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Political Regimes and the Determinants of Terrorism and Counter-terrorism

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Political Regimes and the Determinants of Terrorism and Counter-terrorism

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Abstract

Why do some democratic governments react weakly to terrorism, while apparently similar regimes react harshly? More generally, what are the determinants of governments' reaction to terrorism? And, what are the determinants of terrorism and of its dynamic? In this paper we focus on domestic terrorism and counter-terrorism as affected by economic development, political heterogeneity, citizens' human capital, and government accountability and responsiveness. The empirical research has not reached a consensus on the socioeconomic determinants of terrorism. A possible explanation is that observable data may depend on hidden causal links that are not simply caught by standard regressions.¹ In this paper we argue that terrorism activities are endogenous to the governments' counter-terrorism choices, which in turn does depend on political and socioeconomic factors. Our basic point is that both causes and consequences of terror can only be understood in terms of strategic interaction among political actors, primarily government and citizens. We propose a model that considers human capital, economic development, political heterogeneity, government responsiveness and accountability as possible factors influencing terrorism and the government's response. We show that the game has three possible equilibrium outcomes, uniquely determined by our parameters: a Strong Regime characterized by no terrorism, high counter-terrorism and increasing protests, a Flexible Regime characterized by low terrorism which increase or decrease according to the random reaction of the government, and a Permissive Regime characterized by terrorism activity, no counter-terrorism and no protests. We also show that it is possible for a democratic regime to repress harshly and for an autocratic polity to be permissive.

Keywords: Terrorism, Accountability, Repression.

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1 Introduction

Why do some democratic governments react weakly to terrorism, while apparently similar democratic regimes react harshly? More generally, what are the determinants of governments' reaction to terrorism? And, what are the determinants of terrorism and of its dynamics?

Few subjects evoke as much passion and emotion as terrorism and the issue that surround it. We try to address these thorny problems as objectively as possible. We believe that a clear open analysis aimed to a better and deeper understanding of this phenomenon is the best recipe for making proper sense of the challenges arising from terrorism and fashioning the most effective responses. Scholars are unlikely to agree on a common definition of terrorism, and symmetrically of counter-terrorism, however we may take for granted that almost all definitions of terrorism consider its core **the premeditated use or threat to use violence by individuals or subnational groups to obtain a political or social objective through the intimidation of a large audience beyond that of immediate victims**,² and this is the notion we consider for this work. In particular, we focus on domestic terrorism, i.e. on terrorism homegrown and with consequences for the host country. Of course, terrorism is not all the same, exactly as political regimes are not all the same. This means that we should distinguish among terrorism and among polities. However, a common view is that domestic terrorism is generally affected, among others, by three crucial variables: a country economic development, the political heterogeneity of its citizens and the terrorists' human capital.³ On the other hand, two crucial components that characterize political regimes are accountability and political responsiveness towards citizens. Our model is aimed to analyze the interaction between these government's characteristics and these citizens' factors. Obviously, different tools can be used to pursue the same political aims, for example to induce a change in the government's public policies, citizens might use protests, strikes, etc., instead of violence. Thus, an important topic in studying domestic terrorism is to analyze the reasons behind the use of different ways to try to reach the same aim. In this paper, we explore the connections between the characteristics of polities and the policy choices by citizens and government with respect to terrorism and counter-terrorism. In the literature, some works have focused on counterterrorism policies, however usually at transnational level.⁴

Relating terrorism to social, political, and economic environment on one

²This is the definition proposed in their seminal book by Enders and Sandler (2006).

³Abadie (2006), Blomberg et al. (2004), Caruso - Schneider (2011), Crenshaw (1981), Krieger - Meierrieks (2011), Krueger - Maleckova (2003), Krueger (2007).

⁴E.g. Arce and Sandler (2005), Sandler and Siqueira (2006), Cárceles-Poveda and Taumana (2011), Bandyopadhyay and Sandler (2011) and (2014). See also Brueck and Meierrieks (2015) for a survey.

hand, and counter-terrorism to political characteristics of the government on the other hand, is not an easy task. Hence, it is important empirical and theoretical analysis to complement each other. In this paper we consider a theoretical model to deepen our understanding, and we propose a case study to complement our theoretical model. We now have a large body of data to try to empirically ground the answers to these questions. However, empirical researches have not reached a common consensus on the socioeconomic determinants of terrorism.⁵ Poverty, for example, has often been assumed to be a root cause of terrorism, as people with economic possibilities have a higher opportunity cost to join terrorist organization. However, this thesis flies in the face of most evidence. Obviously, it is important to distinguish socioeconomic characteristics at the individual and at the country level. On one hand, the empirical analysis of the biographies of terrorists shows that education and economic opportunities are either positively correlated to terrorism engagement or at best uncorrelated. On the other hand, low economic development seems not to be correlated to terrorism activity. More generally, there is little or null empirical evidence to support the thesis that deprivation and/or poverty are related to terrorism activity at individual or at national level. How can conclusions based on the clear deductive logic of opportunity costs be wrong? A possible answer is that it may not be. The simple obvious point is that observable data may depend on hidden causal links that are not simply caught by standard regressions. For example, Bueno De Mesquita 2005 argues that the quality of individual terrorists is endogenous to the choices of terrorist organizations; from a different perspective but in the same logic, in this paper we argue that observable terrorism activities are also endogenous to the governments' counter-terrorism choice, which in turn does depend on political and socioeconomic factors. The basic point is that both causes and consequences of terror can only be understood in terms of strategic interaction among political actors, primarily government and citizens. For example, several authors mention the absence of government responsiveness as a factor in the growth of terrorism. In Russia, for example, popular demand for autonomy get no response from the central government and this fact was used to partially explain why some people at some point decided to use terrorism to promote secession. However, in Spain, ETA terrorist violence increased after the advent of democracy and the recognition of autonomy for the Basque region. So terrorism is not simply a reaction to lack of positive response from the government on political issues; the interaction among different factors is complex and difficult to understand without a theoretical help by a model.

This paper is an attempt towards the construction of a general model that considers individual human capital, economic development, political heterogeneity on one hand, and government responsiveness and accountability on the other

⁵See e.g. the essay in Gottlieb (2014).

hand, as possible factors influencing why individual agents might decide to use terrorism as a political strategy to try to reach their political goals and, jointly, the magnitude of government's response. In this work, we assume that terrorists are utility maximizers that use terrorism when the expected political gains minus the expected costs outweigh the net expected benefits of alternative forms of protest. Similarly, the government acts to maximize a weighted sum of citizens' utility and of its own political goal. The observable terrorism and counter-terrorism activities are the equilibrium outcomes of such strategic interaction, that in turn depends on the above socioeconomic and political factors. To the best of our knowledge, our model is one of the few that consider the individual choice of single citizens whether to join a terrorist organization or political activists or, further, no political activity at all. In other words, this paper directly tackles a crucial problem of any model of political engagement, the problem of collective action.⁶ Within this frame, we are able to show that the game has three possible equilibrium outcomes, uniquely determined by our structural parameters:

1. A **Strong Regime** characterized by no terrorism, high counter-terrorism and increasing protests;
2. A **Flexible Regime**⁷ characterized by low terrorism which might increase or decrease according to the random repressive reaction of the government with increasing protests if there is high counter-terrorism;
3. A **Permissive Regime** characterized by terrorism activity, no counter-terrorism and no protests.

In particular, we show that it is perfectly possible for a democratic regime to repress harshly and for an autocratic polity to be permissive.

To stress that these questions and results are relevant, consider four countries variously affected by terrorism in the last 40 years, with different degrees of democratic accountability and responsiveness.⁸ As democratic countries consider Italy and Israel, and as autocracies consider Russia and Nigeria. In the seventies the Italian government's reaction was at first tolerant, then harsh, while Israel always react harshly to any terrorist attack. A significant difference between these two countries is the significant high political heterogeneity between Palestinian activists and Israeli citizens, significantly higher than the political heterogeneity between Italian activists and Italian citizens in the seventies. Similarly, in Nigeria the government's reaction to terrorism has been sometime weak, sometimes harsh, while in Russia the government reaction has

⁶See Apolte (2012) for a clear analysis of this problem within a model of possible revolutions.

⁷This outcome is random because is generated by a mixed strategy equilibrium.

⁸See Crenshaw 2001 and Whittaker 2012.

always been extremely harsh. From the point of view of our explanatory variables, these countries are significantly different even if both are autocratic: low economic development in Nigeria, significant economic development in Russia, intermediate political heterogeneity in Nigeria, high political heterogeneity in Russia, low terrorists' human capital in Nigeria, intermediate in Russia. As our results will show, these structural factors combine to provide an explanation for these different counter-terrorists policies.

An even more puzzling case is provided by the comparison of counter-terrorist policies implemented in Italy and in West Germany in the seventies. Considering the above structural factors - economic development, terrorists' human capital and political heterogeneity - these two countries look pretty similar. However, their counter-terrorism policies were quite different: immediately strong in German, initially weak and only later strong in Italy. And the effects on terrorism were actually different: while terrorism disappeared quite quickly in West Germany, in Italy it affected the country for all the eighties and partly in the nineties too. What can explain such differences? Is it just a matter of different political cultures or there are structural factors at work behind these different choices? In the concluding remarks we will argue that our results provide a partial explanation for this and similar phenomena.

The paper proceeds as follows. The next section presents the structure of the model, then we present the results and the analysis, while the final section concludes explaining the relevance and the prospective of this approach. All the proofs are in an online appendix.

2 The Model

In the following notation the subscript denotes the role of the player while the superscript denotes the time. The model we propose is a three stages sequential game, where within each stage the agents play simultaneously and at the beginning of each stage all the players are informed of all the previous stage players' choices (almost perfect information).

2.1 The Timing of the Game

1. FIRST STAGE Each citizen $i \in \{1, \dots, P\}$ has to decide whether to join one of three different groups, terrorists T^1 , political activists A^1 , or conservatives C^1 . Denote by

$$n_J^\tau = \frac{\sum_{i \in J} I_{\{i \in J\}}}{P} = \frac{N_J^\tau}{P}$$

the proportion of citizens in group $J \in \{T, A, C\}$ at period $\tau \in \{1, 2\}$, where $I_{\{i \in J\}}$ is the indicator function for a citizen $i \in \{1, \dots, P\}$ choice of

group J . Similarly,

$$h_J^\tau = \frac{\sum_{i \in J} H_i^\tau}{N_J^\tau}$$

is the average human capital of citizens within group $J \in \{C, T, A\}$ at period $\tau \in \{1, 2\}$;

2. **SECOND STAGE** The government G observes $(n_T^1, n_A^1, n_C^1) \in [0, 1]^3$ and $(h_T^1, h_A^1, h_C^1) \in [0, 1]^3$, and chooses the amount of counter-terrorism ρ ;
3. **THIRD STAGE** Each citizen $i \in \{1, \dots, P\}$ after observing the government choice, has to decide again whether to join one of the three different groups, terrorists T^2 , political activists A^2 , or conservatives C^2 .
4. **FINAL OUTCOMES** The game ends with two possible final outcomes, Change or Status Quo, with probability, respectively, R and $1 - R$:

$$\tilde{\Pi} = \begin{cases} \text{Change} & \text{probability } R \\ \text{Status quo} & \text{probability } 1 - R \end{cases}$$

The timing of the model is represented in figure 1:

2.2 The Players' Possible Choices

To be a terrorist is full time job, since it mean to get underground and to use all personal capacity using violence to obtain a change in the status quo. Hence we assume that each citizens in the terrorist group use his/her human capital for the terrorism effort:

$$i \in T^\tau \Rightarrow E_i^\tau = H_i.$$

If a citizens wish a change in the status quo, an alternative possibility is to joint political activism, which is not a full time activity, even if it means to use part of the individual human capital for political activities. Hence we assume that each citizens in the group of political activists, use half of his/her human capital for activism:

$$i \in A^\tau \Rightarrow E_i^\tau = \frac{1}{2}H_i.$$

Finally, a citizens might be satisfied with status quo, hence he/she devotes all his/her human capital to work. Hence:

$$i \in C^\tau \Rightarrow E_i^\tau = 0.$$

This means that in each period $\tau \in \{1, 2\}$, each citizens $i \in \{1, \dots, P\}$ has three possible choices $i \in \{T^\tau, A^\tau, C^\tau\}$ with the above consequences.

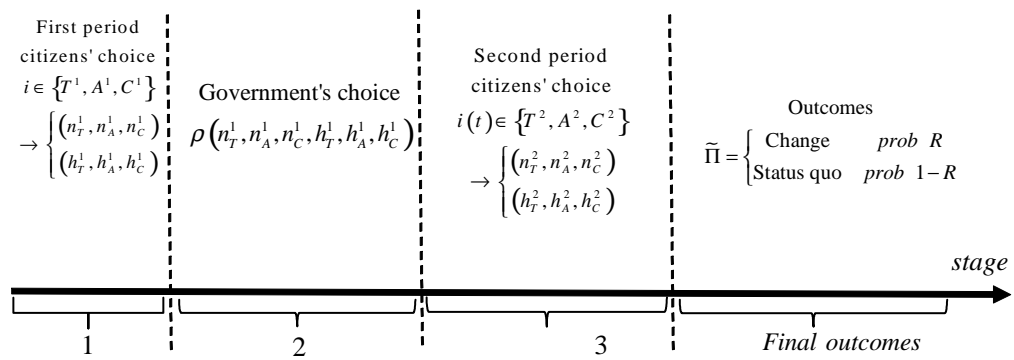


Figure 1: Figure 1.

At stage 2, between period 1 and 2 when the citizens make their choices, the government has the opportunity to decide how much of the second period national income $\sum_{i \in P} Y_i^2$, get used for counterterrorism policies, i.e. $\rho \in [0, \sum_{i \in P} Y_i^2]$. The government choice of the total expenditure on counterterrorism, ρ , is subject to the public budget constraint: $t \sum_{i \in P} Y_i^2 = \rho$, where $t \in [0, 1]$ is the average tax rate. Hence, we might consider as government choice variable

$$t \in [0, 1].$$

2.3 Remarks on the Model

The model is constructed to try to understand the drivers of the choice of each citizens whether to get involved into terrorism or in political activism or abstaining from any political activism. Let us stress that this is one of the few models considering the collective action problem, that is the fact that each agent recognizes how small is his/her contribution towards the realization of the political or social aim.² We believe this is particularly important for a rational choice model of a phenomenon as terrorism. In particular, we wish to connect such individual choice to some structural variables that the literature³ has pinpointed as crucial, as the country economic development, the individual human capital and the country political heterogeneity. Of course, such choice should also be connected to the government choices. Hence, it is also crucial to try to understand the drivers of the government choice on counterterrorism, choice that we connect to the political characteristics of a regime, in particular to accountability and responsiveness towards citizens.⁴

The basic idea behind the timing of the model is that the government reacts to terrorism activity, and in turn the evolution of terrorism is driven by the citizens' tastes for a change and by the government's counterterrorism policy and by its fiscal cost. In turn, terrorism and counterterrorism affect the probability of a change in the social and political status quo. To avoid further complexity, we do not consider the possibility of a learning by doing in citizens choosing to be terrorists, or activists or conservatives.

Let us stress that in this paper we are not interested into the specific activities of terrorist groups, such as bombing, killing and kidnapping and on the

²Among the few works that consider terrorism as an individual choice see Moore (1995), De Mesquita (2010), Faria and Arce (2012), Edmond (2013), Arce and Siqueira (2014), Shadmehr (2015), Hendrickson and Salter (2016).

³See e.g. Thompson (1989), Krueger and Malecukova (2003), Li (2005), Piazza (2006), Benmelech and Berrebi (2007), Gambetta and Hertog (2007), Bandyopadhyaya and Younas (2011), Krieger and Meierrieks (2011) and (2016), Brockhoff et al (2012), Gleditsch and Polo (2016), Jetter and Stadelmann (2017), Abdel Jelil et al (2018).

⁴See Przeworski et al. (1999) for a discussion on the role of these two characteristics to classify political regimes.

victims or on the potential terrorism audience, we just focus on how terrorists' violent activities and activists political activities affect the likelihood of a change in the status quo. This focus allows to analyze the individual choice between terrorism, activism or plain work in terms of individual costs and expected benefits. Similarly, we are not interested in the specific articulation of counterterrorism policies, we just focus on the amount of fiscal revenues devoted to such chapter, as a proxy for its intensity. Thus, we sum up the *effects* of all these choices into a probability of a change in the status quo, as common in conflict models. Note that there is a huge controversy on whether terrorism work,⁵ however the most shared conclusion is that in some situation at some degree with a small but not negligible probability terrorism realizes part of its goals.

2.4 Structural Assumptions

In this section, we propose some simplifying assumptions that will make the payoff functions linear so to allow the precise calculations of the endogenous variables.

2.4.1 The Country Production Function

Each citizen i 's income Y_i is determined by the country production function $f(L_i^\tau) = \alpha L_i^\tau$, where

1. $\alpha \in (0, +\infty)$ is the country labour's productivity, thus is an index of the country economic development
2. L_i^τ is i 's labor supply at time τ , which is equal to his/her human capital H_i minus his/her effort towards a change in the status quo at period τ , E_i^τ .

Thus i 's income at period τ is $Y_i^\tau = \alpha L_i^\tau = \alpha (H_i - E_i^\tau)$.

2.4.2 The Citizens' Payoff Functions

Without loss of generality, the final outcomes are associate to the following public payoff

$$\Pi = \begin{cases} 1 & \text{if change} \\ 0 & \text{if status quo} \end{cases}$$

that in turn affect the players' payoffs linearly. In particular, the citizens' utilities are⁶

$$u_i \left(c_i^1, c_i^2, \tilde{\Pi} \right) = c_i^1 + c_i^2 + \gamma_i \Pi \Rightarrow Eu_i \left(c_i^1, c_i^2, \tilde{\Pi} \right) = c_i^1 + c_i^2 + \gamma_i R$$

⁵See eg. English (2016).

⁶To simplify, the dicount factor is supposed to be equal 1.

where

1. $\gamma_i \sim U \left[-\frac{m}{2}P, \frac{m}{2}P \right]$ is an individual parameter that describes a citizen's attitudes towards changes of status quo that is uniformly distributed on $\left[-\frac{m}{2}P, \frac{m}{2}P \right]$: higher γ_i means a higher propensity for change, while lower γ_i denotes a greater taste for status quo;
2. c_i^τ is i 's consumption at time τ , that we assume is equal to i 's disposable income at time τ , so that $c_i^1 = Y_i^1 = \alpha L_i^1 = \alpha (H_i - E_i^1)$ and $c_i^2 = (1-t)Y_i^2 = \alpha(1-t)L_i^2 = \alpha(1-t)(H_i - E_i^2)$ where t is the tax rate used by the government to finance counterterrorism.

2.4.3 The Government Payoff Function

We assume that the government payoff function is a weighted average of citizens' utilities and of the status quo:

$$\begin{aligned} u^G &= \delta \frac{1}{P} \sum_{i=1}^P u_i + (1-\delta) (1 - \tilde{\Pi}) = \delta \frac{1}{P} \sum_{i=1}^P (c_i^1 + c_i^2) + (1-\delta) (1 - \tilde{\Pi}) \Rightarrow \\ &\Rightarrow EU^G = \delta \frac{1}{P} \sum_{i=1}^P (c_i^1 + c_i^2) + (1-\delta) (1 - R) \end{aligned}$$

where δ is an index of the representativeness of the government: the more δ goes to 1, the more the government is accountable and vice-versa.

2.4.4 The Conflict Technology

A crucial aspect of this model is the specification of how the players' choices affect the probability of a change. We assume the following conflict technology

$$R = \min \left\{ \left(1 - \frac{\rho}{\sum_{i \in P} Y_i^2} \right) \left(\frac{\sum_{i \in T^1} E_i^1 + \sum_{i \in T^2} E_i^2}{P} \right) + a \left(\frac{\sum_{i \in A^1} E_i^1 + \sum_{i \in A^2} E_i^2}{P} \right), 1 \right\}$$

where

1. $\sum_{i \in P} Y_i^2$ is the country second period income, hence $\frac{\rho}{\sum_{i \in P} Y_i^2}$ is the share of national income used for counterterrorism policies;
2. $\sum_{i \in T^1} E_i^1 + \sum_{i \in T^2} E_i^2$ is the terrorists' total effort, i.e. all violent activities used to change the status quo, hence $\frac{\sum_{i \in T^1} E_i^1 + \sum_{i \in T^2} E_i^2}{P}$ is the domestic proportion of such activities;
3. $\sum_{i \in A^1} E_i^1 + \sum_{i \in A^2} E_i^2$ is the total effort of political activists, i.e. all political activities used to change the status quo, hence $\frac{\sum_{i \in A^1} E_i^1 + \sum_{i \in A^2} E_i^2}{P}$ is the domestic proportion of such activities;

4. $a \in (0, 1)$ is a measure of how much citizens' political activism affect the probability of a change in status quo policies.

This conflict technology is quite standard and it says in the simplest way that the probability of a change increases linearly with the percentage of global terrorists' activity and it is reduced by government's counter-terrorism. Also active participation to political protests increases the likelihood of a change, however the effects of such protests are mediated by the parameter $a \in (0, 1)$.

2.4.5 Implications of Structural Assumption

Using the public budget constraint and the previous structural assumptions, we can rewrite the probability of successful revolution as follows

$$R = \min \left\{ (1-t) (n_T^1 h_T^1 + n_T^2 h_T^2) + \frac{1}{2} a (n_A^1 h_A^1 + n_A^2 h_A^2), 1 \right\}.$$

Moreover, a citizen i expected utility can be rewritten separating the effect of the other citizens' choices from the effect of his/her own choice:

$$EU_i (i \in J^1, i \in J^2) = \gamma_i R_{-i} \left(t, \pi, \bar{n}_T^1 \bar{h}_T^1, \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^1 \bar{h}_A^1, \bar{n}_A^2 \bar{h}_A^2 \right) + B_i (i \in J^1, i \in J^2)$$

where

$$\begin{aligned} R_{-i} \left(t, \pi, \bar{n}_T^1 \bar{h}_T^1, \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^1 \bar{h}_A^1, \bar{n}_A^2 \bar{h}_A^2 \right) &= \\ &= \min \left\{ \left[(1-t) \left(\bar{n}_T^1 \bar{h}_T^1 + \bar{n}_T^2 \bar{h}_T^2 \right) + \frac{1}{2} a \left(\bar{n}_A^1 \bar{h}_A^1 + \bar{n}_A^2 \bar{h}_A^2 \right) \right], 1 \right\} \end{aligned}$$

and

$$\bar{n}_J^\tau = \frac{1}{P} \sum_{j \neq i, j=1}^P I_{\{j \in J\}} \quad \text{and} \quad \bar{h}_J^\tau = \frac{1}{P} \sum_{j \neq i, j=1}^P I_{\{j \in J\}} H_j$$

so that \bar{n}_J^τ and \bar{h}_J^τ denote, respectively, the percentage and the average human capital of citizens different from i that choose group $J^\tau = \{T^\tau, A^\tau, C^\tau\}$ in period $\tau \in \{1, 2\}$, and

$$B_i (i \in J^1, i \in J^2) = \begin{cases} \frac{2}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{P} \gamma_i (1 - \pi t) H_i + \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i (1 - \pi t) H_i + \alpha H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \frac{1}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in A^1, i \in T^2 \\ \frac{1}{P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha (1 - t) H_i + \frac{1}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in C^1, i \in T^2 \\ \alpha (1 - t) H_i + \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha (1 - t) H_i + \alpha H_i & \text{if } i \in C^1, i \in C^2 \end{cases}$$

Finally, the government expected payoff can be rewritten as follows

$$EU^G(t) = \delta \left(\left[\frac{1}{2} (1-t) \alpha n_A^1 h_A^1 + \frac{1}{2} \alpha n_A^2 h_A^2 + (1-t) \alpha n_C^1 h_C^1 + \alpha n_C^2 h_C^2 \right] \right) + \\ + (1-\delta) \left(1 - \min \left\{ (1-t) (n_T^1 h_T^1 + n_T^2 h_T^2) + \frac{1}{2} a (n_A^1 h_A^1 + n_A^2 h_A^2), 1 \right\} \right)$$

2.5 Remarks on the structural assumptions

Before going to the results, let us discuss the implications of the structural assumptions for the model and the meaning of our variables.

First of all, let us stress that we use specific linear functions to simplify calculations and to find closed form solutions so as to be as clear as possible on comparative static analysis.

