Disinflation, Inequality and Welfare in a TANK Model

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Disinflation, Inequality and Welfare in a TANK Model

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Abstract

We investigate the redistributive and welfare effects of disinflation in a two-agent New Keynesian (TANK) model characterized by Limited Asset Market Participation (LAMP) and wealth inequality. We highlight two key mechanisms driving our long-run results: i) the cash in advance constraint on firms working capital (CIA); ii) dividends endogeneity. These two channels point in opposite directions. Lower inflation softens the CIA and, by raising labor demand, lowers inequality. But the disinflation also raises dividends and this increases inequality. The disinflation is always welfare-improving for asset holders. We obtain ambiguous results for non-asset holders, who suffer substantial consumption losses during the transition.

JEL Classification System: E31, E5, D3, D6

Keywords: Firms Pricing, Disinflation, Inequality, Welfare Economics

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1 Introduction

This paper investigates the short- and long-run effects of a monetary policy regime change, i.e. a disinflation, on inequality and on the welfare of different households groups.

Recent years have witnessed increasing concern for the distributive effects of monetary policies. A consensus exists that temporary contractionary shocks increase inequality (Romer and Romer, 1998; Coibion et al., 2012; Furceri et al. 2018). By contrast, empirical studies on the long-run effects of monetary regime changes have obtained contradictory results. Coibion et al. (2017) find that a reduction in the Fed inflation target causes strong cumulative effects on consumption and expenditures inequality, but has only temporary adverse effects on incomes and earnings. Their measures of inequality are based on the cross-sectional standard deviations and their data, taken from the Consumer Expenditures Survey, do not include the top one percent of the income distribution which has driven inequality dynamics since 1980. Using a different identification method for permanent inflation shocks and the Gini index which covers the full population, Davtyan (2017) finds that a disinflation lowers inequality in the US. Some earlier country-specific studies document that higher inflation is correlated with a lower income share held by the poorest part of the population (Blejer and Guerrero, 1990; Datt and Ravallion,1998). Several studies document a positive cross country correlation between inflation and inequality over relatively long time spans, suggesting that a permanent disinflation should be associated with a long-run reduction in inequality (Romer and Romer, 1998; Bulíµr, 2001; Easterly and Fisher, 2001; Li and Zou, 2002; Albanesi, 2007, Nantob, 2015). In a sample of ten OECD countries Monnin (2014) finds a U-shaped link between long-run inflation and income inequality, measured as the income share of the top 10% earners.

The effects of monetary policies are typically investigated in DSGE models that incorporate price and nominal wage rigidities and market imperfections such as financial frictions and firms monopoly power. In addition to the traditional short-run effects, where an unexpected monetary contraction causes a fall in GDP, such models allow for two channels of monetary policy non-neutrality in the long run.

The first one is the cost channel of monetary policy. Christiano et al. (2005; CEE henceforth) introduce a cash-in-advance constraint on firms working capital (CIA henceforth), where the real financing cost is determined by the nominal interest rate. In this framework a change in long-run inflation causes a permanent fall in the nominal interest rate which lowers the unit labor cost and raises labor demand. Empirical evidence broadly confirms the relevance of such channel (Barth and Ramey, 2001; Chowdhury et al. 2006; Gaiotti and Secchi, 2006; Ravenna and Walsh, 2006; Rabanal, 2007; Tillmann, 2008; Henzel et al., 2009), but considerable uncertainty exists about its effective strength.

The second one is the inflation sensitivity of real price (and wage) markups. In fact, a positive steady-state inflation rate affects firms price markups because expectations about future inflation affect current price-setting decisions (Ascari,
The two most commonly used formalisms for price and wage setting, i.e. the Calvo (1983) staggered contracts and the Rotemberg (1982) quadratic cost of adjustment, share this feature but predict opposite effects of steady state inflation on markups. Ascarì and Rossi (2012) show that disinflation raises markups under Rotemberg and lowers them under under Calvo. As a result, the differential response of markups implies that the long-run NKPC is negatively (positively) sloped in the Calvo (Rotemberg) model. In this regard, empirical evidence is inconclusive. Berentsen et al. (2011) show positive relationships between the trend components of inflation and unemployment (negatively related to output). Beyer and Farmer (2007) study the low-frequency movements of inflation, unemployment, and the federal funds rate and find that they trend together. Benati (2015) investigates the long-run tradeoff between inflation and the unemployment rate in the US, the Euro area, the UK, Canada and Australia using structural VARs. He cannot reject the null hypothesis of a vertical long-run NKPC for either country. The overall extent of uncertainty is so large that the data are compatible with a comparatively wide range of possible slopes of the long-run trade-off. He also finds that Johansen’s cointegration tests point towards an estimated long-run NKPC trade-off which is negative and sizeable. Thus lower inflation should be associated to higher unemployment and lower output. However he argues that "this evidence should be discounted due to limited power of the Johansen’s procedure".

Ascarì and Ropele (2012, a, b) study the effects of a disinflation policy under the representative agent assumption. To capture the short- and long-run effects of disinflation on inequality we extend their model to account for financial market incompleteness and for wealth inequality.

In our model the short-run effects of disinflation on inequality arise because households differ in their ability to smooth consumption during the transition to the low-inflation steady state. In this regard, recent contributions emphasise the short-run effects of monetary policy on “hand-to-mouth” consumers, who are constrained by large spending commitments relative to their income and liquid assets holdings. This definition encompasses both asset-poor individuals and highly-leveraged holders of illiquid assets, typically residential estate (Kaplan and Violante, 2014; Kaplan et al. 2014; Ampudia et al., 2018). These hand-to-mouth individuals are characterized by a relatively large marginal propensity to consume out of temporary income changes.

