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PIIGS in the Euro Area. An Empirical DSGE Model.*

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Abstract

We build up and estimate a two-region DSGE model of the Euro area, investigating the interactions between the peripheral countries (PIIGS) and the rest of EMU. Our main focus is on the 2008-2009 financial crisis and on the subsequent 2010-sovereign bond crisis. One striking result is that the two crises are characterized by demand shocks in the core Euro area countries, whereas region-specific permanent technology shocks explain most of output growth slowdown in the PIIGS countries. Our results suggest that the capital flows reversals caused important supply-side effects in the Eurozone periphery.

Keywords: *PIIGS, Euro crisis, two-country DSGE, Bayesian estimation*

JEL codes: C11, C13, C32, E21, E32, E37

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1 Introduction

Following the 2008-2009 financial crisis, the sovereign bond crisis severely hit the “peripheral” members of the Euro area countries (Portugal, Ireland, Italy, Greece and Spain, PIIGS henceforth). The most striking feature of the crisis was the apparent disintegration of EMU financial markets (Christiansen, 2014; Sensoy et al., 2015) and the large interest rate spreads in the PIIGS countries, that were also characterized by a sudden contraction in growth and by dramatic increases in unemployment rates in these countries. All this raised concerns of a possible breakup of the Euro area.

The sovereign bond crisis was triggered by disclosure of hitherto hidden public deficits in Greece and by fears of contagion, but it has also been described as a sudden stop in financial flows that caused, among other things, a collapse in the risk-sharing mechanism between PIIGS and the rest of the Eurozone (Kalemli-Ozcan et al., 2014). As pointed out by De Grauwe and Ji (2013), this might have been caused by self-fulfilling expectation of EMU breakup. A complementary view emphasizes the role of underlying balance of payments problems (Giavazzi and Spaventa (2011); Lane, 2012). As a matter of fact, in peripheral countries the early EMU years had been characterized by large capital inflows, decline in domestic real interest rate, real exchange rate appreciation. This could be seen as part of the catching up process triggered by monetary integration (Blanchard and Giavazzi, 2002), but after 2010, in the context of globally deteriorated business cycle conditions, the accumulated real exchange rate appreciations signaled fragility and impossibility to recover the pre-2007 growth rates (Lane, 2012).

The crisis triggered a controversy about which policies should be implemented to restore growth. European institutions and policymakers in the core EMU countries called for a combination of fiscal retrenchment (austerity) and efficiency-enhancing reforms in the PIIGS region. Critics argued that reforms may have contractionary short-run effects, especially if the nominal interest rate is constrained at the zero lower bound, and that austerity would dampen an already depressed domestic demand in the periphery (See Eggertsson et al. 2014, and references cited therein).

We build up and estimate a medium-scale two-region Euro area model, i.e. PIIGS countries and the rest of Euro area countries (CORE countries). This allows us both to estimate the structural interdependence between the regions, and to identify the shocks that caused PIIGS contractions in economic activity during the 2008-2009 and post-2010 crises. One may expect this model to answer questions which are crucial to understand the nature of the crisis and to identify policies that promote regional convergence. Did the disruption of EMU financial markets actually contribute to the slowdown in PIIGS economic activity? Was the crisis driven by demand shocks that brought down consumption and investments in peripheral region, or was it determined by an underlying deterioration in productivity growth? Finally, which role was played by fiscal policies before and during the crisis? To this end we consider a number of region-specific supply and demand shocks. Supply shocks include standard temporary technology shocks and shocks to the productivity growth trend, entailing permanent variations in relative productivity levels between the PIIGS and the CORE countries. Non-policy demand shocks include shocks to the households subjective discount factor, "risk premium" shocks driving a wedge between the return on capital accruing to the households and the price of capital services paid by firms, and a standard investment-specific shock. In addition to that, we account for alternative characterizations of the possibly (dis)integrated EMU financial market, incorporating shocks that impair risk-sharing between households of the two regions. Our model does not explicitly model financial frictions, but the richness of the shocks structure should allow to capture the transmission channels of the two financial crises.

According to our estimates, temporary disruptions to the risk-sharing mechanism that operates through the EMU financial market have potentially large contractionary effects in the periphery, entailing a persistent fall in investment, output and inflation. However, our variance and historical decompositions of output growth suggest that this shock had minimal effects. In addition, on the grounds of the obtained marginal data densities, the model specification accounting for permanent productivity shocks is strongly preferred to the alternative specifications, including the one based on financial market shocks.

The historical decomposition of growth suggests that the output response to the 2008-2009 financial crisis, quantitatively similar in the two regions, was in fact caused by different types of shocks. In the core region demand shocks were more important than in the PIIGS, where the main determinants were permanent and temporary technology shocks. The post-2010 period, when the PIIGS experienced slower recovery and then deeper recession, is mainly explained by adverse permanent technology shocks in the periphery and by demand shocks in the CORE region. Following Khan and Thomas (2013) and Gopinath et al. (2015), we discuss the possible connection between the capital flows reversals that hit the PIIGS countries and the estimated adverse productivity shocks.

Finally, public consumption shocks played a negligible role throughout the EMU years, implying that the post-2008 deterioration in fiscal indicators such as the public-consumption-to-GDP ratio was driven by non-policy shocks. A crucial difference exists between the two regions. In the CORE countries the deterioration of fiscal indicators was mainly caused by temporary shocks, whereas in the PIIGS region permanent shocks were a key driver, so that one cannot expect their reversal when the economy recovers.

The paper is organized as follows. Section 2 describes the model, section 3 introduces the estimation strategy and section 4 presents the results. Section 5 concludes.

2 The monetary union model¹

We assume there are two regions in the European monetary union, respectively corresponding to PIIGS countries (size s) and to the rest of the Euro area, or core region (size $1 - s$). Each region produces both non-tradable and tradable differentiated goods. Tradable good produced in the PIIGS and core regions are respectively indexed by H and F . As in Rabanal (2009), there is no price discrimination across regions, i.e. the law of one price holds. In what follows our focus is on the PIIGS region, as the core region is characterized symmetrically. When needed, variables and parameters referring to the core region are marked by an asterisk.

In each country there is a continuum of households indexed by i . A share $1 - \theta$ of households (Ricardian households, $i = o$) can access financial markets, trade government bonds, accumulate physical capital and rent capital services to firms. The remaining θ households (Non-Ricardian or LAMP households, $i = rt$) do not have access to financial markets and consume all their disposable labor income.

Each household supplies a continuum of size s of differentiated labor inputs that firms demand

$$h_t^i = \left\{ \left(\frac{1}{s} \right)^{\frac{\lambda_t^w}{1+\lambda_t^w}} \int_0^s [h_t^i(j)]^{\frac{1}{1+\lambda_t^w}} dj \right\}^{1+\lambda_t^w} \quad (1)$$

Demand for labor type j is

¹The Appendix contains more details about the equations and the specifications of the model.

$$h_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} h_t^d \quad (2)$$

where W_t^j is type j nominal wage and $W_t = \left[\frac{1}{s} \int_0^s (W_t^j)^{\frac{1}{\lambda_t^w}} dj \right]^{\lambda_t^w}$ is the aggregate nominal wage index.

2.1 Preferences

Households preferences are characterized by non separability between consumption and labor effort (Smets and Wouters, 2005, 2007):

$$U_t^i(c_t^i, h_t^i) = \frac{1}{1-\sigma} \left(\frac{c_t^i}{c_{t-1}^\zeta} \right)^{1-\sigma} \exp \left(\frac{\sigma-1}{1+\phi_l} (h_t^i)^{1+\phi_l} \right) \quad (3)$$

where $c_t^i = \frac{C_t^i}{z_t}$ and $c_t = \frac{C_t}{z_t}$ are individual and total real consumption levels normalized by a labour-augmenting non-stationary technology shifter z_t . The presence of z_t in 3 guarantees that the model has a balanced growth path when productivity is non stationary.

Parameter $0 < \zeta < 1$ measures the degree of external habit in consumption. As in Albonico et al. (2014, 2016), our specification is based on habits in ratios.²

2.2 Production

In each region, perfectly competitive firms produce the consumption good C_t and a final investment good, Q_t^I , using tradable $(C_t^T, Q_t^{I,T})$ and nontradable $(C_t^N, Q_t^{I,N})$ intermediate goods. Tradables incorporate domestic $(C_t^H, Q_t^{I,H})$ and imported $(C_t^F, Q_t^{I,F})$ tradable intermediate goods as inputs. These goods are produced by monopolistically competitive firms. Brands of tradable goods are indexed by $h \in [0, s]$ in the domestic region and by $f \in [s, 1]$ in the foreign region. Indexes $n \in [0, s]$ and $n^* \in [s, 1]$ identify the corresponding notation for nontradable goods .

²The habits-in-ratio specification limits the possibility that a non-negligible share of non-Ricardian households causes indeterminacy. In empirical DSGE model the existence of a relatively large indeterminacy region may bias posterior estimates. See Albonico et al. (2014 and 2015), for an extended discussion.

$$C_t^H = \left[\left(\frac{1}{s} \right)^{\frac{\lambda_t^{p,H}}{1+\lambda_t^{p,H}}} \int_0^s C_t^H(h)^{\frac{1}{1+\lambda_t^{p,H}}} dh \right]^{1+\lambda_t^{p,H}} \quad (4)$$

$$C_t^F = \left[\left(\frac{1}{1-s} \right)^{\frac{\lambda_t^{p,F}}{1+\lambda_t^{p,F}}} \int_s^1 C_t^F(f)^{\frac{1}{1+\lambda_t^{p,F}}} df \right]^{1+\lambda_t^{p,F}} \quad (5)$$

$$Q_t^{I,H} = \left[\left(\frac{1}{s} \right)^{\frac{\lambda_t^{p,H}}{1+\lambda_t^{p,H}}} \int_0^s Q_t^{I,H}(h)^{\frac{1}{1+\lambda_t^{p,H}}} dh \right]^{1+\lambda_t^{p,H}} \quad (6)$$

$$Q_t^{I,F} = \left[\left(\frac{1}{1-s} \right)^{\frac{\lambda_t^{p,F}}{1+\lambda_t^{p,F}}} \int_s^1 Q_t^{I,F}(f)^{\frac{1}{1+\lambda_t^{p,F}}} df \right]^{1+\lambda_t^{p,F}} \quad (7)$$

$$C_t^N = \left[\left(\frac{1}{s} \right)^{\frac{\lambda_t^{p,N}}{1+\lambda_t^{p,N}}} \int_0^s C_t^N(n)^{\frac{1}{1+\lambda_t^{p,N}}} dn \right]^{1+\lambda_t^{p,N}} \quad (8)$$

$$Q_t^{I,N} = \left[\left(\frac{1}{s} \right)^{\frac{\lambda_t^{p,N}}{1+\lambda_t^{p,N}}} \int_0^s Q_t^{I,N}(n)^{\frac{1}{1+\lambda_t^{p,N}}} dn \right]^{1+\lambda_t^{p,N}} \quad (9)$$

The composite price indexes are defined as:

$$P_t^N = \left[\frac{1}{s} \int_0^s P_t^N(n)^{\frac{1}{\lambda_t^{p,N}}} dn \right]^{\lambda_t^{p,N}} \quad (10)$$

$$P_t^H = \left[\frac{1}{s} \int_0^s P_t^H(h)^{\frac{1}{\lambda_t^{p,H}}} dh \right]^{\lambda_t^{p,H}} \quad (11)$$

$$P_t^F = \left[\frac{1}{1-s} \int_s^1 P_t^F(f)^{\frac{1}{\lambda_t^{p,F}}} df \right]^{\lambda_t^{p,F}} \quad (12)$$

where $\lambda_t^{p,H}$, $\lambda_t^{p,F}$, $\lambda_t^{p,N}$ denote time-varying net price markups.

The consumption index C_t is defined as a CES aggregate of tradable C_t^T and nontradable goods C_t^N :

$$C_t = \left[\gamma_c^{\frac{1}{e}} (C_t^T)^{\frac{e-1}{e}} + (1-\gamma_c)^{\frac{1}{e}} (C_t^N)^{\frac{e-1}{e}} \right]^{\frac{e}{e-1}} \quad (13)$$

where γ_c represents the share of tradable goods in the consumption basket at home, $e > 1$ is the elasticity of substitution between tradable and nontradable goods, and C_t^T is defined as

$$C_t^T = \left[\varpi^{\frac{1}{v}} (C_t^H)^{\frac{v-1}{v}} + (1-\varpi)^{\frac{1}{v}} (C_t^F)^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}}; v > 1 \quad (14)$$

The home country consumer price indexes $P_{C,t}$ and P_t^T are:

$$P_{C,t} = \left[\gamma_c (P_t^T)^{1-e} + (1 - \gamma_c) (P_t^N)^{1-e} \right]^{\frac{1}{1-e}} \quad (15)$$

$$P_t^T = \left[\varpi (P_t^H)^{1-v} + (1 - \varpi) (P_t^F)^{1-v} \right]^{\frac{1}{1-v}} \quad (16)$$

CES aggregators also define final investment goods and the price index of tradables:³

$$Q_t^I = \left[\gamma_i^{\frac{1}{e}} \left(Q_t^{I,T} \right)^{\frac{e-1}{e}} + (1 - \gamma_i)^{\frac{1}{e}} \left(Q_t^{I,N} \right)^{\frac{e-1}{e}} \right]^{\frac{e}{e-1}} \quad (17)$$

$$Q_t^{I,T} = \left[\varpi^{\frac{1}{v}} \left(Q_t^{I,H} \right)^{\frac{v-1}{v}} + (1 - \varpi)^{\frac{1}{v}} \left(Q_t^{I,F} \right)^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}} \quad (18)$$

$$P_{I,t} = \left[\gamma_i (P_t^T)^{1-e} + (1 - \gamma_i) (P_t^N)^{1-e} \right]^{\frac{1}{1-e}} \quad (19)$$

Profits maximization leads to the following demand functions:

$$C_t^N = (1 - \gamma_c) \left(\frac{P_t^N}{P_{C,t}} \right)^{-e} C_t \quad (20)$$

$$C_t^H = \varpi \gamma_c \left(\frac{P_t^H}{P_t^T} \right)^{-v} \left(\frac{P_t^T}{P_{C,t}} \right)^{-e} C_t \quad (21)$$

$$C_t^F = (1 - \varpi) \gamma_c \left(\frac{P_t^F}{P_t^T} \right)^{-v} \left(\frac{P_t^T}{P_{C,t}} \right)^{-e} C_t \quad (22)$$

$$Q_t^{I,N} = (1 - \gamma_i) \left(\frac{P_t^N}{P_{I,t}} \right)^{-e} Q_t^I \quad (23)$$

$$Q_t^{I,H} = \varpi \gamma_i \left(\frac{P_t^H}{P_t^T} \right)^{-v} \left(\frac{P_t^T}{P_{I,t}} \right)^{-e} Q_t^I \quad (24)$$

$$Q_t^{I,F} = (1 - \varpi) \gamma_i \left(\frac{P_t^F}{P_t^T} \right)^{-v} \left(\frac{P_t^T}{P_{I,t}} \right)^{-e} Q_t^I \quad (25)$$

Total demand for domestically produced intermediate goods is

$$Y_t^H = C_t^H + Q_t^{I,H} + \frac{1-s}{s} \left(C_t^{H*} + Q_t^{I,H*} \right) \quad (26)$$

$$Y_t^N = C_t^N + Q_t^{I,N} + G_t \quad (27)$$

where $\left(C_t^{H*} + Q_t^{I,H*} \right)$ defines foreign demand for home tradables and G_t is public consumption demand which is assumed to fall entirely on nontradables.⁴

³The home country investment price index $P_{I,t}$ differs from its consumption counterpart $P_{C,t}$ because $\gamma_i \neq \gamma_c$.

⁴We also make the standard assumption that G_t and C_t^N are identically distributed on individual non tradable goods.

2.2.1 Intermediate goods

The representative firm uses the following production technology:

$$Y_t^{int} = \varepsilon_t^{a,int} [u_t^{int} K_t^{int}]^{\alpha_{int}} [z_t h_t^{int}]^{1-\alpha_{int}} - z_t \Phi_{int} \quad (28)$$

where $int = h, f, n, n^*$, Φ_{int} defines fixed costs of production, u_t^{int} is the degree of capacity utilization, K_t^{int} is the capital stock, $\varepsilon_t^{a,int}$ and $z_t = z_{t-1} g_{z,t}$ incorporate temporary and permanent technology shocks *respectively*, modelled as AR(1) processes with i.i.d. Normal innovation terms, η_t^{int} , $\eta_t^{g_z}$:

$$\log(\varepsilon_t^{a,int}) = (1 - \rho_{int}) \log(\varepsilon_t^{a,int}) + \rho_{int} \log(\varepsilon_{t-1}^{a,int}) + \eta_t^{int} \quad (29)$$

$$\log(g_{z,t}) = (1 - \rho_{g_z}) \log(g_z) + \rho_{g_z} \log(g_{z,t-1}) + \eta_t^{g_z} \quad (30)$$

Firms profits are defined as

$$P_t^{int} Y_t^{int} - R_t^k u_t^{int} K_t^{int} - (1 + \tau^{wf}) W_t h_t^{int} \quad (31)$$

where τ^{wf} is a payroll tax. The nominal marginal cost is:

$$MC_t^{int} = \alpha_{int}^{-\alpha_{int}} (1 - \alpha_{int})^{-(1-\alpha_{int})} (\varepsilon_t^{a,int})^{-1} z_t^{-(1-\alpha_{int})} (R_t^k)^{\alpha_{int}} [(1 + \tau^{wf}) W_t]^{1-\alpha_{int}} \quad (32)$$

Price setting Prices are sticky à la Calvo (1983). Firms optimally reset their price with probability $(1 - \xi_p^{int})$. Non-optimizing firms adopt the standard indexation scheme:

$$P_t^{int} = \pi_{int,t-1}^{\chi_p^{int}} \bar{\pi}_t^{1-\chi_p^{int}} P_{t-1}^{int} \quad (33)$$

where $\bar{\pi}_t$ is the monetary union trend inflation rate and $\pi_{int,t} = \frac{P_t^{int}}{P_{t-1}^{int}}$ is the sectorial inflation rate.

The first order condition for the optimizing firm is:

$$E_t \sum_{k=0}^{\infty} (\xi_p^{int})^s \Xi_{t,t+k} Y_{t+k}^{int} \left[\frac{\tilde{P}_t^{int} \pi_{N,t,t+k-1}^{\chi_p^{int}} \bar{\pi}_{t,t+k}^{1-\chi_p^{int}}}{P_{C,t+k}} - (1 + \lambda_{t+k}^{p,int}) \frac{MC_{t+k}^{int}}{P_{C,t+k}} \right] = 0 \quad (34)$$

where Y_{t+k}^{int} defines total demand for goods produced in the sector, $\Xi_{t,t+s}$ is the stochastic discount factor to be defined below and

$$\pi_{int,t,t+k-1} = \begin{cases} 1 & \text{for } k = 0 \\ \pi_{int,t} \cdot \pi_{int,t+1} \cdot \dots \cdot \pi_{int,t+k-1} & \text{for } k = 1, 2, \dots \end{cases} \quad (35)$$

$$\bar{\pi}_{t,t+k} = \begin{cases} 1 & \text{for } k = 0 \\ \bar{\pi}_t \cdot \bar{\pi}_{t+1} \cdot \dots \cdot \bar{\pi}_{t+k} & \text{for } k = 1, 2, \dots \end{cases} \quad (36)$$

The sectorial price index is:

$$P_t^{int} = \left[(1 - \xi_p^{int}) \left(\tilde{P}_t^{int} \right)^{\frac{1}{\lambda_t^{p,int}}} + \xi_p^{int} \left(\pi_{int,t-1}^{\chi_p^{int}} \bar{\pi}_t^{1-\chi_p^{int}} P_{t-1}^{int} \right)^{\frac{1}{\lambda_t^{p,int}}} \right] \lambda_t^{p,int} \quad (37)$$

Note that price-setting decisions are affected by shocks to the elasticity of substitution across goods, that we characterize as net markup shocks, assumed to follow an AR(1) process with i.i.d. Normal error term:

$$\log(\lambda_t^{p,int}) = (1 - \rho_{p,int}) \log(\lambda^{p,int}) + \rho_{p,int} \log(\lambda_{t-1}^{p,int}) + \eta_t^{p,int} \quad (38)$$

2.3 Labor market

In the country-specific labor market j , a monopolistic union sets the nominal wage W_t^j , taking as given firms' demand labor input j . At the given nominal wage, households supply the amount of labor that firms demand. This, in turn, is uniformly split across households, who supply an identical amount of labor services, $h_t = h_t^i$ as in Colciago (2011).