Considering the players, citizens differ because of their attitudes towards changes of the status quo, specifically because of the value of γ_i , i 's marginal utility of a change in the status quo. Since γ_i takes values in the interval $\left[-\frac{m}{2}P, \frac{m}{2}P\right]$, then the length m of this interval is a measure of the dispersion in citizens' values of government policy. Hence, we consider m as a measure of *people political heterogeneity*. Two other important parameters affecting citizens' choices are i 's *human capital* H_i and a country's labor's productivity α , an index of *economic development of the country*. Human capital is relevant because it affects i 's effort both on labor and on political activities, while labor's productivity α affects i 's opportunity cost in joining terrorism or activism. The Government payoff is the weighted average of total citizens' welfare and of the desire to maintain the status quo, i.e. the public policy implemented by the government itself. Since governments are accountable if they are acting in the interest of voters, the weight δ of citizens' welfare can be interpreted as an index of the *accountability of the government*: the greater δ , the more the government is accountable to citizens. The players' payoff is also affected by the conflict technology, i.e. by the probability that the status quo is changed. This probability is a weighted average of the citizens' effort towards changes, which can take two different ways: terrorism (armed struggle) and activism, i.e. all those activities that signal a citizen's desire for a change. The effect of activism on the probability of a change is reduced by a parameter $a \in [0, 1]$. Since "a government is "responsive" if it adopts policies that are signalled as preferred by citizens [and] these signals may include ... various forms of direct political action, including demonstrations, letter campaigns, and the like",⁷ we consider a as a measure of *government responsiveness*. We believe this parameter is actually important to understand both the citizens' decision whether to join terrorism or activism, and the government's reaction. Actually many analysis

⁷Przeworski et al. 1999, p.9.

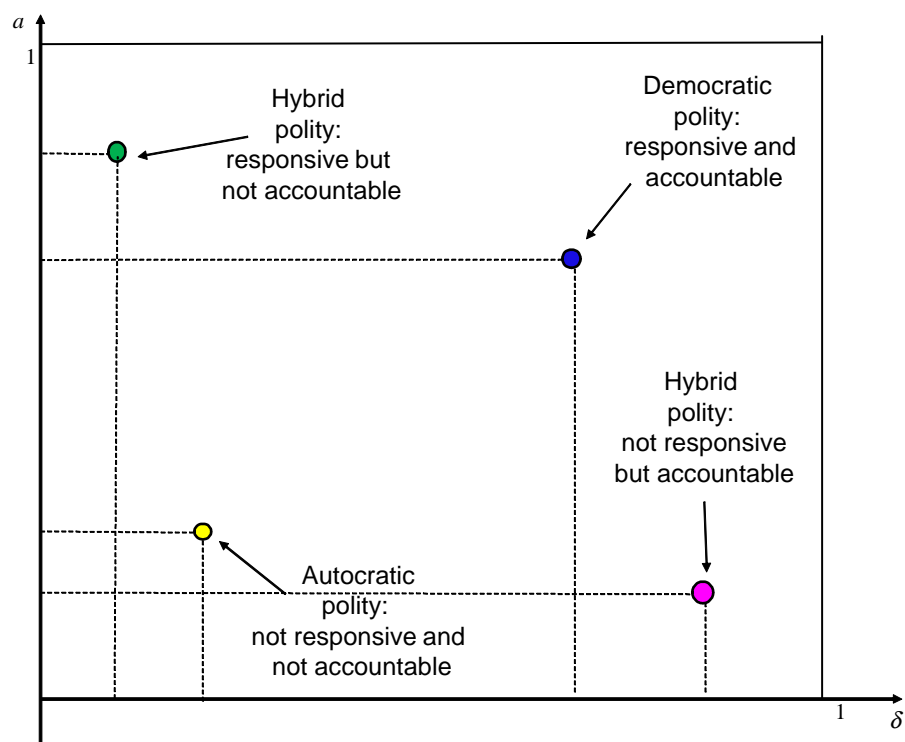


Figure 2: Figure 2.

of terrorism emphasize that disappointment and frustration with nonviolent actions aimed to change the status quo are frequently one of the reasons to turn political activism into armed struggle, both in accountable and non accountable environments. In other words, how responsive is a government to citizens active requests for a change in public policies, seems to be an important determinant for our problem. Note that from the point of view of citizens, accountability is always desirable, while responsiveness is not an unmitigated virtue: "most of us want government to be responsive when it comes to matters like building roads ... But in the administrations of programs in which the dispensing of justice is concerned ... popular responsiveness is not so attractive".⁸ From this point of view, we may characterize a political regime as more or less democratic using together $(a, \delta) \in [0, 1] \times [0, 1]$, as shown by figure 2.

In the following table we sum up the exogenous and endogenous variable of the model, and their meaning

⁸Ferejohn p.131-132 in Przeworski et al. 1999.

| Exogenous variables | Meaning |
|--|--|
| $P \in \mathbb{N}$ | population size |
| $\gamma_i \sim U \left[-\frac{m}{2}P, \frac{m}{2}P \right]$ | citizens' political position |
| $m \in (0, \infty)$ | measure of political heterogeneity |
| $\alpha \in (0, \infty)$ | measure of economic development |
| $h_J^\tau \in [0, 1]$ | average human capital of citizens in group $J \in \{T, A, C\}$ in period $\tau = 1, 2$ |
| $a \in (0, 1)$ | measure of regime's responsiveness |
| $\delta \in [0, 1]$ | measure of government accountability |
| Endogenous variables | Meaning |
| $n_J^\tau \in [0, 1]$ | percentage of citizens in group $J \in \{T, A, C\}$ in period $\tau = 1, 2$ |
| $t \in [0, 1]$ | tax rate used for government counter-terrorism |
| $\pi \in [0, 1]$ | measure of government repressiveness on protests |

3 Solution of the Game

As solution concept, we use Subgame Perfect Equilibria (SPE), thus to solve the game we work backward. To simplify calculations and interpretation, we make the following assumptions.

Conditions 1 1. *Increasing heterogeneity with increasing economic development:*

$$m \geq 2\alpha$$

2. *Stationary:*

$$h_C^1 = h_C^2 = \frac{1}{2}, \quad h_T^1 = h_T^2 = h_A^1 = h_A^2 \equiv h$$

Remarks 2 1. *The first condition means that the ratio of political heterogeneity of the population over productivity is bounded from below by the level of economic development so that their relative changes should be related. The condition is useful to simplify the actual calculus of the proportion of citizens in the different groups, however it is also plausible.*

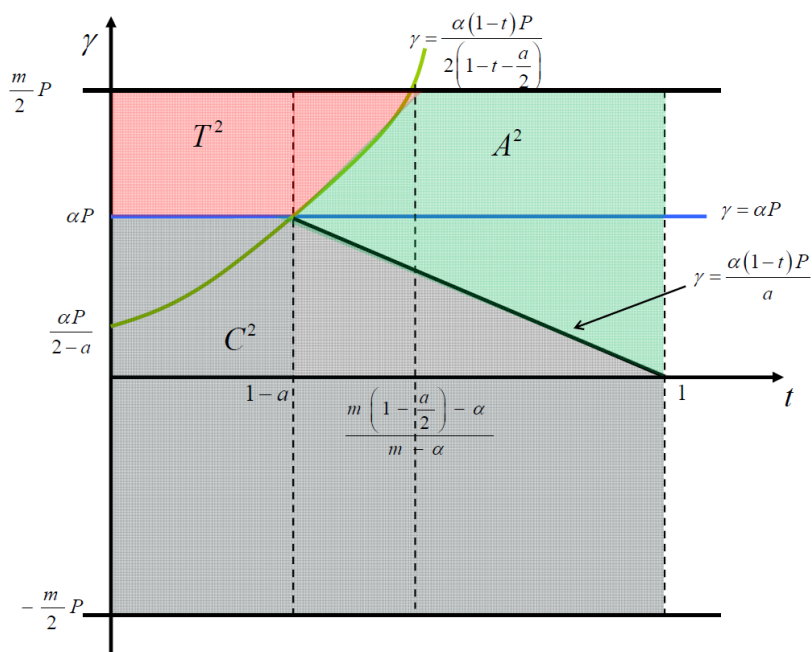


Figure 3: Figure 3.

2. The second condition is extremely useful to get closed form solution, it means that the average human capital of terrorist and activists is the same, stationary and it is greater or smaller than the average human capital of conservatives depending on $h \gtrless \frac{1}{2}$.

3.1 Citizens' Choice in the Second Period

In the Appendix, we find the values of γ_i such that, for given amount of counter-terrorism, a citizens i choose a specific group, as sketch in figure 3.

From this result, using conditions 1 and 2, it is immediate to derive⁹ the percentage of groups in the second period, as a function of the amount of gov-

⁹See the Appendix.

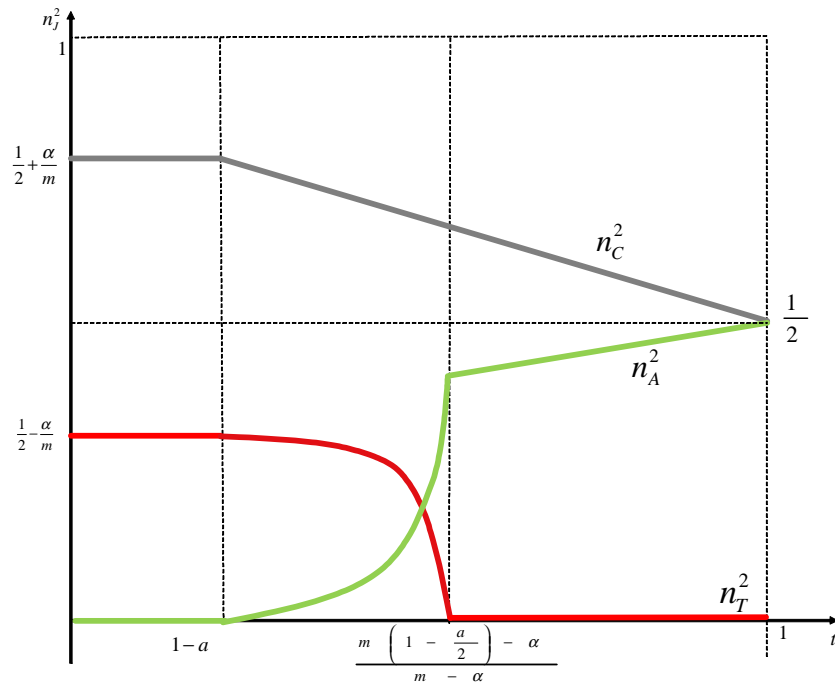


Figure 4: Figure 4.

ernment counter-terrorism and of exogenous parameters:

$$\left\{ \begin{array}{l} n_T^2 = \frac{1}{2} - \frac{\alpha}{m}; n_A^2 = 0; n_C^2 = \frac{1}{2} + \frac{\alpha}{m} \quad t \in [0, 1-a] \\ \\ n_T^2 = \frac{1}{2} - \frac{\alpha(1-t)}{2(1-t-\frac{a}{2})m}; \\ n_A^2 = \frac{\alpha(1-t)[t-(1-a)]}{a(1-t-\frac{a}{2})m}; \\ n_C^2 = \frac{1}{2} + \frac{\alpha(1-t)}{am} \quad t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} \right] \\ \\ n_T^2 = 0; n_A^2 = \frac{1}{2} - \frac{\alpha(1-t)}{am}; \\ n_C^2 = \frac{1}{2} + \frac{\alpha(1-t)}{am} \quad t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right] \end{array} \right.$$

The results is represented in figure 4.

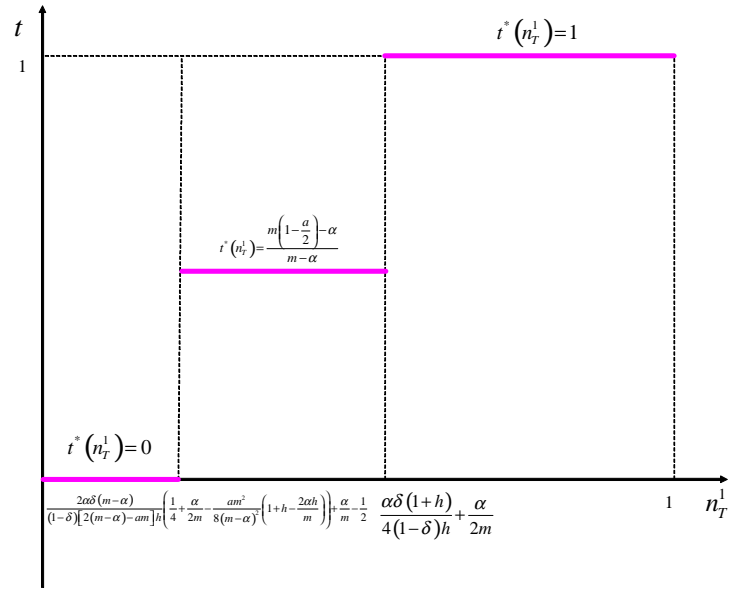


Figure 5: Figure 5.

3.2 Government's Choice

In the Appendix, we derive the following sequential best reply of the government:

$$t^*(n_T^1) = \begin{cases} 0 & n_T^1 \in \langle 1 \rangle \\ \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} & n_T^1 \in \langle 2 \rangle \\ 1 & n_T^1 \in \langle 3 \rangle \end{cases}$$

where

$$\langle 1 \rangle := \left[0, \frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right]$$

$$\langle 2 \rangle :=$$

$$\left[\frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1 + h)}{4(1 - \delta)h} + \frac{\alpha}{2m} \right]$$

$$\langle 3 \rangle := \left[\frac{\alpha\delta(1 + h)}{4(1 - \delta)h} + \frac{\alpha}{2m}, 1 \right]$$

where the intervals can be empty for specific values of the parameters. The result is represented in figure 5.

3.3 First Period Citizens' Choice

In the Appendix, the citizens sequential best reply in the first period is characterized as follows:

1. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[\frac{1}{2} - \frac{\alpha}{m}, \infty \right)$$

there is a pure strategy SPE such that

$$t^* = 0$$

and

$$\begin{aligned} i^* \in T^1 &\Leftrightarrow \gamma_i \in \left[\alpha P, \frac{m}{2} P \right] \\ i^* \in A^1 &\Leftrightarrow \gamma_i \in \emptyset \\ i^* \in C^1 &\Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P; \alpha P \right] \end{aligned}$$

2. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

there is a mixed strategy SPE such that the government randomizes so that

$$t^* = \begin{cases} 0 & \text{probability } 1 - \tau \\ \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} & \text{probability } \tau \end{cases}$$

which implies

$$Et^* = \frac{\left[m \left(1 - \frac{a}{2} \right) - \alpha \right] \tau}{m - \alpha} \in \left[\frac{1}{2}, \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \right]$$

and

$$\begin{aligned} &i^* \in T^1 \Leftrightarrow \\ &\Leftrightarrow \gamma_i \in \left[\left(m - \alpha - \frac{2\alpha\delta m(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) \right) P; \frac{m}{2} P \right] \\ &i^* \in A^1 \Leftrightarrow \\ &\Leftrightarrow \gamma_i \in \left[\alpha P; \left(m - \alpha - \frac{2\alpha\delta m(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) \right) P \right] \\ &i^* \in C^1 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P; \alpha P \right] \end{aligned}$$

3. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in (-\infty, 0]$$

there is SPE such that

$$t^* = \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha}.$$

and

$$\begin{aligned} i^* \in T^1 &\Leftrightarrow \gamma_i \in \emptyset \\ i^* \in A^1 &\Leftrightarrow \gamma_i \in \left[\frac{\alpha}{a}P; \frac{m}{2}P \right] \\ i^* \in C^1 &\Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P; \frac{\alpha}{a}P \right] \end{aligned}$$

4 The Set of Equilibria

Now we are able to characterize the set of Subgame Perfect equilibria outcomes of the terrorist game, as a function of the structural parameters.

Proposition 1 *The terrorist game is characterized by three equilibrium outcomes, that we call "Regimes":*

1. A **Strong Regime** with no terrorism, increasing political activism and strong counterterrorism policy;
2. a **Flexible Regime** with possible increasing or decreasing terrorism, possible decreasing or increasing political activism as a consequence of counterterrorism policy, that randomly can be strong or weak;
3. a **Permissive Regime** with terrorism and no political activism, and weak counterterrorism.

Formally, the parameters regions that are characterized by this players' behavior are

- *When*

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in (-\infty, 0]$$

*the SPE outcome is a **Strong Regime** such that*

$$\begin{aligned} n_T^1 &= n_T^2 = 0 \\ n_A^1 &= \frac{1}{2} - \frac{\alpha}{am} < n_A^2 = \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \end{aligned}$$

$$n_C^1 = \frac{1}{2} + \frac{\alpha}{am} > n_C^2 = \frac{1}{2} + \frac{\alpha}{2(m-\alpha)}.$$

$$t^* = \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}.$$

- when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

the SPE outcome is a **Flexible Regime** such that

$$n_T^{1*} = \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}$$

$$n_T^{2*} = \begin{cases} \frac{1}{2} - \frac{\alpha}{m} > n_T^{1*} & \text{probability } 1 - \tau \\ 0 < n_T^{1*} & \text{probability } \tau \end{cases}$$

$$n_A^{1*} = 1 - \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) - \frac{2\alpha}{m}$$

$$n_A^{2*} = \begin{cases} 0 < n_A^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} > n_A^{1*} & \text{probability } \tau \end{cases}$$

$$n_C^{1*} = \frac{1}{2} + \frac{\alpha}{m}.$$

$$n_C^{2*} = \begin{cases} \frac{1}{2} + \frac{\alpha}{m} = n_C^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} + \frac{\alpha}{2(m-\alpha)} < n_C^{1*} & \text{probability } \tau. \end{cases}$$

$$t^* = \begin{cases} 0 & \text{probability } 1 - \tau \\ \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} & \text{probability } \tau \end{cases}$$

- when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[\frac{1}{2} - \frac{\alpha}{m}, \infty \right)$$

the SPE outcome is a **Permissive Regime** such that

$$n_T^{1*} = n_T^{2*} = \frac{1}{2} - \frac{\alpha}{m}$$

$$n_A^{1*} = n_A^{2*} = 0$$

$$n_C^{1*} = n_C^{2*} = \frac{1}{2} + \frac{\alpha}{m}.$$

$$t^*(n_T^1) = 0.$$

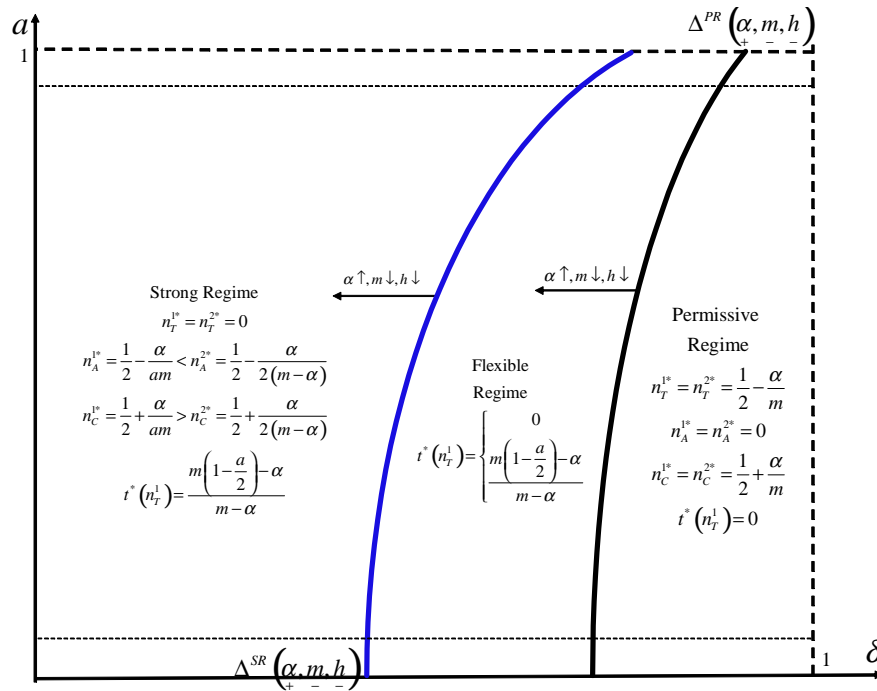


Figure 6: Figure 6.

Note that in a Permissive Regime there is no counter-terrorism, however this has a cost in terms of accepting a certain amount of terrorism activity. More interesting, it is the fact that in equilibrium we might have counter-terrorism even if there is no apparent terrorism, the case of a strong regime: actually it is exactly the government's counter-terrorism that avoids terrorism activities. In other words, it is not necessary to have terrorism in order to trigger counter-terrorism policies. The only situation where we have both terrorism and counter-terrorism is the Flexible Regime, where terrorism activity triggers counter-terrorism with strictly positive probability. Graphically, in the space accountability and responsiveness the situation is represented in figure 6.

Corollary 5 in the Appendix as illustrated by figure 6, shows two important aspects of the relationship between the two characteristics of a political regime, accountability and responsiveness:

1. accountability is by far the most crucial determinant of counter-terrorism policies, in particular
 - when accountability is small enough, i.e. $\delta \leq \frac{2mh-2\alpha h}{2mh-2\alpha h+\alpha m+2\alpha^2}$, for any level of responsiveness the only possible counter-terrorism policy

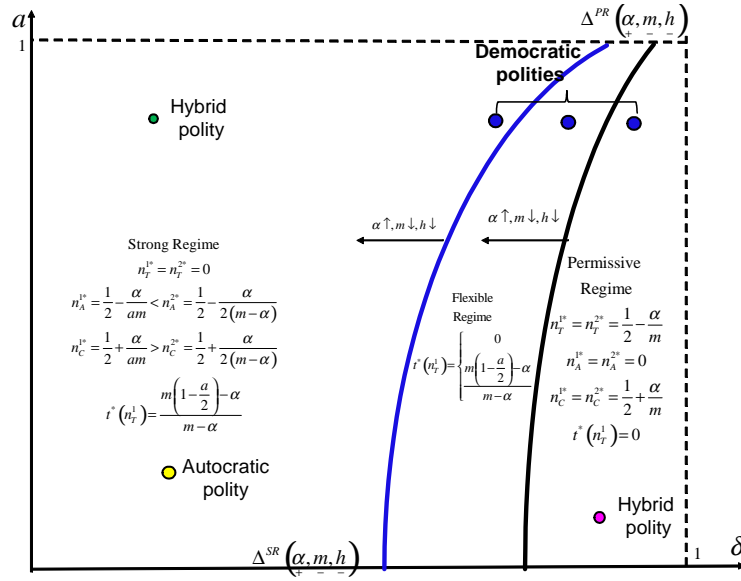


Figure 7: Figure 7.

is the strong one; moreover the region of Strong Regime is shrinking as economic development increases or political heterogeneity decreases or, again, terrorists human capital reduces;

- when accountability is big enough, i.e. $\delta \geq \frac{(m-2\alpha)(1-\frac{2\alpha}{m})h}{\alpha(m-\alpha)(\frac{1}{2}+\frac{\alpha}{m}-\frac{m^2}{4(m-\alpha)^2}(1+h-\frac{2\alpha h}{m}))+(m-2\alpha)(1-\frac{2\alpha}{m})h}$, for any level of responsiveness the only possible counter-terrorism policy is the permissive one; moreover the region of Permissive Regime is shrinking as economic development decreases or political heterogeneity increases or, again, terrorists human capital increases;

2. there is a sort of complementarity between responsiveness and counter-terrorism policies, in the sense that increasing responsiveness increases the region of strong counter-terrorism policies; in other words more responsive regime are also more strong against terrorism activities, ceteris paribus.
3. highly democratic government might react with harsh counter-terrorism policies to an increase in the population political heterogeneity or to a reduction in economic growth or to an increment in the terrorist human capital, as figure 7 shows.

4.1 Responsiveness and political choices

Interesting clues on the role played by responsiveness on players' political choices, can be derived by our results. Therefore, we will consider the effect on polit-

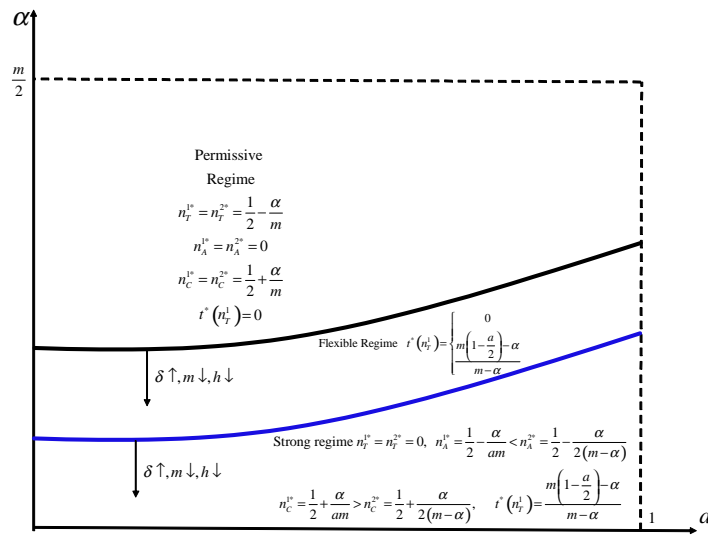


Figure 8: Figure 8.

ical choices of responsiveness w.r.t. in turn economic development, political heterogeneity, and people human capital, ceteris paribus.

