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No. 286 – November 2014
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Alice Albonico† Alessia Paccagnini‡ Patrizio Tirelli§
Università degli Studi di Milano - Bicocca
November 2014

Abstract

We estimate a medium scale DSGE model for the Euro Area to gain intuition on the importance of Limited Asset Market Participation (LAMP). Our results suggest that LAMP is sizeable (39% of households over the 1993-2012 sample) and important to understand EMU business cycle, especially, in the light of the recent financial crisis. In comparison with the representative households counterpart, the LAMP model is preferred on the grounds of both the Bayes factor and the average forecasting performance. Given the tighter credit standards we might expect in the near future, the high proportion of LAMP households is likely to remain an important feature of EMU. We also find that the LAMP model leads to conclusions about the main determinants of EMU business cycle that are substantially different from those obtained under the representative agent hypothesis. Given these results, the LAMP hypothesis should be part and parcel of empirical DSGE models of the Euro area.

Keywords: DSGE, Limited Asset Market Participation, Bayesian Estimation, Euro Area, Business Cycle

JEL codes: C11, C13, C32, E21, E32, E37

*The paper is part of the RastaNews project funded by the European Community’s 7th Framework Programme (FP7). The authors thank Guido Ascarì, Nicola Branzoli, Fabio Canova, Efrem Castelnuovo, Rossana Merola, Tiziano Ropele, Stefania Villa for insightful comments and suggestions. We are also grateful to participants to the Society for Nonlinear Dynamics and Econometrics 22nd Annual Symposium (Baruch College CUNY, New York, New York), the 18th Annual International Conference on Macroeconomic Analysis and International Finance (University of Crete), and the 2nd Macro Banking and Finance Workshop (University of Rome "Tor Vergata"). Alice Albonico and Patrizio Tirelli gratefully acknowledge financial support from EC project 320278-RASTANEWS.

†Email: alice.albonico@unimib.it
‡Email: alessia.paccagnini@unimib.it
§Email: patrizio.tirelli@unimib.it
1 Introduction

The 2007 financial crisis has stimulated the search for new developments in Dynamic Stochastic General Equilibrium (DSGE) models that typically assumed complete financial markets and relied on the representative agent assumption (RA henceforth). One widespread feature in the new wave of DSGE models is the distinction between lenders and borrowers (Christiano et al., 2010; Curdia and Woodford, 2010; Gerali et al., 2010; Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Villa, 2014).

These models are suitable for modelling financial and banking shocks but the interest rate policy of the central bank remains a powerful tool, capable of affecting the intertemporal choices of all households. This assumption seems to be at odds with the empirical wealth distribution and with the microeconomic evidence of household behavior. In fact, according to Iacoviello and Pavan (2013), 40% of US households hold no wealth and no debt. Similar figures are observed in the Euro area (see Cowell et al., 2012 for more details.). Anderson et al. (2013) use US microdata to estimate individual-level impulse responses as well as multipliers for government spending and tax policy shocks. They find that the wealthiest individuals behave according to the predictions of standard DSGE models, but the poorest individuals tend to neglect interest rate changes and adopt consumption patterns that closely follow their current disposable income dynamics. For this reason, they suggest that DSGE models should incorporate the Limited Asset Market Participation hypothesis (LAMP henceforth), where a fraction of Non-Ricardian households do not hold any wealth and entirely consume their disposable labor income in each period.

The implications of the LAMP hypothesis have been investigated in a number of theoretical studies (Galí et al., 2004; Bilbiie, 2008; Motta and Tirelli, 2012, 2013, 2014; Albonico and Rossi, 2014). Other theoretical studies have analyzed the potential role played by LAMP in allowing DSGE models to replicate certain business cycle facts, notably the consumption response to public expenditure shocks (Galí et al., 2007; Colciago, 2011) and to investment shocks (Furlanetto et al., 2013), and the reaction of output, hours and consumption to productivity shocks (Furlanetto and Seneca, 2012).
We incorporate the LAMP hypothesis in a medium scale closed economy DSGE model akin to Smets and Wouters (2003, 2007). Some empirical DSGE models of the Euro area (Coenen and Straub, 2005; Ratto et al., 2008; Forni et al., 2009 and Coenen et al., 2012) do account for the LAMP hypothesis. The justification for reconsidering the relative importance of LAMP in the Euro area is based on four considerations that highlight our innovative contribution. The first one concerns modelling strategies adopted in previous studies where households’ preferences are typically based on external consumption habits in differences. From theoretical studies on the LAMP hypothesis (Motta and Tirelli, 2014), it is well known that under external habits in differences the marginal utility of consumption of Non-Ricardian households is an increasing function of the steady state consumption gap between the two households groups. This, in turn, may induce model indeterminacy for a relatively small share of Non-Ricardian households. Thus, since Bayesian estimation techniques constrain estimated parameters to be consistent with model determinacy, such restrictions might bias estimates of the proportion of Non-Ricardian households. We therefore build on Menna and Tirelli (2014) who consider habits in ratios and obtain model determinacy for a relatively large fraction of LAMP. A closely related issue is that previous studies impose that Non-Ricardian households cannot affect wage-setting decisions. This is a potentially serious shortcoming because wage changes have redistributive effects between the two households groups and wage setting decisions may substantially change if they take into account the interests of Non-Ricardian households (Motta and Tirelli, 2014). The second justification for our empirical analysis is that the relative importance of LAMP might well change over different periods. For instance, Bilbiie and Straub (2012, 2013) forcefully argue that structural changes in the degree of asset market participation explain variations in the monetary policy transmission mechanism in the US. For this reason, we shall devote particular effort to investigate how the proportion of Non-Ricardian households has changed over certain sample periods. The third justification is that, unlike previous studies, we provide a formal comparison of the LAMP and RA models, highlighting the differences in forecasting performance, in the importance of different shocks in determining observed volatility, in the implied monetary policy transmission. To the best of our
knowledge, this is the first analysis that explicitly compares the empirical performance of a LAMP model against the standard RA model. Finally, we shall devote particular attention to the role played by different shocks and by monetary policy in determining the EMU years and particularly during the financial crisis.

Our results in a nutshell. We find that the share of LAMP households is sizable throughout the 1972-2012 sample, about 32%. In comparison with the RA counterpart, the LAMP model is preferred on the grounds of both the Bayes factor and the average forecasting performance. As far as the predictive ability is concerned, the LAMP model has a relative advantage in explaining the dynamics of output, consumption, inflation, and investment during the recent financial crisis. Turning to the analysis of subsample periods, we obtain that the importance of LAMP substantially declines in periods of increasing financial integration and optimism in the European financial markets, such as the apparently successful hard European Monetary System (EMS) period. By contrast, the aftermath of the EMS crisis and the 2007-financial crisis are associated with a surge in LAMP. Over the 1993-2012 period, the fraction of LAMP is as high as 39%, well above the 34% estimated for the turbulent and highly regulated 1972-81 decade and the 25% obtained for the 1972-92 period.

To sharpen our analysis of the EMU years, we then focus on the role of shocks in the model estimated over the 1993-2012 period. The Bayes factor provides even stronger support for the LAMP model. The RA and LAMP models yield similar conclusions about the contractionary role of monetary policy shocks during the 2007-2012 period, when inflation was already below the 2% target. By contrast, conclusions about the role of other shocks are quite different. In the RA model, the risk premium shock is the main driver of output volatility. In the LAMP model, instead, risk premium shocks play a limited role, and the investment-specific shock is relatively more important. To grasp intuition, in the RA model an adverse risk premium shock affects all households, who are then induced to reduce both consumption and investment, whereas an adverse investment shock is associated to capital decumulation and, in contrast with empirical evidence, to an increase in consumption. In the LAMP model, instead, the risk premium shock is cushioned
by LAMP households who do not react to asset returns, whereas the adverse investment shock has a powerful effect on LAMP consumers who suffer from a current disposable income shock. These results emphasize the role of the LAMP hypothesis in DSGE models. LAMP has gained increasing acceptance in fiscal policies analyses, but it has been less frequently used in empirical models focusing on monetary policy. Indeed, we find that under LAMP the stochastic disturbances that matter for monetary policy decisions are quite different from those obtained under RA.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 illustrates the estimation methodology. Section 4 discusses the results of Bayesian estimation. Section 5 concludes.

2 The model

There is a continuum of households indexed by $i \in [0, 1]$. A share $1 - \theta$ of households (Ricardian households, $i = o$) can access financial markets, trade government bonds, accumulate physical capital, and rent capital services to firms. The remaining $\theta$ households (Non-Ricardian or LAMP households, $i = rt$) do not have access to financial markets and consume all their disposable labor income. Each household supplies the bundle of labor services $h_i^t = \left\{ \int_0^1 [h_i^t(j)]^{-1} \lambda \, dj \right\}^{1+\lambda^w}$ that firms demand. For each labor type $j$, the wage setting decision is allocated to a specific labor union. At the given nominal wage $W^j_t$, households supply the amount of labor that firms demand

$$h_i^j = \left( \frac{W^j_t}{W_t} \right)^{-\frac{1+\lambda^w}{\lambda \gamma}} h_i^d$$

(1)

where $h_i^d = \int_0^1 h_i^j \, dj$ is the total labor demand. Demand for labor type $j$ is split uniformly across the households, so that households supply an identical amount of labor services, $h_i^t = h_i^d$ as in Colciago (2011). Combining this expression with (1) we obtain:

$$h_t = h_i^d \int_0^1 \left( \frac{W^j_t}{W_t} \right)^{-\frac{1+\lambda^w}{\lambda \gamma}} \, dj$$

(2)
Labor income is:

\[ W^i_t h^i_t = h^d_t \int_0^1 W^j_t \left( \frac{W^j_t}{W^i_t} \right)^{1 + \lambda^w_t} \, dj \]

Here, the parameter \( \lambda^w_t < 1 \) is inversely related to the intratemporal elasticity of substitution between the differentiated labour services supplied by the households, \( \frac{1 + \lambda^w_t}{\lambda^w_t} \). The parameter \( \lambda^w_t \) is interpreted as a net markup in the household-specific labour market and it is assumed to follow an AR(1) process with i.i.d. Normal error term: \( \log (\lambda^w_t) = (1 - \rho_w) \log (\lambda^w_t) + \rho_w \log (\lambda^w_{t-1}) + \eta^w_t \).