In the long-run, the lower inflation target matters for inequality because it affects returns from accumulated wealth. Following Piketty (2014), there has been increasing concern for the implications of concentration in wealth holdings. Our purpose here is to identify the consequences of inflation-driven variations in firms profits and in the cost of financing their working capital.

To capture the short- and long-run effects of a disinflation while preserving a relatively simple analytical and computational framework, we investigate the distributional effects of disinflation in a medium scale DSGE model augmented for Limited Asset Market Participation (LAMP, henceforth). Under LAMP a fraction of non-Ricardian households (RT hereafter) do not participate in financial markets and do not accumulate wealth. This assumption is associated
to Mankiw’s distinction between savers and spenders (Mankiw, 2000) and is supported by microeconometric studies such as Anderson et al. (2013), who find that in the US the wealthiest individuals behave according to the permanent-income hypothesis, but the poorest individuals disregard interest rate changes and adjust consumption to their disposable income dynamics.

The LAMP hypothesis has been popularized in a number of studies (Galí et al., 2004, 2007; Bilbiie, 2008; Colciago, 2011 Furlanetto and Seneca, 2012; Furlanetto et al., 2013; Motta and Tirelli, 2012, 2014; Albonico and Rossi, 2014, Albonico, Paccagnini and Tirelli 2016, 2017; Ascari et al. 2017). It provides a reasonable approximation to the observed polarization in long-run wealth holdings: Iacoviello and Pavan (2013) document that 40% of the US population has essentially no assets and no debt. Wolff (2010, p. 44) shows that the top quintile of US households own about 90% of total financial wealth. Cowell et al. (2012) provide similar figures for the Euro area.

If one is concerned with households responses to temporary shocks, the LAMP assumption is an admittedly rough-and-ready characterization of households heterogeneity. Havranek and Sokolova (2016) perform a meta-analysis of the excess sensitivity of consumption to income growth, and suggest that it is essentially explained by liquidity constraints. The response of their consumption should therefore be asymmetric to increases and decreases in income, and liquidity constraints should be endogenous to business cycle conditions. These features are captured by HANK models which are based on a detailed description of agents heterogeneity (Kaplan et al. 2016) and account for nominal rigidities.

However, De Bortoli and Galí (2017) show that a simple LAMP model is a tractable framework that captures reasonably well the main predictions of HANK models in response to monetary policy shocks. Furthermore, modelling endogenous borrowing constraints does not seem essential in our deterministic setting where the transition is characterized by a persistent monetary contraction which tightens borrowing constraints and possibly raises the share of constrained households. In this regard, our model seems prone to underestimate the short-run effects of the disinflation on inequality.

Our concern for the long-run redistributive effects of monetary policy calls for a reconsideration of the disinflation modelling strategy adopted in the DSGE literature. In sharp contrast with empirical evidence, disinflations cause a boom in New Keynesian models based on purely forward-looking price setting and rational expectations (Ball, 1994a) because inflation almost immediately jumps to the new long-run level. In fact inflation persistence is potentially inherent to episodes of monetary policy regime change, and it could be treated as a temporary phenomenon potentially explained by several concurring causes such as imperfect credibility (Erceg and Levin, 2003; Goodfriend and King, 2005), inattention, myopia, bounded rationality (Milani, 2012; Branch and McGough, 2009). In our model temporary inflation persistence is obtained by assuming that inflation expectations are partly backward-looking, as in Galí and Gertler (1999).

An apparently convenient alternative, proposed by Ascari and Ropele (2012a) would be to assume that price-setting rules incorporate inflation indexation as
in CEE. We cannot treat inflation indexation as a simple device that allows to capture inflation persistence. In fact indexation limits the response of price markups to inflation regime changes, and therefore crucially affects our results concerning the long-run effects of disinflations on inequality. The widespread use of the indexation assumption in the price-setting equation has been criticized in Benati (2008), who shows that price indexation has become virtually irrelevant since the onset of the Great Moderation period. Thus inflation indexation parameters should not be regarded as structural in the sense of Lucas (1976).

Our results in a nutshell. To sharpen our analysis, we take relative consumption of the two household groups as the preferred measure of inequality. We find that in the long run firms profitability increases irrespective of the price-setting assumptions. This, in turn, implies that consumption inequality increases. By contrast, the lower cost of financing firms working capital unambiguously reduces inequality. The underlying intuition is as follows. Disinflation lowers the long-run nominal interest rate and, due to the reduction in the cost of financing the working capital, it has a powerful effect on labor demand. As a result the real wage increases whereas the real rental cost of physical capital, driven by the Ricardian households’ rate of time preference, remains constant. Thus the long run effect of inflation on inequality depends on the importance of the CIA constraint. Our calculations suggest that under Calvo the disinflation reduces inequality in the long run even if firms must pre-finance only a limited fraction of the wage bill. This happens because the fall in markups limits the increase in profitability determined by the disinflation. By contrast, under Rotemberg the increase in markups raises inequality, and we need that the CIA constraint is relatively far more important to obtain that the disinflation reduces long-run inequality.

Transitional dynamics are quite different for the two household groups. Ricardian households anticipate the beneficial effect of the disinflation on their permanent income by immediately reducing their savings in order to increase their consumption. This requires an investment fall, driving a reduction in labor demand that determines a contraction in the consumption of RT households.

