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Abstract

The present paper analyzes the choice to place an executive in the board of the rival company, within a duopoly where firms with hidden marginal costs of production compete in the product market. Interlocking directorates may emerge as an equilibrium outcome whenever firms gain by exchanging information about their private costs. We show that a unilateral interlocking arises when firms have different degrees of efficiency and the direction of this interlock is affected by the degree of substitutability in the product market. Bilateral interlocking occurs only between similar firms, that is when equally inefficient firms sell substitute products or when equally efficient firms sell complement products. The equilibrium outcome is always welfare increasing for consumers.

JEL classification: D48 (Oligopoly); L22 (Firm Organization and Market Structure); M12 (Executives).

Keywords: Interlocking Directorates; Information Sharing; Oligopoly.

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1 Introduction

Interlocking Directorates occur when an executive of a company sits in another company's board, forming a tie between the two companies. These executives' ties are observed in many industries.

The occurrence of interlocking directorates (ID, hereafter) differs across countries and sectors. For instance Santella et al. (2008) and Baccini and Marroni (2016) provide evidence of a lower density of ID among listed companies in UK and US compared to France, Italy and Germany. Fattobene et al. (2017) observe that in Italy ID are more frequent in telecommunication, financial and consumer services than in other sectors. Also we see ties linking companies that operate in different sectors. Often banks and insurance companies share executives across their boards (e.g. Santella et al., 2008). Frequently we observe unilateral interlocks, that is, an executive of a company (sender) sitting in the board of another company (receiver); sometimes we see bilateral interlocking, when the executive of the receiver company sits also in the sender company's board.

Little is known about the occurrence of these interlockings, about the direction of these ties (unilateral vs. bilateral) and about the involved sectors. Why companies choose to interlock?

Interlocking emerges for many different reasons (see for instance the survey of the literature in Fattobene et al., 2017), but we mainly focus on the fact that sharing information may be strategically important for companies to successfully operate in the market. Learning hidden information about the rival may be therefore the motivation of interlocking.

The aim of this paper is to model and explain the reasons why interlocking occurs as a result of a strategic decision by two companies willing to share hidden information. As a result, we are able to connect the occurrence of ID to the characteristics of firms and product markets.

In this paper we develop a theoretical framework to analyze interlocking as a strategic choice of companies competing in the product market. We introduce two sources of heterogeneity, one at the firm level and another one at the market level according to what follows: i) market are characterized by either complementarity or substitutability among goods; ii) each firm has private information about its type, firms can be efficient or inefficient, depending on their marginal costs. In this framework we introduce the possibility for the manager of each firm to sit in the board of the rival, i.e. interlocking, learning the latter's true type. We model the strategic choice in a two stage game in which firms non-cooperatively decide whether to interlock or not, in the first stage, and in the second stage compete in the product market. Unilateral and bilateral interlocking may arise as Bayesian Nash equilibria of this game depending on the market and firms' characteristics.

In the case of substitute products, bilateral interlocking occurs when both firms are equally inefficient. Interlocking is a commitment to reduce the quantities in case of strategic substitutability. When the two firms are symmetrically efficient the equilibrium outcome is one without interlocking. In the case with asymmetric costs, instead, we find that unilateral in-

terlocking occurs in equilibrium, where only the efficient type interlocks.

We derive results also for the case of complementary products. Again we find that bilateral, unilateral interlocking or no ties are the result of the strategic game, depending on the different combinations of companies' hidden costs. In the paper we prove that our Bayesian Nash equilibria are optimal from a welfare point of view.

We are also aware of the implications of ID for competition policy. The antitrust fears potential collusion among companies sharing executives when they are in the same market (see for instance the ban of ID in the Clayton Act in US in 1914 or the discussion about the ban introduced with the Decreto Salva Italia in 2011 for banks in Italy by Creatini and Main, 2015). Although the implications for the antitrust policy are relevant, as we discuss in the conclusions, we believe they are outside the scope of the present paper.

1.1 Literature review

There is a growing empirical literature on the effects of ID for firms, markets and country performance. Conversely, much less attention has been devoted from the theoretical point of view to the determinants and the effects of ID.

There exists a relevant literature on information sharing in oligopoly to which our paper is naturally related. For instance, Li (1985) and Shapiro (1986) show that in a Cournot model with private information on production costs, the Bayesian Nash equilibrium is characterized by full revelation of information. Conversely, Vives (1984) and Li (1985) show, in the same Cournot framework, but with demand uncertainty, that firms in equilibrium do not share their private information. Furthermore, Vives (1984) and Sakai (1986) investigate how the results about information sharing change across the different oligopoly models. Finally Raith (1996) provides a general framework that encompasses all the oligopoly models with information sharing. In our paper we focus on a specific application of information sharing within oligopoly models: here information sharing derives, as a byproduct, from the decision to share executives through interlocking directorates. In particular, we consider a framework where there is uncertainty about firms' private costs and a market demand that allows us to study competition in either cases of complement and substitute products. Our paper is also related to the models with internal information sharing and vertical relationships, see, for instance, Gal-Or (1991) and Piccolo and Pagnozzi (2013): they focus on how information sharing and product market competition distort communication within the hierarchy. We assume that communication within the company is perfect and concentrate instead on the incentive to share information across companies by means of interlocking in order to affect product market competition. Finally, the paper is only marginally related to the recent literature in game theory, dealing with risk-sharing in networks (see e.g., Bramoulle and Kranton 2007, Fauchamps and Gubert, 2007, Ambrus et al. 2014). However, that literature does not take into account product market competition. In fact it neglects the reasons why firms enter into ID which may be related also upon the characteristics of the firms and markets where companies operate.