Households preferences are

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1 - \sigma} \left( \frac{c^i_t}{c^{t-1}_t} \right)^{b \sigma} \exp \left( \frac{(\sigma - 1)}{1 + \phi_t} (h^i_t)^{1+\phi_t} \right) \right\} \] (3)

where \( c^i_t = \frac{C^i_t}{z_t} \) and \( c_t = \frac{C_t}{z_t} \) are individual and total real consumption levels normalized by a labour-augmenting permanent technology shock, \( z_t \), such that \( \frac{z_t}{z_{t-1}} = g_{z,t} \) which evolves according to:

\[ \log (g_{z,t}) = (1 - \rho_{g_z}) \log (g_z) + \rho_{g_z} \log (g_{z,t-1}) + \eta^g_{t} \]

where \( \eta^g_{t} \) is an i.i.d. Normal innovation term.

Parameter \( 0 < b < 1 \) measures the degree of external habit in consumption. Differently from Smets and Wouters (2007) who use habits in differences, our specification here is based on habits in ratios. According to a popular view, the specification chosen for characterizing consumption habits has little importance in DSGE models based on the representative agent hypothesis (Dennis, 2009). As a matter of fact, an infinite or negative marginal utility of consumption might occur under the subtractive formulation of the consumption-to-habit surplus (Carroll, 2000). This outcome is even more likely in macro models that account for external habits, agents heterogeneity and consumption inequality. More specifically to our context, Motta and Tirelli (2013) show that to avoid indeterminacy in LAMP models a relatively strict upper limit must be imposed onto the
value of $\theta$ and/or to the difference habit parameter. The issue is potentially even more relevant here because the standard Dynare estimation routine forces estimates of the posterior distribution to be located in the determinacy region, potentially excluding large values of $\theta$, and imposing a downward bias on its estimated value.\footnote{Even if priors are imposed to avoid indeterminacy region, whenever an invalid posterior draw is encountered, this proposed draw is discarded and the current entry of the MonteCarlo Markov Chain (MCMC) is set to the previous draw. In technical terms, the proposed draw obtains likelihood 0, is rejected, and the MCMC continues. More details in An and Schorfheide (2007).} According to Menna and Tirelli (2014), indeterminacy turns out to be a lesser problem under the habit-in-ratio specification adopted in (3). Other contributions to this literature constrain habits to be driven by peer-specific consumption levels (Forni et al., 2009; Cogan et al. 2010), and therefore deviate from the "keeping up with the Joneses" hypothesis that is based on observed interactions amongst heterogeneous consumers (Chan and Kogan 2002; Boyce et al. 2010; Frank et al. 2010; Drechsel-Grau and Schmid, 2014). In our context, this choice is open to criticism because it limits the interaction between the two households groups that crucially affects both consumption choices of the Ricardian households and wage-setting decisions (see Motta and Tirelli, 2013).\footnote{Coenen and Straub (2005) and Coenen, Straub and Trabandt (2012) impose that preferences of Non-Ricardian households do not affect wage-setting decisions.}

Right from the outset it is worth noting that our model accounts for tax rates levied on wage and capital incomes and on households consumption, $\tau^w$, $\tau^k$ and $\tau^c$ respectively, and for social contributions levied on labor incomes $\tau^{wh}$. In addition, the model incorporates payroll tax rates on firms, $\tau^{wf}$, nominal lump sum taxes $T^i$ and transfers $TR^i$. Investigating the role of countercyclical fiscal policies is beyond the scope of the paper, therefore we shall maintain that such taxes are held constant at their steady state level.\footnote{The only exception are time-varying lump-sum taxes levied on Ricardian households, necessary to ensure that the government intertemporal budget constraint is satisfied in presence of shocks that affect public debt accumulation.} This is sufficient to incorporate realistic calibrations of the steady-state effect of tax policies on consumption differences between the two consumer groups for any estimated value of $\theta$.\footnote{Motta and Tirelli (2013b) show that steady state redistributive taxation has powerful effects in limiting the indeterminacy region in LAMP models.}
2.1 Ricardian households

The Ricardian household budget constraint is:

\[
(1 + \tau^c) P_t C^o_t + P_t I^o_t + \frac{B^o_{t+1}}{\varepsilon^b_t} = R_{t-1} B^o_t + (1 - \tau^l - \tau^{wh}) W_t h^o_t + P_t D^o_t + \left(1 - \tau^h\right) \left[R^k_t u^o_t - a (u^o_t P_t)\right] K^o_t + \tau^k \delta P_t K^o_t + T R^o_t - T^o_t
\]

Ricardian households allocate their resources between consumption \(C^o_t\), investments \(I^o_t\) and government-issued bonds \(B^o_t\). They receive income from labor services, from dividends \(D^o_t\), from renting capital services \(u^o_t K^o_t\) at the rate \(R^k_t\) and from holding government bonds. Here \(P_t\) is the consumption price index, \(R_t\) is the nominal interest rate, \(K^o_t\) is the physical capital stock and \(u^o_t\) defines capacity utilization. \(T R^o_t\) are transfers Ricardian households and \(T^o_t\) are lump-sum taxes. \(\varepsilon^b_t\) is a risk premium shock that affects the intertemporal margin, creating a wedge between the interest rate controlled by the central bank and the return on assets held by the households. It is assumed to follow a first-order autoregressive process with an i.i.d. Normal error term:

\[
\log (\varepsilon^b_t) = (1 - \rho_b) \log (\varepsilon^b_t) + \rho_b \log (\varepsilon^b_{t-1}) + \eta^b_t
\]

The capital stock evolves as follows:

\[
K^o_{t+1} = (1 - \delta) K^o_t + \varepsilon^i_t \left[1 - S \left(\frac{I^o_t}{I^o_{t-1}}\right)\right] I^o_t
\]

where \(\delta\) is the depreciation rate and \(\varepsilon^i_t\) denotes an investment-specific technology shock that affects the real price of investment. It is assumed to evolve as an AR(1) process with i.i.d. Normal innovation term: \(\log (\varepsilon^i_t) = (1 - \rho_i) \log (\varepsilon^i_t) + \rho_i \log (\varepsilon^i_{t-1}) + \eta^i_t\).

The term \(S \left(\frac{I^o_t}{I^o_{t-1}}\right)\) represents investment adjustment costs. In line with Christoffel et al. (2008, CCW henceforth), the adjustment costs function is:
\[
S \left( \frac{I_t^o}{I_{t-1}^o} \right) = \frac{\gamma_t}{2} \left( \frac{I_t^o}{I_{t-1}^o} - g_z \right)^2
\]

where \( g_z \) is the steady state trend growth rate of the economy. The intensity of utilizing physical capital is subject to a proportional cost, as in Christiano et al. (2005):

\[
a_t^o = \gamma_{u1} (u_t^o - 1) + \frac{\gamma_{u2}}{2} (u_t^o - 1)^2
\]

The Ricardian households maximize (3) with respect to \( C_t^o, B_{t+1}, I_t^o, K_{t+1}^o, u_t^o \), subject to (4), (5), (6) and (7). The first order conditions are:

\[
\frac{(c_t^o)^{\sigma} (c_{t-1}^o)^{(1 - \sigma)}}{(1 + \tau_t^c)} \exp \left( \frac{(\sigma - 1) (h_t^o)^{1 + \phi_t}}{1 + \phi_t} \right) \frac{1}{\pi_t} = \Lambda_t^o
\]

\[
R_t = \pi_{t+1} \frac{\Lambda_t^o}{\beta \varepsilon_t^o \Lambda_{t+1}^o}
\]

\[
1 = Q_t^o \varepsilon_t^o \left\{ 1 - \gamma_I \left( \frac{I_t^o}{I_{t-1}^o} - g_z \right) \frac{I_t^o}{I_{t-1}^o} - \frac{\gamma_I}{2} \left( \frac{I_t^o}{I_{t-1}^o} - g_z \right)^2 \right\}
\]

\[
+ \frac{\Lambda_{t+1}^o}{\Lambda_t^o} Q_{t+1}^o \varepsilon_{t+1}^o \beta \gamma_I \left( \frac{I_{t+1}^o}{I_t^o} - g_z \right) \left( \frac{I_{t+1}^o}{I_t^o} \right)^2
\]

\[
\frac{\Lambda_{t+1}^o}{\Lambda_t^o} \beta \left\{ (1 - \tau^k) \left[ \frac{R_{t+1}^k}{P_{t+1}^k} u_{t+1}^o - a (u_{t+1}^o) \right] + \tau^k \delta + Q_{t+1}^o (1 - \delta) \right\} = Q_t^o
\]

\[
\frac{R_t^k}{P_t^k} = \gamma_{u1} + \gamma_{u2} (u_t^o - 1)
\]

where \( \Lambda_t^o / P_t \) and \( \Lambda_t^o Q_t^o \) are the Lagrange multipliers associated respectively with (4) and (5). \( \Lambda_t^o \) represents the shadow price of a unit of consumption good, thus equation (8) shows the marginal utility of consumption out of income. We define \( \pi_t = \frac{P_t}{P_{t-1}} \) as the gross rate of inflation. Equation
(9) is the Euler equation. $Q_t^o$ measures the shadow price of a unit of investment good. Equations (10) and (11) are the first order conditions for investment and capital respectively. Equation (12) equals the return from capital utilization to its cost. The latter equation implies that $u_t^o$ is identical across Ricardian households, so that $u_t^o = u_t$.

2.2 Non-Ricardian households

LAMP households consume their disposable labor income in each period:

$$
(1 + \tau^c) P_t C_i^{rt} = (1 - \tau^l - \tau^{wh}) W_t^{rt} h_t^{rt} + T R_t^{rt} - T_t^{rt}
$$

(13)

2.3 Wage setting

Nominal wages are staggered à la Calvo (1983). In each period, union $j$ receives permission to optimally reset the nominal wage with probability $(1 - \xi_w)$. Those unions that cannot re-optimize the wage adjust the wage according to the following scheme:

$$
W_t^j = g_{zt} \pi_t^{x_w} \tilde{\pi}_t^{(1-x_w)} W_{t-1}^j
$$

where $\tilde{\pi}_t$ is the exogenous trend inflation rate.