Wages are staggered à la Calvo (1983). Union j receives permission to optimally reset the nominal wage with probability $(1 - \xi_w)$. Non-optimizing unions adjust the wage according to the following scheme:

$$W_t^j = g_{z,t} \pi_{C,t-1}^{\chi_w} \bar{\pi}_t^{1-\chi_w} W_{t-1}^j \quad (39)$$

where $\pi_t = \frac{P_{C,t}}{P_{C,t-1}}$ is the gross rate of consumer price inflation in the region.

Following Colciago (2011), we assume that optimizing unions maximize a weighted average $(1 - \theta, \theta)$ of the two households types' utility functions:⁵

$$\max_{\tilde{W}_t^j} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \{ (1 - \theta) U_t^o(c_{t+k}^o, h_{t+k}^o) + \theta U_t^{rt}(c_{t+k}^{rt}, h_{t+k}^{rt}) \} \quad (40)$$

subject to the budget constraints 47, 48 (to be defined below) and to

$$h_{t+k}^i = \frac{1}{s} h_{t+k}^d \int_0^s \left(\frac{\tilde{W}_t^j g_{z,t,t+k} \pi_{C,t,t+k-1}^{\chi_w} \bar{\pi}_{t,t+k}^{1-\chi_w}}{W_{t+k}} \right)^{-\frac{1+\lambda_{t+k}^w}{\lambda_{t+k}^w}} dj \quad (41)$$

where:

$$\pi_{C,t,t+k-1} = \begin{cases} 1 & \text{for } k = 0 \\ \pi_{C,t} \pi_{C,t+1} \dots \pi_{C,t+k-1} & \text{for } k = 1, 2, \dots \end{cases} \quad (42)$$

$\bar{\pi}_{t,t+k}$ is defined by (36) and $g_{z,t,t+k} = \prod_{k=1}^k g_{z,t,t+k}$.

Then, the first order condition of union is:

$$0 = E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \varepsilon_t^c c_{t+s-1}^{\zeta(\sigma-1)} \exp \left(\frac{\sigma-1}{1+\phi_l} (h_{t+s})^{1+\phi_l} \right) h_{t+s}^j \cdot \left\{ \tilde{W}_t^j \frac{(1-\tau^l - \tau^w h) g_{z,t,t+s} \pi_{C,t,t+s-1}^{\chi_w} \bar{\pi}_{t,t+s}^{1-\chi_w}}{(1+\tau^c) P_{C,t+s} z_{t+s}} \left(1 - \frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w} \right) \left[(1-\theta) (c_{t+s}^o)^{-\sigma} + \theta (c_{t+s}^{rt})^{-\sigma} \right] \right. \\ \left. + \frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w} \left[(1-\theta) (c_{t+s}^o)^{-\sigma} MRS_{t+s}^o + \theta (c_{t+s}^{rt})^{-\sigma} MRS_{t+s}^{rt} \right] \right\} \quad (43)$$

⁵ U_t^o, U_t^{rt} are characterized in (3)

where

$$MRS_t^i = c_t^i h_t^{\phi_i}; i = o, rt \quad (44)$$

The aggregate wage index is

$$W_t = \left[\xi_w \left(g_{z,t} \pi_{C,t-1}^{\chi_w} \bar{\pi}_t^{1-\chi_w} W_{t-1} \right)^{\frac{1}{\lambda_t^w}} + (1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{1}{\lambda_t^w}} \right]^{\lambda_t^w} \quad (45)$$

Note that we incorporate wage markup shocks, because λ_t^w is assumed to follow an AR(1) process with i.i.d. Normal error term:

$$\log(\lambda_t^w) = (1 - \rho_w) \log(\lambda^w) + \rho_w \log(\lambda_{t-1}^w) + \eta_t^w \quad (46)$$

2.4 Non-Ricardian households

Non-Ricardian households consume their current disposable income

$$(1 + \tau^c) P_{C,t} C_t^{rt} = (1 - \tau^l - \tau^{wh}) W_t h_t + TR_t^{rt} - T_t^{rt} \quad (47)$$

where τ^c , τ^l , τ^{wh} , TR_t^{rt} , T_t^{rt} respectively denote consumption and labor income tax rates, social contributions levied on labor incomes, public transfers and lump-sum taxes.

2.5 Ricardian households

Ricardian households allocate their resources between consumption C_t^o , investment in physical capital I_t^o , in public bonds B_{t+1}^o and in a portfolio of financial assets A_t . Their budget constraint is:

$$\begin{aligned} (1 + \tau^c) P_{C,t} C_t^o + P_{I,t} I_t^o + A_t + B_{t+1}^o &= A_{t-1} + R_{t-1} B_t^o + (1 - \tau^l - \tau^{wh}) W_t^o h_t^o + D_t^o \\ &+ (1 - \tau^k) \left[\frac{R_t^k}{\varepsilon_{t-1}^b} u_t^o - a(u_t^o) P_{I,t} \right] K_t^o + \tau^k \delta P_{I,t} K_t^o + TR_t^o - T_t^o \end{aligned} \quad (48)$$

where $P_{I,t}$ is the price of investment, K_t^o is the physical capital stock, D_t^o are dividends and u_t^o defines capacity utilization. TR_t^o are transfers of Ricardian households and T_t^o are lump-sum taxes. ε_t^b is a risk premium shock that creates a wedge between the return on capital accruing to the households and the price of capital paid by firms.⁶ It is assumed to follow a first-order autoregressive process with an i.i.d. Normal error term:

$$\log(\varepsilon_t^b) = (1 - \rho_b) \log(\varepsilon^b) + \rho_b \log(\varepsilon_{t-1}^b) + \eta_t^b \quad (49)$$

The capital accumulation equation is:

$$K_{t+1}^o = (1 - \delta) K_t^o + \varepsilon_t^i \left[1 - S \left(\frac{I_t^o}{I_{t-1}^o} \right) \right] I_t^o \quad (50)$$

where δ is the depreciation rate and ε_t^i denotes an investment-specific technology shock that affects the real price of investment. It is assumed to evolve as an AR(1) process with i.i.d. Normal innovation term:

$$\log(\varepsilon_t^i) = (1 - \rho_i) \log(\varepsilon^i) + \rho_i \log(\varepsilon_{t-1}^i) + \eta_t^i \quad (51)$$

⁶ A similar kind of shock is introduced in Ratto et al. (2008) and Amano and Shukayev (2012).

The term $S\left(\frac{I_t^o}{I_{t-1}^o}\right)$ represents investment adjustment costs. The standard adjustment costs function is:

$$S\left(\frac{I_t^o}{I_{t-1}^o}\right) = \frac{\gamma_I}{2} \left(\frac{I_t^o}{I_{t-1}^o} - g_z\right)^2 \quad (52)$$

Capital utilization costs are defined as in Christiano et al. (2005):

$$a(u_t^o) = \gamma_{u1}(u_t^o - 1) + \frac{\gamma_{u2}}{2}(u_t^o - 1)^2 \quad (53)$$

The Ricardian households maximize

$$E_t \sum_{k=0}^{\infty} \beta^k \varepsilon_{t+k}^c U_k^o(c_{t+k}^o, h_{t+k}^o) \quad (54)$$

with respect to C_t^o , B_{t+1} , I_t^o , K_{t+1}^o , u_t^o , subject to (48), (50), (52) and (53). We define the Lagrange multipliers associated with (48) and (50) respectively as $\Lambda_t^o/P_{C,t}$ and $\Lambda_t^o Q_t^o$. The first order conditions are:

$$\varepsilon_t^c (c_t^o)^{-\sigma} c_{t-1}^{\zeta(\sigma-1)} \exp\left(\frac{\sigma-1}{1+\phi_t} (h_t^o)^{1+\phi_t}\right) \frac{1}{z_t} = \Lambda_t^o (1 + \tau^c) \quad (55)$$

$$R_t = \pi_{C,t+1} \frac{\Lambda_t^o}{\beta \Lambda_{t+1}^o} \quad (56)$$

$$\begin{aligned} \frac{P_{I,t}}{P_{C,t}} = & Q_t^o \varepsilon_t^i \left\{ 1 - \gamma_I \left(\frac{I_t}{I_{t-1}} - g_z \right) \frac{I_t}{I_{t-1}} - \frac{\gamma_I}{2} \left(\frac{I_t}{I_{t-1}} - g_z \right)^2 \right\} \\ & + \Xi_{t,t+1} Q_{t+1}^o \varepsilon_{t+1}^i \gamma_I \left(\frac{I_{t+1}}{I_t} - g_z \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned} \quad (57)$$

$$\Xi_{t,t+1} \left\{ (1 - \tau^k) \left[\frac{R_{t+1}^k}{\varepsilon_{t+1}^b P_{C,t+1}} u_{t+1}^o - \frac{P_{I,t+1}}{P_{C,t+1}} a(u_{t+1}^o) \right] + \tau^k \frac{P_{I,t+1}}{P_{C,t+1}} \delta + Q_{t+1}^o (1 - \delta) \right\} = Q_t^o \quad (58)$$

$$\frac{R_t^k}{\varepsilon_{t-1}^b P_{C,t}} = \frac{P_{I,t}}{P_{C,t}} [\gamma_{u1} + \gamma_{u2}(u_t^o - 1)] \quad (59)$$

Λ_t^o represents the shadow price of a unit of consumption good, thus equation (55) shows the marginal utility of consumption out of income. Q_t^o measures the shadow price of a unit of investment good and $\Xi_{t,t+1} = \beta \frac{\Lambda_{t+1}^o}{\Lambda_t^o}$ is the stochastic discount factor. ε_t^c is a preference shock affecting the subjective discount factor and evolving according to:

$$\log(\varepsilon_t^c) = (1 - \rho_c) \log(\varepsilon^c) + \rho_c \log(\varepsilon_{t-1}^c) + \eta_t^c \quad (60)$$

with $\eta_t^c \sim N(0, \sigma_c^2)$.

Equations (57) and (58) are the first order conditions for investment and capital respectively. Equation (59) equals the return from capital utilization to its cost. The latter equation implies that u_t^o is identical across Ricardian households, so that $u_t^o = u_t$. Further, the sectorial degree of capital utilization is uniform.

To complete the set of first order conditions we need to spell out our assumptions about the functioning of the monetary union financial markets.

2.5.1 Integrated financial markets

If the monetary union financial market are fully integrated, the portfolio A_t is composed of state-contingent securities that allow Ricardian households in the two regions to engage in mutual risk sharing, and of nominally riskless bonds that yield the unique rate of return R_t^{ECB} . This nominal rate is in fact controlled by the central bank, see condition (85) below. In each region the standard Euler equation holds

$$R_t^{ECB} = \pi_{C,t+1} \frac{\Lambda_t^o}{\beta \Lambda_{t+1}^o} = \pi_{C^*,t+1} \frac{\Lambda_t^{o,*}}{\beta \Lambda_{t+1}^{o,*}} \quad (61)$$

With integrated markets, combining the Euler equations in the two countries and iterating backward, we obtain the usual risk sharing condition (see Chari, Kehoe and Mc Grattan (2002)):

$$\frac{P_{C,t}^*}{P_{C,t}} = \frac{P_{C,0}^*}{P_{C,0}} \frac{\Lambda_0^o}{\Lambda_0^{o,*}} \frac{\Lambda_t^{o,*}}{\Lambda_t^o} \quad (62)$$

Then, defining the real effective exchange rate as $RER_t \equiv \frac{P_{C,t}^*}{P_{C,t}}$ we obtain:

$$RER_t = \varepsilon_t^{rsh} \kappa \frac{\Lambda_t^{o,*}}{\Lambda_t^o} \quad (63)$$

where κ defines initial conditions. Condition (63) implies that Ricardian households in the two region, by trading state-contingent assets A_t , commit themselves to international transfer schemes that allow to smooth relative consumption levels unless a variation occurs in their relative price RER_t . The additional term ε_t^{rsh} is a shock meant to capture temporary disruptions in the Euro area financial market, which impair risk sharing.

$$\log(\varepsilon_t^{rsh}) = (1 - \rho_{rsh}) \log(\varepsilon_t^{rsh}) + \rho_{rsh} \log(\varepsilon_{t-1}^{rsh}) + \eta_t^{rsh} \quad (64)$$

Note that our characterization implies that the deviations from full risk sharing affect the relative growth rate of Ricardian households consumption in the two countries, as in Kalemli-Ozcan et al. (2014).

2.5.2 Segmented financial markets

In this case we treat EMU financial markets as if they were segmented, assuming that domestic residents can only trade in domestic bonds whose prices are discounted differently relative to bonds issued in the foreign region, in analogy with a case discussed in Schmitt-Grohé and Uribe (2003). The domestic premium is an increasing function of the region foreign debt holdings, NW_t . Foreign resident trade in both domestic and foreign issued bonds. In this case, the Euler equation for domestic residents is

$$R_t = \pi_{C,t+1} \frac{\Lambda_t^o}{\beta \Lambda_{t+1}^o} \quad (65)$$

where:

$$R_t = \frac{R_t^{ECB}}{\Gamma(NW_t, \varepsilon_t^{rp})} \quad (66)$$

and

$$\Gamma(NW_t, \varepsilon_t^{rp}) = \exp \left\{ \Gamma^b \left(\frac{NW_t}{P_{C,t}Y_t} - \frac{NW}{P_C Y} \right) - \varepsilon_t^{rp} \right\} \quad (67)$$

Note that ε_t^{rp} defines a "confidence" shock following an AR (1) process with a i.i.d. Normal .

$$\log(\varepsilon_t^{rp}) = (1 - \rho_{rp}) \log(\varepsilon^{rp}) + \rho_{rp} \log(\varepsilon_{t-1}^{rp}) + \eta_t^{rp} \quad (68)$$

The law of motion of foreign debt is:

$$\frac{NW_t}{P_{C,t}} = R_{t-1} \frac{NW_{t-1}}{P_{C,t-1}} + NX_t \quad (69)$$

where NX_t stands for domestic net exports (trade balance). As a matter of fact, this characterization differs from the one presented in 2.5.1 because, in addition to shocks, deviations from full risk sharing are determined by an endogenous mechanism of foreign debt accumulation.

2.6 Aggregation

$$C_t = \theta C_t^{rt} + (1 - \theta) C_t^o \quad (70)$$

$$K_t = (1 - \theta) K_t^o \quad (71)$$

$$I_t = (1 - \theta) I_t^o \quad (72)$$

$$B_t = (1 - \theta) B_t^o \quad (73)$$

$$D_t = (1 - \theta) D_t^o \quad (74)$$

$$TR_t = \theta TR_t^{rt} + (1 - \theta) TR_t^o \quad (75)$$

$$T_t = \theta T_t^{rt} + (1 - \theta) T_t^o \quad (76)$$

2.7 Market clearing

The following market clearing conditions obtain. Domestic aggregate GDP in terms of consumption prices:

$$Y_t = \frac{P_t^H}{P_{C,t}} Y_t^H + \frac{P_t^N}{P_{C,t}} Y_t^N \quad (77)$$

Capital services:

$$u_t K_t = u_t^N K_t^N + u_t^H K_t^H \quad (78)$$

Labor input:

$$h_t^d = h_t^N + h_t^H \quad (79)$$

Labor market:

$$h_t = s_{W,t} h_t^d \quad (80)$$

$$s_{W,t} = \frac{1}{s} \int_0^s \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} dj \quad (81)$$

Investment goods:

$$Q_t^I = I_t + a(u_t) K_t \quad (82)$$

2.8 Government

The domestic government budget constraint is:

$$P_{N,t} G_t + R_{t-1} B_t + T R_t = \left\{ \begin{array}{l} B_{t+1} + T_t + \tau^c P_{C,t} C_t + (\tau^l + \tau^{wh} + \tau^{wf}) W_t h_t + \\ + \tau^k [R_t^k u_t - (a(u_t) + \delta) P_{I,t}] K_t \end{array} \right\} \quad (83)$$

where tax variables and public transfers are held constant at their steady state values, public consumption is driven by the stochastic process

$$\log \left(\frac{g_t - g}{y} \right) = \rho_G \log \left(\frac{g_{t-1} - g}{y} \right) + \eta_t^G \quad (84)$$

where lower case letters stand for variables adjusted for growth, i.e. $y_t = Y_t/Z_t$.

2.9 ECB policy

As in Christoffel et al. (2008), the common monetary authority sets the nominal interest rate according to the following log-linear Taylor rule:

$$\begin{aligned} \hat{R}_t^{ECB} &= \phi_R \hat{R}_{t-1}^{ECB} + (1 - \phi_R) (\hat{\pi}_t^{EA} + \phi_\pi (\hat{\pi}_{t-1}^{EA} - \hat{\pi}_t) + \phi_y \hat{y}_t^{EA}) \\ &\quad + \phi_{\Delta\pi} (\hat{\pi}_t^{EA} - \hat{\pi}_{t-1}^{EA}) + \phi_{\Delta y} (\hat{y}_t^{EA} - \hat{y}_{t-1}^{EA}) + \hat{\varepsilon}_t^r \end{aligned} \quad (85)$$

where ‘ $\hat{\cdot}$ ’ denotes log deviations from steady state. $\pi_t^{EA} = P_t^{EA}/P_{t-1}^{EA}$ is Euro area gross inflation rate, which is obtained from $P_t^{EA} = P_{C,t}^s (P_{C,t}^*)^{1-s}$, and $y_t^{EA} = s y_t + (1-s) y_t^*$ is the Euro area aggregate output, where the inflation target $\hat{\pi}_t$ is stochastic shock:

$$\hat{\pi}_t = \rho_{\pi} \hat{\pi}_{t-1} + \eta_t^{\pi} \quad (86)$$

3 Estimation strategy

The model is log-linearized around its steady state and then estimated with Bayesian estimation techniques. We use 19 time series characterizing PIIGS economies and the rest of the Euro area countries. In particular, for both groups, we use: real GDP, real private consumption, consumer price inflation, real investments, real compensation per employee, total employment, government

spending, nontradables inflation and nontradables GDP.⁷ The Euro area short-term nominal interest rate completes the set of observables.⁸ Inflation has been computed as the log difference in the price index (overall HICP index for consumer price inflation and services HICP index for nontradables inflation). Output, consumption, investments, wages and government spending are transformed in log differences; total employment has been detrended with a linear trend. The data sample is 1996Q2-2013Q3, due to data availability.

Following Christoffel et al. (2008), the auxiliary equation

$$\hat{e}_t = \frac{\beta}{1+\beta} E_t \hat{e}_{t+1} + \frac{1}{1+\beta} \hat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e} (\hat{h}_t - \hat{e}_t) \quad (87)$$

relates the employment variable, e_t , to the unobserved hours worked variable, h_t .⁹

We include 19 structural shocks. In all our experiments we consider an interest rate shock and the following country-specific shocks: two transitory sectorial TFP shocks (eq.29), a risk premium shock (eq.49), an investment-specific shock (eq.51), a preference shock (eq.60), price and wage markup shocks (eq.38, 46), and a government spending shock (eq.84). Model A is then closed assuming a permanent labor-augmenting technology shock common to the Euro area (eq. 30) and a Central Bank inflation objective shock (eq.86). In models B and C we remove the inflation objective shock and introduce shocks that generate segmentation in the Eurozone financial markets (eq. 63 and 68, respectively). Finally, in model D we remove the inflation objective shock and allow for two country-specific shocks to the common productivity trend identified in eq. 30.

For non stationary variables we assume a measurement equation of the type:

$$\Delta \ln Y_t = \hat{y}_t - \hat{y}_{t-1} + \bar{\gamma} + \hat{g}_{z,t} \quad (88)$$

where $\bar{\gamma} = 100(g_z - 1)$ denotes a deterministic growth trend common to the real variables while $\hat{g}_{z,t}$ is the stochastic trend component.¹⁰

For inflation variables, the observation equation is the following:

$$\Delta \ln P_t = \pi_t + \bar{\pi}_* \quad (89)$$

where $\bar{\pi}_* = 100(\bar{\pi} - 1)$ is the quarterly steady-state inflation rate.