4.1.1 Responsiveness, economic development and political choices

From corollary 6 in the Appendix, it is immediate to derive the following picture that shows the crucial role of economic development for **any** level of responsiveness, in particular that for **any** level of responsiveness an increase in economic development provokes a shift from the Strong Regime to the Flexible and then to the Permissive one.

4.1.2 Responsiveness, political heterogeneity and political choices

From corollary 7 in the Appendix, it is immediate to derive the following picture that shows the peculiar role of political heterogeneity for any small level of responsiveness. In particular, when economic development is great enough ($\alpha \geq \frac{2(1-\delta)h}{\delta(1-h)}$), then for any level of responsiveness small enough ($a \leq \frac{4(1-\delta)h-2\alpha\delta}{2(1-\delta)h-\alpha\delta(1+h)}$) the Strong Regime is not possible and an increase in political heterogeneity provokes a shift from a Permissive Regime to a Flexible, but never to a Strong one, notwithstanding the amount of political polarization.

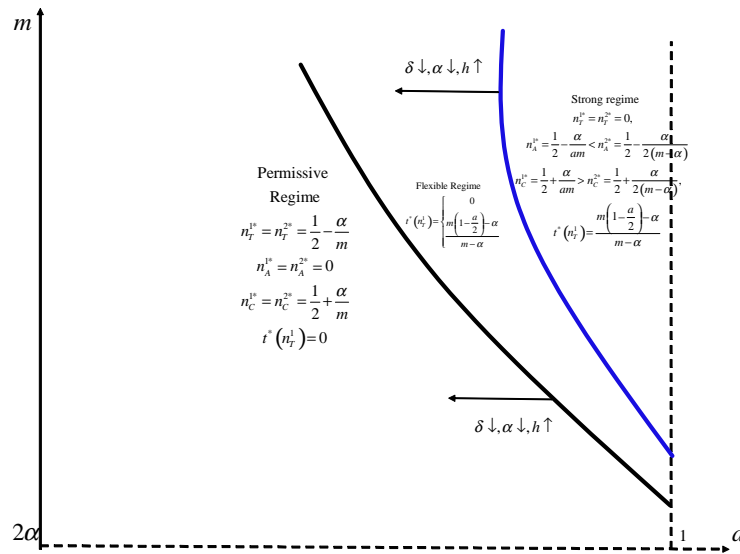


Figure 9: Figure 9.

4.1.3 Responsiveness, terrorists' human capital and political choices

From corollary 8 in the Appendix, it is immediate to derive the following picture that shows the peculiar role of terrorists' human capital for **any** level of responsiveness. In particular, when accountability is great enough $\left(\delta \geq \frac{\frac{1}{2}m - 2\alpha + \frac{\alpha^2}{m}}{\frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m}}\right)$, then for any level of responsiveness the Strong Regime is not possible and an increase in terrorists' human capital provokes a shift from a Permissive Regime to a Flexible, but never to a Strong one.

4.2 Accountability and political choices

Interesting clues on the role played by responsiveness on players' political choices, can be derived by our propositions. Therefore, we will consider the effect on political choices of responsiveness w.r.t. in turn economic development, political heterogeneity, and people human capital, ceteris paribus.

4.2.1 Accountability, economic development and political choices

From corollary 9 in the Appendix, it is immediate to derive the following picture that shows the crucial role of economic development for **any** level of accountability, in particular that for **any** level of accountability an increase in economic development provokes a shift from the Strong Regime to the Flexible and then to the Permissive one.

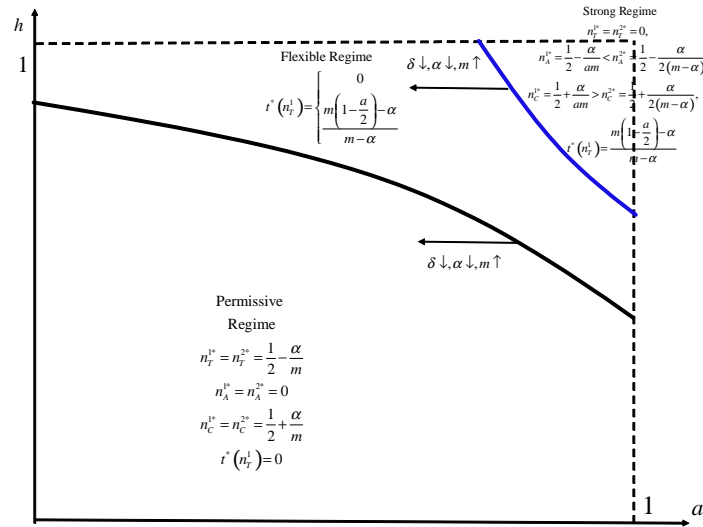


Figure 10: Figure 10.

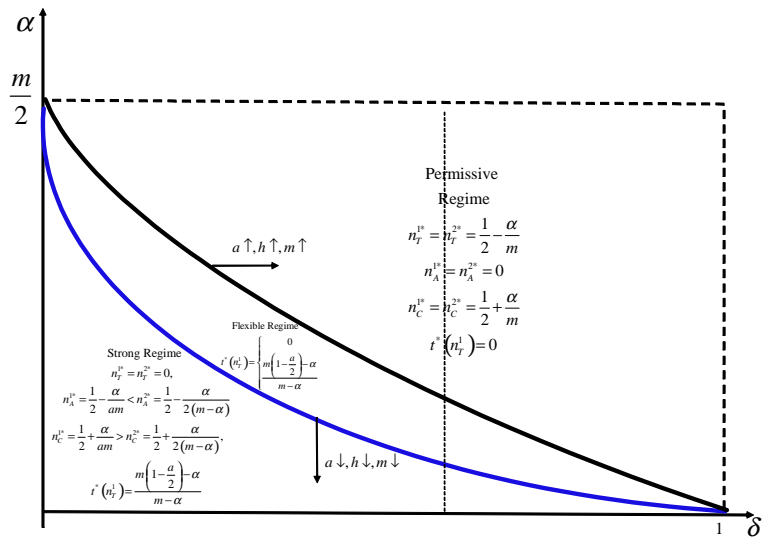


Figure 11: Figure 11.

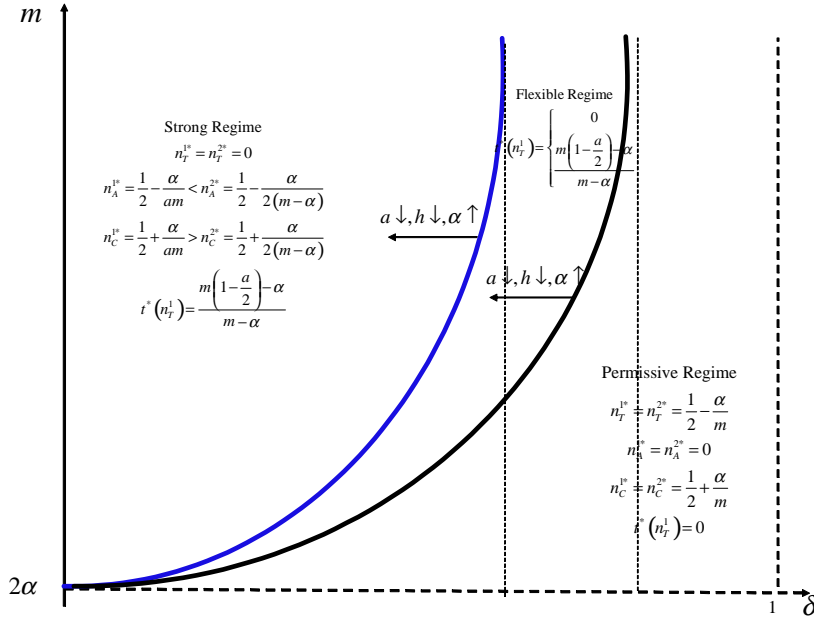


Figure 12: Figure 12.

4.2.2 Accountability, political heterogeneity and political choices

From corollary 10 in the Appendix, it is immediate to derive the following picture that shows the peculiar role of political heterogeneity for high level of accountability. In particular, when accountability is great enough ($\delta \geq \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah}$), then for any level of political heterogeneity the Strong Regime is not possible, while for intermediate level of accountability ($\frac{2(2-a)h}{4h+2\alpha-2ah-\alpha a-\alpha ah} \leq \delta \leq \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah}$), an increase in political heterogeneity provokes a shift from a Permissive Regime to a Flexible, but never to a Strong one, notwithstanding the amount of political polarization. Finally, for small enough level of accountability ($\delta \leq \frac{2(2-a)h}{4h+2\alpha-2ah-\alpha a-\alpha ah}$), then an increase in political heterogeneity provokes a shift from a Permissive Regime to a Flexible, and then to a Strong one.

4.2.3 Accountability, terrorists' human capital and political choices

From corollary 11 in the Appendix, it is immediate to derive the following picture that shows the peculiar role of terrorists' human capital w.r.t. accountability. In particular, when accountability is great enough ($\delta \geq \frac{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am}{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am}$), then for any level of terrorists' human capital, the Strong Regime is not possible, when accountability is intermediate ($\delta \in [\widehat{\delta}^{SR}, \widehat{\delta}^{PR}]$) an increase in h provokes a

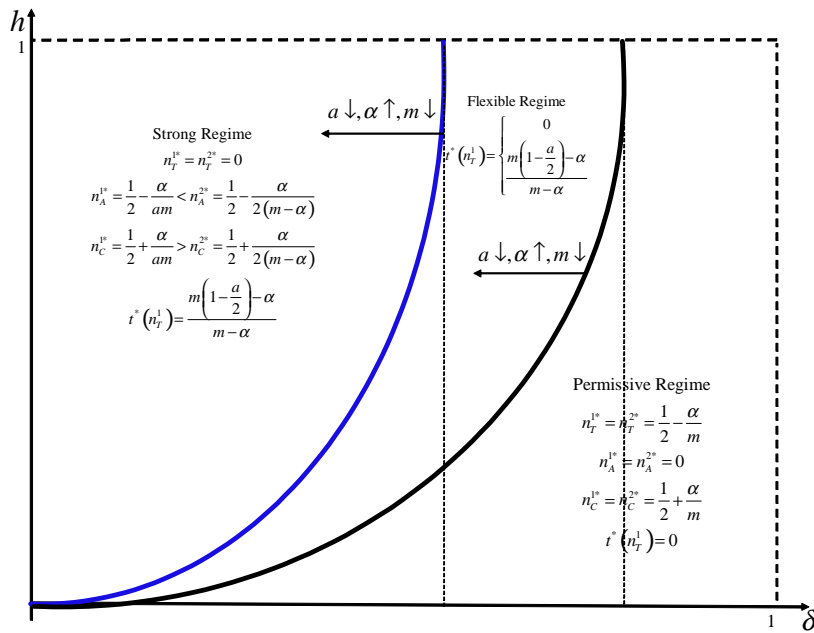


Figure 13: Figure 13.

shift from a Permissive Regime to a Flexible one, but a Strong Regime is possible only for small enough accountability levels ($\delta \leq \frac{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am}{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am}$).

5 Comparative statics

Now we derive some basic comparative statics results relative to the size of these region w.r.t. the exogenous parameters and, within each region, about the political strategic choices.

5.1 The Strong Regime

From corollary 12 in the Appendix, we get the following results:

1. the likelihood of a Strong Regime increases in heterogeneity and in the terrorists's human capital, while it decreases in economic development
2. when there is a Strong Regime, terrorism is fully repressed
3. when there is a Strong Regime, an increment in heterogeneity increases counter-terrorism and activism
4. when there is a Strong Regime, an increment in economic development reduces counter-terrorism and increases conservatism

5. when there is a Strong Regime, an increment in responsiveness increases first period activism and decreases first period conservatism, while has no effect on citizens' choices in the second period, while reduces counter-terrorism.

5.2 The Flexible Regime

From corollary 13 in the Appendix, we get the following results:

1. the likelihood of a Flexible Regime decreases in heterogeneity, is constant in the terrorists's human capital, while it increases in economic development
2. when there is a Flexible Regime, an increment in heterogeneity increases counter-terrorism and both periods' activism, while decreases first period terrorism, however increases second period terrorism
3. when there is a Flexible Regime, an increment in economic development reduces counter-terrorism and increases both period's conservatism, decreasing activism, while it increases first period terrorism and reduces second period terrorism
4. when there is a Flexible Regime, an increment in responsiveness increases first period activism and decreases first period terrorism, while has no effect on citizens' choices in the second period and reduces counter-terrorism
5. when there is a Flexible Regime, an increment in terrorists' and activists' human capital, increases first period activism and decreases first period terrorism, while has no effect on citizens' choices in the second period and on counter-terrorism.

5.3 The Permissive Regime

From corollary 14 in the Appendix, we get the following results:

1. the likelihood of a Permissive Regime decreases in heterogeneity and in the terrorists' human capital, while it increases in economic development
2. when there is a Permissive Regime, an increment in heterogeneity increases terrorism and reduces conservatism
3. when there is a Permissive Regime, an increment in economic development reduces terrorism and increases conservatism
4. when there is a Permissive Regime, an increment in responsiveness has no effect on terrorism or activism or conservatism and on counter-terrorism.

6 Remarks on the results and conclusion

These results help to provide tentative answers to our starting questions. Autocratic regimes have sometimes a weak reaction to terrorism and, conversely, democratic regimes react sometimes harshly because of government's accountability and responsiveness towards citizens. However, accountability and responsiveness are just two determinants of the strength of the reaction, other crucial factors are economic development, terrorists' human capital, and political heterogeneity. Specifically, in our model

1. the greater accountability, the smaller the likelihood of harsh counter-terrorism;
2. the greater responsiveness, the greater the likelihood of harsh counter-terrorism;
3. the more developed is a country, the smaller the likelihood of harsh counter-terrorism;
4. the greater terrorists' human capital, the greater the likelihood of harsh counter-terrorism;
5. the greater political heterogeneity, the greater the likelihood of harsh counter-terrorism.

Finally, in general terrorism activity is increasing with political heterogeneity, decreasing with economic development, and decreasing with government counter-terrorism and responsiveness.

These results are consistent with much of the empirical literature on terrorism. However, as argued in the introduction, the empirical analysis of terrorism almost never is able to find clear cut results connecting specific factors to the strength of terrorism activity. For this reason, we decide to use a case study where our results might illuminate some reasons behind real phenomena. We think that this case shows the potentiality of our results more clearly than a specific econometric study. Of course, case studies are not substitute for rigorous econometric studies, however they might be a useful complementary tool. The comparison of complicated periods of protests and of armed struggle in different countries is not without conceptual challenges and pitfalls, and these discussions cannot even approximate a controlled experiment with clear-cut causal inferences. In particular, they are not intended as a test of the theory but simply as illustrations of the heuristic potentiality of our arguments. Before discussing the specific cases, let us stress that our results should be applied with attention.

For example, if we go back to the four countries we proposed in the introduction, we can collocate these countries in figure 14 in terms of accountability and

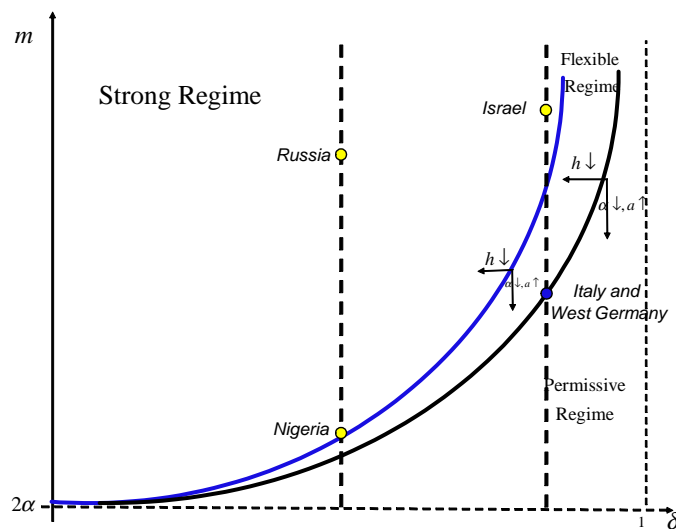


Figure 14: Figure 14.

of political heterogeneity. The yellow points with the country denominations, show how our results can help to explain the differences in their counter-terrorist policies.

However, from these point of views, Italy and West Germany look really similar, however their counter-terrorism policies were extremely different: immediate and strong the German policies, flexible and delayed the Italian one. What factors may explain such a difference? Let us describe the economic, social and political situation in these countries in the seventies.

6.1 Left Wing Terrorism in Italy and in Germany

Italy and Germany in the 1970s and in 1980s faced left terrorist organizations that challenged the state on communist grounds. Terrorism in Italy and Germany was similar in aims and motivation, even if it attracted less popular support in Germany than in Italy, its adherence were fewer, and the magnitude of terrorists violence was lower. However, the Italian government's reaction was first tolerant, and only later harsh, while the response of German government was immediately remarkable in its strength. For example, in Italy the term "terrorism" was not used until midway through the Red Brigades and other groups campaign of violence, although their activities remained essentially the same. This use was actually related to the amount of counter-terrorism effort, that initially was almost nil and that only after some time become significant. On the other hand, in Germany the Red Army Faction was immediately target as

terrorism and the Government reaction was immediately strong. So, what political, economic or social characteristics of the two countries can explain these different counter-terrorism policies?

Let us consider first the economic, social and political environment in these two countries.

6.1.1 Italy

After 1964, Italy maintained for a while a constant growth rate of above 8%. Then, during the late-1960s and most of the 1970s, the economy went stagnant and entered its first recession after that of the late-1940s. This economic recession went on into the early-1980s until a reduction of public costs and spending, tighter budgets and deficits, a steady economic growth, and a lowered inflation rate resulted in Italy left recession by 1983. This plan led to an increasing GDP growth, lower inflation, and increased industrial/agricultural/commercial produce, exports and output, yet made the unemployment rate rise. The 1970s and 1980s was also the period of investment and rapid economic growth in the South, unlike Northern and Central Italy which mainly grew in the 1950s and early 1960s. The "Vanoni Plan" ensured that a new programme to help growth in the South was put in place. Investment was worth billions of US dollars: from 1951–1978, the funds spent in the South was \$11.5 billion for infrastructure, \$13 billion for low-cost loans, and outright grants were worth \$3.2 billion.

Politically, the Socialist joined the conservatives Christian Democrat in the government in 1963. During the first years of the new Centre-Left Government, a wide range of measures were carried out. These included taxation of real estate profits and of share dividends, increases in pensions, a unified secondary school with compulsory attendance up to the age of 14, the nationalization of the electric-power industry, and significant wage rises for workers. The government also made attempts to tackle issues relating to welfare services, hospitals, and education. In particular, social security was extended to previously uncovered categories of the population and entrance to university by examination was abolished. Despite these reforms, however, the reformist drive was soon lost, and the most important problems (including the mafia, social inequalities, inefficient state/social services, North/South imbalance) remained largely un-tackled. The difficult equilibrium of Italian society was challenged by a rising left-wing movement, in the wake of 1968 student unrest. Large numbers of people participate in the protest movements in the late-1960s asking for radical change in the status quo. Widespread labor unrest and student activist groups culminated in the so-called "Hot Autumn" of 1969, a massive series of strikes in factories and industrial centers in Northern Italy. Student strikes and labor strikes, became increasingly common, often deteriorating into clashes between the police and demonstrators composed largely of students, workers, generally

left-wing activists. The political reaction of Italian citizens was mainly conservative, with minor changes in public policies and conservative success at the political elections so to undermine the likelihood of drastic changes in the political status quo. In particular, if we compare the composition of the Chamber of Deputies and of Senate of the Republic after the 1968 and the 1972 elections, the right shift is quite remarkable, even if it was partially reversed in the 1976 elections, even if the Parliament majority was still conservative. One of the peculiar features of the Italian case is the second wave of terrorism after 1976, which coincided with the sudden rise and rapid disappearance of a new wave of protests.

We might conclude that four features of this period seem to be important: the emergence of a new phase of radical protests (using our model this means an increase in m), the economic difficulties (a reduction in α), the existence of a terrorist organization ($n_T^1 > 0$) and the imperviousness of the political system to the requests of the protest movement (small a). If we consider the government counter-terrorism policies, initially they were quite ineffective, only after some time the Italian government approved a significative change in counter-terrorism policies, allocating a significative amount of resources to improve counter-terrorism policies effectiveness.

6.1.2 West Germany

The West German economy did not grow as fast or as consistently in the 1960s as it had during the 1950. Erhard, who had succeeded Konrad Adenauer as chancellor, was voted out of office in December 1966. He was replaced by a Grand Coalition consisting of the Christian Democrat and the Social Democratic under Chancellor Kiesinger of the CDU, formed to deal with economic and political problems. Under the pressure of the slowdown, the new government abandoned the laissez-faire orientation. The new minister for economics, Karl Schiller, argued strongly for legislation that would give the federal government and his ministry greater authority to guide economic policy, in order to give fiscal policy a stronger impact. The grand coalition was important for the introduction of new emergency acts to cope with public order. During the time leading up to the passing of the laws, there was fierce opposition to them, above all by the rising German student movement. All over West Germany, thousands demonstrated against the government policies. The calling in question of the actions and policies of government led to a new climate of debate. The issues of emancipation, colonialism, environmentalism and grass roots democracy were discussed at all levels of society. Anger over the treatment of demonstrators, coupled with growing frustration over the lack of success in achieving their aims led to growing militance among students and their supporters. Several groups set as their objective the aim of radicalizing the industrial workers using violent

instruments and taking an example from activities in Italy of the Red Brigades. The most notorious of the underground groups was the Red Army Faction which began by making bank raids to finance their activities and eventually went underground having killed a number of policemen and several bystanders. In the 1969 election, the socialist headed by Willy Brandt gained enough votes to form a Center-Left coalition government with the Liberal Party. Although Chancellor for only just over four years, Willy Brandt was one of the most popular politicians in the whole period. Brandt began a policy of rapprochement with West Germany's eastern neighbors, Poland, Czechoslovakia and East Germany, implementing left wing policies inside West Germany too. Brandt's contributions to world peace led to his nomination for the Nobel Peace Prize in 1971. The new coalition expanded the West German social security system, substantially increasing the size and cost of the social budget. Social program costs grew by over 10 percent a year during much of the 1970s, introducing into the budget an unalterable obligation that reduced fiscal flexibility. This came back to haunt Schiller as well as every German government since then. Schiller himself had to resign in 1972 when the West German and global economies were in a downturn and when all his ideas did not seem able to revive West German prosperity. After 1973, Germany was hard hit by a worldwide economic crisis, soaring oil prices, and stubbornly high unemployment. The welfare system provided a safety net for the large number of unemployed workers, and many factories reduce their labor force and began to concentrate on high-profit specialty items. A spy scandal forced Brandt to step down as Chancellor while remaining as party leader. He was replaced by Helmut Schmidt, of the SPD, who served as Chancellor from 1974 to 1982. He was intensely interested in economics but also faced great problems, including the dramatic upsurge in oil prices of 1973-74. The debt grew rapidly as he borrowed to cover the cost of the ever more expensive welfare state, GDP in 1975 fell by 1.4%, the first time since the founding of the FRG that it had fallen so sharply. By 1976 the worst was over. West German growth resumed, and the inflation rate began to decline. Although neither reached the favorable levels that had come to be taken for granted during the 1950s and early 1960s, they were accepted as tolerable after the turbulence of the previous years. The government won reelection in 1976, but the economy again turned down and, despite efforts to stimulate growth by government deficits, failed to revive quickly. It was only by mid-1978 that Schmidt and the Bundesbank were able to bring the economy into balance. After that, the economy continued expanding through 1979 and much of 1980, helping Schmidt win reelection in 1980. But the upturn proved to be uneven and unrewarding, as the problems of the mid-1970s rapidly returned. By early 1981, Schmidt faced the worst possible situation: growth fell and unemployment rose, but inflation did not abate.