Following Colciago (2011), we assume that the representative union objective function is a weighted average $(1 - \theta, \theta)$ of the two households types’ utility functions:

$$
\max_{\tilde{W}_t^j} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left\{ \begin{array}{l}
\frac{1-\theta}{1-\sigma} \left( \frac{c_{t+s}^r}{(c_{t+s}+\bar{c}_{t+s})^r} \right)^{1-\sigma} \exp \left( \frac{(\sigma-1)}{1+\phi_t} \left( h_{t+s}^o (h_{t+s}^o)^{1+\phi_t} \right) \right) \\
+ \frac{\theta}{1-\sigma} \left( \frac{c_{t+s}^r}{(c_{t+s}+\bar{c}_{t+s})^r} \right)^{1-\sigma} \exp \left( \frac{(\sigma-1)}{1+\phi_t} \left( h_{t+s}^{rt} (h_{t+s}^{rt})^{1+\phi_t} \right) \right)
\end{array} \right\}
$$

subject to (2), (4) and (13). The corresponding FOC is:
\[ 0 = E_t \sum_{s=0}^{\infty} (\xi_w \beta^s (c_{t+s-1})^{b(\sigma-1)} \exp \left( \frac{(\sigma - 1)}{1 + \phi_l} (h_{t+s})^{1+\phi_l} \right) h_{t+s}^j. \]

\[ \left\{ \begin{array}{l}
  \tilde{W}_t^{j} \left( 1 - r_t^{t+s} - r_{t+s}^{t+s} \right) g_{z,t+t+s} \pi_{t+t+s}^{t+s} \left( 1 - \frac{\lambda_t^{t+s}}{\lambda_{t+s}^{t+s}} \right) \left[ (1 - \theta) (\epsilon_{t+s}^{p})^{1-\sigma} + \theta (\epsilon_{t+s}^{r})^{1-\sigma} \right] \\
  + \frac{1+\lambda_t^{t+s}}{\lambda_{t+s}^{t+s}} \left[ (1 - \theta) (\epsilon_{t+s}^{p})^{1-\sigma} MRS_{t+s}^{p} + \theta (\epsilon_{t+s}^{r})^{1-\sigma} MRS_{t+s}^{r} \right]
\end{array} \right. \]

where:

\[ \pi_{t,t+s-1} = \left\{ \begin{array}{ll}
  1 & \text{for } s = 0 \\
  \pi_t, \pi_{t+1}, \ldots, \pi_{t+s-1} & \text{for } s = 1, 2, \ldots.
\end{array} \right. \]

\[ \bar{\pi}_{t,t+s} = \left\{ \begin{array}{ll}
  1 & \text{for } s = 0 \\
  \bar{\pi}_t, \bar{\pi}_{t+1}, \ldots, \bar{\pi}_{t+s} & \text{for } s = 1, 2, \ldots.
\end{array} \right. \]

\[ MRS_{t}^{p} = -\frac{U_{h}^{p} (c_{t}^{p}, h_{t}^{p})}{U_{c}^{p} (c_{t}^{p}, h_{t}^{p})} = c_{t}^{p} (h_{t}^{p})^{\phi_l} \]

\[ MRS_{t}^{r} = -\frac{U_{h}^{r} (c_{t}^{r}, h_{t}^{r})}{U_{c}^{r} (c_{t}^{r}, h_{t}^{r})} = c_{t}^{r} (h_{t}^{r})^{\phi_l} \]

and \( g_{z,t,t+s} = \prod_{s=1}^{s} g_{z,t+s} \).

The aggregate wage index dynamic equation is:

\[ W_t = \left[ \xi_w \left( g_{z,t} \bar{\pi}_{t-1} \right)_{t-1} W_{t-1} \left( \frac{1}{\lambda_t^{\pi}} \right) + (1 - \xi_w) \left( \tilde{W}_t \right) \right] \frac{1}{\lambda_t^{\pi}} \]

### 2.4 Firms

#### 2.4.1 Final good firms

The final good \( Y_t \) is produced under perfect competition. A continuum of intermediate inputs \( Y_t (z) \) is combined as in Kimball (1995). The final good producers maximize profits:
\[
\max_{Y_t, Y_t^z} P_t Y_t - \int_0^1 P_t^z Y_t^z \, dz
\]

\[
\text{s.t. } \int_0^1 G \left( \frac{Y_t^z}{Y_t} ; \lambda_t^p \right) \, dz = 1
\]

with \( G \) strictly concave and increasing and \( G(1) = 1 \) and \( \lambda_t^p \) is the net price markup, which is assumed to follow an AR(1) process with i.i.d. Normal error term: \( \log (\lambda_t^p) = (1 - \rho_p) \log (\lambda_t^p) + \rho_p \log (\lambda_{t-1}^p) + \eta_t^p \).

From the first order conditions, we obtain:

\[
Y_t^z = Y_t G^{\rho-1} \left[ \frac{P_t^z}{P_t} \int_0^1 G' \left( \frac{Y_t^z}{Y_t} \right) \left( \frac{Y_t^z}{Y_t} \right) \, dz \right]
\]

### 2.4.2 Intermediate good firms

Intermediate firms \( z \) are monopolistically competitive and use as inputs capital and labor services, \( u_t^z K_t^z \) and \( h_t^z \) respectively. The production technology is:

\[
Y_t^z = \varepsilon_t^a [u_t^z K_t^z]^\alpha [z_t h_t^z]^{1-\alpha} - z_t \Phi
\]

where \( \Phi \) are fixed production costs and \( \varepsilon_t^a \) defines a transitory productivity shock:

\[
\varepsilon_t^a = \rho_t^a \varepsilon_{t-1}^a + \eta_t^a
\]

and \( \eta_t^a \) is an i.i.d. Normal innovation term. Profits maximization leads to the following:

\[
\frac{u_t K_t}{h_t} = \frac{\alpha}{(1 - \alpha)} \frac{(1 + \tau^{wf}) W_t}{R_t^k}
\]

(14)

In this framework, the capital-labour ratio is equal across firms and the marginal cost is therefore equal across firms:
\[ MC_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left( \varepsilon_t^a \right)^{-1} z_t^{-(1-\alpha)} \left( \mathcal{R}^*_{t+1} \right)^{\alpha} \left[ (1 + \tau^w) W_t \right]^{1-\alpha} \] (15)

**Price setting** Intermediate goods prices are sticky à la Calvo (1983). Firm \( z \) receives permission to optimally reset its price with probability \( (1 - \xi_p) \). Firms that cannot re-optimize adjust the price according to the following scheme:

\[ P_t^z = \frac{\chi_p}{\pi_{t-1}^{t+1}} \frac{1-\xi_p}{P_{t-1}} P_t^z \]

The representative firm chooses the optimal price \( \tilde{P}_t^z \) that expected maximizes profits:

\[
\max_{\tilde{P}_t^z} E_t \sum_{s=0}^{\infty} \xi_p^{s} \Xi_{t,t+s} \left[ \frac{\tilde{P}_t^z \chi_p}{\pi_{t,t+s-1}^{t+1}} \frac{1-\xi_p}{P_{t+s}} Y_{t+s}^z - MC_{t+s} Y_{t+s}^z \right]
\]

subject to

\[ Y_{t+s}^z = G^{-1} \left( \frac{\tilde{P}_t^z \pi_{t,t+s-1}^{t+1} \chi_p}{P_{t+s}} \right) \frac{1}{0} \int G' \left( \frac{Y_{t+s}^z}{Y_{t,s}^z} \right) \frac{Y_{t+s}^z}{Y_{t,s}^z} dZ \]

where \( MC_t \) is the nominal marginal cost and \( \Xi_{t,t+s} \) is the stochastic discount factor for real payoffs:

\[ \Xi_{t,t+s} = \varepsilon_{t+s}^{b} \beta_{t+s}^{\Lambda_t^{o}} \Lambda_t^{o} \]

Following Smets and Wouters (2007), we define \( \omega_t = \frac{\tilde{P}_t^z}{P_t^z} \int_0^1 G' \left( \frac{Y_t^z}{Y_t^z} \right) \frac{Y_t^z}{Y_t^z} dZ \) and \( x_t = G^{-1} (\omega_t) \), hence the first order condition is:

\[
E_t \sum_{s=0}^{\infty} \xi_p^{s} \Xi_{t,t+s}^{z} \left[ \tilde{P}_t^z \chi_p \pi_{t,t+s-1}^{t+1} \frac{1-\xi_p}{\pi_{t,t+s-1}^{t+1}} - MC_{t+s} \right] \frac{1}{G^{-1} (\omega_{t+s})} \frac{G' (x_{t+s})}{G'' (x_{t+s})} = 0
\]

The aggregate price index dynamic equation is:
\[ P_t = (1 - \xi_p) \frac{\tilde{P}_t}{P_t} G_t^{\alpha} \left( \int_0^1 G' \left( \frac{Y_{t+2}^z}{Y_{t+2}} \right) \frac{Y_{t+2}^z}{Y_{t+2}} dz \right) \]

\[ + \xi_p \tilde{\pi}_{t-1} \pi_t^{1-\theta_p} P_{t-1} G_t^{\alpha-1} \left( \int_0^1 G' \left( \frac{Y_{t+2}^z}{Y_{t+2}} \right) \frac{Y_{t+2}^z}{Y_{t+2}} dz \right) \]

### 2.5 Fiscal policy

The government budget constraint in nominal terms is:

\[ P_t G_t + R_{t-1} B_t + TR = B_{t+1} + T_t + \tau^c P_t C_t + (\tau^l + \tau^{wh} + \tau^{wf}) W_t h_t + \tau^k [R_t^k u_t - (a(u_t) + \delta) P_t] K_t \]

where \( G_t \) is public spending and the adjusted value \( g_t = G_t / z_t \) is assumed to follow an exogenous AR(1) process with i.i.d. Normal innovation.

