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monetary model: A Welfare Analysis**

Maria Ferrara, Patrizio Tirelli

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Disinflation and Inequality in a DSGE monetary model: A Welfare Analysis

Maria Ferrara*
University of Naples "Parthenope"

Patrizio Tirelli†
University of Milano-Bicocca

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Abstract

We investigate the redistributive effects of a disinflation experiment in an otherwise standard medium-scale DSGE model augmented for Limited Asset Market Participation, implying that a fraction of households do not hold any wealth. We highlight two key mechanisms driving consumption and income distribution: i) the cash in advance constraint on firms working capital needs; ii) the response of profit margins to disinflation, which is crucially dependent on the two most used pricing assumptions in the New-Keynesian literature, i.e. Calvo vs Rotemberg. Results show that disinflation softens the cash in advance constraint and raises the real wage in steady state. This, in turn, lowers inequality. While under the Calvo formalism this effect is reinforced by the fall of price markups, under Rotemberg it is more than compensated by the increase of price markups and, therefore, the opposite result obtains.

JEL Classification System: E31, E5

Keywords: Disinflation, Inequality, Welfare, LAMP, Monetary Policy, Calvo Price Adjustment, Rotemberg Price Adjustment

*E-mail: ferraramaria11@gmail.com

†E-mail: patrizio.tirelli@unimib.it

1 Introduction

Recent years have witnessed increasing concern for the distributive effects of monetary policies (see the discussion in Coibion et al., 2012, and references cited therein). Empirical evidence in this regard is relatively limited and has produced partly conflicting results. Romer and Romer (1998) find that contractionary shocks increase inequality. Coibion et al. (2012) obtain identical outcomes for persistent reductions in the Fed inflation target. By the opposite, Romer and Romer (1998), Bulř (2001), Easterly and Fisher (2001) and Albanesi (2007) document a positive cross country correlation between inflation and inequality over relatively long time spans.

This paper investigates the distributional effects of disinflation in an otherwise standard medium scale DSGE model augmented for Limited Asset Market Participation (LAMP, henceforth), where it is assumed that a fraction of households (Rule of Thumb, RT hereafter) do not participate in financial markets and only consume their current labor income. As a matter of fact, the empirical evidence highlights that economies are not populated by representative agents¹, so distributional issues must be taken into account. Our contribution is akin to Ascari and Ropele (2012), henceforth AR, who simulate a disinflation experiment within a standard DSGE model with omogeneous and forward-looking consumers. We depart from their paper in two aspects. First, we assume heterogeneity across households, namely LAMP, in line with the empirical analysis and with a common tradition in the literature². Second, we simulate the disinflation policy exercise comparing the two most commonly used approaches for price and wage setting in the New-Keynesian literature³, i.e. the Calvo (1983) probability of price (wage) adjustment and the Rotemberg (1982) quadratic cost of price (wage) adjustment. As far as we know, this is the first paper analyzing income distribution and welfare effects of disinflation in a DSGE model with LAMP⁴.

Several papers show that Calvo and Rotemberg models imply the same dynamics up to the first order approximation⁵. By contrast, some other contributions are really sceptical about the identity of the two frameworks⁶. Ascari and Rossi (2012) show that once non linearities due to trend inflation are taken

¹The presence of RT households has been detected and estimated by, among others, Campbell and Mankiw (1989), Fuhrer(2000). Moreover, Iacoviello and Pavan (2013) show that the 40% of US households hold no wealth and no debt. Similar figures are observed for the Euro area (Cowell et al., 2013).

²Earlier contributions that investigate the LAMP assumption include, among others, Galí *et al.* (2004, 2007), Bilbiie (2008), Colciago *et al.* (2008), Motta and Tirelli (2012).

³As in Ascari and Rossi (2012) in a simple sticky price model with only labor as productive factor.

⁴Instead, two seminal papers have initiated the debate about the welfare implications of Rotemberg vs Calvo price-setting mechanisms.

⁵See Rotemberg (1987), Roberts (1995), Ball (1994), Mankiw (2001).

⁶Kahn (2005) argues that despite both approaches imply the same reduced form New Keynesian Phillips Curve, the impact of competition on the slope of the NKPC and on the response of inflation and output to shocks differs between the two models.

into account the dynamics of the two specifications are different. Similar results show up in our disinflation experiment under the two price adjustment mechanisms.

Disinflations are typically associated to short-run output contractions. Motta and Tirelli (2012, 2013) document the redistributive effects of monetary contractions, which unambiguously penalize RT households who cannot smooth consumption and are constrained to consume their labor income. The key mechanism behind this result is the combination of reduced labor incomes and increased profit margins that follows a monetary contraction in the short run. Our model replicates this outcome and provides new insights on the long-run effects of disinflations.

In our model two key frictions have distributional implications and are also affected by the long-run inflation rate: firms markups and the cash-in-advance constraint on firms working capital (CIA henceforth)⁷. The CIA constraint implies that asset holders obtain nominal interest rate payments on the funds that firms borrow to finance their working capital needs (see Christiano et al., 2005). Disinflation lowers the long-run nominal inflation rate and has a powerful effect on labor demand. As a result we observe that in the long run both output and the real wage increase, whereas the real rental cost of physical capital remains constant. Thus a disinflation that lowers the cost of borrowing working capital unambiguously reduces inequality. Moreover, the long run effect of disinflation on profit margins depends on assumptions made about the price-setting mechanism, namely Calvo *vs* Rotemberg.

In a nutshell, we find that under Rotemberg framework, disinflation unambiguously increases inequality, whereas results are ambiguous under Calvo pricing according to whether the CIA channel is at work. As a matter of fact, in the Rotemberg model disinflation causes an increase in the average markup that benefits the asset holders and hinders the RT consumers⁸. In the Calvo model disinflation implies a reduction in the average markup that penalizes Ricardian households. In particular, when the CIA channel is in place, not only it works in the same direction of the markup effect - because disinflation determines lower interest payment to the asset holders - but it also produces a powerful increase of labor income, thereby benefiting non Ricardian households. By the opposite, when the CIA channel is shut off the effect of the increase in per-capita capital takes over and increases inequality, although the markup reduction.

Our analysis also focuses on the welfare implications of disinflations. AR obtain that disinflations unambiguously raise welfare. Differently, in our framework redistributive effects might in fact reduce welfare for one of the two groups. It should also be noted that, due to external habits in the households' utility functions, our analysis accounts for the welfare implications of changes in inequality. We find that in the Calvo model disinflation is welfare improving for

⁷The existence of the CIA constraint is controversial. Several contributions in the DSGE literature present models embodying the CIA (see, for instance Christiano, Eichenbaum and Evans, 2005; Schmitt-Grohé and Uribe (2005)) but this is inconsistent with the actual central banks modelling (see, for instance, Christoffel, Coenen and Warne, 2008).

⁸The effects are amplified without CIA.

both household types. However, the larger welfare gain accrues to RT households when the CIA binds while the opposite holds without CIA. In the Rotemberg model, the presence of CIA guarantees a welfare improving disinflation for RT consumers, although the Ricardian households benefit more. Without CIA, disinflation becomes even costly for the liquidity constrained households.

The paper adds to previous contributions on the welfare implications of inflationary regimes which highlight the importance of different portfolio composition of different income groups, where the poor typically hold a relatively large proportion of their wealth in non-interest-bearing assets (Erosa and Ventura, 2002; Albanesi, 2007). Our focus here is clearly different as we investigate the role of two different frictions, that is, price markups and the CIA constraint.

The rest of the paper is organized as follows. The next section describes the main model features in particular focusing on the two price mechanisms. Section 3 focuses on the disinflation experiment and results; section 4 shows some robustness check and section 5 concludes.

2 The Model

Our model is an extended version of the NK-DSGE model developed by Schmitt-Grohé and Uribe (2005), henceforth SGU (2005), and Christiano, Eichenbaum and Evans (2005), henceforth CEE (2005). It embodies both nominal and real frictions. Real frictions include: monopolistic competition in goods and labor markets, external habits in consumption, variable capital utilization and adjustment costs in investment decisions. As for nominal frictions, price and wage stickiness is modelled according to the Calvo and Rotemberg alternative frameworks.

Our model accounts for LAMP, i.e. the economy is populated by two different household types: optimizing (Ricardian) households, who hold assets, and the Rule of Thumb (RT henceforth) households, who consume their current income and do not hold any wealth. Formally, we assume that all households share the same utility function. In particular, their preferences are defined over per capita and total consumption c_t^i and c_t respectively, and per capita labor effort h_t^i :

$$U_t^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln (c_t^i - bc_{t-1}) - \frac{\phi_1}{(1+\phi)} (h_t^i)^{(1+\phi)} \right\} \quad (1)$$

where $i = o, rt$ refers to optimizing and RT consumers respectively.

