array} \right\}$$

In this case notice that the executive enters the second stage with reputation either 1 or 0 or  $\gamma$ . In the first two cases the second period behavior is trivially an equilibrium

one; in the last case the relevant condition is  $c_A \ge \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ , which is implied by the condition  $c_A \ge \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}$ . Therefore the overall condition is:

$$c_A \ge \max \left\{ \begin{array}{c} \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}, \\ \left\{ \begin{array}{c} \frac{1}{(1-\rho)\Gamma}u\left(A,L\right)\left(\frac{1}{2}-\frac{\rho(1-\rho)\Gamma}{4}\right) + \frac{1}{(1-\rho)\Gamma}u\left(A,H\right)\left(-\frac{1}{2}+(1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right) \\ +\overline{Y}+\frac{1}{2}\varepsilon\left(1+\rho\frac{\rho\gamma}{1-\gamma+\rho\gamma}+(1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{array} \right\} \right\}$$

Equilibrium 2. Given the equilibrium behavior of the executive we have that:

$$\begin{split} \widehat{\gamma}\left(A,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=A,s_{1}=H\right)=\gamma,\\ \widehat{\gamma}\left(A,L\right) &= \Pr\left(\theta_{e}=1|g_{1}^{e}=A,s_{1}=L\right)=\gamma,\\ \widehat{\gamma}\left(B,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=H\right)=0,\\ \widehat{\gamma}\left(B,L\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=L\right)=\gamma. \end{split}$$

All the beliefs above are computed using Bayes' rule on the equilibrium path except for  $\widehat{\gamma}(A, L)$ . Given that neither type of executive has a predominant incentive to deviate to (A, L) we assume that  $\widehat{\gamma}(A, L) = \gamma$ .(passive beliefs).

We assume the reputation  $\hat{\gamma}(B, H) = 0$  as in equilibrium 2 of Proposition 4.

-  $\rho < \frac{1}{2-\gamma}$ : In this case the possible deviations are  $g_1^e(L, 1) = A$  and  $g_1^e(H, 0) = B$ . The condition for  $g_1^e(L, 1) = A$  not to be a deviation is the following:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(u(A, H) - c_A + \varepsilon\right) + \frac{1}{2}\left(u(B) + \varepsilon\frac{\gamma}{2 - \gamma}\right)\right)$$

$$\geq \overline{Y} + \left(u(A, L) - c_A\right) + \left(\overline{Y} + \frac{1}{2}\left(u(A, H) - c_A + \varepsilon\right) + \frac{1}{2}\left(u(B) + \varepsilon\frac{\widehat{\gamma}}{2 - \widehat{\gamma}}\right)\right)$$

that is

$$c_A \ge u(A,L) - u(B) + \frac{1}{2}\varepsilon\left(\frac{\widehat{\gamma}}{2-\widehat{\gamma}} - \frac{\gamma}{2-\gamma}\right)$$

which is always satisfied given that  $\widehat{\gamma}(A, L) = \gamma$ .

The condition for  $g_1^e(H,0) = B$  not to be a profitable deviation is the following:

$$\overline{Y} - c_A + \left(\overline{Y} + \varepsilon \frac{\gamma}{2 - \gamma}\right) \ge \overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right)$$

therefore the equilibrium exists only if  $c_A \leq 2\varepsilon \frac{\gamma}{2-\gamma}$ . Remember that the executive enters the second period, in equilibrium, with a reputation equal to  $\gamma$ . Therefore the condition

for the last period behavior to be an equilibrium one is  $c_A \geq \varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right)$ . Overall the equilibrium condition is  $c_A \in \left[\varepsilon \left(\frac{2-2\gamma}{2-\gamma}\right), 2\varepsilon \frac{\gamma}{2-\gamma}\right]$ ; notice that this interval is non-empty only for  $\gamma > \frac{1}{2}$ .