We provide a formal welfare analysis of the disinflation, which is always welfare-improving in the benchmark model, where the CIA constraint is calibrated at the (small) level estimated in Rabanal (2007). Welfare gains accrue to Ricardian households even in the short-run, when the economy contracts. By contrast, RT households suffer a welfare loss during the contraction period. These transitional effects may be alleviated if the monetary policy rule targets the output gap in addition to inflation. In this case it takes more time to disinflate the economy, but the milder output contraction is associated to a smaller consumption loss of RT households. Note that the accommodative monetary policy also stimulates consumption of Ricardian households, and therefore it has negligible impact on short-term inequality. In a way the accommodative policy

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1 Other studies support this conclusion. See, for instance, Shbordone (2006); Coogley and Shbordone (2008); Ascari, Castelmovo, Rossi (2011); Hofman, Peersmann, Straub (2012).
could be seen as the tide that lifts all boats in the short run.

The paper adds to previous contributions on the welfare implications of inflationary regimes, which highlight the importance of different portfolio composition of different income groups, where the poor typically hold a relatively large proportion of their wealth in non-interest-bearing assets and inflation is a substitute for other forms of taxation (Erosa and Ventura, 2002; Albanesi, 2007, Menna and Tirelli, 2017). Our focus here is clearly different as we investigate the distributional implications of inflation regime choice which emerge as a consequence of the endogenous response of financial frictions, i.e. the CIA constraint, and of firms monopoly power, i.e. price markups.

Other studies focus on the distributional effects of monetary shocks in New Keynesian models. Gorneman et al. (2016) focus on matching frictions in the labor market and assume that all households hold identical portfolios of financial assets which provide self-insurance against consumption risk. Luettike (2018) allows for portfolio heterogeneity but imposes that capital is an illiquid asset. Our focus is different because we investigate the distributive effects of an inflation target change in a model of concentrated capital ownership, akin to Lansing (2015), Lansing and Markiewicz (2017), Walsh (2017).

The rest of the paper is organized as follows. The next section describes the main model features in particular focusing on the two price mechanisms. Section 3 focuses on the disinflation experiment and results; section 4 concludes.

2 The Model

Our New Keynesian model embodies both nominal and and real frictions. Real frictions include: monopolistic competition in goods and labor markets, a CIA constraint - such that a fraction of a firm wage bill must payed in advance - and LAMP. To characterize LAMP, we assume that optimizing (Ricardian) households are a fraction $\frac{1}{\Omega}$ of the population, and the remaining $\Omega$ households are RT consumers. As pointed out above, inflation expectations are inertial. Turning to nominal frictions, we investigate the distributional effects of the Calvo and Rotemberg models, that generate quite different dynamics if inflation is positive in steady state (Ascari and Rossi, 2012).

Standard medium-size DSGE models, such as Schmitt-Grohé and Uribe (2005, henceforth SGU), and CEE (2005) typically account for additional frictions such as external habits in consumption, variable capacity utilization, investment adjustment costs. We do not consider them here because their inclusion is un consequential for our qualitative results.\footnote{Proof available upon request. This choice inevitably implies that transitional dynamics become less persistent.}

The structure of the model is summarized in Figure 1.
Households share the same utility function.

\[ U_i^t = E_t \sum_{i=0}^{\infty} \beta^t \left\{ \ln (c_i^t) - \frac{\phi_i^t}{1 + \phi} (h_i^t)^{(1+\phi)} \right\} \]  

(1)

where \( i = o, rt \) defines optimizing and RT households respectively, \( \beta \) is the subjective discount factor, \( c_i^t \) and \( h_i^t \) respectively are two standard Dixit-Stiglitz consumption and labor bundles:

\[ c_i^t = \left[ \int_0^1 c(z) \frac{\theta_{t}^{z-1}}{\theta^t} \ dy \right]^\frac{1}{\theta} \]  

(2)

\[ h_i^t = \left( \int_0^1 (h^t_j) \frac{\eta_w^{n-1}}{\eta_w} \ dy \right)^\frac{\eta_w}{\eta_w-1} \]  

(3)

The two conditions (2, 3) allow to introduce monopolistic competition in the goods and labor markets.

### 2.1 Labour market structure

For each labor type there is a monopolistically competitive market and the wage setting decision is delegated to a union. The representative union \( j \) is confronted with a downward-sloping demand function:

\[ h_j^d = \left( \frac{w_j^t}{w_t} \right)^{-\eta_w} h_t^d \]
where \( w^d_t \) is the real wage for labor type \( j \), \( h^d_t \) is the aggregate labour demand and \( w_t = \left( \int_0^1 \left( w^j_t \right)^{(1-\eta) \omega} dj \right)^{\frac{1}{1-\eta \omega}} \) is the aggregate wage index. Following Galì (2007), the fraction of Ricardian and non-Ricardian households is uniformly distributed across unions and the demand for each labor type is uniformly distributed across households. Households therefore supply the same amount of hours.

### 2.2 Budget constraints

Non-Ricardian agents just consume current labor income and do not accumulate wealth:

\[
e_{rt} = w_t h^d_t \tag{4}
\]

The Ricardian household’s period budget constraint is:

\[
e_t^c + K_{t+1}^o - (1 - \delta) K_t^o + \frac{M_{t+1}^o}{P_t} = r_t^K K_t^o + w_t h^d_t + d_t^o + R_t M_t^o P_t \tag{5}
\]

where \( K^o \), \( r^K \) respectively define the stock of capital and the real rental rate of capital; \( \delta \) is the capital depreciation rate; \( d_t^o \) defines individual holdings of firms dividends; \( M^o \) defines money holdings which are used to finance firms’ wage bills at the nominal gross rate \( R_t \), \( P_t \) is the aggregate price level associated to (2).