The model features a CIA constraint on the wage bill of firms. This implies that, to pay workers in advance of the production, firms must borrow from financial intermediaries⁹ that, in turn, collect money balances from Ricardian

⁹Here we implicitly follow the financial sector characterization adopted in CEE 2005, who assumes that a financial intermediary collects money balances from Ricardian households and from the Central Bank. Such funds are then used to finance the working capital needs of firms, and what is left returns to Ricardian households.

households, . Repayment occurs at the end of each period at the gross nominal interest rate R_t .

2.1 Rotemberg (1982) and Calvo (1983)

Monopolistic competition in good and labor markets respectively entails that firms and unions are price and wage setters, thereby enjoying markets power. The Calvo specification assumes that each period each firm faces a constant probability of being able to reoptimize prices. Therefore monopolistic firms are assumed to have different prices in different periods. A crucial implication of that is a relative-price dispersion leading to a misallocation of resources. In particular, price dispersion appears as an inefficiency loss in aggregate production. As stressed in SGU (2005), the higher is the dispersion, the more labor is needed to produce a given level of output.

The Rotemberg specification produces a symmetric equilibrium in which all firms (unions) reoptimize the same price (wage) at each point in time, bearing a quadratic adjustment cost.

2.2 Monetary Policy

We assume that monetary policy follows the non-linear rule:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \quad (2)$$

where R_t , R , π_t , π respectively denote the current nominal interest rate, interest rate target, current inflation and inflation target. $\phi_\pi > 1$ is the parameter governing the inflation stabilization.

2.3 Calibration

Structural parameter calibration follows CEE (2005)¹⁰, with two exceptions: the CIA and the indexation degrees parameters. CEE (2005) assumes that the full wage bill is subject to the CIA constraint. Differently, we follow Rabanal (2007) who finds that only 15% of workers must be paid in advance of the production. Moreover, we depart from CEE (2005) to calibrate the price and wage indexation degrees. As a matter of fact, they simply assume full indexation to past period inflation in order to match the inflation persistence found in the post world war II U.S. data. This is in contrast with a large body of empirical evidence¹¹ showing that indexation is at best partial. Therefore, we allow for partial indexation in line with the range of values found in the empirical evidence setting the price indexation at 0.50 and the wage indexation at 0.75¹².

¹⁰Table B1 in Appendix B shows the parameters values with description. Notice that the parameter governing the habit persistence under Rotemberg is calibrated at 0.6 following the literature in the field, rather than 0.65 (as in CEE (2005)), to avoid model instability problems.

¹¹See, for instance, Sbordone (2006); Smets and Wouters (2007); Benati (2008), (2009); Coogley and Sbordone (2008); Ascari, Castelnovo, Rossi (2011); Hofman, Peersmann, Straub (2012)

¹²See Smets and Wouters (2003).

We fix the share of Rule of Thumb consumers at $RT = 0.3$, according to a conservative parametrisation of LAMP behavior.

Furthermore, in order to compare the two models (Calvo *vs* Rotemberg), we refer to the paper by Keen and Wang (2007) providing the relationship between the price (wage) adjustment cost parameter and the constant fraction of reoptimizing firms (unions).

3 The Disinflation Experiment

The disinflation experiment entails a transition from high- to low-inflation steady state.¹³ In particular, following AR (2012), we assume that the central bank implements a credible cold-turkey disinflation aiming to disinflate the economy from the annual level of 5% (π_{old}^*) to the target of 2%¹⁴ (π_{new}^*). Table 1 reports the steady state percentage variations of output (y), consumption (c), average firms markup (μ^p), real wage (w), hours (h), capital (K), price (s) and wage (s_w) dispersion under Calvo and price (ξ_p) and wage (ξ_w) quadratic adjustment costs under Rotemberg. In addition to the baseline calibration according to which indexation is partial, the analysis takes also into account the full indexation case where the markup effect vanishes and we can pinpoint the CIA effect.

Table 1 - Steady state percentage variations

<i>Aggregate Variables</i>	<i>Partial Indexation</i>		<i>Full Indexation</i>
	Calvo	Rotemberg	Calvo/Rotemberg
y	0.11	-0.02	0.02
c	0.08	0.07	0.02
μ^p	-0.03	0.02	0
w	0.28	0.17	0.20
h	0.02	-0.01	0.01
K	0.93	-0.25	0.19
i	0.02	-0.01	0.01
Price rigidity (s, ξ_p)	-0.04	-0.03	0
Wage rigidity (s_w, ξ_w)	-0.05	-0.04	0

Disinflation under Calvo reduces price dispersion that results as an inefficiency loss in aggregate production:

$$y = \frac{AK^\alpha h^{(1-\alpha)}}{s}$$

¹³This explains why the model cannot be analyzed by log-linearizing it around one of the two steady states (see Ascari and Ropele (2012)). For the same reason, Ascari and Merkl (2009) show that the use of log-linear approximation to study disinflation may imply misleading results.

¹⁴Therefore this experiment takes into account trend inflation values consistent with the post-war history of the industrialized countries.

As a result, output and consumption increase following the rise in capital and hours. Under Rotemberg, the policy comes with an output and employment fall. In fact disinflation reduces the quadratic adjustment costs that appear in the aggregate resource constraint:

$$y = \frac{c + i + G + \xi_w h}{\xi_p}$$

In turn, since investments (i) fall and public spending (G) is held constant, this reduces the wedge between output and consumption. This explains why consumption increases. In fact, the reduction in the fraction of output wasted for adjusting prices and wages more than compensates the decrease in output¹⁵. Hence, disinflation implies output and consumption to move in opposite directions.

Importantly, the average price markup decreases under Calvo and increases under Rotemberg. To make the point clear, the average markup in the Calvo model reads as:

$$\mu^p = \frac{1}{MC^r} = \left[p^* \left(\frac{\eta - 1}{\eta} \right) \left(\frac{1 - \alpha \beta \pi^{\eta(1-\chi)}}{1 - \alpha \beta \pi^{(\eta-1)(1-\chi)}} \right) \right]^{-1} \quad (3)$$

or, in other terms:

$$\mu^p = (1 - \alpha) \mu^* + \alpha \mu^{old} \quad (4)$$

where α is the price stickiness parameter, μ^* and μ^{old} respectively denote the markup of optimizing firms and the markup of firms that are not able to reset prices. In particular, $\mu^* = \frac{p^*}{MC^r} = \frac{1}{MC^r} \left(\frac{1 - \alpha \pi^{(\eta-1)(1-\chi)}}{(1-\alpha)} \right)^{\left(\frac{1}{1-\eta} \right)}$. The increase in the marginal cost is more than compensated for by the reduction of p^* , therefore μ^* reduces and μ^p as well.

In the Rotemberg model the average markup is defined as:

$$\mu^p = \frac{1}{MC^r} = \left[\left(\frac{\eta - 1}{\eta} \right) + (1 - \beta) \xi_p \left(\pi^{(1-\chi)} - 1 \right) \pi^{(1-\chi)} \right]^{-1} \quad (5)$$

and it's straightforward to figure out that it increases after the disinflation policy.

Under full indexation the two mechanisms are equivalent. Under Calvo there is no dispersion and under Rotemberg there is no adjustment cost because all prices and wages are indexed to past inflation. Therefore we observe positive variations of the aggregate variables and a constant markup, i.e. $\mu^p = \left(\frac{\eta-1}{\eta} \right)$.

Figure 1 shows the transition dynamics¹⁶ of the macroeconomic variables defined above starting off from the initial annual level of $\pi_{old}^* = 5\%$ achieving the policy target $\pi_{new}^* = 2\%$.

¹⁵See Ascari and Rossi (2012).

¹⁶The transition paths are expressed in percentage deviations from the first steady state.

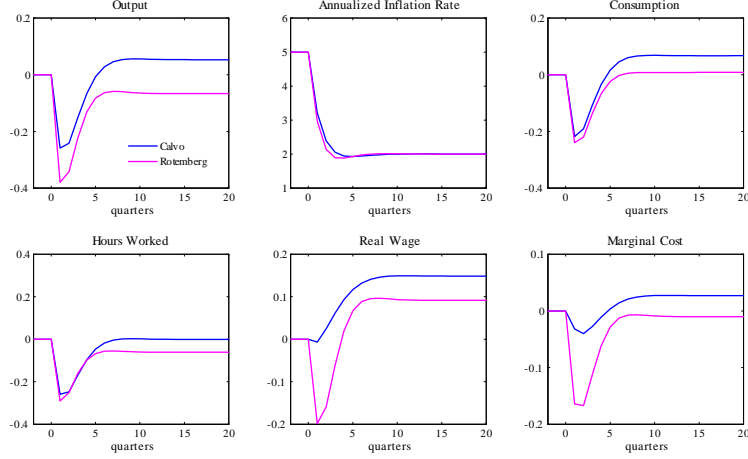


Fig. 1 - Disinflation under Calvo and Rotemberg frameworks

During the transition, disinflation implies a deep recession and a reduction of both consumption and employment under both price mechanisms. It's important to notice the real wage path that under Calvo almost at once jumps to the new and higher steady state. As a matter of fact, in this case the positive long-run effect of the policy takes over the short-run dynamics.