-  $\rho > \frac{1}{2-\gamma}$ : In this case, given that  $\widehat{\gamma}(A, L) = \gamma > \frac{2\rho-1}{\rho}$ , the condition that ensures that  $g_1^e(L, 1) = A$  is not a profitable deviation for the congruent executive is the following one:

$$\overline{Y} + u(B) + \left(\overline{Y} + \frac{1}{2}\left(u(A, H) - c_A + \varepsilon\right) + \frac{1}{2}\left(\begin{array}{c}\rho\left(u(B) + \varepsilon\frac{\rho\gamma}{1 - \gamma + \rho\gamma}\right)\\ + (1 - \rho)\left(u(A, L) - c_A + \varepsilon\frac{(1 - \rho)\gamma}{1 - \rho\gamma}\right)\end{array}\right)\right)$$

$$\geq \overline{Y} + \left(u(A, L) - c_A\right) + \left(\overline{Y} + \frac{1}{2}\left(u(A, H) - c_A + \varepsilon\right) + \frac{1}{2}\left(u(B) + \varepsilon\frac{\hat{\gamma}}{2 - \hat{\gamma}}\right)\right)$$

The condition reduces to:

$$c_{A} \ge u\left(A,L\right) - u\left(B\right) + \varepsilon \frac{\left(\frac{\widehat{\gamma}}{2-\widehat{\gamma}} - \frac{\rho^{2}\gamma}{1-\gamma+\gamma\rho} - \frac{(1-\rho)^{2}\gamma}{1-\rho\gamma}\right)}{(1+\rho)};$$

this condition is always satisfied given that  $\frac{\left(\frac{\hat{\gamma}}{2-\hat{\gamma}}-\frac{\rho^2\gamma}{1-\gamma+\gamma\rho}-\frac{(1-\rho)^2\gamma}{1-\rho\gamma}\right)}{(1+\rho)} < 1.$ 

The condition for the non-congruent not to find profitable to deviate to  $g_1^e(0, H) = B$  is the following one:

$$\overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A + \frac{1}{2}\varepsilon\left(\frac{\rho\gamma}{1 - \gamma + \rho\gamma} + \frac{(1 - \rho)\gamma}{1 - \rho\gamma}\right)\right) \ge \overline{Y} + \left(\overline{Y} - \frac{1}{2}c_A\right)$$

Therefore the condition becomes

$$c_A \le \frac{\varepsilon}{2} \left( \frac{\rho \gamma}{1 - \gamma + \gamma \rho} + \frac{(1 - \rho) \gamma}{1 - \rho \gamma} \right)$$

Remember moreover that, given that the executive's reputation in equilibrium is  $\gamma$  at the beginning of the second period, the last period behavior is an equilibrium behavior iff

$$c(A) \ge \varepsilon \left( 1 - \rho \gamma \left( 1 - \rho \right) \left( \frac{1}{1 - \rho \gamma} + \frac{1}{1 - \gamma + \gamma \rho} \right) \right)$$

Therefore such equilibrium exists only for

$$c_A \in \left[\varepsilon \left(1 - \rho\gamma \left(1 - \rho\right) \left(\frac{1}{1 - \rho\gamma} + \frac{1}{1 - \gamma + \gamma\rho}\right)\right), \frac{\varepsilon}{2} \left(\frac{\rho\gamma}{1 - \gamma + \gamma\rho} + \frac{(1 - \rho)\gamma}{1 - \rho\gamma}\right)\right]$$

when this interval is non-empty.<sup>5</sup>

**Equilibrium 3.** As shown in Proposition 5, given this equilibrium behavior, congruent legislators always follow their signal in the first period as long as  $\rho \geq \frac{1}{2} + \frac{\gamma}{6}$ . Moreover, such equilibrium is perfectly separating. Therefore, after the first period action, each executive is "recognized" as congruent or non-congruent. Therefore each executive can enter the second stage either with  $\hat{\gamma} = 0$  or with  $\hat{\gamma} = 1$ , as follows:

$$\begin{split} \widehat{\gamma}\left(A,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=A,s_{1}=H\right)=1,\\ \widehat{\gamma}\left(A,L\right) &= \Pr\left(\theta_{e}=1|g_{1}^{e}=A,s_{1}=L\right)=1,\\ \widehat{\gamma}\left(B,H\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=H\right)=0,\\ \widehat{\gamma}\left(B,L\right) &= \Pr\left(\theta^{e}=1|g_{1}^{e}=B,s_{1}=L\right)=0. \end{split}$$

If  $\rho \geq \frac{1}{2} + \frac{\gamma}{6}$  and  $g_0 = A$  the equilibrium conditions are the following ones.