### 2.3 Firms

#### 2.3.1 Retail firms

Perfectly competitive retail firms assemble the wholesale goods into the final bundle which is used for either consumption or investment in physical capital. Their demand for goods produced by the wholesale producer \( z \) is

\[
y_t \left( z^W \right) = \left( \frac{P_t^W (z^W)}{P_t^W} \right)^{-\eta} y_t^d \tag{6}
\]

where \( y_t^d \) defines the amount of final goods that retail firms supply in the final goods market at the retail price \( P_t^W \) and \( P_t^W = \left[ \int_0^1 \left( P_t \left( z^W \right) \right)^{1-\eta \omega} dz \right]^{\frac{1}{1-\eta \omega}} \) is the
wholesale price index. Right from the outset, note that the zero profit condition requires
\[ P_t^W y_t^W dz = P_t^R y_t^d \] (7)
where \( P_t^R \) defines the price in the retail market.

### 2.3.2 Wholesale firms

The representative wholesale firm produces good \( z \) using a standard Cobb-Douglas technology:
\[ y_t(z) = (K_t(z))^\phi (h_t^d(z))^{(1-\phi)} \] (8)

Given the CIA constraint, financing needs are defined as \( \nu w_t h_t^d \) where \( \nu \) is the fraction of the wage bill which is paid in advance. Following SGU, real marginal costs, defined in terms of the final bundle price, are:
\[ mc_t = \left( \frac{r_t}{\partial_l} \right)^\phi \left( \frac{w_t \left( 1 + \nu \left( 1 - \frac{1}{K_t} \right) \right)}{1 - \phi} \right)^{1-\phi} \] (9)

### Nominal rigidities

#### Calvo pricing

Under the Calvo specification a fraction \((1 - \alpha)\) of firms choose the optimal price \( P_t^* \) and the remaining \( \alpha \) firms hold their price constant. The wholesale price index is:
\[ P_t^W = \left( (1 - \alpha) \left( P_t^{W*} \right)^{1-\eta} + \alpha \left( P_t^{W} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \]

A crucial implication of Calvo pricing is that relative-price dispersion causes resource misallocation which impacts on firms profits.

As shown in SGU, integrating 6 over all firms yields:
\[ y_t^W = y_t^d \left( s_t^{Calvo} \right) \] (10)

where \( y_t^d = C_t + I_t \) and
\[ s_t^{Calvo} = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\eta} dz = \]
\[ = \alpha (\pi_t)^{\eta} s_{t-1}^{Calvo} + \left( 1 - \alpha \right) \left[ \frac{1 - \alpha \pi_t^{\eta-1}}{(1 - \alpha)} \right]^{\frac{-\eta}{1-\eta}} \]

\( s_t \) has a lower bound at 1 and that it matters up to first order when inflation is non-zero in steady state. From 10 it is easy to see that \( s_t \) drives a wedge between the resources available for final use and the resources that firms must
utilize to satisfy any given level of aggregate demand. This output loss causes a reduction in aggregate dividends. In fact from 7 and we get

$$\frac{P_t^{W}}{P_t^{R}} = \frac{1}{s_t^{Calvo}}$$

therefore dividends of wholesale firms amount to:

$$d_t^{Calvo} = (P_t^{W} - MC_t) y_t s_t^{Calvo} =$$

$$= \left( \frac{P_t^{R}}{s_t^{Calvo}} - MC_t \right) y_t s_t^{Calvo} =$$

$$= \left( 1 - s_t^{Calvo} \frac{MC_t}{P_t^{R}} \right) y_t =$$

$$= \left( 1 - \frac{s_t^{Calvo} \mu_t^{P,Calvo}}{\mu_t^{P,Calvo}} \right) y_t =$$

$$= \left( \frac{\mu_t^{P,Calvo} - s_t^{Calvo}}{\mu_t^{P,Calvo}} \right) y_t$$

(12)

and are crucial for the analysis of income inequality.

**Rotemberg pricing**  In each period all firms can choose the optimal price subject to an adjustment cost:

$$Q_t^P = \frac{\xi_p}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 y_t.$$  

(13)

and dividends are:

$$d_t^{Rotemberg} = \left( \frac{\mu_t^{P,Rotemberg} - \frac{\xi_p}{2} (\pi_t - 1)^2}{\mu_t^{P,Rotemberg}} \right) y_t$$

(14)

### 2.3.3 Labor Unions

Our modelling strategy here is characterized by two key assumption. First, labor unions maximize a weighted average of agents’ intertemporal utilities (Colciago, 2012):

$$E_t \sum_{s=0}^{\infty} (\beta)^s \left[ (1 - \theta) U_{t+s}^o + \theta U_{t+s}^t \right]$$

Second, our characterization of wage dynamics will incorporate a moderate but non-negligible amount of wage indexation, as documented in Hofmann et al. (2012) and De Schryder et al (2014).
Under Calvo we therefore assume that in each period \( (1 - \alpha_w) \) unions re-optimize the wage rate \( W^j_t \). The remaining \( \alpha_w \) unions index it to past inflation:

\[ W^j_t = W^j_{t-1} \pi_{t-1}^{X_w} \]

To model wage stickiness under Rotemberg we posit that for each labor type \( j \) the wage adjustment cost is:

\[ Q^w_t = \frac{\xi^w}{2} \left( \frac{W^j_t}{W^j_{t-1} (\pi_{t-1}^{X_w})} - 1 \right)^2 h_t \tag{15} \]

### 2.4 Monetary Policy

We assume that monetary policy follows the standard rule:

\[ \frac{R_t}{R_t} = \left( \frac{\pi_t}{\pi_{t-1}^{R}} \right) \phi^r \left( \frac{y_t}{y^*} \right) \phi^y \tag{16} \]

where \( R, y_t, y^*, \pi_t, \pi^* \), respectively denote the steady state gross nominal interest rate, the current and steady state output levels, the current and target gross inflation rates.