The growing empirical literature on ID focuses on the network of interlocked companies

and its evolution over time. Several papers propose different measures of these interconnections and their relative weights (Mrizak 2009; Santella et al. 2010; van Veen and Kratzer, 2011; Heemskerk et al. 2013; Bellenzier and Grassi, 2014; Di Bartolomeo and Canofari, 2015; Fattobene et al. 2017 to cite some). Some of these papers measure the impact of interlocking ties on the performance of companies or on the degree of concentration of the industry (e.g., Pomboa and Gutierrez, 2011; Croci and Grassi, 2012; Kaczmarek et al., 2014; Drago et al., 2015). Some other papers study the evolution of the ties, as for instance Carbonai and Di Bartolomeo (2007) do for the Italian insurance industry. Fattobene et al. (2017) measure the impact of the Italian ban of interlocking among banks in 2011, Santella et al. (2008) the last corporate governance reform in Italy on company's performance, while Helmers et al. (2017) the corporate governance reform in India on innovation by listed companies.

The paper is also related to the managerial and sociological literature on organization and strategic behavior of firms. Although there is little mathematical formalization in that literature, we have been inspired by their work when modeling interlock as the result of individual company's strategic choice. For instance Brennecke and Rank (2017) describe the interlock tie as a strategic choice of individual companies and distinguish between unilateral and bilateral interlocking. Also Zajac (1988) focuses on the matching between individual companies.

Finally, the only paper that marginally touches the corporate finance perspective is Fich and White (2005) by focusing on bilateral interlocks.

The remainder of the paper is organized as follows. In section 2 we develop a model in which firms with asymmetric hidden costs compete as rival in the same product market; in section 3 we analyze competition in the product market, while in section 4 the choice to interlock. Then we turn to the case of firms with asymmetric costs in section 5. Section 6 derives and compares the social welfare in the different equilibria. In Section 7 we discuss possible testable implications of the model. Section 8 concludes the paper.

2 The model

There are two firms, firm 1 and firm 2, competing in the product market. Consumers value each product according to the following utility function (see Singh and Vives, 1984; Brander and Spencer, 2015; and Amir et al., 2017)

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - bq_1q_2 + M. \quad (1)$$

The inverse demand function for company selling product i , while the rival sells product j , is easily derived from (2):

$$D_i^{-1} : P_i(q_i, q_j) = a - q_i - bq_j \quad (2)$$

where the parameter b allows for differentiation between the two products. This parameter can take positive values ($b > 0$), denoting substitutability between the two goods or negative

values ($b < 0$), indicating complementarity. Finally, notice that $b = 0$ captures the case of two separate monopolists and $b = 1$ is the case of competition *à la* Cournot.

The two firms can be either efficient or inefficient. The degree of efficiency is captured by their marginal costs of production, that can be either c_i^L (efficient) or c_i^H (inefficient) with $c_i^H > c_i^L$. The marginal cost is private information for all insiders within a company (executives and owners).

As anticipated in the Introduction, by interlocking we mean that one company places its executive in the board of the rival company. This allows the "sender" company to retrieve the information about the marginal cost of the "receiver" rival company. Interlocking is modeled here as a strategic choice by each firm and it is studied as a non-cooperative simultaneous moves game by the two companies. Furthermore we distinguish between unilateral (the sender places an executive in the rival's board) and bilateral (also the receiver company places an executive in the board of the sender company) ID.

We solve the model as a two-stage game, where each firm decides non-cooperatively whether to send its executive in the competitor's board, by anticipating the outcome of the product market competition. The equilibrium is computed by backward induction: first we solve for product market competition (second stage) given a specific interlocking combination, and then for interlocking (first stage). Notice that, depending on the information setting, the solution concept of the game is either Sub-Game Perfect equilibrium (when both companies interlock) or Bayesian equilibrium (if at least one of the companies interlocks).

3 Product market competition

In this section we solve the second stage of the game where each company sets the quantity to be sold in the product market. There are three possible cases to be analyzed:

- Bilateral interlocking (ID,ID): both companies have their executive sitting in the rival's board, hence the marginal costs are common information.
- Unilateral interlocking (N,ID) or the symmetric case (ID,N): one of the two companies has its executive sitting in the rival's board, but not viceversa. Therefore the interlocking company ("sender") knows the marginal cost of the rival ("receiver"), but the other does not.
- No interlocking (N,N): none of the companies' executive sits in the rival's board, therefore marginal costs remain private information.

We will analyze competition in quantities for each case. Furthermore we assume that the two companies have asymmetric costs: without loss of generality, we posit that Firm 1 is efficient, while Firm 2 is inefficient, that is $c_1 = c_1^L$ and $c_2 = c_2^H$.

The two companies set their quantities $\{q_1, q_2\}$ by maximizing their profits:

$$\Pi_1(q_1, q_2) = [P_1(q_1, q_2) - c_1^L] q_1$$

$$\Pi_2(q_1, q_2) = [P_2(q_1, q_2) - c_2^H] q_2$$

where the two inverse demand functions are:

$$P_1(q_1, q_2) = a - q_1 - bq_2 \tag{3}$$

$$P_2(q_1, q_2) = a - q_2 - bq_1$$

3.1 Bilateral interlocking

In the case of bilateral interlocking (ID, ID) there is complete information, since each company knows not only its own marginal cost, but also the marginal cost of the rival.

From the FOCs we derive the two reaction functions:

$$q_1^{IDID}(q_2^{IDID}) = \frac{a - c_1^L}{2} - \frac{b}{2}q_2^{IDID} \tag{4}$$

$$q_2^{IDID}(q_1^{IDID}) = \frac{a - c_2^H}{2} - \frac{b}{2}q_1^{IDID}$$

The solution of the system yields the quantities in the Nash equilibrium :

$$q_1^{IDID} \equiv q_1^{IDID}(c_1^L, c_2^H) = A - \frac{2}{B}c_1^L + \frac{b}{B}c_2^H \tag{5}$$

$$q_2^{IDID} \equiv q_2^{IDID}(c_1^L, c_2^H) = A - \frac{2}{B}c_2^H + \frac{b}{B}c_1^L$$

where the condition $b \in (-2, 2)$ ensures that the equilibrium quantities are positive and $B = (2 + b)(2 - b) = 4 - b^2$ and $A = \frac{a}{2+b}$. Since $c_1^L < c_2^H$ it follows that $q_1^{IDID} > q_2^{IDID}$ for any value of $b \in (-2, 2)$.

