



DEPARTMENT OF ECONOMICS,
MANAGEMENT AND STATISTICS
UNIVERSITY OF MILAN – BICOCCA

DEMS WORKING PAPER SERIES

**Can a DSGE Model Explain a Costly
Disinflation?**

Maria Ferrara, Patrizio Tirelli

No. 306 – July 2015

Can a DSGE Model Explain a Costly Disinflation?

Maria Ferrara*
University of Naples "Parthenope"

Patrizio Tirelli†
University of Milano-Bicocca

July, 2015

Abstract

This paper shows that a medium-scale DSGE model is able to explain a contemporaneous reduction of output and consumption during a disinflation policy, as it is in the empirical evidence. To this aim, we introduce Rotemberg (1982) adjustment costs and the limited asset market participation assumption.

JEL Classification System: E31, E5

Keywords: Disinflation, Rotemberg price adjustment, Monetary Policy

*ferraramaria11@gmail.com

†patrizio.tirelli@unimib.it

1 Introduction

The apparent inability to match empirical evidence about the short-run contractions associated to disinflation has been identified as a weak spot of Dynamic Stochastic General Equilibrium (DSGE). Based on the the post-70s reduction in inflation, the empirical literature has shown that disinflations require a short-run output "sacrifice" (Gordon and King, 1982; Ball 1994; Cecchetti and Rich, 2001), and that this happens even under inflation targeting regimes (Durand et al., 2007; Corbo, Landerretche, and Schmidt-Hebbel, 2001; Goncalves and Carvalho, 2009). Merkl (2013) has pointed out that disinflationary booms are in fact a robust feature of small-scale DSGE models under inflation targeting. By contrast, Ascari and Ropele (2012a, b; AR henceforth) show that in the work-horse medium-scale DSGE model (Christiano et al. 2005; CEE henceforth) a credible cold-turkey disinflation causes a deep and prolonged recession. The Merkl and AR models differ in one crucial aspect, concerning the degree of inflation indexation, respectively set at zero and one. In fact a large body of empirical evidence shows that inflation indexation has been very low since the beginning of the Great moderation period (Benati, 2008, 2009; Ascari, Castelnovo and Rossi, 2011; Hofmann, Peersman and Straub, 2010) and possibly zero if the central bank pursues a time-varying inflation target (Cogley and Sbordone, 2008). Ascari and Rossi (2012) document that under Rotemberg (1982) pricing disinflations cause short-run output losses even for low inflation indexation.

This paper reconsiders the the issue of disinflation in a medium scale DSGE model with Rotemberg pricing, showing that it can produce a short run consumption loss under Limited Asset Market Participation (LAMP henceforth), that is, when a fraction of consumers do not hold any wealth and entirely consume their current labor income in each period (as in Galí *et al.*, 2004,2007; Bilbiie, 2008). The reason for this is rather simple. Under Rotemberg (1982) framework the output loss associated to disinflation implies a reduction in wages and in labor incomes. The liquidity-constrained households cannot smooth consumption in response to the disinflation, and aggregate consumption therefore falls if the share of constrained households is sufficiently large.

The paper is organized as follows. The next section briefly presents the main features of the model. Section 2 explains the disinflation experiment. Section 3 reports the results and Section 4 finally concludes.

2 The Model

Our standard medium scale NK-DSGE model embodies both nominal and real frictions (see the Appendix for details). The former concern both price- and wage-setters. The latter include monopolistic competition in goods and labor markets, internal habit in consumption, variable capital utilization and adjustment cost in investment decisions. We augment the model by introducing the LAMP assumption, implying that a fraction of households just consume current labor income.

3 The Disinflation Experiment

3.1 Monetary Policy Rule

We follow Smets and Wouters (2005) assuming that the central bank sets the nominal interest rate according to the following non-linear rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^\rho \left[\left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{y_t}{y^*} \right)^{\phi_y} \right]^{(1-\rho)} \quad (1)$$

where R_t , R_{t-1} , and R^* respectively denote the actual, past and target nominal interest rate. π_t and π^* are the actual inflation and inflation target. y_t and y^* are the actual output and the final steady state output. The parameter ρ indicates the lag in the interest rate and ϕ_y represents the output stabilization. Moreover $\phi_\pi > 1$ is the parameter denoting the central bank's concern with the inflation stabilization around the target. We assume that the central bank credibly implements a cold-turkey disinflation from the annual level of 5% to the target of 2%.

