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Optimal Inflation to Reduce Inequality[*]

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Abstract

A popular argument in favour of price stability is that the inflation-tax burden would disproportionately fall on the poor because wealth is unevenly distributed and portfolio composition of poorer households is skewed towards a larger share of money holdings. We reconsider the issue in a DSGE model characterized by limited participation to the market for interest bearing assets (LAMP). We show that a combination of higher inflation and lower income taxes reduces inequality. When we calibrate the share of constrained agents to fit the wealth Gini index for the US, the optimal inflation rate is above 4%. This result is robust to alternative foundations of money demand equations.

Jel codes: E52, E58, J51, E24.

Keywords: inflation, monetary and fiscal policy, Ramsey plan, inequality.

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1 Introduction

Over the last 30 years income and wealth inequality have increased in developed economies and the issue of wealth ownership concentration has come to the forefront (Saez and Zucman (2014)). Concern has also grown for the distributive effects of monetary policies (Galbraith et al (2007), Coibion et al (2012)) and for the apparently adverse effects of Central Banks actions on inequality.[1]

A widely popular argument in favour of price stability is the asymmetric incidence of the inflation tax when wealth is unevenly distributed and portfolio composition of poorer households is skewed towards a larger share of money holdings, so that the inflation tax burden would disproportionately fall on the poor (Erosa and Ventura (2002); Boel and Camera (2009); Schmitt-Grohé and Uribe (2011)). In fact, this is the key justification for endorsing price stability as a contribution to reducing inequality and poverty.[2]

In the paper we reconsider the issue of inflation optimality in a model where distributional issues arise because wealth holdings are concentrated in the hands of few households. We modify a standard DSGE model by introducing Limited Asset Market Participation (LAMP henceforth), in the form of a distinction between holders of interest bearing assets (unconstrained agents) and agents who only own money (constrained agents), as in Coenen et al (2008).

Heterogeneity in the access to the market for interest bearing assets is a salient feature of the data. While the majority of US households (92.5%) own transaction accounts (including checking, savings, money market deposit accounts and money market mutual funds), only a small minority hold other financial assets, such as stocks, bonds, investment funds and other managed assets (which are held by less than the 20% of households).[3] The major long term saving vehicle for US households are retirement accounts, held by the 50.4% of families. Excluding such important

[1]See, e.g., "Inequality. A monetary policy for the 1%", The Economist, London, Jul 5th 2012, and Belotti and Farley, "Fed policies: Income inequality has been one of the results", San Josè Mercury News, April 4th 2014.

[2]For instance, in a speech at the International Day for the Eradication of Poverty, Intergroup "Extreme Poverty and Human Rights, Fourth World Committee" event, held on October, 17th 2012 at the European Parliament in Brussels Benoît Cœuré, Member of the Executive Board of the ECB, stated that "*...poorer households tend to hold a larger fraction of their financial wealth in cash, implying that both expected and unexpected increases in inflation make them even poorer. In addition, monetary policy shocks and surprise inflation can have an impact on inequality through other sources of income. Income from labour and the unemployment of less-skilled workers tend to be adversely affected to a disproportionate degree during recessions. All in all, recent studies suggest that a higher inflation rate is accompanied by greater income inequality*".

[3]These statistics refer to the 2010 Survey of Consumer Finances.

differences in wealth holdings from macroeconomic models implies that the distributional effects of policies and shocks are also ignored.

Assuming wealth inequality is not sufficient to identify a potential role for monetary policy and inflation. In principle, a redistributive scheme taxing wealth returns to subsidize constrained households should make monetary policy redundant. To obtain non-trivial results, we assume that the policymaker set of fiscal tools is incomplete. Further, in line with previous contributions on Ramsey-optimal monetary and fiscal policies, we posit that firms monopoly profits are not taxed. In representative agent models such as Schmitt-Grohé and Uribe (2004a) this assumption is made because distortionary taxation does not warrant an increase in the optimal inflation rate unless factor incomes are suboptimally taxed (see Schmitt-Grohé and Uribe (2011); and references cited therein). Maintaining this assumption here seems appropriate for several reasons. First, anecdotal evidence suggests that large multinationals such as Apple and Google are subject to a negligible tax burden on their worldwide profits (Fuest et al (2013)). This has become a primary concern for international agencies (OECD (2013a), OECD (2013b) and European Commission (2012)). Second, a large literature documents that tax evasion and tax avoidance are related to firms rents.[4] Third, emphasys on firms ability to escape profit taxation is particularly relevant in our context, where concern for inequality motivates the Ramsey planner's decisions.

Optimal inflation analysis crucially depends on underlying assumptions concerning economic incentives to hold money balances. The classical approach assumes that real money balances are proportional to consumption, so that money holdings and consumption should be equally distributed among households (Schmitt-Grohé and Uribe (2004a)).

Erosa and Ventura (2002) adopt instead a cash/credit-transaction approach, where a fraction of transactions are undertaken using a credit technology that exhibits economies of scale. In this case inflation increases inequality because poor households realize a larger percentage of cash transactions relative to rich households.

The classical approach implies that the distribution of money holdings replicates distribution of consumption levels, whereas the cash/credit-transaction approach implies that money holdings are more equally distributed than consumption because richer households consume more but hold less money balances. According to Ragot (2014), these results are in sharp contrast with empirical evidence in countries

[4] Piketty and Saez (2012) point out that firms can escape taxation by increasing fringe benefits or by allowing managers/entrepreneurs consumption through the use of firms goods or the use of tax heavens. For a broader discussion see Slemrod and Yitzhaki (2002), Saez et al (2012) and Saez (2004).

like the US and Italy, where the observed distribution of money across households is similar to the distribution of financial wealth, suggesting that the transaction motive for holding money plays a limited role relative to financial motives. In fact Ragot (2014) shows that the Gini indexes for money and financial wealth holdings are replicated in an heterogeneous agent model where consumption transactions are subject to a cash in advance constraint and financial transactions are subject to fixed costs à la Baumol–Tobin. We label this characterization of money demand as the infrequent trading approach. To assess the implications of the infrequent trading approach to money demand, we follow Alvarez et al (2009), and assume that unconstrained agents can trade in financial markets only infrequently. Consistently with the facts discussed by Ragot (2014), this allows to obtain a distribution of money holding similar to that of financial wealth.

In our analysis, we compute the Ramsey steady state solution for our model. This adds to Schmitt-Grohé and Uribe (2004a) because the identification of the optimal financing mix (inflation and income tax) for a given level of public consumption takes into account the planner's concern for redistribution as a determinant of inflation. We take into account the alternative money demand specifications based on the consumption technologies discussed above, and account for unconstrained households' infrequent trading in the markets for interest-bearing assets.

We obtain the striking result that, irrespective of the money demand specification adopted in the model, the planner chooses a steady state inflation rate which is higher than under the representative agent assumption. In fact we obtain optimal inflation rates which are always above 4%. In contrast with received wisdom, the fundamental reason underlying this result is that expected inflation shifts the fiscal burden towards asset holders. This happens because, even if inflation is a regressive tax that falls proportionally more on the wealth holdings of the poor, it allows to tax consumption out of monopoly profits by exploiting the fact that money holdings are larger for wealthier individuals, as implied by the Gini index for money holdings. Thus, shifting the financing mix towards higher inflation allows to shift the overall burden of taxation towards asset holders. For reasons explained in the text, our results are strengthened when we assume that the Planner can levy distinct labor and capital income taxes.