We can now compute the profits at the Nash equilibrium. First, notice that the mark-up computed at the equilibrium, $(P_i^{IDID} - c_i)$, collapses to the equilibrium quantity. Therefore the equilibrium profits are given by:

$$\Pi_1^{IDID}(c_1^L, c_2^H) = \left(A - \frac{2}{B}c_1^L + \frac{b}{B}c_2^H \right)^2 = (q_1^{IDID})^2 \tag{6}$$

$$\Pi_2^{IDID}(c_1^L, c_2^H) = \left(A - \frac{2}{B}c_2^H + \frac{b}{B}c_1^L \right)^2 = (q_2^{IDID})^2$$

Notice that the more efficient company is also more profitable, i.e. $\Pi_1^{IDID}(c_1^L, c_2^H) > \Pi_2^{IDID}(c_1^L, c_2^H)$ for any value of $b \in (-2, 2)$. In conclusion, the more efficient firm produces more and gains more than its rival.

3.2 No interlocking

In the case without interlockings (N,N) both types are private information. Hence both firms formulate expectations by assigning a probability to the opponent's type. Firm 1 expects Firm 2 to be efficient with probability $\mu \in [0, 1]$, while Firm 2 expects Firm 1 to be efficient with probability $\nu \in [0, 1]$. Notice that when $\mu = 0$ and $\nu = 1$, we are back to the (ID, ID) case, since their expectations are correct.

Let's start by analyzing the problem of Firm 1. The quantity q_1 is set with the objective to maximize Firm 1's profit:

$$\Pi_1(q_1, \bar{q}_2(\mu)) = [P_1(q_1, \bar{q}_2(\mu)) - c_1^L] q_1$$

with $\bar{q}_2(\mu) = \mu q_2(c_2^L) + (1 - \mu)q_2(c_2^H)$. From the FOC we derive the best reply functions for Firm 1 in the two cases, respectively, efficient and inefficient:

$$q_{1|c^L}^{NN}(\bar{q}_2(\mu)) = \frac{(a - c_1^L)}{2} - \frac{b}{2}\bar{q}_2(\mu) \quad (7)$$

$$q_{1|c^H}^{NN}(\bar{q}_2(\mu)) = \frac{(a - c_1^H)}{2} - \frac{b}{2}\bar{q}_2(\mu) \quad (8)$$

Analogously for Firm 2, the quantity q_2 is set with the objective to maximize Firm 2's profit:

$$\Pi_2(\bar{q}_1(\nu), q_2) = [P_2(\bar{q}_1(\nu), q_2) - c_2^H] q_2$$

with $\bar{q}_1(\nu) = \nu q_1(c_1^L) + (1 - \nu)q_1(c_1^H)$. From the FOC we derive the best reply functions for Firm 2 in the two cases, respectively, efficient and inefficient:

$$q_{2|c^H}^{NN}(\bar{q}_1(\nu)) = \frac{a - c_2^H}{2} - \frac{b}{2}\bar{q}_1(\nu) \quad (9)$$

$$q_{2|c^L}^{NN}(\bar{q}_1(\nu)) = \frac{a - c_2^L}{2} - \frac{b}{2}\bar{q}_1(\nu) \quad (10)$$

Solving the system of the first order conditions (7)-(10), we derive the equilibrium quantities:

$$q_1^{NN} \equiv q_1^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) = q_1^{IDID} - \frac{b^2}{2B}(1 - \nu)(c_1^H - c_1^L) - \frac{b\mu}{B}(c_2^H - c_2^L) \quad (11)$$

$$q_2^{NN} \equiv q_2^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) = q_2^{IDID} + \frac{b}{B}(1 - \nu)(c_1^H - c_1^L) + \frac{b^2}{2B}\mu(c_2^H - c_2^L)$$

where q_1^{IDID} and q_2^{IDID} are defined in (5). Therefore the equilibrium profits in the case without interlocking are again the squared values of equilibrium quantities:

$$\begin{aligned} \Pi_1^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) &= (q_1^{NN})^2 \\ \Pi_2^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) &= (q_2^{NN})^2 \end{aligned} \quad (12)$$

3.3 Unilateral interlocking

Notice that all the other can be derived as special cases of (N,N) by selecting opportune values for the probabilities μ and ν . For instance when $\mu = 0$ (Firm 1 expects Firm 2 to be inefficient) and $\nu = 1$ (Firm 2 expects Firm 1 to be efficient), we are back at the (ID,ID) case and the quantities are defined in (5).

- In the (ID,N) case Firm 1's executive sits in the board of the rival company, but not viceversa. Therefore, Firm 2 does not observe Firm 1's type, while Firm 1 has complete information. We derive this case from the general case (N,N) by setting $\mu = 0$ (Firm 1 expects Firm 2 to be inefficient) and $\nu \neq 1$ (Firm 2 does not know Firm 1's type).