The disinflation experiment is an unanticipated permanent decrease in the money growth rate and it entails a move from one steady state to another. As in AR, the perfect foresight transition paths are obtained by numerically solving the non linear model¹. Therefore such an experiment concerns trend inflation values consistent with the post-war history of the industrialized countries.

3.2 Calibration

With some exceptions, the parameters calibration follows Smets and Wouters (2005), (SW henceforth) who use bayesian techniques to estimate DSGE models for US and EURO area. In particular we consider the US economy in the sample period 1983-2002. Table 1 displays the parameters value. As it is mentioned above, the indexation parameter calibration is a debated issue in the empirical literature. Therefore, we first follow SW. In particular $\chi_w = 0.75$ and $\chi = 0.34$. We then verify that result still holds for different indexation degrees. In particular, nothing changes considering different wage indexation degrees². As for price indexation, we follow the view of a very low inflation indexation to check the robustness of our results. Therefore, we set the price indexation parameter equal to 0.33³ as in Ascari, Castelnuovo, Rossi (2011) and equal to 0.2 as in Acocella, Di Bartolomeo, Tirelli (2014). Next section shows that the result is robust.

¹Ascari and Merkl (2009) show that the use of log-linear approximation to study disinflation may generate misleading results. Transition paths are obtained using the software platform DYNARE.

²Results are available upon request.

³Results are basically the same as the baseline case where the parameter is set at 0.34. Results are available upon request.

Relatively to RT households' share, first we set the parameter equal to 0.3, according to a conservative parametrization in DSGE literature⁴. Then, we fix the parameter at 0.5 to show the stronger effect.

Finally, in order to calibrate Rotemberg quadratic adjustment costs, we refer to the paper by Keen and Wang (2007) providing the relationship between the price (wage) adjustment cost parameter and the constant fraction of reoptimizing firms (unions).

Table 1: Parameter Values

<i>Parameter</i>	<i>Value</i>	<i>Description</i>
<i>Households</i>		
β	0.99	Subjective discount factor
b	0.44	Degree of habit persistence
ϕ	1	Inverse of Frisch elasticity
ϕ_1	1	Disutility of work
η_w	21	Wage elasticity of demand for a specific labor variety
Ω	0.30 - 0.50	Share of Rule of Thumb consumers
χ_w	0.75	Wage indexation
<i>Firms</i>		
ϑ	0.24	Share of capital in value added
δ	0.025	Depreciation rate of capital
η	6	Price elasticity of demand for a specific good variety
χ	0.34 ⁵ - 0.2	Price indexation
<i>Monetary Authority</i>		
ϕ_π	1.49	Inflation stabilization in Taylor rule
ϕ_y	0.09	Output stabilization in Taylor rule
ρ	0.90	Lag of interest rate

4 Results

Figure 1 displays the transition paths of inflation rate, output, nominal and ex-ante real interest rates⁶ according to different shares of rule of thumbers populating the economy.

Results show that with homogeneous and forward-looking households (blue line) the model cannot explain the consumption fall after disinflation⁷. As a matter of fact consumption increases achieving the new and higher steady state level. Differently, we observe a contemporaneous reduction of output and consumption if liquidity constrained households populate the economy. The reason is as follows. RT consumers cannot smooth consumption over time and therefore entirely bear the disinflation costs. In turn, the aggregate consumption falls.

⁴See, among others, Coenen and Straub, 2005; Forni *et al.*, 2009; Ratto *et al.* (2009).

⁵Results do not change setting the price indexation parameter at 0.33, according to Ascari, Castelnuovo, Rossi (2011).

⁶The output path, the ex-ante real interest rate and the nominal interest rate are expressed in percentage deviations from the new steady state.

⁷The same result is reported by Ascari and Rossi (2012) in a simple sticky price model.

The cyan and magenta lines respectively denote a share of 30% and 50% of RT households showing that the higher is the fraction of non Ricardian consumers the stronger is the effect. In fact, the bottom value of aggregate consumption is about -4.21 in the former case and -6 in the latter one⁸.