A type  $\theta^e = 0$  could deviate and choose  $g_1^e(0, L) = A$  or  $g_1^e(0, H) = A$ , because since  $g_0 = A$  this would ensure being in power in period 2. He has the greatest incentive to deviate when  $s_1 = H$  because of the additional gain in reputation.

For  $g_{1}^{e}(0, H) = A$  not to be a profitable deviation the following must hold:

$$\overline{Y} - \rho \Gamma c_A + (1 - \rho \Gamma) \left( \overline{Y} - \frac{1}{2} c_A \right) \ge \overline{Y} - c_A + \overline{Y} + \varepsilon$$

that is  $c_A \geq \frac{2\rho\Gamma\overline{Y}+2\varepsilon}{1-\rho\Gamma}$ .

When  $g_0 = A$  a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1, L) = B$ . For  $g_1^e(1, L) = B$  not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + (u(A, L) - c_A) + \left(\overline{Y} + \frac{1}{2}(u(A, H) - c_A) + \frac{1}{2}u(B)\right) + \varepsilon \\ \ge \quad \overline{Y} + (1 - (1 - \rho)\Gamma)u(B) + ((1 - \rho)\Gamma)(u(A, L) - c_A) + \\ + (1 - (1 - \rho)\Gamma)\left(\overline{Y} + \frac{1}{2}(u(A, H) - c_A) + \frac{1}{2}(\rho(u(B)) + ((1 - \rho))(u(A, L) - c_A))\right) \end{aligned}$$

This condition becomes

$$c_{A} \leq \frac{2}{1+\rho-\rho\Gamma+\rho^{2}\Gamma} \left\{ \begin{array}{c} (1-\rho)\,\Gamma\left(\overline{Y}+\frac{1}{2}u\,(A,H)\right) \\ -\left(\frac{1}{2}+\frac{\rho}{2}-\left(1+\frac{\rho}{2}\right)\,(1-\rho)\,\Gamma\right)u(B) \\ +\left(\frac{1}{2}+\frac{\rho}{2}-(1-\rho)\,\Gamma\left(\frac{1}{2}+\frac{\rho}{2}\right)\right)u\,(A,L)+\varepsilon \end{array} \right\}$$

Moreover we replace  $u(B) = \frac{u(A,H)+u(A,L)}{2} - c_A$ , therefore the above condition be-

<sup>&</sup>lt;sup>5</sup>The interval is non-empty for some values of  $\rho$  and  $\gamma$ . We verified it graphically.

comes:

$$c_{A} \leq \left\{ \begin{array}{c} \frac{1}{(1-\rho)\Gamma} u\left(A,L\right) \left(\frac{1+\rho}{4} - \frac{\rho(1-\rho)\Gamma}{4}\right) + \frac{1}{(1-\rho)\Gamma} u\left(A,H\right) \left(-\frac{1+\rho}{4} + (1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right) \\ + \overline{Y} + \frac{\varepsilon}{(1-\rho)\Gamma} \end{array} \right\}$$

Notice that both executive's types enter the second stage by being recognized as being either congruent or non-congruent. Therefore, there is no updating in the final reputation in the second period. As a consequence, the only equilibrium behavior in the second period is  $g_2^e(1,L) = B$ ,  $g_2^e(1,H) = A$ ,  $g_2^e(0,L) = g_2^e(0,H) = B$ , without any additional condition. Therefore the overall existence condition is:

$$c_A \in \left[ \frac{2\rho\Gamma\overline{Y} + 2\varepsilon}{1 - \rho\Gamma}, \left\{ \begin{array}{c} \frac{1}{(1 - \rho)\Gamma}u\left(A, L\right)\left(\frac{1 + \rho}{4} - \frac{\rho(1 - \rho)\Gamma}{4}\right) + \frac{1}{(1 - \rho)\Gamma}u\left(A, H\right)\left(-\frac{1 + \rho}{4} + (1 - \rho)\Gamma\left(1 + \frac{\rho}{4}\right)\right) \\ + \overline{Y} + \frac{\varepsilon}{(1 - \rho)\Gamma} \end{array} \right\} \right]$$

If  $\rho < \frac{1}{2} + \frac{\gamma}{6}$  then the equilibrium does not exist as  $g_1^e(0, H) = A$  is a profitable deviation. In fact:

$$\overline{Y} - c_A < \overline{Y} - c_A + \overline{Y} + \varepsilon$$

Equilibrium 4. First of all notice that the first period actions of each executive are not state dependent. Therefore, we assume that the legislators follow their signal in the first period if they observe a deviation to B. Notice that the described equilibrium no longer exists if both types of legislators approve B in the first period, as the congruent executive always has an incentive to deviate to (B, L).