6.1.3 Comparing Italy and West Germany

If we compare, the political reactions to the protest movements in Italy and West Germany, a thing appears clearly, the different responsiveness of the two countries: in Italy the response was conservative, in Germany the status quo was partially changed. On the other hand, we may say that both countries share qualitatively the same degree of accountability, as well as the same level of terrorists' human capital, of political heterogeneity and of economic development. However, since the two countries display different degree of responsiveness, low in Italy, high in West Germany, according to our results this difference can explain the different magnitude of counter-terrorist policies, as summed up by the following table.

| Parameters | West Germany | Italy |
|----------------------------|---------------------|---------------------|
| Econ. Devel. α | low | low |
| Terr. human cap. h | high | high |
| Pol. heterogeneity m | intermediate | intermediate |
| Accountability δ | high | high |
| <i>Responsiveness a</i> | <i>high</i> | <i>intermediate</i> |
| Endogenous variable | | |
| <i>Counter-terrorism t</i> | <i>high</i> | <i>Flexible</i> |
| Table 3 | | |

This table lead to figure 15

According to our results and to this quick review of the political, social and economic situations in these two countries, we might conclude that in Italy the refusal by the majority of the ruling class to integrate the new demands voiced by the activists, yielded terrorists' activities that was much more enduring than the corresponding phenomenon in West Germany, together with a delayed counter-terrorism policy.

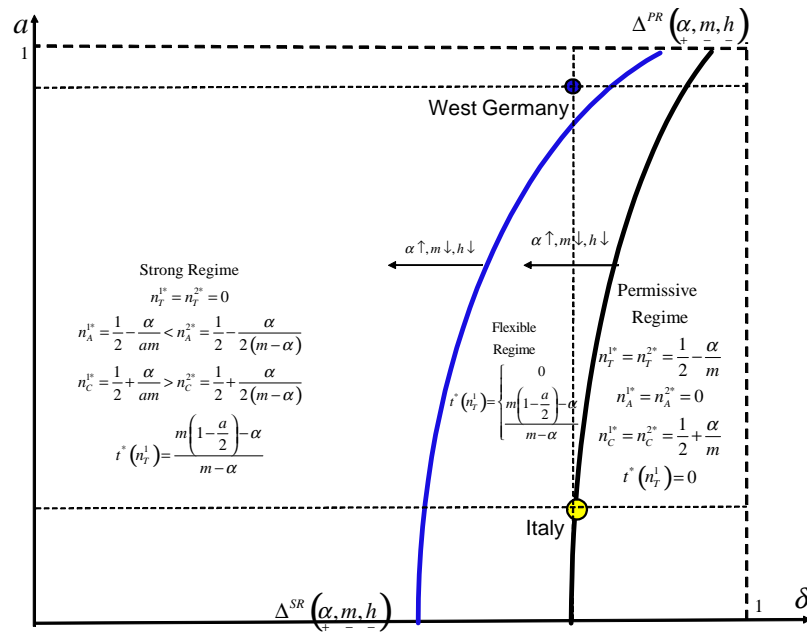


Figure 15: Figure 15.

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Part I

Appendix

6.1.4 Structural Assumptions

The Conflict Technology A crucial aspect of this model is the specification of how the players' choices affect the probability of a change. We assume the following conflict technology

$$R = \min \left\{ \left(1 - \frac{\pi \rho}{\sum_{i \in P} Y_i^1} \right) \left(\frac{\sum_{i \in T^1} E_i^1 + \sum_{i \in T^2} E_i^2}{P} \right) + a \left[1 - \frac{(1 - \pi) \rho}{\sum_{i \in P} Y_i^1} \right] \left(\frac{\sum_{i \in A^1} E_i^1 + \sum_{i \in A^2} E_i^2}{P} \right), 1 \right\}$$

where

- R is the probability of a drastic change in the status quo
- ρ is the total amount of counter-terrorism
- π is the share of such counter-terrorism amount devote to counter-terrorism activity, hence $(1 - \pi)$ is the share of resources used to counter protest
- $\sum_{i \in P} Y_i^1$ is the first period country income
- E_i^τ is citizen $i \in P$ effort at time $\tau \in \{1, 2\}$ not used to produce income
- P is the set of citizens and, with the obvious abuse of notation, its numerosity
- a is a measure of how much citizens' political activism affect the probability of a change in status quo policies.

This conflict technology is quite standard and it is a revised version of the famous Tullock technology: it says in the simplest way that the probability of a change increases linearly with the percentage of global terrorists' activity, but its is reduced by governments counter-terrorism. Also active participation to protests increases the likelihood of a revolution, however the effects of such protests is reduced by the parameter a .

Some change in notation will help the following calculus. So let define

1.

$$n_J^\tau = \frac{\sum_{i \in J} I_{\{i \in J\}}}{P} = \frac{N_J^\tau}{P}$$

where $I_{\{i \in J\}}$ is the indicator function for a citizen $i \in \{1, \dots, P\}$ choice of group J , hence n_J^τ is the proportion of citizens in group $J \in \{T, A, C\}$ at period $\tau \in \{1, 2\}$;

2.

$$e_J^\tau = \frac{\sum_{i \in J} E_i^\tau}{N_J^\tau}$$

as the average effort of citizens within group $J \in \{C, T, A\}$ at period $\tau \in \{1, 2\}$, so that

$$\sum_{i \in J} E_i^\tau = P n_J^\tau e_J^\tau$$

3.

$$h_J^\tau = \frac{\sum_{i \in J} H_i^\tau}{N_J^\tau}$$

as the average human capital of citizens within group $J \in \{C, T, A\}$ at period $\tau \in \{1, 2\}$, so that

$$\sum_{i \in J} H_i^\tau = P n_J^\tau h_J^\tau$$

Using this notation, we can rewrite the public budget constraint as follows:

$$t = \frac{\rho}{\sum_{i \in P} Y_i^1} = \frac{\rho}{\alpha \left[\sum_{i \in C^1} H_i + \frac{1}{2} \sum_{i \in A^1} H_i \right]} = \frac{\rho}{\alpha P [n_C^1 h_C^1 + \frac{1}{2} n_A^1 h_A^1]} = \frac{\rho}{\alpha P l^1}$$

where l^1 is the (endogenous) average labour supply at period $\tau = 1$ and thus αl^1 is the average income.

Then, we can rewrite the probability of successful revolution as follows

$$\begin{aligned} R &= \\ &= \min \left\{ \left(1 - \frac{\pi \rho}{\sum_{i \in P} Y_i^1} \right) \left(\frac{\sum_{i \in T^1} E_i^1 + \sum_{i \in T^2} E_i^2}{P} \right) + a \left[1 - \frac{(1 - \pi) \rho}{\sum_{i \in P} Y_i^1} \right] \left(\frac{\sum_{i \in A^1} E_i^1 + \sum_{i \in A^2} E_i^2}{P} \right), 1 \right\} \\ &= \min \left\{ (1 - \pi t) (n_T^1 h_T^1 + n_T^2 h_T^2) + \frac{1}{2} a (1 + \pi t - t) (n_A^1 h_A^1 + n_A^2 h_A^2), 1 \right\}. \end{aligned}$$

The Players' Payoff Functions The final outcomes are associate to the following public payoff

$$\Pi = \begin{cases} 1 & \text{if change} \\ 0 & \text{if status quo} \end{cases}$$

that in turn affect the players' payoffs linearly, as follows:

1. the citizens' utilities are

$$u_i(C_i^1, C_i^2, \tilde{\Pi}) = c_i^1 + c_i^2 + \gamma_i \Pi \Leftrightarrow Eu_i(C_i^1, C_i^2, \tilde{\Pi}) = c_i^1 + c_i^2 + \gamma_i R$$

where

$$\gamma_i \in \left[-\frac{m}{2} P, \frac{m}{2} P \right]$$

is an individual parameter that describes a citizen's attitudes towards changes of status quo: higher γ_i means a high propensity for dramatic change, while lower γ_i denotes a taste for status quo. A citizen i expected utility can be rewritten as follows

$$EU_i (i \in J^1, i \in J^2) = \gamma_i R_{-i} \left(t, \pi, \bar{n}_T^1 \bar{h}_T^1, \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^1 \bar{h}_A^1, \bar{n}_A^2 \bar{h}_A^2 \right) + B_i (i \in J^1, i \in J^2)$$

where

$$\begin{aligned} & R_{-i} \left(t, \pi, \bar{n}_T^1 \bar{h}_T^1, \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^1 \bar{h}_A^1, \bar{n}_A^2 \bar{h}_A^2 \right) = \\ & = \min \left\{ \left[(1 - \pi t) \left(\bar{n}_T^1 \bar{h}_T^1 + \bar{n}_T^2 \bar{h}_T^2 \right) + \frac{1}{2} a (1 + \pi t - t) \left(\bar{n}_A^1 \bar{h}_A^1 + \bar{n}_A^2 \bar{h}_A^2 \right) \right], 1 \right\} \\ & \bar{n}_J^\tau = \frac{1}{P} \sum_{j \neq i, j=1}^P I_{\{j \in J\}} \quad \text{and} \quad \bar{h}_J^\tau = \frac{1}{P} \sum_{j \neq i, j=1}^P I_{\{j \in J\}} H_j \end{aligned}$$

where $I_{\{j \in J\}}$ is the indicator function for a citizen $j \in \{1, \dots, P\}$ choice of group J , hence \bar{n}_J^τ and \bar{h}_J^τ denote, respectively, the percentage and the average human capital of citizens different from i that choose group $J^\tau = \{T^\tau, A^\tau, C^\tau\}$ in period $\tau \in \{1, 2\}$. and

$$\begin{aligned} & B_i (i \in J^1, i \in J^2) = \\ & = \begin{cases} \frac{2}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{P} \gamma_i (1 - \pi t) H_i + \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i (1 - \pi t) H_i + \alpha H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \frac{1}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in A^1, i \in T^2 \\ \frac{1}{P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i + \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha (1 - t) H_i + \frac{1}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in C^1, i \in T^2 \\ \alpha (1 - t) H_i + \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha (1 - t) H_i + \alpha H_i & \text{if } i \in C^1, i \in C^2 \end{cases} \end{aligned}$$

2. the government objective function is

$$u^G (U_i, \Pi) = \delta \frac{1}{P} \sum_{i=1}^P (c_i^1 + c_i^2) + (1 - \delta) (1 - \tilde{\Pi})$$

where δ is an index of the representativeness of the government: the more δ goes to 1, the more the government is democratic and vice-versa. This payoff can be rewritten as follows

$$\begin{aligned} & EU^G (t, \pi) = \delta \frac{1}{P} \sum_{i=1}^P (c_i^1 + c_i^2) + (1 - \delta) (1 - R) = \\ & = \delta \left(\left[\frac{1}{2} (1 - t) \alpha n_A^1 h_A^1 + \frac{1}{2} \alpha n_A^2 h_A^2 + (1 - t) \alpha n_C^1 h_C^1 + \alpha n_C^2 h_C^2 \right] \right) + \\ & + (1 - \delta) \left(1 - \min \left\{ (1 - \pi t) \left(n_T^1 h_T^1 + n_T^2 h_T^2 \right) + \frac{1}{2} a (1 + \pi t - t) \left(n_A^1 h_A^1 + n_A^2 h_A^2 \right), 1 \right\} \right) \end{aligned}$$

Before solving the game, we make the following hypotheses on the model's parameters.

Condition 3 *The parameters of the model satisfy the following conditions*

$P \in \{1, \dots, \infty\}$, $h_J^\tau \in [0, 1]$, $\alpha \in (0, +\infty)$, $a \in (0; 1)$, $\gamma_i \sim U \left[-\frac{m}{2}P, \frac{m}{2}P \right]$, $m \in (0, +\infty)$

where

1. $P \in \{1, \dots, \infty\}$ is the population size;
2. $h_J^\tau := \frac{1}{N_J} \sum_{i \in J^\tau} H_i \in [0, 1]$ is the average human capital at period $\tau = 1, 2$ within group $J \in \{T, A, C\}$;
3. $\alpha \in (0, \infty)$ is a measure of productivity which is correlated to the general economic situation;
4. $a \in (0; 1)$ is a measure of the government responsiveness to people activism to change the status quo;
5. γ_i is citizen's i marginal utility of the revolution;
6. $U \left[-\frac{m}{2}P, \frac{m}{2}P \right]$ is the uniform distribution on $\left[-\frac{m}{2}P, \frac{m}{2}P \right]$, thus $G(\gamma_i) = \frac{1}{mP}\gamma_i + \frac{1}{2}$, hence
7. $m \in (0, +\infty)$ is a measure of the political heterogeneity in the population.

7 Solution of the Game

To solve the game we work backward, thus it is important to consider the players' continuation payoffs:

Third stage: Citizen's i 's continuation payoff is

$$EV_i^2 \left(i \in J^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) = B_i \left(i \in J^2 \right) = \begin{cases} \frac{1}{P} \gamma_i (1 - \pi t) H_i & \text{if } i \in T^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi t - t) H_i + \frac{1}{2} \alpha (1 - t) H_i & \text{if } i \in A^2 \\ \alpha (1 - t) H_i & \text{if } i \in C^2 \end{cases}$$

Second stage: Government continuation payoff is

$$EV^G \left(t, \pi | n_T^1 h_T^1, n_A^1 h_A^1, n_C^1 h_C^1 \right) = \delta \left(\left[\frac{1}{2} \alpha n_A^1 h_A^1 + \frac{1}{2} \alpha (1 - t) n_A^{2*} h_A^2 + \alpha n_C^1 h_C^1 + \alpha (1 - t) n_C^{2*} h_C^2 \right] \right) + (1 - \delta) \left(1 - \min \left\{ (1 - \pi t) (n_T^1 h_T^1 + n_T^{2*} h_T^2) + \frac{1}{2} a (1 + \pi t - t) (n_A^1 h_A^1 + n_A^{2*} h_A^2), 1 \right\} \right)$$

where n_J^{2*} is the sequential best reply at the fifth stage;

First stage: Citizen's $i \in T^1$ continuation payoff is

$$\begin{aligned}
EV_i^1 (i \in J^1) &= \\
&= \gamma_i R_{-i} \left(t^*, \pi^*, \bar{n}_T^1 \bar{h}_T^1, \bar{n}_T^{2*} \bar{h}_T^{2*}, \bar{n}_A^1 \bar{h}_A^1, \bar{n}_A^{2*} \bar{h}_A^{2*} \right) + B_i (i \in J^1, i^* \in J^2) = \\
&= \gamma_i \min \left\{ (1 - \pi^* t^*) \left(\bar{n}_T^1 \bar{h}_T^1 + \bar{n}_T^{2*} \bar{h}_T^{2*} \right) + \frac{1}{2} a (1 + \pi^* t^* - t^*) \left(\bar{n}_A^1 \bar{h}_A^1 + \bar{n}_A^{2*} \bar{h}_A^{2*} \right), 1 \right\} + \\
&\quad + \begin{cases} \frac{2}{P} \gamma_i (1 - \pi^* t^*) H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{P} \gamma_i (1 - \pi^* t^*) H_i + \frac{1}{2P} \gamma_i a (1 + \pi^* t^* - t^*) H_i + \frac{1}{2} \alpha (1 - t^*) H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i (1 - \pi^* t^*) H_i + \alpha (1 - t^*) H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi^* t^* - t^*) H_i + \frac{1}{2} \alpha H_i + \frac{1}{P} \gamma_i (1 - \pi^* t^*) H_i & \text{if } i \in A^1, i \in T^2 \\ \frac{1}{P} \gamma_i a (1 + \pi^* t^* - t^*) H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha (1 - t^*) H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a (1 + \pi^* t^* - t^*) H_i + \frac{1}{2} \alpha H_i + \alpha (1 - t^*) H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \frac{1}{P} \gamma_i (1 - \pi^* t^*) H_i & \text{if } i \in C^1, i \in T^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a (1 + \pi^* t^* - t^*) H_i + \frac{1}{2} \alpha (1 - t^*) H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha H_i + \alpha (1 - t^*) H_i & \text{if } i \in C^1, i \in C^2. \end{cases}
\end{aligned}$$

7.1 Citizens' Choice in the Second Period

Condition 4 To simplify, for the time being let assume

$$\pi = 1.$$

In the third stage (second period), a citizen i will choose group J^2 depending on his continuation payoff:

$$= \begin{cases} \frac{1}{P} \gamma_i (1 - t) H_i & \text{if } i \in T^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1 - t) H_i & \text{if } i \in A^2 \\ \alpha (1 - t) H_i & \text{if } i \in C^2 \end{cases}$$

Hence

$$\begin{aligned}
&i \in T^2 \Leftrightarrow \\
&\Leftrightarrow \begin{cases} EV_i^2 \left(i \in T^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in A^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \\ EV_i^2 \left(i \in T^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in C^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \end{cases} \Leftrightarrow \\
&\Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i (1 - t) H_i \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1 - t) H_i \\ \frac{1}{P} \gamma_i (1 - t) H_i \geq \alpha (1 - t) H_i \end{cases} \Leftrightarrow \begin{cases} 2\gamma_i (1 - t) \geq \gamma_i a + \alpha (1 - t) P \\ \gamma_i \geq \alpha P \end{cases} \Leftrightarrow \\
&\Leftrightarrow \begin{cases} 2 \left(1 - t - \frac{a}{2} \right) \gamma_i \geq \alpha (1 - t) P \\ \gamma_i \geq \alpha P. \end{cases}
\end{aligned}$$

First, note that

$$1 - t - \frac{a}{2} \leq 0 \Leftrightarrow t \geq 1 - \frac{a}{2}$$

hence

$$t \geq 1 - \frac{a}{2} \Leftrightarrow \begin{cases} 2\gamma_i(1-t-\frac{a}{2}) \geq \alpha(1-t)P \\ \gamma_i \geq \alpha P \end{cases} \Leftrightarrow \gamma_i \in \emptyset \Leftrightarrow T^2 = \emptyset$$

on the other hand, let suppose

$$t < 1 - \frac{a}{2}$$

then

$$i \in T^2 \Leftrightarrow \begin{cases} \gamma_i \geq \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \\ \gamma_i \geq \alpha P \end{cases}$$

hence we have to compare $\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}$ and αP when $t < 1 - \frac{a}{2}$:

$$\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \geq \alpha P \Leftrightarrow 1-t \geq 2-2t-a \Leftrightarrow t \geq 1-a.$$

Finally, we can conclude

$$i \in T^2 \Leftrightarrow \begin{cases} \gamma_i \in [\alpha P, \frac{m}{2}P] & \text{if } t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}, \frac{m}{2}P \right] & \text{if } t \in \left[1-a, 1-\frac{a}{2} \right) \\ \gamma_i \in \emptyset & \text{if } t \in \left[1-\frac{a}{2}, 1 \right] \end{cases} \quad (T^2)$$

Similarly

$$\begin{aligned} & i \in A^2 \Leftrightarrow \\ & \Leftrightarrow \begin{cases} EV_i^2 \left(i \in A^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in T^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \\ EV_i^2 \left(i \in A^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in C^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t) H_i \geq \frac{1}{P} \gamma_i (1-t) H_i \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t) H_i \geq \alpha (1-t) H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i a + \alpha (1-t) P \geq 2\gamma_i (1-t) \\ \gamma_i a \geq \alpha (1-t) P \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} 2\gamma_i \left(1-t-\frac{a}{2} \right) \leq \alpha (1-t) P \\ \gamma_i \geq \frac{\alpha (1-t) P}{a} \end{cases} \end{aligned}$$

First, note that

$$1-t-\frac{a}{2} \leq 0 \Leftrightarrow t \geq 1-\frac{a}{2}$$

hence

$$t \geq 1 - \frac{a}{2} \Rightarrow \left[i \in A^2 \Leftrightarrow \begin{cases} 2\gamma_i \left(1-t-\frac{a}{2} \right) \leq \alpha (1-t) P \\ \gamma_i \geq \frac{\alpha (1-t) P}{a} \end{cases} \Leftrightarrow \gamma_i \geq \frac{\alpha (1-t) P}{a} \right]$$

on the other hand, let suppose

$$t < 1 - \frac{a}{2}$$

so that

$$t < 1 - \frac{a}{2} \Rightarrow \left[i \in A^2 \Leftrightarrow \begin{cases} \gamma_i \leq \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \\ \gamma_i \geq \frac{\alpha(1-t)P}{a} \end{cases} \right]$$

note that

$$\frac{\alpha(1-t)P}{a} \leq \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \Leftrightarrow 2-2t-a \leq a \Leftrightarrow t \geq 1-a$$

hence

$$i \in A^2 \Leftrightarrow \begin{cases} \gamma_i \in \emptyset & \text{if } t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \right] & \text{if } t \in \left[1-a, 1-\frac{a}{2} \right) \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{m}{2}P \right] & \text{if } t \in \left[1-\frac{a}{2}, 1 \right] \end{cases} \quad (A^2)$$

Finally

$$\begin{aligned} & i \in C^2 \Leftrightarrow \\ \Leftrightarrow & \begin{cases} EV_i^2 \left(i \in C^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in T^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \\ EV_i^2 \left(i \in C^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \geq EV_i^2 \left(i \in A^2 | \bar{n}_T^2 \bar{h}_T^2, \bar{n}_A^2 \bar{h}_A^2; t, \pi, n_T^1 h_T^1, n_A^1 h_A^1 \right) \end{cases} \Leftrightarrow \\ & = \begin{cases} \frac{1}{P} \gamma_i (1-t) H_i & \text{if } i \in T^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t) H_i & \text{if } i \in A^2 \\ \alpha (1-t) H_i & \text{if } i \in C^2 \end{cases} \\ \Leftrightarrow & \begin{cases} \alpha (1-t) H_i \geq \frac{1}{P} \gamma_i (1-t) H_i \\ \alpha (1-t) H_i \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t) H_i \end{cases} \Leftrightarrow \begin{cases} \alpha \geq \frac{1}{P} \gamma_i \\ \alpha (1-t) P \geq a \gamma_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \leq \frac{\alpha P}{a} \\ \gamma_i \leq \frac{\alpha (1-t) P}{a} \end{cases} \end{aligned}$$

note that

$$\alpha P \leq \frac{\alpha(1-t)P}{a} \Leftrightarrow a \leq 1-t \Leftrightarrow t \leq 1-a$$

hence

$$i \in C^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[-\frac{m}{2}P, \alpha P \right] & \text{if } t \in [0, 1-a] \\ \gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha(1-t)P}{a} \right] & \text{if } t \in [1-a, 1] \end{cases} \quad (C^2)$$

Before formally stating our first result, since

$$\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \xrightarrow[t \rightarrow (1-\frac{a}{2})^-]{} +\infty$$

we need to consider the condition

$$\left[\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}, \frac{m}{2}P \right] \neq \emptyset \Leftrightarrow$$

$$\Leftrightarrow \alpha - \alpha t \leq m - mt - \frac{a}{2}m \Leftrightarrow (m - \alpha)t \leq m \left(1 - \frac{a}{2}\right) - \alpha \Leftrightarrow t \leq \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}$$

So, we establish the following result

Lemma 1 *The citizens that join the different groups in the second period are characterized as follows:*

$$i \in T^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[\alpha P, \frac{m}{2}P \right] & t \in [0, 1 - a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}, \frac{m}{2}P \right] & t \in \left[1 - a, \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} \right] \\ \gamma_i \in \emptyset & t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right] \end{cases}$$

$$i \in A^2 \Leftrightarrow \begin{cases} \gamma_i \in \emptyset & t \in [0, 1 - a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \right] & t \in \left[1 - a, \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} \right] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{m}{2}P \right] & t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right] \end{cases}$$

$$i \in C^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[-\frac{m}{2}P, \alpha P \right] & t \in [0, 1 - a] \\ \gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha(1-t)P}{a} \right] & t \in [1 - a, 1]. \end{cases}$$

Condition 5 *From now on, assume that*

$$m \geq 2\alpha.$$

This condition means that the ratio of political heterogeneity of the population over productivity is bounded from below by the level of economic development so that their relative changes should be related. The condition is useful to simplify the actual calculus of the proportion of citizens in the different groups, however it is also plausible.

