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OPTIMAL OWNERSHIP REGIME IN THE PRESENCE OF INVESTMENT SPILLOVERS

Valeria Gattai* and Piergiovanna Natale*[†]

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Abstract In the context of the property rights theory of the firm, we study the role of investment spillovers in shaping the efficiency ranking of ownership regimes. In our model, spillovers arise from asset-embodied investment and footloose investment. Under the former, the benefits of investment can be appropriated only through asset control; under the latter, the benefits of investment can be appropriated independently of asset control. Our model predicts that asset-embodied investment favors the adoption of non-integration, while joint ownership may prevail in the presence of footloose investment.

JEL: D23; D86; L24

Keywords: Incomplete contracts; Property rights; Investment spillovers; Joint-control

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1. Introduction

The seminal contribution of Grossman and Hart (1986) [henceforth GH] established that asset ownership matters because it affects the incentive of trading parties to undertake relationship-specific investments. A party's investment responds to her share in the trade surplus. The latter, in turn, depends on the party's outside option. Asset control improves the outside option and it strengthens a party's incentive to invest. An efficiency ranking of ownership regimes is obtained: Control rights should be assigned to the party whose investment is more important in the generation of trade surplus, while non-integration is optimal when both parties' investments are important. Furthermore, it is never optimal to assign the parties veto power over the usage of the assets. Joint-control cannot do better than integration: it reduces the incentive to invest for the party losing control, without increasing it for the non-controlling party.

Several contributions have assessed the robustness of the GH ranking with respect to alternative assumptions on the nature of investment. In GH, investment positively affects the trade surplus and the outside option of the investing party. However, investments may have spillovers: Investment by one party may benefit the other even if they fail to trade. Consider investment in physical capital. The asset-controlling party needs not to cooperate with the investing party to benefit from the investment. This reduces the incentive to invest for the non-controlling party. Hart (1995) suggests that when there is just one asset and investment is in physical capital, joint-control can be optimal.

In the literature, investment in physical capital has come to epitomize investment spillovers and reversal of regime ranking.¹ In this paper, we examine closely the role of spillovers in shaping the regime ranking. We show that investment in physical capital is neither a sufficient nor a necessary condition for the optimality of joint-control.

Investment in physical capital is an instance of asset-embodied investment: its benefits are fully appropriated by any party controlling the asset. Thus, there are investment spillovers only under integration.² With more than one asset, non-integration is preferred to joint-control. As with joint-control, non-integration neutralizes spillovers, but it preserves the incentive to invest that stems from one's outside option. Besides, because it neutralizes spillovers, non-integration is preferred to integration even if one party is much more productive than the other. Our result appears in contrast with the literature on

¹ See Segal and Whinston (2012). For a survey, see Gattai and Natale (2015).

² See De Meza and Lockwood (2004).

investment in physical capital.³ However, in this literature non-integration is ruled out by design, as it is standard to assume that there is just one asset or assets are strict complements.⁴

Spillovers are not confined to asset-embodied investments. Consider a multinational company (MNC) cooperating with a local supplier. Investments by the MNC improving the quality of the final product raise the supplier's reputation for input reliability.⁵ Likewise, the development of common best practices may favor the supplier in alternative business relationships.⁶ On the other hand, workforce training by the supplier improves the quality of the local labor pool and the MNC's prospects if it internalized outsourced operations. By footloose investments, we refer to investments the benefits of which are appropriated independently of asset control. Footloose spillovers affect non-integration and integration alike. Joint-control only neutralizes them and it is indeed optimal when investment cross-effects exceed own-effects.⁷ In a nutshell, our model predicts that asset-embodied investments favor the adoption of non-integration, while joint-control may prevail in the presence of footloose investments. The exact nature of spillovers is crucial to establish the optimality of joint-control when non-integration is a viable option.

³ An exception being Schmitz (2013), showing that if parties have different productivity, joint-control is optimal only when relationship-specificity is small.

⁴ See Guriev (2003), Schmitz (2013) and Segal and Whinston (2012).

⁵ See Midler (2005): "...Chinese factory owners liked the idea of being able to say that they are a supplier to Wal-Mart, because Wal-Mart's reputation for supplier audits was so strong..." (p. 237).

⁶ The literature on FDI backward linkages provides several examples. See Javorcik (2004): "After a Czech producer...signed its first contract with a multinational customer, the staff from the multinational would visit the Czech firm's premises....to work on improving the quality of the control system. Subsequently, the Czech firm applied these improvements to its other production lines (not serving this particular customer)..." (p. 608).

⁷ In Rosenkranz and Schmitz (1999), joint-control of a non-excludable asset is optimal when parties investing in human capital disclose information affecting the trade surplus and the partner's outside option. Our model complements their results as we consider excludable assets and the empirically relevant case of unintended transmission of information.

2. The model

Consider two parties. M_1 produces a good, by means of an input –a widget– and asset a_1 . M_2 produces widgets by means of asset a_2 . M_1 and M_2 expect to trade and can increase the trade surplus by relationship-specific investments. Because of contract incompleteness, they are unable to commit to any investment or trade price.⁸ However, they can sign a contract assigning control rights over a_1 and a_2 .