With regard to employment variables, we have the following measurement equation:

$$\ln e_t = \hat{e}_t + \bar{e} \quad (90)$$

with \bar{e} being a constant normalized at zero.

Finally, the nominal interest rate is defined as:

$$\ln R_t^* = \hat{R}_t^* + \bar{R} \quad (91)$$

with $\bar{R} = 100(\beta^{-1}g_z\bar{\pi} - 1)$, corresponding to the steady state nominal interest rate.

⁷Following Kolasa (2009), nontradables inflation is obtained by services HICP. Following Rabanal (2009), we use services GDP as a proxy for nontradables GDP.

⁸We use quarterly data from the Eurostat database. The group series are based on the authors' computations.

⁹Parameter ξ_e determines the sensitivity of employment with respect to worked hours.

¹⁰We allow for a measurement error in nontradables GDP equations.

3.1 Calibration and priors

We calibrate a number of parameters (Table 1). The common discount factor β is fixed at 0.9988, which is consistent with a steady state growth rate of 2% per year (as in Christoffel et al. 2008) and a 2.5% real interest rate. Some parameters are calibrated to be equal across countries. In particular, the steady-state depreciation rate δ is 0.025, corresponding to a 10% depreciation rate per year. The capital shares in each sector of production and country is set at 0.3. The steady state net price markup is fixed at 35% in both countries and sectors as well as the steady state wage markup which is set at 30%. We assume that redistributive transfers between the two households groups determine a steady state consumption ratio $c^{rt}/c^o = 0.8$.

The remaining parameters are calibrated using average data over the sample. The size of PIIGS countries, s , is 35%, as measured by the HICP weights. The share of tradable consumption goods for the PIIGS (γ_c) and the rest of the Euro area (γ_c^*) is set to 0.624 and 0.597 respectively. These correspond to sample-average shares of goods in the HICP basket. The shares of investment goods (γ_i, γ_i^*) are measured by the share of non-construction investments over total investment expenditures and are set to 0.43 for PIIGS and 0.49 for the rest of the Euro area. We set the share of home produced goods in the tradable index ϖ equal to 0.91 for the PIIGS. This is obtained by subtracting the average ratio between total bilateral imports and GDP from 1. The rest of the Euro area counterpart, ϖ^* , is obtained endogenously through the steady state (and gives a value of 0.96, which is in line with the data).

The constant tax rates are obtained again from sample average. In particular, $\tau^c, \tau^{c,*}$ are the result of the ratio between total indirect tax revenues and consumption expenditures. Similarly, $\tau^l, \tau^{l,*}$ correspond to the ratio between total direct taxes of households over total households' wages and salaries. The capital tax rates $\tau^k, \tau^{k,*}$ are computed as the ratio between corporate tax revenues and corporations income. Social security contributions tax rates are obtained from the ratio of total social security contributions over wages and salaries. As a proxy for employees and employers social security contributions we suppose that 1/3 of contributions are paid by the households while 2/3 of contributions are paid by firms. We use average ratios for calculating government spending to GDP and debt to GDP ratios. We derive the difference between aggregate transfers and taxes to GDP ratios ($tr/y - t/y$) as a residual from the steady state government budget constraint.

The remaining parameters are estimated with Bayesian techniques. Priors, reported in Table 2, are set in line with the literature on Euro area models (see Christoffel et al. (2008), Smets and Wouters (2003, 2005), Rabanal (2009), Kolasa (2009)). All the parameters priors are set symmetrically for the two countries, so that the data can tell about possible asymmetries. In particular, parameters measuring the persistence of the shocks are set to be Beta distributed, with mean 0.5 and standard deviation 0.1 and the standard errors of the innovations are assumed to follow Gamma distributions, similarly to Rabanal (2009). The parameters governing price and wage setting, habits, utilization elasticity, interest rate smoothing and the steady state fraction of LAMP are also Beta distributed. The fraction of LAMP θ, θ^* are assumed to be Beta distributed with mean 0.4 and standard deviation 0.1, in line with the recent results obtained for the Euro area by Albonico et al. (2014). We estimate the monetary authority's long-run (net) quarterly inflation objective $100(\bar{\pi} - 1)$ assuming a prior mean of 0.5% (2% in annual terms), consistent with the ECB's quantitative definition of price stability. Also the trend growth rate of the economy is estimated with a Normal distribution with mean 0.6 (corresponding to 2.4% in annual terms) and 0.1 standard deviation. The priors for the elasticity of substitutions in the consumption indices (e, v) are set in line with Rabanal (2009) as Normal(1, 0.5) distributions.

Risk aversion, the inverse of Frisch elasticity and the parameters of the Taylor rule are Normally

Table 1: Calibration of parameters

parameter	value
β	0.9988
δ, δ^*	0.025
$\alpha_N, \alpha_H, \alpha_{N,*}, \alpha_F$	0.3
λ_p, λ_p^*	0.35
λ_w, λ_w^*	0.3
$\frac{c^{rt}}{c^o}, \frac{c^{rt,*}}{c^{o,*}}$	0.8
s	0.35
γ_c	0.624
γ_c^*	0.597
γ_i	0.43
γ_i^*	0.49
ϖ	0.91
τ_c	0.215
τ_c^*	0.23
τ_l	0.303
τ_l^*	0.229
τ_k	0.206
τ_k^*	0.159
τ_{wh}	0.127
τ_{wh}^*	0.131
τ_{wf}	0.254
τ_{wf}^*	0.262
$\frac{b}{y}$	0.912*4
$\frac{b^*}{y^*}$	0.669*4
$\frac{g}{y}$	0.186
$\frac{g^*}{y^*}$	0.207

distributed, whereas the parameter defining investment adjustment costs is Gamma distributed.

Table 2: Prior distributions of parameters

parameters	shape	mean	std dev
$\bar{\gamma}$	norm	0.6	0.1
$\bar{\pi}_*$	norm	0.5	0.1
e, v	gamma	1	0.5
σ, σ^*	norm	2.5	0.375
ζ, ζ^*	beta	0.6	0.1
ϕ_l, ϕ_l^*	norm	2.5	0.4
θ, θ^*	beta	0.4	0.1
γ_I, γ_I^*	gamma	4	0.5
σ_u, σ_u^*	beta	0.5	0.15
$\chi_p^N, \chi_p^{N,*}, \chi_p^H, \chi_p^F$	beta	0.75	0.1
$\xi_p^N, \xi_p^{N,*}, \xi_p^H, \xi_p^F$	beta	0.75	0.1
χ_w, χ_w^*	beta	0.75	0.1
ξ_w, ξ_w^*	beta	0.75	0.1
ξ_e, ξ_e^*	beta	0.5	0.15
ϕ_r	beta	0.9	0.05
ϕ_π	norm	1.7	0.1
ϕ_y	norm	0.12	0.05
$\phi_{\Delta y}$	norm	0.063	0.05
$\phi_{\Delta \pi}$	norm	0.3	0.1
shocks persistences (ρ)	beta	0.5	0.1
$\sigma^{gz}, \sigma^{gz,*}, \sigma^{a,N}, \sigma^{a,H}, \sigma^{a,N,*}, \sigma^{a,F}$	gamma	0.7	0.3
$\sigma^b, \sigma^{b,*}, \sigma^r, \sigma^\pi, \sigma^{rsh}$	gamma	0.4	0.2
$\sigma^c, \sigma^{c,*}, \sigma^i, \sigma^{i,*}, \sigma^w, \sigma^{w,*}, \sigma^p, \sigma^{p,*}, \sigma^g, \sigma^{g,*}$	gamma	1	0.5

4 Results

4.1 Posterior estimates

Tables 3 and 4 show the posterior mean estimates of structural parameters and shocks under the different specifications of the model. The four specifications yield fairly similar posterior estimates for the common parameters, which appear to be well identified.¹¹ Model D is strongly preferred on the grounds of the marginal data density (-870), followed by model A (-915).

All models yield a larger posterior estimate for the share of non-Ricardian households in the core countries region than in the PIIGS region. This result survives also if we estimate the model for a shorter sample which excludes the financial crisis (1996q2-2007q3). Under specification D, in the PIIGS the fraction is 25% (29.7% with the full sample) while in the core region is 30.6% (increasing to 41.6% considering the crisis period). This latter result is also in line with previous findings for the Euro area in Albonico et al. (2014), where the authors show how this fraction has increased due to the financial retrenchment during the recent crisis. The finding that the share

¹¹Visual diagnostics of the estimation results are available upon request. The posterior distributions are computed considering 4 Monte Carlo Markov chains of 250,000 draws each, with 20% draws being discarded as burn-in draws. The average acceptance rate is comprised between 23 and 35 percent.

of non-Ricardian households is larger in the core region is not entirely new in the literature. Di Bartolomeo et. al. (2011) estimate a larger fraction of LAMP consumers in France (44%) than in Italy (9%). These authors argue that it could be due to differences in savings rates and/or to different structures of the financial markets. We checked the average savings rates for the two group of countries and we could not find such a significant difference between the two regions, indeed the savings rate is higher in the core countries. More recently, Kaplan et al. (2014) estimated the fraction of LAMP using survey data. Their theoretical approach is a bit different from ours. In fact, they allow for the possibility that the total fraction of LAMP is composed by "poor hand-to-mouth" consumers, who do not hold any type of assets, and "wealthy hand-to-mouth" consumers, i.e. consumers holding illiquid assets, such as housing, but whose consumption is still strongly correlated with their income. They show that the presence of these "wealthy hand-to-mouth" consumers can significantly influence the estimates of the total LAMP fraction. In particular, for a cross-section of 2010 they find that, although the fraction of "poor hand-to-mouth" consumers in Germany and Italy is not very different (7.4% versus 8.3% respectively¹²), the gap between the two countries is reversed and increases considerably if "wealthy hand-to-mouth" consumers are taken into account. In this case, in fact, Germany's total fraction of LAMP is found to be 32.2%, while being only 23.8% in Italy and 19.6% in Spain. Our results appear then in line with this finding.

In Table 5 we report the variance decomposition obtained under model D. The first striking result is that technology and demand shocks have different importance in the two regions. In the PIIGS region demand shocks play a lesser role than in the rest of the Euro area. The second striking result is that shocks originating in one region have a minimal impact onto the rest of the Eurozone.

In Table 6 in the Appendix we check the variance decomposition of model B, to get some intuition about the importance of the risk sharing shock in explaining macro variables volatilities. The role of the risk sharing is negligible.

4.2 Output growth determinants during the post-2007 crises

In this section, we present the historical decomposition of output growth to gain intuition about which shocks caused the recent crises in the Euro area.¹³ This analysis is also useful to detect possible spillover effects and transmission of shocks across the two regions.

Figure 1 shows that in the CORE countries the 2008-2009 global crisis has been mainly characterized by adverse demand shocks. In this model the demand shocks include investment-specific and risk premium shocks. A contractionary investment-specific shock entails a drop in investment and consumption (the latter effect being due to the presence of LAMP consumers, as discussed in Furlanetto et al., 2013). Similarly, in case of a negative risk premium shock the wedge between the return on capital accruing to households and the price of capital paid by firms causes a decrease in capital demand and thus investment. The weaker demand pushes the economic system into recession. Differently, in the PIIGS region (Figure 2) the financial crisis has been determined by a combination of demand and technology shocks. The contribution of the interest rate policy was contractionary in both regions.

Asymmetries between the two region become more important after the onset of the Greek crisis in 2010. In the core region the initial faint recovery has been due to favorable demand shocks and to the ECB accommodative interest rate policy. The subsequent slowdown has been determined

¹²Note that the value for Spain is even lower (4.4%), although also for France a value of 3.2% is found.

¹³We show the results obtained under model D. However, the other specifications give similar results.

Table 3: Posterior mean estimates of parameters

parameters	MODEL			
	A	B	C	D
$\bar{\gamma}$	0.369	0.366	0.342	0.349
π_*	0.392	0.470	0.438	0.395
e	0.565	0.565	0.497	0.564
v	1.404	2.205	1.404	1.966
σ	2.287	2.339	2.572	2.436
σ^*	2.548	2.405	2.442	2.623
b	0.673	0.660	0.705	0.621
b^*	0.597	0.598	0.581	0.645
ϕ_l	2.741	2.744	2.752	2.879
ϕ_l^*	2.730	2.795	2.807	2.763
θ	0.311	0.302	0.323	0.297
θ^*	0.488	0.511	0.508	0.416
γ_I	4.804	4.353	4.629	4.615
γ_I^*	4.415	4.472	4.304	4.697
σ_u	0.354	0.395	0.338	0.183
σ_u^*	0.463	0.414	0.403	0.467
χ_p^N	0.961	0.817	0.816	0.883
$\chi_p^{N,*}$	0.894	0.848	0.846	0.859
χ_p^H	0.282	0.462	0.512	0.255
χ_p^F	0.433	0.352	0.369	0.345
ξ_p^N	0.892	0.889	0.892	0.663
$\xi_p^{N,*}$	0.547	0.531	0.536	0.557
ξ_p^H	0.888	0.539	0.499	0.613
ξ_p^F	0.493	0.478	0.483	0.502
χ_w	0.712	0.745	0.740	0.717
χ_w^*	0.738	0.678	0.759	0.641
ξ_w	0.840	0.866	0.875	0.770
ξ_w^*	0.832	0.831	0.829	0.849
ξ_e	0.658	0.682	0.672	0.619
ξ_e^*	0.549	0.534	0.538	0.563
ϕ_r	0.872	0.874	0.898	0.879
ϕ_π	1.602	1.640	1.612	1.682
ϕ_y	0.034	0.030	0.029	0.023
$\phi_{\Delta y}$	0.163	0.164	0.149	0.167
$\phi_{\Delta\pi}$	0.249	0.206	0.171	0.216
Γ_b			0.018	
MDD	-914.5	-946.2	-985.3	-869.7

Table 4: Posterior mean estimates of shocks

parameters	MODEL			
	A	B	C	D
$\rho_{a,H}$	0.888	0.864	0.873	0.636
$\rho_{a,F}$	0.575	0.621	0.618	0.636
$\rho_{a,N}$	0.722	0.894	0.920	0.621
$\rho_{a,N,*}$	0.596	0.594	0.607	0.584
ρ_c	0.935	0.924	0.879	0.512
$\rho_{c,*}$	0.437	0.489	0.409	0.591
ρ_b	0.836	0.838	0.809	0.795
$\rho_{b,*}$	0.820	0.809	0.802	0.836
ρ_i	0.519	0.547	0.579	0.476
$\rho_{i,*}$	0.627	0.676	0.832	0.555
ρ_p	0.494	0.839	0.875	0.939
$\rho_{p,*}$	0.949	0.948	0.944	0.948
ρ_w	0.633	0.516	0.466	0.729
$\rho_{w,*}$	0.781	0.732	0.685	0.721
ρ_g	0.919	0.917	0.908	0.442
$\rho_{g,*}$	0.880	0.856	0.877	0.789
ρ_{gz}	0.483	0.515	0.480	0.544
$\rho_{gz,*}$				0.485
$\rho_s(A), \rho_{rp}(C)$		0.491	0.668	
$\sigma_{a,H}$	1.547	1.447	1.384	1.325
$\sigma_{a,F}$	1.003	0.975	0.950	1.046
$\sigma_{a,N}$	0.807	1.710	1.487	1.123
$\sigma_{a,N,*}$	0.697	0.669	0.694	0.739
σ_c	2.302	2.383	2.696	1.053
$\sigma_{c,*}$	1.425	1.318	1.481	1.656
σ_b	1.919	2.055	1.908	1.715
$\sigma_{b,*}$	2.183	2.045	2.072	2.261
σ_i	0.178	0.186	0.174	0.186
$\sigma_{i,*}$	0.179	0.175	0.147	0.193
σ_p	0.168	0.092	0.085	0.270
$\sigma_{p,*}$	0.298	0.326	0.319	0.285
σ_w	0.274	0.302	0.300	0.299
$\sigma_{w,*}$	0.099	0.123	0.135	0.115
σ_g	0.238	0.242	0.248	0.142
$\sigma_{g,*}$	0.108	0.102	0.103	0.097
σ_{gz}	0.318	0.294	0.297	0.604
$\sigma_{gz,*}$				0.274
σ_r	0.107	0.095	0.076	0.104
$\sigma_{pi}(A), \sigma_s(B), \sigma_{rp}(C)$	0.656	0.435	0.050	
σ_{me}	0.533	0.535	0.539	0.519
$\sigma_{me,*}$	0.437	0.433	0.431	0.433

Table 5: Variance decomposition of selected variables

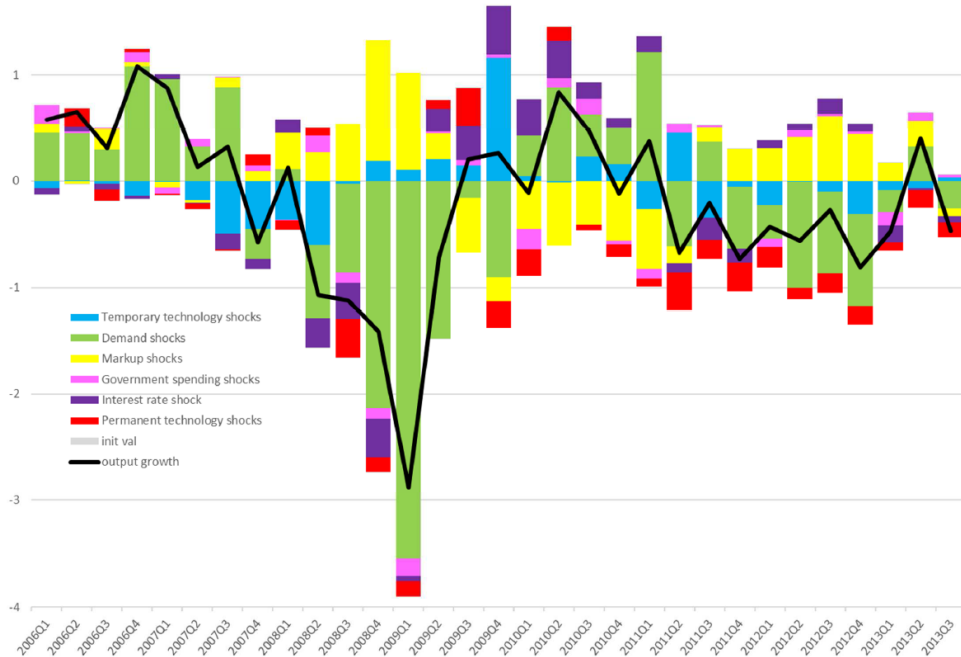
	Δy	Δc	Δi	π_c	π^N	Δc^{rt}	Δc^o	Δex	Δim	nx	Δy^h	Δy^n
Temporary technology shocks												
Home	16.86	11.88	13.73	24.53	18.39	38.48	5.89	19.07	37.08	4.53	17.06	15.24
Foreign	0.04	0.02	0.05	0.96	0.59	0.04	0.03	19.89	14.37	3.13	0.51	0.03
Permanent technology shocks												
Home	36.14	55.42	1.50	2.68	2.60	33.58	56.82	16.31	4.67	28.78	34.34	27.97
Foreign	0.18	0.30	0.20	1.15	1.05	0.09	0.36	1.48	0.43	21.34	0.18	0.15
Demand shocks												
Home	12.39	0.96	13.92	4.91	4.96	1.67	0.81	1.94	6.12	1.87	10.60	11.02
Foreign	0.17	0.29	0.32	0.83	0.68	0.08	0.35	9.60	2.30	4.75	0.21	0.16
Markup shocks												
Home	25.71	15.18	66.44	55.57	63.57	22.75	14.77	19.36	18.59	24.12	29.15	24.36
Foreign	0.36	0.44	0.41	3.15	2.52	0.21	0.50	5.42	5.96	9.61	0.74	0.36
Government shocks												
Home	2.81	0.19	0.05	0.07	0.07	0.80	0.07	0.04	0.42	0.04	0.30	5.37
Foreign	0.00	0.01	0.01	0.01	0.01	0.00	0.02	0.14	0.07	0.06	0.00	0.00
Monetary shocks												
Interest rates	1.82	2.52	1.54	4.79	4.21	0.82	2.99	0.81	2.75	0.17	2.13	1.40
Measurement error												11.83
	Δy^*	Δc^*	Δi^*	π_c^*	$\pi^{N,*}$	$\Delta c^{rt,*}$	$\Delta c^{o,*}$	Δy^f	$\Delta y^{n,*}$	r	rer	tt
Temporary technology shocks												
Home	0.40	0.04	0.07	0.45	0.31	0.02	0.07	0.48	0.02	9.94	3.96	5.48
Foreign	15.47	21.51	15.46	32.59	25.25	44.73	7.61	17.54	10.43	11.82	3.78	5.76
Permanent technology shocks												
Home	1.43	6.46	0.95	0.28	0.28	0.55	10.59	2.09	0.69	5.42	28.15	28.63
Foreign	9.98	11.53	3.06	3.63	3.63	15.18	6.29	7.25	10.36	13.35	24.98	25.41
Demand shocks												
Home	0.16	0.13	0.19	0.21	0.17	0.03	0.18	0.14	0.08	3.09	1.20	1.13
Foreign	34.79	2.69	37.11	10.49	10.75	5.06	1.69	32.55	25.24	17.92	3.32	3.32
Markup shocks												
Home	1.59	0.85	1.50	3.40	2.96	0.29	1.04	1.42	0.59	13.82	23.17	19.75
Foreign	22.81	17.46	37.55	43.41	51.37	28.98	10.89	24.03	16.75	11.87	10.74	9.85
Government shocks												
Home	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.44	0.01	0.01
Foreign	2.16	0.27	0.19	0.14	0.14	0.82	0.05	0.17	4.81	0.91	0.07	0.07
Monetary shocks												
Interest rates	2.99	4.08	3.67	5.06	4.80	1.16	5.25	3.56	1.90	6.53	0.21	0.20
Measurement error									24.63			

by negative demand shocks, partly offset by favorable markup shocks. Differently, the largest part of the double dip in the PIIGS is explained by adverse permanent technology shocks.