Lemma 2 *The percentages of the groups in the second period are:*

$$n_J^2(t) = \begin{cases} n_T^2 = \frac{1}{2} - \frac{\alpha}{m}; n_A^2 = 0; n_C^2 = \frac{1}{2} + \frac{\alpha}{m} & t \in [0, 1-a] \\ \left. \begin{aligned} n_T^2 &= \frac{1}{2} - \frac{\alpha(1-t)}{2(1-t-\frac{a}{2})m}; \\ n_A^2 &= \frac{\alpha(1-t)[t-(1-a)]}{a(1-t-\frac{a}{2})m}; \\ n_C^2 &= \frac{1}{2} + \frac{\alpha(1-t)}{am} \end{aligned} \right\} & t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}\right] \\ \left. \begin{aligned} n_T^2 &= 0; n_A^2 = \frac{1}{2} - \frac{\alpha(1-t)}{am}; \\ n_C^2 &= \frac{1}{2} + \frac{\alpha(1-t)}{am} \end{aligned} \right\} & t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1\right] \end{cases}$$

or equivalently

$$n_J^2(a) = \begin{cases} n_T^2 = \frac{1}{2} - \frac{\alpha}{m}; n_A^2 = 0; n_C^2 = \frac{1}{2} + \frac{\alpha}{m} & a \in [0, 1-t] \\ \left. \begin{aligned} n_T^2 &= \frac{1}{2} - \frac{\alpha(1-t)}{2(1-t-\frac{a}{2})m}; \\ n_A^2 &= \frac{\alpha(1-t)[t-(1-a)]}{a(1-t-\frac{a}{2})m}; \\ n_C^2 &= \frac{1}{2} + \frac{\alpha(1-t)}{am} \end{aligned} \right\} & a \in [1-t, 2(1-t)(1-\frac{\alpha}{m})] \\ \left. \begin{aligned} n_T^2 &= 0; n_A^2 = \frac{1}{2} - \frac{\alpha(1-t)}{am}; \\ n_C^2 &= \frac{1}{2} + \frac{\alpha(1-t)}{am} \end{aligned} \right\} & t \in [2(1-t)(1-\frac{\alpha}{m}), 1] \end{cases}$$

7.2 Government's Choice

The Government continuation payoff is

$$\begin{aligned} EV^G(t, \pi | n_T^1 h_T^1, n_A^1 h_A^1, n_C^1 h_C^1) &= \\ &= \delta \left(\left[\frac{1}{2} \alpha n_A^1 h_A^1 + \frac{1}{2} \alpha (1-t) n_A^{2*} h_A^2 + \alpha n_C^1 h_C^1 + \alpha (1-t) n_C^{2*} h_C^2 \right] \right) + \\ &+ (1-\delta) \left(1 - \min \left\{ (1-\pi t) (n_T^1 h_T^1 + n_T^{2*} h_T^2) + \frac{1}{2} a (1+\pi t-t) (n_A^1 h_A^1 + n_A^{2*} h_A^2), 1 \right\} \right) \end{aligned}$$

where n_J^{2*} is the sequential best reply at the fifth stage. First preliminary let we assume

$$(1-\pi t) (n_T^1 h_T^1 + n_T^{2*} h_T^2) + \frac{1}{2} a (1+\pi t-t) (n_A^1 h_A^1 + n_A^{2*} h_A^2) < 1,$$

a condition that we will check after finding the solution.

Moreover, to get closed form solution we make the following hypothesis

Condition 6 To simplify the analysis let suppose

$$a = \frac{1}{2}, \quad h_C^1 = h_C^2 = \frac{1}{2}, \quad h_T^1 = h_T^2 = h_A^1 = h_A^2 \equiv h.$$

Moreover, since the Government's choice variable is t , then, we might write its payoff as follows

$$\begin{aligned} & EV^G(t|n_T^1, n_A^1, n_C^1) = \\ &= \delta \left[\frac{1}{2} \alpha n_A^1 h_A^1 + \frac{1}{2} \alpha (1-t) n_A^{2*} h_A^2 + \alpha n_C^1 h_C^1 + \alpha (1-t) n_C^{2*} h_C^2 \right] + \\ & \quad + (1-\delta) \left[1 - (1-t) (n_T^1 h_T^1 + n_T^{2*} h_T^2) - \frac{1}{2} a (n_A^1 h_A^1 + n_A^{2*} h_A^2) \right] = \\ &= \begin{cases} \delta \left[\frac{1}{2} \alpha n_A^1 h + \frac{1}{2} \alpha (1-t) n_A^{2*} h + \frac{1}{2} \alpha n_C^1 + \frac{1}{2} \alpha (1-t) n_C^{2*} \right] + & t \in [0, 1-a] \\ + (1-\delta) \left[1 - (1-t) (n_T^1 h + n_T^{2*} h) - \frac{1}{2} a (n_A^1 h + n_A^{2*} h) \right] \\ \\ \delta \left[\frac{1}{2} \alpha n_A^1 h + \frac{1}{2} \alpha (1-t) n_A^{2*} h + \frac{1}{2} \alpha n_C^1 + \frac{1}{2} \alpha (1-t) n_C^{2*} \right] + & t \in \left[1-a, \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} \right] \\ + (1-\delta) \left[1 - (1-t) (n_T^1 h + n_T^{2*} h) - \frac{1}{2} a (n_A^1 h + n_A^{2*} h) \right] \\ \\ \delta \left[\frac{1}{2} \alpha n_A^1 h + \frac{1}{2} \alpha (1-t) n_A^{2*} h + \frac{1}{2} \alpha n_C^1 + \frac{1}{2} \alpha (1-t) n_C^{2*} \right] + & t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right] \\ + (1-\delta) \left[1 - (1-t) (n_T^1 h + n_T^{2*} h) - \frac{1}{2} a (n_A^1 h + n_A^{2*} h) \right] \\ \\ \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1-\delta) - \frac{1}{2} a (1-\delta) n_A^1 h + & t \in [0, 1-a] \\ + \frac{1}{2} \alpha \delta (1-t) n_A^{2*} h + \frac{1}{2} \alpha \delta (1-t) n_C^{2*} - (1-\delta) (1-t) n_T^1 h + \\ - (1-\delta) (1-t) n_T^{2*} h - \frac{1}{2} a (1-\delta) n_A^{2*} h \\ \\ \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1-\delta) - \frac{1}{2} a (1-\delta) n_A^1 h + & t \in \left[1-a, \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} \right] \\ + \frac{1}{2} \alpha \delta (1-t) n_A^{2*} h + \frac{1}{2} \alpha \delta (1-t) n_C^{2*} - (1-\delta) (1-t) n_T^1 h + \\ - (1-\delta) (1-t) n_T^{2*} h - \frac{1}{2} a (1-\delta) n_A^{2*} h \\ \\ \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1-\delta) - \frac{1}{2} a (1-\delta) n_A^1 h + & t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right] \\ + \frac{1}{2} \alpha \delta (1-t) n_A^{2*} h + \frac{1}{2} \alpha \delta (1-t) n_C^{2*} - (1-\delta) (1-t) n_T^1 h + \\ - (1-\delta) (1-t) n_T^{2*} h - \frac{1}{2} a (1-\delta) n_A^{2*} h \end{cases} = \end{aligned}$$

$$= \left\{ \begin{array}{l}
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
+ \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta)(1-t)n_T^1 h + \\
- (1-\delta)(1-t)\left(\frac{1}{2} - \frac{\alpha}{m}\right)h \quad t \in [0, 1-a] \\
\\
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h - (1-\delta)(1-t)n_T^1 h + \\
+ \frac{1}{2}\alpha\delta(1-t)\left(\frac{\alpha(1-t)(t-1+a)}{a(1-t-\frac{a}{2})m}\right)h + \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha(1-t)}{am}\right) + \\
- (1-\delta)(1-t)\left(\frac{1}{2} - \frac{\alpha(1-t)}{2(1-t-\frac{a}{2})m}\right)h - \frac{1}{2}a(1-\delta)\left(\frac{\alpha(1-t)(t-1+a)}{a(1-t-\frac{a}{2})m}\right)h \quad t \in \left[1-a, \frac{m\left(1-\frac{a}{2}\right)}{m-\alpha}\right] \\
\\
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
+ \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} - \frac{\alpha(1-t)}{am}\right)h + \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha(1-t)}{am}\right) + \\
- (1-\delta)(1-t)n_T^1 h - \frac{1}{2}a(1-\delta)\left(\frac{1}{2} - \frac{\alpha(1-t)}{am}\right)h \quad t \in \left[\frac{m\left(1-\frac{a}{2}\right)}{m-\alpha}, 1\right] \\
\\
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
+ \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta)(1-t)\left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right)h + \quad t \in [0, 1-a] \\
\\
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h - (1-\delta)(1-t)n_T^1 h + \quad t \in \left[1-a, \frac{m\left(1-\frac{a}{2}\right) - \alpha}{m-\alpha}\right] \\
+ \frac{\alpha^2\delta(1-t)^2(t-1+a)h}{2a(1-t-\frac{a}{2})m} + \frac{1}{4}\alpha\delta(1-t) + \frac{\alpha^2\delta(1-t)^2}{2am} + \\
- \frac{1}{2}(1-\delta)(1-t)h + \frac{\alpha(1-\delta)(1-t)^2h}{2(1-t-\frac{a}{2})m} - \frac{\alpha(1-\delta)(1-t)(t-1+a)h}{2(1-t-\frac{a}{2})m} \\
\\
\frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h - (1-\delta)(1-t)n_T^1 h + \quad t \in \left[\frac{m\left(1-\frac{a}{2}\right) - \alpha}{m-\alpha}, 1\right] \\
+ \frac{1}{4}\alpha\delta(1-t)h - \frac{\alpha^2\delta(1-t)^2h}{2am} + \frac{1}{4}\alpha\delta(1-t) + \frac{\alpha^2\delta(1-t)^2}{2am} + \\
- \frac{1}{4}a(1-\delta)h + \frac{\alpha(1-\delta)(1-t)h}{2m}
\end{array} \right.$$

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
& + \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta)(1-t)\left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h + \\
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
& + \frac{\alpha^2\delta(1-t)^2(t-1+a)h}{2a(1-t-\frac{a}{2})m} + \frac{1}{4}\alpha\delta(1-t)\left(1 + \frac{2\alpha(1-t)}{am}\right) + \\
& - (1-\delta)(1-t)\left(n_T^1 + \frac{1}{2}\right)h + \frac{\alpha(1-\delta)(1-t)h}{2(1-t-\frac{a}{2})m}(1-t-t+1-a) \\
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h - \frac{1}{4}a(1-\delta)h + \\
& + \frac{1}{4}\alpha\delta(1-t)(1+h) + \frac{\alpha^2\delta(1-t)^2(1-h)}{2am} + \\
& - (1-\delta)(1-t)\left(n_T^1 - \frac{\alpha}{2m}\right)h
\end{aligned} \right\} \begin{aligned}
& t \in [0, 1-a] \\
& t \in \left[1-a, \frac{m\left(1-\frac{a}{2}\right)-\alpha}{m-\alpha}\right] \\
& t \in \left[\frac{m\left(1-\frac{a}{2}\right)-\alpha}{m-\alpha}, 1\right]
\end{aligned} \\
& \left. \begin{aligned}
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
& + \frac{1}{2}\alpha\delta(1-t)\left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta)(1-t)\left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h + \\
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \\
& + \frac{\alpha^2\delta(1-t)^2(t-1+a)h}{2a(1-t-\frac{a}{2})m} + \frac{1}{4}\alpha\delta(1-t)\left(1 + \frac{2\alpha(1-t)}{am}\right) + \\
& - (1-\delta)(1-t)\left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right)h \\
& \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h - \frac{1}{4}a(1-\delta)h + \\
& + \frac{1}{4}\alpha\delta(1-t)(1+h) + \frac{\alpha^2\delta(1-t)^2(1-h)}{2am} + \\
& - (1-\delta)(1-t)\left(n_T^1 - \frac{\alpha}{2m}\right)h
\end{aligned} \right\} \begin{aligned}
& t \in [0, 1-a] \\
& t \in \left[1-a, \frac{m\left(1-\frac{a}{2}\right)-\alpha}{m-\alpha}\right] \\
& t \in \left[\frac{m\left(1-\frac{a}{2}\right)-\alpha}{m-\alpha}, 1\right]
\end{aligned}
\end{aligned}$$

Hence

$$\frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} =$$

$$\begin{aligned}
&= \left\{ \begin{array}{ll} (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h - \frac{1}{2} \alpha \delta \left(\frac{1}{2} + \frac{\alpha}{m} \right) & t \in [0, 1-a] \\ \frac{\alpha^2 \delta \left\{ \left[-2(1-t)(t-1+a) + (1-t)^2 \right] (1-t - \frac{a}{2}) + (1-t)^2 (t-1+a) \right\} h}{2a(1-t - \frac{a}{2})^2 m} + & t \in \left[1-a, \frac{m(1-\frac{a}{2})}{m-\alpha} \right] \\ -\frac{1}{4} \alpha \delta \left(1 + \frac{2\alpha(1-t)}{am} \right) - \frac{1}{4} \alpha \delta (1-t) \frac{2\alpha}{am} + & \\ + (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h & \\ \\ (1-\delta) \left(n_T^1 - \frac{\alpha}{2m} \right) h - \frac{1}{4} \alpha \delta (1+h) - \frac{\alpha^2 \delta (1-t)(1-h)}{am} & t \in \left[\frac{m(1-\frac{a}{2}) - \alpha}{m-\alpha}, 1 \right] \end{array} \right. \\
&= \left\{ \begin{array}{ll} (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h - \frac{1}{2} \alpha \delta \left(\frac{1}{2} + \frac{\alpha}{m} \right) & t \in [0, 1-a] \\ \frac{\alpha^2 \delta (1-t) \left\{ (3-3t-2a)(1-t - \frac{a}{2}) + (1-t)(t-1+a) \right\} h}{2a(1-t - \frac{a}{2})^2 m} + & t \in \left[1-a, \frac{m(1-\frac{a}{2}) - \alpha}{m-\alpha} \right] \\ -\frac{1}{4} \alpha \delta \left(1 + \frac{4\alpha(1-t)}{am} \right) + & \\ + (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h & \\ \\ (1-\delta) \left(n_T^1 - \frac{\alpha}{2m} \right) h - \frac{1}{4} \alpha \delta (1+h) - \frac{\alpha^2 \delta (1-t)(1-h)}{am} & t \in \left[\frac{m(1-\frac{a}{2}) - \alpha}{m-\alpha}, 1 \right] \end{array} \right. =
\end{aligned}$$

Thus, the situation is as follows:

1. if $t \in [0, 1-a]$

$$\frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \geq 0 \Leftrightarrow (1-\delta) \underbrace{\left[n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right]}_{>0 \text{ since } m > 2\alpha} h \geq \alpha \delta \left(\frac{1}{4} + \frac{\alpha}{2m} \right)$$

2. if $t \in \left[1-a, \frac{m(1-\frac{a}{2}) - \alpha}{m-\alpha} \right]$

$$\begin{aligned}
&\frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \geq 0 \Leftrightarrow \\
&\Leftrightarrow (1-\delta) \underbrace{\left[n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right]}_{>0 \text{ since } m > 2\alpha} h \geq \alpha \delta \underbrace{\left[\frac{1}{4} + \frac{\alpha(1-t)}{am} - \frac{\alpha(1-t) \left[(3-3t-2a)(1-t - \frac{a}{2}) + (1-t)(t-1-a) \right]}{2a(1-t - \frac{a}{2})^2 m} \right]}_{\geq \frac{1}{4} + \frac{\alpha}{2m}}
\end{aligned}$$

$$3. \text{ if } t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right]$$

$$\begin{aligned} & \frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \geq 0 \Leftrightarrow \\ & \Leftrightarrow (1 - \delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \geq \alpha \delta \left[\frac{1}{4} (1 + h) + \frac{2\alpha (1 - t) (1 - h)}{m} \right] \Leftrightarrow \end{aligned}$$

Lemma 3 We get the following chain of logical implications

$$\frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \Big|_{t \in \left[\frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1 \right]} \geq 0 \Rightarrow \begin{cases} \frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \Big|_{t \in [0, 1-a]} \geq 0 \\ \frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \Big|_{t \in \left[1-a, \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} \right]} \geq 0. \end{cases}$$

Moreover, note that

$$\begin{aligned} & EV^G(t|n_T^1, n_A^1, n_C^1) \Big|_{t=0} = \\ & = \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1 - \delta) - \frac{1}{2} a (1 - \delta) n_A^1 h + \frac{1}{2} \alpha \delta \left(\frac{1}{2} + \frac{\alpha}{m} \right) - (1 - \delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h = \end{aligned}$$

while

$$\begin{aligned} & EV^G(t|n_T^1, n_A^1, n_C^1) \Big|_{t = \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}} = \\ & = \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1 - \delta) - \frac{1}{2} a (1 - \delta) n_A^1 h + \\ & + \frac{\alpha^2 \delta \left(\frac{am}{2(m-\alpha)} \right)^2 \left(\frac{a(m-2\alpha)}{2(m-\alpha)} \right) h}{2a \left(\frac{\alpha a}{2(m-\alpha)} \right) m} + \frac{1}{4} \alpha \delta \left(\frac{am}{2(m-\alpha)} \right) \left(1 + \frac{2\alpha \frac{am}{2(m-\alpha)}}{am} \right) + \\ & - (1 - \delta) \left(\frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h = \\ & = \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1 - \delta) - \frac{1}{2} a (1 - \delta) n_A^1 h + \\ & + \alpha^2 \delta \left(\frac{am}{2(m-\alpha)} \right)^2 \left(\frac{a(m-2\alpha)}{2(m-\alpha)} \right) h \frac{m-\alpha}{\alpha a^2 m} + \frac{1}{4} \alpha \delta \left(\frac{am}{2(m-\alpha)} \right) \left(1 + \frac{\alpha am}{(m-\alpha) am} \right) + \\ & - (1 - \delta) \left(\frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h = \\ & = \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1 - \delta) - \frac{1}{2} a (1 - \delta) n_A^1 h + \\ & + \delta \frac{\alpha a (m-2\alpha) mh}{8(m-\alpha)^2} + \delta \frac{\alpha am^2}{8(m-\alpha)^2} - (1 - \delta) \left(\frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h = \\ & = \frac{1}{2} \alpha \delta n_A^1 h + \frac{1}{2} \alpha \delta n_C^1 + (1 - \delta) - \frac{1}{2} a (1 - \delta) n_A^1 h + \end{aligned}$$

$$\delta \frac{\alpha am^2}{8(m-\alpha)^2} \left(1+h-\frac{2\alpha h}{m}\right) - (1-\delta) \left(\frac{am}{2(m-\alpha)}\right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h$$

since

$$1-t \Big|_{t=\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}} = 1 - \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} = \frac{m-\alpha-m+\frac{ma}{2}+\alpha}{m-\alpha} = \frac{am}{2(m-\alpha)}$$

$$t-1+a \Big|_{t=\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}} = \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} - 1 + a = \frac{m-\frac{am}{2}-\alpha-m+\alpha}{m-\alpha} + a = \frac{-am+2am-2\alpha a}{2(m-\alpha)} = \frac{a(m-2\alpha)}{2(m-\alpha)}$$

$$1-t-\frac{a}{2} \Big|_{t=\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}} = \frac{am}{2(m-\alpha)} - \frac{a}{2} = \frac{am-am+\alpha a}{2(m-\alpha)} = \frac{\alpha a}{2(m-\alpha)}$$

which gives the following result:

Lemma 4

$$EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=0} \geq EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=\frac{3m-4\alpha}{4(m-\alpha)}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h + \frac{1}{2}\alpha\delta \left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h \geq$$

$$\geq \frac{1}{2}\alpha\delta n_A^1 h + \frac{1}{2}\alpha\delta n_C^1 + (1-\delta) - \frac{1}{2}a(1-\delta)n_A^1 h +$$

$$+ \delta \frac{\alpha am^2}{8(m-\alpha)^2} \left(1+h-\frac{2\alpha h}{m}\right) - (1-\delta) \left(\frac{am}{2(m-\alpha)}\right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\alpha\delta \left(\frac{1}{2} + \frac{\alpha}{m}\right) - (1-\delta) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h \geq$$

$$\geq \delta \frac{\alpha am^2}{8(m-\alpha)^2} \left(1+h-\frac{2\alpha h}{m}\right) - (1-\delta) \left(\frac{am}{2(m-\alpha)}\right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h-\frac{2\alpha h}{m}\right)\right) \geq (1-\delta) \left(1 - \frac{am}{2(m-\alpha)}\right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m}\right) h$$

Then, these two lemmas allow us to conclude with following result:

Corollary 1 *There are three possible solutions*

$$t^* = 0;$$

$$t^* = \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}$$

$$t^* = 1.$$

In particular there are three possible cases:

1. when

$$\frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \Big|_{t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1\right]} \geq 0 \Leftrightarrow t^* = 1$$

2. when

$$\left. \frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \right|_{t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right]} \leq 0$$

and

$$EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=0} \leq EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}} \Rightarrow t^* = \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}$$

3. when

$$\left. \frac{\partial EV^G(t|n_T^1, n_A^1, n_C^1)}{\partial t} \right|_{t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right]} \leq 0$$

and

$$EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=0} \geq EV^G(t|n_T^1, n_A^1, n_C^1)|_{t=\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}} \Rightarrow t^* = 0.$$

Using the explicit expressions for these conditions, we have:

1. when

$$\forall t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right]$$

$$(1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \geq \alpha \delta \left[\frac{1}{4}(1+h) + \frac{2\alpha(1-t)(1-h)}{m} \right] \Leftrightarrow t^* = 1$$

that thus taking $t = 1$ can be written as

$$(1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \geq \alpha \delta \frac{1}{4}(1+h) \Leftrightarrow t^* = 1$$

2. when

$$\begin{cases} \alpha \delta \frac{1}{4}(1+h) \geq (1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \\ \alpha \delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} (1+h - \frac{2\alpha h}{m}) \right) \leq (1-\delta) \left(1 - \frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \end{cases} \Leftrightarrow$$

$$\Leftrightarrow t^* = \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}$$

3. when

$$\begin{cases} \alpha \delta \frac{1}{4}(1+h) \geq (1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \\ \alpha \delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} (1+h - \frac{2\alpha h}{m}) \right) \geq (1-\delta) \left(1 - \frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \end{cases} \Leftrightarrow$$

$$\Leftrightarrow t^* = 0$$

So, let we consider these cases and make explicit the range of n_T^1 .

Case 1 $t^* = 1$

$$\begin{aligned} &\Leftrightarrow (1 - \delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \geq \alpha \delta \left[\frac{1}{4} (1 + h) \right] \Leftrightarrow \\ &\Leftrightarrow n_T^1 \geq \frac{\alpha \delta (1 + h)}{4(1 - \delta) h} + \frac{\alpha}{2m} \Leftrightarrow \\ &n_T^1 \in \left[\frac{\alpha \delta (1 + h)}{4(1 - \delta) h} + \frac{\alpha}{2m}, 1 \right] \Leftrightarrow t^* = 1 \end{aligned}$$

Case 2 $t^* = \frac{m(1 - \frac{a}{2}) - \alpha}{m - \alpha}$

$$\begin{cases} \alpha \delta \left[\frac{1}{4} (1 + h) \right] \geq (1 - \delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \\ \alpha \delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) \leq (1 - \delta) \left(1 - \frac{am}{2(m - \alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \end{cases} \Leftrightarrow$$

$$\Leftrightarrow t^* = \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha}$$

Let we start with the first condition

$$\begin{aligned} &(1 - \delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \leq \alpha \delta \left[\frac{1}{4} (1 + h) \right] \Leftrightarrow \\ &\Leftrightarrow n_T^1 \leq \frac{\alpha \delta (1 + h)}{4(1 - \delta) h} + \frac{\alpha}{2m} \Leftrightarrow \\ &n_T^1 \in \left[0, \frac{\alpha \delta (1 + h)}{4(1 - \delta) h} + \frac{\alpha}{2m} \right] \end{aligned}$$

This condition should be coupled with the second

$$\begin{aligned} &(1 - \delta) \left(\frac{2(m - \alpha) - am}{2(m - \alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \geq \alpha \delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) \Leftrightarrow \\ &\Leftrightarrow n_T^1 \geq \frac{2\alpha \delta (m - \alpha)}{(1 - \delta) [2(m - \alpha) - am] h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \Leftrightarrow \\ &n_T^1 \in \left[\frac{2\alpha \delta (m - \alpha)}{(1 - \delta) [2(m - \alpha) - am] h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, 1 \right] \end{aligned}$$

Hence, we might conclude that

$$\begin{aligned} &n_T^1 \in \left[0, \frac{\alpha \delta (1 + h)}{4(1 - \delta) h} + \frac{\alpha}{2m} \right] \cap \\ &\cap \left[\frac{2\alpha \delta (m - \alpha)}{(1 - \delta) [2(m - \alpha) - am] h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, 1 \right] \Leftrightarrow \\ &\Leftrightarrow t^* = \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \end{aligned}$$

Case 3 $t^* = 0$

$$\left\{ \begin{array}{l} \alpha\delta \left[\frac{1}{4}(1+h) \right] \geq (1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \\ \alpha\delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) \geq (1-\delta) \left(1 - \frac{am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \end{array} \right. \Leftrightarrow$$

$$\Leftrightarrow t^* = 0$$

Let us start with the first condition

$$(1-\delta) \left[n_T^1 - \frac{\alpha}{2m} \right] h \leq \alpha\delta \left[\frac{1}{4}(1+h) \right] \Leftrightarrow$$

$$\Leftrightarrow n_T^1 \leq \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \Leftrightarrow$$

$$n_T^1 \in \left[0, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \right]$$

This condition should be coupled with the second

$$(1-\delta) \left(\frac{2(m-\alpha) - am}{2(m-\alpha)} \right) \left(n_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h \leq \alpha\delta \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) \Leftrightarrow$$

$$\Leftrightarrow n_T^1 \leq \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \Leftrightarrow$$

$$n_T^1 \in \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right]$$

Hence, we might conclude that

$$n_T^1 \in \left[0, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \right] \cap$$

$$\cap \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right] \Leftrightarrow$$

$$\Leftrightarrow t^* = 0$$

Note that

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}$$

Then, we get the following sequential best reply of the government.