Our results are related to those of Jin (2009) that considers a growth model with inequality in both wealth and skill levels. Interestingly, that paper finds that inflation tends to reduce inequality when the latter depends on wealth inequality, a result compatible with ours. Another paper related to ours is Da Costa and Werning (2008), who analyse the optimality of the Friedman rule in a model in which agents have different private-knowledge labor productivities and the planner has access to nonlinear labor income taxes. In that framework, zero inflation is always on the Pareto optimal frontier, which suggests that heterogeneity in non-labor incomes is necessary to introduce a role for inflation as a redistributive tool.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 defines the competitive equilibrium and the Ramsey optimal policy. Section 4 describes the results. Section 6 concludes.

2 The model

We consider an infinite-horizon production economy populated by a continuum of households $i \in [0;1]$. A mass $\theta \in [0,1]$ of agents (constrained agents, henceforth c) hold money balances but does not participate in the market for interest bearing assets, while a mass $1-\theta$ of agents (uncontrained agents, henceforth u) benefit from full participation to financial markets and own firms. As discussed in the introduction, we allow for the possibility that interest bearing assets are traded infrequently.

We assume that consumption purchases are subject to transaction costs, S^i. As mentioned above, we consider two characterizations of such costs. In the classical case (Sims (1994), Guerron-Quintana (2009), Altig et al (2011)) it is assumed that transaction costs

$$S_t^i = c_{t,i} s(\frac{P_{t,i}c_{t,i}}{M_{t,i}}), \quad s'(\frac{P_{t,i}c_{t,i}}{M_{t,i}}) > 0 \text{ for } \frac{P_{t,i}c_{t,i}}{M_{t,i}} > v_i^* \tag{1}$$

depend on money velocity, $v_i = \frac{c_{t,i}}{m_{t,i}}$. $c_{t,i}$ defines individual consumption, $m_{t,i} = \frac{M_{t,i}}{P_{t,i}}$ defines real money balances, and $P_{t,i}$ and $M_{t,i}$ respectively define the consumption price level and nominal money balances. The features of $s(\frac{P_{t,i}c_{t,i}}{M_{t,i}})$ are such that a satiation level of money balances ($v^* > 0$) exists where the transaction cost vanishes and, simultaneously, a finite demand for money is associated to a zero nominal interest rate. Following Schmitt-Grohé and Uribe (2004a) the transaction cost is parameterized as follows

$$s(\frac{P_t c_{t,i}}{M_{t,i}}) = A\frac{P_t c_{t,i}}{M_{t,i}} + \frac{B}{\frac{P_t c_{t,i}}{M_{t,i}}} - 2\sqrt{AB} \tag{2}$$

The alternative characterization based on the cash/credit-transaction approach (Erosa and Ventura (2002) and Albanesi (2007)), posits instead that a credit technology exists, and that each consumer chooses the share of purchases made using the credit technology, a_t^i, taking into account that credit transactions imply a cost

$$S_t^i = s\left(a_t^i\right) = \int_0^{a_t^i} \gamma_1 \left(\frac{j}{1-j}\right)^{\gamma_2} dj. \tag{3}$$

whereas a cash-in-advance constraint exists for the remaining monetary transactions:

$$(1-a_t^i)c_t^i \leq m_t^i \tag{4}$$

As shown by Erosa and Ventura (2002), this formulation implies that the consumption money velocity is decreasing in consumption and the transaction technology exhibits economies of scale.

Monopolistic competition and nominal rigidities characterize product and labor markets.

The government finances an exogenous stream of expenditures by levying distortionary income taxes and by printing money. Optimal policy is set according to a Ramsey plan.

2.1 Households

All households share a KPR utility function of the form

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, l_t^i\right); \quad u\left(c_t^i, l_t^i\right) = \ln c_t^i + \eta \ln\left(1 - l_t^i\right) \tag{5}$$

where $\beta \in (0,1)$ is the intertemporal discount rate, l_t^i denotes worked hours and c_t^i is consumption. Each household supplies a differentiated labor type.

2.2 Labor Packers

Labor packers buy the differentiated labor services from households and produce the aggregate labor bundle, l_t^d, that is then rented to firms. They operate under perfect competition and solve the following problem:

$$\max \; w_t l_t^d - \int_0^1 w_t^i l_t^i di$$

s.t.

$$l_t^d = \left(\int_0^1 \left(l_t^i\right)^{\rho_w} di\right)^{\frac{1}{\rho_w}} \tag{6}$$

where $\rho_w > 1$ is the steady state inverse wage mark-up that prevails if wages are flexible. This generates a downward sloping demand function for labor type i

$$l_t^i = l_t^d \left(\frac{w_t^i}{w_t} \right)^{\frac{1}{\rho_w - 1}} \tag{7}$$

and the wage index

$$w_t = \left(\int_0^1 \left(w_t^i \right)^{\frac{\rho_w}{\rho_w - 1}} di \right)^{\frac{\rho_w - 1}{\rho_w}}. \tag{8}$$

As we show below, all agents of the same type work the same number of hours and receive the same wage. For this reason equation (8) can be rewritten as follows

$$w_t = \left[(1-\theta) \left(w_t^u \right)^{\frac{\rho_w}{\rho_w - 1}} + \theta \left(w_t^c \right)^{\frac{\rho_w}{\rho_w - 1}} \right]^{\frac{\rho_w - 1}{\rho_w}}$$

Similarly, equation (6) can be rewritten as

$$l_t^d = \left[(1-\theta) \left(l_t^u \right)^{\rho_w} + \theta \left(l_t^c \right)^{\rho_w} \right]^{\frac{1}{\rho_w}}. \tag{9}$$

2.3 Unconstrained consumers

Our modelling strategy for capturing the effects of infrequent trading on money demand is based on the inventory-theoretic model of money demand developed in Alvarez et al (2009). Unconstrained households are assumed to possess a bank and a brokerage account. In the bank account they hold money balances and receive monetary payment for their wage bill. In the brokerage account they hold all other types of wealth. Consumption decisions involving monetary transactions can only occur by withdrawing money balances from the bank account. Transfers of funds between the two accounts occur every N periods, so that in each period only a share $1/N$ of unconstrained agents can transfer funds. Indexing each agent by $p_t \in [0, N-1]$, i.e. the number of periods left at time t before a transfer can be made, for each type p_t the bank account evolves as follows:[5]

$$\begin{gathered} c_t^u(p_t) + S_t^u(p_t) + m_t^u(p_t) = \\ (1-\tau_t) \left[w_t^u l_t^u \right] + \frac{m_{t-1}^u(p_t)}{\pi_t} - \frac{\xi_w}{2} l_t^u \left(\frac{w_t^u \pi_t}{w_{t-1}^u} - 1 \right)^2 + x_t(p_t) \end{gathered} \tag{10}$$

[5]Notice that agents of type p at time t (p_t) were of type $p+1$ at time t-1, which implies that p_t and $(p+1)_{t-1}$ index the same agent. This is true for all agents apart from type N-1 agents, for whom the type was 0 at time t-1.

where τ_t is the income tax rate; $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, parameter ξ_w defines standard nominal wage adjustment costs (Rotemberg), and $x_t(p_t)$ is the transfer between the brokerage account and the bank account.[6] Note that, due to infrequent trading, $x_t(p_t)$ is constrained to zero for all $p_t \neq 0$.

Similarly, the brokerage account evolves as follows:

$$k_t(p_t) + b_t(p_t) = \left\{ \begin{array}{c} (1-\tau_t)\left(r_t^k k_{t-1}(p_t)\right) + (1-\delta)\, k_{t-1}(p_t) + \\ +\tau_t \delta k_{t-1}(p_t) + d_t + \frac{R_{t-1} b_{t-1}(p_t)}{\pi_t} - x_t(p_t) \end{array} \right\} \tag{11}$$

where d_t are real firms profits; R_t is the gross nominal interest rate, b_t^u is the real amount of a nominally riskless bond that pays one unit of currency in period $t+1$.[7] k_t^u denotes the capital stock , r_t^k is the real rental rate of capital and δ is the depreciation rate.[8] In expression (11) τ_t is replaced by τ_t^k when considering the model with distinct capital and labor taxes.