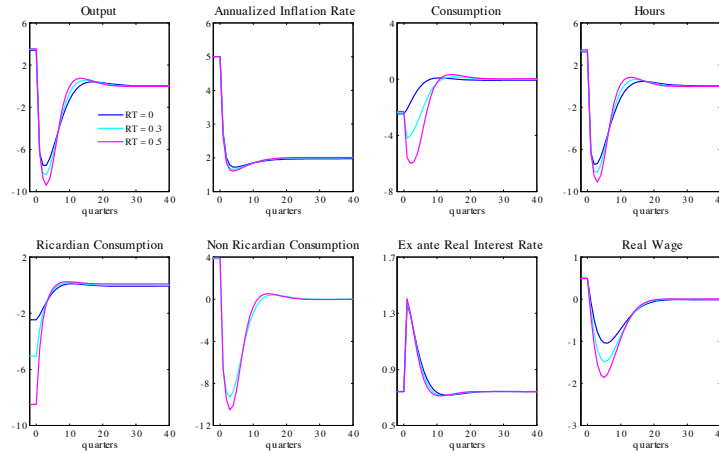


Fig.1 - Short-run effects of disinflation (Baseline indexation calibration)

As Figure 2 shows, the previous result still holds if we reduce the price indexation parameter, following Acocella, Di Bartolomeo, Tirelli (2014).

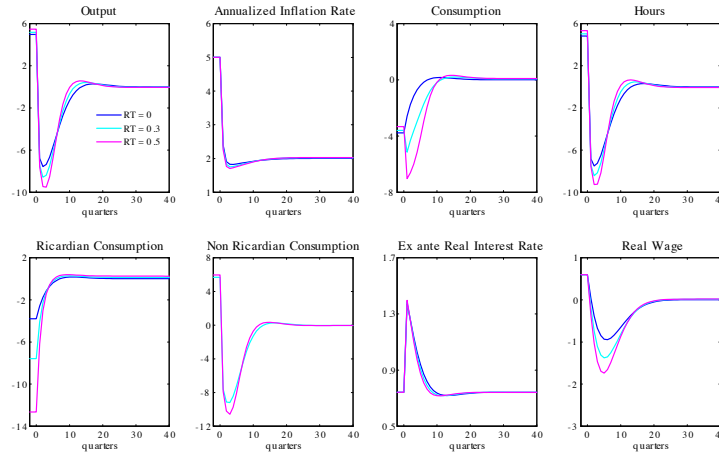


Fig. 2 - Short run effects of disinflation (Robustness check)

In this case the consumption bottom value is about -5.15 for 30% of RT households and about -6 if the half of population is liquidity constrained.

⁸The bottom values are expressed in percentage deviations from the new steady state.

5 Conclusions

In this work we simulate a disinflation experiment in a standard medium scale DSGE model. We show that, introducing Rotemberg (1982) staggered price framework and the LAMP assumption, the model is able to match the empirical evidence. In fact a disinflation policy entails a contemporaneous reduction of output and consumption after a disinflation policy.

References

- [1] **Ascari, Guido & Castelnuovo, Efrem & Rossi, Lorenza, 2011.** "Calvo vs. Rotemberg in a trend inflation world: An empirical investigation," *Journal of Economic Dynamics and Control*, Elsevier, vol. 35(11), pages 1852-1867.
- [2] **Ascari, Guido & Ropele, Tiziano, 2012 a.** "Disinflation in a DSGE perspective: Sacrifice ratio or welfare gain ratio?," *Journal of Economic Dynamics and Control*, Elsevier, vol. 36(2), pages 169-182.
- [3] **Ascari, Guido & Ropele, Tiziano, 2012 b.** "Sacrifice Ratio in a Medium-Scale New Keynesian Model," *Journal of Money, Credit and Banking*, Blackwell Publishing, vol. 44, pages 457-467, 03.
- [4] **Ascari, Guido & Rossi, Lorenza, 2012.** "Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo," *The Economic Journal*, Royal Economic Society, vol. 122(563), pages 1115-1141, 09.
- [5] **Ball, Laurence, 1994.** "What determines the sacrifice ratio?" *NBER Chapters, in: Monetary Policy, pages 155-193 National Bureau of Economic Research, Inc.* In N. Gregory Mankiw, ed; *Monetary Policy*, pp. 155-182, The University of Chicago Press (1994).
- [6] **Benati, Luca, 2008.** "Investigating Inflation Persistence Across Monetary Regimes," *The Quarterly Journal of Economics*, MIT Press, vol. 123(3), pages 1005-1060, August.
- [7] **Benati, Luca, 2009.** "Are 'intrinsic inflation persistence' models structural in the sense of Lucas (1976)?," *Working Paper Series 1038*, European Central Bank.
- [8] **Bilbiie, Florin O., 2008.** "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of Economic Theory*, Elsevier, vol. 140(1), pages 162-196, May.
- [9] **Blanchard, Olivier, Dell'Ariccia Giovanni, and Mauro Paolo, 2010.** "Rethinking macroeconomic policy". SPN/10/03, *IMF Staff Position Note*