3.1 Disinflation and Inequality

In our relatively simple framework, it is possible to obtain an analytical characterization of inequality focusing on the relative consumption and income shares of RT households, $\frac{c^{rt}}{c}$ and $\frac{wh}{y}$ respectively¹⁷:

$$\frac{c^{rt}}{c} = \frac{wh}{y} \left(\frac{c}{y}\right)^{-1} = \left(1 - \frac{G}{y} - \frac{i}{y}\right)^{-1} (1 - \vartheta) \left\{ \mu^p \left[1 + \nu \left(1 - \frac{1}{R}\right) \right] \right\}^{-1} \quad (6)$$

$$\frac{wh}{y} = (1 - \vartheta) \left\{ \mu^p \left[1 + \nu \left(1 - \frac{1}{R}\right) \right] \right\}^{-1} \quad (7)$$

where the investment ratio is given by $\frac{i}{y} = \delta \left(\frac{k}{h}\right)^{1-\vartheta}$, $\frac{G}{y}$ is the public spending ratio and the rental rate of capital is $r^k = \frac{1}{\beta} - 1 + \delta$. Moreover ν denotes the fraction of wage bill that firms must pay in advance of the production and R is the nominal interest rate at which repayment to the asset holders occurs.

From (6) and (7) it is obvious that both $\frac{c^{rt}}{c}$ and $\frac{wh}{y}$ are affected by disinflation through its effects on markups and the CIA constraint. Disinflation reduces the nominal interest rate. Therefore the lower interest rate unambiguously raises the real wage, boosting income and consumption ratios of RT households. A

¹⁷Derivation of inequality measures is laid out in Appendix C.

fall in markups would provide similar effects on $\frac{c^{rt}}{c}$ and $\frac{wh}{y}$, but from Table 1 we already know that the Calvo and Rotemberg price setting mechanisms have opposite effects on markups. Table 2 presents numerical computations for our inequality measures.

Table 2 - Inequality measures, steady state percentage variations

<i>Inequality measures</i>	<i>Partial Indexation</i>		<i>Full Indexation</i>
	Calvo	Rotemberg	Calvo/Rotemberg
$\frac{c^{rt}}{c}$	0.07	-0.04	0.1
$\frac{wh}{y}$	0.05	0.05	0.06
c^o	0.06	0.08	-0.01
c^{rt}	0.11	0.04	0.07

Results suggest that disinflation decreases consumption inequality under Calvo and increases it under Rotemberg. In fact Calvo price setting strongly benefits non-Ricardian households and to a much lesser extent favors the asset holders. Under Rotemberg the positive variation of optimizers' consumption is slightly larger than RT's consumption increase. Analysis of the full indexation case, when the markup remains constant, allows to discriminate the different consequences of CIA and markup effects. Lower returns on loans to firms determine a powerful increase of real wages that benefits all households, but Ricardian households loose interest payments from firms. Finally, Calvo and Rotemberg have identical impact on the labor income share although different mechanisms are at work in generating this outcome. In particular, note that $\frac{wh}{y} = \frac{c^{rt}}{c} \frac{c}{y}$. Under Rotemberg the relative consumption of the rule of thumbers ($\frac{c^{rt}}{c}$) falls but this is more than compensated for by the increase in the consumption ratio ($\frac{c}{y}$). The net effect is a positive variation of ($\frac{wh}{y}$) equal to 0.05%. By contrast, under Calvo the increase of ($\frac{c^{rt}}{c}$) is accompanied by a modest increase of ($\frac{c}{y}$) still determining the same positive variation of ($\frac{wh}{y}$).

Figure 2 shows the transition paths of Ricardian and non Ricardian consumption¹⁸. During the disinflation process, the rule of thumb consumers bear the disinflation costs under both price settings. In fact, while Ricardian consumption achieves the bottom value at -0.20 under Calvo and -0.15 under Rotemberg, non Ricardian consumption achieves the bottom at -0.27 under Calvo and -0.49 under Rotemberg¹⁹. This is because the rule of thumb consumers cannot smooth consumption and their labor income falls more under Rotemberg.

¹⁸The transition paths are expressed in percentage deviations from the first steady state.

¹⁹The bottom value is expressed in percentage deviation from the first steady state.

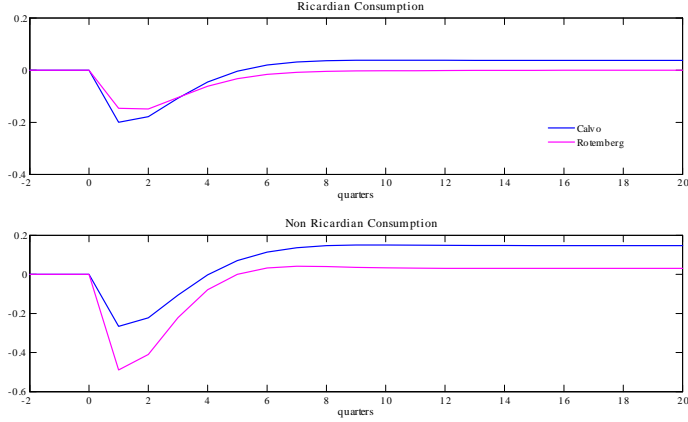


Fig.2 - Ricardian and non Ricardian Consumption

3.2 Welfare Effects of Disinflation

We carry out the welfare analysis under the two price- and wage-setting mechanisms. We also analyse the welfare implications of the external habit assumption (catching up with the Jones) by considering the alternative internal habit hypothesis, where concern for relative income disappears. We define V_0^i and V_{old}^i as the expected values of (1), respectively at time zero (when disinflation experiment is actually implemented) and at the initial steady state (before the disinflation experiment).

Determining V_{old}^i is straightforward:

$$V_{old}^i = \frac{1}{(1-\beta)} \left[\ln(1-b) c_{old}^i - \frac{\phi_1}{(1+\phi)} (h_{old}^i)^{(1+\phi)} \right]; i = o, rt$$

where c_{old}^i and h_{old}^i are respectively the initial steady state values of consumption and hours. Note that V_0^i accounts for both the new steady state and for the transition phase.

V_{old}^i and V_0^i allow to compute the welfare-based ratio, WR^i :

$$WR^i = - \left(\frac{V_0^i - V_{old}^i}{\pi_{old}^* - \pi_{new}^*} \right)$$

where the denominator $\pi_{old}^* - \pi_{new}^*$ allows to weigh the welfare change by the size of disinflation.

Since the utility function is not cardinal, the numerator of the ratio needs to be transformed in a measure which actually can “quantify” the welfare cost (or gain) of disinflation. Following Ascari and Ropele (2012 a, b), the analysis is based on the *consumption equivalent measure* which is defined as the constant fraction of consumption that households must give up to permanently reduce inflation. The *consumption equivalent measure* reads as:

$$\gamma^i = 1 - \exp [(1 - \beta) (V_0^i - V_{old}^i)] \quad (8)$$

and the welfare-based ratio is:

$$WR^i = \left(\frac{\gamma^i}{\pi_{old}^* - \pi_{new}^*} \right) \quad (9)$$

Since γ^i denotes a welfare cost, disinflation is welfare improving when the welfare-based ratio is negative. Therefore, we read the negative values as welfare gains.

Table 3 reports the results.

Table 3 - Welfare Effects

<i>Calvo</i>	WR^i
Ricardian H.	-1.41
Non Ricardian H.	-11.44
<i>Rotemberg</i>	
Ricardian H.	-6.63
Non Ricardian H.	-0.42
<i>Full Indexation</i>	
Ricardian H.	3.5
Non Ricardian H.	-9

All the values are expressed in percentage terms

Although disinflation is welfare improving for both household types, the larger welfare gain accrues to RT consumers under Calvo while the opposite occurs under Rotemberg²⁰. In fact, the welfare gain is equivalent to only an extra 1.41% of consumption for Ricardian households and to an extra 11.44% of consumption for rule of thumbers. However, consistently with the inequality effects, disinflation becomes costly for Ricardian households under the full indexation case where the CIA channel alone benefits RT households and prejudices the asset holders' welfare. In fact, while Ricardian households give up an extra 3.5% of consumption, liquidity constrained agents gain an extra 9% of consumption.

3.3 The Role of the CIA Channel

In this section we shut off the CIA channel and compute the inequality measures and the welfare effects.²¹ Inequality and welfare results are reported in Table 4 and 5, respectively.

²⁰Results hold even under the internal habit specification.

²¹Current DSGE models developed by the central banks do not embody any cash in advance constraint (see, for instance, Christoffel, Coenen and Warne, 2008).