In this case the quantities are as follows:

$$q_1^{IDN} \equiv q_1^{IDN}(c_1^L, c_1^H, c_2^H) = q_1^{IDID} - \frac{b^2}{2B}(1-\nu)(c_1^H - c_1^L) \quad (13)$$

$$q_2^{IDN} \equiv q_2^{IDN}(c_1^L, c_1^H, c_2^H) = q_2^{IDID} + \frac{b}{B}(1-\nu)(c_1^H - c_1^L)$$

- In the (N,ID) case Firm 2's executive sits in the board of the rival company, but not viceversa. Therefore, Firm 1 does not observe Firm 2's type, while Firm 2 has complete information. We derive this case from the general case (N,N) by setting $\mu \neq 0$ (Firm 1 does not observe Firm 2's type) and $\nu = 1$ (Firm 2 expects Firm 1 to be efficient). In this case the quantities are as follows:

$$q_1^{NID} \equiv q_1^{NID}(c_1^L, c_2^L, c_2^H) = q_1^{IDID} - \frac{b}{B}\mu(c_2^H - c_2^L) \quad (14)$$

$$q_2^{NID} \equiv q_2^{NID}(c_1^L, c_2^L, c_2^H) = q_2^{IDID} + \frac{b^2}{B}\mu(c_2^H - c_2^L)$$

It is useful to establish the following ranking of equilibrium quantities (and thus equilibrium profits) among the different cases:

For $b > 0$:

$$q_1^{IDID} > q_1^{NID} > q_1^{NN}$$

$$q_1^{IDID} > q_1^{IDN} > q_1^{NN}$$

and

$$q_2^{NN} > q_2^{NID} > q_2^{IDID}$$

$$q_2^{NN} > q_2^{IDN} > q_2^{IDID}$$

For $b < 0$, instead for any of the two firms $i = 1, 2$ it is:

$$q_i^{NID} > q_i^{IDID} > q_i^{IDN}$$

$$q_i^{NID} > q_i^{NN} > q_i^{IDN}$$

4 The choice to interlock

We focus now on the outcome of the first stage of the game, namely on the choice to interlock. We model interlocking as a strategic choice that a firm makes whenever it maximizes profits. In particular, by interlocking choice we mean that there has been an invitation from the board of an opponent firm, which can be accepted (ID) or rejected (N). It could be that both companies decide to interlock, giving rise to a bilateral interlocking, or that only one chooses to interlock, i.e. unilateral interlocking. We analyze the choice of interlocking therefore as a game where firms decide non-cooperatively and simultaneously whether to interlock or not. The interlocking decision is taken in the first stage of the game, by anticipating the strategic implication of this choice on the product market competition (second stage) according to the results in the previous section.

In the first stage of the game we compute the Nash equilibrium according to the payoffs in the following table.

| | | Firm 2 | |
|--------|-----------|------------------------------|----------------------------|
| | | <i>ID</i> | <i>N</i> |
| Firm 1 | <i>ID</i> | $\Pi_1^{IDID}, \Pi_2^{IDID}$ | Π_1^{IDN}, Π_2^{IDN} |
| | <i>N</i> | Π_1^{NID}, Π_2^{NID} | Π_1^{NN}, Π_2^{NN} |

We can now state the equilibrium of the first stage of the game for different product market characteristics.

Proposition 1 *The following unilateral interlocking is the unique Bayesian Nash equilibrium with $\mu \in [0, 1]$ and $\nu \in [0, 1]$:*

- *(ID,N) when $b > 0$;*
- *(N,ID) when $b < 0$.*

Proof. Given that the equilibrium profits are increasing in the equilibrium quantities, we can rank the profit levels distinguishing between two cases according to the sign of b .

- Case $b > 0$: given that

$$\begin{aligned} \Pi_1^{IDID}(c_1^L, c_2^H) &> \Pi_1^{NID}(c_1^L, c_2^L, c_2^H) > \Pi_1^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) \\ \Pi_1^{IDID}(c_1^L, c_2^H) &> \Pi_1^{IDN}(c_1^L, c_1^H, c_2^H) > \Pi_1^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) \end{aligned}$$

and

$$\begin{aligned} \Pi_2^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) &> \Pi_2^{NID}(c_1^L, c_2^L, c_2^H) > \Pi_2^{IDID}(c_1^L, c_2^H) \\ \Pi_2^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) &> \Pi_2^{IDN}(c_1^L, c_1^H, c_2^H) > \Pi_2^{IDID}(c_1^L, c_2^H) \end{aligned}$$

it is easy to prove that the unique NE of the game is (ID,N).

- Case $b < 0$: given have that for the two firms $i = 1, 2$ it is

$$\begin{aligned}\Pi_i^{NID}(c^L, c^H) &> \Pi_i^{IDID}(c^L, c^H) > \Pi_i^{IDN}(c^L, c^H) \\ \Pi_i^{NID}(c^L, c^H) &> \Pi_i^{NN}(c^L, c^H) > \Pi_i^{IDN}(c^L, c^H)\end{aligned}$$

it is easy to prove that the unique NE of the game is (N,ID).

■

The above Proposition states that there are two cases in which unilateral interlocking emerges as the equilibrium of the game. First, when the two goods are substitutes, just the efficient firm chooses to interlock. Second, when the two goods are complements, only the inefficient firm engages in interlocking. The intuition of these opposite equilibrium behaviors rests on the information value of the ID. In fact, through interlocking the firm observes the true type of its competitor and learns the level of efficiency.

When products are substitutes, $b > 0$, Firm 2, the inefficient one, has no incentives to know the true type of Firm 1, given that its profit is decreasing in the rival's quantity. Therefore Firm 2 prefers to pretend that the rival is inefficient, producing less, and giving some positive probability to this case, instead of knowing the rival's true type. Conversely, the efficient type, Firm 1, benefits from knowing the opponent's type. In fact, given that it is efficient it produces a larger quantity, obtaining higher profits. However, without knowing its opponent type, Firm 1 should attach some positive probability to the fact that also Firm 2 is efficient, but, given that its profit is decreasing in the rival's quantity, this reduces its equilibrium quantity and profit. Therefore, the efficient firm, Firm 1, is better off by observing the true type of its competitor. To wrap up the efficient firm prefers to know the true type of the rival, while the inefficient firm is better off ignoring the type of the competitor. In other words, the efficient firm has more to gain from the information, when profits are decreasing in the opponent's quantity.