Following Alvarez et al (2009), we assume that the initial financial wealth distribution among unconstrained agent types is such that the marginal value of a dollar delivered on the brokerage account in the initial period is the same for everyone, which greatly helps aggregation because, as we remark below, it is no longer necessary to keep track of wealth distribution to determine equilibrium outcomes.[9] Infrequent trading, however, still implies that allocations in each period are influenced by the consumption and money holdings decisions, that will differ across the N p_t-type agents. Another simplifying assumption is made concerning wage-setting behavior. Our characterization of the labor market, where each household supplies a differentiated labor type, implies that the individual marginal rate of substitutions, and therefore individual wages, may be different. When trading is frequent, $N = 1$, it is intuitively obvious that unconstrained households wage-setting decisions are identical. This cannot happen under infrequent trading, because consumption decisions of p_t-type agents are different.[10] To limit computational problems and to facilitate comparison with the classical and cash/ credit technology cases, we assume that individual wage setting decisions of unconstrained households are delegated to

[6] As we explain below, hours and wages are not indexed by p because all unconstrained agents work the same number of hours and receive the same wage.

[7] To facilitate comparison with previous contributions (Schmitt-Grohé and Uribe (2004a)), nominal riskless bonds are not taxed.

[8] We assume that the government grants a depreciation tax allowance.

[9] In addition, Alvarez et al (2009) also assume that unconstrained households can invest in state-contingent assets, so that they can insulate their accumulation of wealth in the brokerage account from idyosincratic shocks. Given the deterministic nature of our model, we do not need contingent assets.

[10] Alvarez et al (2009) assume an exogenous labor (income) endowment for each household.

a labor union that maximises a weighted average of (5) for the N p_t-type agents. As a result the wage rate is unique and unconstrained households supply the same number of hours.[11]

Each unconstrained household maximizes (5) subject to the bank- and brokerage-account constraints (10, 11), to (7), and to the consumption transaction technology. In addition, the following inequalities must be satified

$$c_t^u(p_t) \geq 0, M_t^u(p_t) \geq 0, k_t^u\,(p_t) \geq 0, l_t^u \in [0,1]$$

Given the functional form of the utility function, the production function and the functional form of the transaction technologies, the non-negativity constraints on consumption, capital and money balances are always non-binding and we can ignore them. Finally, we impose on the standard no-Ponzi game condition on the accumulation of bonds:

$$\lim_{T\to\infty} \beta^{T-t} b_T^u(p_t) \geq 0. \tag{12}$$

2.3.1 FOCs under the classical approach

In the classical case asset trading occurs every period ($N = 1$, that is $p = 0$ for all unconstrained agents) and the consumption transactions technology is defined in (2). In this case, the Lagrange multiplier λ^u on the bank account constraint, (10), is always equal to that on the brokerage account constraint, (11).[12] The first order conditions for the representative unconstrained agent are:

$$\lambda_t^u = \frac{u_c\left(c_t^u, l_t^u\right)}{1 + s(\frac{c_t^u}{m_t^u}) + \frac{c_t^u}{m_t^u} s'(\frac{c_t^u}{m_t^u})} \tag{13}$$

$$\lambda_t^u = \beta\left(\frac{\lambda_{t+1}^u R_t}{\pi_{t+1}}\right) \tag{14}$$

$$\begin{gathered}\frac{\left(w_t^u \rho_w + mrs_t^u\right)\left(\frac{w_t^u}{w_t}\right)^{\frac{1}{\rho_w - 1}}}{1-\rho_w} + \xi_w \frac{w_t^u \pi_t}{w_{t-1}^u}\left(\frac{w_t^u \pi_t}{w_{t-1}^u} - 1\right) = \\ \beta\left[\frac{l_{t+1}^u \lambda_{t+1}^u}{l_t^u \lambda_t^u}\xi_w\left[\frac{w_{t+1}^u \pi_{t+1}}{w_t^u}\left(\frac{w_{t+1}^u \pi_{t+1}}{w_t^u} - 1\right)\right]\right]\end{gathered} \tag{15}$$

[11] As suggested by one referee, replacing the utility function (5) with a GHH preference function would remove this problem. We do not follow this alternative route here in order to facilitate comparison with previous contributions.

[12] This can be easily seen by taking the first order condition with respect to $x(p)$.

$$\lambda_t^u = \beta \left\{ \lambda_{t+1}^u \left[(1 - \tau_{t+1}) \left(r_{t+1}^k - \delta \right) + 1 \right] \right\} \tag{16}$$

$$1 - \left[\frac{\beta}{\pi_{t+1}} \frac{\lambda_{t+1}^u}{\lambda_t^u} \right] = s'(\frac{c_t^u}{m_t^u}) \left(\frac{c_t^u}{m_t^u} \right)^2 \tag{17}$$

As in Schmitt-Grohé and Uribe (2004a) condition (13) states that the transaction cost introduces a wedge between the marginal utility of consumption, $u_c\left(c_t^u, l_t^u\right)$, and the marginal utility of wealth, λ_t^u, that vanishes only if $\frac{c_t^u}{m_t^u} = v^*$. Equation (15) is the wage-setting equation. Absent wage stickiness the real wage is a mark-up over the marginal rate of substitution between consumption and leisure, $mrs_t^u = \frac{u_l(c_t^u, l_t^u)}{(1-\tau_t)\lambda_t^u}$. In the presence of wage stickiness, the real wage also depends on actual and expected nominal wage growth. Equation (14) is a standard Euler condition, while equation (16) is a standard Euler condition for capital. The income tax distorts capital accumulation at the margin, while this is not the case for inflation since the rental rate of capital is not set in advance. Equation (17) implicitly defines the money demand function. Taking into account (14), condition (17) takes the familiar form

$$1 - \frac{1}{R_t} = s'(\frac{c_t^u}{m_t^u}) \left(\frac{c_t^u}{m_t^u} \right)^2$$

where the nominal interest rate defines the opportunity cost of holding money.[13] Notice that in equation (16) τ_t is replaced by τ_t^k when considering distinct taxes on capital and labor.

2.3.2 FOCs under the cash/credit-transaction approach

Mantaining the assumption of frequent trading, $N = 1$ and $p = 0$ for all unconstrained households, the only difference with the classical case is that the transaction technology is now defined by conditions (3) and (4). Assuming that constraint (4) is always binding, the first order conditions are:

$$\lambda_t^u = \frac{1}{c_t^u} - \phi_t^u (1 - a_t^u) \tag{18}$$

$$\phi_t^u c_t^u = \lambda_t^u \gamma_1 \left(\frac{a_t^u}{1 - a_t^u} \right)^{\gamma_2} \tag{19}$$

[13] Notice that the nominal net interest rate must be non-negative, i.e. $R_t \geq 1$, $\forall t$.

$$\phi_t^u = \lambda_t^u - \beta\left[\frac{\lambda_{t+1}^u}{\pi_{t+1}}\right] \tag{20}$$

where ϕ^u is the Lagrange multiplier on the cash in advance constraint (4). Equations (18), (19) and (20) are the first order condition with respect to consumption, c^u, the share of purchases made with credit, a^u, and money holdings, m^u, respectively. Additional first order conditions have the same form as in the classical case, (14), (15) and (16).