Proposition 2 *The sequential best reply of the government is*

$$t^*(n_T^1) = \begin{cases} 0 & n_T^1 \in \langle 1 \rangle \\ \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} & n_T^1 \in \langle 2 \rangle \\ 1 & n_T^1 \in \langle 3 \rangle \end{cases}$$

where

$$\begin{aligned} \langle 1 \rangle &:= \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right] \\ \langle 2 \rangle &:= \left[\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \right] \\ \langle 3 \rangle &:= \left[\frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}, 1 \right] \end{aligned}$$

and the intervals can be empty for specific values of the parameters.

7.3 First Period Citizens' Choice

Citizen's $i \in J^1$ continuation payoff is

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i R_{-i} (t^*, \bar{n}_T^1 h, \bar{n}_T^2 h, \bar{n}_A^1 h, \bar{n}_A^2 h) + B_i (i \in J^1, i^* \in J^2) = \\ &= \gamma_i \min \left\{ (1-t^*) (\bar{n}_T^1 + \bar{n}_T^2) h + \frac{1}{2} a (\bar{n}_A^1 + \bar{n}_A^2) h, 1 \right\} + \\ &+ \begin{cases} \frac{2}{P} \gamma_i (1-t^*) H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{P} \gamma_i (1-t^*) H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t^*) H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i (1-t^*) H_i + \alpha (1-t^*) H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{P} \gamma_i (1-t^*) H_i & \text{if } i \in A^1, i \in T^2 \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha (1-t^*) H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha (1-t^*) H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \frac{1}{P} \gamma_i (1-t^*) H_i & \text{if } i \in C^1, i \in T^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha (1-t^*) H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha H_i + \alpha (1-t^*) H_i & \text{if } i \in C^1, i \in C^2. \end{cases} \end{aligned}$$

7.3.1 Case 1

Suppose

$$\bar{n}_T^1 \in \left[\frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}, 1 \right]$$

then

$$t^* = 1$$

and

$$n_T^{2*} = 0; n_A^{2*} = \frac{1}{2}; n_C^{2*} = \frac{1}{2}$$

hence $i \in J^1$ continuation payoff is

$$EV_i^1 (i \in J^1) =$$

$$= \gamma_i \frac{1}{4} \left(\bar{n}_A^1 + \frac{1}{2} \right) h +$$

$$+ \begin{cases} \frac{1}{2P} \gamma_i a H_i & \text{if } i \in T^1, i \in A^2 \\ 0 & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha H_i & \text{if } i \in C^1, i \in C^2 \end{cases}$$

Moreover, in this case

$$i^* \in T^2 \Leftrightarrow \gamma_i \in \emptyset$$

$$i^* \in A^2 \Leftrightarrow \gamma_i \in \left[0, \frac{m}{2} P \right]$$

$$i^* \in C^2 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P, 0 \right]$$

1. Suppose

$$\gamma_i \in \left[-\frac{m}{2} P, 0 \right] \Leftrightarrow i^* \in C^2$$

then

$$EV_i^1 (i \in J^1) =$$

$$= \gamma_i \frac{1}{4} \left(\bar{n}_A^1 + \frac{1}{2} \right) h +$$

$$+ \begin{cases} 0 & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i & \text{if } i \in C^1, i \in C^2 \end{cases}$$

hence

$$i \in T^1 \Leftrightarrow \begin{cases} 0 \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \\ 0 \geq \alpha H_i \end{cases}$$

which is clearly impossible. Hence in this case

$$i \in T^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Similarly

$$i \in A^1 \Leftrightarrow \begin{cases} \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \geq 0 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \geq \alpha H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \geq -\frac{\alpha}{a} P \\ \gamma_i \geq \frac{\alpha^a}{a} P \end{cases}$$

which is not acceptable when

$$\gamma_i \in \left[-\frac{m}{2} P, 0 \right].$$

Hence

$$i \in A^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Thus, we might conclude that in this case

$$i \in C^1 \cap C^2 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P, 0 \right]$$

2. **Suppose**

$$\gamma_i \in \left[0, \frac{m}{2}P\right] \Leftrightarrow i \in A^2$$

then

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i \frac{1}{2} a \left(\bar{n}_A^1 + \frac{1}{2} \right) h + \\ &+ \begin{cases} \frac{1}{2P} \gamma_i a H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i & \text{if } i \in A^1, i \in A^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a H_i & \text{if } i \in C^1, i \in A^2 \end{cases} \end{aligned}$$

hence

$$i \in T^1 \Leftrightarrow \begin{cases} \frac{1}{2P} \gamma_i a H_i \geq \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \\ \frac{1}{2P} \gamma_i a H_i \geq \alpha H_i + \frac{1}{2P} \gamma_i a H_i \end{cases} \Leftrightarrow \begin{cases} 0 \geq \frac{1}{2P} a \gamma_i + \frac{1}{2} \alpha \\ 0 \geq \alpha \end{cases}$$

which is impossible, hence,

$$i \in T^1 \cap A^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Similarly

$$i \in A^1 \Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \geq \frac{1}{2P} \gamma_i a H_i \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i \geq \alpha H_i + \frac{1}{2P} \gamma_i a H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \geq -\frac{\alpha}{a} P \\ \gamma_i \geq \frac{\alpha^a}{a} P \end{cases}$$

Hence,

$$i \in A^1 \cap A^2 \Leftrightarrow \gamma_i \in \left[\frac{\alpha}{a} P; \frac{m}{2} P \right]$$

and

$$i \in C^1 \cap A^2 \Leftrightarrow \gamma_i \in \left[0; \frac{\alpha}{a} P \right].$$

Therefore we might conclude with the following lemma.

Lemma 5 *There is no equilibrium when*

$$t^* = 1$$

because it implies

$$n_T^{1*} = 0$$

however

$$t^* = 1$$

requires

$$n_T^1 \in \left[\frac{\alpha \delta (1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}, 1 \right] \Rightarrow n_T^1 > 0.$$

7.3.2 Case 2

Suppose

$$n_T^1 \in \left[\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \right]$$

then

$$t^* = \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}$$

Moreover, in this case, since

$$1 - t = 1 - \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} = \frac{m - \alpha - m + \frac{am}{2} + \alpha}{m - \alpha} = \frac{am}{2(m - \alpha)}$$

then

$$\begin{aligned} n_T^2 &= 0 \\ n_A^2 &= \frac{1}{2} - \frac{\alpha(1-t)}{am} = \frac{1}{2} - \frac{\alpha \frac{am}{2(m-\alpha)}}{am} = \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} = \frac{m-2\alpha}{2(m-\alpha)}; \\ n_C^2 &= \frac{1}{2} + \frac{\alpha(1-t)}{am} = \frac{1}{2} + \frac{\alpha \frac{am}{2(m-\alpha)}}{am} = \frac{1}{2} + \frac{\alpha}{2(m-\alpha)} = \frac{m}{2(m-\alpha)} \end{aligned}$$

i.e.

$$n_T^{2*} = 0; n_A^{2*} = \frac{m-2\alpha}{2(m-\alpha)}; n_C^{2*} = \frac{m}{2(m-\alpha)}.$$

This means that in this case

$$\begin{aligned} i^* \in T^2 &\Leftrightarrow \gamma_i \in \emptyset \\ i^* \in A^2 &\Leftrightarrow \gamma_i \in \left[\frac{\alpha m}{2(m-\alpha)}P, \frac{m}{2}P \right] \\ i^* \in C^2 &\Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha m}{2(m-\alpha)}P \right] \end{aligned}$$

Finally, $i \in J^1$ continuation payoff is

$$\begin{aligned} EV_i^1(i \in J^1) &= \\ &= \gamma_i \frac{am}{2(m-\alpha)} \bar{n}_T^1 h + \frac{1}{2} a \left(\bar{n}_A^1 + \frac{m-2\alpha}{2(m-\alpha)} \right) h + \\ &+ \begin{cases} \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in A^1, i \in A^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in C^1, i \in A^2 \\ \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in C^1, i \in C^2. \end{cases} \end{aligned}$$

1. **Suppose**

$$\gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha m}{2(m-\alpha)}P \right] \Leftrightarrow i^* \in C^2$$

then

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i \frac{am}{2(m-\alpha)} \bar{n}_T^1 h + \frac{1}{4} \left(\bar{n}_A^1 + \frac{m-2\alpha}{2(m-\alpha)} \right) h + \\ &+ \begin{cases} \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in C^1, i \in C^2 \end{cases} \end{aligned}$$

hence

$$\begin{aligned} i \in T^1 \Leftrightarrow & \begin{cases} \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \alpha \frac{am}{2(m-\alpha)} H_i \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i \\ \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \alpha \frac{am}{2(m-\alpha)} H_i \geq \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i \left(\frac{\alpha a}{2(m-\alpha)} \right) \geq \frac{1}{2} \alpha \\ \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} \geq \alpha \end{cases} \Leftrightarrow \begin{cases} \gamma_i \geq \frac{(m-\alpha)P}{a} \\ \gamma_i \geq \frac{2\alpha(m-\alpha)}{am} P \end{cases} \end{aligned}$$

Note that $m - \alpha > \frac{am}{2}$ since $m > 2\alpha$ by hypothesis. Hence

$$i \in T^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Similarly

$$\begin{aligned} i \in A^1 \Leftrightarrow & \begin{cases} \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i \geq \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \alpha \frac{am}{2(m-\alpha)} H_i \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i \geq \alpha H_i + \alpha \frac{am}{2(m-\alpha)} H_i \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} \alpha P \geq \gamma_i \frac{am-am+\alpha a}{(m-\alpha)} \\ \gamma_i a \geq \alpha P \end{cases} \Leftrightarrow \begin{cases} \gamma_i \leq \frac{(m-\alpha)P}{a} \\ \gamma_i \geq \frac{\alpha}{a} P \end{cases} \end{aligned}$$

Note that

$$\frac{\alpha}{a} \leq \frac{\alpha m}{2(m-\alpha)} \Leftrightarrow 2(m-\alpha) \leq am$$

which is impossible because $m > 2\alpha$ by hypothesis. Hence

$$i \in A^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Thus, we might conclude that

$$i \in C^1 \cap C^2 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha m}{2(m-\alpha)}P \right]$$

2. **Suppose**

$$\gamma_i \in \left[\frac{\alpha m}{2(m-\alpha)}P, \frac{m}{2}P \right] \Leftrightarrow i \in A^2$$

then

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i \frac{am}{2(m-\alpha)} \bar{n}_T^1 h + \frac{1}{4} \left(\bar{n}_A^1 + \frac{m-2\alpha}{2(m-\alpha)} \right) h + \\ &+ \begin{cases} \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in T^1, i \in A^2 \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in A^1, i \in A^2 \\ \alpha H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i & \text{if } i \in C^1, i \in A^2 \end{cases} \end{aligned}$$

hence

$$\begin{aligned} i \in T^1 &\Leftrightarrow \\ \Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \geq \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \\ \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \geq \alpha H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \end{cases} &\Leftrightarrow \\ \Leftrightarrow \begin{cases} \gamma_i \frac{am-am+\alpha a}{(m-\alpha)} \geq \alpha P \\ \gamma_i \frac{am}{2(m-\alpha)} \geq \alpha P \end{cases} &\Leftrightarrow \begin{cases} \gamma_i \geq \frac{m-\alpha}{a} P \\ \gamma_i \geq \frac{2\alpha(m-\alpha)}{am} P \end{cases} \end{aligned}$$

since $\frac{m-\alpha}{a} > \frac{m}{2}$ because $m > 2\alpha$, hence,

$$i \in T^1 \cap A^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Similarly

$$\begin{aligned} i \in A^1 &\Leftrightarrow \\ \Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \geq \frac{1}{P} \gamma_i \frac{am}{2(m-\alpha)} H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \\ \frac{1}{P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \geq \alpha H_i + \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha \frac{am}{2(m-\alpha)} H_i \end{cases} &\Leftrightarrow \\ \Leftrightarrow \begin{cases} \alpha P \geq \gamma_i \frac{am-am+\alpha a}{(m-\alpha)} \\ \frac{1}{2P} \gamma_i a \geq \frac{1}{2} \alpha \end{cases} &\Leftrightarrow \begin{cases} \gamma_i \leq \frac{m-\alpha}{a} P \\ \gamma_i \geq \frac{\alpha}{P} \end{cases} \end{aligned}$$

Hence, since $\frac{\alpha}{a} > \frac{\alpha m}{2(m-\alpha)}$ because $m > 2\alpha$,

$$i \in A^1 \cap A^2 \Leftrightarrow \gamma_i \in \left[\frac{\alpha}{a} P; \frac{m}{2} P \right]$$

and

$$i \in C^1 \cap A^2 \Leftrightarrow \gamma_i \in \left[\frac{\alpha m}{2(m-\alpha)} P; \frac{\alpha}{a} P \right].$$

Thus, we can conclude with the following result:

Lemma 6 *When*

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0$$

there is SPE such that

$$\begin{aligned} n_T^1 &= n_T^2 = 0 \\ n_A^1 &= \frac{1}{2} - \frac{\alpha}{am} < n_A^2 = \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \\ n_C^1 &= \frac{1}{2} + \frac{\alpha}{am} > n_C^2 = \frac{1}{2} + \frac{\alpha}{2(m-\alpha)}. \\ t^* &= \frac{m \left(1 - \frac{\alpha}{2} \right) - \alpha}{m - \alpha}. \end{aligned}$$

7.3.3 Case 3

Suppose

$$n_T^1 \in \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right]$$

then

$$t^* = 0$$

and

$$n_T^{2*} = \frac{1}{2} - \frac{\alpha}{m}; n_A^{2*} = 0; n_C^{2*} = \frac{1}{2} + \frac{\alpha}{m}$$

hence $i \in J^1$ continuation payoff is

$$\begin{aligned} EV_i^1(i \in J^1) &= \\ &= \gamma_i \left[\left(\bar{n}_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h + \frac{1}{2} a \bar{n}_A^1 h \right] + \\ &+ \begin{cases} \frac{2}{P} \gamma_i H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{P} \gamma_i H_i + \alpha H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{P} \gamma_i H_i & \text{if } i \in A^1, i \in T^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \frac{1}{P} \gamma_i H_i & \text{if } i \in C^1, i \in T^2 \\ \alpha H_i + \alpha H_i & \text{if } i \in C^1, i \in C^2. \end{cases} \end{aligned}$$

Moreover, in this case

$$i \in T^2 \Leftrightarrow \gamma_i \in \left[\alpha P, \frac{m}{2} P \right]$$

$$i \in A^2 \Leftrightarrow \gamma_i \in \emptyset$$

$$i \in C^2 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P, \alpha P \right]$$

1. **Suppose**

$$\gamma_i \in \left[-\frac{m}{2}P, \alpha P\right] \Leftrightarrow i \in C^2$$

then

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i \left[\left(\bar{n}_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h + \frac{1}{2} a \bar{n}_A^1 h \right] + \\ &+ \begin{cases} \frac{1}{P} \gamma_i H_i + \alpha H_i & \text{if } i \in T^1, i \in C^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha H_i & \text{if } i \in A^1, i \in C^2 \\ \alpha H_i + \alpha H_i & \text{if } i \in C^1, i \in C^2 \end{cases} \end{aligned}$$

hence

$$i \in T^1 \Leftrightarrow \begin{cases} \frac{1}{P} \gamma_i H_i + \alpha H_i \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha H_i \\ \frac{1}{P} \gamma_i H_i + \alpha H_i \geq \alpha H_i + \alpha H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \geq \frac{\alpha}{2-a} P \\ \gamma_i \geq \alpha P \end{cases}$$

which is impossible. Hence

$$i \in T^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Similarly

$$i \in A^1 \Leftrightarrow \begin{cases} \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha H_i \geq \frac{1}{P} \gamma_i H_i + \alpha H_i \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \alpha H_i \geq \alpha H_i + \alpha H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \leq \frac{\alpha}{2-a} P \\ \gamma_i \geq \frac{\alpha}{a} P \end{cases}$$

which is impossible. Hence

$$i \in A^1 \cap C^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Thus, we might conclude that

$$i \in C^1 \cap C^2 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P, \alpha P\right]$$

2. **Suppose**

$$\gamma_i \in \left[\alpha P, \frac{m}{2}P\right] \Leftrightarrow i \in T^2$$

then

$$\begin{aligned} EV_i^1 (i \in J^1) &= \\ &= \gamma_i \left[\left(\bar{n}_T^1 + \frac{1}{2} - \frac{\alpha}{m} \right) h + \frac{1}{2} a \bar{n}_A^1 h \right] + \\ &+ \begin{cases} \frac{2}{P} \gamma_i H_i & \text{if } i \in T^1, i \in T^2 \\ \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{P} \gamma_i H_i & \text{if } i \in A^1, i \in T^2 \\ \alpha H_i + \frac{1}{P} \gamma_i H_i & \text{if } i \in C^1, i \in T^2 \end{cases} \end{aligned}$$

hence

$$i \in T^1 \Leftrightarrow \begin{cases} \frac{2}{P} \gamma_i H_i \geq \frac{1}{2P} \gamma_i a H_i + \frac{1}{2} \alpha H_i + \frac{1}{P} \gamma_i H_i \\ \frac{2}{P} \gamma_i H_i \geq \alpha H_i + \frac{1}{P} \gamma_i H_i \end{cases} \Leftrightarrow \begin{cases} \gamma_i \geq \frac{\alpha}{2-a} P \\ \gamma_i \geq \alpha P \end{cases}$$

which is always satisfied. Hence,

$$i \in T^1 \cap T^2 \Leftrightarrow \gamma_i \in \left[\alpha P, \frac{m}{2} P \right].$$

Then, we might conclude that

$$i \in A^1 \cap T^2 \Leftrightarrow \gamma_i \in \emptyset$$

and

$$i \in C^1 \cap T^2 \Leftrightarrow \gamma_i \in \emptyset.$$

Thus we might conclude with the following lemma:

Lemma 7 *When*

$$\begin{aligned} \frac{1}{2} - \frac{\alpha}{m} &\leq \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \Leftrightarrow \\ \Leftrightarrow 1 - \frac{2\alpha}{m} &\leq \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) \end{aligned}$$

there is a pure strategy SPE such that

$$t^* = 0$$

and

$$\begin{aligned} n_T^{1*} &= n_T^{2*} = \frac{1}{2} - \frac{\alpha}{m} \\ n_A^{1*} &= n_A^{2*} = 0 \\ n_C^{1*} &= n_C^{2*} = \frac{1}{2} + \frac{\alpha}{m}. \end{aligned}$$

When

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

there is a mixed strategy SPE such that the government randomizes so that

$$t^* = \begin{cases} 0 & \text{probability } 1 - \tau \\ \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} & \text{probability } \tau \end{cases}$$

which implies

$$Et^* = \frac{\left[m \left(1 - \frac{a}{2} \right) - \alpha \right] \tau}{m - \alpha} \in \left[\frac{1}{2}, \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \right]$$

$$n_T^{1*} = \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}$$

$$n_T^{2*} = \begin{cases} \frac{1}{2} - \frac{\alpha}{m} > n_T^{1*} & \text{probability } 1 - \tau \\ 0 < n_T^{1*} & \text{probability } \tau \end{cases}$$

$$n_A^{1*} = 1 - \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) - \frac{2\alpha}{m}$$

$$n_A^{2*} = \begin{cases} 0 < n_A^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} > n_A^{1*} & \text{probability } \tau \end{cases}$$

$$n_C^{1*} = \frac{1}{2} + \frac{\alpha}{m}.$$

$$n_C^{2*} = \begin{cases} \frac{1}{2} + \frac{\alpha}{m} = n_C^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} + \frac{\alpha}{2(m-\alpha)} < n_C^{1*} & \text{probability } \tau. \end{cases}$$

8 The Set of Equilibria

Now we are able to characterize the set of Subgame Perfect equilibria of the terrorist game, as a function of the structural parameters.