### 2.5 Inflation Expectations

Following our discussion in the introduction, inflation persistence is modelled by assuming that inflation expectations are partly backward-looking.

\[ \tilde{E}_t \{ \pi_{t+1} \} = (1 - \Psi) E_t \{ \pi_{t+1} \} + \Psi \pi_{t-1} \tag{17} \]

where \( E_t \{ \pi_{t+1} \} \) defines the rational expectation of \( \pi_{t+1} \). This characterization allows to obtain that the disinflation causes short-run output losses consistent with estimated sacrifice ratios.

### 2.6 Calibration

We calibrate the model at quarterly frequency. All parameter values are reported in Table 1. A number of parameters are borrowed from CEE (2005): the discount factor \( \beta \) is set to to obtain a 3% real interest rate per annum; the capital income share parameter \( \vartheta \) is set at 36%; the inverse of the Frisch elasticity, \( \phi \), is 1; the capital depreciation rate per quarter is 2.5%. The elasticities of substitution \( \eta = 6 \) and \( \eta_w = 21 \) imply that at zero inflation the steady state price and wage markups are 20% and 5% respectively. In CEE (2005) the \( \nu \) parameter is set at 1. Rabanal (2007) estimates that \( \nu \) has a posterior mean of 15%, with a large standard deviation, 13%. In the paper we study two cases, when \( \nu \) is either 15% (full model) or zero.

Empirical DSGE-LAMP models estimate a substantial share of RT households. Earlier studies for the EMU obtain estimates for \( \Omega \) in a range between
24% and 37% (Coenen and Straub, 2005; Forni et al., 2009). Albonico et al. (2016, 2017) estimate a fraction of RT consumers at 50% in both the EMU and the US. Gali et al. (2007) calibrate $\Omega$ at 0.5. We choose a conservative benchmark calibration by setting $\Omega = 0.3^\dagger$. The Taylor rule parameters take standard values $\phi_p = 1.5$, $\phi_y = 0.1$. The preference parameter $\phi_1$ is calibrated to obtain that worked hours amount to 25% in the initial steady state.

Let us now turn to the calibration strategy adopted for the parameters that characterize nominal rigidities. In CEE the Calvo price and wage parameters, $\alpha$ and $\alpha_w$ respectively are 0.6 and 0.64 and full inflation indexation is assumed for non-optimizing firms and labor unions. In our benchmark exercise we maintained the CEE values for $\alpha$ and $\alpha_w$. To obtain comparable inflation dynamics under the two price-setting mechanisms, we impose that the Calvo and Rotemberg formalisms yield identical slopes of the loglinearized price and wage Phillips curves up to first order approximation. Following Keen and Wang (2007), this requires that

$$\xi_p = \frac{(\eta - 1) \alpha}{(1 - \alpha)(1 - \beta \alpha)} = 18.5$$

$$\xi_w = \frac{(\eta_w - 1) \alpha_w}{(1 - \alpha_w)(1 - \beta \alpha_w)} = 97.4$$

The wage indexation parameter is set at 0.5. This calibration falls in the mid-range of the cross-country estimates in López-Villavicencio and Saglio (2017). Parameter $\Psi$, which captures inertia in inflation expectations, is calibrated to obtain under Calvo pricing a sacrifice ratio of 1.31, in the lower range of the empirically plausible values documented in Ascari and Ropele (2012a).

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$^\dagger$Our results are robust for a larger share of RT households, namely, $\Omega = 0.5$. 

12
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.03</td>
<td>Subjective discount factor</td>
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<tr>
<td>$\phi$</td>
<td>1</td>
<td>Inverse of Frisch elasticity</td>
</tr>
<tr>
<td>$\eta_{lv}$</td>
<td>21</td>
<td>labor elasticity of substitution</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.3</td>
<td>Share of non Ricardian households</td>
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<tr>
<td>$\alpha_w$</td>
<td>0.64</td>
<td>Calvo wage parameter</td>
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<tr>
<td>$\xi_{sw}$</td>
<td>97.4</td>
<td>Rotemberg wage parameter</td>
</tr>
<tr>
<td>$\chi_{sw}$</td>
<td>0.5</td>
<td>Wage Indexation</td>
</tr>
<tr>
<td>Firms</td>
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<tr>
<td>$\theta$</td>
<td>0.36</td>
<td>Capital share</td>
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<td>$\delta$</td>
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<td>Capital depreciation</td>
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<tr>
<td>$\eta$</td>
<td>6</td>
<td>Goods elasticity of substitution</td>
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<td>Calvo price parameter</td>
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<td>$\xi_p$</td>
<td>18.5</td>
<td>Rotemberg price parameter</td>
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<tr>
<td>$\nu$</td>
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<td>CIA parameter</td>
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<td>Monetary Authority</td>
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<tr>
<td>$\phi_{in}$</td>
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<td>Inflation feedback</td>
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<td>$\phi_{y}$</td>
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<td>output gap feedback</td>
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<td>$\Psi$</td>
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<td>Inertia in inflation expectations</td>
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</tbody>
</table>

3 The Disinflation Experiment

The disinflation experiment entails a transition from high- to low-inflation steady state, respectively defined as $\pi_{old}$ and $\pi_{new}$. Following Ascari and Ropele (2012), we assume that the Central Bank inflation target is reduced from $\pi_{old} = 1.05$ to $\pi_{new} = 1.02$.\(^6\)

The log-run consequences of disinflation may be decomposed into efficiency effects that relate to average variables, and redistributive effects which affect relative consumption levels. We shall also account for the effects of disinflation during the transition, when RT households cannot exploit accumulated wealth to smooth consumption.

3.1 Efficiency effects of disinflation

Table 2 reports the steady state percentage variations of output ($y$), consumption ($c$), average firms markup ($\mu^p$) and dividends ($d$), real wage ($w$), hours ($h$), capital ($K$), consumption - output ratio ($c/y$).