2.3.3 FOCs under the infrequent trading approach

To facilitate comparison with the classical approach case, we maintain that the consumption transactions technology is defined in (2). Relative to the classical case, in the infrequent trading approach the traditional Lagrange multiplier λ_t^u is replaced by two multipliers on the bank (eq. 10) and brokerage (eq. 11) accounts, $\lambda_t^u(p_t)$ and ζ_t^u respectively. It can be easily shown that the multiplier ζ_t^u is the same for all types, and the following first order conditions must hold for each unconstrained agent, irrespective of the time left before he can access his brokerage account:[14]

$$\zeta_t^u = \beta\left(\frac{\zeta_{t+1}^u R_t}{\pi_{t+1}}\right) \tag{21}$$

$$\zeta_t^u = \beta\left\{\zeta_{t+1}^u\left[\left(1-\tau_{t+1}\right)\left(r_{t+1}^k - \delta\right)+1\right]\right\} \tag{22}$$

Then, N first order conditions, one for each type, identify the lagrange multipliers on the bank accounts

$$\lambda_t^u(p_t) = \frac{u_c\left(c_t^u(p_t), l_t^u\right)}{1 + s\left(\frac{c_t^u(p_t)}{m_t^u(p_t)}\right) + \frac{c_t^u(p_t)}{m_t^u(p_t)} s'\left(\frac{c_t^u(p_t)}{m_t^u(p_t)}\right)} \tag{23}$$

Note that $\lambda^u(p_t) \neq \zeta^u$ for $p_t > 0$, but for agents who have immediate access to the brokerage account, i.e. who are characterized by $p_t = 0$, the two multipliers are equal. In this case an additional first order condition for type 0, taken with respect to $x_t(p)$ in (10) and in (11) yields

$$\lambda_t^u(p_t = 0) = \zeta_t^u \tag{24}$$

N first order conditions govern individual money demand of unconstrained agents:

[14] In particular, the first order condition with respect to bonds is $\zeta_t^u(p_t) = \beta\zeta_{t+1}^u(p_t)\frac{R_t}{\pi_{t+1}}$. Given our assumptions regarding the initial wealth distribution, $\zeta_0^u(p_0) = \zeta_0^u$ for all p_0, and, as a consequence, the bond euler equation implies that $\zeta_t^u(p_t) = \zeta_t^u$ for all t.

$$1 - \left[\frac{\beta}{\pi_{t+1}} \frac{\lambda_{t+1}^u(p_t)}{\lambda_t^u(p_t)}\right] = s'(\frac{c_t^u(p_t)}{m_t^u(p_t)}) \left(\frac{c_t^u(p_t)}{m_t^u(p_t)}\right)^2 \tag{25}$$

Average values for consumption, money holdings and marginal utility of unconstrained households are as follows:

$$c_t^u = \sum_{p=0}^{N} \frac{1}{N} c_t^u(p)$$

$$m_t^u = \sum_{p=0}^{N} \frac{1}{N} m_t^u(p)$$

$$\lambda_t^u = \sum_{p=0}^{N} \frac{1}{N} \lambda_t^u(p) \tag{26}$$

Using (26), the wage setting equation for unconstrained households is the same as in equation (15). Also in equation (22) τ_t is replaced by τ_t^k when considering distinct taxes on capital and labor. Furthermore, notice that in the classical as well as in the cash/credit and in the infrequent trading cases, optimality for unconstrained consumers also include the transversality condition for wealth holdings, which guarantees zero wealth as time approaches infinity.

2.4 Constrained consumers

The representative constrained household maximizes (5) subject to the flow budget constraint[15]

$$c_t^c \left(1 + s(\frac{P_t c_t^c}{M_t^c})\right) + \frac{M_t^c}{P_t} = (1 - \tau_t)\, w_t l_t^c + \frac{M_{t-1}^c}{P_t} - \frac{\xi_w}{2} l_t^c \left(\frac{w_t^c \pi_t}{w_{t-1}^c} - 1\right)^2 \tag{27}$$

and to (7).

2.4.1 FOCs under the classical approach

Under the consumption transactions technology defined in (2), the first-order conditions are:

[15] For the reasons discussed above the constraints $c_t^c \geq 0, M_t^c \geq 0$ and $l_t^u \in [0, 1]$ are non binding.

$$\lambda_t^c = \frac{u_c\left(c_t^c, l_t^c\right)}{1 + s(\frac{c_t^c}{m_t^c}) + \frac{c_t^c}{m_t^c} s'(\frac{c_t^c}{m_t^c})} \tag{28}$$

$$\frac{\left(w_t^c \rho_w + \frac{u_l(c_t^c, l_t^c)}{(1-\tau_t)\lambda_t^c}\right)\left(\frac{w_t^c}{w_t}\right)^{\frac{1}{\rho_w - 1}}}{1-\rho_w} + \xi_w \frac{w_t^c \pi_t}{w_{t-1}^c}\left(\frac{w_t^c \pi_t}{w_{t-1}^c} - 1\right) = \tag{29}$$

$$\beta\left[\frac{l_{t+1}^c \lambda_{t+1}^c}{l_t^c \lambda_t^c} \xi_w \left[\frac{w_{t+1}^c \pi_{t+1}}{w_t^c}\left(\frac{w_{t+1}^c \pi_{t+1}}{w_t^c} - 1\right)\right]\right]$$

$$1 - \left[\frac{\beta}{\pi_{t+1}} \frac{\lambda_{t+1}^c}{\lambda_t^c}\right] = s'(\frac{c_t^c}{m_t^c})\left(\frac{c_t^c}{m_t^c}\right)^2 \tag{30}$$

Constrained households' money demand is a negative function of expected inflation and a positive function of the expected increase in the marginal utility of wealth (eq. 30). Two important remarks are in order here. First, under the classical case for unconstrained households, the functional forms in (30) and (17) are identical because both households types define their current money-to-consumption ratio taking into account the discounted payoff from carrying money into the next period. However the implied money-to-consumption ratios for the two types are identical only in steady state, when $\frac{\lambda_{t+1}^u}{\lambda_t^u} = \frac{\lambda_{t+1}^c}{\lambda_t^c} = 1$ and $R = \frac{\pi}{\beta}$. Outside the steady state the money-to-consumption ratio of unconstrained households manage to equalize discounted returns on money and discounted returns on bonds, whereas this possibility is precluded to constrained households. As a result consumption dynamics of uncontrained agents depend on the expected real interest rate, whereas constrained households react to expected inflation rate.

2.4.2 FOCs under the cash/credit-transaction approach

Similarly to the unconstrained households case, the first order conditions in the cash/credit case are:

$$\lambda_t^c = \frac{1}{c_t^c} - \phi_t^c(1 - a_t^c)$$

$$\phi_t^c c_t^c = \lambda_t^c \gamma_1 \left(\frac{a_t^c}{1 - a_t^c}\right)^{\gamma_2}$$

$$\phi_t^c = \lambda_t^c - \beta\left[\frac{\lambda_{t+1}^c}{\pi_{t+1}}\right]$$

The wage setting equation is the same as in the classical case, (29).

2.5 Intermediate Firms

The representative intermediate firm produces a differentiated good $z \in (0,1)$ under a standard Cobb-Douglas technology

$$y_t(z) = l_t(z)^{\alpha} k_{t-1}(z)^{1-\alpha} \tag{31}$$

and faces a downward sloping demand function,

$$y_t(z) = y_t^d \left(\frac{P_t(z)}{P_t} \right)^{\frac{1}{\rho-1}} \tag{32}$$

We assume a sticky price specification based on a Rotemberg (1982) quadratic cost of nominal price adjustment:

$$\frac{\xi_p}{2} y_t^d \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 \tag{33}$$

where $\xi_p > 0$ is a measure of price stickiness. In line with Ascari et al (2011), we assume that the re-optimization cost is proportional to output.