Table 4 - Inequality measures, steady state percentage variations

<i>Inequality measures</i>	<i>Partial Indexation</i>		<i>Full Indexation</i>
	Calvo	Rotemberg	Calvo/Rotemberg
$\frac{c^{rt}}{c}$	-0.03	-0.14	0
$\frac{wh}{y}$	-0.001	-0.001	0
c^o	0.13	0.09	0
c^{rt}	0.04	-0.03	0

Relative to Table 2, results confirm that under Rotemberg disinflation increases inequality while we reject the conclusion that under Calvo inequality falls. As a matter of fact, when the CIA channel is closed other effects take over, apart from the markup behavior. In particular, in the Calvo model, the lower markup is associated to greater output and to an increase in the stock of capital and in interest payments that entirely accrue to optimizing households. In the Rotemberg model, the markup increase still implies a drop in the consumption of the rule of thumbers and an increase in the asset holders' consumption. However, with respect to the Calvo model, the output fall causes also a drop in the per-capita capital, thereby restraining the Ricardian consumption growth.

Table 5 - Welfare Effects (CIA=0)

Calvo	WR^i
Ricardian H.	-5.50
Non Ricardian H.	-1.76
Rotemberg	
Ricardian H.	-10.79
Non Ricardian H.	9.12
Full Indexation	
Ricardian H.	-0.55
Non Ricardian H.	0.64

All the values are expressed in percentage terms

Compared to the case with CIA, Table 5 shows that under Calvo the welfare gain of Ricardian households increases while the welfare gain of RT households notably falls. Contrary to the case with CIA, the disinflation policy benefits more the optimizers than the liquidity constrained households. Under Rotemberg, disinflation strongly benefits asset holders and is very costly for non-Ricardian households.

4 Conclusions

This paper investigates the distributional welfare effects of disinflation in an otherwise standard medium scale DSGE model augmented for LAMP, taking into account the two most commonly used approaches to set prices and wages

in the New Keynesian literature, i.e. Calvo (1983) staggered price scheme and Rotemberg (1982) price framework.

Results can be summarized as follows. A disinflation policy unambiguously increases inequality under Rotemberg mechanism. By contrast, under Calvo this effect may obtain only if the CIA constraint doesn't bind firms ability to finance their working capital.

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5 Appendix A: The Model

In this Appendix we lay out the full model structure.

5.1 Households

There is a continuum of households indexed by i , $i \in [0, 1]$. RT (rt) and Ricardian (o) agents are respectively defined over the intervals $[0, \Omega]$ and $[\Omega, 1]$. All households share the same utility function. Each household has preferences defined over consumption c and labour effort h . Hence, the period household's utility function is:

$$U_t^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln (c_t^i - bc_{t-1}) - \frac{\phi_1}{(1+\phi)} (h_t^i)^{(1+\phi)} \right\} \quad (\text{A1})$$

where c_t^i denotes total individual consumption, b represents the degree of external habit formation in consumption, h_t^i denotes individual labor supply of a differentiated labor bundle. As for preference parameters, ϕ is the inverse of the intertemporal elasticity of substitution of labour and ϕ_1 accounts for the relative importance of disutility of work and utility of consumption in the total utility.

5.2 Consumption bundles

The consumption good is assumed to be a composite good produced with a continuum of differentiated goods c_t^i via the Dixit-Stiglitz consumption basket of household i :

$$c_t^i = \left[\int_0^1 c(z)_t^{\frac{\eta-1}{\eta}} dz \right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 1$ denotes the elasticity of substitution across different varieties of goods.

In particular, the household decides how to allocate its consumption expenditures among different goods. This requires that the consumption index c_t^i is maximized for any given level of expenditures $X_t = \int_0^1 P(z)_t c(z)_t dz$. Solving the intratemporal goods allocation problem, the set of demand equation is:

$$c(z)_t = \left(\frac{P(z)_t}{P_t} \right)^{-\eta} c_t$$

where

$$P_t = \left(\int_0^1 p(z)_t^{(1-\eta)} dz \right)^{\frac{1}{1-\eta}}$$

is the aggregate price consumption index.

5.3 Labour market structure

It is assumed a continuum of differentiated labour inputs indexed by j , $j \in [0, 1]$. Following Schmitt-Grohé and Uribe (2005), household i supplies all labour inputs. Moreover, labor type-specific unions indexed by $j \in [0, 1]$ have some monopoly power in the labour market and make wage-setting decisions. Given the wage W_t^j fixed by union j , households are assumed to supply enough labour h_t^j to satisfy demand. That is,

$$h_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} h_t^d$$

where $\eta_w > 1$ is the elasticity of substitution across different labour inputs, h_t^d is the aggregate labour demand and $W_t = \left(\int_0^1 (W_t^j)^{(1-\eta_w)} dj \right)^{\frac{1}{(1-\eta_w)}}$ is the aggregate wage index. As in Galí (2007), it's assumed that the fraction of Ricardian and non-Ricardian households is uniformly distributed across unions and the aggregate demand for each labor type is uniformly distributed across households. Therefore optimizers and rule of thumbers work for the same amount of work. Therefore the labour supply, which is common across households, must satisfy the resource constraint $h_t^s = \int_0^1 h_t^j dj$. Combining the latter with equation (5) we get:

$$h_t^s = h_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} dj$$

Therefore, the common labour income is denoted by $h_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} dj$.

5.4 Ricardian Households

Ricardian agents are assumed to have access to market for physical capital and to contingent nominal assets. In particular, each period asset holders can purchase any state-contingent nominal payment X_{t+1} in period $t+1$ at the cost $E_t r_{t,t+1} X_{t+1}$ where $r_{t,t+1}$ is a stochastic discount factor between periods t and $t+1$.

Therefore, the ricardian household's period budget constraint in real terms reads as:

$$\begin{aligned} E_t r_{t,t+1} x_{t+1} + c_t^o + i_t^o &= \frac{x_t}{\pi_t} + [r_t^k u_t - a(u_t)] K_t^o + \\ &+ q_t \delta K_t^o + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj + d_t^o + d_t^{WH} \end{aligned}$$

where $\frac{x_t}{\pi_t} \equiv \frac{X_t}{P_t}$ is the real payoff in period t of the nominal state contingent assets purchased at $t-1$. i_t^o denotes the real purchases investment goods at time t .

It is assumed that ricardian households own physical capital K_t^o , accumulate it and then rent it out the firms at a real interest rate r_t^k . Moreover, the optimizers can control the intensity u_t at which the capital is utilized. Hence, the cost of capital depends upon the degree of utilization $a(u_t)$ and it is defined as $a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$. Following CEE (2005) the function satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. Ricardian households also receive firms dividends, d_t^o , and returns from financing working capital of firms²², d_t^{WH} . The gross rate of inflation is $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

The capital stock evolves according to the following law of motion:

$$K_{t+1}^o = (1 - \delta)K_t^o + i_t^o \left[1 - S \left(\frac{i_t^o}{i_{t-1}^o} \right) \right]$$

where δ is the depreciation rate of capital. The function S introduces the adjustment costs on investment and satisfies the following properties: $S(1) = S'(1) = 0$, $S''(1) > 0$.

Hence, the Lagrangean to the maximization problem, with Lagrange multipliers $\beta^t \lambda_t (1 - \tau_{t+s}^h) w_t / \mu_t$, $\beta^t \lambda_t$ and $\beta^t q_t \lambda_t$ respectively associated to the constraints (6), (7) and (8), reads as:

$$L = E_t \sum_{s=0}^{\infty} \left\{ \begin{array}{l} + \lambda_{t+s}^o \left[\begin{array}{l} \frac{U(c_{t+s}^o(i) - bc_{t+s-1}^o; h_{t+s}(i)) +}{\pi_{t+s}} + [r_{t+s}^k u_{t+s} - a(u_{t+s})] K_{t+s}^o + \\ + qt_{t+s} \delta K_{t+s}^o + \\ + h_{t+s}^d \int_0^1 w_{t+s}^j \left(\frac{w_{t+s}^j}{w_{t+s}} \right)^{-\eta_w} dj + \\ + d_{t+s}^o - R_{t+s, t+s+1} x_{t+s+1} - c_{t+s}^o - i_{t+s}^o \end{array} \right] + \\ + \lambda_{t+s}^o q_{t+s} \left[(1 - \delta) K_{t+s}^o + i_{t+s}^o \left[1 - S \left(\frac{i_{t+s}^o}{i_{t+s-1}^o} \right) \right] - K_{t+s+1}^o \right] + \\ + \frac{\lambda_{t+s}^o (1 - \tau_{t+s}^h) w_{t+s}}{\mu_{t+s}} \left[h_{t+s}^s - h_{t+s}^d \int_0^1 \left(\frac{w_{t+s}^j}{w_{t+s}} \right)^{-\eta_w} \right] \end{array} \right\}$$

The Ricardian household's first order conditions with respect to c_t^o , x_{t+1} , K_t^o , i_t^o , and u_t are respectively:

$$\frac{1}{c_t^o - bc_{t-1}} = \lambda_t^o \quad (\text{A2})$$

$$\lambda_t^o = \beta R_{t, t+1} \frac{\lambda_{t+1}^o}{\pi_{t+1}} \quad (\text{A3})$$

$$q_t = \beta \frac{\lambda_{t+1}^o}{\lambda_t^o} [q_{t+1} (1 - \delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})] \quad (\text{A4})$$