Such equilibrium is perfectly pooling, and B is never observed as a first period offer. We assume that the reputation after (B, H) is 0 as the congruent executive never has an incentive to deviate to B in H. Moreover we assume that  $\widehat{\gamma}(B, L) = \gamma$  that is not wlog but it does simplify the subsequent comparative statics analysis. Therefore each executive can enter the second stage either with  $\widehat{\gamma} = 0$  or with  $\widehat{\gamma} = \gamma$ , as follows:

$$\widehat{\gamma} (A, H) = \Pr \left( \theta^e = 1 | g_1^e = A, s_1 = H \right) = \gamma,$$
  

$$\widehat{\gamma} (A, L) = \Pr \left( \theta_e = 1 | g_1^e = A, s_1 = L \right) = \gamma,$$
  

$$\widehat{\gamma} (B, H) = \Pr \left( \theta^e = 1 | g_1^e = B, s_1 = H \right) = 0,$$
  

$$\widehat{\gamma} (B, L) = \Pr \left( \theta^e = 1 | g_1^e = B, s_1 = L \right) = \gamma.$$

-  $\rho > \frac{1}{2-\gamma}$ : a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1,L) = B$ . For  $g_1^e(1,L) = B$ 

not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + (u(A,L) - c_A) + \left(\overline{Y} + \frac{1}{2}(u(A,H) - c_A) + \frac{1}{2}(\rho u(B) + (1-\rho)(u(A,L) - c_A))\right) \\ + \frac{1}{2}\varepsilon \left(1 + \rho \frac{\rho\gamma}{1 - \gamma + \rho\gamma} + (1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma + (1-\gamma)}\right) \\ \ge \ \overline{Y} + (1 - (1-\rho)\Gamma)u(B) + (1-\rho)\Gamma(u(A,L) - c_A) + \\ + (1 - (1-\rho)\Gamma)\left(\overline{Y} + \frac{1}{2}(u(A,H) - c_A) + \frac{1}{2}(\rho u(B) + (1-\rho)(u(A,L) - c_A))\right) \\ + (1 - (1-\rho)\Gamma)\frac{1}{2}\varepsilon \left(1 + \rho \frac{\rho\gamma}{1 - \gamma + \rho\gamma} + (1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma + (1-\gamma)}\right) \end{aligned}$$

this condition becomes

$$c_{A} \leq \frac{2}{2 - \rho (1 - \rho) \Gamma} \begin{cases} (1 - (1 - \rho) \Gamma) (u (A, L) - u (B)) \\ + (1 - \rho) \Gamma \left(\overline{Y} + \frac{1}{2}u (A, H) + \frac{1}{2} (\rho u (B) + (1 - \rho) u (A, L))\right) \\ + (1 - \rho) \Gamma \frac{1}{2} \varepsilon \left(1 + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} + (1 - \rho) \frac{(1 - \rho) \gamma}{(1 - \rho) \gamma + (1 - \gamma)}\right) \end{cases}$$

Moreover we replace  $u(B) = \frac{u(A,H)+u(A,L)}{2} - c_A$ , therefore the above condition becomes:

$$c_A \leq \begin{cases} \frac{1}{(1-\rho)\Gamma} u\left(A,L\right) \left(\frac{1}{2} - \frac{\rho(1-\rho)\Gamma}{4}\right) + \frac{1}{(1-\rho)\Gamma} u\left(A,H\right) \left(-\frac{1}{2} + (1-\rho)\Gamma\left(1+\frac{\rho}{4}\right)\right) \\ + \overline{Y} + \frac{1}{2}\varepsilon \left(1 + \rho\frac{\rho\gamma}{1-\gamma+\rho\gamma} + (1-\rho)\frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right) \end{cases}$$