Figures 3 and 4 show that each region country-specific shocks drove output growth volatility. Each region is only marginally affected by shocks originating in the other group. All in all, permanent technology shocks seem to have played a key role in explaining growth differentials in the Eurozone.

Finally, in Figures 5 and 6 we present the historical decompositions of the growth in the Public-Consumption-to-GDP ratios. We have already shown that public consumption shocks were not particularly important to explain output growth volatility. Here we observe important differences in the post-2007 drivers of the two ratios. In the CORE region the ratio almost exclusively reacted to domestic shocks, implying that public consumption had a stabilizing effect. By contrast, in the PIIGS region the ratio is more responsive to fiscal shocks. However, it is difficult to identify a pattern in these discretionary actions.

Figure 1: Rest of EA output growth historical decomposition by nature of shocks.



4.2.1 Inspecting the black box: the transmission mechanism and the possible interpretations of shocks to PIIGS productivity growth rate

The analysis of the historical decomposition of output growth stresses the importance of permanent technology shocks in the PIIGS region. This section aims at better understanding the effects of this shock. A temporary slowdown in productivity growth has a widespread contractionary effect in the PIIGS region. All households reduce consumption, investment output and hours worked fall, along with the real wage and inflation. There are two key differences with respect to a temporary technology shock: the first is that Ricardian consumers now react to the permanent income reduction and do not smooth consumption, the second is that the effects of the permanent shock accumulate in the long run. On impact, we observe a limited positive transmission to output, consumption and hours worked in the CORE region (Figure 8). The permanent shock to PIIGS

Figure 2: PIIGS output growth historical decomposition by nature of shocks.

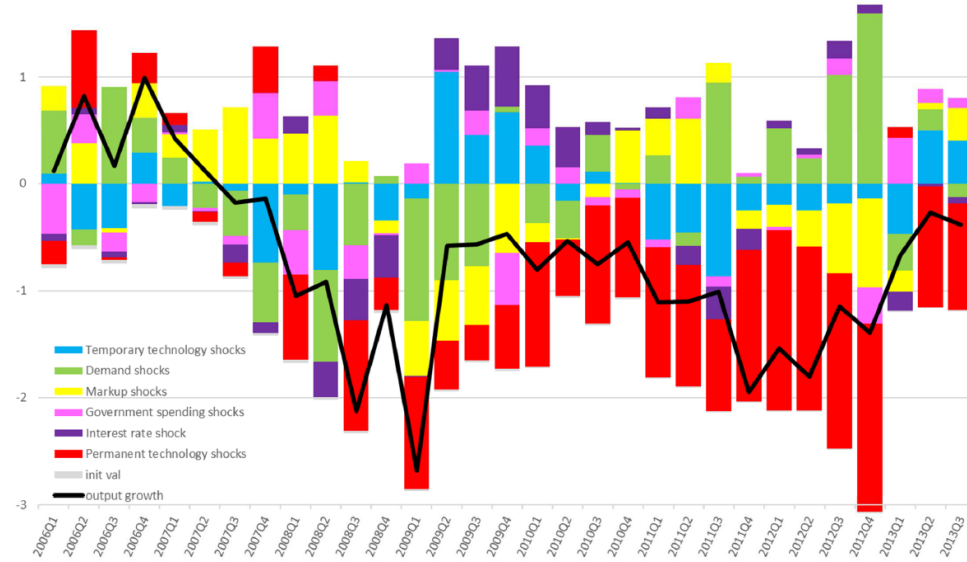


Figure 3: Rest of EA output growth historical decomposition by country origin of shocks.

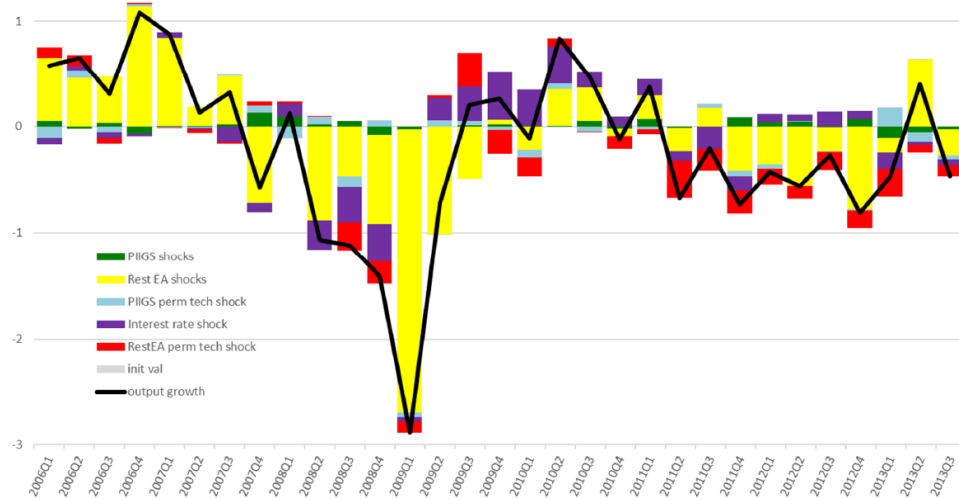


Figure 4: PIIGS output growth historical decomposition by country origin of shocks.

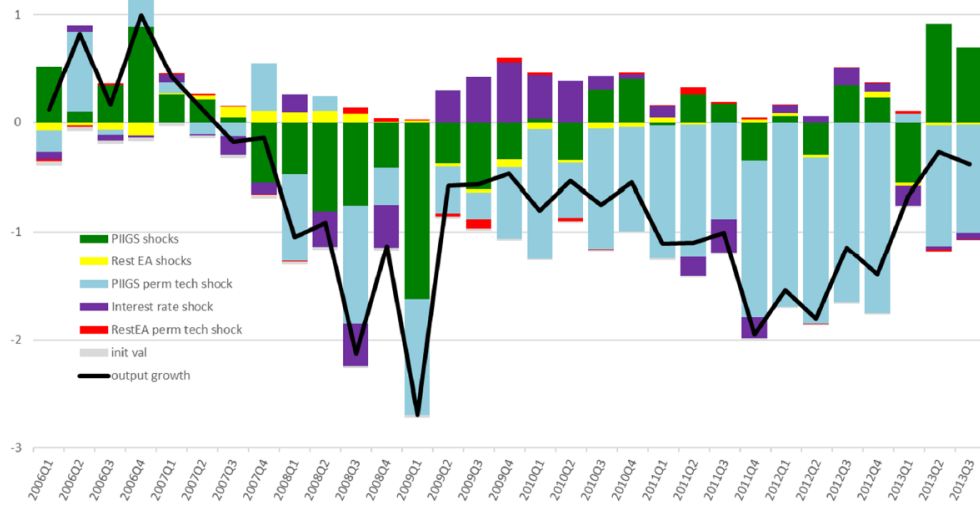


Figure 5: PIIGS G/Y growth historical decomposition.

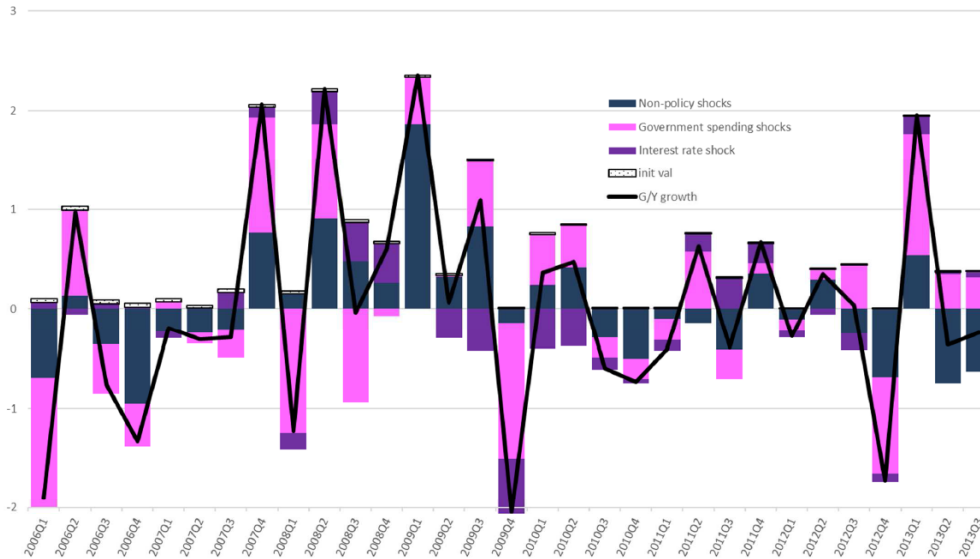
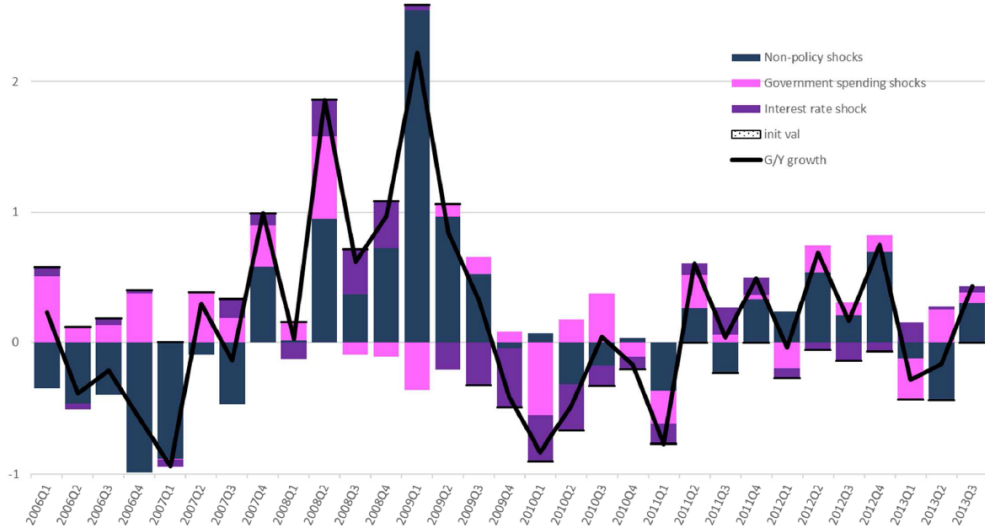
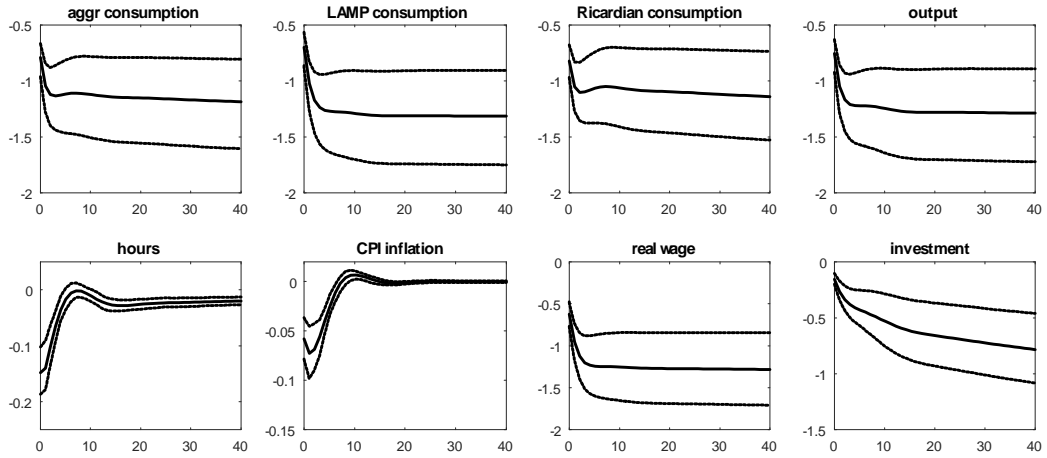


Figure 6: Rest of the EA G/Y growth historical decomposition.



productivity level has no statistically significant long-run effects in the CORE region. Considering the aggregate Euro area, we observe a temporary fall in inflation and a permanent output loss.

Figure 7: PIIGS IRFs to a negative growth rate shock in the PIIGS under model D.



But how can we interpret the sequence of adverse permanent technology shocks that started in 2008 and persisted until the end of our sample? According to a popular view (Shambaugh, 2012), the financial shocks and the weak growth rate are distinct and mutually reinforcing factors in the Eurozone crisis. Our estimates suggest that neither the risk sharing shock nor the "confidence" shock in (67) seem to have played any role during the crises,¹⁴ nor the PIIGS growth slowdown was caused by standard domestic shocks such as the risk premium and the investment-specific shocks.¹⁵

¹⁴Note that these shocks typically impact on Ricardian households demand and saving decisions but have no direct effect on the supply side.

¹⁵According to Justiniano et al. (2011), an adverse investment-specific shock might be interpreted as a loss of efficiency in the financial intermediation process that allows to transform savings into future capital inputs.

Figure 8: Core countries IRFs to a negative growth rate shock in the PIIGS under model D.

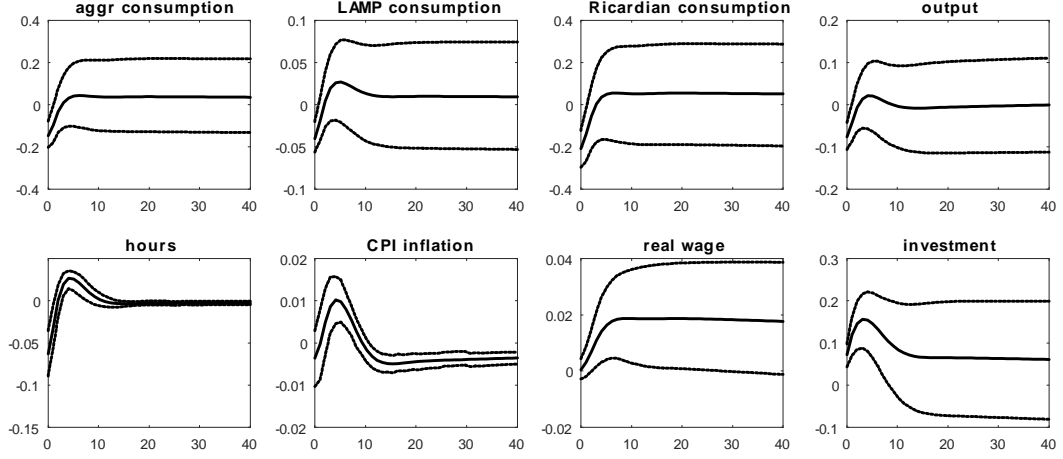
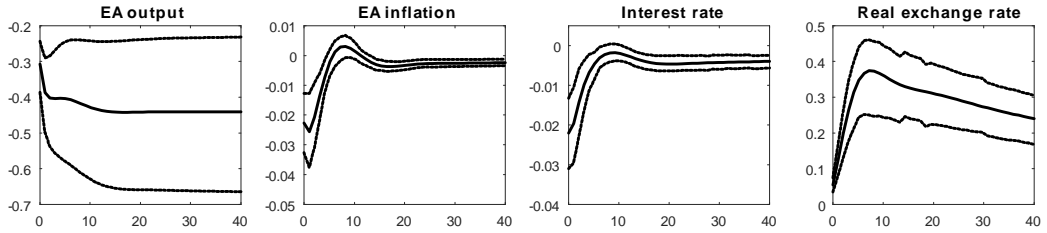


Figure 9: Other variables IRFs to a negative growth rate shock in the PIIGS under model D.



In fact, the shocks to productivity growth are crucial to characterize PIIGS performance. Bearing in mind that we cannot find evidence of adverse productivity shocks in the rest of the Eurozone during the same period, it is tempting to interpret them as a consequence of specific features of the financial crises in the Eurozone periphery.

The Post 2007 financial crisis in the PIIGS region is a multifaceted event. Figure 10 shows that a reversal in total capital flows took place only after 2011, and was compensated by an increase in public capital inflows. However, commercial banks channel the bulk of firms credit in these countries and Figure 11 documents the post-2008 reversal in the external funds accruing to banks in the PIIGS region. Bonds spreads (Figure 12) also document that in the region the relative cost of credit strongly increased since 2010.¹⁶ If one takes the ratio of non-performing to total gross loans as a proxy for the availability of bank credit (Nkusu, 2011), there is little doubt that the situation in the PIIGS region was certainly worse than in the rest of the Eurozone (see Figure 13).

Khan and Thomas (2013) provide a theoretical explanation of the reason why a credit crunch may adversely affect productivity. In their model collateralized borrowing constraints and partial investment irreversibilities imply that firms net worth, as opposed to productivity, drives external finance flows and firms capital accumulation. Adverse financial shocks cause large and persistent disruptions to the cross-sectional distribution of capital, penalizing firms characterized by relatively high productivity but relatively low net worth. This, in turn, implies large and persistent reductions in aggregate total factor productivity. A growing body of empirical evidence provides support for this view. Furceri and Mourougane (2012) find that the occurrence of a financial crisis hits negatively and permanently potential output in a panel of OECD countries. Benigno et al. (2015) identify 155 episodes of large capital inflows and find that larger inflows of foreign capital are associated with a deeper fall in TFP, measured by the Solow residual, at the end of the episode. Gopinath et al. (2015) provide microeconomic evidence on manufacturing firms total factor productivity in a number of Eurozone countries. They find that after 2008 an increase in the dispersion in firm-specific productivity shocks was associated to a downward trend in TFP in manufacturing sectors in Spain and Italy. These phenomena were not observed for Germany and France. Identifying TFP with the Solow residual without controlling for varying factor utilization, as these studies do, is open to criticism (Basu et al. 2006), but our results seem to provide further indirect support for the Khan and Thomas (2013) argument.

¹⁶Gunn and Johri (2013) identify the transmission channel from sovereign to bank loans spreads.

Figure 10: PIIGS: public and private capital flows (blue and red line repectively). Source: Gros and Alcidi (2013).

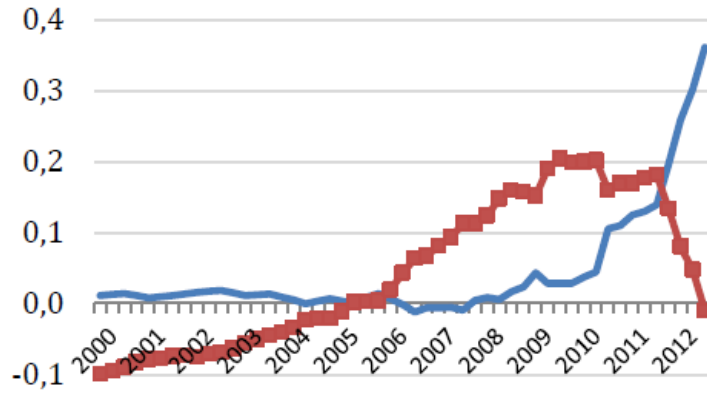


Figure 11: BIS-reporting banks external claims vis-à-vis PIIGS countries (Index, base year Dec. 1999=100). Source: Gros and Alcidi (2013).