There are four ownership regimes $A = \{NI; T_1; T_2; JC\}$:

Non-integration (NI): M_1 owns a_1 and M_2 owns a_2

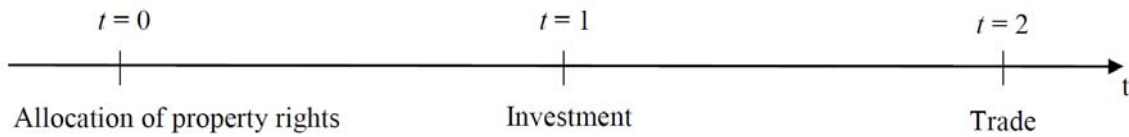
Type 1 integration (T_1): M_1 owns a_1 and a_2

Type 2 integration (T_2): M_2 owns a_1 and a_2

Joint-control: (JC): both M_1 and M_2 have veto power over the use of a_1 and a_2

At $t = 0$, M_1 and M_2 select the ownership regime. At $t = 1$, they select non-cooperatively investments e_1 and e_2 , at cost $C(e_1) = \frac{1}{2}e_1^2$ and $C(e_2) = \frac{1}{2}e_2^2$. At $t = 2$, they negotiate over the exchange of a widget.

Figure 1: Time line



If they agree to trade, they realize $S = e_1 + \chi e_2$, with $\chi > 0$, with χ capturing the relative productivity of each party's investment. If negotiations fail, M_1 and M_2 turn to the market. M_1 's and M_2 's outside option are $s_1^A = \lambda_{11}^A e_1 + \lambda_{12}^A \chi e_2$ and $s_2^A = \lambda_{21}^A e_1 + \lambda_{22}^A \chi e_2$, respectively. Parameter λ_{ii}^A captures investment own-effects, i.e. the marginal change in M_i 's outside option due to her own investment. Parameter λ_{ij}^A captures investment spillovers or cross-effects, i.e. the marginal change in M_i 's outside option due to M_j 's investment. Because of relationship-specificity, the marginal benefit of investment is larger inside the relationship than outside it. Moreover, we rule out negative own- and cross effects.⁹ Thus: $0 \leq \lambda_{ii}^A < 1$ $0 \leq \lambda_{ij}^A < 1$.

⁸ Contract incompleteness entails the inability to write state-contingent contracts. This impairs commitment if the features of the widget or the nature of the investments are not defined in advance.

⁹ Non-negative own-effects are standard in the literature on the property rights theory (PRT) of the firm, Rajan and Zingales (1998) being the only exception. We discuss negative spillovers in fn.20.

Information is symmetric. Ex-post negotiations are governed by the Nash Bargaining Solution, with parties having equal bargaining power. In the event of agreement, M_i receives her outside option plus $\frac{1}{2}$ of the surplus from trade with M_j . Investment being relationship-specific, the parties are better off trading. M_1 and M_2 select investment to maximize their payoff:

$$\max_{e_1} \pi_1^A = \lambda_{11}^A e_1 + \lambda_{12}^A \chi e_2 + \frac{1}{2} [e_1 + \chi e_2 - (\lambda_{11}^A e_1 + \lambda_{12}^A \chi e_2) - (\lambda_{21}^A e_1 + \lambda_{22}^A \chi e_2)] - \frac{1}{2} e_1^2 \quad (1)$$

$$\max_{e_2} \pi_2^A = \lambda_{21}^A e_1 + \lambda_{22}^A \chi e_2 + \frac{1}{2} [e_1 + \chi e_2 - (\lambda_{11}^A e_1 + \lambda_{12}^A \chi e_2) - (\lambda_{21}^A e_1 + \lambda_{22}^A \chi e_2)] - \frac{1}{2} e_2^2 \quad (2)$$

Substituting the equilibrium investments $\hat{e}_1^A = \frac{1}{2}(1 + \lambda_{11}^A - \lambda_{21}^A)$ and $\hat{e}_2^A = \frac{1}{2}\chi(1 + \lambda_{22}^A - \lambda_{12}^A)$ into π_1^A and π_2^A , we obtain the equilibrium payoffs, $\hat{\pi}_1^A$ and $\hat{\pi}_2^A$. Absent liquidity constraints, the parties select the ownership regime to maximize the joint surplus:

$$\hat{S}^A = \hat{\pi}_1^A + \hat{\pi}_2^A = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left\{ (\lambda_{11}^A - \lambda_{21}^A) \left[1 - \frac{1}{2}(\lambda_{11}^A - \lambda_{21}^A) \right] + \chi^2 (\lambda_{22}^A - \lambda_{12}^A) \left[1 - \frac{1}{2}(\lambda_{22}^A - \lambda_{12}^A) \right] \right\} \quad (3)$$

If parties could select investments cooperatively, the joint surplus would be $S^* = \frac{1}{2}(1 + \chi^2)$.

Since $0 \leq \lambda_{ii}^A < 1$ and $0 \leq \lambda_{ij}^A < 1$, $\hat{S}^A < S^*$: No ownership regime achieves the first best; however, not all regimes are equally inefficient. As inspection of \hat{S}^A reveals, the ranking of ownership regimes depends on the magnitude of investment own- and cross-effects.