- [10] **Cecchetti, Stephen G & Rich, Robert W, 2001.** "Structural Estimates of the U.S. Sacrifice Ratio," *Journal of Business & Economic Statistics*, *American Statistical Association*, vol. 19(4), pages 416-27, October.
- [11] **Christiano, Lawrence J. & Eichenbaum, Martin & Evans, Charles L., 2005.** "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, University of Chicago Press, vol. 113(1), pages 1-45, February.
- [12] **Cogley, Timothy and Sbordone Argia M., (2008).** "Trend inflation, indexation and inflation persistence in the New Keynesian Phillips Curve". *American Economic Review* 98 (5), 2101—2126.
- [13] **Corbo, Vittorio, Oscar Landerretche, and Klaus Schmidt-Hebbel, 2001.** "Assessing Inflation Targeting after a Decade of World Experience." *International Journal of Finance and Economics*, 6, 343–68
- [14] **Durand Jean-Jacques & Huchet-Bourdon Marilyne & Licheron Julien, 2008.** "Sacrifice ratio dispersion within the Euro Zone: what can be learned about implementing a single monetary policy?," *International Review of Applied Economics*, *Taylor & Francis Journals*, vol. 22(5), pages 601-621.
- [15] **Galí, Jordi, J. David López-Salido and Javier Vallés, 2004.** "Rule-of-Thumb Consumers and the Design of Interest Rate Rules," NBER Working Papers 10392, National Bureau of Economic Research, Inc.
- [16] **Galí, Jordi, J. David López-Salido and Javier Vallés, 2007.** "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, MIT Press, vol. 5(1), pages 227-270, 03.
- [17] **Goncalves, Carlos Eduardo. (2009).** "Inflation Targeting Matters: Evidence from OECD Economies Sacrifice Ratios." *Journal of Money, Credit and Banking*, 40, 233–43.
- [18] **Gordon, Robert J. and King Stephen R., 1982.** "The output cost of disinflation in traditional and vector autoregressive models." *Brooking Papers on Economic Activity* 1, 205-242.
- [19] **Hofmann, Boris & Peersman, Gert & Straub, Roland, 2012.** "Time variation in U.S. wage dynamics," *Journal of Monetary Economics*, Elsevier, vol. 59(8), pages 769-783.
- [20] **Keen, Benjamin & Wang, Yongsheng, 2007.** "What is a realistic value for price adjustment costs in New Keynesian models?," *Applied Economics Letters*, Taylor and Francis Journals, vol. 14(11), pages 789-793.
- [21] **Merkel, Christian, 2013.** "Disinflationary booms?," *Economics Letters*, Elsevier, vol. 121(1), pages 105-109.

- [22] **Rotemberg, Julio J, 1982.** "Sticky Prices in the United States," *Journal of Political Economy*, University of Chicago Press, vol. 90(6), pages 1187-1211, December.
- [23] **Smets Frank and Wouters Raf, 2005.** "Comparing shocks and frictions in US and Euro area business cycles: a bayesian DSGE approach". *Journal of Applied Econometrics*, vol. 20, pages 161 - 183.

6 Appendix A: The Model

In this Appendix we lay out the full model structure.

6.1 Households

There is a continuum of households indexed by i , $i \in [0, 1]$. RT (rt) and Ricardian (o) agents are respectively defined over the intervals $[0, \Omega]$ and $[\Omega, 1]$. All households share the same utility function. Each household has preferences defined over consumption c and labour effort h . Hence, the period household's utility function is:

$$U_t^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln (c_t^i - b c_{t-1}^i) - \frac{\phi_1}{(1 + \phi)} (h_t^i)^{(1+\phi)} \right\} \quad (\text{A1})$$

where c_t^i denotes total individual consumption, b represents the degree of external habit formation in consumption, h_t^i denotes individual labor supply of a differentiated labor bundle. As for preference parameters, ϕ is the inverse of the intertemporal elasticity of substitution of labour and ϕ_1 accounts for the relative importance of disutility of work and utility of consumption in the total utility.