\(^6\)We simulate the non-linear first order conditions because approximating transitions with log-linear first-order conditions may bias results (Ascari and Merkl, 2009). The model is numerically solved using DYNARE.: http://www.cepremap.cnrs.fr/dynare/
Table 2 - Steady state percentage variations

<table>
<thead>
<tr>
<th>Aggregate Variables</th>
<th>$\nu = 0$</th>
<th>$\nu = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.49</td>
<td>0.53</td>
</tr>
<tr>
<td>$c$</td>
<td>0.50</td>
<td>0.54</td>
</tr>
<tr>
<td>$\mu^p$</td>
<td>-0.14</td>
<td>-0.14</td>
</tr>
<tr>
<td>$d$</td>
<td>0.63</td>
<td>0.67</td>
</tr>
<tr>
<td>$w$</td>
<td>0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>$h^d$</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>$K$</td>
<td>0.46</td>
<td>0.50</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

To rationalize these results, we focus on the steady state real marginal cost and the capital-labor ratio, which are pinned down by the price markup:

$$mc = \left( \frac{r^k}{\vartheta} \right) \left[ \frac{w (1 + \nu (1 - \frac{1}{\vartheta}))}{1 - \vartheta} \right]^{1-\vartheta} = \frac{1}{\mu^p.X}; X = Calvo, Rotemberg \quad (18)$$

$$\frac{K}{h} = \left( \frac{\mu^p.X r^k}{\vartheta} \right)^{\frac{1}{1-\vartheta}}. \quad (19)$$

where $r^k = \frac{1}{\vartheta} - 1 + \delta$ and $R = \frac{\vartheta}{\vartheta}.$

The inflation effect on dividends is twofold. On the one hand it affects price markups. On the other hand it generates either price-adjustment costs or price-dispersion losses that reduce dividends distributed to households. Consider first the case of a non-binding CIA, i.e. $\nu = 0.$

3.1.1 Calvo pricing

Under Calvo pricing the steady state average markup is:

$$\mu^{p,Calvo} = \frac{\eta}{\eta - 1} \left( \frac{1 - \beta \alpha \pi^{-(1-\eta)}}{1 - \beta \alpha \pi^{\eta}} \right) \left[ \frac{1}{(1 - \alpha)} \right]^{\frac{1}{1-\eta}} (1 - \alpha \pi^{\eta}) \quad (20)$$

Calculations show that the disinflation experiment considered here entails a markup reduction.$^7$ As a result, the real wage unambiguously increases (see 18). From 19 it is easy to see that this occurs because lower markups are associated to an increase in the capital-labor ratio. Furthermore, disinflation reduces price dispersion:

$$s = \left( \frac{1 - \alpha}{1 - \alpha \pi^{\eta}} \right) \left[ \frac{1 - \alpha \pi^{1-\eta}}{(1 - \alpha)} \right]^{\frac{1}{1-\eta}} \quad (21)$$

$^7$Ascari and Rossi (2012) document the positive effect of moderate steady state inflation on the markup.
In the Appendix we document that the reduction in wage dispersion and wage markups is associated to an increase in the labor supply. As a result output and consumption increase. In spite of lower markups, the smaller output losses due to price dispersion cause an increase in firms profitability and in aggregate dividends.

3.1.2 Rotemberg pricing

Disinflation unambiguously reduces price and wage adjustment costs. This, in turn, leaves room for an increase in consumption at any given level of aggregate supply:

\[
y^d_{\text{Rotemberg}} = \frac{y^d}{1 - \frac{\xi_p}{2} [\pi - 1]^2 - \frac{\xi_p}{2} [\pi - 1]^2 (\frac{K}{n})^{-\sigma}}
\]  

(22)

By contrast, the price markup

\[
\mu^p_{\text{Rotemberg}} = \frac{\eta}{(\eta - 1) + \eta (1 - \beta) \xi_p (\pi - 1) \pi}
\]  

(23)

unambiguously increases, causing a supply reduction. The markup increase explains why under Rotemberg the disinflation has less favourable effects on consumption and on the real wage. The ratio \( \frac{\xi}{y} \) increases because the higher price markup reduces the capital-labor ratio and the investment share in steady state. The combination of higher markups and smaller price adjustment costs raises firms dividends.

Consider now the case of a binding CIA, i.e. \( \nu = 0.15 \).

By holding the price markup constant in 18 it is straightforward to determine the effects of disinflation that occur through the CIA channel. The reduction in the interest payments on loans financing the wage bill is entirely absorbed by a real wage increase. This, in turn, stimulates a labor supply expansion which is matched by an increase in the capital stock (see 19). Given these results it is therefore obvious that both output and consumption must increase when the CIA binds.

3.2 Long-run redistributive effects of disinflation

In this section we discuss closed-form solutions and Table 3 reports numerical calculations which support intuition when theoretical results are ambiguous. Using 8, 18, 19 we obtain the labor income share:

\[
\frac{w_h}{y} = \frac{1 - \vartheta}{(\mu^p X) [1 + \nu (1 - \vartheta \pi)]}.
\]

Lower interest payments on the wage bill unambiguously raise \( \frac{w_h}{y} \). Under Calvo this latter effect is strengthened by the fall in price markups, whereas under Rotemberg the markup increase works in the opposite direction.