The magnitude of own-effects captures asset complementarity. Assets are complements [independent] when the marginal benefit of investment is increasing [constant] in the number of controlled assets, normalizing it to zero in case of no asset. As the number of assets a party controls varies across ownership regimes, the magnitude of own-effects varies too: $\lambda_{ii}^{T_i} \geq \lambda_{ii}^{N_i} \geq \lambda_{ii}^{T_j} = \lambda_{ii}^{J_C} = 0$.¹⁰

The magnitude of cross-effects depends on the nature of investment. In GH, the parties invest in human capital. M_i benefits of M_j 's investment only if they cooperate ex-post: $\lambda_{ij}^A = 0, \forall A$. However, investment can be asset-embodied. The asset-controlling party fully appropriates the return of any investment by the non-controlling one: $\lambda_{ij}^{T_i} > \lambda_{ij}^{N_i} = \lambda_{ij}^{T_j} = \lambda_{ij}^{J_C} = 0$.¹¹ If investment is asset-embodied, there are spillovers only under integration.¹²

¹⁰ We rule out asset substitutability. This is standard in the PRT, Bel (2013) being the only exception.

¹¹ Like for own-effects, cross-effects are normalized to zero in case of no asset.

¹² Our taxonomy of investments cuts across the distinction between tangible and intangible investments developed in the literature on dissipation of proprietary advantages (Markusen, 1995). In our paper, R&D and advertising expenditures are examples of asset-embodied investments.

However, spillovers are not confined to asset-embodied investments. By footloose investments, we refer to investments the benefits of which are appropriated independently of asset control. M_i 's footloose investment improves M_j 's outside option also when M_i maintains control of asset a_i . Footloose spillovers affect integration and non-integration: $\lambda_{ij}^{T_i} \geq \lambda_{ij}^{NI} > \lambda_{ij}^{T_j} = \lambda_{ij}^{JC} = 0$.¹³

In the following section, we study the ranking of ownership regimes in case of: i) no spillovers; ii) asset-embodied spillovers; iii) footloose spillovers.

3. Results

Case 1: No spillovers

Consider \hat{S}^A evaluated at $\lambda_{ij}^A = 0, \forall A$.

Proposition 1:

i) Non-integration is preferred to integration iff χ is such that:

$$\chi_{T_1NI} \equiv \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right) - \lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI}\right)}{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)} \right]^{1/2} < \chi < \left[\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)} \right]^{1/2} \equiv \chi_{T_2NI} \quad (4)$$

Sufficient condition for $\chi_{T_1NI} < \chi_{T_2NI}$ is:

$$\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI}\right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)} + \frac{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} > 1 \quad (5)$$

ii) Type 2 integration is preferred to type 1 integration iff:

$$\chi > \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{T_1T_2} \quad (6)$$

iii) Joint control is never optimal.

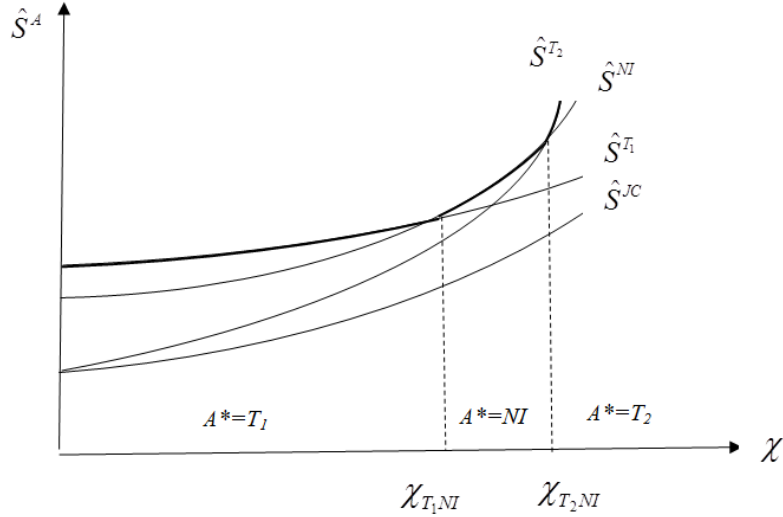
Proof: Solving $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ for χ establishes i). Solving $\hat{S}^{T_2} > \hat{S}^{T_1}$ for χ establishes ii).

Comparison of \hat{S}^{JC} and \hat{S}^{T_i} establishes iii). Details of the proof are in the Appendix.

Figure 2 illustrates Proposition 1.

¹³ As for own-effects, cross-effects are non-decreasing in the number of controlled assets.

Figure 2: No spillovers



In line with GH, Proposition 1 establishes that non-integration can be optimal only if asset complementary is not too strong. In fact, inspection of (5) reveals that the set of χ supporting non-integration is not empty only when λ_{ii}^{NI} and $\lambda_{ii}^{T_i}$ are relatively close, i.e. asset complementarity is not strong. When asset complementarity is strong, the marginal return of investment outside the relationship and thus the incentive to invest are larger under integration than non-integration.

Furthermore, non-integration is optimal when the parties' contributions are both "important".¹⁴ Asset complementary favors integration. However, taking away assets from a party reduces her investment and the trade surplus. The more productive is the party losing control, the stronger is the adverse effect of integration on the surplus. It follows that non-integration is optimal when the parties are similarly productive.

Type 2 integration is preferred to type 1 integration when χ is above the threshold $\chi_{T_1 T_2}$. If asset complementarity is the same across parties,¹⁵ $\chi_{T_1 T_2} = 1$. As in GH, control goes to the most productive party. Finally, joint-control is never optimal as integration is preferred to it. Switching from joint-control to type i integration leaves unaffected M_j 's incentive to invest while increasing M_i 's one.