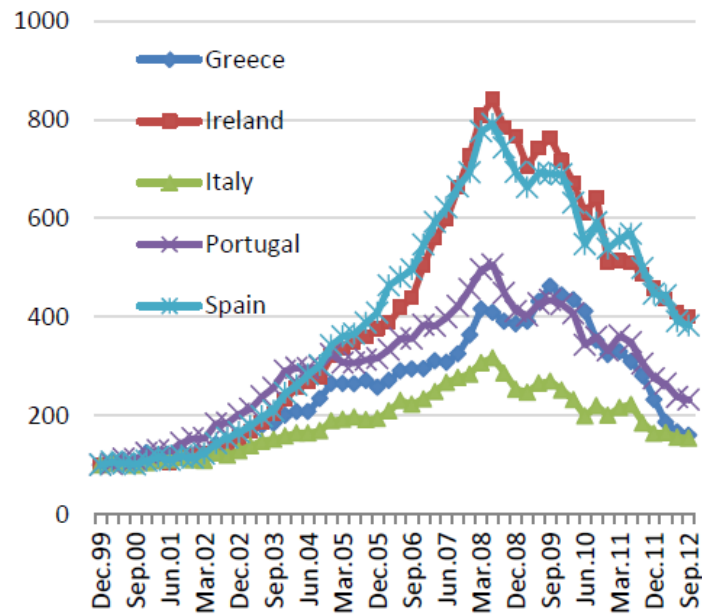


Figure 12: Spreads 10-year government bond rates eurozone.

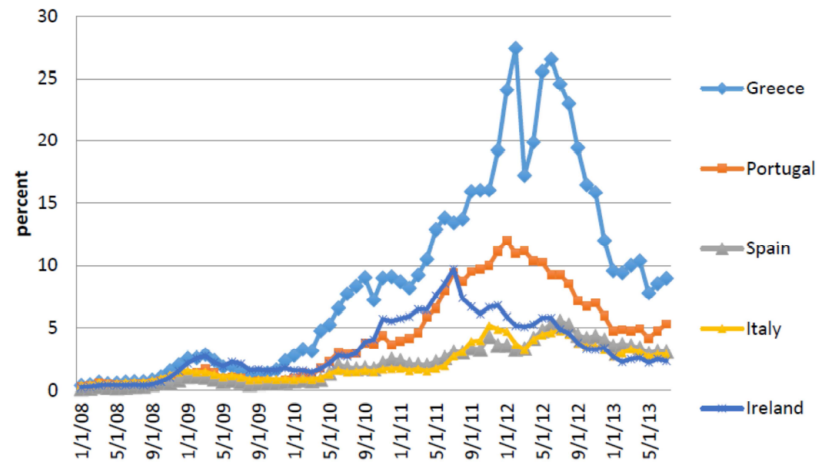
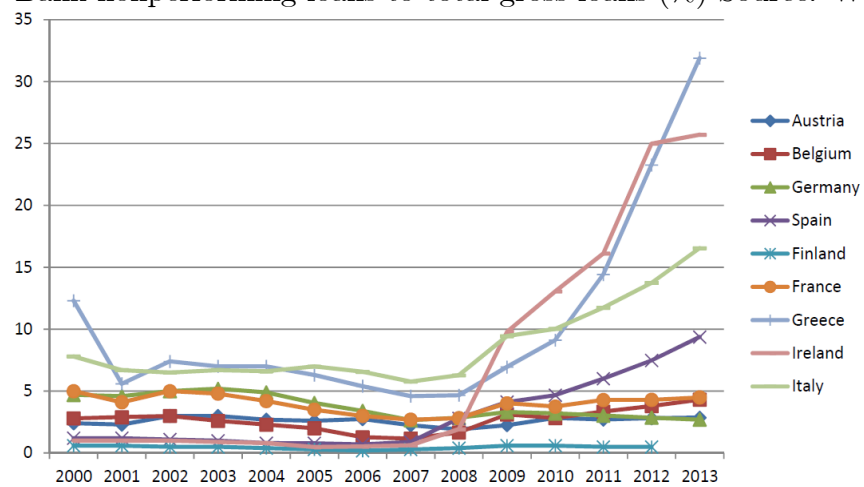


Figure 13: Bank nonperforming loans to total gross loans (%) Source: World Bank.



5 Conclusions

In this paper we build a medium-scale two country model for the European monetary union. The aim is in fact to understand the interactions between the PIIGS and the CORE regions of the Eurozone.

We consider different specifications of the model, concerning the structure of shocks and segmentation of financial markets. We obtain similar estimates for structural parameters in the two regions, but the shocks that caused the post-2007 crises appear to be quite different. Permanent adverse productivity shocks explain the worse performance of the PIIGS region.

Our analysis has also shown that in both regions public consumption did not particularly contribute to output growth volatility either before or after 2007. We do observe, however, important differences in the post-2007 drivers of the public consumption-to-GDP ratios. In the CORE region the ratio almost exclusively reacted to domestic shocks, implying that relative public consumption stability had a stabilizing effect. By contrast, in the PIIGS region the ratio is more responsive to fiscal shocks. However, it is difficult to interpret such shocks as attempts to stabilize the economy in bad times or, to the contrary, as the consequence of austerity imposed by EMU institutions. This latter result questions the effectiveness of EMU institutions and PIIGS governments in designing consistent national fiscal policies.

Our results suggest that achieving cyclical recovery will not be sufficient to restore the relative income level that PIIGS had achieved before 2007, and productivity-enhancing reforms should be at the forefront of political debate. This has important implications for EMU fiscal rules, which should foster convergence and macroeconomic stability. In fact, austerity should be abandoned in favor of accommodating fiscal policies that should limit the short-run adverse effects of reforms. To the extent that the slow down in productivity growth in the PIIGS region was caused by the capital flows reversal and by the ensuing credit crunch, macroeconomic policies should target credit availability and external financing conditions for innovative firms.

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A Appendix

A.1 Variance decomposition under model B

Table 6: Variance decomposition of selected variables

	Δy	Δc	Δi	π_c	π^N	Δc^{rt}	Δc^o	Δex	Δim	nx	Δy^h	Δy^n
Technology shocks												
Home	35.99	25.10	33.02	53.12	21.91	63.64	13.83	45.34	36.71	31.99	42.36	23.00
Foreign	0.54	0.11	0.10	0.75	0.19	0.23	0.08	14.79	14.35	2.65	2.33	0.19
aggregate	4.50	5.26	0.40	1.39	1.02	5.20	4.23	1.00	0.60	0.52	3.48	3.55
Demand shocks												
Home	17.95	0.72	14.87	3.13	0.84	1.89	0.50	1.99	2.50	1.79	13.41	12.64
Foreign	0.45	0.48	0.59	0.70	0.33	0.19	0.51	2.42	1.66	1.40	0.58	0.41
Risk sharing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Markup shocks												
Home	17.20	8.60	40.59	25.64	66.78	20.88	7.01	20.97	20.18	21.98	15.52	27.30
Foreign	1.79	0.83	2.12	6.13	3.06	0.98	0.67	5.57	10.60	16.04	3.20	1.66
Government shocks												
Home	4.22	0.18	0.36	0.32	0.18	1.13	0.06	0.37	0.29	0.63	0.28	8.73
Foreign	0.02	0.04	0.02	0.02	0.01	0.01	0.05	0.05	0.06	0.13	0.01	0.02
Monetary shocks												
Interest rates	3.34	3.83	3.44	3.86	2.56	1.26	4.16	0.53	1.23	0.20	3.20	2.61
Measurement error												11.64
	Δy^*	Δc^*	Δi^*	π_c^*	$\pi^{N,*}$	$\Delta c^{rt,*}$	$\Delta c^{o,*}$	Δy^f	$\Delta y^{n,*}$	r	rer	tt
Technology shocks												
Home	1.35	0.15	0.33	2.81	2.15	0.16	0.29	2.15	0.09	19.49	29.72	38.70
Foreign	13.09	16.78	9.12	21.82	15.23	40.63	5.07	14.87	8.45	5.74	2.75	3.33
aggregate	5.82	8.78	0.64	1.77	1.62	11.23	5.26	4.11	6.22	10.06	0.54	0.39
Demand shocks												
Home	0.46	0.41	0.54	0.45	0.38	0.16	0.50	0.47	0.28	3.51	1.81	1.94
Foreign	25.67	2.95	22.22	7.15	6.91	5.57	1.87	23.24	20.18	9.40	2.14	1.58
Risk sharing	0.22	1.09	0.00	0.00	0.00	0.11	1.76	0.26	0.13	0.04	0.00	0.00
Markup shocks												
Home	0.96	0.45	2.00	1.33	1.18	0.28	0.46	1.22	0.55	11.31	12.12	15.21
Foreign	40.57	27.29	60.45	60.59	68.79	36.07	18.92	41.28	30.93	19.15	32.11	22.26
Government shocks												
Home	0.04	0.13	0.08	0.03	0.03	0.03	0.18	0.04	0.05	1.00	0.74	0.67
Foreign	1.25	0.10	0.13	0.08	0.08	0.60	0.01	0.08	3.40	0.61	0.16	0.13
Monetary shocks												
Interest rates	3.47	5.30	3.34	3.27	2.98	1.63	6.76	3.85	2.44	7.56	0.05	0.14
Measurement error									22.54			

A.2 Non linear equations

The model is adjusted for growth, to obtain a balanced growth equilibrium. Thus, all growing variables are divided by the level of technology. Lower case letters stand for detrended variables, for example, $y_t = \frac{Y_t}{z_t}$. We define $\lambda_t^o = \Lambda_t^o z_t$ (see Christoffel, Coenen and Warne (2008)). c_t^o and c_t are already expressed as stationary variables. We also define $r_t^k = \frac{R_t^k}{P_{C,t}}$, $w_t = \frac{W_t}{z_t P_{C,t}}$, $tr_t^{rt} = \frac{TR_t^{rt}}{z_t P_{C,t}}$, $t_t^{rt} = \frac{T_t^{rt}}{z_t P_{C,t}}$. In this way it is also possible to compute the steady state of the model. For each country, lower letters price variables with a tilde " \sim " stand for the optimal price relative to aggregate price of the sector, for example: $\frac{\tilde{P}_t^n}{P_t^N} = \tilde{p}_t^n$.

Moreover, it is possible to express all the equilibrium equations as functions of relative prices. In particular, we will adopt the following definitions.

Terms of trade:

$$tt_t = \frac{P_t^F}{P_t^H} = \frac{P_t^{F,*}}{P_t^{H,*}}$$

where the second equality comes from the law of one price assumption.

"Internal" exchange rates:

$$x_t = \frac{P_t^N}{P_t^T}$$

$$x_t^* = \frac{P_t^{N*}}{P_t^{T*}}$$

All prices are expressed in terms of tt_t , x_t and x_t^* .

A.2.1 Relative prices

Relative investment prices Home country:

$$\frac{P_{I,t}}{P_{C,t}} = \left[\frac{\gamma_i + (1 - \gamma_i)(x_t)^{1-e}}{\gamma_c + (1 - \gamma_c)(x_t)^{1-e}} \right]^{\frac{1}{1-e}}$$

Foreign country:

$$\frac{P_{I,t}^*}{P_{C,t}^*} = \left[\frac{\gamma_i^* + (1 - \gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}} \right]^{\frac{1}{1-e}}$$

Relative non tradable prices Home country:

$$\frac{P_t^N}{P_{C,t}} = \frac{x_t}{[\gamma_c + (1 - \gamma_c)(x_t)^{1-e}]^{\frac{1}{1-e}}}$$

Foreign country:

$$\frac{P_t^{N,*}}{P_{C,t}^*} = \frac{x_t^*}{[\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}]^{\frac{1}{1-e}}}$$

Relative tradables prices Home country:

$$\begin{aligned}\frac{P_t^H}{P_{C,t}^H} &= \left\{ [\varpi + (1 - \varpi) (tt_t)^{1-v}]^{\frac{1}{1-v}} [\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{\frac{1}{1-e}} \right\}^{-1} \\ \frac{P_t^H}{P_t^T} &= [\varpi + (1 - \varpi) (tt_t)^{1-v}]^{-\frac{1}{1-v}} \\ \frac{P_t^F}{P_t^T} &= \left[\varpi \left(\frac{1}{tt_t} \right)^{1-v} + (1 - \varpi) \right]^{-\frac{1}{1-v}} \\ \frac{P_t^T}{P_{C,t}^T} &= [\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{-\frac{1}{1-e}}\end{aligned}$$

Foreign country:

$$\begin{aligned}\frac{P_t^{F,*}}{P_{C,t}^{F,*}} &= \left\{ \left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{\frac{1}{1-v}} [\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e}]^{\frac{1}{1-e}} \right\}^{-1} \\ \frac{P_t^{H,*}}{P_t^{T,*}} &= [(1 - \varpi^*) + \varpi^* (tt_t)^{1-v}]^{-\frac{1}{1-v}} \\ \frac{P_t^{F,*}}{P_t^{T,*}} &= \left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{-\frac{1}{1-v}} \\ \frac{P_t^{T,*}}{P_{C,t}^{T,*}} &= [\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e}]^{-\frac{1}{1-e}}\end{aligned}$$

A.2.2 Households

Home country:

$$\varepsilon_t^c (c_t^o)^{-\sigma} c_{t-1}^{\zeta(\sigma-1)} \exp \left(\frac{(\sigma-1)}{1+\phi_l} (h_t)^{1+\phi_l} \right) = \lambda_t^o (1 + \tau^c) \quad (92)$$

$$R_t = \pi_{C,t+1} g_{z,t+1} \frac{\lambda_t^o}{\beta \lambda_{t+1}^o} \quad (93)$$

$$\begin{aligned} \left[\frac{\gamma_i + (1 - \gamma_i) (x_t)^{1-e}}{\gamma_c + (1 - \gamma_c) (x_t)^{1-e}} \right]^{\frac{1}{1-e}} &= Q_t^o \varepsilon_t^i \left\{ 1 - \gamma_I \left(g_{z,t} \frac{i_t}{i_{t-1}} - g_z \right) g_{z,t} \frac{i_t}{i_{t-1}} - \frac{\gamma_I}{2} \left(g_{z,t} \frac{i_t}{i_{t-1}} - g_z \right)^2 \right\} \\ &\quad + \frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^o}{\lambda_t^o} Q_{t+1}^o \varepsilon_{t+1}^i \beta \gamma_I \left(g_{z,t+1} \frac{i_{t+1}}{i_t} - g_z \right) \left(g_{z,t+1} \frac{i_{t+1}}{i_t} \right)^2 \end{aligned} \quad (94)$$

$$\frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^o}{\lambda_t^o} \beta \left\{ (1 - \tau^k) \left[\frac{r_{t+1}^k}{\varepsilon_t^b} u_{t+1} - \left[\frac{\gamma_i + (1 - \gamma_i) (x_{t+1})^{1-e}}{\gamma_c + (1 - \gamma_c) (x_{t+1})^{1-e}} \right]^{\frac{1}{1-e}} a(u_{t+1}) \right] \right. \\ \left. + \tau^k \delta \left[\frac{\gamma_i + (1 - \gamma_i) (x_{t+1})^{1-e}}{\gamma_c + (1 - \gamma_c) (x_{t+1})^{1-e}} \right]^{\frac{1}{1-e}} + Q_{t+1}^o (1 - \delta) \right\} = Q_t^o \quad (95)$$

$$\frac{r_t^k}{\varepsilon_{t-1}^b} = \left[\frac{\gamma_i + (1 - \gamma_i)(x_t)^{1-e}}{\gamma_c + (1 - \gamma_c)(x_t)^{1-e}} \right]^{\frac{1}{1-e}} [\gamma_{u1} + \gamma_{u2}(u_t - 1)] \quad (96)$$

$$k_{t+1} = (1 - \delta) \frac{k_t}{g_{z,t}} + \varepsilon_t^i \left[1 - \frac{\gamma_I}{2} \left(g_{z,t} \frac{i_t}{i_{t-1}} - g_z \right)^2 \right] i_t \quad (97)$$

$$(1 + \tau^c) c_t^{rt} = (1 - \tau^l - \tau^{wh}) w_t h_t + tr_t^{rt} - t_t^{rt} \quad (98)$$

$$c_t = \theta c_t^{rt} + (1 - \theta) c_t^o \quad (99)$$

$$tr_t = \theta tr_t^{rt} + (1 - \theta) tr_t^o \quad (100)$$

$$t_t = \theta t_t^{rt} + (1 - \theta) t_t^o \quad (101)$$

$$MRS_t^o = c_t^o \varepsilon_t^l (h_t)^{\phi_l} \quad (102)$$

$$MRS_t^{rt} = c_t^{rt} \varepsilon_t^l (h_t)^{\phi_l} \quad (103)$$

Foreign country:

$$\varepsilon_t^{c,*} (c_t^{o,*})^{-\sigma} (c_{t-1}^*)^{\zeta^*(\sigma^*-1)} \exp \left(\frac{(\sigma^* - 1)}{1 + \phi_l^*} (h_t^*)^{1+\phi_l^*} \right) = \lambda_t^{o,*} (1 + \tau^{c,*}) \quad (104)$$

$$\begin{aligned} \left[\frac{\gamma_i^* + (1 - \gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}} \right]^{\frac{1}{1-e}} &= Q_t^{o,*} \varepsilon_t^{i,*} \left\{ 1 - \gamma_I^* \left(g_{z,t} \frac{i_t^*}{i_{t-1}^*} - g_z \right) g_{z,t} \frac{i_t^*}{i_{t-1}^*} - \frac{\gamma_I^*}{2} \left(g_{z,t} \frac{i_t^*}{i_{t-1}^*} - g_z \right)^2 \right\} \\ &+ \frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^{o,*}}{\lambda_t^{o,*}} Q_{t+1}^{o,*} \varepsilon_{t+1}^{i,*} \beta \gamma_I^* \left(g_{z,t+1} \frac{i_{t+1}^*}{i_t^*} - g_z \right) \left(g_{z,t+1} \frac{i_{t+1}^*}{i_t^*} \right)^2 \end{aligned} \quad (105)$$

$$\frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^{o,*}}{\lambda_t^{o,*}} \beta \left\{ (1 - \tau^{k,*}) \left[r_{t+1}^{k,*} u_{t+1}^* - \left[\frac{\gamma_i^* + (1 - \gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}} \right]^{\frac{1}{1-e}} a(u_{t+1}^*) \right] \right. \\ \left. + \tau^{k,*} \delta^* \left[\frac{\gamma_i^* + (1 - \gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}} \right]^{\frac{1}{1-e}} + Q_{t+1}^{o,*} (1 - \delta^*) \right\} = Q_t^{o,*} \quad (106)$$

$$r_t^{k,*} = \left[\frac{\gamma_i^* + (1 - \gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*)(x_t^*)^{1-e}} \right]^{\frac{1}{1-e}} [\gamma_{u1}^* + \gamma_{u2}^*(u_t^* - 1)] \quad (107)$$

$$k_{t+1}^* = (1 - \delta^*) \frac{k_t^*}{g_{z,t}} + \varepsilon_t^{i,*} \left[1 - \frac{\gamma_I^*}{2} \left(g_{z,t} \frac{i_t^*}{i_{t-1}^*} - g_z \right)^2 \right] i_t^* \quad (108)$$

$$(1 + \tau^{c,*}) c_t^{rt,*} = (1 - \tau^{l,*} - \tau^{wh,*}) w_t^* h_t^* + tr_t^{rt,*} - t_t^{rt,*} \quad (109)$$