### 2.6 Monetary policy

Following CCW, the monetary authority sets the nominal interest rate according to a log-linear Taylor rule:

\[ \hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) \left( \hat{\pi}_t + \phi_\pi \left( \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \phi_y \hat{y}_t \right) + \phi_{\Delta \pi} \left( \hat{\pi}_t - \hat{\pi}_{t-1} \right) + \phi_{\Delta y} \left( \hat{y}_t - \hat{y}_{t-1} \right) + \hat{\varepsilon}_t^r \]

where the hatted variables define log-deviations from steady state. In particular, \( \hat{y}_t = \hat{Y}_t / z_t \) is the logarithmic deviation of observed output from the trend output level implied by the permanent technology component. Variable \( \hat{y}_t \) is also interpreted as the output gap measure. \( \varepsilon_t^r \) is a monetary shock that follows a first-order autoregressive process with an i.i.d. Normal error term.
\[
\log (\varepsilon_i^r) = (1 - \rho_r) \log (\varepsilon^r) + \rho_r \log (\varepsilon_{i-1}^r) + \eta_i^r
\]

### 2.7 Aggregation

The relationship between aggregate and individual variables is:

\[C_t = \theta C_t^* + (1 - \theta) C_t^o\]
\[K_t = (1 - \theta) K_t^o\]
\[I_t = (1 - \theta) I_t^o\]
\[B_t = (1 - \theta) B_t^o\]
\[d_t = (1 - \theta) d_t^o\]

### 2.8 Market clearing

The aggregate resource constraint:

\[Y_t = C_t + G_t + I_t + a(u_t) K_t\]

Labor market clearing:

\[h_t = \int_0^1 h_t^d dj\]
\[= h_t^d \int_0^1 \left( \frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_t}} dj\]
\[= s_{W,t} h_t^d\]

---

Footnote: Aggregate and average variables here coincide. For this reason, wealth holdings of Ricardian households are larger than the corresponding aggregates.
where $s_{W,t} = \int_0^1 \left( \frac{W_j}{W_t} \right)^{- \frac{1 + \lambda^w}{\lambda^w}} dz$ is the wage dispersion across the differentiated labor services.

Capital market:

$$u_t K_t = u_t \int_0^1 K^*_t dz$$

Firms’ aggregate demand for labor input:

$$h^d_t = \int_0^1 h^*_t dz$$

Good market:

$$\int_0^1 Y^*_t dz = \int_0^1 \left( \frac{P^*_t}{P_t} \right)^{- \frac{1 + \lambda^p}{\lambda^p}} dz Y_t = s_{P,t} Y_t$$

where $s_{P,t} = \int_0^1 \left( \frac{P^*_t}{P_t} \right)^{- \frac{1 + \lambda^p}{\lambda^p}} dz$ is the price dispersion across differentiated goods.

Note that both $s_{W,t}$ and $s_{P,t}$ vanish in the log-linearized version of the model.

### 3 Estimation strategy

Our observables are seven Euro area time series: inflation, the short term nominal interest rate, employment and the real levels of output, private consumption, investments and compensation per employee\(^6\). Inflation has been calculated as the log difference in the GDP deflator. Output, consumption, investments, and wages are transformed in log differences; instead, total employment has been detrended with a linear trend. The data sample is 1972Q2-2012Q4.

Following CCW, the auxiliary equation is as follows:

$$\hat{e}_t = \frac{\beta}{1 + \beta} E_t \hat{e}_{t+1} + \frac{1}{1 + \beta} \hat{e}_{t-1} + \frac{(1 - \xi_e)(1 - \beta \xi_e)}{(1 + \beta) \xi_e} \left( \hat{h}_t - \hat{e}_t \right)$$

\(^6\)We use quarterly data from the AWM database (Fagan, Henry and Mestre, 2001, 13\(^{th}\) update).
relates the employment variable, \( e_t \), to the unobserved worked-hours variable, \( h_t \).

The corresponding measurement equations are:

\[
Y_t = \begin{bmatrix}
\Delta \ln y_t \\
\Delta \ln c_t \\
\Delta \ln i_t \\
\Delta \ln w_t \\
\ln e_t \\
\Delta \ln P_t \\
\ln R^e_t 
\end{bmatrix} = \begin{bmatrix}
\gamma \\
\gamma \\
\gamma \\
\gamma \\
\bar{\varepsilon} \\
\pi_* \\
\bar{\pi} 
\end{bmatrix} + \begin{bmatrix}
y_t - y_{t-1} \\
c_t - c_{t-1} \\
i_t - i_{t-1} \\
w_t - w_{t-1} \\
e_t \\
\pi_t \\
r_t 
\end{bmatrix}
\]

where \( \ln \) denotes 100 times log and \( \Delta \ln \) refers to the log difference. \( \bar{\gamma} = 100(g_z - 1) + g_{z,t} \) is the common quarterly trend growth rate to real GDP, consumption, investment and wages, where \( g_{z,t} \) is the permanent technology shock.\(^8\) Further, \( \pi_* = 100(\pi - 1) \) is the quarterly steady-state inflation rate, \( \bar{\pi} = 100(\beta^{-1}g_z\pi - 1) \) is the steady-state nominal interest rate, and \( \bar{\varepsilon} \) is the steady-state employment, which is normalized at zero.

To parameterize and evaluate DSGE models, several econometric procedures have been proposed. Kydland and Prescott (1982) use calibration, Christiano and Eichenbaum (1992) consider the generalized method of moments (GMM) estimation of equilibrium relationships, while Rotemberg and Woodford (1996) and Christiano et al. (2005) use the minimum distance estimation based on the discrepancy among VAR and DSGE model impulse response functions. Moreover, the full-information likelihood-based estimation is considered by Altug (1989), McGrattan (1994), Leeper and Sims (1994) and Kim (2000). In last years, Bayesian estimation became very popular. As stressed by An and Schorfheide (2007), there are essentially three main characteristics. First, the Bayesian estimation is system-based and fits the solved DSGE model to a vector of aggregate time series, as opposed to the GMM which is based on equilibrium relationships, such as the Euler

\(^7\)Parameter \( \xi_e \) determines the sensitivity of employment with respect to worked hours.

\(^8\)Note that we are not estimating this shock, so that we consider only a deterministic trend, similarly to Smets and Wouters (2007).
equation for the consumption or the monetary policy rule. Second, it is based on the likelihood function generated by the DSGE model rather than the discrepancy between DSGE responses and VAR impulse responses. Third, prior distributions can be used to incorporate additional information into the parameter estimation.

On a theoretical level, the Bayesian estimation takes the observed data as given, and treats the parameters of the model as random variables. In general terms, the estimation procedure involves solving the linear rational expectations model described in the previous Section. The solution can be written in a state space form, i.e. as a reduced form state equation augmented by the observation (measurement) equations. At the next step, the Kalman Filter is applied to construct the likelihood function. Prior distributions are important to estimate DSGE models. According to An and Schorfheide (2007), priors might downweigh regions of the parameter space that are at odds with observations which are not contained in the estimation sample. Priors could add curvature to a likelihood function that is (nearly) flat for some parameters, given a strong influence to the shape of the posterior distribution. Posterior distribution of the structural parameters is formed by combining the likelihood function of the data with a prior density, which contains information about the model parameters obtained from the other sources (microeconometrics, calibration, and cross-country evidence), thus allowing to extend the relevant data beyond the time series that are used as observables. Numerical methods such as Monte-Carlo Markov-Chain (MCMC) are used to characterize the posterior with respect to the model parameters.\footnote{See Smets and Wouters (2003, 2007), Dynare Manual and An and Schorfheide (2007) for more details on Bayesian estimation of DSGE models.}

### 3.1 Calibration and priors

Following the recent medium scale DSGE models, we calibrate a number of parameters (Table 1). In particular, the discount factor $\beta$ is fixed at 0.99, in line with a steady-state real interest rate of about 4%. The steady-state depreciation rate $\delta$ is 0.025, corresponding to a 10% depreciation rate per year. The capital share $\alpha$ is set at 0.3. The monetary authority’s long-run (net) annualized inflation objective $\bar{p} - 1$ is 1.9%, consistent with the ECB’s quantitative definition of price stability.
The steady state growth rate \( g_z \) is set at 2% in annual terms, in line with CCW. The elasticity of the demand for goods is set at 6, which implies a 20% net price markup in steady state. The steady state wage markup is also set at 20%. The ratios of fiscal variables to GDP and the steady state tax rates are borrowed from Coenen et al. (2012) and are collected in Table 1. In particular, government spending to GDP ratio is fixed at 21.5%, in line with the sample average, and public-debt-to-GDP ratio is set at 60% in annual terms, in line with the Maastricht objective. We derive the difference between aggregate transfers and taxes to GDP ratios \( (t_r/y - t/y) \) as a residual from the steady state government budget constraint. Similarly to Coenen et al. (2012), transfers to Non-Ricardian households are calibrated to obtain a steady state consumption ratio between the two groups \( (c^t/c^o) \) around 0.8 at the prior mean.

The remaining parameters are estimated with Bayesian techniques. Priors, reported in Table 2, are set in line with the literature on Euro area model estimation (see CCW, Coenen et al. (2012) and Smets and Wouters (2003, 2005)). In particular, parameters measuring the persistence of the shocks are set to be Beta distributed, with mean 0.5 and standard deviation 0.1 and the standard errors of the innovations are assumed to follow an Inverse-gamma distribution. The parameters governing price and wage setting, habits, utilization elasticity, interest rate smoothing and the steady state fraction of LAMP are also Beta distributed. The fraction of LAMP \( \theta \) is assumed to be Beta distributed with mean 0.3 and standard deviation 0.1, in line with Coenen et al. (2012).

Risk aversion, the inverse of Frisch elasticity and the parameters of the Taylor rule are Normally distributed, whereas the parameter defining investment adjustment costs is Gamma distributed.