¹⁴ Proposition 1 sets bounds on the values of χ supporting non-integration.

¹⁵ That is: $\lambda_{11}^{T_1} = \lambda_{22}^{T_2}$.

Case 2: Asset-embodied spillovers

Consider \hat{S}^A evaluated at $\lambda_{ij}^{T_i} > \lambda_{ij}^{NI} = \lambda_{ij}^{T_j} = \lambda_{ij}^{JC} = 0$.

Proposition 2:

i) Non-integration is preferred to integration iff χ is such that:

$$\chi_{T_1NI} \equiv \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - \lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right) \right]^{1/2}}{\left[\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]} < \chi < \frac{\left[\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right) \right]} \equiv \chi_{T_2NI} \quad (7)$$

Sufficient condition for $\chi_{T_1NI} < \chi_{T_2NI}$ is:

$$\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right) \left[\frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} + \frac{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right) \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right)} \right]}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \right]}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \left[\frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} + \frac{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right) \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right)} \right]}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \right]} > 1 - \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \lambda_{12}^{T_1} \left(1 - \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \quad (8)$$

ii) Type 2 integration is preferred to type 1 integration iff:

$$\chi > \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]} \equiv \chi_{T_1T_2} \quad (9)$$

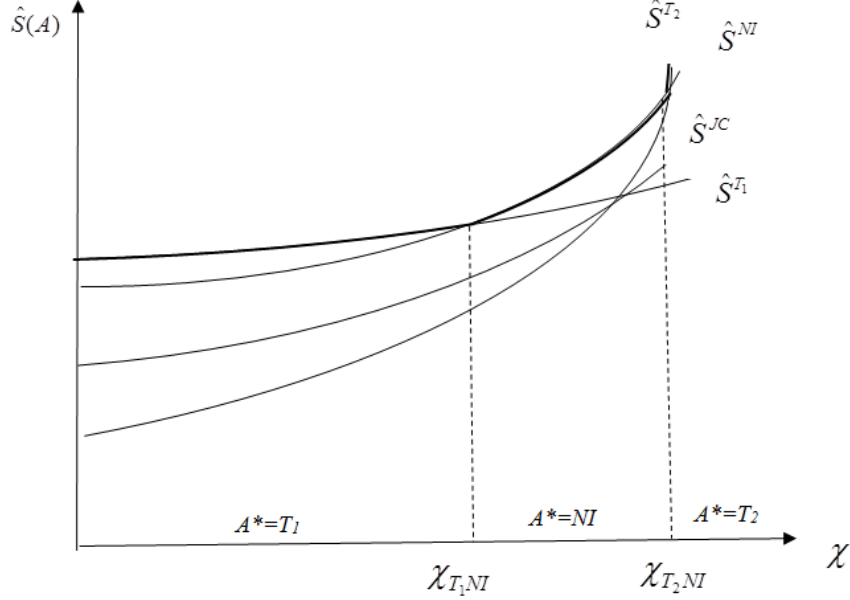
iii) Joint control is never optimal.

Proof: Solving $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ for χ establishes i). Solving $\hat{S}^{T_2} > \hat{S}^{T_1}$ for χ establishes ii).

Comparison of \hat{S}^{JC} and \hat{S}^{NI} establishes iii). Details of the proof are in the Appendix.

Figure 3 illustrates Proposition 2.

Figure 3: Asset-embodied spillovers



Proposition 2 shows that in the presence of asset-embodied spillovers, non-integration is optimal for a larger set of χ than predicted in GH.¹⁶ Because of spillovers' distortionary effects, the parties renounce the benefits from asset complementarity and choose non-integration also when one of them is much less productive than the other. Moreover, in the presence of asset-embodied spillovers, non-integration can be optimal also when asset complementarity is strong. Inspection of (8) reveals that the L.H.S. is increasing whereas the R.H.S. is decreasing in investment cross-effects λ_{ij}^A . It follows that (8) holds for lower values of the ratio $\lambda_{ii}^{NI} / \lambda_{ii}^T$ than (5).

Type 2 integration is preferred to type 1 integration also when M_2 is less productive than M_1 . Assume asset complementarity to be the same across parties, but M_2 's investment spillovers to be stronger than M_1 's. In this case, M_2 has little incentive to invest under type 1 integration. Finally, joint-control is never optimal as non-integration is preferred to it. Non-integration neutralizes the adverse effects of spillovers, but it also preserves the incentive to invest that stems from one's outside option.

Our result is in contrast with the literature on investment in physical capital. However, in this literature joint-control is proved optimal under the assumption that there is just one asset or assets are strictly complements. Non-integration is ruled out by design.

¹⁶ This result holds also for one-sided spillovers, i.e. $\lambda_{ij}^T > 0$ and $\lambda_{ji}^T = 0$.

Case 3: Footloose spillovers

Consider \hat{S}^A evaluated at $\lambda_{ij}^{T_i} \geq \lambda_{ij}^{NI} > \lambda_{ij}^{T_j} = \lambda_{ij}^{JC} = 0$.