In a symmetrical equilibrium the price adjustment rule satisfies:

$$\frac{(\rho - mc_t)}{1-\rho} + \xi_p \pi_t (\pi_t - 1) = \beta \left[\frac{y_{t+1}\lambda_{t+1}^u}{y_t \lambda_t^u} \xi_p \left[\pi_{t+1} (\pi_{t+1} - 1) \right] \right] \tag{34}$$

where mc_t are the real marginal costs.

Cost minimization implies that the following two equations hold:

$$w_t = \alpha mc_t \left(\frac{l_t}{k_{t-1}} \right)^{\alpha-1} \tag{35}$$

$$r_t^k = (1-\alpha) mc_t \left(\frac{l_t}{k_{t-1}} \right)^{\alpha} \tag{36}$$

From (32) it would be straightforward to show that $\frac{1}{\rho} = \mu^p$ defines the price markup that obtains under flexible prices. Firm profits are

$$d_t = l_t^{\alpha} k_{t-1}^{1-\alpha} - w_t l_t^d - r_t^k k_{t-1} - \frac{\xi_p}{2} l_t^{\alpha} k_{t-1}^{1-\alpha} (\pi_t - 1)^2 \tag{37}$$

2.6 Final Good Firms

Final good firms buy differentiated goods from intermediate firms and produce an aggregated good, y_t^d, which can be used both for private and public consumption and for investment. They operate under perfect competition and solve the following problem:

$$\max \; P_t y_t^d - \int_0^1 P_t(z)\, y_t(z)\, dz$$

s.t.

$$y_t^d = \left(\int_0^1 y_t(z)^\rho \, dz \right)^{\frac{1}{\rho}}$$

The optimality conditions are equation (32) and the price index $P_t = \left(\int_0^1 P_t(z)^{\frac{\rho}{\rho-1}} \, dz \right)^{\frac{\rho-1}{\rho}}$.

2.7 Government budget

The government supplies an exogenous and unproductive amount of public good g_t. Government financing is obtained through an income tax, money creation and issuance of one-period, nominally risk free bonds. The government flow budget constraint is then given by

$$R_{t-1} \frac{b_{t-1}}{P_t} + g_t + t_t = \tau_t \left(w_t l_t^d + r_t^k k_{t-1} \right) - \tau_t \delta k_{t-1} + \frac{M_t - M_{t-1}}{P_t} + \frac{b_t}{P_t} \tag{38}$$

2.8 Aggregation

Equations (39)-(44) define aggregate consumption, aggregate hours, aggregate real money balances, bonds, profits, aggregate capital and total output:

$$c_t = (1-\theta)\, c_t^u + \theta c_t^c \tag{39}$$

$$m_t = (1-\theta)\, m_t^u + \theta m_t^c \tag{40}$$

$$b_t^u = \frac{b_t}{1-\theta} \tag{41}$$

$$d_t^u = \frac{d_t}{1-\theta} \tag{42}$$

$$k_t^u = \frac{k_t}{1-\theta} \tag{43}$$

$$y_t^d = (1-\theta)\, c_t^u + (1-\theta) S_t^u + \theta c_t^c + \theta S_t^c + k_t - (1-\delta)\, k_{t-1} + \tag{44}$$
$$g_t + \frac{\xi_p}{2} y_t \left(\pi_t - 1\right)^2 + (1-\theta)\, \frac{\xi_w}{2} l_t^u \left(\frac{w_t^u \pi_t}{w_{t-1}^u} - 1\right)^2 + \theta \frac{\xi_w}{2} l_t^c \left(\frac{w_t^c \pi_t}{w_{t-1}^c} - 1\right)^2$$

Note that the concentration of capital and public bond holdings and of profits is increasing in θ. Thus incomes and consumption inequality also increases in θ.

3 Equilibrium and Ramsey policy

3.1 Competitive Equilibrium

Definition 1 *A competitive equilibrium is a set of plans*

$$\left\{c_t^u(p), c_t^c, c_t, l_t^u, l_t^c, l_t, \lambda_t^u(p), \lambda_t^c, mc_t, \pi_t, w_t, w_t^u, w_t^c, m_t^u(p), m_t^c, m_t, y_t, b_t, R_t, k_t, r_t^k, \tau_t\right\}_{t=0}^{\infty},$$

that, given initial values $\left\{m_{-1}^u(p), m_{-1}^c, m_{-1}, b_{-1}, k_{-1}\right\}$*, satisfies the no-Ponzi game condition (12), the non-negativity constraint* $R_t \geq 1$ *and the competitive equilibrium conditions associated with each case we study reported in section 2.*

3.2 Ramsey Optimal Policy

Private sector choices only identify a continuum of competitive equilibria indexed by the policy plan $\{R_t, \tau_t\}_{t=0}^{\infty}$. The Ramsey planner sets the policy plan to maximise the social welfare function characterized below, under the constraint that the resulting allocation is itself a competitive equilibrium or, in other words, that the private sector equilibrium conditions are satisfied.[16]

Definition 2 *A Ramsey optimal policy is the policy plan* $\{R_t, \tau_t\}_{t=0}^{\infty}$ *that attains the maximum of the following additive social welfare function*

$$W = \sum_{t=0}^{\infty} \beta^t \left(\left(1 - X^R\right) \frac{1}{N} \sum_{p=0}^{N} u\left(c_t^u(p_t), l_t^u\right) + X^R u\left(c_t^c, l_t^c\right) \right) \tag{45}$$

[16]The fact that the competitive equilibrium conditions must be satisfied is what distinguish the Ramsey problem from the social planner problem, in which allocations are decided by the planner directly, and no private sector decision has to be satisfied.

where X^R is the weight given to constrained households utility, under the constraint that the resulting allocation is a competitive equilibrium.[17]

The plan will satisfy the no Ponzi game condition:

$$\lim_{T\to\infty} \beta^{T-t} b_T \leq 0 \tag{46}$$

The Ramsey program is non-stationary, in the sense that in the initial period the Ramsey planner has an incentive to generate surprise movements in inflation or taxes. We neglect these non-stationary transitory components and concentrate on the time-invariant long run outcome, the Ramsey steady state. This procedure is common in the literature (see for instance Schmitt-Grohé and Uribe, 2004a). The first order conditions of the Ramsey problem are reported in a technical appendix available on-line. [18]

3.3 Calibration

The time unit is a year[19] and we set the subjective discount rate β at 0.96 to be consistent with a steady-state real rate of return of 4 percent per year. Following Schmitt-Grohé and Uribe (2004a), public consumption is 19% of GDP. Calibration of price markups is crucial to define the amount of untaxed profits. Jaimovich and Floetotto (2008) report that estimates of gross markups in value added data range between 1.2 and 1.4. We set ρ to obtain a price markup of 1.2. The wage markup is set at the same level, which is conservative relative to previous studies for the US (Smets and Wouters (2007)).[20] Parameter η is calibrated to obtain that both households groups work 20% of their time in steady state irrespective of the assumption made about financial and monetary frictions. We set α to 64% and δ to 8%,

[17] When $X^R = \theta$, the planner is utilitarian. We will consider this case throughout the paper. We also consider the cases in which X^R is 0.9 and 0.1, when discussing results.

[18] Since the analitycal derivation of the first order conditions of the Ramsey plan is cumbersome, we compute them using symbolic Matlab routines. The steady state of the Ramsey program is obtained using the OLS approach suggested in Schmitt-Grohé and Uribe (2011).