²²Here we implicitly follow the financial sector characterization adopted in CEE 2005, who assume that a financial intermediary collects money balances from ricardian households and from the central bank. Such funds are then used to finance the working capital needs of firms, and what is left returns to ricardian households. Given that the Central Bank follows an interest rate policy, explicit modelling of the money market is unnecessary.

$$\begin{aligned} \lambda_t^o &= q_t \lambda_t^o \left[1 - S \left(\frac{i_t^o}{i_{t-1}^o} \right) - \left[S_i \left(\frac{i_t^o}{i_{t-1}^o} \right) \right] i_t^o \right] + \\ &\quad - \beta q_{t+1} \lambda_{t+1}^o S_i \left(\frac{i_{t+1}^o}{i_t^o} \right) i_{t+1}^o \end{aligned} \quad (\text{A5})$$

$$a_u(u_t) = r_t^k \quad (\text{A6})$$

Following CEE (2005), the adjustment cost function and the capital utilization function are given by:

$$\begin{aligned} S \left(\frac{i_t}{i_{t-1}} \right) &= \frac{k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \\ a(u_t) &= \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \end{aligned}$$

5.5 Rule of thumb households

As pointed out above, Non-Ricardian agents just consume current labor income because they cannot save neither invest. Since they don't have access to capital markets, they only pay taxes on labor income and receive transfers from the government. Therefore:

$$c_t^{rt} = w_t h_t^d \quad (\text{A7})$$

The marginal utility of consumption for rule of thumbers is:

$$\frac{1}{c_t^{rt} - b c_{t-1}} = \lambda_t^{rt} \quad (\text{A8})$$

5.6 Wage Setting: Calvo framework

According to the Calvo (1983) framework, each period a union faces a constant probability $(1 - \alpha_w)$ of being able to reoptimize wages. In other words, α_w denotes the degree of wage stickiness. The unions which are not able to reoptimize the wage index it to past inflation according to the following rule:

$$W_t^j = W_{t-1}^j \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\chi_w} = W_{t-1}^j \pi_{t-1}^{\chi_w}$$

where the parameter $\chi_w \in [0, 1]$ is the indexation parameter.

Unions, in choosing the optimal wage w_t^* , have to take into account that they might not be able to do the same after s periods. If this is the case, taking into account that all unions resetting at time t choose the same wage, the real wage at the generic period $t + s$ will be:

$$w_{t+s} = w_t^* \prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}}$$

Hence, to derive the households' first order conditions with respect to the optimal wage, it is possible to pull out the part of the Lagrangean which is useful for this purpose. In particular, a weighted average of the two households types utility function is maximized by the optimizing union which will take into account of not being able to reoptimize in the future. Therefore the union objective is:

$$E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s \left\{ \begin{array}{l} \lambda_{t+s} \left[h_{t+s}^d w_{t+s}^{\eta_w} \left(w_t^* \prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right)^{(1-\eta_w)} \right] + \\ -\lambda_{t+s} \left[h_{t+s}^d w_{t+s}^{(1+\eta_w)} \left(w_t^* \prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right)^{(-\eta_w)} \right] \end{array} \right\}$$

where, importantly,

$$\lambda_{t+s} = [(1 - \Omega) \lambda_{t+s}^o + \Omega \lambda_{t+s}^{rt}] \quad (\text{A9})$$

is the average marginal utility between the ricardian and non ricardian's marginal utilities.

The first order condition with respect to the optimal wage is:

$$\begin{aligned} E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s h_{t+s}^d \left(\frac{w_t^*}{w_{t+s}} \right)^{(-\eta_w)} \left(\prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right)^{(-\eta_w)} \lambda_{t+s} \times \\ \times \left\{ \begin{array}{l} \frac{(\eta_w - 1)}{\eta_w} w_t^* \left(\prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right) + \\ + \frac{U_{t+s}^n}{\lambda_{t+s}} \end{array} \right\} = 0 \end{aligned}$$

The term $\frac{(\eta_w - 1)}{\eta_w}$ is the markup which would prevail in absence of wage stickiness²³.

It's now convenient to write the wage setting equation in recursive form by defining:

$$\begin{aligned} f_{1t} \equiv & \left[\frac{(\eta_w - 1)}{\eta_w} w_t^* E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s h_{t+s}^d \left(\frac{w_t^*}{w_{t+s}} \right)^{(-\eta_w)} \right. \\ & \left. \lambda_{t+s} \left(\prod_{k=1}^s \frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right)^{(1-\eta_w)} \right] \end{aligned}$$

²³In the deterministic steady state it also denotes the wage markup in absence of trend inflation or in case of full indexation, (this is the case in this paper).

and

$$f_{2t} \equiv -w_t^{*(-\eta_w)} E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s h_{t+s}^d w_{t+s}^{\eta_w} U_{n_{t+s}} \left(\prod_{k=1}^s \frac{\pi_{t+k}^{X_w}}{\pi_{t+k}} \right)^{(-\eta_w)}$$

In recursive form:

$$\begin{aligned} f_{1t} &= \frac{(\eta_w - 1)}{\eta_w} w_t^* h_t^d \left(\frac{w_t}{w_t^*} \right)^{(\eta_w)} \lambda_t + \\ &+ \beta \alpha_w E_t \left(\frac{w_{t+1}^*}{w_t^*} \right)^{(\eta_w - 1)} \left(\frac{\pi_t^{X_w}}{\pi_{t+1}} \right)^{(1 - \eta_w)} f_{1_{t+1}} \end{aligned} \quad (\text{A10})$$

and

$$f_{2t} = - \left(\frac{w_t}{w_t^*} \right)^{(\eta_w)} h_t^d U_{n_t} + \beta \alpha_w E_t \left(\frac{w_{t+1}^*}{w_t^*} \right)^{(\eta_w)} \left(\frac{\pi_t^{X_w}}{\pi_{t+1}} \right)^{(-\eta_w)} f_{2_{t+1}} \quad (\text{A11})$$

Hence, the wage setting equation reads as:

$$f_{1t} = f_{2t} \quad (\text{A12})$$

5.7 Wage Setting: Rotemberg framework

In choosing the optimal wage, the unions have to take into account to face a quadratic adjustment cost of the form:

$$\frac{\xi_w}{2} \left(\frac{W_t^j}{(\pi_{t-1}^{X_w}) W_{t-1}^j} - 1 \right)^2 h_t$$

where ξ_w denotes the degree of nominal wage rigidity.

The union objective function in real terms reads as:

$$\mathcal{L}^w = E_t \sum_{t=0}^{\infty} \left\{ \begin{aligned} & - \frac{\phi_1}{(1+\phi)} \left[h_t \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{(-\eta_w)} \right]^{(1+\phi)} + \\ & - \lambda_t \left[- \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{(1-\eta_w)} w_t h_t + \frac{\xi_w}{2} \left(\frac{w_t^j}{w_{t-1}^j} \frac{\pi_t}{\pi_{t-1}^{X_w}} - 1 \right)^2 h_t \right] \end{aligned} \right\}$$

Since unions choose the same wage, they face the same problem. Therefore a symmetric equilibrium takes place.

From the first order condition the wage setting equation comes out:

$$mrs_t = \left\{ \begin{aligned} & \frac{\eta_w - 1}{\eta_w} w_t + \frac{\xi_w}{\eta_w} \left(\frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{X_w}} - 1 \right) \frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{X_w}} + \\ & - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\xi_w}{\eta_w} \left(\frac{w_{t+1}}{w_t} \frac{\pi_{t+1}}{\pi_t^{X_w}} - 1 \right) \frac{h_{t+1}}{h_t} \frac{w_{t+1}}{w_t} \frac{\pi_{t+1}}{\pi_t^{X_w}} \end{aligned} \right\} \quad (\text{A13})$$

where, importantly:

$$\lambda_t = [(1 - \Omega) \lambda_t^o + \Omega \lambda_t^{rt}] \quad (\text{A14})$$

5.8 Firms

Intermediate firms compete monopolistically by producing good z according to the following technology:

$$y_t(z) = (K_t(z))^\vartheta (h_t(z))^{(1-\vartheta)}$$

where $K_t(z)$ is the physical capital stock that firms rent by ricardian households and $h_t(z)$ is the labor input used by each firm z . In particular it is defined as:

$$h_t(z) = \left(\int_0^1 (h_t^j(z))^{\frac{\eta_w-1}{\eta_w}} dj \right)^{\frac{\eta_w}{\eta_w-1}}$$

Firms must pay the wage bill in advance of the production. In other words they are subject to a cash in advance constraint of the form:

$$m_{zt}^f = \nu w_t h_{zt} \quad (\text{A15})$$

where m_{zt}^f denotes the real money balances by firm z and ν is the fraction of wage which is paid in advance. The wage is lent by ricardian households which at the end of the period receive back money at the gross nominal interest rate.