The opposite reasoning works for the case of complements, $b < 0$, in which the inefficient Firm 2 chooses the interlocking strategy. Notice that in this case the profit is increasing not only in its own quantity, but also in the competitor's quantity. Given that Firm 2 is inefficient, it would supply a little quantity earning a low profit. However, Firm 2 can be stimulated by an efficient competitor that produces a large quantity, increasing also its supply and profit. The worst case for the inefficient Firm 2 is to face an inefficient competitor. Therefore, Firm 2 prefers to know the true type of Firm 1, attaching a positive probability to Firm 1 being efficient. Conversely, Firm 1 would produce less when aware of facing an inefficient rival. Hence Firm 1 is better off not knowing the true type of the opponent and pretending the rival is an efficient one. To wrap up the inefficient firm prefers to know the true type of the rival, while the efficient firm is better off ignoring the type of the competitor. In other words, the inefficient firm has more to gain from the information, when the profits are increasing in the opponent's quantity.

5 The case of symmetric marginal costs

We now analyze the case of symmetry in the degree of efficiency among firms. Both firms are efficient and their marginal costs are $c_1 = c_2 = c^L$, or both firms are inefficient and their marginal costs are $c_1 = c_2 = c^H$. It is quite cumbersome to compute all the quantities and profits in the equilibria that arise in this context, hence we collect all the implications within one Proposition.

Proposition 2 *When both companies are efficient ($c_1 = c_2 = c^L$) the unique Bayesian Nash equilibrium when $\mu \in [0, 1]$ and $\nu \in [0, 1]$ is:*

- (N, N) in the case of $b > 0$; (ID, ID) in the case of $b < 0$.

Conversely, when both companies are inefficient ($c_1 = c_2 = c^H$) the unique Bayesian Nash equilibrium when $\mu \in [0, 1]$ and $\nu \in [0, 1]$ is:

- (ID, ID) in the case of $b > 0$; (N, N) in the case of $b < 0$.

Proof. Appendix A.1. ■

The above Proposition 2 shows that bilateral interlocking emerges as an equilibrium outcome in two cases: first, in the case where both firms are efficient and compete on complementary products; second, when both firms are inefficient and compete on substitute products. Notice that, due to symmetry in the marginal costs, firms' heterogeneity does not matter. It is crucial to focus on the market determinants, on the profit value determined by the efficiency level and, more precisely, on the shape of the best reply functions.

When both firms are efficient, $c_i = c^L$ for $i = 1, 2$, and the best reply functions are negatively sloped, $b > 0$, each firm would like to expand the production at the expenses of the rival's output. Therefore any informative advantage disappears. It would be better to pretend that the opponent is an inefficient type. Conversely, when the best reply functions are positively sloped, $b < 0$, each efficient firm would like to expand the production following the increase in the rival's output. Therefore, there is a clear informative advantage to know the true type of the competitor. Given the symmetry in costs, both firms behave in the same way and the equilibrium with a bilateral interlocking (ID, ID) arises. In the case of inefficient firms, the reverse result holds. In fact, when both firms are inefficient, $c_i = c^H$ for $i = 1, 2$, and the products are substitutes, there is a gain from knowing the true type of the opponent. Therefore at the Bayesian equilibrium a bilateral interlocking emerges. Conversely, when the products are complementary, the firm does not have any advantage in discovering that the opponent is inefficient. Therefore the unique equilibrium is one without interlocking.

In the following tables we summarize all the equilibria obtained for all the possible combinations of marginal costs and types of products.

| | | Firm 2 | |
|--------|-------|-----------|------------|
| | | c^L | c^H |
| Firm 1 | c^L | (N, N) | (ID, N) |
| | c^H | (N, ID) | (ID, ID) |

| For $b < 0$ | | Firm 2 | |
|-------------|-------|----------|---------|
| | | c^L | c^H |
| Firm 1 | c^L | (ID, ID) | (N, ID) |
| | c^H | (ID, N) | (N, N) |

- With substitute products ($b > 0$), bilateral interlocking prevails whenever firms are equally inefficient; unilateral interlocking prevails, in which the sender is the most efficient company while the receiver is the inefficient type.
- With complement products ($b < 0$), bilateral interlocking prevails when firms are equally efficient; unilateral interlocking prevails in which the sender is the inefficient company and the receiver is the most efficient one.

6 Welfare analysis

We now compute the consumer surplus and derive implications for the social welfare at the different Bayesian equilibria. Following Singh and Vives (1984) total surplus, TS , for a given pair of quantities (q_1, q_2) , is:

$$TS(q_1, q_2) = (U(q_1, q_2) - p_1 q_1 - p_2 q_2) + \Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) \quad (15)$$

Therefore, the consumer surplus, CS , becomes:

$$\begin{aligned} CS(q_1, q_2) &= TS(q_1, q_2) - \Pi_1(q_1, q_2) - \Pi_2(q_1, q_2) = U(q_1, q_2) - p_1 q_1 - p_2 q_2 \\ &= \frac{1}{2}(q_1^2 + q_2^2) + b q_1 q_2 = \frac{1}{2}(\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2)) + b q_1 q_2 \end{aligned} \quad (16)$$

Let us consider first the case where the two firms have symmetric marginal costs. According to the Bayesian equilibria in Section 5, we compare the benefits of the consumers in the case of bilateral interlocking (ID, ID) with the case without interlocking (N, N). The following Proposition illustrates the result:

Proposition 3 *Bilateral interlockings maximize consumer surplus when: i) firms are efficient (c^L) and products are complements ($b > 0$) or ii) firms are inefficient (c^H) and products are substitutes. Otherwise consumer surplus is higher without interlockings.*

Proof. See Appendix A.2. ■

Let us consider the case in which the two firms have asymmetric marginal costs, namely $c_1 = c_1^L$ and $c_2 = c_2^H$. From Proposition (1) we know that there are two possible Bayesian equilibria, both with unilateral interlocking, according to the sign of b . Analogously to the previous symmetric case, we investigate if at the equilibrium with interlocking consumer surplus is larger compared to the equilibrium without interlocking. The following Proposition illustrates the result:

Proposition 4 *Unilateral interlocking of the efficient firm (ID,N), with $0.89443 < b < 1.4142$, and unilateral interlocking of the inefficient firm (N,ID) with $b < 0$, maximize consumer surplus relatively to the case without interlocking (N,N).*

Proof. See Appendix A.3. ■

In both cases, given specific values of the parameter b , the Bayesian equilibria with ID are Pareto optimal from a social welfare perspective compared to the case without interlocking.