[19] In setting the time unit to be a year, we follow the literature. See, e.g., Schmitt-Grohé and Uribe (2004a). Tax rate adjustments require a political process that may take time. As a consequence, it may be difficult to change them at quarterly frequency. We feel one year is a much more realistic time lenght. No fundamental result of the paper depends on this assumption.

[20] Note that in our framework profit and wage markups play quite different roles. In fact the former are necessary to obtain the untaxed profits that eventually generate the planner's incentive to inflate. To this end, wage markups are unconsequential because labor incomes are fully taxed.

as in Erosa and Ventura (2002). Finally, we set the annualized Rotemberg price and wage adjustment costs (ξ_p and ξ_w) to 4.375, as Schmitt-Grohé and Uribe (2004a). As explained in the latter, these values are obtained from the estimation of a linear New-Keynesian Phillips curve for the US economy (see Sbordone (2002)).

As usual in Ramsey problems where the government issues non-state contingent nominally riskless debt, the Ramsey steady state is indeterminate. This is easily seen by taking the derivative of the Lagrangian of the Ramsey problem with respect to public debt, b_t, that is

$$\phi_{b,t} = \beta \frac{\phi_{b,t+1} R_t}{\pi_{t+1}} \tag{47}$$

where $\phi_{b,t}$ is the Lagrange multiplier on the government budget constraint. Evaluated at the steady state, both (47) and (14) become

$$\frac{1}{R} = \frac{\beta}{\pi}$$

As a consequence, it is not possible to pin down a steady state public debt level and we must calibrate it. In fact, while the transition from the initial level of public debt to its steady state is endogenous, the latter is not independent from the former, which implies that there is a continuum of steady states for each choice of initial public debt level. Following Cogan et al (2013), we set steady state public debt at 60% of GDP. To highlight the effect of steady state public debt on the optimal rate of inflation, we also consider the case where the debt-to-GDP ratio is 80%. $X^R = \theta$ is our benchmark assumption, but we also discuss the cases $X^R = 0.9$ and $X^R = 0.1$.

Given our emphasis on alternative money demand characterizations, calibrations of transaction costs and of frequency in access to brokerage accounts require a detailed discussion. To begin with, note that consumption money velocity has been 4.58 on annual basis in the US since 1947 when taking into account M1 and 1.04 in the case of M2.[21] Simple regressions show that the semi-elasticity of money demand to the nominal interest rate is around -0.05 in a univariate regression and around -0.01 when controlling for income, with little difference between M1 and M2. Over the period 1947-2014, the average annual inflation rate measured as the rate of change in the CPI index has been 3.92%.[22] Our approach consists in choosing parameter values that allow to obtain a steady state money velocity between 1.04

[21] In the case of M2, the data go back to 1980

[22] All data are taken from FRED. Money velocity was computed using data on personal consumption expenditures on non-durables and services. As for the money demand elasticity regression, we used the secondary market interest rate on three months T bills as a measure of the nominal interest rate.

and 4.58 and a steady state money demand elasticity between -0.05 and -0.01, when trend inflation is 3.92%.

Parameters A and B in (2) are set at 0.011 and 0.075 respectively, as in Schmitt-Grohé and Uribe (2004a). As shown in Table 2, under the classical approach, this allows to obtain a consumption money velocity of 3.71 when we set the fraction of constrained agents, $\theta = 0.8$, to replicate the wealth Gini index of the US economy (see Quadrini and Ríos-Rull (1997)). In this case, the model implies that the money demand semi-elasticity to the nominal interest rate is -0.027.

The empirical evidence discussed in Alvarez et al (2009) shows that agents withdraw funds from the brokerage account every 24 to 36 months on average. Given the annual frequency chosen for our model, this implies that we consider $N = 2$ and $N = 3$, to be evaluated against the case $N = 1$ that holds under the classical and under the cash/credit transaction models. To ease comparison with the classical money demand case, we calibrate parameters A and B to 0.011 and 0.075 also in the infrequent trading case. Table 2 shows that in this experiment, money velocity and money demand elasticity remain within the range of values found in the data.[23] The two figures are somewhat lower than in the classical case.

The cash/credit technology is defined by parameters γ_1 and γ_2. We set them respectively to 0.0007 and 3.13 to obtain a money velocity and money demand elasticity equal to 3.71 and -0.027, as under the classical approach. With this calibration, we obtain that the fractions of monetary transactions amount to 22% and 32% for unconstrained and constrained agents repectively.

Note that also under the infrequent trading and the cash/credit experiments, the share of constrained agents that allows to fit the wealth Gini index is 0.8.

4 Ramsey Steady State

To understand steady state results one should bear in mind that in this model inflation has several effects: i) it redistributes wealth across households due to heterogeneity in money holdings, ii) it induces price and wage nominal adjustment costs, iii) it allows to reduce distortionary taxation for any given amount of public consumption. Our effort here is to clarify the relevance of each of these effects. To identify the importance of redistributive effects we benchmark our results against representative agent version of our models. We shall also present our results at different levels of steady state debt (See Table 3).

[23]This holds also for the case N=2, which is not shown in Table 2.

Parameters	Values	Description
β	0.96	Discount Factor
α	0.64	Capital Share
δ	0.08	Depreciation Rate
A	0.011	Trans. Cost Parameter
B	0.075	Trans. Cost Parameter
ρ	1/1.2	Inverse Price Mark-up
ρ_w	1/1.2	Inverse Wage Mark-up
ξ_p	4.375	Rotemberg Par. on Prices
ξ_w	4.375	Rotemberg Par. on Wages
$\frac{g}{y}$	0.19	Public Consumption over GDP
$\frac{b}{y}$	0.6-0.8	Public Debt over GDP
N	1-2-3	Frequency of financial trading
γ_1	0.0007	Credit technology
γ_2	3.13	Credit technology

Table 1: Calibration

Statistics	US Data	Classical	Cash/Credit	Infr Trading N=3
Velocity	1.04-4.58	3.71	3.71	1.74
Gini Wealth	0.78	0.78	0.78	0.78
Gini Money	0.78	0.35	0.25	0.64
Money Demand Elasticity	-0.05/-0.01	-0.027	-0.027	-0.016

Table 2: Data and model statistics

Classical case Optimal inflation is almost nil when the redistribution motive is absent ($\theta = 0$),[24] and it reaches 4.3% when we match the weath Gini index for the US. Higher inflation has no direct effect on capital accumulation in steady state (see *16*), but it allows to reduce the distortionary effect of the income tax. Wages grow due to the productivity gains associated to the increase in the capital stock and to the direct effect of the tax reduction, notwithstanding the (limited) reduction in wage mark-ups due to inflation. Note that price and nominal wage adjustment costs limit the planner's incentive to exploit inflation in order to reduce the tax rate.[25] Implementing the optimal combination of inflation and income tax rate instead of the optimal solution for $\theta = 0$ implies that $\Delta c^u = -0.16\%$ and $\Delta c^c = 0.92\%$. Unconstrained agents hold 70.29% of the money they would hold under the Friedman rule, and constrained agents 71.62%.[26]

An increase in steady state debt ($\frac{b}{y} = 0.8$) induces the planner to raise inflation and to lower tax distortions even when $\theta = 0$.[27] However the incentive to inflate increases in θ because a larger debt stock generates a fiscal redistribution and an increase in consumption inequality that is larger when wealth holdings are more concentrated, i.e. when θ increases.