$$c_t^* = \theta^* c_t^{rt,*} + (1 - \theta^*) c_t^{o,*} \quad (110)$$

$$tr_t^* = \theta^* tr_t^{rt,*} + (1 - \theta^*) tr_t^{o,*} \quad (111)$$

$$t_t^* = \theta^* t_t^{rt,*} + (1 - \theta^*) t_t^{o,*} \quad (112)$$

$$MRS_t^{o,*} = c_t^{o,*} \varepsilon_t^{l,*} (h_t^*)^{\phi_l^*} \quad (113)$$

$$MRS_t^{rt,*} = c_t^{rt,*} \varepsilon_t^{l,*} (h_t^*)^{\phi_l^*} \quad (114)$$

A.2.3 Risk sharing condition

$$RER_t = \kappa \frac{\lambda_t^{o,*}}{\lambda_t^o} \quad (115)$$

A.2.4 Wages

$$\begin{aligned} 0 = & E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \varepsilon_t^c c_{t+s-1}^{\zeta(\sigma-1)} \exp \left(\frac{(\sigma-1)}{1+\phi_l} (h_{t+s})^{1+\phi_l} \right) \left(\frac{\pi_{C,t,t+s-1}^{\chi_w} \bar{\pi}_{t,t+s}^{1-\chi_w}}{w_{t+s} \pi_{C,t,t+s}} \right)^{-\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}} h_{t+s}^d \cdot \\ & \cdot \left\{ \begin{aligned} & \tilde{w}_t \frac{(1-\tau^l - \tau^{wh}) \pi_{C,t,t+s-1}^{\chi_w} \bar{\pi}_{t,t+s}^{1-\chi_w}}{(1+\tau^c) \pi_{C,t,t+s}} \left[(1-\theta) (c_{t+s}^o)^{-\sigma} + \theta (c_{t+s}^{rt})^{-\sigma} \right] \\ & - (1+\lambda_{t+s}^w) \left[(1-\theta) (c_{t+s}^o)^{-\sigma} MRS_{t+s}^o + \theta (c_{t+s}^{rt})^{-\sigma} MRS_{t+s}^{rt} \right] \end{aligned} \right\} \quad (116) \end{aligned}$$

$$\begin{aligned} 0 = & E_t \sum_{s=0}^{\infty} (\xi_w^* \beta)^s \varepsilon_t^{c,*} (c_{t+s-1}^*)^{\zeta(\sigma-1)} \exp \left(\frac{(\sigma^*-1)}{1+\phi_l} (h_{t+s}^*)^{1+\phi_l} \right) h_{t+s}^{d,*} \left(\frac{(\pi_{C,t,t+s-1}^*)^{\chi_w^*} \bar{\pi}_{t,t+s}^{1-\chi_w^*}}{w_{t+s}^* \pi_{C,t,t+s}^*} \right)^{-\frac{1+\lambda_{t+s}^{w,*}}{\lambda_{t+s}^{w,*}}} \\ & \cdot \left\{ \begin{aligned} & \tilde{w}_t^* \frac{(1-\tau^l - \tau^{wh}) (\pi_{C,t,t+s-1}^*)^{\chi_w^*} \bar{\pi}_{t,t+s}^{1-\chi_w^*}}{(1+\tau^c) \pi_{C,t,t+s}^*} \left[(1-\theta^*) (c_{t+s}^{o,*})^{-\sigma^*} + \theta^* (c_{t+s}^{rt,*})^{-\sigma^*} \right] \\ & - (1+\lambda_{t+s}^{w,*}) \left[(1-\theta^*) (c_{t+s}^{o,*})^{-\sigma^*} MRS_{t+s}^{o,*} + \theta^* (c_{t+s}^{rt,*})^{-\sigma^*} MRS_{t+s}^{rt,*} \right] \end{aligned} \right\} \quad (117) \end{aligned}$$

$$1 = \xi_w \left(\frac{\pi_{C,t-1}^{\chi_w} \bar{\pi}_t^{1-\chi_w}}{\pi_{C,t}} \frac{w_{t-1}}{w_t} \right)^{\frac{1}{\lambda_t^w}} + (1 - \xi_w) \left(\frac{\tilde{w}_t}{w_t} \right)^{\frac{1}{\lambda_t^w}} \quad (118)$$

$$1 = \xi_w^* \left(\frac{(\pi_{C,t-1}^*)^{\chi_w^*} \bar{\pi}_t^{1-\chi_w^*}}{\pi_{C,t}^*} \frac{w_{t-1}^*}{w_t^*} \right)^{\frac{1}{\lambda_t^{w,*}}} + (1 - \xi_w^*) \left(\frac{\tilde{w}_t^*}{w_t^*} \right)^{\frac{1}{\lambda_t^{w,*}}} \quad (119)$$

A.2.5 Production

Non-tradable goods Home country:

$$\frac{u_t k_t^N}{h_t^N g_{z,t}} = \frac{\alpha_N}{(1 - \alpha_N)} \frac{(1 + \tau_t^{wf}) w_t}{r_t^k} \quad (120)$$

$$mc_t^N = \alpha_N^{-\alpha_N} (1 - \alpha_N)^{-(1-\alpha_N)} (\varepsilon_t^{a,N})^{-1} (r_t^k)^{\alpha_N} [(1 + \tau^{wf}) w_t]^{1-\alpha_N} \quad (121)$$

$$s_{P,t}^N y_t^N = \varepsilon_t^{a,N} \left(\frac{u_t^N k_t^N}{g_{z,t}} \right)^{\alpha_N} (h_t^N)^{1-\alpha_N} - \Phi_N \quad (122)$$

$$s_{P,t}^N = \frac{1}{s} \int_0^s \left(\frac{P_t^n}{P_t^N} \right)^{-\frac{1+\lambda_t^{p,N}}{\lambda_t^{p,N}}} dn \quad (123)$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta \xi_p^N)^s \lambda_{t+s}^o y_{t+s}^N \left(\frac{\pi_{N,t,t+s-1}^{\chi_p^N} \bar{\pi}_{t,t+s}^{1-\chi_p^N}}{\pi_{N,t,t+s}} \right)^{-\frac{1+\lambda_{t+s}^{p,N}}{\lambda_{t+s}^{p,N}}} \quad (124)$$

$$\cdot \left[\tilde{p}_t^n \frac{\pi_{N,t,t+s-1}^{\chi_p^N} \bar{\pi}_{t,t+s}^{1-\chi_p^N}}{\pi_{N,t,t+s}} \frac{x_{t+s}}{[\gamma_c + (1 - \gamma_c) (x_{t+s})^{1-e}]^{\frac{1}{1-e}}} - \left(1 + \lambda_{t+s}^{p,N} \right) mc_{t+s}^N \right]$$

$$1 = (1 - \xi_p^N) (\tilde{p}_t^n)^{\frac{1}{\lambda_t^{p,N}}} + \xi_p^N \left(\frac{\pi_{N,t-1}^{\chi_p^N} \bar{\pi}_t^{1-\chi_p^N}}{\pi_{N,t}} \right)^{\frac{1}{\lambda_t^{p,N}}} \quad (125)$$

Foreign country:

$$\frac{u_t k_t^{N,*}}{h_t^{N,*} g_{z,t}} = \frac{\alpha_N^*}{(1 - \alpha_N^*)} \frac{(1 + \tau^{wf,*}) w_t^*}{r_t^{k,*}} \quad (126)$$

$$mc_t^{N,*} = (\alpha_N^*)^{-\alpha_N^*} (1 - \alpha_N^*)^{-(1-\alpha_N^*)} (\varepsilon_t^{a,N*})^{-1} (r_t^{k,*})^{\alpha_N^*} [(1 + \tau^{wf,*}) w_t^*]^{1-\alpha_N^*} \quad (127)$$

$$s_{P,t}^{N,*} y_t^{N,*} = \varepsilon_t^{a,N*} \left(\frac{u_t^{N,*} k_t^{N,*}}{g_{z,t}} \right)^{\alpha_N^*} (h_t^{N,*})^{1-\alpha_N^*} - \Phi_N^* \quad (128)$$

$$s_{P,t}^{N,*} = \frac{1}{1-s} \int_s^1 \left(\frac{P_t^{n,*}}{P_t^{N,*}} \right)^{-\frac{1+\lambda_t^{p,N*}}{\lambda_t^{p,N*}}} dn \quad (129)$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta \xi_p^{N,*})^s \lambda_{t+s}^{o,*} y_{t+s}^{N,*} \left(\frac{(\pi_{N,t,t+s-1}^{N,*})^{\chi_p^{N,*}} \bar{\pi}_{t,t+s}^{1-\chi_p^{N,*}}}{\pi_{N,t,t+s}^{N,*}} \right)^{-\frac{1+\lambda_{t+s}^{p,N*}}{\lambda_{t+s}^{p,N*}}} \quad (130)$$

$$\cdot \left[\tilde{p}_t^{n,*} \frac{(\pi_{N,t,t+s-1}^{N,*})^{\chi_p^{N,*}} \bar{\pi}_{t,t+s}^{1-\chi_p^{N,*}}}{\pi_{N,t,t+s}^{N,*}} \frac{x_{t+s}^*}{[\gamma_c^* + (1 - \gamma_c^*) (x_{t+s}^*)^{1-e}]^{\frac{1}{1-e}}} - \left(1 + \lambda_{t+s}^{p,N*} \right) mc_{t+s}^{N,*} \right]$$

$$1 = (1 - \xi_p^{N,*}) (\tilde{p}_t^{n,*})^{\frac{1}{\lambda_t^{p,N*}}} + \xi_p^{N,*} \left(\frac{(\pi_{N,t-1}^*)^{\chi_p^{N,*}} \bar{\pi}_t^{1-\chi_p^{N,*}}}{\pi_{N,t}^*} \right)^{\frac{1}{\lambda_t^{p,N*}}} \quad (131)$$

Tradable goods Home country:

$$\frac{u_t k_t^H}{h_t^H g_{z,t}} = \frac{\alpha_H}{(1 - \alpha_H)} \frac{(1 + \tau^{wf}) w_t}{r_t^k} \quad (132)$$

$$mc_t^H = \alpha_H^{-\alpha_H} (1 - \alpha_H)^{-(1-\alpha_H)} (\varepsilon_t^{a,H})^{-1} (r_t^k)^{\alpha_H} [(1 + \tau^{wf}) w_t]^{1-\alpha_H} \quad (133)$$

$$s_{P,t}^H y_t^H = \varepsilon_t^{a,H} \left(\frac{u_t^H k_t^H}{g_{z,t}} \right)^{\alpha_H} (h_t^H)^{1-\alpha_H} - \Phi_H \quad (134)$$

$$s_{P,t}^H = \frac{1}{s} \int_0^s \left(\frac{P_t^h}{P_t^H} \right)^{-\frac{1+\lambda_t^{p,H}}{\lambda_t^{p,H}}} dh \quad (135)$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta \xi_p^H)^s \lambda_{t,t+s}^o y_{t+s}^H \left(\frac{\pi_{H,t,t+s-1}^{\chi_p^H} \bar{\pi}_{t,t+s}^{1-\chi_p^H}}{\pi_{H,t,t+s}} \right)^{-\frac{1+\lambda_{t,t+s}^{p,H}}{\lambda_{t,t+s}^{p,H}}} \cdot \left[\tilde{p}_t^h \frac{\pi_{H,t,t+s-1}^{\chi_p^H} \bar{\pi}_{t,t+s}^{1-\chi_p^H}}{\pi_{H,t,t+s}} \left\{ [\varpi + (1 - \varpi) (tt_{t+s})^{1-v}]^{\frac{1}{1-v}} [\gamma_c + (1 - \gamma_c) (x_{t+s})^{1-e}]^{\frac{1}{1-e}} \right\}^{-1} - (1 + \lambda_{t,t+s}^{p,H}) mc_{t+s}^H \right] \quad (136)$$

$$1 = (1 - \xi_p^H) (\tilde{p}_t^h)^{\frac{1}{\lambda_t^{p,H}}} + \xi_p^H \left(\frac{\pi_{H,t-1}^{\chi_p^H} \bar{\pi}_t^{1-\chi_p^H}}{\pi_{H,t}} \right)^{\frac{1}{\lambda_t^{p,H}}} \quad (137)$$

Foreign country:

$$\frac{u_t k_t^F}{h_t^F g_{z,t}} = \frac{\alpha_F}{(1 - \alpha_F)} \frac{(1 + \tau^{wf,*}) w_t^*}{r_t^{k,*}} \quad (138)$$

$$mc_t^F = \alpha_F^{-\alpha_F} (1 - \alpha_F)^{-(1-\alpha_F)} (\varepsilon_t^{a,F})^{-1} (r_t^{k,*})^{\alpha_F} [(1 + \tau^{wf,*}) w_t^*]^{1-\alpha_F} \quad (139)$$

$$s_{P,t}^F y_t^F = \varepsilon_t^{a,F} \left(\frac{u_t^F k_t^F}{g_{z,t}} \right)^{\alpha_F} (h_t^F)^{1-\alpha_F} - \Phi_F \quad (140)$$

$$s_{P,t}^F = \frac{1}{1-s} \int_s^1 \left(\frac{P_t^f}{P_t^F} \right)^{-\frac{1+\lambda_t^{p,F}}{\lambda_t^{p,F}}} df \quad (141)$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta \xi_p^F)^s \lambda_{t,t+s}^{o,*} y_{t+s}^F \left(\frac{\pi \chi_p^F}{\pi_{F,t,t+s-1} \bar{\pi}_{t,t+s}^{1-\chi_p^F}} \right)^{-\frac{1+\lambda_{t,t+s}^{p,F}}{\lambda_{t,t+s}^{p,F}}} \quad (142)$$

$$\cdot \left[\tilde{p}_t^f \frac{\pi \chi_p^F}{\pi_{F,t,t+s}} \left\{ \left[(1 - \varpi^*) \left(\frac{1}{tt_{t+s}} \right)^{1-v} + \varpi^* \right]^{\frac{1}{1-v}} \left[\gamma_c^* + (1 - \gamma_c^*) (x_{t+s}^*)^{1-e} \right]^{\frac{1}{1-e}} \right\}^{-1} \right. \\ \left. - \left(1 + \lambda_{t,t+s}^{p,F} \right) m c_{t+s}^f \right]$$

$$1 = (1 - \xi_p^F) \left(\tilde{p}_t^f \right)^{\frac{1}{\lambda_{t,t+s}^{p,F}}} + \xi_p^F \left(\frac{\pi \chi_p^F}{\pi_{F,t,t+s-1} \bar{\pi}_{t,t+s}^{1-\chi_p^F}} \right)^{\frac{1}{\lambda_{t,t+s}^{p,F}}} \quad (143)$$

A.2.6 Demand functions

Home country:

$$c_t^N = (1 - \gamma_c) (x_t)^{-e} [\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{\frac{e}{1-e}} c_t \quad (144)$$

$$c_t^H = \varpi \gamma_c [\varpi + (1 - \varpi) (tt_t)^{1-v}]^{\frac{v}{1-v}} [\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{\frac{e}{1-e}} c_t \quad (145)$$

$$c_t^F = (1 - \varpi) \gamma_c \left[\varpi \left(\frac{1}{tt_t} \right)^{1-v} + (1 - \varpi) \right]^{\frac{v}{1-v}} [\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{\frac{e}{1-e}} c_t \quad (146)$$

$$q_t^{I,N} = (1 - \gamma_i) (x_t)^{-e} [\gamma_i + (1 - \gamma_i) (x_t)^{1-e}]^{\frac{e}{1-e}} q_t^I \quad (147)$$

$$q_t^{I,H} = \varpi \gamma_i [\varpi + (1 - \varpi) (tt_t)^{1-v}]^{\frac{v}{1-v}} [\gamma_i + (1 - \gamma_i) (x_t)^{1-e}]^{\frac{e}{1-e}} q_t^I \quad (148)$$

$$q_t^{I,F} = (1 - \varpi) \gamma_i \left[\varpi \left(\frac{1}{tt_t} \right)^{1-v} + (1 - \varpi) \right]^{\frac{v}{1-v}} [\gamma_i + (1 - \gamma_i) (x_t)^{1-e}]^{\frac{e}{1-e}} q_t^I \quad (149)$$

Foreign country:

$$c_t^{N*} = (1 - \gamma_c^*) (x_t^*)^{-e} [\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} c_t^* \quad (150)$$

$$c_t^{H*} = (1 - \varpi^*) \gamma_c^* [(1 - \varpi^*) + \varpi^* (tt_t)^{1-v}]^{\frac{v}{1-v}} [\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} c_t^* \quad (151)$$

$$c_t^{F*} = \varpi^* \gamma_c^* \left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{\frac{v}{1-v}} [\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} c_t^* \quad (152)$$

$$q_t^{I,N*} = (1 - \gamma_i^*) (x_t^*)^{-e} [\gamma_i^* + (1 - \gamma_i^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} q_t^{I,*} \quad (153)$$

$$q_t^{I,H*} = (1 - \varpi^*) \gamma_i^* [(1 - \varpi^*) + \varpi^* (tt_t)^{1-v}]^{\frac{v}{1-v}} [\gamma_i^* + (1 - \gamma_i^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} q_t^{I,*} \quad (154)$$

$$q_t^{I,F*} = \varpi^* \gamma_i^* \left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{\frac{v}{1-v}} [\gamma_i^* + (1 - \gamma_i^*) (x_t^*)^{1-e}]^{\frac{e}{1-e}} q_t^{I,*} \quad (155)$$

A.2.7 Relative price of non tradable goods

$$\frac{x_t}{x_{t-1}} = \frac{\pi_t^N}{\pi_t^T} \quad (156)$$

$$\frac{x_t^*}{x_{t-1}^*} = \frac{\pi_t^{N,*}}{\pi_t^{T,*}} \quad (157)$$

A.2.8 Tradables inflation

Home country:

$$\pi_t^T = [\varpi + (1 - \varpi) (tt_t)^{1-v}]^{\frac{1}{1-v}} [\varpi + (1 - \varpi) (tt_{t-1})^{1-v}]^{-\frac{1}{1-v}} \pi_t^H \quad (158)$$

Foreign country:

$$\pi_t^T = \left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{\frac{1}{1-v}} \left[(1 - \varpi^*) \left(\frac{1}{tt_{t-1}} \right)^{1-v} + \varpi^* \right]^{-\frac{1}{1-v}} \pi_t^F \quad (159)$$

A.2.9 Market clearing

Home country:

$$y_t^N = c_t^N + q_t^{I,N} + g_t \quad (160)$$

$$y_t^H = c_t^H + \frac{1-s}{s} c_t^{H*} + q_t^{I,H} + \frac{1-s}{s} q_t^{I,H*} \quad (161)$$

$$y_t = \frac{[\varpi + (1 - \varpi) (tt_t)^{1-v}]^{-\frac{1}{1-v}} y_t^H + x_t y_t^N}{[\gamma_c + (1 - \gamma_c) (x_t)^{1-e}]^{\frac{1}{1-e}}} \quad (162)$$

$$k_t = k_t^N + k_t^H \quad (163)$$

$$h_t^d = h_t^N + h_t^H \quad (164)$$

$$h_t = s_{W,t} h_t^d \quad (165)$$

$$s_{W,t} = \frac{1}{s} \int_0^s \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} dj \quad (166)$$

$$q_t^I = i_t + a(u_t) \frac{k_t}{g_{z,t}} \quad (167)$$

Foreign country:

$$y_t^{N*} = c_t^{N*} + q_t^{I,N,*} + g_t^* \quad (168)$$

$$y_t^F = \frac{S}{1-s} c_t^F + c_t^{F*} + \frac{S}{1-s} q_t^{I,F} + q_t^{I,F*} \quad (169)$$

$$y_t^* = \frac{\left[(1 - \varpi^*) \left(\frac{1}{tt_t} \right)^{1-v} + \varpi^* \right]^{-\frac{1}{1-v}} y_t^F + x_t^* y_t^{N*}}{\left[\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e} \right]^{\frac{1}{1-e}}} \quad (170)$$

$$k_t^* = k_t^{N,*} + k_t^F \quad (171)$$

$$h_t^{d,*} = h_t^{N,*} + h_t^F \quad (172)$$

$$h_t^* = s_{W,t}^* h_t^{d,*} \quad (173)$$

$$s_{W,t}^* = \frac{1}{1-s} \int_s^1 \left(\frac{W_t^{j,*}}{W_t^*} \right)^{-\frac{1+\lambda_t^{w*}}{\lambda_t^{w*}}} dj \quad (174)$$

$$q_t^{I,*} = i_t^* + a(u_t^*) \frac{k_t^*}{g_{z,t}} \quad (175)$$

A.2.10 CPI Inflation

Home country:

$$\pi_{C,t} = \left[\frac{\gamma_c (\pi_t^T)^{1-e} + (1 - \gamma_c) (\pi_t^N)^{1-e} (x_{t-1})^{1-e}}{\gamma_c + (1 - \gamma_c) (x_{t-1})^{1-e}} \right]^{\frac{1}{1-e}} \quad (176)$$

Foreign country:

$$\pi_{C,t}^* = \left[\frac{\gamma_c^* (\pi_t^{T,*})^{1-e} + (1 - \gamma_c^*) (\pi_t^{N,*})^{1-e} (x_{t-1}^*)^{1-e}}{\gamma_c^* + (1 - \gamma_c^*) (x_{t-1}^*)^{1-e}} \right]^{\frac{1}{1-e}} \quad (177)$$

A.2.11 Real exchange rate

$$RER_t = \frac{\left[\gamma_c^* + (1 - \gamma_c^*) (x_t^*)^{1-e} \right]^{\frac{1}{1-e}} \left[(1 - \varpi^*) + \varpi^* (tt_t)^{1-v} \right]^{\frac{1}{1-v}}}{\left[\gamma_c + (1 - \gamma_c) (x_t)^{1-e} \right]^{\frac{1}{1-e}} \left[\varpi + (1 - \varpi) (tt_t)^{1-v} \right]^{\frac{1}{1-v}}} \quad (178)$$

$$\frac{RER_t}{RER_{t-1}} = \frac{\frac{P_{C,t}^*}{P_{C,t}}}{\frac{P_{C,t-1}^*}{P_{C,t-1}}} = \frac{\pi_{C,t}^*}{\pi_{C,t}} \quad (179)$$

A.2.12 Resource constraints

$$\begin{aligned}
y_t &= c_t + \frac{P_{I,t}}{P_{C,t}} q_t^I + \frac{P_t^N}{P_{C,t}} g_t + \frac{P_t^H}{P_{C,t}} \frac{1-s}{s} \left(c_t^{H,*} + q_t^{I,H,*} \right) - \frac{P_t^F}{P_{C,t}} \left(c_t^F + q_t^{I,F} \right) \\
&= c_t + \frac{P_{I,t}}{P_{C,t}} q_t^I + \frac{P_t^N}{P_{C,t}} g_t + \frac{P_t^H}{P_{C,t}} \frac{1-s}{s} ex_t - \frac{P_t^F}{P_{C,t}} im_t
\end{aligned}$$

where ex_t stands for exports and im_t for imports.