7 Empirical implications

The model has different implications that can be taken to the data, such as heterogeneity of interlocking and the determinants of the choice to interlock.

- **Heterogeneity of interlocking.** The model suggests that we should observe interlocking ties to be unevenly spread across companies and sectors. The incentive to interlock is in fact determined by the different level of efficiency of companies; moreover the degree of substitutability or complementarity among products sold by a couple of companies affects their incentive to interlock. In the model the type of tie, reciprocal or unilateral, is affected by companies and product market characteristics. We could summarize the implications that could be tested:
 - *bilateral interlocking* (the company sending a director into the rival company's board, hosts in its board a director of the receiving company) occurs only within couples of companies with a similar level of efficiency;
 - *unilateral interlocking* (a company sends its director into the rival company's board, without hosting the director of the receiver company) occurs within couples of companies with different levels of efficiency, operating in markets with competition on complements or with substitutes;
 - *no interlocking* occurs within companies with a similar levels of efficiency. However the level of efficiency differs according to the sectors in which the companies are: efficient companies producing substitute products or inefficient companies producing complement products.
- **Interlocking as a strategic choice.** Companies evaluate their strategic gain from interlocking. In other words, interlocking is an individual company decision, but it takes into account the characteristics of the two companies, in addition to the markets where they operate, for each candidate interlocking tie.

The implications of the model could be tested on a population of companies, with either companies with interlocking ties and companies without ties, where each observation is the tie between couples of companies. The empirical model is a discrete choice model where the dependent variable takes three possible values: "bilateral", "unilateral" or "no" interlocking.

The probability of occurrence of one specification of this choice should be related to the characteristics of the two companies forming the couple and to the characteristics of the sectors to which the two companies belong.

There are very few papers in the empirical literature on interlocking investigating both the characteristics of the markets and the features of the companies involved. The literature distinguishes between interlocked companies and not interlocked to study the effect of interlocking on different measures of performance, without studying the incentives of each single company to join the interlock. The only paper we know, on the determinants of the choice to interlock, is Fich and White (2005) that studies whether companies rather than CEOs characteristics affect the choice of a bilateral interlocking. They find that company characteristics do not affect the probability of a bilateral interlocking. Their evidence could be explained with our result that only similar companies choose to enter a bilateral interlocking: the probability of a bilateral tie should be inversely related to the difference in efficiency among interlocked companies and market characteristics rather than factors pertaining to each single interlocked company.

8 Conclusions

In this paper we provide a model for the choice to interlock within a duopoly in which companies compete in the product market without observing the rival's marginal cost. We show under which conditions, related to differences in efficiency and product market characteristics, firms may choose to interlock either bilaterally or unilaterally. Firms may also choose not to interlock. Hence we derive interlocking as an endogenous outcome of the model.

We discuss here some of the limitations of the analysis. We have assumed that the receiving company could not refuse to include the outside director within its board of directors. We model the choice to interlock as a unilateral non negotiable decision by the sender company within a non-cooperative game. When a tie arises, it is a non-cooperative equilibrium of the game, therefore neither of the two companies have incentives to deviate. However one could also try to model interlocking as a decision to form a coalition and study the stability of the network with the tools of the cooperative game theory.

Antitrust considerations may also be important. Within an exchange of ID, firms share hidden information: this may facilitate collusion in the future. Since our model is one shot, there is no repeated interaction in the product market, learning hidden marginal costs does not benefit firms willing to collude. Instead by introducing repeated interaction in the model, it could be possible to treat interlocking within a cartel framework and see whether learning private information leads to greater incentives to collude. Buch-Hansen (2014) study this relation from an empirical point of view: they scrutinize cartel cases pursued by the EU antitrust and look for evidence that in these cases there was previously an interlocking tie between companies involved in the cartel. Their findings seem to reject this hypothesis. Still, given the antitrust concerns about ID and the reasons motivating the ban of interlocking among companies in horizontal markets, it would be advisable to study this possible

relation.

There are also possible avenues for future research. First of all, it is possible to test empirically the implications of the model, along the lines discussed in Section 7.

Furthermore, it would be interesting to add an endogenous effort and study the impact of the different interlocking equilibria on incentives. In our model welfare is enhanced by interlocking whenever interlocking arises as an equilibrium outcome. This may not be necessary the case when also effort incentives are endogenous.