6.2 Consumption bundles

The consumption good is assumed to be a composite good produced with a continuum of differentiated goods c_t^i via the Dixit-Stiglitz consumption basket of household i :

$$c_t^i = \left[\int_0^1 c(z)_t^{\frac{\eta-1}{\eta}} dz \right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 1$ denotes the elasticity of substitution across different varieties of goods.

In particular, the household decides how to allocate its consumption expenditures among different goods. This requires that the consumption index c_t^i is maximized for any given level of expenditures $X_t = \int_0^1 P(z)_t c(z)_t dz$. Solving the intratemporal goods allocation problem, the set of demand equation is:

$$c(z)_t = \left(\frac{P(z)_t}{P_t} \right)^{-\eta} c_t$$

where

$$P_t = \left(\int_0^1 p(z)_t^{(1-\eta)} dz \right)^{\frac{1}{1-\eta}}$$

is the aggregate price consumption index.

6.3 Labour market structure

It is assumed a continuum of differentiated labour inputs indexed by j , $j \in [0, 1]$. Following Schmitt-Grohé and Uribe (2005), household i supplies all labour inputs. Moreover, labor type-specific unions indexed by $j \in [0, 1]$ have some monopoly power in the labour market and make wage-setting decisions. Given the wage W_t^j fixed by union j , households are assumed to supply enough labour h_t^j to satisfy demand. That is,

$$h_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} h_t^d$$

where $\eta_w > 1$ is the elasticity of substitution across different labour inputs, h_t^d is the aggregate labour demand and $W_t = \left(\int_0^1 (W_t^j)^{(1-\eta_w)} dj \right)^{\frac{1}{(1-\eta_w)}}$ is the aggregate wage index. As in Galí (2007), it's assumed that the fraction of Ricardian and non-Ricardian households is uniformly distributed across unions and the aggregate demand for each labor type is uniformly distributed across households. Therefore optimizers and rule of thumbers work for the same amount of work. Therefore the labour supply, which is common across households, must satisfy the resource constraint $h_t^s = \int_0^1 h_t^j dj$. Combining the latter with equation (5) we get:

$$h_t^s = h_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} dj$$

Therefore, the common labour income is denoted by $h_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\eta_w} dj$.

6.4 Ricardian Households

Ricardian agents are assumed to have access to market for physical capital and to contingent nominal assets. In particular, each period asset holders can purchase any state-contingent nominal payment X_{t+1} in period $t+1$ at the cost $E_t r_{t,t+1} X_{t+1}$ where $r_{t,t+1}$ is a stochastic discount factor between periods t and $t+1$.

Therefore, the ricardian household's period budget constraint in real terms reads as:

$$\begin{aligned} E_t r_{t,t+1} x_{t+1} + c_t^o + i_t^o &= \frac{x_t}{\pi_t} + [r_t^k u_t - a(u_t)] K_t^o + \\ &+ q_t \delta K_t^o + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\eta_w} dj + d_t^o + d_t^{WH} \end{aligned}$$

where $\frac{x_t}{\pi_t} \equiv \frac{X_t}{P_t}$ is the real payoff in period t of the nominal state contingent assets purchased at $t-1$. i_t^o denotes the real purchases investment goods at time t .

It is assumed that ricardian households own physical capital K_t^o , accumulate it and then rent it out the firms at a real interest rate r_t^k . Moreover, the optimizers can control the intensity u_t at which the capital is utilized. Hence, the cost of capital depends upon the degree of utilization $a(u_t)$ and it is defined as $a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$. The function satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. Ricardian households also receive firms dividends, d_t^o , and returns from financing working capital of firms⁹, d_t^{WH} . The gross rate of inflation is $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

The capital stock evolves according to the following law of motion:

$$K_{t+1}^o = (1 - \delta)K_t^o + i_t^o \left[1 - S \left(\frac{i_t^o}{i_{t-1}^o} \right) \right]$$

where δ is the depreciation rate of capital. The function S introduces the adjustment costs on investment and satisfies the following properties: $S(1) = S'(1) = 0$, $S''(1) > 0$.