Proposition 3

Assume the condition $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) > 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) > 0$ holds:

i) Non-integration is preferred to integration iff χ is such that:

$$\chi_{T_1NI} = \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] \right]^{1/2}}{\left[(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]} < \chi < \frac{\left[(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] \right]} \equiv \chi_{T_2NI} \quad (10)$$

Sufficient condition for $\chi_{T_1NI} < \chi_{T_2NI}$ is:¹⁷

$$\frac{(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left(1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right) \left[\frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \right]}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \left[1 + \frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \right]} + \frac{\left[(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left(1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right) \right] \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right)} \right]}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \left[1 + \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right)} \right]} > 1 - \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right)} \quad (11)$$

ii) Type 2 integration is preferred to type 1 integration iff:

$$\chi > \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]} \equiv \chi_{T_1T_2} \quad (12)$$

iii) Joint control is never optimal.

Assume the condition $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) > 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) > 0$ is violated:

iv) Joint control can be optimal.

Proof: Solving $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ for χ establishes i). Solving $\hat{S}^{T_2} > \hat{S}^{T_1}$ for χ establishes ii).

Comparison of \hat{S}^{JC} and \hat{S}^{NI} establishes iii). To prove iv), assume $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) < 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) < 0$.

¹⁷ Inspection reveals that the L.H.S. in (11) is decreasing in investment cross-effects materializing under non-integration, λ_{ij}^{NI} . It follows that (11) is satisfied only for weaker asset complementarity than (8), i.e for higher values of the ratio $\lambda_{ii}^{NI} / \lambda_{ii}^{T_i}$. Non-integration can be optimal when asset complementarity is relatively strong in the presence asset-embodied spillovers, but not footloose spillovers. Nonetheless, non-integration can be optimal in the presence of footloose spillovers even if it would not be so absent spillovers. Inspection of (5) and (11) reveals that the R.H.S. in (11) is smaller than the R.H.S in (5), whereas the L.H.S. in (11) is larger than the L.H.S. in (5) for large value of $\lambda_{ij}^{T_i}$.

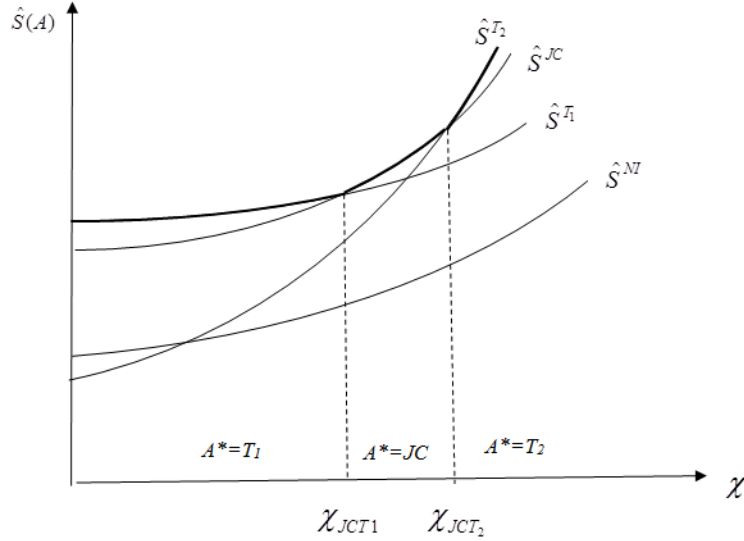
Comparison of \hat{S}^{JC} and \hat{S}^{NI} establishes $\hat{S}^{JC} > \hat{S}^{NI}$. $\hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$ holds for any χ such

$$\chi_{JC T_1} \equiv \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)}{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2} < \chi < \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{JC T_2}$$

Details of the proof are in the Appendix.

Figure 4 illustrates Proposition 3.

Figure 4: Footloose spillovers



Proposition 3 shows that under footloose spillovers joint-control may indeed be optimal, albeit under quite strict conditions. Joint-control may dominate non-integration only if cross-effects λ_{ij}^{NI} are larger than own-effects λ_{ii}^{NI} for at least one party.¹⁸ Moreover, joint-control may dominate integration only if each party's investment has cross-effects, as we assume.¹⁹ Otherwise, ownership by the party whose investment has no spillovers is preferred to joint-control. In brief, when cross-effects are larger than own-effects it makes sense to sacrifice the incentive to invest that stems from one's outside option to limit distortionary effects due to spillovers.²⁰

¹⁸ Under integration, investment cross-effects are larger than own-effects by design for the non-controlling party.

¹⁹ The set of χ supporting joint-control is not empty iff:

$$\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) < \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right) \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right).$$

²⁰ Were spillovers negative, no trade-off arises as own and cross-effects cooperate in promoting investment. In our view, the case of negative spillovers should be addressed in a model featuring over-investment.

4. Conclusions

When investment is relationship-specific, positive spillovers exacerbate the underinvestment affecting all ownership regimes. The optimal ownership regime severs the link between a party's investment and her partner's outside option. However, which regime is optimal depends on the exact nature of spillovers.

We show that asset-embodied spillovers favor the adoption of non-integration, while joint-control may prevail in case of footloose spillovers. Our model innovates with respect to the literature in suggesting that i) non-integration is likelier to be adopted than predicted in GH; ii) absent strict complementarity, joint control prevails only if spillovers are footloose.

Our results rest on two pillars: the magnitude of spillover effects and the distinction between asset-embodied and footloose spillovers. The magnitude is likely to be large and the distinction meaningful when there is little overlapping between the parties' capabilities, as in the case of international joint-ventures.