4 Results

4.1 The full sample estimates

Table 2 shows the posterior estimates of the structural parameters and coefficients governing shock processes.\(^\text{10}\) Visual diagnostics of the estimation results can be found in Figure 6 in the Technical

\(^{10}\)All the marginal posterior distributions are unimodal, MCMC’s convergence criteria are satisfied. As robustness check, Metropolis-Hastings convergence graphs suggest a fast and efficient convergence for all parameters. The
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>6</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\bar{\pi} - 1$</td>
<td>0.0047</td>
</tr>
<tr>
<td>$g_z$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\frac{b}{y}$</td>
<td>2.4</td>
</tr>
<tr>
<td>$\frac{y}{y}$</td>
<td>0.215</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.223</td>
</tr>
<tr>
<td>$\tau^f$</td>
<td>0.116</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau^{wh}$</td>
<td>0.127</td>
</tr>
<tr>
<td>$\tau^{wf}$</td>
<td>0.232</td>
</tr>
</tbody>
</table>

Appendix, where we plot prior and posterior distributions, that are substantially different for most parameters. We obtain an estimate for the fraction of LAMP households, $\theta = 0.317$, that is close to the 0.3 prior. We check for the robustness of this prior by re-estimating the model with a flat prior on $\theta$ (Uniform (0.01, 0.99)), and we obtain $\theta = 0.342$.\footnote{See the Appendix for more details.} In this case, posterior distributions for the remaining parameters remain close to our benchmark estimates. We also experiment with a prior based on a beta distribution (mean=0.5, std dev=0.2) obtaining $\theta = 0.336$.\footnote{The marginal data density in these two cases was $-734.4$ and $-733.7$ respectively.}

In this case, posterior distributions for the remaining parameters remain close to our benchmark estimates. We also experiment with a prior based on a beta distribution (mean=0.5, std dev=0.2) obtaining $\theta = 0.336$.\footnote{For more technical details about the marginal data density, see An and Schorfheide (2007) and Bekiros and Paccagnini (2014a and 2014b).}

In Table 2, we also present the estimates for the RA model. The LAMP and RA models are characterized by similar posterior distributions for the common parameters, with the notable exceptions of productivity shocks standard deviation, $\sigma^a$, and the risk aversion coefficient $\sigma$: both are significantly larger in the RA case. The marginal data density (MDD in the Table 2) is -732 for the LAMP model and -740 for the RA model, which can be translated into a Bayes factor of posterior distributions are based on four Markov chains with 250,000 draws, with 50,000 draws being discarded as burn-in draws. The average acceptance rate is roughly 25 percent.
exp[8] in favor of a better fit produced by the LAMP model. We can interpret the magnitude of the Bayes factor using the Kass and Raftery (1995) criterion, that multiplies the log of the Bayes factor by two, as recently proposed by Curdia et al. (2014) and Merola (2014). In our case, the Kass and Raftery criterion amounts to 16, suggesting a strong evidence in favor of the LAMP model. Moreover, Table 3 shows that both the output standard deviation and the cross-correlations with output obtained with the LAMP model are always closer to the data moments, thus enforcing the evidence in favor of the LAMP model.

Finally, we perform a forecasting comparison between the two models. Figure 1 plots the difference between the root mean squared errors (RMSE) of the LAMP and RA models for h=4 step-ahead forecasts.14 Both models are recursively estimated over the period 1972:Q2 to 2001:Q4, the out-of-sample evaluation is from 2002:Q1 to 2012:Q4; for each observation, the RMSEs are computed using the 12 previous quarters (see Del Negro and Schorfheide (2012) for more details). On average, the LAMP model has a better forecasting performance, but the ranking between the two models is not stable over time. For output and consumption, the LAMP model is the best model in terms of prediction after 2007. The LAMP hypothesis appears to be important to explain the dynamics of key macroeconomic series such as output, consumption, inflation, and investment during the recent financial crisis. As the short term interest rate concerns, the RA model outperforms the LAMP model. This result is apparently not surprising because until recently, in the ECB projections on the New Area Wide Model (NAWM), as introduced in CCW, the LAMP is not considered (ECB, 2008).15

4.2 LAMP over time

Our estimated fraction of LAMP households is substantially larger than the fraction found in Coenen, Straub and Trabandt (2012), $\theta = 0.18$ for the sample 1985:Q1 to 2010:Q2, and well in the

---

14 We generate unconditional forecasts taking each 20th draw from the final 150,000 parameter draws (with the first 30,000 draws used as burn-in period) produced by the Metropolis-Hastings algorithm, which gives us 6,000 draws from the posterior distribution. The point forecasts are calculated as means of these draws. For more technical details, see Kolasa et al. (2012) and Kolasa and Rubaszek (2014).

15 The LAMP hypothesis has been introduced only recently in a calibrated version of the NAWM by Coenen et al. (2008).
Table 2: Prior and posterior distributions of estimated parameters (1972:Q2-2012Q4)

<table>
<thead>
<tr>
<th>parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>shape</td>
<td>mean</td>
<td>std dev</td>
</tr>
<tr>
<td>σ</td>
<td>norm</td>
<td>1</td>
<td>0.375</td>
</tr>
<tr>
<td>b</td>
<td>beta</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>φ_t</td>
<td>norm</td>
<td>2</td>
<td>0.75</td>
</tr>
<tr>
<td>θ</td>
<td>beta</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>γ_I</td>
<td>gamma</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>σ_u</td>
<td>beta</td>
<td>0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>χ_p</td>
<td>beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>ξ_p</td>
<td>beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>χ_w</td>
<td>beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>ξ_w</td>
<td>beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>ξ_e</td>
<td>beta</td>
<td>0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>φ_r</td>
<td>beta</td>
<td>0.9</td>
<td>0.05</td>
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<tr>
<td>φ_π</td>
<td>norm</td>
<td>1.7</td>
<td>0.1</td>
</tr>
<tr>
<td>φ_y</td>
<td>norm</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>φ_Δ_y</td>
<td>norm</td>
<td>0.063</td>
<td>0.05</td>
</tr>
<tr>
<td>φ_Δ_π</td>
<td>norm</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>(y + Φ)/y</td>
<td>norm</td>
<td>1.45</td>
<td>0.25</td>
</tr>
<tr>
<td>ρ_a</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_b</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_i</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_r</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_p</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_w</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>ρ_g</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>σ_a</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_b</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_i</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_r</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_p</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_w</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>σ_g</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
</tr>
</tbody>
</table>

MDD: -731.9

range reported in Coenen and Straub (2005), 0.24 < θ < 0.37 for an estimation over the sample 1980:Q1-1999:Q4.\(^{16}\) Furthermore, in Forni et al. (2009), the fraction is estimated in a range of 0.34-0.37 for the sample 1980:Q1-2005:Q4. However, these results are obtained under different conditions.

\(^{16}\)Moreover, in Coenen and Straub (2005), the estimated marginal data density for the LAMP model is always smaller than the one in the corresponding RA model.
Table 3: Key variables: data and model estimated moments

<table>
<thead>
<tr>
<th>Sample 1972-2012</th>
<th>DATA</th>
<th>LAMP</th>
<th>RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation output</td>
<td>0.646</td>
<td>0.819</td>
<td>0.846</td>
</tr>
</tbody>
</table>

Correlations with output:

<table>
<thead>
<tr>
<th>Variable</th>
<th>DATA</th>
<th>LAMP</th>
<th>RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.136</td>
<td>-0.079</td>
<td>-0.147</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.699</td>
<td>0.710</td>
<td>0.759</td>
</tr>
<tr>
<td>Investment</td>
<td>0.811</td>
<td>0.747</td>
<td>0.643</td>
</tr>
<tr>
<td>Short term interest rate</td>
<td>0.059</td>
<td>-0.114</td>
<td>-0.128</td>
</tr>
<tr>
<td>Wage</td>
<td>0.340</td>
<td>0.197</td>
<td>0.111</td>
</tr>
<tr>
<td>Employment</td>
<td>-0.072</td>
<td>-0.077</td>
<td>-0.071</td>
</tr>
</tbody>
</table>

Figure 1: Forecast comparison: LAMP vs RA model. (A value greater than zero indicates that the LAMP model attains a lower RMSE.)

Our empirical analysis shows an estimated model for a longer time span, encompassing the turbulent 1970s, the great moderation period, and the financial crisis. To shed light on a possibly changing role of LAMP, we re-estimate the model for selected subsamples. The sample 1972-81 coincides with the Great Inflation period and with a phase where financial markets were tightly regulated. Then, the sub-sample 1972-92 incorporates the disinflation period and the "hard EMS" phase. Finally, we concentrate on the post-Maastricht period that led to EMU inception and to the theoretical assumptions as we have already discussed, and for different sample periods.\textsuperscript{17} Our empirical analysis shows an estimated model for a longer time span, encompassing the turbulent 1970s, the great moderation period, and the financial crisis. To shed light on a possibly changing role of LAMP, we re-estimate the model for selected subsamples. The sample 1972-81 coincides with the Great Inflation period and with a phase where financial markets were tightly regulated. Then, the sub-sample 1972-92 incorporates the disinflation period and the "hard EMS" phase. Finally, we concentrate on the post-Maastricht period that led to EMU inception and to the financial crisis. To shed light on a possibly changing role of LAMP, we re-estimate the model for selected subsamples. The sample 1972-81 coincides with the Great Inflation period and with a phase where financial markets were tightly regulated. Then, the sub-sample 1972-92 incorporates the disinflation period and the "hard EMS" phase. Finally, we concentrate on the post-Maastricht period that led to EMU inception and to the financial crisis.

\textsuperscript{17}Moreover, the estimated models have different features and observed variables. For example, in Forni et al. (2009) the DSGE model includes fiscal variables, and in Coenen, Straub and Trabandt (2012) the model discusses an open economy with fiscal variables.
financial crisis years.

Table 4 shows that significant variations in the posterior estimates seem to concern only a limited subset of parameters, i.e., relative to the full sample estimates, in the 1970s the fractions of non-optimizing wage and price setters, \( \xi_w \) and \( \xi_p \), were relatively smaller, whereas the inflation indexation parameters \( \chi_p \) and \( \chi_w \) were relatively larger. This is in line with the interpretations of the "great moderation" period that emphasizes the importance of the adjustment to a low inflation environment. We also observe clear evidence of "great moderation" for the post Maastricht sample in both real and nominal shocks, with the notable exceptions of larger (but less persistent) risk premium shocks and of larger and more persistent investment specific shocks.

Given our full sample estimate, where \( \theta = 0.317 \) (HPD interval 0.224-0.417), we find that the point estimate for the fraction of LAMP is relatively larger in the 1972-81 period, \( \theta = 0.34 \) (HPD interval 0.182-0.497), and it substantially decreases in the 1972-1992 sample with \( \theta = 0.247 \) (HPD interval 0.144-0.351). Finally, the estimated posterior mean for the LAMP parameter in the 1993-2012 period, \( \theta = 0.39 \), (HPD interval 0.316-0.466) is strikingly larger than in the full sample case. These results do not fit well with a conventional interpretation of the great moderation as a period when credit availability increased and access to financial markets was easier. In fact, the fall in the importance of LAMP appears to be a feature of the 1981-1992 period when several countries benefited from large capital inflows and from a reduction in domestic interest rate spreads as a consequence of the membership in the (increasingly) hard EMS. The post-92 crisis phase might have been characterized by a financial retrenchment. To check for this point, we re-estimate the model over the 1972-1998 period, obtaining \( \theta = 0.36 \) (HPD interval 0.262-0.465). In addition, when we restrict the post-1992 sample excluding the financial crisis years, we obtain \( \theta = 0.36 \) (HPD interval 0.258-0.449). This latter result with the contribution of the LAMP hypothesis to the post-2007 forecasts of output, consumption and investment, suggest an intriguing analogy between the EMS 1992 collapse and the recent financial crisis.
Table 4: Prior mean estimates of the LAMP model over different subsamples.