A type  $\theta^e = 0$  could deviate and choose  $g_1^e(0, L) = B$ . For  $g_1^e(0, L) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + \left(\overline{Y} - \frac{1}{2}c_A + \varepsilon \frac{1}{2} \left( \frac{\rho\gamma}{1 - \gamma + \rho\gamma} + \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)} \right) \right)$$
  

$$\geq \overline{Y} - (1 - \rho)\Gamma c_A + \left(1 - (1 - \rho)\Gamma\right) \left(\overline{Y} - \frac{1}{2}c_A + \varepsilon \frac{1}{2} \left( \frac{\rho\gamma}{1 - \gamma + \rho\gamma} + \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)} \right) \right)$$

this condition becomes:

$$c_A \le \frac{2(1-\rho)\Gamma}{2-(1-\rho)\Gamma} \left(\overline{Y} + \varepsilon \frac{1}{2} \left(\frac{\rho\gamma}{1-\gamma+\rho\gamma} + \frac{(1-\rho)\gamma}{(1-\rho)\gamma+(1-\gamma)}\right)\right)$$

Notice that both executive's types enter the second stage with reputation  $\gamma$  therefore the second period equilibrium exists if  $c_A \geq \varepsilon \left(1 - \rho \gamma \left(1 - \rho\right) \left(\frac{1}{(1-\rho)\gamma + (1-\gamma)} + \frac{1}{1-\gamma + \gamma \rho}\right)\right)$ .

Then equilibrium exists if:

$$c_{A} \in \left[ \begin{array}{c} \varepsilon \left(1 - \rho \gamma \left(1 - \rho\right) \left(\frac{1}{(1 - \rho)\gamma + (1 - \gamma)} + \frac{1}{1 - \gamma + \gamma \rho}\right)\right), \\ \frac{2(1 - \rho)\Gamma}{2 - (1 - \rho)\Gamma} \left(\overline{Y} + \varepsilon \frac{1}{2} \left(\frac{\rho \gamma}{1 - \gamma + \rho \gamma} + \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)}\right)\right) \\ \left\{ \begin{array}{c} \frac{1}{(1 - \rho)\Gamma} u \left(A, L\right) \left(\frac{1}{2} - \frac{\rho(1 - \rho)\Gamma}{4}\right) + \frac{1}{(1 - \rho)\Gamma} u \left(A, H\right) \left(-\frac{1}{2} + (1 - \rho)\Gamma\left(1 + \frac{\rho}{4}\right)\right) \\ + \overline{Y} + \frac{1}{2}\varepsilon \left(1 + \rho \frac{\rho \gamma}{1 - \gamma + \rho \gamma} + (1 - \rho) \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + (1 - \gamma)}\right) \end{array} \right\} \right\}$$

-  $\rho < \frac{1}{2-\gamma}$ : a type  $\theta^e = 1$  could deviate and choose  $g_1^e(1, L) = B$ . For  $g_1^e(1, L) = B$  not to be a profitable deviation the following must hold:

$$\begin{aligned} \overline{Y} + (u(A,L) - c_A) + \left(\overline{Y} + \frac{1}{2}(u(A,H) - c_A) + \frac{1}{2}u(B)\right) + \frac{1}{2}\varepsilon\left(1 + \frac{\gamma}{2 - \gamma}\right) \\ \geq & \overline{Y} + (1 - (1 - \rho)\Gamma)u(B) + (1 - \rho)\Gamma(u(A,L) - c_A) + \\ & + (1 - (1 - \rho)\Gamma)\left(\overline{Y} + \frac{1}{2}(u(A,H) - c_A) + \frac{1}{2}u(B) + \frac{1}{2}\varepsilon\left(1 + \frac{\gamma}{2 - \gamma}\right)\right) \end{aligned}$$

this condition becomes:

$$c_A \leq \frac{2}{2 - (1 - \rho)\Gamma} \left\{ \begin{array}{l} \left(1 - (1 - \rho)\Gamma\right)u\left(A, L\right) - \left(1 - \frac{(1 - \rho)\Gamma}{2}\right)u\left(B\right) \\ + \left(1 - \rho\right)\Gamma\left(\overline{Y} + \frac{1}{2}u\left(A, H\right) + \frac{1}{2}\varepsilon\left(1 + \frac{\gamma}{2 - \gamma}\right)\right) \end{array} \right\}$$

Moreover we replace  $u(B) = \frac{u(A,H)+u(A,L)}{2} - c_A$ , therefore the above condition becomes:

$$\left(u\left(A,H\right)-u\left(A,L\right)\right)\left(\frac{3}{4}\left(1-\rho\right)\Gamma-\frac{1}{2}\right)+\left(1-\rho\right)\Gamma\left(\overline{Y}+\frac{1}{2}\varepsilon\left(1+\frac{\gamma}{2-\gamma}\right)\right)>0$$

which is either satisfied  $\forall c_A$  or not.