Hence, the Lagrangean to the maximization problem, with Lagrange multipliers $\beta^t \lambda_t (1 - \tau_{t+s}^h) w_t / \mu_t$, $\beta^t \lambda_t$ and $\beta^t q_t \lambda_t$ respectively associated to the constraints (6), (7) and (8), reads as:

$$L = E_t \sum_{s=0}^{\infty} \left\{ \begin{aligned} & U(c_{t+s}^o(i) - bc_{t+s-1}^o(i); h_{t+s}(i)) + \\ & \frac{x_{t+s}}{\pi_{t+s}} + [r_{t+s}^k u_{t+s} - a(u_{t+s})] K_{t+s}^o + \\ & + \lambda_{t+s}^o \left[\begin{aligned} & + q_{t+s} \delta K_{t+s}^o + \\ & + h_{t+s}^d \int_0^1 w_{t+s}^j \left(\frac{w_{t+s}^j}{w_{t+s}} \right)^{-\eta_w} dj + \\ & + d_{t+s}^o - R_{t+s, t+s+1} x_{t+s+1} - c_{t+s}^o - i_{t+s}^o \end{aligned} \right] + \\ & + \lambda_{t+s}^o q_{t+s} \left[(1 - \delta) K_{t+s}^o + i_{t+s}^o \left[1 - S \left(\frac{i_{t+s}^o}{i_{t+s-1}^o} \right) \right] - K_{t+s+1}^o \right] + \\ & + \frac{\lambda_{t+s}^o (1 - \tau_{t+s}^h) w_{t+s}}{\mu_{t+s}} \left[h_{t+s}^s - h_{t+s}^d \int_0^1 \left(\frac{w_{t+s}^j}{w_{t+s}} \right)^{-\eta_w} \right] \end{aligned} \right\}$$

The Ricardian household's first order conditions with respect to c_t^o , x_{t+1} , K_t^o , i_t^o , and u_t are respectively:

$$\frac{1}{c_t^o - bc_{t-1}^o} - \frac{b\beta}{c_{t+1}^o - bc_t^o} = \lambda_t^o \quad (\text{A2})$$

$$\lambda_t^o = \beta R_{t, t+1} \frac{\lambda_{t+1}^o}{\pi_{t+1}} \quad (\text{A3})$$

$$q_t = \beta \frac{\lambda_{t+1}^o}{\lambda_t^o} [q_{t+1} (1 - \delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})] \quad (\text{A4})$$

⁹ Here we implicitly follow the financial sector characterization adopted in CEE 2005, who assume that a financial intermediary collects money balances from ricardian households and from the central bank. Such funds are then used to finance the working capital needs of firms, and what is left returns to ricardian households. Given that the Central Bank follows an interest rate policy, explicit modelling of the money market is unnecessary.

$$\begin{aligned}\lambda_t^o &= q_t \lambda_t^o \left[1 - S \left(\frac{i_t^o}{i_{t-1}^o} \right) - \left[S_i \left(\frac{i_t^o}{i_{t-1}^o} \right) \right] i_t^o \right] + \\ &\quad - \beta q_{t+1} \lambda_{t+1}^o S_i \left(\frac{i_{t+1}^o}{i_t^o} \right) i_{t+1}^o\end{aligned}\tag{A5}$$

$$a_u(u_t) = r_t^k\tag{A6}$$

The adjustment cost function and the capital utilization function are given by:

$$\begin{aligned}S \left(\frac{i_t}{i_{t-1}} \right) &= \frac{k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \\ a(u_t) &= \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2\end{aligned}$$

6.5 Rule of thumb households

As pointed out above, Non-Ricardian agents just consume current labor income because they cannot save neither invest. Since they don't have access to capital markets, they only pay taxes on labor income and receive transfers from the government. Therefore:

$$c_t^{rt} = w_t h_t^d\tag{A7}$$

The marginal utility of consumption for rule of thumbers is:

$$\frac{1}{c_t^{rt} - b c_{t-1}^{rt}} = \lambda_t^{rt}\tag{A8}$$

6.6 Wage Setting

In choosing the optimal wage, the unions have to take into account to face a quadratic adjustment cost of the form:

$$\frac{\xi_w}{2} \left(\frac{W_t^j}{(\pi_{t-1}^{\chi_w}) W_{t-1}^j} - 1 \right)^2 h_t$$

where ξ_w denotes the degree of nominal wage rigidity.