---

8 An identical result obtains for the wage markup.
In our relatively simple framework, it is possible to obtain an analytical characterization of steady state inequality by focusing on consumption levels of the two household types:

\[ c^t = wh \left( 1 + \frac{\nu}{1 - \Omega} \left( 1 - \frac{1}{R} \right) \right) + \frac{(r^k - \delta)}{1 - \Omega} K + \frac{d}{1 - \Omega} ; R = \frac{\pi}{\beta} \]

Consumption inequality is determined by the concentration of wealth holdings in the hands of Ricardian households. In addition to their labor income, in steady state they consume the net real return on physical capital \( \frac{(r^k - \delta)K}{1 - \Omega} \), the net interest payments on real money holdings which finance firms’ wage bills \( \frac{\nu}{\pi} (1 - \frac{1}{\pi}) wh \), and individual holdings of dividends, \( \frac{d}{1 - \Omega} \). As shown in the Appendix,

\[ \frac{c^t}{c^o} = \left( 1 + \frac{\nu}{1 - \Omega} \left( 1 - \frac{1}{\pi} \right) \right) + \left( \frac{(r^k - \delta)}{\pi - 1 - \sigma_\pi} \right) \theta + \left( \frac{\nu}{\pi - 1 - \sigma_\pi} \right) (\pi - 1) \frac{(1 + \nu (1 - \frac{1}{\pi}))}{(1 - \Omega) (1 - \theta)} \]  

(24)

The fall in inflation reduces the importance of the CIA, and the relative consumption of RT households unambiguously increases for this reason. The dividend effect on relative consumption depends on the specific features of the price-setting mechanism. Under Rotemberg, the lower inflation rate raises dividends because the price markup increases (see eq. 23) and because inflation adjustment costs fall. Under Calvo, disinflation has ambiguous effects because the price markup falls but the reduction in price dispersion has beneficial effects on dividends. Our calculations show that even in this latter case disinflation is associated with an increase in dividends that, in turn, raises consumption inequality.

Our calibrated model predicts that under Rotemberg a relatively strongly binding CIA constraint, namely 33%, is needed to nullify the inequality between the two groups of households, whereas under Calvo a fall in inequality occurs only if at least 5% of the wage bill must be pre-financed.

<table>
<thead>
<tr>
<th>Table 3 - Inequality measures, percentage variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inequality Measures</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( c^t / c^o )</td>
</tr>
<tr>
<td>( wh / y )</td>
</tr>
</tbody>
</table>

3.3 Short-run dynamics and inequality

In Figures 2a,b we report transitions under the Calvo and Rotemberg price-setting mechanisms when the output gap feedback is either 0 or 1. Results are broadly consistent with the empirical findings reported in Ascari and Ropele.
(2012a): the disinflation causes short-run output losses. This outcome is driven by the permanent income effect of the disinflation and by the real interest rate increase caused by inflation inertia, which induce Ricardian households to reduce investment in physical capital and to raise their consumption. RT households suffer a loss of disposable income due to the fall in both wages and worked hours. As a result we observe a sharp deterioration in RT consumption levels. The output contraction and the fall in RT consumption are less sharp if the Taylor rule incorporates a feedback on the output gap. In this case the milder recession is obtained at the cost of slowing down the pace of inflation convergence to the new target. In spite of the substantial degree of wage inertia imposed with our calibration, the transition to the low inflation steady state is always characterized by a sharp increase in price markups.

Figure 2a: Short-run dynamics and inequality; full model ($\nu = 0.15$); $\phi_y = 0$. 
To measure the costs of disinflation we calculate sacrifice ratios, $SR_X$, for output and RT consumption.

$$SR_X = \frac{1}{\pi_{old}^* - \pi_{new}^*} \sum_{t=0}^{T} \left( \frac{X_t - X_{old}^*}{X_{old}^*} \right)$$ (25)

where $X_{old}^* = y_{old}^*, c_{old}^{rot,*}$ defines output and RT consumption in the high inflation steady state, $\pi_{old}^* - \pi_{new}^*$ is the disinflation in percentage points, and $T$ is the number of periods necessary for output to return to $y_{old}^*$ after the initial contraction.\(^9\) Losses for RT consumers are much larger than conventional measures of output sacrifice ratios.

<table>
<thead>
<tr>
<th>Table 4 - Sacrifice Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SR_y^d$</td>
</tr>
<tr>
<td>Calvo</td>
</tr>
<tr>
<td>$\nu = 0.15; \phi_y = 0$</td>
</tr>
<tr>
<td>$\nu = 0.15; \phi_y = 0.1$</td>
</tr>
<tr>
<td>Rotemberg</td>
</tr>
<tr>
<td>$\nu = 0.15; \phi_y = 0$</td>
</tr>
<tr>
<td>$\nu = 0.15; \phi_y = 0.1$</td>
</tr>
</tbody>
</table>

\(^9\)To facilitate comparison between the two price setting mechanisms, $T$ is the number of "sacrifice periods" observed under Calvo.
3.4 Welfare effects of disinflation

The intertemporal welfare function in recursive form is

\[ V_i^t = \ln (c_i^t) - \frac{\phi_1}{(1 + \phi)} (h_i^t)^{(1+\phi)} + \beta E_t V_i^{t+1}, \tag{26} \]

we define

\[ V_{old}^i = \frac{1}{1-\beta} \left[ \ln (c_{old}^i) - \frac{\phi_1}{(1 + \phi)} (h_{old}^i)^{(1+\phi)} \right]; i = o, rt \tag{27} \]

as the pre-disinflation steady state value of \( V_i \), and \( V_0^i \) as the value of (26) at time zero, when the disinflation is implemented. Since the utility function is not cardinal, the numerator of the ratio needs to be transformed in a measure which can “quantify” the welfare cost (or gain) of disinflation. This is a standard methodology for measuring the welfare effects of business cycles in terms of a consumption equivalent measure (Lucas, 1987, Krusell et al. 2009). Following Ascari and Ropele (2012 a) and Ascari et al. (2018), this consumption equivalent measure is defined here as the constant fraction of consumption that households must give up to permanently reduce inflation:

\[ \frac{1}{1-\beta} \left[ \ln (c_{old}^i (1 - \gamma^i)) - \frac{\phi_1}{(1 + \phi)} (h_{old}^i)^{(1+\phi)} \right] = V_0^i \]

\[ \gamma^i = 1 - \exp \left[ (1 - \beta) (V_0^i - V_{old}^i) \right] \tag{28} \]