Similarly, for the foreign country we obtain:

$$\begin{aligned}
y_t^* &= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} \left(c_t^F + q_t^{I,F} \right) - \frac{P_t^H}{P_{C,t}^*} \left(c_t^{H,*} + q_t^{I,H,*} \right) \\
&= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} ex_t^* - \frac{P_t^H}{P_{C,t}^*} im_t^* \\
&= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} im_t - \frac{P_t^H}{P_{C,t}^*} ex_t
\end{aligned}$$

where ex_t^* stands for the foreign country exports and im_t^* for the foreign country imports and $ex_t^* = im_t$, $im_t^* = ex_t$.

Definition of exports and imports

$$ex_t = c_t^{H,*} + q_t^{I,H,*}$$

$$im_t = c_t^F + q_t^{I,F}$$

A.3 Steady state

We assume that exogenous shocks are equal to one in steady state. For utilization, $u = 1$ so that $a(u) = 0$.

We make some simplifying assumptions, which enable us to find the steady state analytically.

In each country, we impose the same price markup ($\lambda^{p,H} = \lambda^{p,N} = \lambda_p$, $\lambda^{p,F} = \lambda^{p,N,*} = \lambda_p^*$) and the same shares of capital in production ($\alpha_H = \alpha_N$, $\alpha_F = \alpha_{N,*}$) in both sectors. This implies that $P^H = P^N$ and $P^F = P^{N,*}$ in steady state. We set the fixed costs so that steady state profits are zero, which implies also that $\frac{y^{N+\Phi_N}}{y^N} = \frac{y^{H+\Phi_H}}{y^H} = 1 + \lambda_p$.

Moreover, we impose that in steady state quantities of exports and imports correspond in steady state, so that home tradable prices are equal to foreign tradable prices ($P^H = P^F$) and finally steady state net exports are equal to zero. Thus:

$$\frac{\frac{1-s}{s} (c_t^{H,*} + q_t^{I,H,*})}{(c_t^F + q_t^{I,F})} = \frac{P^F}{P^H} = 1$$

This in turn implies that $P^F = P^H = P^N = P^{N,*} = P^T = P^{T,*} = P_C = P_C^* = P_I = P_I^*$, thus all relative prices are equal to 1 in steady state. Thus, also tt_t , x_t , x_t^* are 1 in steady state.

A.4 Log-linearized equations

A.4.1 Households

Home country:

$$\begin{aligned}\hat{c}_t^o &= \hat{c}_{t+1}^o + \frac{(1-\sigma)\zeta}{\sigma} (\hat{c}_t - \hat{c}_{t-1}) - \frac{1}{\sigma} \left(\hat{R}_t - \hat{\pi}_{C,t+1} - \hat{g}_{z,t+1} + \hat{\varepsilon}_{t+1}^c - \hat{\varepsilon}_t^c \right) \\ &\quad + \frac{(1-\sigma)h^{1+\phi_l}}{\sigma} (\hat{h}_{t+1} - \hat{h}_t)\end{aligned}\tag{180}$$

$$\begin{aligned}\hat{i}_t &= \frac{1}{\gamma_I g_z^2 (1+\beta)} \left(\hat{Q}_t^o + \hat{\varepsilon}_t^i \right) - \frac{1}{1+\beta} \hat{g}_{z,t} + \frac{1}{1+\beta} \hat{i}_{t-1} \\ &\quad + \frac{\beta}{1+\beta} \hat{i}_{t+1} + \frac{\beta}{1+\beta} \hat{g}_{z,t+1} - \frac{\gamma_c - \gamma_i}{\gamma_I g_z^2 (1+\beta)} \hat{x}_t\end{aligned}\tag{181}$$

$$\hat{Q}_t^o = -\hat{R}_t + \hat{\pi}_{t+1} + \frac{\beta}{g_z} (1 - \tau^k) \gamma_{u1} \hat{r}_{t+1}^k + \frac{\beta}{g_z} (1 - \delta) \hat{Q}_{t+1}^o + \frac{\beta}{g_z} \tau^k \delta (\gamma_c - \gamma_i) \hat{x}_{t+1}\tag{182}$$

$$\hat{r}_t^k = \frac{\sigma_u}{1 - \sigma_u} \hat{u}_t + (\gamma_c - \gamma_i) \hat{x}_t\tag{183}$$

where $\frac{\sigma_u}{1 - \sigma_u} \equiv \frac{\gamma_{u2}}{\gamma_{u1}} = \frac{a''(u)}{a'(u)}$.

$$\hat{k}_{t+1} = \frac{(1-\delta)}{g_z} \hat{k}_t + \frac{i}{k} \hat{i}_t - \frac{(1-\delta)}{g_z} \hat{g}_{z,t} + \frac{i}{k} \hat{\varepsilon}_t^i\tag{184}$$

$$(1 + \tau^c) \frac{c^{rt}}{c} \hat{c}_t^{rt} = (1 - \tau^l - \tau^{wh}) \frac{wh}{c} (\hat{w}_t + \hat{h}_t) + \frac{y}{c} \hat{tr}_t^{rt} - \frac{y}{c} \hat{t}_t^{rt}\tag{185}$$

$$\hat{c}_t = \theta \frac{c^{rt}}{c} \hat{c}_t^{rt} + (1 - \theta) \frac{c^o}{c} \hat{c}_t^o\tag{186}$$

$$\hat{tr}_t = \theta \hat{tr}_t^{rt} + (1 - \theta) \hat{tr}_t^o\tag{187}$$

$$\hat{t}_t = \theta \hat{t}_t^{rt} + (1 - \theta) \hat{t}_t^o\tag{188}$$

$$\widehat{MRS}_t^o = \hat{c}_t^o + \phi_l \hat{h}_t + \hat{\varepsilon}_t^l\tag{189}$$

$$\widehat{MRS}_t^{rt} = \hat{c}_t^{rt} + \phi_l \hat{h}_t + \hat{\varepsilon}_t^l\tag{190}$$

Foreign country:

$$\begin{aligned}\hat{i}_t^* &= \frac{1}{\gamma_I^* g_z^2 (1+\beta)} \left(\hat{Q}_t^{o,*} + \hat{\varepsilon}_t^{i,*} \right) - \frac{1}{1+\beta} \hat{g}_{z,t} + \frac{1}{1+\beta} \hat{i}_{t-1}^* \\ &\quad + \frac{\beta}{1+\beta} \hat{i}_{t+1}^* + \frac{\beta}{1+\beta} \hat{g}_{z,t+1} - \frac{\gamma_c^* - \gamma_i^*}{\gamma_I^* g_z^2 (1+\beta)} \hat{x}_t\end{aligned}\tag{191}$$

$$-\hat{R}_t + \hat{\pi}_{C,t+1}^* + \frac{\beta}{g_z} (1 - \tau^{k,*}) r^{k,*} \hat{r}_{t+1}^{k,*} + \frac{\beta}{g_z} (1 - \delta^*) \hat{Q}_{t+1}^{o,*} + \frac{\beta}{g_z} \tau^k \delta (\gamma_c^* - \gamma_i^*) \hat{x}_{t+1}^* = \hat{Q}_t^{o,*} \quad (192)$$

$$\hat{r}_t^{k,*} = \frac{\sigma_u^*}{1 - \sigma_u^*} \hat{u}_t^* + (\gamma_c^* - \gamma_i^*) \hat{x}_t \quad (193)$$

where $\frac{\sigma_u^*}{1 - \sigma_u^*} \equiv \frac{\gamma_{u2}^*}{\gamma_{u1}^*} = \frac{a''(u^*)}{a'(u^*)}$.

$$\hat{k}_{t+1}^* = \frac{(1 - \delta^*)}{g_z} \hat{k}_t^* + \frac{i^*}{k^*} \hat{i}_t^* - \frac{(1 - \delta^*)}{g_z} \hat{g}_{z,t} + \frac{i^*}{k^*} \hat{\varepsilon}_t^{i,*} \quad (194)$$

$$(1 + \tau^{c,*}) \frac{c^{rt,*}}{c^*} \hat{c}_t^{rt,*} = (1 - \tau^{l,*} - \tau^{wh,*}) \frac{w^* h^*}{c^*} (\hat{w}_t^* + \hat{h}_t^*) + \frac{y^*}{c^*} \hat{tr}_t^{rt,*} - \frac{y^*}{c^*} \hat{t}_t^{rt,*} \quad (195)$$

$$\hat{c}_t^* = \theta^* \frac{c^{rt,*}}{c^*} \hat{c}_t^{rt,*} + (1 - \theta^*) \frac{c^{o,*}}{c^*} \hat{c}_t^{o,*} \quad (196)$$

$$\hat{tr}_t^* = \theta^* \hat{tr}_t^{rt,*} + (1 - \theta^*) \hat{tr}_t^{o,*} \quad (197)$$

$$\hat{t}_t^* = \theta^* \hat{t}_t^{rt,*} + (1 - \theta^*) \hat{t}_t^{o,*} \quad (198)$$

$$\widehat{MRS}_t^{o,*} = \hat{c}_t^{o,*} + \phi_l^* \hat{h}_t^* + \hat{\varepsilon}_t^* \quad (199)$$

$$\widehat{MRS}_t^{rt,*} = \hat{c}_t^{rt,*} + \phi_l^* \hat{h}_t^* + \hat{\varepsilon}_t^{l,*} \quad (200)$$

A.4.2 Risk sharing condition

$$\begin{aligned} \widehat{RER}_t &= \hat{\varepsilon}_t^{c,*} - \hat{\varepsilon}_t^c + \sigma \hat{c}_t^o - \sigma^* \hat{c}_t^{o,*} + (1 - \sigma) \zeta \hat{c}_{t-1} - (1 - \sigma^*) \zeta^* \hat{c}_{t-1}^* \\ &\quad + (1 - \sigma) h^{1+\phi_l} \hat{h}_t - (1 - \sigma^*) (h^*)^{1+\phi_l^*} \hat{h}_t^* \end{aligned} \quad (201)$$

A.4.3 Wages

Home country:

$$\begin{aligned} \hat{w}_t &= -\frac{(1 - \xi_w)(1 - \xi_w \beta)}{(1 + \beta) \xi_w} \hat{w}_t + \frac{(1 - \xi_w)(1 - \xi_w \beta)}{(1 + \beta) \xi_w} \frac{\lambda^w}{1 + \lambda^w} \hat{\lambda}_t^w \\ &\quad + \frac{(1 - \xi_w)(1 - \xi_w \beta)}{(1 + \beta) \xi_w (\omega + 1)} \left\{ \left[\frac{\sigma \varrho \left(\frac{c^{rt}}{c^o} - 1 \right)}{(\varrho + 1)} + 1 \right] \widehat{MRS}_t^o + \left[\omega - \frac{\sigma \varrho \left(\frac{c^{rt}}{c^o} - 1 \right)}{(\varrho + 1)} \right] \widehat{MRS}_t^{rt} \right\} \\ &\quad + \frac{(1 - \chi_w)}{1 + \beta} \hat{\pi}_t - \frac{\beta}{1 + \beta} (1 - \chi_w) \hat{\pi}_{t+1} + \frac{\beta}{1 + \beta} \hat{w}_{t+1} + \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\chi_w}{1 + \beta} \hat{\pi}_{C,t-1} \\ &\quad - \frac{(1 + \beta \chi_w)}{1 + \beta} \hat{\pi}_{C,t} + \frac{\beta}{1 + \beta} \hat{\pi}_{C,t+1} \end{aligned} \quad (202)$$

$$\begin{aligned}
\hat{w}_t^* = & -\frac{(1-\xi_w^*)(1-\xi_w^*\beta)}{(1+\beta)\xi_w^*}\hat{w}_t^* + \frac{(1-\xi_w^*)(1-\xi_w^*\beta)}{(1+\beta)\xi_w^*}\frac{\lambda_w^*}{1+\lambda_w^*}\hat{\lambda}_t^{w,*} \\
& + \frac{(1-\xi_w^*)(1-\xi_w^*\beta)}{(1+\beta)\xi_w^*(\omega^*+1)}\left\{\left[\frac{\sigma^*\varrho^*\left(\frac{c^{rt,*}}{c^{o,*}}-1\right)}{(\varrho^*+1)}+1\right]\widehat{MRS}_t^{o,*}+\left[\omega^*-\frac{\sigma^*\varrho^*\left(\frac{c^{rt,*}}{c^{o,*}}-1\right)}{(\varrho^*+1)}\right]\widehat{MRS}_t^{rt,*}\right\} \\
& + \frac{(1-\chi_w^*)}{1+\beta}\hat{\pi}_t - \frac{\beta}{1+\beta}(1-\chi_w^*)\hat{\pi}_{t+1} + \frac{\beta}{1+\beta}\hat{w}_{t+1}^* + \frac{1}{1+\beta}\hat{w}_{t-1}^* + \frac{\chi_w^*}{1+\beta}\hat{\pi}_{C,t-1}^* \\
& - \frac{(1+\beta\chi_w^*)}{1+\beta}\hat{\pi}_{C,t}^* + \frac{\beta}{1+\beta}\hat{\pi}_{C,t+1}^*
\end{aligned} \tag{203}$$

where $\varrho = \frac{\theta}{1-\theta}\left(\frac{c^{rt}}{c^o}\right)^{-\sigma}$ and $\omega = \frac{\theta}{1-\theta}\left(\frac{c^{rt}}{c^o}\right)^{1-\sigma} = \varrho\frac{c^{rt}}{c^o}$ and, symmetrically, $\varrho^* = \frac{\theta^*}{1-\theta^*}\left(\frac{c^{rt,*}}{c^{o,*}}\right)^{-\sigma^*}$ and $\omega^* = \frac{\theta^*}{1-\theta^*}\left(\frac{c^{rt,*}}{c^{o,*}}\right)^{1-\sigma^*} = \varrho^*\frac{c^{rt,*}}{c^{o,*}}$.

A.4.4 Production

Capital labor ratio Home country:

$$\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t^k \tag{204}$$

Foreign country:

$$\hat{u}_t^* + \hat{k}_t^* - \hat{h}_t^* - \hat{g}_{z,t} = \hat{w}_t^* - \hat{r}_t^{k,*} \tag{205}$$

Labor input Home country:

$$\hat{h}_t = \alpha(\hat{r}_t^k - \hat{w}_t) + \frac{y^N}{y}\left(\frac{1}{\frac{y^{N,*}+\Phi_N}{y^N}}\hat{y}_t^N - \hat{\varepsilon}_t^{a,N}\right) + \frac{y^H}{y}\left(\frac{1}{\frac{y^{N,*}+\Phi_N}{y^N}}\hat{y}_t^H - \hat{\varepsilon}_t^{a,H}\right) \tag{206}$$

Foreign country:

$$\hat{h}_t^* = \alpha^*(\hat{r}_t^{k,*} - \hat{w}_t^*) + \frac{y^{N,*}}{y^*}\left(\frac{1}{\frac{y^{N,*}+\Phi_N^*}{y^{N,*}}}\hat{y}_t^{N,*} - \hat{\varepsilon}_t^{a,N,*}\right) + \frac{y^F}{y^*}\left(\frac{1}{\frac{y^{N,*}+\Phi_N^*}{y^{N,*}}}\hat{y}_t^F - \hat{\varepsilon}_t^{a,F}\right) \tag{207}$$

Non-tradables Home country:

$$\widehat{mc}_t^N = -\hat{\varepsilon}_t^{a,N} + \alpha_N\hat{r}_t^k + (1-\alpha_N)\hat{w}_t \tag{208}$$

$$\begin{aligned}
(1+\beta\chi_p^N)\hat{\pi}_{N,t} = & \frac{(1-\xi_p^N)(1-\beta\xi_p^N)}{\xi_p^N}\left(\widehat{mc}_t^N - \gamma_c\hat{x}_t + \frac{\lambda^{p,N}}{1+\lambda^{p,N}}\hat{\lambda}_t^{p,N}\right) \\
& + \chi_p^N\hat{\pi}_{N,t-1} + \beta\hat{\pi}_{N,t+1} + (1-\chi_p^N)\hat{\pi}_t - \beta(1-\chi_p^N)\hat{\pi}_{t+1}
\end{aligned} \tag{209}$$

Foreign country:

$$\widehat{mc}_t^{N,*} = -\widehat{\varepsilon}_t^{a,N*} + \alpha_N^* \widehat{r}_t^{k,*} + (1 - \alpha_N^*) \widehat{w}_t^* \quad (210)$$

$$\begin{aligned} (1 + \beta \chi_p^{N,*}) \widehat{\pi}_{N,t}^* &= \frac{(1 - \xi_p^{N,*})(1 - \beta \xi_p^{N,*})}{\xi_p^{N,*}} \left(\widehat{mc}_t^{N,*} + \frac{\lambda^{p,N*}}{1 + \lambda^{p,N*}} \widehat{\lambda}_t^{p,N*} - \gamma_c^* \widehat{x}_t^* \right) \\ &+ \chi_p^{N,*} \widehat{\pi}_{N,t-1}^* + \beta \widehat{\pi}_{N,t+1}^* + (1 - \chi_p^{N,*}) \widehat{\pi}_t - \beta (1 - \chi_p^{N,*}) \widehat{\pi}_{t+1} \end{aligned} \quad (211)$$

Tradables Home country:

$$\widehat{mc}_t^H = -\widehat{\varepsilon}_t^{a,H} + \alpha_H \widehat{r}_t^k + (1 - \alpha_H) \widehat{w}_t \quad (212)$$

$$\begin{aligned} (1 + \beta \chi_p^H) \widehat{\pi}_{H,t} &= \frac{(1 - \xi_p^H)(1 - \beta \xi_p^H)}{\xi_p^H} \left(\widehat{mc}_t^H + \frac{\lambda^{p,H}}{1 + \lambda^{p,H}} \widehat{\lambda}_t^{p,H} \right. \\ &\quad \left. + (1 - \varpi) \widehat{tt}_t + (1 - \gamma_c) \widehat{x}_t \right) \\ &+ \chi_p^H \widehat{\pi}_{H,t-1} + \beta \widehat{\pi}_{H,t+1} + (1 - \chi_p^H) \widehat{\pi}_t - \beta (1 - \chi_p^H) \widehat{\pi}_{t+1} \end{aligned} \quad (213)$$