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Appendix

Recall the joint surplus function for regime A, $A = \{NI; T_1; T_2; JC\}$:

$$\hat{S}^A = \hat{\pi}_1^A + \hat{\pi}_2^A = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left\{ (\lambda_{11}^A - \lambda_{21}^A) \left[1 - \frac{1}{2}(\lambda_{11}^A - \lambda_{21}^A) \right] + \chi^2 (\lambda_{22}^A - \lambda_{12}^A) \left[1 - \frac{1}{2}(\lambda_{22}^A - \lambda_{12}^A) \right] \right\} \quad (A.1)$$

Proof of Proposition 1:

Substituting $\lambda_{ij}^A = 0, \forall A$ in (A.1), ownership regimes give the following surplus functions:

$$\hat{S}^{NI} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right) + \chi^2 \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right) \right] \quad (A.2)$$

$$\hat{S}^{T_1} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \right] \quad (A.3)$$

$$\hat{S}^{T_2} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\chi^2 \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \right] \quad (A.4)$$

$$\hat{S}^{JC} = \frac{3}{8}(1 + \chi^2) \quad (A.5)$$

i) Non-integration is preferred to integration iff $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$

$$\text{Comparing (A.2) with (A.3): } \hat{S}^{NI} > \hat{S}^{T_1} \text{ iff } \chi > \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - \lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right)}{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right)} \right]^{1/2}$$

$$\text{Comparing (A.2) and (A.4): } \hat{S}^{NI} > \hat{S}^{T_2} \text{ iff } \chi < \left[\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right)} \right]^{1/2}$$

Therefore, $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ iff:

$$\chi_{T_1NI} \equiv \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - \lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right)}{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right)} \right]^{1/2} < \chi < \left[\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI} \right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI} \right)} \right]^{1/2} \equiv \chi_{T_2NI}$$

Sufficient condition for the set of χ values supporting non-integration to be not empty. i.e. $\chi_{T_1NI} < \chi_{T_2NI}$,
is²¹

$$\frac{\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI}\right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)} + \frac{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} > 1 \quad (\text{A.6})$$

ii) Type 2 integration is preferred to type 1 integration iff $\hat{S}^{T_2} > \hat{S}^{T_1}$

$$\text{Comparing (A.3) and (A.4): } \hat{S}^{T_2} > \hat{S}^{T_1} \text{ iff } \chi > \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{T_1T_2}$$

iii) Comparing (A.3) with (A.5): $\hat{S}^{T_1} > \hat{S}^{JC} \forall \chi$

Being less preferred than integration for every parameter value, joint control is never optimal.

Proof of Proposition 2:

Substituting $\lambda_{ij}^{T_i} > \lambda_{ij}^{NI} = \lambda_{ij}^{T_j} = \lambda_{ij}^{JC} = 0$ in (A.1), ownership regimes give the following surplus functions:

$$\hat{S}^{NI} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\lambda_{11}^{NI} \left(1 - \frac{1}{2} \lambda_{11}^{NI}\right) + \chi^2 \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right) \right] \quad (\text{A.7})$$

$$\hat{S}^{T_1} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right) - \chi^2 \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right) \right] \quad (\text{A.8})$$

$$\hat{S}^{T_2} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[-\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right) + \chi^2 \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) \right] \quad (\text{A.9})$$

$$\hat{S}^{JC} = \frac{3}{8}(1 + \chi^2) \quad (\text{A.10})$$

²¹ Figure 2 is drawn under condition (A.6).

i) Non-integration is preferred to integration iff $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$

$$\text{Comparing (A.7) with (A.8): } \hat{S}^{NI} > \hat{S}^{T_1} \text{ iff } \chi > \left[\frac{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right) - \lambda_{41}^{NI} \left(1 - \frac{1}{2} \lambda_{41}^{NI}\right)}{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2}$$

$$\text{Comparing (A.7) and (A.9): } \hat{S}^{NI} > \hat{S}^{T_2} \text{ iff } \chi < \left[\frac{\lambda_{41}^{NI} \left(1 - \frac{1}{2} \lambda_{41}^{NI}\right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)} \right]^{1/2}$$

Therefore, $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ iff

$$\chi_{T_1NI} \equiv \left[\frac{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right) - \lambda_{41}^{NI} \left(1 - \frac{1}{2} \lambda_{41}^{NI}\right)}{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2} < \chi < \left[\frac{\lambda_{41}^{NI} \left(1 - \frac{1}{2} \lambda_{41}^{NI}\right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) - \lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)} \right]^{1/2} \equiv \chi_{T_2NI}$$

Sufficient condition for the set of χ values supporting non-integration to be not empty, i.e. $\chi_{T_1NI} < \chi_{T_2NI}$, is²²

$$\frac{\lambda_{41}^{NI} \left(1 - \frac{1}{2} \lambda_{41}^{NI}\right)}{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right)} \left[1 + \frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right] + \frac{\lambda_{22}^{NI} \left(1 - \frac{1}{2} \lambda_{22}^{NI}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \left[1 + \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right)}{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right)} \right] > 1 - \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right) \lambda_{12}^{T_1} \left(1 - \frac{1}{2} \lambda_{12}^{T_1}\right)}{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \quad (\text{A.11})$$

ii) Type 2 integration is preferred to type 1 integration iff $\hat{S}^{T_2} > \hat{S}^{T_1}$

$$\text{Comparing (A.8) and (A.9): } \hat{S}^{T_2} > \hat{S}^{T_1} \text{ iff } \chi > \left[\frac{\lambda_{41}^{T_1} \left(1 - \frac{1}{2} \lambda_{41}^{T_1}\right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2} \equiv \chi_{T_1T_2}$$

iii) Comparing (A.7) with (A.10): $\hat{S}^{NI} > \hat{S}^{JC} \forall \chi$

Being less preferred than non-integration for every parameter value, joint control is never optimal.