LAMP generates large differences in preferred inflation and income tax rates between unconstrained and constrained households. In fact our calculations show that a planner who cares predominantly about unconstrained agents' welfare ($X^R = 0.1$) sets $\pi_{X^R=0.1} = -1.01\%$, $\tau_{X^R=0.1} = 32.7\%$, whereas a planner who cares predominantly about constrained agents' welfare ($X^R = 0.9$) sets $\pi_{X^R=0.1} = 17.66\%$, $\tau_{X^R=0.1} = 29.88\%$.

While fitting the wealth Gini index of the US, the benchmark model with $\theta = 0.8$ implies that our other measures of inequality are lower than in the data. A comparison with the statistics presented in Ragot (2014) is useful here. Our calibration implies a money Gini index equal to 0.35, whereas the corresponding figure in the data is 0.8. The model correctly predicts that the wealthy hold more money than the poor in absolute terms, but less money as a share of total wealth. The ratio of money to total wealth is one for constrained agents, but only 1.59% for unconstrained

[24]In a similar model, Schmitt-Grohé and Uribe (2006) obtain that optimal inflation under the representative agent assumpton is 0.5%.

[25]However, we verified in a separate experiment (not reported here) that while eliminating nominal rigidities would increase the optimal inflation rate, it would not affect substantially the redistributive effects of inflation.

[26]Constrained agents reduce their money holdings less because higher inflation (with lower taxes) is associated to an increase in their consumption.

[27]Di Bartolomeo et al (2015) explain why public transfer, such as debt service payments, induce an increase in the optimal inflation rate under the repreentative agent assumption.

households. Even with this minimal share of wealth allocated to money, the model predicts that inflation should be used as a redistributive tool.

Cash/credit-transaction case Results under the cash/credit technology (and continuous trading, N=1) may look surprising. In the representative agent case, in fact, the optimal inflation rate is 1.3%, larger than under the classical case. This happens because increasing inflation raises the number of credit goods and total transaction costs but, due to economies of scale in the transaction technology, marginal transaction costs fall. This in turn, tilts the planner's decision in favor of a higher inflation rate than in the classical case. Contrary to what one could expect on the grounds of the findings in Erosa and Ventura (2002), even in this model the optimal steady state inflation rate is increasing in the share of constrained agents: when $\theta = 0.8$ it rises to 4.76%.[28] To rationalize this result one should bear in mind that in Erosa and Ventura (2002) monopoly profits are nil and inflation is a regressive tax, i.e. it falls by more on the poor who hold a larger fraction of their wealth in the form of money holdings. As a result, raising inflation to lower the labor tax increases inequality. In our model inflation still is a regressive tax, but it allows to tax consumption out of monopoly profits. Therefore exploiting inflation to lower taxes allows to redistribute consumption in favor of constrained households because through the inflation tax unconstrained households contribute an amount of fiscal revenues that is larger than the rebate they receive on the labor tax.

To highlight this mechanism, consider how the contribution of unconstrained and constrained agents to per-capita government revenues is affected by higher inflation when the price mark-up over marginal costs is either 20% or 1%. First, consider the case where inflation is zero and price markups are 20%. Assuming 100 dollars per-capita government revenues from income and inflation taxes, each unconstrained (constrained) agent pays 176.31 (80.92) dollars. If inflation is set at the optimal level, 4.76%, each unconstrained (constrained) agent pays 179 (80.24) dollars. The effect of raising inflation when mark-ups are 1% is just the opposite. In fact, under a zero inflation steady state, each unconstrained (constrained) agent pays 212.72 (71.8)dollars. When inflation is 4.76%, each unconstrained (constrained) agent pays 212.45 (71.88) dollars. In this case, the inflation tax on profits is irrelevant and inflation penalizes constrained households.

Preferred inflation and income tax rates between the two household group remain quite different. A planner who cares predominantly about unconstrained agents' welfare ($X^R = 0.1$) sets $\pi_{X^R=0.1} = -0.26\%$, $\tau_{X^R=0.1} = 32.46\%$, whereas a planner who

[28] In this case the Gini index for money holdings is 0.28, quite far from what we observe in the data. Notice, however, that individual money holdings are still larger for unconstrained agents.

cares predominantly about constrained agents' welfare ($X^R = 0.9$) sets $\pi_{X^R=0.1} = 18\%$, $\tau_{X^R=0.1} = 29.77\%$.

Infrequent trading case Let us consider first the case where $\theta = 0$. Figure 1 decribes the steady state distribution of money holdings and consumption decisions for the N p-type agents. Infrequent access to the brokerage account induces households to keep an inventory of money to pay for purchases for all the periods left before their next access the brokerage account again. On average, unconstrained agents hold more money than if they had access to the brokerage account in every period. In this case the optimal inflation rate is 1.9% and 2.44% for N equal 2 and 3 respectively. Relative to the classical case under the RA assumption, inflation is now higher because infrequent trading raises money holdings, i.e. the inflation tax base, for any given level of consumption transaction costs.

Figure 1: Money and consumption distribution under the infrequent trading model with $\theta = 0$. Circles indicate the corresponding distribution under the classical case.

By setting $\theta = 0.8$, $N = 3$ we obtain a steady state solution that matches the wealth Gini index for the US and raises the corresponding money Gini index from 0.35 (classical case) to 0.64.[29] Figure 2 shows the distribution of individual money

[29] It is interesting to note that $N = 3$ is exactly the benchmark value chosen in Alvarez et al (2009).

holdings and consumption decisions in steady state. The optimal inflation rate is 7.8%.[30] Coeteris paribus, infrequent trading unambiguously raises optimal inflation because it concentrates money holdings in the hands of unconstrained households, thereby strengthening the redistribution motive to generate inflation.

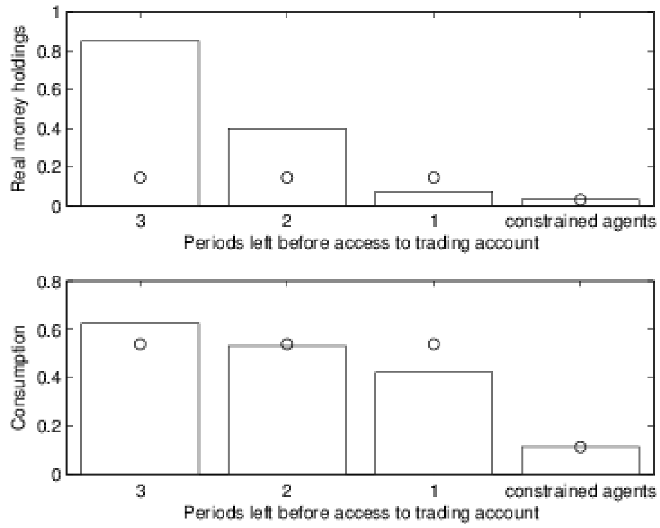

Figure 2: Money and consumption distribution under the infrequent trading model with $\theta = 0.8$. Circles indicate the corresponding distribution under the classical case.

Even in this case LAMP generates large differences in preferred inflation and income tax rates between unconstrained and constrained households. In fact our calculations show that a planner who cares predominantly about unconstrained agents' welfare ($X^R = 0.1$) sets $\pi_{X^R=0.1} = -2.67\%$, $\tau_{X^R=0.1} = 34.02\%$, whereas a planner who cares predominantly about constrained agents' welfare ($X^R = 0.9$) sets $\pi_{X^R=0.1} = 10.07\%$, $\tau_{X^R=0.1} = 28.23\%$.[31]

5 Effects of a capital income tax

So far we have assumed that the Planner uses the same linear tax schedule for labor and capital income. This might be regarded as too strong an assumption, and we

[30] Optimal inflation with N=2 is around 7.4%.

[31] Results refer to the case $N = 3$.