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A Appendix

A.1 Proof of Proposition 2

- When $c_1 = c_2 = c^L$, the equilibrium quantities become:

$$q^{IDID}(c^L) = q_1^{IDID}(c^L) = q_2^{IDID}(c^L) = \frac{a - c^L}{(2 + b)} \quad (17)$$

$$\begin{aligned} q_{1|c^L}^{NN}(c^L, c^H) &= q^{IDID}(c^L) + \left(\frac{b}{B}(1 - \mu) - \frac{b^2}{2B}(1 - \nu) \right) (c^H - c^L) \\ q_{2|c^L}^{NN}(c^L, c^H) &= q^{IDID}(c^L) + \left(\frac{b}{B}(1 - \nu) - \frac{b^2}{2B}(1 - \mu) \right) (c^H - c^L) \end{aligned} \quad (18)$$

$$\begin{aligned} q_{1|c^L}^{NID}(c^L, c^H) &= q^{IDID}(c^L) + \frac{b(1 - \mu)}{B}(c^H - c^L) \\ q_{2|c^L}^{NID}(c^L, c^H) &= q^{IDID}(c^L) - \frac{b^2(1 - \mu)}{2B}(c^H - c^L) \end{aligned} \quad (19)$$

$$\begin{aligned} q_{1|c^L}^{IDN}(c^L, c^H) &= q^{IDID}(c^L) - \frac{b^2(1 - \nu)}{2B}(c^H - c^L) \\ q_{2|c^L}^{IDN}(c^L, c^H) &= q^{IDID}(c^L) + \frac{b(1 - \nu)}{B}(c^H - c^L) \end{aligned} \quad (20)$$

Analogously to the asymmetric costs case the equilibrium profits are given by the squared values of the equilibrium quantities. With $b > 0$, the ranking of profits is given by: $\Pi_1^{NID}(c^L) > \Pi_1^{NN}(c^L) > \Pi_1^{IDID}(c^L) > \Pi_1^{IDN}(c^L)$; $\Pi_2^{IDN}(c^L) > \Pi_2^{IDID}(c^L) > \Pi_2^{NN}(c^L) > \Pi_2^{NID}(c^L)$. According to the ranking of profits, the unique Bayesian equilibrium is (N, N) . While in the case of $b < 0$ the ranking of profits becomes: $\Pi_1^{IDID}(c^L) > \Pi_1^{NID}(c^L) > \Pi_1^{NN}(c^L) > \Pi_1^{IDN}(c^L)$ and $\Pi_2^{IDID}(c^L) > \Pi_2^{NID}(c^L) > \Pi_2^{NN}(c^L) > \Pi_2^{IDN}(c^L)$. Therefore, the only Bayesian equilibrium is (ID, ID) .

- When instead $c_1 = c_2 = c^H$, the equilibrium quantities become:

$$q^{IDID}(c^H) = q_1^{IDID}(c^H) = q_2^{IDID}(c^H) = \frac{a - c^H}{(2 + b)} \quad (21)$$

$$\begin{aligned} q_{1|c^H}^{NN}(c^L, c^H) &= q^{IDID}(c^H) + \frac{b^2}{2B}(c^H - c^L)\nu - \frac{b}{B}\mu(c^H - c^L) \\ q_{2|c^H}^{NN}(c^L, c^H) &= q^{IDID}(c^H) + \frac{b^2}{2B}(c^H - c^L)\mu - \frac{b}{B}\nu(c^H - c^L) \end{aligned} \quad (22)$$

$$\begin{aligned} q_{1|c^H}^{NID}(c^L, c^H) &= q^{NID}(c^H) - \frac{b}{B}\mu(c^H - c^L) \\ q_{2|c^H}^{NID}(c^L, c^H) &= q^{NID}(c^H) + \frac{b^2}{B}\mu(c^H - c^L) \end{aligned} \quad (23)$$

$$\begin{aligned} q_{1|c^H}^{IDN}(c^L, c^H) &= q^{IDID}(c^H) + \frac{b^2}{2B}\nu(c^H - c^L) \\ q_{2|c^H}^{IDN}(c^L, c^H) &= q^{IDID}(c^H) - \frac{b}{B}\nu(c^H - c^L) \end{aligned} \quad (24)$$

The equilibrium profits are given by the squared values of the equilibrium quantities. In particular, with $b > 0$ the ranking of profits is the following: $\Pi_1^{IDN}(c^H) > \Pi_1^{NN}(c^H)$ and $\Pi_1^{IDN}(c^H) > \Pi_1^{IDID}(c^H) > \Pi_1^{NID}(c^H)$; and $\Pi_2^{NID}(c^H) > \Pi_2^{NN}(c^H) > \Pi_2^{IDN}(c^H)$ and

$\Pi_2^{NID}(c^H) > \Pi_2^{IDID}(c^H) > \Pi_1^{IDN}(c^H)$. Therefore the unique Bayesian equilibrium is (ID, ID) . While in the case $b < 0$ the ranking of profits is the following: $\Pi_1^{NN}(c^H) > \Pi_1^{NID}(c^H) > \Pi_1^{IDID}(c^H)$ and $\Pi_1^{NN}(c^H) > \Pi_1^{IDN}(c^H) > \Pi_1^{IDID}(c^H)$; $\Pi_2^{NN}(c^H) > \Pi_2^{NID}(c^H) > \Pi_2^{IDID}(c^H)$ and $\Pi_2^{NN}(c^H) > \Pi_2^{IDN}(c^H) > \Pi_2^{IDID}(c^H)$ the only Bayesian equilibrium becomes (N, N) .

A.2 Proof of Proposition 3

- When both firms are efficient, $c_1 = c_2 = c^L$, the Bayesian equilibrium is without interlocking (N, N) when the products are substitutes ($b > 0$), while is (ID, ID) when the products are complements ($b < 0$). In the (N, N) case the consumer surplus is:

$$CS^{NN}(c^L) = \frac{1}{2} \left(\left(q_{1|c^L}^{NN}(c^L, c^H) \right)^2 + \left(q_{2|c^L}^{NN}(c^L, c^H) \right)^2 \right) + b q_{1|c^L}^{NN}(c^L, c^H) q_{2|c^L}^{NN}(c^L, c^H) \quad (25)$$

with $q_{1|c^L}^{NN}(c^L, c^H)$ and $q_{2|c^L}^{NN}(c^L, c^H)$ as in (18). While the consumer surplus with bilateral interlocking (ID, ID) is:

$$CS^{IDID}(c^L) = \frac{1}{2} \left(\left(q^{IDID}(c^L) \right)^2 + \left(q^{IDID}(c^L) \right)^2 \right) + b \left(q^{IDID}(c^L) \right)^2 \\ = (1+b) \left(\frac{a-c^L}{2+b} \right)^2 \quad (26)$$

with $q^{IDID}(c^L)$ as in (17). Comparison between $CS^{NN}(c^L)$ (25) and $CS^{IDID}(c^L)$ (26) reduces to the comparison between the respective quantities,

$$q_{1|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) = \left(\frac{b}{B}(1-\mu) - \frac{b^2}{2B}(1-\nu) \right) (c^H - c^L) \\ q_{2|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) = \left(\frac{b}{B}(1-\nu) - \frac{b^2}{2B}(1-\mu) \right) (c^H - c^L)$$