Therefore the marginal costs the firms have to face reads as:

$$mc_t = \left(\frac{r_t^k}{\vartheta} \right)^\vartheta w_t \left[1 + \nu \left(1 - \frac{1}{R} \right) \right] \quad (\text{A16})$$

5.9 Price Setting: Calvo framework

According to the Calvo (1983) framework, each period a firm faces a constant probability $(1 - \alpha)$ of being able to reoptimize prices. In other words, α denotes the degree of price stickiness. The firms which are not able to reoptimize the price index it to past period inflation according to the following rule:

$$P_t(z) = P_{t-1}(z) \left(\frac{P_{t-1}}{P_{t-2}} \right)^\chi \pi^{(1-\chi)} = P_{t-1}(z) \pi_{t-1}^\chi$$

where the parameter $\chi \in [0, 1]$ is the degree of price indexation.

The firms in choosing the optimal price P_t^* have to take into account that they might not be able to do the same after s periods. If this is the case, by taking into account that all firms resetting at time t choose the same price, at the generic period $t + s$ it will be:

$$P_{t,t+s} = P_t^* \prod_{k=1}^s \pi_{t+k-1}^\chi$$

The optimal price P_t^* is chosen in order to maximize the discounted value of expected future profits. Moreover, it's important to remind here that only ricardian households own firms. Hence, the firms' maximization problem is:

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \frac{P_t \lambda_{t+s}^o}{\lambda_t^o P_{t+s}} \left(P_t^* \prod_{k=1}^s \pi_{t+k-1}^\chi - P_{t+s} mc_{t+s} \right) y_{t,t+s}(z)$$

subject to:

$$y_{t,t+s}(z) = \left(\frac{P_t^* \prod_{k=1}^s \pi_{t+k-1}^\chi}{P_{t+s}} \right)^{(-\eta)} y_{t+s}^d$$

where y_t^d is the aggregate demand and $\frac{\lambda_{t+s}^o}{\lambda_t^o}$ denotes the stochastic discount factor of ricardian households.

The first order condition with respect to P_t^* is:

$$E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \frac{\lambda_{t+s}^o}{\lambda_t^o} \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^\chi}{\prod_{k=1}^s \pi_{t+k}} \right)^{(-\eta)} y_{t+s}^d \left(\frac{P_t^*}{P_t} \right)^{(-\eta-1)} \left[\left(\frac{P_t^*}{P_t} \right) \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^\chi}{\prod_{k=1}^s \pi_{t+k}} \right) + \frac{\eta}{(1-\eta)} mc_{t+s} \right] = 0$$

The term $\frac{(\eta-1)}{\eta}$ is the markup which would prevail in absence of price stickiness²⁴.

It's useful to write the price setting equation in recursive form by defining:

$$x_{1t} \equiv \left(\frac{P_t^*}{P_t} \right)^{(-\eta-1)} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \frac{\lambda_{t+s}^o}{\lambda_t^o} \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^\chi}{\prod_{k=1}^s \pi_{t+k}} \right)^{(-\eta)} y_{t+s}^d mc_{t+s}$$

and

$$x_{2t} \equiv \left(\frac{P_t^*}{P_t} \right)^{(-\eta)} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \frac{\lambda_{t+s}^o}{\lambda_t^o} \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^\chi}{\prod_{k=1}^s \pi_{t+k}} \right)^{(1-\eta)} y_{t+s}^d$$

By writing recursively:

²⁴In the deterministic steady state it also denotes the price markup in absence of trend inflation or in case of full indexation, (this is the case in this paper).

$$x_{1t} = p_t^{*(-\eta-1)} y_t^d m c_t + E_t \left\{ (\beta \alpha) \frac{\lambda_{t+1}^o}{\lambda_t^o} \left(\frac{\pi_t^\chi}{\pi_{t+1}} \right)^{(-\eta)} \left(\frac{p_t^*}{p_{t+1}^*} \right)^{(-\eta-1)} x_{1t+1} \right\} \quad (\text{A17})$$

and

$$x_{2t} = p_t^{*(-\eta)} y_t^d + E_t \left\{ (\beta \alpha) \frac{\lambda_{t+1}^o}{\lambda_t^o} \left(\frac{\pi_t^\chi}{\pi_{t+1}} \right)^{(1-\eta)} \left(\frac{p_t^*}{p_{t+1}^*} \right)^{(-\eta)} x_{2t+1} \right\} \quad (\text{A18})$$

It's possible to rewrite the price setting equation as:

$$x_{2t} = \frac{\eta}{\eta-1} x_{1t} \quad (\text{A19})$$

5.10 Price Setting: Rotemberg framework

The Rotemberg price setting assumes that each intermediate firm pays an increasing and convex cost measured in terms of aggregate output. This cost is given by:

$$\frac{\xi_p}{2} \left(\frac{P_t(z)}{\pi_{t-1}^\chi P_{t-1}(z)} - 1 \right)^2 y_t$$

where $\phi_p > 0$ measures the degree of nominal price rigidity.

Therefore each firm maximizes its present discounted value of profits for its owners (Ricardian households, i.e. o):

$$\begin{aligned} \max_{P_t(z)} E_t \left[\sum_{s=0}^{\infty} \beta^s \lambda_{t+s}^o \frac{D_{t+s}(z)}{P_{t+s}} \right] \\ \text{s.t. } y_{t+s}(z) = \left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{-\eta} y_{t+s} \end{aligned}$$

where

$$\frac{D_{t+s}(z)}{P_{t+s}} = \frac{P_{t+s}(z)}{P_{t+s}} y_{t+s}(z) - m c_{t+s} y_{t+s}(z) - \frac{\xi_p}{2} \left(\frac{P_{t+s}(z)}{\pi_{t+s-1}^\chi P_{t+s-1}(z)} - 1 \right)^2 y_{t+s}$$

Substituting the constraint to the maximization problem into the objective function:

$$\begin{aligned} E_t \sum_{s=0}^{\infty} \beta^s \lambda_{t+s}^o \left[\left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{1-\eta} y_{t+s} + \right. \\ \left. - m c_{t+s} \left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{-\eta} y_{t+s} - \frac{\xi_p}{2} \left(\frac{P_{t+s}(z)}{\pi_{t+s-1}^\chi P_{t+s-1}(z)} - 1 \right)^2 y_{t+s} \right] \end{aligned}$$

The FOC to the problem is:

$$\begin{aligned}
0 &= (1 - \eta) \lambda_t^\circ \left(\frac{P_t(z)}{P_t} \right)^{-\eta} \frac{y_t}{P_t} + \eta \lambda_t^\circ m c_t \left(\frac{P_t(z)}{P_t} \right)^{-\eta-1} \frac{y_t}{P_t} + \\
&\quad - \xi_p \lambda_t^\circ \left(\frac{P_t(z)}{\pi_{t-1}^\chi P_{t-1}(z)} - 1 \right) \frac{y_t}{\pi_{t-1}^\chi P_{t-1}(z)} + \\
&\quad + \beta E_t \left[\xi_p \lambda_{t+1}^\circ \left(\frac{P_{t+1}(z)}{\pi_t^\chi P_t(z)} - 1 \right) \frac{P_{t+1}(z) y_{t+1}}{\pi_t^\chi P_t(z)^2} \right]
\end{aligned}$$

Given the symmetric equilibrium:

$$\begin{aligned}
0 &= (1 - \eta) \lambda_t^\circ \frac{y_t}{P_t} + \eta \lambda_t^\circ m c_t \frac{y_t}{P_t} - \phi_p \lambda_t^\circ \left(\frac{P_t}{\pi_{t-1}^\chi P_{t-1}} - 1 \right) \frac{y_t}{\pi_{t-1}^\chi P_{t-1}} + \\
&\quad + \beta E_t \left[\xi_p \lambda_{t+1}^\circ \left(\frac{P_{t+1}}{\pi_t^\chi P_t} - 1 \right) \frac{P_{t+1} y_{t+1}}{\pi_t^\chi P_t^2} \right]
\end{aligned}$$

Multiplying by P_t :

$$\begin{aligned}
0 &= (1 - \eta) \lambda_t^\circ y_t + \eta \lambda_t^\circ m c_t y_t - \xi_p \lambda_t^\circ \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) y_t \frac{\pi_t}{\pi_{t-1}^\chi} + \\
&\quad + \beta E_t \xi_p \lambda_{t+1}^\circ \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} y_{t+1}
\end{aligned}$$

Dividing by y_t :

$$0 = (1 - \eta) \lambda_t^\circ + \eta \lambda_t^\circ m c_t - \xi_p \lambda_t^\circ \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) \frac{\pi_t}{\pi_{t-1}^\chi} + \beta E_t \xi_p \lambda_{t+1}^\circ \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} \frac{y_{t+1}}{y_t}$$