Proof of Proposition 3

²² Figure 3 is drawn under condition (A.11).

Substituting $\lambda_{ij}^{T_i} \geq \lambda_{ij}^{NI} > \lambda_{ij}^{T_j} = \lambda_{ij}^{JC} = 0$ in (A.1), ownership regimes give the following surplus functions:

$$\hat{S}^{NI} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left\{ (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2}(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] + \chi^2 (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2}(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] \right\} \quad (\text{A.12})$$

$$\hat{S}^{T_1} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - \chi^2 \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right] \quad (\text{A.13})$$

$$\hat{S}^{T_2} = \frac{3}{8}(1 + \chi^2) + \frac{1}{4} \left[-\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) + \chi^2 \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \right] \quad (\text{A.14})$$

$$\hat{S}^{JC} = \frac{3}{8}(1 + \chi^2) \quad (\text{A.15})$$

Assume the following condition holds: $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) > 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) > 0$

i) Non-integration is preferred to integration iff $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$

Comparing (A.12) with (A.13): $\hat{S}^{NI} > \hat{S}^{T_1}$ iff $\chi > \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] \right]^{1/2}}{\left(\lambda_{22}^{NI} - \lambda_{12}^{NI} \right) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)} \equiv \chi_{T_1NI}$

Comparing (A.12) with (A.14): $\hat{S}^{NI} > \hat{S}^{T_2}$ iff

$$\chi < \frac{\left[(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right]} \equiv \chi_{T_2NI}$$

Therefore, $\hat{S}^{NI} > \hat{S}^{T_1} \wedge \hat{S}^{NI} > \hat{S}^{T_2}$ iff

$$\chi_{T_1NI} = \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) - (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] \right]^{1/2}}{\left(\lambda_{22}^{NI} - \lambda_{12}^{NI} \right) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] + \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right)} < \chi < \frac{\left[(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] + \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) - (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right]} \equiv \chi_{T_2NI}$$

Sufficient condition for the set of χ values supporting non-integration to be not empty, i.e. $\chi_{T_1NI} < \chi_{T_2NI}$,

is

$$\frac{(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left(1 - \frac{1}{2}(\lambda_{11}^{NI} - \lambda_{21}^{NI})\right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2}\lambda_{11}^{T_1}\right)} \left[\frac{\lambda_{12}^{T_1} \left(1 + \frac{1}{2}\lambda_{12}^{T_1}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2}\lambda_{22}^{T_2}\right)} \right] + \frac{(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2}(\lambda_{22}^{NI} - \lambda_{12}^{NI})\right]}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2}\lambda_{22}^{T_2}\right)} \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2}\lambda_{21}^{T_2}\right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2}\lambda_{11}^{T_1}\right)} \right] > 1 - \frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2}\lambda_{21}^{T_2}\right) \lambda_{12}^{T_1} \left(1 - \frac{1}{2}\lambda_{12}^{T_1}\right)}{\lambda_{11}^{T_1} \left(1 - \frac{1}{2}\lambda_{11}^{T_1}\right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2}\lambda_{22}^{T_2}\right)} \quad (\text{A.16})$$

ii) Type 2 integration is preferred to type 1 integration iff $\hat{S}^{T_2} > \hat{S}^{T_1}$

$$\text{Comparing (A.13) with (A.14): } \hat{S}^{T_2} > \hat{S}^{T_1} \text{ iff } \chi > \frac{\left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2}\lambda_{11}^{T_1}\right) + \lambda_{21}^{T_2} \left(1 + \frac{1}{2}\lambda_{21}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2}\lambda_{22}^{T_2}\right) + \lambda_{12}^{T_1} \left(1 + \frac{1}{2}\lambda_{12}^{T_1}\right)} \right]^{1/2}}{\equiv \chi_{T_1T_2}}$$

iii) Comparing (A.12) with (A.15): $\hat{S}^{NI} > \hat{S}^{JC} \forall \chi$

Being less preferred than non-integration for every parameter value, joint control is never optimal.

Assume the condition $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) > 0 \wedge$ is violated. This gives rise to three alternative cases.²³

Case a: $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) < 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) > 0$

Joint control is optimal iff $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$

$$\text{Comparing (A.12) with (A.15): } \hat{S}^{JC} > \hat{S}^{NI} \text{ iff } \chi < \frac{\left[-(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2}(\lambda_{11}^{NI} - \lambda_{21}^{NI})\right] \right]^{1/2}}{\left[(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2}(\lambda_{22}^{NI} - \lambda_{12}^{NI})\right] \right]} \equiv \chi_{JCNI}$$

$$\text{Comparing (A.13) with (A.15): } \hat{S}^{JC} > \hat{S}^{T_1} \text{ iff } \chi > \frac{\left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2}\lambda_{11}^{T_1}\right)}{\lambda_{12}^{T_2} \left(1 + \frac{1}{2}\lambda_{12}^{T_2}\right)} \right]^{1/2}}{\equiv \chi_{JCT_1}}$$

²³ For the sake of brevity, in the text we present only case c.