<table>
<thead>
<tr>
<th>parameters</th>
<th>LAMP 72-Q2-81:Q4</th>
<th>LAMP 72-Q2-92:Q4</th>
<th>LAMP 93:Q2-12:Q4</th>
<th>RA 93:Q2-12:Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>1.655</td>
<td>1.445</td>
<td>2.157</td>
<td>1.827</td>
</tr>
<tr>
<td>b</td>
<td>0.649</td>
<td>0.713</td>
<td>0.741</td>
<td>0.749</td>
</tr>
<tr>
<td>φ₁</td>
<td>2.217</td>
<td>2.753</td>
<td>2.217</td>
<td>2.321</td>
</tr>
<tr>
<td>θ</td>
<td>0.341</td>
<td>0.247</td>
<td>0.390</td>
<td>-</td>
</tr>
<tr>
<td>γ₁</td>
<td>4.178</td>
<td>3.452</td>
<td>4.018</td>
<td>3.817</td>
</tr>
<tr>
<td>σ_u</td>
<td>0.403</td>
<td>0.282</td>
<td>0.229</td>
<td>0.224</td>
</tr>
<tr>
<td>χₚ</td>
<td>0.601</td>
<td>0.837</td>
<td>0.895</td>
<td>0.896</td>
</tr>
<tr>
<td>ξₚ</td>
<td>0.722</td>
<td>0.786</td>
<td>0.621</td>
<td>0.480</td>
</tr>
<tr>
<td>χₜ</td>
<td>0.805</td>
<td>0.925</td>
<td>0.919</td>
<td>0.920</td>
</tr>
<tr>
<td>ξₑ</td>
<td>0.753</td>
<td>0.857</td>
<td>0.795</td>
<td>0.800</td>
</tr>
<tr>
<td>φ_r</td>
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<td>0.774</td>
<td>0.876</td>
<td>0.840</td>
</tr>
<tr>
<td>φₚ</td>
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<td>1.820</td>
<td>1.725</td>
<td>1.764</td>
</tr>
<tr>
<td>φ_y</td>
<td>-0.072</td>
<td>0.203</td>
<td>0.152</td>
<td>0.132</td>
</tr>
<tr>
<td>φ_Dₙy</td>
<td>0.108</td>
<td>0.129</td>
<td>0.137</td>
<td>0.127</td>
</tr>
<tr>
<td>φ_Dₚy</td>
<td>0.338</td>
<td>0.223</td>
<td>0.146</td>
<td>0.158</td>
</tr>
<tr>
<td>(y + Φ)/y</td>
<td>1.443</td>
<td>1.424</td>
<td>1.554</td>
<td>1.385</td>
</tr>
<tr>
<td>ρ_u</td>
<td>0.893</td>
<td>0.949</td>
<td>0.938</td>
<td>0.938</td>
</tr>
<tr>
<td>ρ_b</td>
<td>0.698</td>
<td>0.939</td>
<td>0.388</td>
<td>0.942</td>
</tr>
<tr>
<td>ρ_i</td>
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<td>0.307</td>
<td>0.827</td>
<td>0.576</td>
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<tr>
<td>ρ_r</td>
<td>0.415</td>
<td>0.458</td>
<td>0.512</td>
<td>0.491</td>
</tr>
<tr>
<td>ρ_p</td>
<td>0.468</td>
<td>0.789</td>
<td>0.538</td>
<td>0.532</td>
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<tr>
<td>ρ_w</td>
<td>0.607</td>
<td>0.688</td>
<td>0.829</td>
<td>0.812</td>
</tr>
<tr>
<td>ρ_q</td>
<td>0.840</td>
<td>0.917</td>
<td>0.908</td>
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</tr>
<tr>
<td>σ^a</td>
<td>1.393</td>
<td>1.493</td>
<td>0.506</td>
<td>0.661</td>
</tr>
<tr>
<td>σ^b</td>
<td>0.276</td>
<td>0.166</td>
<td>0.286</td>
<td>0.102</td>
</tr>
<tr>
<td>σ^i</td>
<td>0.481</td>
<td>0.530</td>
<td>0.535</td>
<td>0.444</td>
</tr>
<tr>
<td>σ^r</td>
<td>0.308</td>
<td>0.229</td>
<td>0.084</td>
<td>0.082</td>
</tr>
<tr>
<td>σ^p</td>
<td>0.356</td>
<td>0.104</td>
<td>0.093</td>
<td>0.105</td>
</tr>
<tr>
<td>σ^w</td>
<td>0.300</td>
<td>0.160</td>
<td>0.062</td>
<td>0.066</td>
</tr>
<tr>
<td>σ^g</td>
<td>0.503</td>
<td>0.400</td>
<td>0.283</td>
<td>0.289</td>
</tr>
<tr>
<td>MDD</td>
<td>-230.6</td>
<td>-243.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3 A LAMP model for the EMU years

Between 1993 and 1999, the Maastricht Treaty forced EMU accession candidates to seek nominal convergence to the German levels, and there is ample evidence of continuity between the Bundesbank and the ECB in its early years (Issing et al., 2011). Thus our estimates for the post-1992 period may well characterize a model for the EMU years.

Turning to a comparison between the LAMP and RA models (see Table 4), we find that for
this sample the marginal data density is -231 in the LAMP model, and -244 in the RA model. The Bayes factor, approximately exp[13], and the Kass and Raftery criterion, around 26, are now larger than in the full sample case, showing a very strong evidence in favor of the LAMP model. Under LAMP, we estimate more volatile and far less persistent risk-premium shocks, and more volatile and persistent investment-specific shocks. Technology shocks are less volatile and equally persistent in the LAMP model.

Table 5 reports the variance decomposition for the LAMP and RA models. It is easy to see that the bulk of output growth volatility in the LAMP model is caused by investment-specific shocks, whereas in the RA model the risk premium shock has a predominant role. We also obtain that in the RA model the risk premium shock is almost the only source of consumption volatility. By contrast, in the LAMP model, risk premium and investment specific shocks have similar weights in explaining consumption volatility, followed by interest rates and productivity shocks. Turning to inflation, both models assign a minuscule weight to monetary shocks and a very important role to wage markup (LAMP model) and to productivity shocks (RA model).

<table>
<thead>
<tr>
<th></th>
<th>LAMP</th>
<th>RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta^a )</td>
<td>13.01</td>
<td>3.93</td>
</tr>
<tr>
<td>( \eta^b )</td>
<td>30.04</td>
<td>74.46</td>
</tr>
<tr>
<td>( \eta^i )</td>
<td>22.23</td>
<td>2.73</td>
</tr>
<tr>
<td>( \eta^r )</td>
<td>14.98</td>
<td>13.32</td>
</tr>
<tr>
<td>( \eta^p )</td>
<td>7.82</td>
<td>3.32</td>
</tr>
<tr>
<td>( \eta^w )</td>
<td>11.50</td>
<td>1.71</td>
</tr>
<tr>
<td>( \eta^g )</td>
<td>0.42</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Summing up, in the RA model the risk premium shock is the main driver of output, consumption and interest rates. Risk premium shocks play a limited role in the LAMP model, where the investment specific shock is relatively more important (but for consumption).

To understand the different effects of these shocks, Figures 2 and 3 report IRFs to 1% risk premium and investment specific shocks for the two models.

Consider the risk premium shock. In line with previous estimates for the Euro area (Smets
Figure 2: Impulse responses to a 1% risk premium shock. Solid lines: LAMP model. Dotted lines: RA model. Structural parameters and shock persistences are set at the posterior mean values for each specification. Estimation sample: 1993:Q2-2012:Q4.

and Wouters, 2005) in the RA model, all households reduce consumption. Investment also falls because households anticipate a prolonged real interest rate decline. Consumption and investment decisions are sufficient to drive dynamics of all remaining variables. In the LAMP model, only a fraction of households is directly affected by the shock: LAMP households consumption decisions cushion total demand and output from the shock. In addition, the estimated relative risk aversion coefficient in the LAMP model is much larger, limiting the Ricardian households sensitivity to the shock. These effects, in turn, call for a modest reduction in real interest rate. As a result Ricardian households are also induced to limit the contraction in investment. The global effect of the shock is indeed minor.

Turning to the investment shocks, we see that the LAMP model exhibits a much stronger positive correlation between total consumption and output. The specific role of LAMP in explaining the co-movements of consumption with investment and output, observed in the data, was first discussed in Furlanetto et al. (2013). When the economy is hit by an investment specific shock, agents who trade in financial markets cut consumption to finance investment. LAMP households,
Figure 3: Impulse responses to a 1% investment specific shock. Solid lines: LAMP model. Dotted lines: RA model. Structural parameters and shock persistences are set at the posterior mean values for each specification. Estimation sample: 1993:Q2-2012:Q4.

instead, increase their consumption because the investment specific shock increases hours worked and wages, thus rising labor income. Hence, for a sufficiently high share of LAMP, aggregate consumption may increase. In our setting, this aspect may explain why the investment shock plays such an important role in driving output and consumption volatility, whereas this cannot happen in the RA model.

To better understand the role of LAMP we also investigate the counterfactual responses of key macroeconomic variables to a stochastic simulation where we impose $\theta = 0$ (see Table 6).\textsuperscript{18} Results unambiguously show that predicted standard deviations are considerably different from those obtained under the model with a positive fraction of LAMP. With the notable exception of consumption, the standard deviations of output, inflation, the real wage and investment fall substantially when $\theta = 0$. Under LAMP, shocks have redistributive effects between the two consumers groups, and such effects almost cancel out in the aggregate consumption variable.

However, consumption and saving decisions of Ricardian households are now more sensitive to

\textsuperscript{18}Simulations are based on the posterior estimates for the sample 1993:Q2-2012:Q4 (LAMP model), reported in Table 4.
shocks and this affects aggregate volatility through their investment decisions.