Proposition 3 1. When

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0$$

there is SPE such that

$$i^* \in T^1 \Leftrightarrow \gamma_i \in \emptyset$$

$$i^* \in A^1 \Leftrightarrow \gamma_i \in \left[\frac{\alpha}{a}P; \frac{m}{2}P \right]$$

$$i^* \in C^1 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P; \frac{\alpha}{a}P \right]$$

$$t^*(n_T^1) = \begin{cases} 0 & n_T^1 \in \langle 1 \rangle \\ \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} & n_T^1 \in \langle 2 \rangle \\ 1 & n_T^1 \in \langle 3 \rangle \end{cases}$$

where

$$\langle 1 \rangle := \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right]$$

$$\langle 2 \rangle := \left[\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m} \right]$$

$$\langle 3 \rangle := \left[\frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}, 1 \right]$$

$$\begin{aligned}
i \in T^2 &\Leftrightarrow \begin{cases} \gamma_i \in \left[\alpha P, \frac{m}{2} P \right] & t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}, \frac{m}{2} P \right] & t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} \right] \\ \gamma_i \in \emptyset & t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right] \end{cases} \\
i \in A^2 &\Leftrightarrow \begin{cases} \gamma_i \in \emptyset & t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \right] & t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} \right] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{m}{2} P \right] & t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right] \end{cases} \\
i \in C^2 &\Leftrightarrow \begin{cases} \gamma_i \in \left[-\frac{m}{2} P, \alpha P \right] & t \in [0, 1-a] \\ \gamma_i \in \left[-\frac{m}{2} P, \frac{\alpha(1-t)P}{a} \right] & t \in [1-a, 1]. \end{cases}
\end{aligned}$$

2. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

there is a mixed strategy SPE such that

$$\begin{aligned}
&i^* \in T^1 \Leftrightarrow \\
&\Leftrightarrow \gamma_i \in \left[\left(m - \alpha - \frac{2\alpha\delta m(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) \right) P; \frac{m}{2} P \right] \\
&i^* \in A^1 \Leftrightarrow \\
&\Leftrightarrow \gamma_i \in \left[\alpha P; \left(m - \alpha - \frac{2\alpha\delta m(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) \right) P \right] \\
&i^* \in C^1 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2} P; \alpha P \right] \\
t^*(n_T^1) &= \begin{cases} 0 & n_T^1 \in \langle 1 \rangle \\ \begin{cases} 0 & \text{prob } 1-\tau \\ \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} & \text{prob } \tau \end{cases} & n_T^1 \in \langle 1 \cap 2 \rangle \\ \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} & n_T^1 \in \langle 2 \rangle \\ 1 & n_T^1 \in \langle 3 \rangle \end{cases}
\end{aligned}$$

where

$$\langle 1 \rangle := \left[0, \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \right]$$

$$\langle 1 \cap 2 \rangle := \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}$$

$$\langle 2 \rangle := \left[\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2} \right]$$

$$\langle 3 \rangle := \left[\frac{\alpha\delta(1+h)}{4(1-\delta)h} + \frac{\alpha}{2m}, 1 \right]$$

$$i \in T^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[\alpha P, \frac{m}{2} P \right] & t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})}, \frac{m}{2} P \right] & t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} \right] \\ \gamma_i \in \emptyset & t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right] \end{cases}$$

$$i \in A^2 \Leftrightarrow \begin{cases} \gamma_i \in \emptyset & t \in [0, 1-a] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{\alpha(1-t)P}{2(1-t-\frac{a}{2})} \right] & t \in \left[1-a, \frac{m(1-\frac{a}{2})-\alpha}{m-\alpha} \right] \\ \gamma_i \in \left[\frac{\alpha(1-t)P}{a}, \frac{m}{2} P \right] & t \in \left[\frac{m(1-\frac{a}{2})-\alpha}{m-\alpha}, 1 \right] \end{cases}$$

$$i \in C^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[-\frac{m}{2} P, \alpha P \right] & t \in [0, 1-a] \\ \gamma_i \in \left[-\frac{m}{2} P, \frac{\alpha(1-t)P}{a} \right] & t \in [1-a, 1]. \end{cases}$$

3. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \geq \frac{1}{2} - \frac{\alpha}{m}$$

there is a pure strategy SPE such that

$$i^* \in T^1 \Leftrightarrow \gamma_i \in \left[\alpha P, \frac{m}{2} P \right]$$

$$i^* \in A^1 \Leftrightarrow \gamma_i \in \emptyset$$

$$i^* \in C^1 \Leftrightarrow \gamma_i \in \left[-\frac{m}{2}P; \alpha P\right]$$

$$t^*(n_T^1) = \begin{cases} 0 & n_T^1 \in \langle 1 \rangle \\ \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} & n_T^1 \in \langle 2 \rangle \\ 1 & n_T^1 \in \langle 3 \rangle \end{cases}$$

where

$$\langle 1 \rangle := \left[0, \frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m}\right)\right) + \frac{\alpha}{m} - \frac{1}{2}\right]$$

$$\langle 2 \rangle := \left[\frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m}\right)\right) + \frac{\alpha}{m} - \frac{1}{2}, \frac{\alpha\delta(1 + h)}{4(1 - \delta)h} + \frac{\alpha}{m}\right]$$

$$\langle 3 \rangle := \left[\frac{\alpha\delta(1 + h)}{4(1 - \delta)h} + \frac{\alpha}{2m}, 1\right]$$

$$i \in T^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[\alpha P, \frac{m}{2}P\right] & t \in [0, 1 - a] \\ \gamma_i \in \left[\frac{\alpha(1 - t)P}{2\left(1 - t - \frac{a}{2}\right)}, \frac{m}{2}P\right] & t \in \left[1 - a, \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}\right] \\ \gamma_i \in \emptyset & t \in \left[\frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1\right] \end{cases}$$

$$i \in A^2 \Leftrightarrow \begin{cases} \gamma_i \in \emptyset & t \in [0, 1 - a] \\ \gamma_i \in \left[\frac{\alpha(1 - t)P}{a}, \frac{\alpha(1 - t)P}{2\left(1 - t - \frac{a}{2}\right)}\right] & t \in \left[1 - a, \frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}\right] \\ \gamma_i \in \left[\frac{\alpha(1 - t)P}{a}, \frac{m}{2}P\right] & t \in \left[\frac{m\left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha}, 1\right] \end{cases}$$

$$i \in C^2 \Leftrightarrow \begin{cases} \gamma_i \in \left[-\frac{m}{2}P, \alpha P\right] & t \in [0, 1 - a] \\ \gamma_i \in \left[-\frac{m}{2}P, \frac{\alpha(1 - t)P}{a}\right] & t \in [1 - a, 1]. \end{cases}$$

Corollary 2 *The terrorist game is characterized by three equilibrium outcomes, that we call "Regimes":*

1. When

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0$$

the SPE outcome is a **Strong Regime** such that

$$\begin{aligned} n_T^1 &= n_T^2 = 0 \\ n_A^1 &= \frac{1}{2} - \frac{\alpha}{am} < n_A^2 = \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \\ n_C^1 &= \frac{1}{2} + \frac{\alpha}{am} > n_C^2 = \frac{1}{2} + \frac{\alpha}{2(m-\alpha)}. \\ t^* &= \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha}. \end{aligned}$$

2. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

the SPE outcome is a **Flexible Regime** such that

$$\begin{aligned} n_T^{1*} &= \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \\ n_T^{2*} &= \begin{cases} \frac{1}{2} - \frac{\alpha}{m} > n_T^{1*} & \text{probability } 1 - \tau \\ 0 < n_T^{1*} & \text{probability } \tau \end{cases} \\ n_A^{1*} &= 1 - \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) - \frac{2\alpha}{m} \\ n_A^{2*} &= \begin{cases} 0 < n_A^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} > n_A^{1*} & \text{probability } \tau \end{cases} \\ n_C^{1*} &= \frac{1}{2} + \frac{\alpha}{m}. \\ n_C^{2*} &= \begin{cases} \frac{1}{2} + \frac{\alpha}{m} = n_C^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} + \frac{\alpha}{2(m-\alpha)} < n_C^{1*} & \text{probability } \tau. \end{cases} \\ t^* &= \begin{cases} 0 & \text{probability } 1 - \tau \\ \frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} & \text{probability } \tau \end{cases} \end{aligned}$$

3. when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \geq \frac{1}{2} - \frac{\alpha}{m}$$

the SPE outcome is a **Permissive Regime** such that

$$n_T^{1*} = n_T^{2*} = \frac{1}{2} - \frac{\alpha}{m}$$

$$n_A^{1*} = n_A^{2*} = 0$$

$$n_C^{1*} = n_C^{2*} = \frac{1}{2} + \frac{\alpha}{m}.$$

$$t^*(n_T^1) = 0.$$

Corollary 3 When

$$\delta \leq \Delta^{SR}(\alpha, m, h) := \frac{2mh - 2\alpha h}{2mh - 2\alpha h + \alpha m + 2\alpha^2} \in (0, 1)$$

for any $a \in [0, 1]$, only the Strong Regime is possible, and this region is shrinking as α increases, or m and h decrease.

Moreover, when

$$\Leftrightarrow \delta \geq \Delta^{PR}(\alpha, m, h) := \frac{(m-2\alpha) \left(1 - \frac{2\alpha}{m} \right) h}{\alpha(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + (m-2\alpha) \left(1 - \frac{2\alpha}{m} \right) h} \in (0, 1)$$

for any $a \in [0, 1]$, only the Permissive Regime is possible, and this region is shrinking as α decreases, or m and h increase.

Proof. Let calculate the intersection of the boundary of the implicit function $\delta^{SR}(a)$ when $a = 0$:

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \Big|_{a=0} \leq 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{\alpha\delta}{(1-\delta)(m-\alpha)h} \left(\frac{1}{4} + \frac{\alpha}{2m} \right) \leq \frac{1}{2} - \frac{\alpha}{m} \Leftrightarrow \frac{\alpha\delta}{(1-\delta)h} \leq \frac{m-\alpha}{2m} \frac{4m}{m+2\alpha} \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m+2\alpha) \leq (2-2\delta)(mh-\alpha h) \Leftrightarrow \alpha\delta m + 2\alpha^2\delta \leq 2mh - 2\alpha h - 2\delta mh + 2\alpha\delta h \Leftrightarrow$$

$$\Leftrightarrow (2mh - 2\alpha h + \alpha m + 2\alpha^2)\delta \leq 2mh - 2\alpha h \Leftrightarrow \delta \leq \frac{2mh - 2\alpha h}{2mh - 2\alpha h + \alpha m + 2\alpha^2} \in (0, 1).$$

Moreover

$$\frac{\partial \Delta^{SR}(\alpha, m, h)}{\partial \alpha} = \frac{-2h(2mh - 2\alpha h + \alpha m + 2\alpha^2) - (-2h + m + 4\alpha)(2mh - 2\alpha h)}{(2mh - 2\alpha h + \alpha m + 2\alpha^2)^2} < 0$$

$$\frac{\partial \Delta^{SR}(\alpha, m, h)}{\partial m} = \frac{2h(2mh - 2\alpha h + \alpha m + 2\alpha^2) - (2h + \alpha)(2mh - 2\alpha h)}{(2mh - 2\alpha h + \alpha m + 2\alpha^2)^2} > 0$$

$$\frac{\partial \Delta^{SR}(\alpha, m, h)}{\partial h} = \frac{2(m - \alpha)(2mh - 2\alpha h + \alpha m + 2\alpha^2) - 2(m - \alpha)(2mh - 2\alpha h)}{(2mh - 2\alpha h + \alpha m + 2\alpha^2)^2} > 0$$

Let calculate the intersection of the boundary of the implicit function $\delta^{PR}(a)$ when $a = 1$:

$$\frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - \alpha m]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{\alpha m^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{2\alpha}{m} - 1 \Big|_{a=1} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - m]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{m^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) = 1 - \frac{2\alpha}{m} \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m - \alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) = (1 - \delta)(m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m - \alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \delta(m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h = (m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h \Leftrightarrow$$

$$\Leftrightarrow \delta \left[\alpha(m - \alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + (m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h \right] = (m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h$$

$$\Leftrightarrow \delta = \frac{(m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h}{\alpha(m - \alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + (m - 2\alpha) \left(1 - \frac{2\alpha}{m} \right) h} \in (0, 1)$$

Moreover

$$\frac{\partial \Delta^{PR}(\alpha, m, h)}{\partial \alpha} < 0$$

$$\frac{\partial \Delta^{PR}(\alpha, m, h)}{\partial m} > 0$$

$$\frac{\partial \Delta^{PR}(\alpha, m, h)}{\partial h} > 0$$

■

8.1 Responsiveness and political choices

Corollary 4 *The function $\alpha(a)$ such that there is a Strong regime for $a = 1$ has an ordinate smaller than $\frac{m}{2}$*

$$\alpha(1) \in \left[0, \frac{m}{2} \right]$$

hence

$$\forall a \in [0, 1]$$

an increase in α provokes a shift from the Strong Regime to the Flexible and then to the Permissive one.

Proof. let consider the implicit function $\alpha(a)$ as $a = 1$:

$$\begin{aligned} & \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} = 0 \Big|_{a=1} \Leftrightarrow \\ \Leftrightarrow & \frac{2\hat{\alpha}\delta(m-\hat{\alpha})}{(1-\delta)[2(m-\hat{\alpha})-m]h} \left(\frac{1}{4} + \frac{\hat{\alpha}}{2m} - \frac{m^2}{8(m-\hat{\alpha})^2} \left(1+h - \frac{2\hat{\alpha}h}{m} \right) \right) + \frac{\hat{\alpha}}{m} - \frac{1}{2} = 0 \Leftrightarrow \end{aligned}$$

has a significative solution $\hat{\alpha}$ such that

$$\hat{\alpha}(\delta, h, m) \in \left(0, \frac{m}{2} \right).$$

■

Corollary 5 For any

$$\alpha \geq \frac{2(1-\delta)h}{\delta(1+h)}$$

the function $m(a)$ such that there is a Strong Regime has a vertical asymptote for

$$a = \frac{4(1-\delta)h - 2\alpha\delta}{2(1-\delta)h - \alpha\delta(1+h)} \in [0, 1]$$

hence

$$\forall a \leq \frac{4(1-\delta)h - 2\alpha\delta}{2(1-\delta)h - \alpha\delta(1+h)} \in [0, 1]$$

the Strong Regime is not possible.

Proof. let consider the implicit function $m(a)$ as $m \rightarrow \infty$:

$$\begin{aligned} & \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} = 0 \xrightarrow{m \rightarrow \infty} \frac{2\alpha\delta}{(1-\delta)[2-a]h} \left(\frac{1}{4} - \frac{a}{8}(1+h) \right) \\ \Leftrightarrow & \frac{2\alpha\delta}{(1-\delta)[2-a]h} \left(\frac{1}{4} - \frac{a}{8}(1+h) \right) = \frac{1}{2} \Leftrightarrow 4\alpha\delta \left(\frac{1}{4} - \frac{a}{8}(1+h) \right) = (1-\delta)[2h-ah] \Leftrightarrow \\ \Leftrightarrow & \alpha\delta - \frac{1}{2}\alpha a\delta - \frac{1}{2}\alpha a\delta h = 2h-ah-2\delta h+a\delta h \Leftrightarrow 2\alpha\delta - \alpha a\delta - \alpha a\delta h = 4h-2ah-4\delta h+2a\delta h \Leftrightarrow \\ \Leftrightarrow & a[2(1-\delta)h - \alpha\delta(1+h)] = 4(1-\delta)h - 2\alpha\delta \Leftrightarrow a = \frac{4(1-\delta)h - 2\alpha\delta}{2(1-\delta)h - \alpha\delta(1+h)} \end{aligned}$$

where

$$\begin{aligned} a = \frac{4(1-\delta)h - 2\alpha\delta}{2(1-\delta)h - \alpha\delta(1+h)} \geq 0 & \Leftrightarrow \begin{cases} 4(1-\delta)h - 2\alpha\delta \geq 0 \\ 2(1-\delta)h - \alpha\delta(1+h) \geq 0 \end{cases} \vee \begin{cases} 4(1-\delta)h - 2\alpha\delta \leq 0 \\ 2(1-\delta)h - \alpha\delta(1+h) \leq 0 \end{cases} \Leftrightarrow \\ \Leftrightarrow & \begin{cases} \alpha \leq \frac{2(1-\delta)h}{\delta} \\ \alpha \leq \frac{2(1-\delta)h}{\delta(1+h)} \end{cases} \vee \begin{cases} \alpha \geq \frac{2(1-\delta)h}{\delta} \\ \alpha \geq \frac{2(1-\delta)h}{\delta(1+h)} \end{cases} \Leftrightarrow a \notin \left(\frac{2(1-\delta)h}{\delta(1+h)}, \frac{2(1-\delta)h}{\delta} \right) \end{aligned}$$

and

$$a = \frac{4(1-\delta)h - 2\alpha\delta}{2(1-\delta)h - \alpha\delta(1+h)} \leq 1 \Leftrightarrow 4(1-\delta)h - 2\alpha\delta \leq 2(1-\delta)h - \alpha\delta(1+h) \Leftrightarrow$$

$$\Leftrightarrow 2(1-\delta)h \leq \alpha\delta(1-h) \Leftrightarrow \alpha \geq \frac{2(1-\delta)h}{\delta(1-h)}.$$

■

Corollary 6 *When*

$$\delta \geq \frac{\frac{1}{2}m - 2\alpha + \frac{\alpha^2}{m}}{\frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m}}$$

the function $h(a)$ such that there is a Strong Regime when $a = 1$ has an ordinate $\hat{h} > 1$, hence

$$\forall a \in [0, 1] \quad \text{and} \quad \forall h \in [0, 1]$$

the Strong Regime is not possible: an increase in h provokes a shift from a Permissive Regime to a Flexible one, but a Strong Regime is not possible even for $a = 1$.

Proof. let consider the implicit function $h(a)$ as $a = 1$:

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} = 0 \Big|_{a=1} \Leftrightarrow$$

$$\Leftrightarrow \frac{\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-m]\hat{h}} \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} - \hat{h} \frac{m^2}{4(m-\alpha)^2} \left(1 - \frac{2\alpha}{m} \right) \right) = \frac{1}{2} - \frac{\alpha}{m} \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} - \hat{h} \frac{m^2}{4(m-\alpha)^2} \left(1 - \frac{2\alpha}{m} \right) \right) = \hat{h}(1-\delta)(m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} \right) - \hat{h} \frac{\alpha\delta m^2}{4(m-\alpha)} \left(1 - \frac{2\alpha}{m} \right) = \hat{h}(1-\delta)(m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} \right) = \hat{h} \left[\frac{\alpha\delta m^2}{2(m-\alpha)} + (1-\delta)(m-2\alpha) \right] \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

hence

$$\hat{h} \geq 1 \Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} \right) \geq \left[\frac{\alpha\delta m^2}{2(m-\alpha)} + (1-\delta)(m-2\alpha) \right] \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{4(m-\alpha)^2} - \frac{m^2}{2(m-\alpha)^2} \left(\frac{1}{2} - \frac{\alpha}{m} \right) \right) \geq (1-\delta)(m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

$$\Leftrightarrow \alpha\delta(m-\alpha) \left(\frac{1}{2} + \frac{\alpha}{m} - \frac{m^2}{2(m-\alpha)^2} + \frac{\alpha m}{2(m-\alpha)^2} \right) \geq (1-\delta)(m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow$$

$$\begin{aligned}
&\Leftrightarrow \delta \left(\frac{\alpha(m-\alpha)}{2} + \frac{\alpha^2(m-\alpha)}{m} - \frac{\alpha m}{2} \right) \geq (m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) - \delta(m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) \Leftrightarrow \\
&\Leftrightarrow \delta \left(-\frac{\alpha^2}{2} + \alpha^2 - \frac{\alpha^3}{m} \right) \geq (m-2\alpha) \left(\frac{1}{2} - \frac{\alpha}{m} \right) - \delta \left(\frac{1}{2}m - \alpha - \alpha + \frac{2\alpha^2}{m} \right) \Leftrightarrow \\
&\Leftrightarrow \delta \left(\frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m} \right) \geq \frac{1}{2}m - \alpha - \alpha + \frac{\alpha^2}{m} \Leftrightarrow \\
&\Leftrightarrow \delta \geq \frac{\frac{1}{2}m - 2\alpha + \frac{\alpha^2}{m}}{\frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m}}
\end{aligned}$$

where

$$\begin{aligned}
\frac{\frac{1}{2}m - 2\alpha + \frac{\alpha^2}{m}}{\frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m}} \leq 1 &\Leftrightarrow \frac{1}{2}m - 2\alpha + \frac{2\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m} \geq \frac{1}{2}m - 2\alpha + \frac{\alpha^2}{m} \Leftrightarrow \\
&\Leftrightarrow \frac{\alpha^2}{m} + \frac{\alpha^2}{2} - \frac{\alpha^3}{m} \geq 0 \Leftrightarrow \frac{1}{m} + \frac{1}{2} - \frac{\alpha}{m} \geq 0 \Leftrightarrow m \geq 2\alpha - 2
\end{aligned}$$

which is always satisfied. ■

8.2 Accountability and political choices

Corollary 7 *The functions $\alpha^{SR}(\delta)$ and $\alpha^{PR}(\delta)$ such that there is a Strong Regime and a Permissive Regime for $\delta = 1$ have the same ordinate equal to zero and for $\delta = 0$ have the same ordinate equal to $\frac{m}{2}$, hence*

$$\forall \delta \in [0, 1]$$

an increase in α provokes a shift from the Strong Regime to the Flexible and then to the Permissive one.

Proof. let consider the implicit function $\alpha^{SR}(\delta)$ as $\delta = 0$:

$$\begin{aligned}
\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} &= 0 \Big|_{\delta=0} \Leftrightarrow \\
&\Leftrightarrow \frac{\alpha}{m} - \frac{1}{2} = 0 \Leftrightarrow \alpha = \frac{m}{2};
\end{aligned}$$

Similarly, let consider the implicit function $\alpha^{PR}(\delta)$ as $\delta = 0$:

$$\begin{aligned}
\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} &= \frac{1}{2} - \frac{\alpha}{m} \Big|_{\delta=0} \Leftrightarrow \\
&\Leftrightarrow \frac{\alpha}{m} - \frac{1}{2} = \frac{1}{2} - \frac{\alpha}{m} \Leftrightarrow \alpha = \frac{m}{2}.
\end{aligned}$$

Again, let consider let consider the implicit function $\alpha^{SR}(\delta)$ as $\delta = 1$:

$$2\alpha\delta(m-\alpha)\left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2}\left(1+h - \frac{2\alpha h}{m}\right)\right) + (1-\delta)[2(m-\alpha) - am]h\left(\frac{\alpha}{m} - \frac{1}{2}\right) = 0 \Big|_{\delta=1} \Leftrightarrow$$

$$\Leftrightarrow 2\alpha(m-\alpha)\left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2}\left(1+h - \frac{2\alpha h}{m}\right)\right) = 0 \Leftrightarrow \alpha = 0$$

Similarly, let consider the implicit function $\alpha^{PR}(\delta)$ as $\delta = 1$:

$$2\alpha\delta(m-\alpha)\left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2}\left(1+h - \frac{2\alpha h}{m}\right)\right) + (1-\delta)[2(m-\alpha) - am]h\left(\frac{2\alpha}{m} - 1\right) = 0 \Big|_{\delta=1} \Leftrightarrow$$

$$\Leftrightarrow 2\alpha(m-\alpha)\left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2}\left(1+h - \frac{2\alpha h}{m}\right)\right) = 0 \Leftrightarrow \alpha = 0.$$

■

Corollary 8 *The function $m(\delta)$ such that there is a Permissive Regime has a vertical asymptote for*

$$\delta = \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah} \in [0, 1]$$

hence

$$\delta \geq \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah} \in [0, 1]$$

the Strong Regime is not possible.

Proof. let consider the implicit function $m^{PR}(a)$ as $m \rightarrow \infty$:

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha) - am]h}\left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2}\left(1+h - \frac{2\alpha h}{m}\right)\right) + \frac{2\alpha}{m} - 1 = 0 \xrightarrow{m \rightarrow \infty} \frac{2\alpha\delta}{(1-\delta)[2-a]h}\left(\frac{1}{4} - \frac{a}{8}(1+h)\right) = 1$$

$$\Leftrightarrow \frac{2\alpha\delta}{(1-\delta)[2-a]h}\left(\frac{1}{4} - \frac{a}{8}(1+h)\right) = 1 \Leftrightarrow 2\alpha\delta\left(\frac{1}{4} - \frac{a}{8}(1+h)\right) = (1-\delta)[2h - ah] \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\alpha\delta - \frac{1}{4}\alpha a\delta - \frac{1}{4}\alpha a\delta h = 2h - ah - 2\delta h + a\delta h \Leftrightarrow 2\alpha\delta - \alpha a\delta - \alpha a\delta h = 8h - 4ah - 8\delta h + 4a\delta h \Leftrightarrow$$

$$\Leftrightarrow (8h + 2\alpha - 4ah - \alpha a - \alpha ah)\delta = 4(2-a)h \Leftrightarrow \delta = \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah}$$

where

$$\delta = \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah} \geq 0 \Leftrightarrow 8h+2\alpha-4ah-\alpha a-\alpha ah \geq 0 \Leftrightarrow$$

$$\Leftrightarrow 8h+2\alpha \geq 4ah+\alpha a+\alpha ah \Leftrightarrow a \leq \frac{8h+2\alpha}{4h+\alpha(1+h)}$$

which is always satisfied since $\frac{8h+2\alpha}{4h+\alpha(1+h)} > 1$, moreover

$$\delta = \frac{4(2-a)h}{8h+2\alpha-4ah-\alpha a-\alpha ah} \leq 1 \Leftrightarrow 8h-4ah \leq 8h+2\alpha-4ah-\alpha a-\alpha ah \Leftrightarrow 2\alpha \geq \alpha a(1+h)$$

which again is always satisfied.