The union objective function in real terms reads as:

$$\mathcal{L}^w = E_t \sum_{t=0}^{\infty} \left\{ \begin{aligned} & - \frac{\phi_1}{(1+\phi)} \left[h_t \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{(-\eta_w)} \right]^{(1+\phi)} + \\ & - \lambda_t \left[- \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{(1-\eta_w)} w_t h_t + \frac{\xi_w}{2} \left(\frac{w_t^j}{w_{t-1}^j} \frac{\pi_t}{\pi_{t-1}^{\chi_w}} - 1 \right)^2 h_t \right] \end{aligned} \right\}$$

Since unions choose the same wage, they face the same problem. Therefore a symmetric equilibrium takes place.

From the first order condition the wage setting equation comes out:

$$mrs_t = \left\{ \begin{array}{l} \frac{\eta_w - 1}{\eta_w} w_t + \frac{\xi_w}{\eta_w} \left(\frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\chi_w}} - 1 \right) \frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\chi_w}} + \\ -\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\xi_w}{\eta_w} \left(\frac{w_{t+1}}{w_t} \frac{\pi_{t+1}}{\pi_t^{\chi_w}} - 1 \right) \frac{h_{t+1}}{h_t} \frac{w_{t+1}}{w_t} \frac{\pi_{t+1}}{\pi_t^{\chi_w}} \end{array} \right\} \quad (\text{A9})$$

where, importantly:

$$\lambda_t = [(1 - \Omega) \lambda_t^o + \Omega \lambda_t^{rt}] \quad (\text{A10})$$

6.7 Firms

Intermediate firms compete monopolistically by producing good z according to the following technology:

$$y_t(z) = (K_t(z))^\vartheta (h_t(z))^{(1-\vartheta)}$$

where $K_t(z)$ is the physical capital stock that firms rent by ricardian households and $h_t(z)$ is the labor input used by each firm z . In particular it is defined as:

$$h_t(z) = \left(\int_0^1 (h_t^j(z))^{\frac{\eta_w - 1}{\eta_w}} dj \right)^{\frac{\eta_w}{\eta_w - 1}}$$

6.8 Price Setting

The Rotemberg price setting assumes that each intermediate firm pays an increasing and convex cost measured in terms of aggregate output. This cost is given by:

$$\frac{\xi_p}{2} \left(\frac{P_t(z)}{\pi_{t-1}^\chi P_{t-1}(z)} - 1 \right)^2 y_t$$

where $\phi_p > 0$ measures the degree of nominal price rigidity.

Therefore each firm maximizes its present discounted value of profits for its owners (Ricardian households, i.e. o):

$$\begin{aligned} \max_{P_t(z)} E_t \left[\sum_{s=0}^{\infty} \beta^s \lambda_{t+s}^o \frac{D_{t+s}(z)}{P_{t+s}} \right] \\ \text{s.t. } y_{t+s}(z) = \left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{-\eta} y_{t+s} \end{aligned}$$

where

$$\frac{D_{t+s}(z)}{P_{t+s}} = \frac{P_{t+s}(z)}{P_{t+s}} y_{t+s}(z) - mc_{t+s} y_{t+s}(z) - \frac{\xi_p}{2} \left(\frac{P_{t+s}(z)}{\pi_{t+s-1}^\chi P_{t+s-1}(z)} - 1 \right)^2 y_{t+s}$$

Substituting the constraint to the maximization problem into the objective function:

$$E_t \sum_{s=0}^{\infty} \beta^s \lambda_{t+s}^o \left[\left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{1-\eta} y_{t+s} + \right. \\ \left. - mc_{t+s} \left(\frac{P_{t+s}(z)}{P_{t+s}} \right)^{-\eta} y_{t+s} - \frac{\xi_p}{2} \left(\frac{P_{t+s}(z)}{\pi_{t+s-1}^\chi P_{t+s-1}(z)} - 1 \right)^2 y_{t+s} \right]$$