	Classical	*Infrequent trading*$(N=3)$	*Cash/credit technology*
$\theta = 0$	$\pi = 0.45\%$ $\tau = 32.26\%$	$\pi = 2.44\%$ $\tau = 31.27\%$	$\pi = 1.3\%$ $\tau = 32.01\%$
LAMP	$\pi = 4.3\%$ $\tau = 31.42\%$	$\pi = 7.8\%$ $\tau = 28.9\%$	$\pi = 4.76\%$ $\tau = 31.29\%$
relative to $\theta = 0$ case	$\Delta c^u = -0.16\%$ $\Delta l^u = 3.67\%$ $\Delta c^c = 0.92\%$; $\Delta l^c = 0.91\%$	$\Delta c^u = 0\%$ $\Delta l^u = 13.84\%$ $\Delta c^c = 3.18\%$ $\Delta l^c = 2\%$	$\Delta c^u = -0.3\%$ $\Delta l^u = 3.67\%$ $\Delta c^c = 0.74\%$ $\Delta l^c = 1.25\%$
$\frac{b}{y} = 0.8$	$\pi = 5.91\%$ $\tau = 32.24\%$	$\pi = 8.43\%$ $\tau = 29.83\%$	$\pi = 6.92\%$ $\tau = 32.1\%$
relative to $\frac{b}{y} = 0.6$ case	$\Delta c^u = -0.32\%$ $\Delta l^u = 2.06\%$ $\Delta c^c = 0.25\%$ $\Delta l^c = 0.67\%$	$\Delta c^u = -0.13\%$ $\Delta l^u = 1.69\%$ $\Delta c^c = 0.3\%$ $\Delta l^c = 0.34\%$	$\Delta c^u = -0.36\%$ $\Delta l^u = 3.11\%$ $\Delta c^c = 0.28\%$ $\Delta l^c = 1.1\%$

Table 3: relative to $\theta = 0$ case refers to the consumption and hours differences computed as the change in consumption and hours that takes place when inflation is increased from the level that would be optimal under $\theta = 0$ to the optimal level under $\theta = 0.8$. Similarly, relative to $\frac{b}{y} = 0.6$ case refers to the consumption and hours differences computed as the change in consumption and hours that takes place when inflation is increased from the level that would be optimal under $\frac{b}{y} = 0.6$ to the optimal level under $\frac{b}{y} = 0.8$

now consider the case where the Planner can levy distinct labor and capital income taxes.[32]

The literature on optimal dynamic taxation under perfectly competitive goods markets suggests that public expenditures should not be financed by capital taxes, which in fact should be zero even if some households have no wealth and the policymaker cares about them (Judd (1985); Chamley (1986); Atkeson et al. (1999)). Judd (1987) shows that the steady-state optimal tax on capital income can be negative for imperfectly competitive product markets. Our contribution here is twofold. On the one hand we highlight the inflation tax role under the representative agent and LAMP models. On the other hand we assess how sensitive the results are to different money demand specifications.

Results are shown in Table 4. In line with previous contributions, under the representative agent assumption the optimal capital income tax is negative. Relative to the uniform income tax case, we observe an increase in steady state inflation. This suggests an intriguing interpretation: the inflation tax on consumption out of monopoly profits is used as a substitute for the unavailable tax on profits and, to offset the monopolistic distortion, its proceedings are used to subsidize production. Under LAMP, the planner's concern for redistribution induces the policymaker to raise both inflation and the capital income tax, and to reduce the labor tax. The optimal capital income tax rate, however, remains below the optimal labor income tax rate. The alternative money demand specifications yield results consistent with those obtained under a uniform income tax: inflation grows when we move from the classical case to the cash/credit technology and to the infrequent trading models.

6 Conclusions

A relatively large body of empirical research has pointed out that inflation is particularly harmful for the poor (Easterly et al (2001)) and high inflation and inequality are positively related (Albanesi (2007)). We show that this need not be the case when monetary and fiscal policies are optimally designed.

Untaxed monopoly profits generate a potential incentive to inflate in order to reduce income taxes. This incentive is unambiguously stronger when such profits are concentrated in the hands of few wealthy individuals. We find that for different characterization of money demand functions, the optimal inflation rate is larger than 4%, well above the 2% target adopted by central banks like the Fed and the ECB.

[32] In Erosa and Ventura (2002) the inflation tax is only used to reduce the labor income tax because the capital income tax is held constant.

	Classical	*Infrequent trading(N = 3)*	*Cash/credit technology*
$\theta = 0$	$\pi = 1.06\%$ $\tau = 41.05\%$ $\tau^k = -5.94\%$	$\pi = 4.56\%$ $\tau = 39.81\%$ $\tau^k = -11.77\%$	$\pi = 2.64\%$ $\tau = 40.64\%$ $\tau^k = -6.52\%$
LAMP	$\pi = 5.52\%$ $\tau = 35.27\%$ $\tau^k = 15.9\%$	$\pi = 8.94\%$ $\tau = 33.41\%$ $\tau^k = 8.99\%$	$\pi = 6.27\%$ $\tau = 35.12\%$ $\tau^k = 16.03\%$
relative to $\theta = 0$ case	$\Delta c^u = -2.42\%$ $\Delta l^u = 21.92\%$ $\Delta c^c = 4.41\%$; $\Delta l^c = 1.25\%$	$\Delta c^u = -2.17\%$ $\Delta l^u = 27.32\%$ $\Delta c^c = 5.42\%$ $\Delta l^c = 2.05\%$	$\Delta c^u = -2.35\%$ $\Delta l^u = 21.34\%$ $\Delta c^c = 4.06\%$ $\Delta l^c = 1.58\%$
$\frac{b}{y} = 0.8$	$\pi = 8.16\%$ $\tau = 35.92\%$ $\tau^k = 17.37\%$	$\pi = 9.65\%$ $\tau = 34.34\%$ $\tau^k = 10.09\%$	$\pi = 9.44\%$ $\tau = 35.7\%$ $\tau^k = 17.54\%$
relative to $\frac{b}{y} = 0.6$ case	$\Delta c^u = -1.03\%$ $\Delta l^u = 5.83\%$ $\Delta c^c = 0.57\%$ $\Delta l^c = 1.33\%$	$\Delta c^u = -0.34\%$ $\Delta l^u = 2.89\%$ $\Delta c^c = 0.45\%$ $\Delta l^c = 0.44\%$	$\Delta c^u = -1.18\%$ $\Delta l^u = 7.39\%$ $\Delta c^c = 0.54\%$ $\Delta l^c = 2.04\%$

Table 4: relative to $\theta = 0$ case refers to the consumption and hours differences computed as the change in consumption and hours that takes place when inflation and the capital tax are increased from the level that would be optimal under $\theta = 0$ to the optimal level under $\theta = 0.8$. Similarly, relative to $\frac{b}{y} = 0.6$ case refers to the consumption and hours differences computed as the change in consumption and hours that takes place when inflation and the capital tax are increased from the level that would be optimal under $\frac{b}{y} = 0.6$ to the optimal level under $\frac{b}{y} = 0.8$

Empirical evidence on the relationship between inflation and inequality suggests that the two are positively correlated across countries (Albanesi (2007), Easterly et al (2001), Erosa and Ventura (2002), Beetsma and Van der Ploeg (1996)). Albanesi (2007) adopts a cash/credit-transaction approach and interprets this result as arising from a self-fulfilling process in which elites in high inequality countries exert pressure on the political process to obtain higher inflation, which, due to economies of scale in transactions, worsens inequality. Our model suggests that inflation could reduce inequality and predicts that inflation should increase with inequality, but for reasons that are opposite to those put forward in Albanesi (2007).

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