Solving for marginal cost:

$$m c_t = \left(\frac{\eta - 1}{\eta} \right) + \frac{\xi_p}{\eta} \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) \frac{\pi_t}{\pi_{t-1}^\chi} - \beta E_t \frac{\xi_p}{\eta} \lambda_{t+1}^\circ \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} \frac{y_{t+1}}{y_t} \quad (\text{A20})$$

5.11 Aggregation

The aggregate production function is:

$$y_t = (u_t K_t)^\vartheta h_t^{d(1-\vartheta)} \quad (\text{A21})$$

and the aggregate absorption is:

$$y_t^d = c_t + i_t + g_t + a(u_t) K_t \quad (\text{A22})$$

where:

$$c_t = (1 - \Omega) c_t^\circ + \Omega c_t^{\tau t} \quad (\text{A23})$$

$$i_t = (1 - \Omega) i_t^o \quad (\text{A24})$$

$$K_t = (1 - \Omega) K_t^o \quad (\text{A25})$$

5.12 Market clearing

5.12.1 Goods market equilibrium under Calvo

The expression warranting the equilibrium in the good market is:

$$y_t = s_t y_t^d \quad (\text{A26})$$

where s_t denotes the resource cost due to relative price dispersion in the Calvo model. It evolves according to:

$$s_t = (1 - \alpha) p_t^{*(-\eta)} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^\chi} \right)^\eta s_{t-1} \quad (\text{A27})$$

where p_t^* , in the light of the aggregate price index, must satisfy:

$$\alpha \pi_t^{(\eta-1)} (\pi_{t-1}^\chi)^{(1-\eta)} + (1 - \alpha) p_t^{*(1-\eta)} = 1$$

5.12.2 Goods market equilibrium under Rotemberg

The aggregate resource constraint accounts for price and nominal wage adjustment costs, therefore:

$$y_t = c_t + i_t + g_t + a(u_t) K_t + \frac{\xi_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right)^2 y_t + \frac{\xi_w}{2} \left(\frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\chi_w}} - 1 \right)^2 h_t \quad (\text{A28})$$

5.12.3 Labour market equilibrium under Calvo

The equilibrium on the labour market is given by:

$$h_t^s = \tilde{s}_t h_t^d \quad (\text{A29})$$

where \tilde{s}_t denotes the resource cost due to relative wage dispersion in the Calvo model. It evolves according to:

$$\tilde{s}_t = (1 - \alpha_w) \left(\frac{w_t^*}{w_t} \right)^{(-\eta_w)} + \alpha_w \left(\frac{w_{t-1}}{w_t} \right)^{(-\eta_w)} \left(\frac{\pi_t}{\pi_{t-1}^{\chi_w}} \right)^{\eta_w} \tilde{s}_{t-1} \quad (\text{A30})$$

where it must hold that:

$$w_t^* = \left(\frac{w_t^{(1-\eta_w)} - \alpha_w w_{t-1}^{(1-\eta_w)} \left(\frac{\pi_{t-1}^{\chi_w}}{\pi_t} \right)^{(1-\eta_w)}}{(1-\alpha_w)} \right)^{\frac{1}{(1-\eta_w)}} \quad (\text{A31})$$

5.13 Fiscal Authority

Public spending is financed through seigniorage:

$$g_t = m_t - \frac{m_{t-1}}{\pi_t}$$

where m_t denotes real money balances and π_t is the actual gross inflation rate. Government minimizes the costs of purchasing the composite good. Therefore, government's absorption of a single type of good is $g_{zt} = \left(\frac{P_{zt}}{P_t} \right)^{-\eta} g_t$.

5.14 Monetary Authority

Monetary authority sets the nominal interest rate according to the non-linear rule described above:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi}$$

6 Appendix B: Calibration

Baseline calibration of the structural parameter values follows CEE (2005).

Table B1: Parameter Values

<i>Parameter</i>	<i>Value</i>	<i>Description</i>
<i>Households</i>		
β	$1.03^{(-1/4)}$	Subjective discount factor
b	0.65	Degree of habit persistence
ϕ	1	Inverse of intertemporal substitution of labor
ϕ_1	1.1196	Disutility of work
η_w	21	Wage elasticity of demand for a specific labor variety
Ω	0.30	Share of Rule of Thumb consumers
α_w	0.64	Calvo wage
χ_w	0.75*	Wage indexation
<i>Firms</i>		
ϑ	0.36	Share of capital in value added
δ	0.025	Depreciation rate of capital
η	6	Price elasticity of demand for a specific good variety
α	0.6	Calvo price
χ	0.50*	Price indexation
ν	0.15*	Cash in advance parameter
<i>Monetary Authority</i>		
ϕ_π	1.5	Inflation stabilization

7 Appendix C: Steady State Solution and Inequality Measures

7.1 Steady State in Calvo

From the wage setting equations A10 and A11 we get:

$$f_1 = \left(\frac{\eta_w - 1}{\eta_w} \right) \left(\frac{w^* h AMU}{1 - \alpha_w \beta \pi^{(\eta_w - 1)(1 - \chi_w)}} \right) \left(\frac{w}{w^*} \right)^{\eta_w} \quad (C1)$$

and

$$f_2 = - \left(\frac{AMDh}{1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}} \right) \left(\frac{w}{w^*} \right)^{\eta_w} \quad (C2)$$

where the Average Marginal Utility is $AMU = (1 - RT)\lambda^o + RT\lambda^{rt}$ and $\lambda^o = \frac{1}{c^o - bc}$ and $\lambda^{rt} = \frac{1}{c^{rt} - bc}$. Therefore:

$$AMU = \frac{1 - RT}{c^o - bc} + \frac{RT}{c^{rt} - bc} \quad (C3)$$

The Average Marginal Disutility is:

$$AMD = -\phi_1 \tilde{s}h \quad (C4)$$

In equilibrium A12 holds. Imposing equality between C1 and C2 we get:

$$\frac{\phi_1 \tilde{s}h}{1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}} = \left(\frac{\eta_w - 1}{\eta_w} \right) \left[\frac{w^* \left(\frac{1 - RT}{c^o - bc} + \frac{RT}{c^{rt} - bc} \right)}{1 - \alpha_w \beta \pi^{(\eta_w - 1)(1 - \chi_w)}} \right]$$

Notice that:

$$c^{rt} = wh \quad (C5)$$

Therefore:

$$\frac{\phi_1 \tilde{s}h}{1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}} = \left(\frac{\eta_w - 1}{\eta_w} \right) \left[\frac{w^* \left(\frac{1 - RT}{c^o - bc} + \frac{RT}{wh - bc} \right)}{1 - \alpha_w \beta \pi^{(\eta_w - 1)(1 - \chi_w)}} \right]$$

Dividing by $\left(\frac{\eta_w - 1}{\eta_w} \right) w^*$ and multiplying by $1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}$:

$$\frac{\phi_1 \tilde{s}h}{\left(\frac{\eta_w - 1}{\eta_w} \right) w^*} = \left(\frac{1 - RT}{c^o - bc} + \frac{RT}{wh - bc} \right) \left(\frac{1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}}{1 - \alpha_w \beta \pi^{(\eta_w - 1)(1 - \chi_w)}} \right)$$

Defining: $\Gamma_1 \equiv \left(\frac{\eta_w - 1}{\eta_w} \right) w^*$ and $\Gamma \equiv \left(\frac{1 - \alpha_w \beta \pi^{\eta_w(1 - \chi_w)}}{1 - \alpha_w \beta \pi^{(\eta_w - 1)(1 - \chi_w)}} \right)$, we rewrite:

$$\frac{\phi_1 \tilde{s}h}{\Gamma_1} = \left(\frac{1 - RT}{c^o - bc} + \frac{RT}{wh - bc} \right) \Gamma \quad (C6)$$

Defining $A1 \equiv \left(\frac{k}{h}\right)$, the average consumption $c = \frac{c}{y}y$, where $y = k^\vartheta h^{(1-\vartheta)} = \left(\frac{k}{h}\right)^\vartheta h = A1^\vartheta h$ and $\frac{c}{y} = 1 - \frac{G}{y} - \frac{I}{y} = 1 - g - \frac{\delta k}{A1^\vartheta h} = 1 - g - \delta A1^{(1-\vartheta)}$, reads as:

$$c = A1^\vartheta h \left(1 - g - \delta A1^{(1-\vartheta)}\right) \quad (C7)$$

Rewriting C6 and defining $\Gamma_3 \equiv 1 - g - \delta A1^{(1-\vartheta)}$:

$$\frac{\phi_1 \tilde{s} h}{\Gamma_1} = \left(\frac{1 - RT}{c^\circ - b(A1^\vartheta h \Gamma_3)} + \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right) \Gamma$$

Rearranging terms, we can get now an equation for c° as a function of h :

$$c^\circ = \frac{(1 - RT) + bA1^\vartheta h \Gamma_3 \left(\frac{\phi_1 \tilde{s} h}{\Gamma_1} - \frac{RT}{wh - bA1^\vartheta h \Gamma_3} \right)}{\left(\frac{\phi_1 \tilde{s} h}{\Gamma_1} - \frac{RT}{wh - bA1^\vartheta h \Gamma_3} \right)} \quad (C8)$$