The FOC to the problem is:

$$0 = (1-\eta) \lambda_t^o \left(\frac{P_t(z)}{P_t} \right)^{-\eta} \frac{y_t}{P_t} + \eta \lambda_t^o mc_t \left(\frac{P_t(z)}{P_t} \right)^{-\eta-1} \frac{y_t}{P_t} + \\ - \xi_p \lambda_t^o \left(\frac{P_t(z)}{\pi_{t-1}^\chi P_{t-1}(z)} - 1 \right) \frac{y_t}{\pi_{t-1}^\chi P_{t-1}(z)} + \\ + \beta E_t \left[\xi_p \lambda_{t+1}^o \left(\frac{P_{t+1}(z)}{\pi_t^\chi P_t(z)} - 1 \right) \frac{P_{t+1}(z) y_{t+1}}{\pi_t^\chi P_t(z)^2} \right]$$

Given the symmetric equilibrium:

$$0 = (1-\eta) \lambda_t^o \frac{y_t}{P_t} + \eta \lambda_t^o mc_t \frac{y_t}{P_t} - \xi_p \lambda_t^o \left(\frac{P_t}{\pi_{t-1}^\chi P_{t-1}} - 1 \right) \frac{y_t}{\pi_{t-1}^\chi P_{t-1}} + \\ + \beta E_t \left[\xi_p \lambda_{t+1}^o \left(\frac{P_{t+1}}{\pi_t^\chi P_t} - 1 \right) \frac{P_{t+1} y_{t+1}}{\pi_t^\chi P_t^2} \right]$$

Multiplying by P_t :

$$0 = (1-\eta) \lambda_t^o y_t + \eta \lambda_t^o mc_t y_t - \xi_p \lambda_t^o \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) y_t \frac{\pi_t}{\pi_{t-1}^\chi} + \\ + \beta E_t \xi_p \lambda_{t+1}^o \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} y_{t+1}$$

Dividing by y_t :

$$0 = (1-\eta) \lambda_t^o + \eta \lambda_t^o mc_t - \xi_p \lambda_t^o \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) \frac{\pi_t}{\pi_{t-1}^\chi} + \beta E_t \xi_p \lambda_{t+1}^o \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} \frac{y_{t+1}}{y_t}$$

Solving for marginal cost:

$$mc_t = \left(\frac{\eta-1}{\eta} \right) + \frac{\xi_p}{\eta} \left(\frac{\pi_t}{\pi_{t-1}^\chi} - 1 \right) \frac{\pi_t}{\pi_{t-1}^\chi} - \beta E_t \frac{\xi_p}{\eta} \lambda_{t+1}^o \left(\frac{\pi_{t+1}}{\pi_t^\chi} - 1 \right) \frac{\pi_{t+1}}{\pi_t^\chi} \frac{y_{t+1}}{y_t} \quad (\text{A11})$$

6.9 Aggregation

The aggregate production function is:

$$y_t = (u_t K_t)^\vartheta h_t^{d(1-\vartheta)} \quad (\text{A12})$$

and the aggregate absorption is:

$$y_t^d = c_t + i_t + g_t + a(u_t) K_t \quad (\text{A13})$$

where:

$$c_t = (1 - \Omega) c_t^o + \Omega c_t^{rt} \quad (\text{A14})$$

$$i_t = (1 - \Omega) i_t^o \quad (\text{A15})$$

$$K_t = (1 - \Omega) K_t^o \quad (\text{A16})$$

6.10 Market clearing

The aggregate resource constraint accounts for price and nominal wage adjustment costs, therefore:

$$y_t = c_t + i_t + g_t + a(u_t) K_t + \frac{\xi_p}{2} \left(\frac{\pi_t}{\pi_{t-1}^x} - 1 \right)^2 y_t + \frac{\xi_w}{2} \left(\frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{x_w}} - 1 \right)^2 h_t \quad (\text{A17})$$

6.11 Fiscal Authority

Public spending is financed through seigniorage:

$$g_t = m_t - \frac{m_{t-1}}{\pi_t}$$

where m_t denotes real money balances and π_t is the actual gross inflation rate. Government minimizes the costs of purchasing the composite good. Therefore, government's absorption of a single type of good is $g_{zt} = \left(\frac{P_{zt}}{P_t} \right)^{-\eta} g_t$.