<table>
<thead>
<tr>
<th>Table 6: Simulated standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
</tr>
<tr>
<td>full model</td>
</tr>
<tr>
<td>$\theta = 0$</td>
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</table>

### 4.3.1 Historical decomposition of output growth and inflation

We are now interested to analyze how shocks contributed to output growth and inflation over the business cycle in the EMU years. We first concentrate on the historical decomposition of output growth for the post-1999 period, that is, the period of ECB operational activity. Figure 4 presents results for the LAMP and RA models.

Both models capture expansionary interest rate shocks after the burst of the IT bubble in 2001-2002, but the verdict is caustic if we look at more recent years. In fact, we observe a negative contribution of the interest rate shocks to economic growth during 2008-2009 recession. According to both models, interest rate shocks contributions to the recession in these years were significant. This is broadly in line with popular beliefs about the late response of the ECB to the crisis. Indeed, the ECB kept the interest rate on the main refinancing operations fixed at 4% from June 2007 till July 2008, when it even increased interest rates by 25 basis points. Interest rates in the Euro area started decreasing gradually only from October 2008.

The trough of output growth in 2008-2009 is also affected by adverse government spending shock, thus indicating a procyclical fiscal discretionary policy.\(^1^9\)

The two models offer different interpretations of the non-policy shocks contributions to the crisis. According to the RA model, the risk premium shock played a dominant role, whereas according to the LAMP model the investment shock was the key driver.\(^2^0\) The intuition behind this result is that probably financial conditions do not matter so much if a large number of households

\(^1^9\)We do not emphasize this result, as the fiscal structure of the model affects only the steady state, government spending is assumed to be exogenous and public consumption data are excluded from our observables.

\(^2^0\)Both models evidence that these non-monetary shocks became important after the third quarter of 2008.
Figure 4: Historical decomposition of output growth (estimated sample: 1993:Q2-2012:Q4), LAMP model: upper panel, RA model: lower panel.
have no access to financial markets. Therefore, real shocks in times of lower financial markets participation regain relevance in explaining macroeconomic fluctuations.

Turning to the historical decomposition of inflation (Figure 5), we observe that the two models seem to agree about the deflationary role of monetary policy between 2008:Q2 and 2012:Q4, a period where inflation was already below the 2% official objective. By contrast, they lead to radically different conclusions about the determinants of inflation. According to the LAMP model, the adverse investment shocks contributed to raise inflation as capital accumulation slowed down. This was more than compensated by a sequence of negative wage markup shocks. In the RA model, the wage markup shocks played a lesser role and the risk premium shocks that depressed demand also disciplined inflation.

\footnote{Note that according to the LAMP model investment shocks were mainly adverse throughout the post-1999 period.}
Figure 5: Historical decomposition of inflation (estimated sample: 1993:Q2-2012:Q4), LAMP model: upper panel, RA model: lower panel.

Figure 6:
5 Conclusions

The LAMP hypothesis is important to understand EMU business cycle, especially in the aftermath of the recent financial crisis. Given the tighter credit standards, we might expect in the near future, the relatively large proportion of LAMP households is likely to remain an important feature of EMU.

Our results call for a reconsideration of ECB policies that should account for households heterogeneity. In this regard, theoretical analyses have shown that monetary policies and shocks can have powerful redistributive effects. In this regard, theoretical LAMP models typically pave the way for fiscal stabilization policies that should openly interact with central bank actions. Given our findings about the size of LAMP, ECB actions should take into account the "non conventional effects" of fiscal policies under LAMP. Finally, our estimates downplay the importance of risk premium shocks as a determinant of the output losses during the financial crisis. It would be interesting to assess the empirical effects of LAMP in models that explicitly account for financial frictions and for a banking sector. The analysis of these issues is left for future research.
References


A Technical Appendix

A.1 Non-linear equations

After deriving the first order conditions for Ricardian agents, unions and firms, we adjust all growing variables for growth to obtain a stationary equilibrium. In this case, lower case letters stand for "adjusted" variables, for example, $y_t = \frac{Y_t}{z_t}$. Notice that $w_t = \frac{W_t}{P_{z_t}}$ and $\lambda^o_t = \lambda^o_t z_t$. We end up the following set of non-linear equations:

\[
(c^o_t)^{-\sigma} \frac{b'(\sigma-1)}{c^o_{t-1}} \exp \left( \frac{(\sigma - 1)\varepsilon^1_t}{1 + \phi_t} h^1_t + \phi_t \right) = \lambda^o_t \left( 1 + \tau^o \right)
\]

\[
R_t = \pi_{t+1} g_{z.t+1} \frac{\lambda^o_t}{\beta^{z_t} \lambda^o_{t+1}}
\]

\[
1 = Q^o_t \varepsilon^1_t \left\{ 1 - \gamma_t \left( g_{z.t} \frac{i_t}{i_{t-1}} - g_z \right) g_{z.t} \frac{i_t}{i_{t-1}} + \frac{\gamma_t}{2} \left( g_{z.t} \frac{i_t}{i_{t-1}} - g_z \right)^2 \right\}
\]

\[
+ g_{z.t+1} \frac{\lambda^o_{t+1}}{\lambda^o_t} Q^o_{t+1} \varepsilon^1_{t+1} \beta \gamma_t \left( g_{z,t+1} \frac{i_{t+1}}{i_t} - g_z \right) \left( \frac{i_{t+1}}{i_t} \right)^2
\]

\[
\frac{1}{g_{z.t+1}} \frac{\lambda^o_{t+1}}{\lambda^o_t} \beta \left\{ (1 - \tau^k) \left[ r^k_{t+1} u_{t+1} - a (u_{t+1}) \right] + \tau^k \delta + Q^o_{t+1} (1 - \delta) \right\} = Q^o_t
\]

\[
r^k_t = \gamma_{u1} + \gamma_{u2} (u_t - 1)
\]

\[
k_{t+1} = (1 - \delta) \frac{k_t}{g_{z.t}} + \varepsilon^i_t \left[ 1 - \frac{\gamma_t}{2} \left( g_{z.t} \frac{i_t}{i_{t-1}} - g_z \right)^2 \right] i_t
\]

\[
(1 + \tau^o) c^t_r = (1 - \tau^l - \tau^{wh}) w_t h_t + t r^t_r - t^r_r
\]

\[
g_t + \frac{R_{t-1}}{\pi_t} \frac{b_t}{g_{z.t}} + t r_t = b_{t+1} + t + \tau^c c_t + \left( \tau^l + \tau^{wh} + \tau^{wf} \right) w_t h_t + \tau^k \left[ r^k_t u_t - (a (u_t) + \delta) \right] \frac{k_t}{g_{z.t}}
\]

\[
y_t = c_t + g_t + i_t + \frac{a (u_t) k_t}{g_{z.t}}
\]

\[
c_t = \theta c^t_r + (1 - \theta) c^o_t
\]
\[ 0 = \sum_{s=0}^{\infty} (\xi_w e^{\beta})^s c_{t+s+1} \exp \left( \frac{(\sigma - 1)}{1 + \phi_t} (h_{t+s} \phi_t) \right) \left( \frac{1 + \lambda_t w}{\lambda_t} \right) \left( \frac{\xi_w \chi_w}{\chi_t} \right) \left( \frac{1 - \lambda_t w}{\lambda_t} \right) h_t^d. \]

\[
\left\{ \begin{array}{l}
\tilde{w}_t \frac{(1 - \tau)^{\chi_t}}{(1 + \tau)^{\chi_t} \pi_{t+s+1} \pi_{t+s+1} \pi_{t+s}} \left( 1 - \frac{1 + \lambda_t w}{\lambda_t} \right) \left[ (1 - \theta) (c_t \phi_t)^{-\sigma} + \theta (c_t \phi_t)^{-\sigma} \right] \\
+ \frac{1 + \lambda_t w}{\lambda_t} \left[ (1 - \theta) (c_t \phi_t)^{-\sigma} \text{MRS}_t^\phi \theta (c_t \phi_t)^{-\sigma} \text{MRS}_t^\phi \right]
\end{array} \right\}
\] (28)

\[ w_t = \left[ \frac{\xi_w}{\pi_t} \left( \frac{\pi_t - \pi_{t-1} \chi_t}{\pi_t} w_{t-1} \right) \right]^{\frac{1}{\lambda_t}} + (1 - \xi_w) \left( \tilde{w}_t \right)^{\frac{1}{\lambda_t}} \] (29)

\[ \frac{u_t k_t}{h_t g_{z,t}} = \frac{\alpha}{1 - \alpha} \frac{1 + \tau w f}{r_t^k} \] (30)

\[ mc_t = \alpha^{-\alpha} (1 - \alpha)^{-(1 - \alpha)} (c_t)^{-\alpha} (e_t)^{(1 + \tau w f)} w_t \] (31)

\[ s_{t, t} y_t = c_t \left( \frac{u_t k_t}{g_{z,t}} \right) \] (32)

\[ E_t \sum_{s=0}^{\infty} (\xi_p e^{\beta})^s c_{t+s} \pi_t^o \pi_{t+s} y_{t+s} \left[ \frac{\pi_t^o \chi_p}{\pi_t^o \chi_{t+s}} \left( 1 + \frac{1}{G^{t-1} (\omega_{t+s})} \right) G' (x_{t+s}) \right] - mc_{t+s} \frac{1}{G^{t-1} (\omega_{t+s})} G'' (x_{t+s}) = 0 \] (33)

\[ 1 = (1 - \xi_p) \tilde{p}^z G' \int_0^1 G' \left( \frac{y_t^z}{y_t} \right) y_t^z dz \] (34)

\[ + \xi_p \chi_p \pi_t^{-1} \pi_{t+s}^{-1} G' \left( \frac{\chi_p}{\pi_t} \right) \pi_t^{-1} \pi_{t+s}^{-1} \int_0^1 G' \left( \frac{y_t^z}{y_t} \right) y_t^z dz \]

\[ t_r = \theta t_r^t + (1 - \theta) t_r^o \] (35)

\[ t_t = \theta t_t^t + (1 - \theta) t_t^o \] (36)

\[ h_t = s_{W,t} h_t^d \] (37)

\[ s_{W,t} = \int_0^1 \left( \frac{W_t^j}{W_t} \right)^{-\frac{1 + \lambda_t^p}{\lambda_t^p}} dj \] (38)

\[ s_{P,t} = \int_0^1 \left( \frac{P_t^z}{P_t} \right)^{-\frac{1 + \lambda_t^p}{\lambda_t^p}} dz \] (39)
\[ MRS_t^o = c_t^o h_t^{\phi_t} \]  
\[ MRS_t^{rt} = c_t^{rt} h_t^{\phi_t} \]  