Recall that the aggregate resource constraint reads as:

$$A1^\vartheta h = s [c + I + G + a(u)k] \quad (C9)$$

From the aggregate resource constraint it's now possible to obtain steady state hours, knowing that:

$$c = (1 - RT)c^\circ + RTc^{rt} \quad (C10)$$

$$I = \delta k = \delta \frac{k}{h} h = \delta A1 h \quad (C11)$$

$$a(u) = 0$$

Therefore, rewrite C9 taking into account C10, C8, C5 and C11:

$$A1^\vartheta h = s \left\{ (1 - RT) \left[\frac{(1 - RT) + bA1^\vartheta h \Gamma_3 \left(\frac{\phi_1 \tilde{s} h}{\Gamma_1} - \frac{RT}{wh - bA1^\vartheta h \Gamma_3} \right)}{\left(\frac{\phi_1 \tilde{s} h}{\Gamma_1} - \frac{RT}{wh - bA1^\vartheta h \Gamma_3} \right)} \right] + \right\}$$

In rearranging and collecting terms, rewrite $\frac{s\phi_1 h G}{\Gamma_1}$ as $\frac{s\phi_1 \tilde{s} h^2 g A1^\vartheta}{\Gamma_1}$ and $\frac{sRTG}{wh - bA1^\vartheta h \Gamma_3}$ as $\frac{sRTgA1^\vartheta}{w - bA1^\vartheta h \Gamma_3}$, having defined $g \equiv \frac{G}{y}$.
Therefore:

$$h = \left(\frac{\Gamma_7}{\Gamma_6} \right)^{\frac{1}{2}} \quad (C12)$$

where:

$$\begin{aligned} \Gamma_7 &\equiv s(1 - RT)^2 + \frac{A1^\vartheta RT}{\Gamma_4} - \frac{s(1 - RT)bA1^\vartheta \Gamma_3 RT}{\Gamma_4} - \frac{sRTgA1^\vartheta}{\Gamma_4} - \frac{sRT\Gamma_5}{\Gamma_4} \\ \Gamma_6 &\equiv \frac{A1^\vartheta \phi_1 \tilde{s}}{\Gamma_1} - \frac{s(1 - RT)bA1^\vartheta \Gamma_3 \phi_1 \tilde{s}}{\Gamma_1} - \frac{s\phi_1 \tilde{s} g A1^\vartheta}{\Gamma_1} - \frac{s\phi_1 \tilde{s} \Gamma_5}{\Gamma_1} \\ \Gamma_5 &\equiv RTw + \delta A1 \\ \Gamma_4 &\equiv w - bA1^\vartheta \Gamma_3 \end{aligned}$$

7.2 Steady State in Rotemberg

The marginal rate of substitution in steady state is:

$$\frac{\phi_1 h^\phi}{AMU} = \left(\frac{\eta_w - 1}{\eta_w} \right) w + \frac{(1 - \beta)}{\eta_w} \xi_w \left(\pi^{(1 - \chi_w)} - 1 \right) \pi^{(1 - \chi_w)} \quad (C13)$$

Recalling C3 and rearranging terms, we rewrite C13 as:

$$\phi_1 h^\phi = \left(\frac{1 - RT}{c^o - bc} + \frac{RT}{c^{rt} - bc} \right) \Omega$$

where $\Omega \equiv \left(\frac{\eta_w - 1}{\eta_w} \right) w + \frac{(1 - \beta)}{\eta_w} \xi_w \left(\pi^{(1 - \chi_w)} - 1 \right) \pi^{(1 - \chi_w)}$.

Recalling C5 and C7, we rewrite:

$$\phi_1 h^\phi = \left[\frac{1 - RT}{c^o - b(A1^\vartheta h \Gamma_3)} + \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right] \Omega$$

Rearranging terms, we can get now an equation for c^o as a function of h :

$$c^o = \frac{(1 - RT) + bA1^\vartheta h \Gamma_3 \left(\frac{\phi_1 h^\phi}{\Omega} - \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right)}{\left(\frac{\phi_1 h^\phi}{\Omega} - \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right)} \quad (C14)$$

Recall that the aggregate resource constraint reads as:

$$y = c + I + G + a(u)k + \frac{\xi_p}{2} \left(\pi^{(1 - \chi)} - 1 \right)^2 y + \frac{\xi_w}{2} \left(\pi^{(1 - \chi_w)} - 1 \right)^2 h \quad (C15)$$

Recalling that:

$$\begin{aligned} y &= A1^\vartheta h \\ c &= (1 - RT)c^o + RTc^{rt} \\ I &= \delta A1h \\ a(u) &= 0 \\ \phi &= 1 \end{aligned}$$

Rewrite C15:

$$\begin{aligned} A1^\vartheta h Z_1 &= (1 - RT) \left[\frac{(1 - RT) + bA1^\vartheta h \Gamma_3 \left(\frac{\phi_1 h}{\Omega} - \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right)}{\left(\frac{\phi_1 h}{\Omega} - \frac{RT}{wh - b(A1^\vartheta h \Gamma_3)} \right)} \right] + \\ &+ RTwh + \delta A1h + G + ZZh \end{aligned}$$

where: $Z_1 \equiv (1 - Z)$; $Z \equiv \frac{\xi_p}{2} \left(\pi^{(1 - \chi)} - 1 \right)^2$ and $ZZ \equiv \frac{\xi_w}{2} \left(\pi^{(1 - \chi_w)} - 1 \right)^2$.

In rearranging and collecting terms, rewrite $\frac{\phi_1 h G}{\Omega}$ as $\frac{\phi_1 h^2 g A1^\vartheta}{\Omega}$ and $\frac{RTG}{wh - b(A1^\vartheta h \Gamma_3)}$ as $\frac{RTg A1^\vartheta}{w - b(A1^\vartheta \Gamma_3)}$.

Therefore:

$$h = \left(\frac{Z_5}{Z_6} \right)^{\frac{1}{2}} \quad (\text{C16})$$

where:

$$\begin{aligned} Z_5 &\equiv \frac{A1^\vartheta Z_1 RT}{\Gamma_4} + (1 - RT)^2 - \frac{(1 - RT) RT b A1^\vartheta \Gamma_3}{\Gamma_4} - \frac{RT^2 w}{\Gamma_4} - \frac{RT \delta A1}{\Gamma_4} + \\ &\quad - \frac{RT g A1^\vartheta}{\Gamma_4} - \frac{RT Z Z}{\Gamma_4} \\ Z_6 &\equiv \frac{A1^\vartheta Z_1 \phi_1}{\Omega} - \frac{(1 - RT) b A1^\vartheta \Gamma_3 \phi_1}{\Omega} - \frac{\phi_1 RT w}{\Omega} - \frac{\phi_1 \delta A1}{\Omega} - \frac{\phi_1 g A1^\vartheta}{\Omega} - \frac{\phi_1 Z Z}{\Omega} \end{aligned}$$

7.3 Inequality Measures

In this Appendix we derive the following inequality measures: $\frac{c^{rt}}{c}$ and $\frac{wh}{y}$ under the Calvo price mechanism.

The relative consumption of rule of thumbers can be written as:

$$\frac{c^{rt}}{c} = \frac{wh}{y} \left(\frac{c}{y} \right)^{-1} = \left(1 - \frac{G}{y} - \frac{I}{y} \right)^{-1} w \left(\frac{h}{k} \right)^\vartheta$$

From the cost minimization problem the marginal cost reads as:

$$MC = \left(\frac{r^k}{\vartheta} \right)^\vartheta \left[\frac{w \left[1 + \nu \left(1 - \frac{1}{R} \right) \right]}{(1 - \vartheta)} \right]^{(1 - \vartheta)}$$

Recalling also that the average markup is defined as:

$$\mu^p = \frac{1}{MC}$$

the real wage is:

$$w = (1 - \vartheta) \left(\frac{k}{h} \right)^\vartheta \left\{ \mu^p \left[1 + \nu \left(1 - \frac{1}{R} \right) \right] \right\}^{-1}$$

Therefore:

$$\frac{c^{rt}}{c} = \frac{wh}{y} \left(\frac{c}{y} \right)^{-1} = \left(1 - \frac{G}{y} - \frac{I}{y} \right)^{-1} (1 - \vartheta) \left\{ \mu^p \left[1 + \nu \left(1 - \frac{1}{R} \right) \right] \right\}^{-1}$$

where $\frac{I}{y} = \delta \left(\frac{k}{h} \right)^{1 - \vartheta}$ and $\frac{k}{h} = \left(\frac{r^k}{\vartheta MC} \right)^{\left(\frac{1}{\vartheta - 1} \right)}$.

It's now straightforward to obtain:

$$\frac{wh}{y} = (1 - \vartheta) \left\{ \mu^p \left[1 + \nu \left(1 - \frac{1}{R} \right) \right] \right\}^{-1}$$