Now, consider let consider the implicit function $m^{SR}(a)$ as $m \rightarrow \infty$:

$$\begin{aligned} & \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} = 0 \xrightarrow{m \rightarrow \infty} \frac{2\alpha\delta}{(1-\delta)[2-a]h} \left(\frac{1}{4} \right. \\ & \Leftrightarrow \frac{4\alpha\delta}{(1-\delta)[2-a]h} \left(\frac{1}{4} - \frac{a}{8}(1+h) \right) = 1 \Leftrightarrow 4\alpha\delta \left(\frac{1}{4} - \frac{a}{8}(1+h) \right) = (1-\delta)[2h-ah] \Leftrightarrow \\ & \Leftrightarrow \alpha\delta - \frac{1}{2}\alpha a\delta - \frac{1}{2}\alpha a\delta h = 2h-ah-2\delta h+a\delta h \Leftrightarrow 2\alpha\delta - \alpha a\delta - \alpha a\delta h = 4h-2ah-4\delta h+2a\delta h \Leftrightarrow \\ & \Leftrightarrow (4h+2\alpha-2ah-\alpha a-\alpha ah)\delta = 2(2-a)h \Leftrightarrow \delta = \frac{2(2-a)h}{4h+2\alpha-2ah-\alpha a-\alpha ah} \end{aligned}$$

where

$$\begin{aligned} \delta &= \frac{2(2-a)h}{4h+2\alpha-2ah-\alpha a-\alpha ah} \geq 0 \Leftrightarrow 4h+2\alpha-2ah-\alpha a-\alpha ah \geq 0 \Leftrightarrow \\ & \Leftrightarrow 4h+2\alpha \geq 2ah+\alpha a+\alpha ah \Leftrightarrow a \leq \frac{4h+2\alpha}{2h+\alpha(1+h)} \end{aligned}$$

which is always satisfied since $\frac{4h+2\alpha}{2h+\alpha(1+h)} > 1$, moreover

$$\delta = \frac{2(2-a)h}{4h+2\alpha-2ah-\alpha a-\alpha ah} \leq 1 \Leftrightarrow 4h-2ah \leq 4h+2\alpha-2ah-\alpha a-\alpha ah \Leftrightarrow 2\alpha \geq \alpha a(1+h)$$

which again is always satisfied. ■

Corollary 9 *The functions $h^{SR}(\delta)$ and $h^{PR}(\delta)$ such that there is, respectively, a Strong and a Permissive Regime when $h = 1$, have the abscissas*

$$\begin{aligned} \widehat{\delta}^{SR} &= \frac{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am}{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am} \in [0, 1] \\ \widehat{\delta}^{PR} &= \frac{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am}{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am} \in [0, 1] \end{aligned}$$

hence

$$\forall \delta \geq \widehat{\delta}^{PR} = \frac{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am}{4m+4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am} \quad \text{and} \quad \forall h \in [0, 1]$$

only the Permissive Regime is possible,

$$\forall \delta \in \left[\widehat{\delta}^{SR}, \widehat{\delta}^{PR} \right] \quad \text{and} \quad \forall h \in [0, 1]$$

an increase in h provokes a shift from a Permissive Regime to a Flexible one, but a Strong Regime is possible only for

$$\delta \leq \widehat{\delta}^{SR} = \frac{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am}{2m+2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am}.$$

Proof. let consider the implicit function $h^{SR}(\delta)$ as $h = 1$:

$$\begin{aligned} & \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} = 0 \Big|_{h=1} \Leftrightarrow \\ & \Leftrightarrow 2\alpha\delta(m-\alpha) \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(\frac{2(m-\alpha)}{m} \right) \right) + (1-\delta)[2(m-\alpha)-am] \left(\frac{\alpha}{m} - \frac{1}{2} \right) = 0 \Leftrightarrow \\ & \Leftrightarrow 2\alpha\delta(m-\alpha) \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am}{4(m-\alpha)} \right) + [2(m-\alpha)-am] \left(\frac{\alpha}{m} - \frac{1}{2} \right) + [a\delta m - 2\delta(m-\alpha)] \left(\frac{\alpha}{m} - \frac{1}{2} \right) = 0 \Leftrightarrow \\ & \Leftrightarrow \frac{1}{2}\alpha\delta(m-\alpha) + \frac{\alpha^2\delta(m-\alpha)}{m} - \frac{1}{2}\alpha a\delta m + \frac{2\alpha(m-\alpha)}{m} - \alpha a - (m-\alpha) + \frac{1}{2}am + \alpha a\delta - \frac{2\alpha\delta(m-\alpha)}{m} + -\frac{1}{2}a\delta m + \delta(m-\alpha) = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta(m-\alpha) + \frac{2\alpha^2\delta(m-\alpha)}{m} - \alpha a\delta m + \frac{4\alpha(m-\alpha)}{m} - 2\alpha a - 2(m-\alpha) + am + 2\alpha a\delta - \frac{4\alpha\delta(m-\alpha)}{m} - a\delta m + 2\delta(m-\alpha) = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta m - \alpha^2\delta + 2\alpha^2\delta - \frac{2\alpha^3\delta}{m} - \alpha a\delta m + 4\alpha - \frac{4\alpha^2}{m} - 2\alpha a - 2m + 2\alpha + am + 2\alpha a\delta - 4\alpha\delta + \frac{4\alpha^2\delta}{m} - a\delta m + 2\delta m - 2\alpha\delta = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta m + \alpha^2\delta - \frac{2\alpha^3\delta}{m} - \alpha a\delta m + 6\alpha - \frac{4\alpha^2}{m} - 2\alpha a - 2m + am + 2\alpha a\delta - 6\alpha\delta + \frac{4\alpha^2\delta}{m} - a\delta m + 2\delta m = 0 \Leftrightarrow \\ & \Leftrightarrow \left(2m + 2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am + \alpha m + \alpha^2 - \frac{2\alpha^3}{m} - \alpha am \right) \delta = 2m + 2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am \Leftrightarrow \\ & \Leftrightarrow \widehat{\delta}^{SR} = \frac{2m + 2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am}{2m + 2\alpha a + \frac{4\alpha^2}{m} - 6\alpha - am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am} \in [0, 1] \end{aligned}$$

Similarly, let consider the implicit function $h^{PR}(\delta)$ as $h = 1$:

$$\begin{aligned} & \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{2\alpha}{m} - 1 = 0 \Big|_{h=1} \Leftrightarrow \\ & \Leftrightarrow 2\alpha\delta(m-\alpha) \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(\frac{2(m-\alpha)}{m} \right) \right) + (1-\delta)[2(m-\alpha)-am] \left(\frac{2\alpha}{m} - 1 \right) = 0 \Leftrightarrow \\ & \Leftrightarrow 2\alpha\delta(m-\alpha) \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am}{4(m-\alpha)} \right) + [2(m-\alpha)-am] \left(\frac{2\alpha}{m} - 1 \right) + [a\delta m - 2\delta(m-\alpha)] \left(\frac{2\alpha}{m} - 1 \right) = 0 \Leftrightarrow \\ & \Leftrightarrow \frac{1}{2}\alpha\delta(m-\alpha) + \frac{\alpha^2\delta(m-\alpha)}{m} - \frac{1}{2}\alpha a\delta m + \frac{4\alpha(m-\alpha)}{m} - 2\alpha a - 2(m-\alpha) + am + 2\alpha a\delta - \frac{4\alpha\delta(m-\alpha)}{m} - a\delta m + 2\delta(m-\alpha) = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta(m-\alpha) + \frac{2\alpha^2\delta(m-\alpha)}{m} - \alpha a\delta m + \frac{8\alpha(m-\alpha)}{m} - 4\alpha a - 4(m-\alpha) + 2am + 4\alpha a\delta - \frac{8\alpha\delta(m-\alpha)}{m} - 2a\delta m + 4\delta(m-\alpha) = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta m - \alpha^2\delta + 2\alpha^2\delta - \frac{2\alpha^3\delta}{m} - \alpha a\delta m + 8\alpha - \frac{8\alpha^2}{m} - 4\alpha a - 4m + 4\alpha + 2am + 4\alpha a\delta - 8\alpha\delta + \frac{8\alpha^2\delta}{m} - 2a\delta m + 4\delta m - 4\alpha\delta = 0 \Leftrightarrow \\ & \Leftrightarrow \alpha\delta m + \alpha^2\delta - \frac{2\alpha^3\delta}{m} - \alpha a\delta m + 12\alpha - \frac{8\alpha^2}{m} - 4\alpha a - 4m + 2am + 4\alpha a\delta - 12\alpha\delta + \frac{8\alpha^2\delta}{m} - 2a\delta m + 4\delta m = 0 \Leftrightarrow \\ & \Leftrightarrow \left(4m + 4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am + \alpha m + \alpha^2 - \frac{2\alpha^3}{m} - \alpha am \right) \delta = 4m + 4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am \Leftrightarrow \\ & \Leftrightarrow \widehat{\delta}^{PR} = \frac{4m + 4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am}{4m + 4\alpha a + \frac{8\alpha^2}{m} - 12\alpha - 2am + \alpha m + \alpha^2 - \frac{2\alpha^3\delta}{m} - \alpha am} \in [0, 1] \end{aligned}$$

■

9 Comparative statics

Now we derive some basic comparative statics results relative to the size of these region w.r.t. the exogenous parameters and, within each region, about the political strategic choices.

9.1 The Strong Regime

From the characterization of the possible equilibria, we know that there is a String Regime if and only if

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0.$$

This condition implicitly defines a region $\dot{A}(\text{Strong})$ in the space $(\delta, a) \in [0, 1] \times [0, 1]$:

$$\begin{aligned} \dot{A}(\text{Strong}) &:= \\ := \left\{ (\delta, a) \in [0, 1] \times [0, 1] : \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0 \right\} \end{aligned}$$

Then, it easy to derive the following result.

Corollary 10 *Let define $\dot{A}(\text{Strong})$ as the region of the (δ, a) region where there is a strong regime. Then*

$$\begin{aligned} \frac{\partial \dot{A}(\text{Strong})}{\partial m} &> 0, & \frac{\partial \dot{A}(\text{Strong})}{\partial \alpha} &< 0, & \frac{\partial \dot{A}(\text{Strong})}{\partial h} &> 0 \\ \frac{\partial n_T^{1*}}{\partial m} &= \frac{\partial n_T^{1*}}{\partial \alpha} = \frac{\partial n_T^{1*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial m} = \frac{\partial n_T^{2*}}{\partial \alpha} = \frac{\partial n_T^{2*}}{\partial a} &= 0 \\ \frac{\partial n_A^{1*}}{\partial m} &> 0, & \frac{\partial n_A^{1*}}{\partial \alpha} &< 0, & \frac{\partial n_A^{1*}}{\partial a} &> 0, & \frac{\partial n_A^{2*}}{\partial m} &> 0, & \frac{\partial n_A^{2*}}{\partial \alpha} &< 0, & \frac{\partial n_A^{2*}}{\partial a} &= 0 \\ \frac{\partial n_C^{1*}}{\partial m} &< 0, & \frac{\partial n_C^{1*}}{\partial \alpha} &> 0, & \frac{\partial n_C^{1*}}{\partial a} &< 0, & \frac{\partial n_C^{2*}}{\partial m} &< 0, & \frac{\partial n_C^{2*}}{\partial \alpha} &> 0, & \frac{\partial n_C^{2*}}{\partial a} &= 0 \\ \frac{\partial t^*(n_T^1)}{\partial m} &> 0, & \frac{\partial t^*(n_T^1)}{\partial \alpha} &< 0, & \frac{\partial t^*(n_T^1)}{\partial a} &< 0. \end{aligned}$$

Proof. We get a repressive regime when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \leq 0$$

It is easy to check that the (δ, a) region where this condition is satisfied is shrinking as α increases or m and h decrease. Moreover

$$n_T^{1*} = n_T^{2*} = 0 \Rightarrow \frac{\partial n_T^{1*}}{\partial m} = \frac{\partial n_T^{1*}}{\partial \alpha} = \frac{\partial n_T^{1*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial m} = \frac{\partial n_T^{2*}}{\partial \alpha} = \frac{\partial n_T^{2*}}{\partial a} = 0$$

$$\begin{aligned} \frac{\partial n_A^{1*}}{\partial m} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{am} \right)}{\partial m} = \frac{\alpha}{am^2} > 0 \\ \frac{\partial n_A^{1*}}{\partial \alpha} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{am} \right)}{\partial \alpha} = -\frac{1}{ma} < 0 \\ \frac{\partial n_A^{1*}}{\partial a} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{am} \right)}{\partial m} = \frac{\alpha}{a^2m} > 0 \\ \frac{\partial n_A^{2*}}{\partial m} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \right)}{\partial m} = \frac{\alpha}{2(m-\alpha)^2} > 0 \\ \frac{\partial n_A^{2*}}{\partial \alpha} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \right)}{\partial \alpha} = -\frac{2(m-\alpha) + 2\alpha}{4(m-\alpha)^2} = -\frac{2m}{4(m-\alpha)^2} < 0 \\ \frac{\partial n_A^{2*}}{\partial a} &= \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{2(m-\alpha)} \right)}{\partial \alpha} = 0 \\ \frac{\partial n_C^{1*}}{\partial m} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{am} \right)}{\partial m} = -\frac{\alpha}{am^2} < 0 \\ \frac{\partial n_C^{1*}}{\partial \alpha} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{am} \right)}{\partial \alpha} = \frac{1}{am} > 0 \\ \frac{\partial n_C^{1*}}{\partial a} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{am} \right)}{\partial m} = -\frac{\alpha}{a^2m} < 0 \\ \frac{\partial n_C^{2*}}{\partial m} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{2(m-\alpha)} \right)}{\partial m} = -\frac{\alpha}{2(m-\alpha)^2} < 0 \\ \frac{\partial n_C^{2*}}{\partial \alpha} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{2(m-\alpha)} \right)}{\partial \alpha} = \frac{2(m-\alpha) + 2\alpha}{4(m-\alpha)^2} = \frac{m}{2(m-\alpha)^2} > 0 \\ \frac{\partial n_C^{2*}}{\partial a} &= \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{2(m-\alpha)} \right)}{\partial \alpha} = 0 \\ \frac{\partial t^* (n_T^1)}{\partial m} &= \frac{\partial \left(\frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \right)}{\partial m} = \frac{\left(1 - \frac{a}{2} \right) (m - \alpha) - m \left(1 - \frac{a}{2} \right) + \alpha}{(m - \alpha)^2} \\ &= \frac{\left(1 - \frac{a}{2} \right) (-\alpha) + \alpha}{(m - \alpha)^2} = \frac{\alpha a}{2(m - \alpha)^2} > 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial t^*(n_T^1)}{\partial \alpha} &= \frac{\partial \left(\frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \right)}{\partial \alpha} = \frac{-(m - \alpha) + m \left(1 - \frac{a}{2} \right) - \alpha}{(m - \alpha)^2} = \\ &= \frac{-\frac{am}{2}}{(m - \alpha)^2} = -\frac{am}{2(m - \alpha)^2} < 0 \\ \frac{\partial t^*(n_T^1)}{\partial a} &= \frac{\partial \left(\frac{m \left(1 - \frac{a}{2} \right) - \alpha}{m - \alpha} \right)}{\partial a} = -\frac{m}{2(m - \alpha)} < 0. \end{aligned}$$

■

9.2 The Flexible Regime

From the characterization of the possible equilibria, we know that there is a String Regime if and only if

$$\frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right].$$

This condition implicitly defines a region $\dot{A}(Flex)$ in the space $(\delta, a) \in [0, 1] \times [0, 1]$:

$$\begin{aligned} \dot{A}(Flex) &:= \\ := \left\{ (\delta, a) \in [0, 1] \times [0, 1] : \frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \right. \end{aligned}$$

Then, it easy to derive the following result.

Corollary 11 *Let define $\dot{A}(Flex)$ as the region of the (δ, a) region where there is a Flexible Regime. Then*

$$\begin{aligned} \frac{\partial \dot{A}(Flex)}{\partial m} < 0, \quad \frac{\partial \dot{A}(Flex)}{\partial \alpha} > 0, \quad \frac{\partial \dot{A}(Flex)}{\partial h} = 0 \\ \frac{\partial n_T^{1*}}{\partial m} < 0, \quad \frac{\partial n_T^{1*}}{\partial \alpha} > 0, \quad \frac{\partial n_T^{1*}}{\partial a} < 0, \quad \frac{\partial n_T^{1*}}{\partial h} < 0 \\ \frac{\partial n_T^{2*}}{\partial m} \geq 0, \quad \frac{\partial n_T^{2*}}{\partial \alpha} \leq 0, \quad \frac{\partial n_T^{2*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial h} = 0 \\ \frac{\partial n_A^{1*}}{\partial m} > 0, \quad \frac{\partial n_A^{1*}}{\partial \alpha} < 0, \quad \frac{\partial n_A^{1*}}{\partial a} > 0, \quad \frac{\partial n_A^{1*}}{\partial h} > 0 \\ \frac{\partial n_A^{2*}}{\partial m} \geq 0, \quad \frac{\partial n_A^{2*}}{\partial \alpha} \leq 0, \quad \frac{\partial n_A^{2*}}{\partial a} = \frac{\partial n_A^{2*}}{\partial h} = 0 \\ \frac{\partial n_C^{1*}}{\partial m} < 0, \quad \frac{\partial n_C^{1*}}{\partial \alpha} > 0, \quad \frac{\partial n_C^{1*}}{\partial a} = \frac{\partial n_C^{1*}}{\partial h} = 0 \end{aligned}$$

$$\frac{\partial n_C^{2*}}{\partial m} < 0, \frac{\partial n_C^{2*}}{\partial \alpha} > 0, \frac{\partial n_C^{2*}}{\partial a} = \frac{\partial n_C^{2*}}{\partial h} = 0$$

$$\frac{\partial t^*(n_T^1)}{\partial m} > 0, \frac{\partial t^*(n_T^1)}{\partial \alpha} < 0, \frac{\partial t^*(n_T^1)}{\partial a} < 0.$$

Proof. Since we get a repressive regime when

$$\frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \in \left[0, \frac{1}{2} - \frac{\alpha}{m} \right]$$

then

$$\dot{A}(Flex) = \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{2\alpha}{m} - 1 +$$

$$- \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) - \frac{\alpha}{m} + \frac{1}{2} =$$

$$= \frac{\alpha}{m} - \frac{1}{2}$$

It is easy to check that the (δ, a) region where this condition is satisfied is shrinking as α decreases or m increase, while h has no effect. Moreover

$$n_T^{1*} = \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{\partial n_T^{1*}}{\partial m} < 0, \frac{\partial n_T^{1*}}{\partial \alpha} > 0, \frac{\partial n_T^{1*}}{\partial a} < 0, \frac{\partial n_T^{1*}}{\partial h} < 0$$

$$n_T^{2*} = \begin{cases} \frac{1}{2} - \frac{\alpha}{m} > n_T^{1*} & \text{probability } 1 - \tau \\ 0 < n_T^{1*} & \text{probability } \tau \end{cases} \Rightarrow$$

$$\Rightarrow \frac{\partial n_T^{2*}}{\partial m} \geq 0, \frac{\partial n_T^{2*}}{\partial \alpha} \leq 0, \frac{\partial n_T^{2*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial h} = 0$$

$$n_A^{1*} = 1 - \frac{2\alpha\delta(m-\alpha)}{(1-\delta)[2(m-\alpha)-am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m-\alpha)^2} \left(1+h - \frac{2\alpha h}{m} \right) \right) - \frac{2\alpha}{m} \Rightarrow$$

$$\Rightarrow \frac{\partial n_A^{1*}}{\partial m} > 0, \frac{\partial n_A^{1*}}{\partial \alpha} < 0, \frac{\partial n_A^{1*}}{\partial a} > 0, \frac{\partial n_A^{1*}}{\partial h} > 0$$

$$n_A^{2*} = \begin{cases} 0 < n_A^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} - \frac{\alpha}{2(m-\alpha)} > n_A^{1*} & \text{probability } \tau \end{cases} \Rightarrow$$

$$\Rightarrow \frac{\partial n_A^{2*}}{\partial m} \geq 0, \frac{\partial n_A^{2*}}{\partial \alpha} \leq 0, \frac{\partial n_A^{2*}}{\partial a} = \frac{\partial n_A^{2*}}{\partial h} = 0$$

$$n_C^{1*} = \frac{1}{2} + \frac{\alpha}{m} \Rightarrow$$

$$\Rightarrow \frac{\partial n_C^{1*}}{\partial m} < 0, \frac{\partial n_C^{1*}}{\partial \alpha} > 0, \frac{\partial n_C^{1*}}{\partial a} = \frac{\partial n_C^{1*}}{\partial h} = 0$$

$$\begin{aligned}
n_C^{2*} &= \begin{cases} \frac{1}{2} + \frac{\alpha}{m} = n_C^{1*} & \text{probability } 1 - \tau \\ \frac{1}{2} + \frac{\alpha}{2(m - \alpha)} < n_C^{1*} & \text{probability } \tau \end{cases} \Rightarrow \\
&\Rightarrow \frac{\partial n_C^{2*}}{\partial m} < 0, \frac{\partial n_C^{2*}}{\partial \alpha} > 0, \frac{\partial n_C^{2*}}{\partial a} = \frac{\partial n_C^{2*}}{\partial h} = 0 \\
t^* &= \begin{cases} 0 & \text{probability } 1 - \tau \\ \frac{m \left(1 - \frac{a}{2}\right) - \alpha}{m - \alpha} & \text{probability } \tau \end{cases} \Rightarrow \\
&\Rightarrow \frac{\partial t^* (n_T^1)}{\partial m} > 0, \frac{\partial t^* (n_T^1)}{\partial \alpha} < 0, \frac{\partial t^* (n_T^1)}{\partial a} < 0.
\end{aligned}$$

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9.3 The Permissive Regime

From the characterization of the possible equilibria, it easy to derive the following result.

Corollary 12 *Let define \dot{A} (Permissive) as the measure of the (δ, α) region where there is a repressive regime. Then*

$$\begin{aligned}
\frac{\partial \dot{A} (\text{Permissive})}{\partial m} < 0, \quad \frac{\partial \dot{A} (\text{Permissive})}{\partial \alpha} > 0, \quad \frac{\partial \dot{A} (\text{Permissive})}{\partial h} < 0 \\
\frac{\partial n_T^{1*}}{\partial m} = \frac{\partial n_T^{2*}}{\partial m} > 0, \quad \frac{\partial n_T^{1*}}{\partial \alpha} = \frac{\partial n_T^{2*}}{\partial \alpha} < 0, \quad \frac{\partial n_T^{1*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial a} = 0 \\
\frac{\partial n_A^{1*}}{\partial m} = \frac{\partial n_A^{1*}}{\partial \alpha} = \frac{\partial n_A^{2*}}{\partial m} = \frac{\partial n_A^{2*}}{\partial \alpha} = \frac{\partial n_A^{1*}}{\partial a} = \frac{\partial n_A^{2*}}{\partial a} = 0 \\
\frac{\partial n_C^{1*}}{\partial m} = \frac{\partial n_C^{2*}}{\partial m} < 0, \quad \frac{\partial n_C^{1*}}{\partial \alpha} = \frac{\partial n_C^{2*}}{\partial \alpha} > 0, \quad \frac{\partial n_C^{1*}}{\partial a} = \frac{\partial n_C^{2*}}{\partial a} = 0 \\
\frac{\partial t^* (n_T^1)}{\partial m} = \frac{\partial t^* (n_T^1)}{\partial \alpha} = \frac{\partial t^* (n_T^1)}{\partial a} = 0.
\end{aligned}$$

Proof. We get a Permissive Regime when

$$\frac{2\alpha\delta(m - \alpha)}{(1 - \delta)[2(m - \alpha) - am]h} \left(\frac{1}{4} + \frac{\alpha}{2m} - \frac{am^2}{8(m - \alpha)^2} \left(1 + h - \frac{2\alpha h}{m} \right) \right) + \frac{\alpha}{m} - \frac{1}{2} \geq \frac{1}{2} - \frac{\alpha}{m}$$

It is easy to check that the (δ, a) region where this condition is satisfied is increasing as α increases or m and h decrease. Moreover

$$\begin{aligned}
n_T^{1*} = n_T^{2*} &= \frac{1}{2} - \frac{\alpha}{m} \\
n_A^{1*} = n_A^{2*} &= 0 \\
n_C^{1*} = n_C^{2*} &= \frac{1}{2} + \frac{\alpha}{m}.
\end{aligned}$$

$$t^*(n_T^1) = 0.$$

Hence

$$\frac{\partial n_T^{1*}}{\partial m} = \frac{\partial n_T^{2*}}{\partial m} = \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{m} \right)}{\partial m} = \frac{\alpha}{m^2} > 0$$

$$\frac{\partial n_T^{1*}}{\partial \alpha} = \frac{\partial n_T^{2*}}{\partial \alpha} = \frac{\partial \left(\frac{1}{2} - \frac{\alpha}{m} \right)}{\partial \alpha} = -\frac{1}{m} < 0$$

$$\frac{\partial n_T^{1*}}{\partial a} = \frac{\partial n_T^{2*}}{\partial a} = 0$$

$$n_A^{1*} = n_A^{2*} = 0 \Rightarrow \frac{\partial n_A^{1*}}{\partial m} = \frac{\partial n_A^{1*}}{\partial \alpha} = \frac{\partial n_A^{1*}}{\partial a} = \frac{\partial n_A^{2*}}{\partial m} = \frac{\partial n_A^{2*}}{\partial \alpha} = \frac{\partial n_A^{2*}}{\partial a} = 0$$

$$\frac{\partial n_C^{1*}}{\partial m} = \frac{\partial n_C^{2*}}{\partial m} = \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{m} \right)}{\partial m} = -\frac{\alpha}{m^2} < 0$$

$$\frac{\partial n_C^{1*}}{\partial \alpha} = \frac{\partial n_C^{2*}}{\partial \alpha} = \frac{\partial \left(\frac{1}{2} + \frac{\alpha}{m} \right)}{\partial \alpha} = \frac{1}{m} > 0$$

$$\frac{\partial n_C^{1*}}{\partial a} = \frac{\partial n_C^{2*}}{\partial a} = 0$$

$$\frac{\partial t^*(n_T^1)}{\partial m} = \frac{\partial t^*(n_T^1)}{\partial \alpha} = \frac{\partial t^*(n_T^1)}{\partial a} = 0.$$

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