**A.2 Set of log-linearized equations**

After log-linearizing the model around its non-stochastic steady state and making some algebra, we obtain a system composed by 16 equation and 16 endogenous variables. Hatted variables stand for variables in log deviation from their steady state, for example: \( \hat{y}_t = \log \left( \frac{y_t}{y} \right) \). Notice also that fiscal variables, such as government spending, have been defined in deviation from steady state output, for example: \( \hat{g}_t = \frac{g_t - g}{y} \).

\[ \hat{c}_t^o = \hat{c}_{t+1}^o + \frac{(1-\sigma)b}{\sigma} (\hat{c}_t - \hat{c}_{t-1}) - \frac{1}{\sigma} (\hat{\varepsilon}_t^b + \hat{R}_t - \hat{\pi}_{t+1} - \hat{g}_{z,t+1}) \]
\[ + \frac{(1-\sigma)h_t^{1+\phi_t}}{\sigma} (\hat{h}_{t+1} - \hat{h}_t) \]  
\[ \hat{h}_t = \frac{1}{\gamma_1 g_z^2 (1+\beta)} \left( \hat{Q}_t^o + \hat{\varepsilon}_t^i \right) - \frac{1}{1+\beta} \hat{g}_{z,t} + \frac{1}{1+\beta} \hat{h}_{t-1} \]
\[ + \frac{\beta}{1+\beta} \hat{h}_{t+1} + \frac{\beta}{1+\beta} \hat{g}_{z,t+1} \]  
\[ - \hat{R}_t - \hat{\varepsilon}_t^b + \hat{\pi}_{t+1} + \frac{\beta}{g_z} (1-\tau^k) r^k \hat{r}_{t+1}^k \]
\[ + \frac{\beta}{g_z} (1-\delta) \hat{Q}_{t+1}^o = \hat{Q}_t^o \]  
\[ \hat{r}_t^k = \frac{\gamma_{w^2}}{r^k} \hat{u}_t = \frac{\sigma_u}{1-\sigma_u} \hat{u}_t \]  
\[ \hat{k}_{t+1} = \frac{(1-\delta)}{g_z} \hat{k}_t + \frac{i}{k} \hat{t}_t - \frac{(1-\delta)}{g_z} \hat{g}_{z,t} + \frac{i}{k} \hat{e}_t^i \]
\[ (1+\tau^c) \frac{c}{c} \hat{c}_t^{rt} = (1-\tau^l - \tau^{wh}) \frac{w}{c} \left( \hat{w}_t + \hat{h}_t \right) \]
\[
\begin{align*}
0 &= \frac{c}{y} \hat{c}_t + \hat{g}_t + \frac{i}{y} \hat{r}_t - \hat{\gamma}_t + \frac{y w}{g_z} \hat{w}_t \\
\hat{c}_t &= \theta \frac{e^t}{c} \hat{c}_t + (1 - \theta) \frac{c}{c} \hat{c}_t \\
(1 + \beta \chi_p) \hat{\pi}_t &= \chi_p \hat{\pi}_{t-1} + \beta \hat{\pi}_{t+1} - \beta (1 - \chi_p) \hat{\pi}_{t+1} + (1 - \chi_p) \hat{\pi}_t + A (1 - \beta \xi_p) (1 - \xi_p) \left( \tilde{mc}_t + \hat{\lambda}_t \right)
\end{align*}
\]

\[
\begin{align*}
\hat{w}_t &= -\frac{(1 - \xi_w) (1 - \xi_w) \beta}{1 + \beta} \hat{w}_t + \frac{(1 - \xi_w) (1 - \xi_w) \beta}{1 + \beta} \frac{\lambda w}{\xi_w} \hat{x}_w \\
+ &\frac{(1 - \xi_w)(1 - \xi_w, \beta)}{(1 + \beta) \xi_w (\omega + 1)} \left\{ \left[ \left[ \frac{c x}{c w} - 1 \right] + 1 \right] \overline{MRS} - \left[ \left[ \frac{c x}{c w} - 1 \right] + 1 \right] \overline{MRS} \right\} \\
+ &\frac{\beta}{1 + \beta} \hat{w}_{t+1} + \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\chi w}{1 + \beta} \hat{\pi}_{t-1} - \frac{(1 + \beta \chi w)}{1 + \beta} \hat{\pi}_t + \frac{\beta}{1 + \beta} \hat{\pi}_{t+1} + \frac{(1 - \chi w)}{1 + \beta} \hat{\pi}_t - \frac{\beta}{1 + \beta} (1 - \chi w) \hat{\pi}_{t+1}
\end{align*}
\]

\[
\overline{MRS} = \overline{c}_t^\pi + \phi_t \hat{h}_t
\]

\[
\overline{MRS}^t = \overline{c}_t^p + \phi_t \hat{h}_t
\]

\[
\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t
\]

\[
\tilde{mc}_t = -\hat{\epsilon}_t + \alpha \hat{r}_t + (1 - \alpha) \hat{w}_t
\]

\[
\hat{y}_t = \frac{y + \Phi}{y} \hat{\epsilon}_t + \alpha \left( \frac{y + \Phi}{y} \right) \hat{k}_t + \alpha \left( \frac{y + \Phi}{y} \right) \hat{u}_t + \frac{(1 - \alpha) (y + \Phi)}{y} \hat{h}_t - \frac{y + \Phi}{y} \hat{g}_{z,t}
\]

\[
\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) \left( \hat{\pi}_t + \phi \left( \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \phi \hat{\gamma}_t \right) + \phi \Delta \left( \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \phi \Delta \left( \hat{\gamma}_t - \hat{\gamma}_{t-1} \right) + \hat{\epsilon}_t
\]

\[
\text{with } A = \frac{1 + \sigma^2(x)}{2 + \sigma^2(x)} = \frac{1}{\lambda^p} (\lambda^p) = \frac{1}{\lambda^p} \sigma \left( \frac{e^t}{c^w} \right)^{-\sigma} \text{ and } \omega = \frac{\sigma}{c^w}.
\]

The estimated shocks are:
\[
\begin{align*}
\dot{\varepsilon}_t^a &= \rho_a \dot{\varepsilon}_{t-1}^a + \eta_t^a \\
\dot{\varepsilon}_t^i &= \rho_i \dot{\varepsilon}_{t-1}^i + \eta_t^i \\
\dot{\varepsilon}_t^r &= \rho_r \dot{\varepsilon}_{t-1}^r + \eta_t^r \\
\dot{\lambda}_t^p &= \rho_p \dot{\lambda}_{t-1}^p + \eta_t^p \\
\dot{\lambda}_t^w &= \rho_w \dot{\lambda}_{t-1}^w + \eta_t^w \\
\dot{\varepsilon}_t^b &= \rho_b \dot{\varepsilon}_{t-1}^b + \eta_t^b \\
\dot{\gamma}_t &= \rho_g \dot{\gamma}_{t-1} + \eta_t^g
\end{align*}
\]
A.3 Priors and posteriors distributions

Figure 7: Prior (dotted line) and posterior (solid line) distributions of estimated parameters and standard deviations, 1972:Q2-2012:Q4.
## A.4 Robustness on the prior for $\theta$: a uniform distribution

Table 7: Posterior estimates for the sample 1972:Q2-2012:Q4 with a uniform distribution for $\theta$

<table>
<thead>
<tr>
<th>parameters</th>
<th>post. mean</th>
<th>90% HPD interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.378</td>
<td>1.171 1.578</td>
</tr>
<tr>
<td>$b$</td>
<td>0.773</td>
<td>0.647 0.901</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>3.073</td>
<td>2.234 3.960</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.342</td>
<td>0.223 0.460</td>
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<tr>
<td>$\gamma_I$</td>
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<td>3.196 4.583</td>
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<tr>
<td>$\sigma_u$</td>
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<td>0.889 0.975</td>
</tr>
<tr>
<td>$\chi_p$</td>
<td>0.146</td>
<td>0.107 0.183</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.897</td>
<td>0.894 0.900</td>
</tr>
<tr>
<td>$\chi_w$</td>
<td>0.681</td>
<td>0.508 0.845</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.929</td>
<td>0.911 0.947</td>
</tr>
<tr>
<td>$\xi_e$</td>
<td>0.841</td>
<td>0.821 0.860</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.832</td>
<td>0.794 0.874</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.676</td>
<td>1.551 1.797</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.248</td>
<td>0.202 0.294</td>
</tr>
<tr>
<td>$\phi_{\Delta y}$</td>
<td>0.148</td>
<td>0.110 0.185</td>
</tr>
<tr>
<td>$\phi_{\Delta \pi}$</td>
<td>0.161</td>
<td>0.106 0.214</td>
</tr>
<tr>
<td>$(y + \Phi)/y$</td>
<td>1.458</td>
<td>1.325 1.588</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>0.952</td>
<td>0.950 0.953</td>
</tr>
<tr>
<td>$\rho_b$</td>
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<td>0.943 0.953</td>
</tr>
<tr>
<td>$\rho_i$</td>
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<tr>
<td>$\rho_r$</td>
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<td>$\rho_p$</td>
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<td>0.649 0.841</td>
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<td>$\rho_w$</td>
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<td>0.722 0.851</td>
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<tr>
<td>$\rho_g$</td>
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<td>0.931 0.953</td>
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<td>$\sigma^a$</td>
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<tr>
<td>$\sigma^b$</td>
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<td>0.148 0.196</td>
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<tr>
<td>$\sigma^i$</td>
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<td>0.415 0.555</td>
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<td>$\sigma^c$</td>
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<td>$\sigma^w$</td>
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<td>0.078 0.125</td>
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<td>$\sigma^g$</td>
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<td>0.326 0.398</td>
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<td>-734.4</td>
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</tbody>
</table>
Figure 8: Prior (dotted line) and posterior (solid line) distributions of estimated parameters and standard deviations, 1972:Q2-2012:Q4, uniform distribution for $\theta$. 