Foreign country:

$$\widehat{mc}_t^F = -\widehat{\varepsilon}_t^{a,F} + \alpha_F \widehat{r}_t^{k,*} + (1 - \alpha_F) \widehat{w}_t^* \quad (214)$$

$$\begin{aligned} (1 + \beta \chi_p^F) \widehat{\pi}_{F,t} &= \frac{(1 - \xi_p^F)(1 - \beta \xi_p^F)}{\xi_p^F} \left(\widehat{mc}_t^F + \frac{\lambda^{p,F}}{1 + \lambda^{p,F}} \widehat{\lambda}_t^{p,F} \right. \\ &\quad \left. + (1 - \varpi^*) \widehat{tt}_t + (1 - \gamma_c^*) \widehat{x}_t^* \right) \\ &+ \chi_p^F \widehat{\pi}_{F,t-1} + \beta \widehat{\pi}_{F,t+1} + (1 - \chi_p^F) \widehat{\pi}_t - \beta (1 - \chi_p^F) \widehat{\pi}_{t+1} \end{aligned} \quad (215)$$

Demand functions Home country:

$$\widehat{c}_t^N = -e \gamma_c \widehat{x}_t + \widehat{c}_t \quad (216)$$

$$\widehat{c}_t^H = v(1 - \varpi) \widehat{tt}_t + \widehat{c}_t + e(1 - \gamma_c) \widehat{x}_t \quad (217)$$

$$\widehat{c}_t^F = -v \varpi \widehat{tt}_t + \widehat{c}_t + e(1 - \gamma_c) \widehat{x}_t \quad (218)$$

$$\widehat{q}_t^{I,N} = -e \gamma_i \widehat{x}_t + \widehat{q}_t^I \quad (219)$$

$$\widehat{q}_t^{I,H} = v(1 - \varpi) \widehat{tt}_t + \widehat{q}_t^I + e(1 - \gamma_i) \widehat{x}_t \quad (220)$$

$$\widehat{q}_t^{I,F} = -v \varpi \widehat{tt}_t + \widehat{q}_t^I + e(1 - \gamma_i) \widehat{x}_t \quad (221)$$

Foreign country:

$$\widehat{c}_t^{N,*} = -e \gamma_c^* \widehat{x}_t^* + \widehat{c}_t^* \quad (222)$$

$$\hat{c}_t^{H,*} = v\varpi^* \hat{t}t_t + e(1 - \gamma_c^*) \hat{x}_t^* + \hat{c}_t^* \quad (223)$$

$$\hat{c}_t^{F,*} = -v(1 - \varpi^*) \hat{t}t_t + e(1 - \gamma_c^*) \hat{x}_t^* + \hat{c}_t^* \quad (224)$$

$$\hat{q}_t^{I,N,*} = -e\gamma_i^* \hat{x}_t^* + \hat{q}_t^{I,*} \quad (225)$$

$$\hat{q}_t^{I,H,*} = v\varpi^* \hat{t}t_t + \hat{q}_t^{I,*} + e(1 - \gamma_i^*) \hat{x}_t^* \quad (226)$$

$$\hat{q}_t^{I,F,*} = -v(1 - \varpi^*) \hat{t}t_t + \hat{q}_t^{I,*} + e(1 - \gamma_i^*) \hat{x}_t^* \quad (227)$$

A.4.5 Fiscal policy

$$\begin{aligned} & \frac{g}{y} p^N \hat{p}_t^N + p^N \hat{g}_t + \frac{b}{y} \frac{R}{\pi_C g_z} \hat{R}_{t-1} + \frac{R}{\pi_C g_z} \hat{b}_t - \frac{b}{y} \frac{R}{\pi_C g_z} \hat{g}_{z,t} - \frac{b}{y} \frac{R}{\pi_C g_z} \hat{\pi}_{C,t} + \hat{t}r_t \\ = & \hat{b}_{t+1} + \hat{t}_t + \frac{c}{y} \tau^c \hat{c}_t + \frac{wh}{c} \frac{c}{y} (\tau^l + \tau^{wh} + \tau^{wf}) (\hat{w}_t + \hat{h}_t) \\ & + \frac{k}{y} \frac{\tau^k}{g_z} \left[\gamma_{u1} \hat{r}_t^k + (\gamma_{u1} - \delta) (\hat{k}_t - \hat{g}_{z,t}) - \delta (\gamma_c - \gamma_i) \hat{x}_t \right] \end{aligned} \quad (228)$$

$$\begin{aligned} & \frac{g^*}{y^*} p^{N,*} \hat{p}_t^{N,*} + p^{N,*} \hat{g}_t^* + \frac{b^*}{y^*} \frac{R}{\pi_C^* g_z} \hat{R}_{t-1} + \frac{R}{\pi_C^* g_z} \hat{b}_t^* - \frac{b^*}{y^*} \frac{R}{\pi_C^* g_z} \hat{g}_{z,t} - \frac{b^*}{y^*} \frac{R}{\pi_C^* g_z} \hat{\pi}_{C,t}^* + \hat{t}r_t^* \\ = & \hat{b}_{t+1}^* + \hat{t}_t^* + \frac{c^*}{y^*} \tau^{c,*} \hat{c}_t^* + \frac{w^* h^*}{c^*} \frac{c^*}{y^*} (\tau^{l,*} + \tau^{wh,*} + \tau^{wf,*}) (\hat{w}_t^* + \hat{h}_t^*) \\ & + \frac{k^*}{y^*} \frac{\tau^{k,*}}{g_z} \left[\gamma_{u1}^* \hat{r}_t^{k,*} + (\gamma_{u1}^* - \delta^*) (\hat{k}_t^* - \hat{g}_{z,t}^*) - \delta^* (\gamma_c^* - \gamma_i^*) \hat{x}_t^* \right] \end{aligned} \quad (229)$$

A.4.6 Market clearing

Home country

$$\hat{y}_t^N = \frac{c^N}{y^N} \hat{c}_t^N + \frac{q^{I,N}}{y^N} \hat{q}_t^{I,N} + \frac{y}{y^N} \hat{g}_t \quad (230)$$

$$\hat{y}_t^H = \frac{c^H}{y^H} \hat{c}_t^H + \frac{i^H}{y^H} \hat{q}_t^{I,H} + \frac{1-s}{s} \frac{c^{H*}}{y^H} \hat{c}_t^{H*} + \frac{1-s}{s} \frac{i^{H*}}{y^H} \hat{q}_t^{I,H,*} \quad (231)$$

$$\hat{y}_t = \frac{y^H}{y} \hat{y}_t^H + \frac{y^N}{y} \hat{y}_t^N - (1 - \varpi) \frac{y^H}{y} \hat{t}t_t + \left[\gamma_c \frac{y^N}{y} - (1 - \gamma_c) \frac{y^H}{y} \right] \hat{x}_t \quad (232)$$

$$\hat{q}_t^I = \hat{u}_t + \frac{a'(1)}{g_z} \frac{k}{y} \frac{y}{i} \hat{u}_t \quad (233)$$

Foreign country:

$$\hat{y}_t^{N,*} = \frac{c^{N,*}}{y^{N,*}} \hat{c}_t^{N,*} + \frac{q^{I,N,*}}{y^{N,*}} \hat{q}_t^{I,N,*} + \frac{y^*}{y^{N,*}} \hat{g}_t^* \quad (234)$$

$$\hat{y}_t^F = \frac{s}{1-s} \frac{c^F}{y^F} \hat{c}_t^F + \frac{s}{1-s} \frac{q^{I,F}}{y^F} \hat{q}_t^{I,F} + \frac{c^{F*}}{y^F} \hat{c}_t^{F*} + \frac{q^{I,F,*}}{y^F} \hat{q}_t^{I,F,*} \quad (235)$$

$$\begin{aligned} \hat{y}_t &= \frac{y^F}{y^*} \hat{y}_t^F + \frac{y^{N,*}}{y^*} \hat{y}_t^{N,*} \\ &\quad - (1 - \varpi^*) \frac{y^F}{y^*} \hat{t}t_t + \left[\gamma_c^* \frac{y^{N,*}}{y^*} - (1 - \gamma_c^*) \frac{y^F}{y^*} \right] \hat{x}_t^* \end{aligned} \quad (236)$$

$$\hat{q}_t^{I,*} = \hat{i}_t^* + \frac{a'(1)}{g_z} \frac{k^*}{y^*} \frac{y^*}{i^*} \hat{u}_t^* \quad (237)$$

Definition of exports and imports

$$\widehat{ex}_t = \frac{\gamma_c^* \frac{c^*}{y^*}}{\gamma_c^* \frac{c^*}{y^*} + \gamma_i^* \frac{i^*}{y^*}} \hat{c}_t^{H,*} + \frac{\gamma_i^* \frac{i^*}{y^*}}{\gamma_c^* \frac{c^*}{y^*} + \gamma_i^* \frac{i^*}{y^*}} \hat{q}_t^{I,H,*} \quad (238)$$

$$\widehat{im}_t = \frac{\gamma_c \frac{c}{y}}{\gamma_c \frac{c}{y} + \gamma_i \frac{i}{y}} \hat{c}_t^F + \frac{\gamma_i \frac{i}{y}}{\gamma_c \frac{c}{y} + \gamma_i \frac{i}{y}} \hat{q}_t^{F,I} \quad (239)$$

A.4.7 CPI inflation

Home country:

$$\hat{\pi}_{C,t} = \gamma_c \hat{\pi}_t^T + (1 - \gamma_c) \hat{\pi}_t^N \quad (240)$$

Foreign country:

$$\hat{\pi}_{C,t}^* = \gamma_c^* \hat{\pi}_t^{T,*} + (1 - \gamma_c^*) \hat{\pi}_t^{N,*} \quad (241)$$

A.4.8 Tradables inflation

Home country:

$$\hat{\pi}_t^T = (1 - \varpi) \hat{t}t_t - (1 - \varpi) \hat{t}t_{t-1} + \hat{\pi}_t^H \quad (242)$$

Foreign country:

$$\hat{\pi}_t^{T*} = -(1 - \varpi^*) \hat{t}t_t + (1 - \varpi^*) \hat{t}t_{t-1} + \hat{\pi}_t^{F*} \quad (243)$$

A.4.9 Relative price of non tradable goods

$$\hat{x}_t - \hat{x}_{t-1} = \hat{\pi}_t^N - \hat{\pi}_t^T \quad (244)$$

$$\hat{x}_t^* - \hat{x}_{t-1}^* = \hat{\pi}_t^{N,*} - \hat{\pi}_t^{T,*} \quad (245)$$

A.4.10 Real exchange rate

$$\widehat{RER}_t = \frac{(1 - \gamma_c^*)}{RER} \hat{x}_t^* - \frac{(1 - \gamma_c)}{RER} \hat{x}_t + \frac{(\varpi^* + \varpi - 1)}{RER} \hat{t}t_t \quad (246)$$

$$\widehat{RER}_t - \widehat{RER}_{t-1} = \hat{\pi}_{C,t}^* - \hat{\pi}_{C,t} \quad (247)$$

A.4.11 Monetary authority

$$\begin{aligned} \hat{R}_t^{ECB} &= \phi_R \hat{R}_{t-1}^{ECB} + (1 - \phi_R) (\hat{\pi}_t^{EA} + \phi_\pi (\hat{\pi}_{t-1}^{EA} - \hat{\pi}_t) + \phi_y \hat{y}_t^{EA}) \\ &\quad + \phi_{\Delta\pi} (\hat{\pi}_t^{EA} - \hat{\pi}_{t-1}^{EA}) + \phi_{\Delta y} (\hat{y}_t^{EA} - \hat{y}_{t-1}^{EA}) + \hat{\varepsilon}_t^r \end{aligned} \quad (248)$$

$$\hat{\pi}_t^{EA} = s \hat{\pi}_{C,t} + (1 - s) \hat{\pi}_{C,t}^* \quad (249)$$

$$\hat{y}_t^{EA} = s \hat{y}_t + (1 - s) \hat{y}_t^* \quad (250)$$

A.4.12 Shocks

The shocks included in all the specifications of the model are:

Transitory technology shocks:

$$\hat{\varepsilon}_t^{a,N} = \rho_{a,N} \hat{\varepsilon}_{t-1}^{a,N} + \eta_t^{a,N} \quad (251)$$

$$\hat{\varepsilon}_t^{a,N*} = \rho_{a,N*} \hat{\varepsilon}_{t-1}^{a,N*} + \eta_t^{a,N*} \quad (252)$$

$$\hat{\varepsilon}_t^{a,H} = \rho_{a,H} \hat{\varepsilon}_{t-1}^{a,H} + \eta_t^{a,H} \quad (253)$$

$$\hat{\varepsilon}_t^{a,F} = \rho_{a,F} \hat{\varepsilon}_{t-1}^{a,F} + \eta_t^{a,F} \quad (254)$$

Preference shocks:

$$\hat{\varepsilon}_t^c = \rho_c \hat{\varepsilon}_{t-1}^c + \eta_t^c \quad (255)$$

$$\hat{\varepsilon}_t^{c,*} = \rho_{c,*} \hat{\varepsilon}_{t-1}^{c,*} + \eta_t^{c,*} \quad (256)$$

Risk premium shocks:

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + \eta_t^b \quad (257)$$

$$\hat{\varepsilon}_t^{b,*} = \rho_{b,*} \hat{\varepsilon}_{t-1}^{b,*} + \eta_t^{b,*} \quad (258)$$

Investment specific shocks:

$$\hat{\varepsilon}_t^i = \rho_i \hat{\varepsilon}_{t-1}^i + \eta_t^i \quad (259)$$

$$\hat{\varepsilon}_t^{i,*} = \rho_{i,*} \hat{\varepsilon}_{t-1}^{i,*} + \eta_t^{i,*} \quad (260)$$

Price markup shocks:

$$\hat{\lambda}_t^p = \rho_p \hat{\lambda}_{t-1}^p + \eta_t^p \quad (261)$$

$$\hat{\lambda}_t^{p,*} = \rho_{p,*} \hat{\lambda}_{t-1}^{p,*} + \eta_t^{p,*} \quad (262)$$

Wage markup shocks:

$$\hat{\lambda}_t^w = \rho_w \hat{\lambda}_{t-1}^w + \eta_t^w \quad (263)$$

$$\hat{\lambda}_t^{p,*} = \rho_{p,*} \hat{\lambda}_{t-1}^{p,*} + \eta_t^{p,*} \quad (264)$$

Government spending shocks:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \eta_t^g \quad (265)$$

$$\hat{g}_t^* = \rho_{g,*} \hat{g}_{t-1}^* + \eta_t^{g,*} \quad (266)$$

Interest rate shock:

$$\hat{\varepsilon}_t^r = \rho_r \hat{\varepsilon}_{t-1}^r + \eta_t^r \quad (267)$$

Model A In Model A we add the following shocks.

An inflation objective shock:

$$\hat{\pi}_t = \rho_{\pi} \hat{\pi}_{t-1} + \eta_t^{\pi} \quad (268)$$

A permanent labor augmenting technology shock:

$$\hat{g}_{z,t} = \rho_{g_z} \hat{g}_{z,t-1} + \eta_t^{g_z} \quad (269)$$

Model B Model B is characterised by the following shocks.

A risk sharing condition shock:

$$\hat{\varepsilon}_t^{rsh} = \rho_{rsh} \hat{\varepsilon}_{t-1}^{rsh} + \eta_t^{rsh} \quad (270)$$

A permanent labor augmenting technology shock:

$$\hat{g}_{z,t} = \rho_{g_z} \hat{g}_{z,t-1} + \eta_t^{g_z} \quad (271)$$

Model C Model specification C entails also differences concerning the model equations. The FOC for consumption of the foreign country is included:

$$R_t^{ECB} = \frac{\pi_{C,t+1}^* g_{z,t+1} \varepsilon_t^{c,*} (c_t^{o,*})^{-\sigma^*} (c_{t-1}^*)^{\zeta^*(\sigma^*-1)} \exp\left(\frac{(\sigma^*-1)}{1+\phi_l^*} (h_t^*)^{1+\phi_l^*}\right)}{\beta^* \varepsilon_{t+1}^{c,*} (c_{t+1}^{o,*})^{-\sigma^*} (c_t^*)^{\zeta^*(\sigma^*-1)} \exp\left(\frac{(\sigma^*-1)}{1+\phi_l^*} (h_{t+1}^*)^{1+\phi_l^*}\right)}$$

Then, the following equations are also needed:

$$\begin{aligned} R_t^{ECB} &= R_t \Gamma(nw_t, \varepsilon_t^{rp}) \\ nw_t &= R_{t-1} \frac{nw_{t-1}}{g_{z,t}} + nx_t \\ \Gamma(nw_t, \varepsilon_t^{rp}) &= \exp\left\{\Gamma^b\left(\frac{nw_t}{y_t} - \frac{nw}{y}\right) - \varepsilon_t^{rp}\right\} \\ nx_t &= \frac{P_t^H}{P_{C,t}} \frac{1-s}{s} ex_t - \frac{P_t^F}{P_{C,t}} im_t \\ ex_t &= c_t^{H,*} + q_t^{I,H,*} \\ im_t &= c_t^F + q_t^{I,F} \end{aligned}$$

In log-linear terms we obtain:

$$\begin{aligned} \hat{c}_t^{o,*} &= \hat{c}_{t+1}^{o,*} + \frac{(1-\sigma^*)b^*}{\sigma^*} (\hat{c}_t^* - \hat{c}_{t-1}^*) - \frac{1}{\sigma^*} \left(\hat{R}_t^{ECB} - \hat{\pi}_{C,t+1}^* - \hat{g}_{z,t+1} + \hat{\varepsilon}_{t+1}^{c,*} - \hat{\varepsilon}_t^{c,*} \right) \\ &\quad + \frac{(1-\sigma^*)(h^*)^{1+\phi_l^*}}{\sigma^*} (\hat{h}_{t+1}^* - \hat{h}_t^*) \end{aligned} \quad (272)$$

For the following the equations, note that we imposed $nx = nw = 0$ in steady state.

$$\begin{aligned} \hat{R}_t^{ECB} &= \hat{R}_t + \hat{\Gamma}_t \\ \widehat{nw}_t &= \frac{R}{g_z} \widehat{nw}_{t-1} - \widehat{nx}_t \\ \hat{\Gamma}_t &= \Gamma^b \widehat{nw}_t - \varepsilon_t^{rp} \\ \widehat{nx}_t &= \frac{1-s}{s} \frac{y^*}{y} \left[(1-\varpi^*) \gamma_c^* \frac{c^*}{y^*} + (1-\varpi^*) \gamma_i^* \frac{i^*}{y^*} \right] [\widehat{ex}_t - (1-\gamma_c) \hat{x}_t - (1-\varpi) \widehat{tt}_t] \\ &\quad - \left[(1-\varpi) \gamma_c \frac{c}{y} + (1-\varpi) \gamma_i \frac{i}{y} \right] [\widehat{im}_t - (1-\gamma_c) \hat{x}_t + \varpi \widehat{tt}_t] \end{aligned}$$

where we define $\widehat{nw}_t = \frac{nw_t}{y}$ and $\widehat{nx}_t = \frac{nx_t}{y}$.

The shocks included in this specification are the following.

A shock to the risk premium function:

$$\varepsilon_t^{rp} = \rho_{rp} \varepsilon_{t-1}^{rp} + \eta_t^{rp} \quad (273)$$

A permanent labor augmenting technology shock:

$$\hat{g}_{z,t} = \rho_{g_z} \hat{g}_{z,t-1} + \eta_t^{g_z} \quad (274)$$

Model D We suppose country specific shocks to the trend growth rate:

$$\hat{g}_{z,t} = \rho_{g_z} \hat{g}_{z,t-1} + \eta_t^{g_z} \quad (275)$$

$$\hat{g}_{z,t}^* = \rho_{g_z,*} \hat{g}_{z,t-1}^* + \eta_t^{g_z,*} \quad (276)$$

The deterministic growth trend g_z remains common to both economies. In the equations characterizing the foreign region, $\hat{g}_{z,t}$ is replaced by $\hat{g}_{z,t}^*$.