Notice that for $b \in (0, 2)$, $0 \leq \mu < 1$ and $0 \leq \nu < 1$

$$\left(q_{1|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) \right) + \left(q_{2|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) \right) = -\frac{b}{2}(2-\nu-\mu) + (2-\nu-\mu) > 0$$

therefore $CS^{NN}(c^L) > CS^{IDID}(c^L)$.

While for $b < 0$, $0 \leq \mu < 1$ and $0 \leq \nu < 1$,

$$q_{1|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) = \frac{b}{B}(1-\mu) - \frac{b^2}{2B}(1-\nu) < 0 \\ q_{2|c^L}^{NN}(c^L, c^H) - q^{IDID}(c^L) = \frac{b}{B}(1-\nu) - \frac{b^2}{2B}(1-\mu) < 0$$

therefore $CS^{IDID}(c^L) > CS^{NN}(c^L)$.

- Analogously, when both firms are inefficient, $c_1 = c_2 = c^H$, the consumer surplus in

(N,N) becomes:

$$CS^{NN}(c^H) = \frac{1}{2} \left(\left(q_{1|c^H}^{NN}(c^L, c^H) \right)^2 + \left(q_{2|c^H}^{NN}(c^L, c^H) \right)^2 \right) + bq_{1|c^H}^{NN}(c^L, c^H) q_{2|c^H}^{NN}(c^L, c^H) \quad (27)$$

with $q_{1|c^H}^{NN}(c^L, c^H)$ and $q_{2|c^H}^{NN}(c^L, c^H)$ as in (22). While the consumer surplus with bilateral interlocking (ID, ID) is:

$$CS^{IDID}(c^H) = \frac{1}{2} \left((q^{IDID}(c^H))^2 + (q^{IDID}(c^H))^2 \right) + b (q^{IDID}(c^H))^2 = (1+b) \left(\frac{a-c^H}{(2+b)} \right)^2 \quad (28)$$

with $q^{IDID}(c^H)$ as in (21). As in the previous case, comparison between $CS^{NN}(c^H)$ (27) and $CS^{IDID}(c^H)$ (28) reduces to the comparison between the respective quantities. Therefore, if $b \in (0, 2)$, consumer surplus is higher with bilateral interlocking, $CS^{IDID}(c^H) > CS^{NN}(c^H)$; while if $b < 0$, consumer surplus is higher without interlocking, $CS^{NN}(c^H) > CS^{IDID}(c^H)$.

A.3 Proof of Proposition 4

- When $c_1 = c_1^L$ and $c_2 = c_2^H$ and the products are substitutes, $b > 0$, the consumer surplus in the equilibrium (ID, N) becomes:

$$CS^{IDN}(c_1^L, c_1^H, c_2^H) = \frac{1}{2} (\Pi_1^{IDN} + \Pi_2^{IDN}) + bq_1^{IDN} q_2^{IDN} \quad (29)$$

While the consumer surplus in the case without interlockings is:

$$CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) = \frac{1}{2} (\Pi_1^{NN} + \Pi_2^{NN}) + bq_1^{NN} q_2^{NN} \quad (30)$$

To compare $CS^{IDN}(c_1^L, c_1^H, c_2^H)$ (29) and $CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H)$ (30) we rewrite:

$$CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) - CS^{IDN}(c_1^L, c_1^H, c_2^H) = \frac{1}{2} (\Pi_1^{NN} + \Pi_2^{NN}) + bq_1^{NN} q_2^{NN} - \frac{1}{2} (\Pi_1^{IDN} + \Pi_2^{IDN}) - bq_1^{IDN} q_2^{IDN}$$

After some calculations, we obtain:

$$CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) - CS^{IDN}(c_1^L, c_1^H, c_2^H) = -\frac{b}{B} \left(\frac{2-b^2}{2} q_1 + \frac{3b}{2} q_2 + \frac{b(4b^2-B)}{8B} \mu C_2 \right) \mu C_2 - \frac{b^3}{4B^2} (4+b^2) (1-\nu) \mu C_1 C_2 \quad (31)$$

with $C_1 = (c_1^H - c_1^L)$ and $C_2 = (c_2^H - c_2^L)$. Sufficient condition for

$$CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) - CS^{IDN}(c_1^L, c_1^H, c_2^H) < 0$$

being $(2-b^2) > 0$ and $b(4b^2-B) > 0$, with $0.89443 < b < 1.4142$.

- When $c_1 = c_1^L$ and $c_2 = c_2^H$ and the products are complements, $b < 0$, the consumer surplus in the equilibrium (N,ID) becomes:

$$CS^{NID}(c_1^L, c_2^L, c_2^H) = \frac{1}{2} (\Pi_1^{NID} + \Pi_2^{NID}) + bq_1^{NID}q_2^{NID} \quad (32)$$

Comparing with the previous expression (30) and considering the following disequalities, $q_i^{NN} < q_i^{NID}$ and $\Pi_i^{NN} > \Pi_i^{IDN}$ for $i = 1, 2$, as in Section (3.3), we obtain:

$$CS^{NN}(c_1^L, c_1^H, c_2^L, c_2^H) - CS^{NID}(c_1^L, c_2^L, c_2^H) < 0$$