A type  $\theta^e = 0$  could deviate and choose  $g_1^e(0, L) = B$ . For  $g_1^e(0, L) = B$  not to be a profitable deviation the following must hold:

$$\overline{Y} - c_A + \left(\overline{Y} + \varepsilon \frac{\gamma}{2 - \gamma}\right)$$

$$\geq \overline{Y} - (1 - \rho) \Gamma c_A + (1 - (1 - \rho) \Gamma) \left(\overline{Y} + \varepsilon \frac{\gamma}{2 - \gamma}\right)$$

this condition becomes:

$$c_A \le \frac{(1-\rho)\,\Gamma}{1-(1-\rho)\,\Gamma} \left(\overline{Y} + \varepsilon \frac{\gamma}{2-\gamma}\right)$$

Notice that both executive's types enter the second stage with reputation  $\gamma$  therefore the second period equilibrium exists if  $c_A \geq \varepsilon \frac{2-2\gamma}{2-\gamma}$ . Then equilibrium exists if:

$$c_A \in \left[\varepsilon \frac{2-2\gamma}{2-\gamma}, \frac{(1-\rho)\Gamma}{1-(1-\rho)\Gamma} \left(\overline{Y} + \varepsilon \frac{\gamma}{2-\gamma}\right)\right]$$

and

$$\left(u\left(A,H\right)-u\left(A,L\right)\right)\left(\frac{3}{4}\left(1-\rho\right)\Gamma-\frac{1}{2}\right)+\left(1-\rho\right)\Gamma\left(\overline{Y}+\frac{1}{2}\varepsilon\left(1+\frac{\gamma}{2-\gamma}\right)\right)>0$$

**Full characterization.** There is no equilibrium in which  $g_1^e(H, 1) = B$  because by deviating to  $g_1^e(H, 1) = A$  the congruent executive increases his expected payoff since the efficient policy is always implemented and this is enough to compensate the possible loss in reputation. In addition neither  $g_1^e(s, 1) = A$  and  $g_1^e(s, 0) = s$  nor  $g_1^e(s, 1) = s$  and  $g_1^e(s, 0) = A$  can be equilibria because in both cases B would be approved with probability one from the assembly and therefore one of the two types of executive would like to deviate to B (in particular in the first case the congruent would offer B in L while in the second one the non-congruent would offer B in each state).

**Proof of Proposition 7.** We have shown in Proposition 5 that when  $g^0 = B$  the best response of any type of legislators to the two described equilibrium proposals of the executive is to approve A, moreover the assembly vote is not required when the executive wishes to implement B. Therefore in both equilibria we never observe an election after the first period; hence the executive's incentives to deviate never differ from those analyzed in Proposition 4 for the Presidential System with  $g^0 = B$ . This implies that the conditions for existence are exactly the same.

These are the only two pure-strategy equilibrium behavior of the executive. We cannot have in fact any equilibrium in which  $g_1^e(s_1, 1) = A$ , because by deviating to  $g_1^e(L, 1) = B$  the congruent executive increases his expected payoff because the efficient policy is always implemented and this is enough to compensate the possible loss in reputation. Moreover the following cannot be equilibria:  $g_1^e(s_1, 1) = g^*(s_1)$  and  $g_1^e(s_1, 0) = A$ ,  $g_1^e(s_1, 1) = B$  and  $g_1^e(s_1, 0) = A$ ,  $g_1^e(s_1, 0) = g^*(s_1)$ ; the reason being that in all these cases there is at least a state of the world in which the congruent will offer B while the non-congruent is supposed to offer A and this creates an incentive to deviate to B increasing both first period payoff and reputation. Finally  $g_1^e(s_1, 1) = g_1^e(s_1, 0) = B$  cannot be an equilibrium under the mild assumption that a deviation to A is performed by the congruent executive when the state is high.