Comparing (A.14) with (A.15): $\hat{S}^{JC} > \hat{S}^{T_2}$ iff $\chi < \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{22}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{JCT_2}$

Therefore, $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$ iff:²⁴

$$\chi_{JCT_1} \equiv \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)}{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2} < \chi < \min \left\{ \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{22}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{JCT_2}; \left[\frac{-\left(\lambda_{11}^{NI} - \lambda_{21}^{NI}\right) \left[1 - \frac{1}{2} \left(\lambda_{11}^{NI} - \lambda_{21}^{NI}\right)\right]}{\left(\lambda_{22}^{NI} - \lambda_{12}^{NI}\right) \left[1 - \frac{1}{2} \left(\lambda_{22}^{NI} - \lambda_{12}^{NI}\right)\right]} \right]^{1/2} \equiv \chi_{JCN1} \right\}$$

Case b: $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) > 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) < 0$

Joint control is optimal iff $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$

Comparing (A.12) with (A.15): $\hat{S}^{JC} > \hat{S}^{NI}$ iff $\chi > \left[\frac{-\left(\lambda_{11}^{NI} - \lambda_{21}^{NI}\right) \left[1 - \frac{1}{2} \left(\lambda_{11}^{NI} - \lambda_{21}^{NI}\right)\right]}{\left(\lambda_{22}^{NI} - \lambda_{12}^{NI}\right) \left[1 - \frac{1}{2} \left(\lambda_{22}^{NI} - \lambda_{12}^{NI}\right)\right]} \right]^{1/2} \equiv \chi_{JCN1}$

Comparing (A.13) with (A.15): $\hat{S}^{JC} > \hat{S}^{T_1}$ iff $\chi > \left[\frac{\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1}\right)}{\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1}\right)} \right]^{1/2} \equiv \chi_{JCT_1}$

Comparing (A.14) with (A.15): $\hat{S}^{JC} > \hat{S}^{T_2}$ iff $\chi < \left[\frac{\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{22}^{T_2}\right)}{\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2}\right)} \right]^{1/2} \equiv \chi_{JCT_2}$

Therefore, $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$ iff:²⁵

²⁴ The set of χ supporting joint-control is not empty if and only if $\chi_{JCT_1} < \min\{\chi_{JCT_1}; \chi_{JCN1}\}$.

²⁵ The set of χ supporting joint-control is not empty if and only if $\max\{\chi_{JCT_1}; \chi_{JCN1}\} < \chi_{JCT_2}$.

$$\max \left\{ \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \right]^{1/2}}{\left[\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]^{1/2}} \right\} \equiv \chi_{JCT_1}; \left\{ \frac{\left[-(\lambda_{11}^{NI} - \lambda_{21}^{NI}) \left[1 - \frac{1}{2} (\lambda_{11}^{NI} - \lambda_{21}^{NI}) \right] \right]^{1/2}}{\left[(\lambda_{22}^{NI} - \lambda_{12}^{NI}) \left[1 - \frac{1}{2} (\lambda_{22}^{NI} - \lambda_{12}^{NI}) \right] \right]^{1/2}} \right\} \equiv \chi_{JCNl} < \chi < \left\{ \frac{\left[\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \right]^{1/2}} \right\} \equiv \chi_{JCT_2}$$

Case c: $(\lambda_{11}^{NI} - \lambda_{21}^{NI}) < 0 \wedge (\lambda_{22}^{NI} - \lambda_{12}^{NI}) < 0$

Joint control is optimal iff $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$

Comparing (A.12) with (A.15): $\hat{S}^{JC} > \hat{S}^{NI} \forall \chi$

Comparing (A.13) with (A.15): $\hat{S}^{JC} > \hat{S}^{T_1}$ iff $\chi > \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \right]^{1/2}}{\left[\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]^{1/2}} \equiv \chi_{JCT_1}$

Comparing (A.14) with (A.15): $\hat{S}^{JC} > \hat{S}^{T_2}$ iff $\chi < \frac{\left[\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \right]^{1/2}} \equiv \chi_{JCT_2}$

Therefore, $\hat{S}^{JC} > \hat{S}^{NI} \wedge \hat{S}^{JC} > \hat{S}^{T_1} \wedge \hat{S}^{JC} > \hat{S}^{T_2}$ iff:

$$\chi_{JCT_1} \equiv \frac{\left[\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \right]^{1/2}}{\left[\lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \right]^{1/2}} < \chi < \frac{\left[\lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \right]^{1/2}}{\left[\lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) \right]^{1/2}} \equiv \chi_{JCT_2}$$

Sufficient condition for the set of χ values supporting joint-control to be not empty, i.e. $\chi_{T_1NI} < \chi_{T_2NI}$, is²⁶

$$\lambda_{11}^{T_1} \left(1 - \frac{1}{2} \lambda_{11}^{T_1} \right) \lambda_{22}^{T_2} \left(1 - \frac{1}{2} \lambda_{22}^{T_2} \right) < \lambda_{21}^{T_2} \left(1 + \frac{1}{2} \lambda_{21}^{T_2} \right) \lambda_{12}^{T_1} \left(1 + \frac{1}{2} \lambda_{12}^{T_1} \right) \quad (A.17)$$

²⁶ Figure 4 is drawn under condition (A.17).