Disinflation is welfare improving when the welfare-based ratio is negative, and we read the negative values as welfare gains. Tables 5a,b report our results\(^{10}\), where we also compute the consumption-equivalent measure associated to the welfare losses incurred during the \( T \) periods of output sacrifice, \( \gamma_{SR}^i \):

\[ \gamma_{SR}^i = 1 - \exp \left[ \frac{V_{SR}^i}{A} - V_{OLD}^i (1 - \beta) \right] \tag{29} \]

where \( A = \sum_{t=1}^{T} \beta^t, V_{SR}^i = \sum_{t=1}^{T} \beta^t \left\{ \ln (c_i^t) - \frac{\phi_1}{(1 + \phi)} (h_i^t)^{(1+\phi)} \right\} \).

Note that \( \gamma^i \) is always negative, and Ricardian households are relatively better off under Rotemberg whereas the opposite results obtains under Calvo. This result is determined by the differenter markup responses that we observe in the long run under the two price-setting mechanisms. The short-run welfare effects of the disinflation are instead quite different for the two groups. This cannot be a surprise given the different consumption dynamics discussed above. Table 5b shows that a more accomodative monetary policy stance can alleviate short run losses for RT consumers.

Consistently with the inequality results, our calibrated model predicts that under Rotemberg the difference in the total welfare gain of the two groups

\(^{10}\)Results are expressed in percentage valus.
of households is nil with a relatively strongly binding CIA constraint, namely $\nu = 35\%$, whereas under Calvo the total welfare gain is the same for ricardian and non asset holders households when $\nu = 7\%$.

<table>
<thead>
<tr>
<th>Table 5a - Welfare analysis ($\phi_y = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Equivalent Measure during the sacrifice period</strong></td>
</tr>
<tr>
<td>Calvo</td>
</tr>
<tr>
<td>$\gamma_{SR}$</td>
</tr>
<tr>
<td>$\gamma_{SR}^{*}$</td>
</tr>
<tr>
<td><strong>Total Consumption Equivalent Measure</strong></td>
</tr>
<tr>
<td>Calvo</td>
</tr>
<tr>
<td>$\gamma_{SR}^{**}$</td>
</tr>
<tr>
<td>$\gamma_{SR}^{***}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5b - Welfare analysis ($\phi_y = 0.1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Equivalent Measure during the sacrifice period</strong></td>
</tr>
<tr>
<td>Calvo</td>
</tr>
<tr>
<td>$\gamma_{SR}$</td>
</tr>
<tr>
<td>$\gamma_{SR}^{*}$</td>
</tr>
<tr>
<td><strong>Total Consumption Equivalent Measure</strong></td>
</tr>
<tr>
<td>Calvo</td>
</tr>
<tr>
<td>$\gamma_{SR}^{**}$</td>
</tr>
<tr>
<td>$\gamma_{SR}^{***}$</td>
</tr>
</tbody>
</table>

4 Conclusions

This paper investigates the distributional and welfare effects of disinflation in a TANK model where monetary policy non-neutrality is due to a cash-in-advance constraint on firms wage bill and to the endogeneity of firm dividends.

Our theoretical conclusions boil down to two simple predictions. In the long run a disinflation unambiguously raises firms dividends, thus it can be associated to a reduction in inequality only if the cost channel of monetary policy, i.e. the CIA effect, is sufficiently strong. Transitions to the lower inflation rate are temporarily characterized by a strong increase in inequality.

Our welfare analysis suggests that the overall effect of the disinflation is always beneficial. However the price-setting mechanism and the ensuing long-run effect on price markups is crucial to determine the distribution of benefits. In fact under Rotemberg pricing the Ricardian households are relatively better off, whereas the opposite conclusion holds under Calvo pricing.

In all cases considered in the paper, short run dynamics heavily penalize RT consumers. Shifting monetary policy towards a more accommodating stance can alleviate short run losses but has negligible impact on inequality. Thus, if inequality is a source of political concern, the policy implication of the paper is that fiscal tools should be exploited to compensate loosers during the disinflation process. We leave this for future research.
References


[37] **Cowell F. A., Karagiannaki E. and McKnight, A., 2013.** Accounting for cross-country differences in wealth inequality. *CASE Papers /168, Centre for Analysis of Social Exclusion, LSE.*

[38] **Davtyan, K. (2017).** The distributive effect of monetary policy: The top one percent makes the difference. Economic Modelling, 65, 106-118.


5 Appendix A: The Model

5.1 Households

There is a continuum of households indexed by $i$, $i \in [0,1]$. RT ($rt$) and Ricardian ($o$) agents are respectively defined over the intervals $[0,\Omega]$ and $[\Omega,1]$.

The household’s utility function is:

$$U_i^t = E_t \sum_{t=0}^\infty \beta^t \left\{ \ln \left( c_i^t \right) \right\}$$

where $c_i^t$ denotes consumption, $h_i^t$ denotes labor supply of a differentiated labor bundle.

5.2 Consumption bundles

The consumption good is characterized by Dixit-Stiglitz preferences:

$$c_i^t = \int_0^1 c(z) \left( \frac{\eta}{\eta-1} \right) \frac{dz}{P_t(z)}$$

where $\eta > 1$ denotes the elasticity of substitution across different varieties of goods.

Demand for good $z$ is:

$$c_i^t(z) = \left( \frac{P(z)_t}{P_t} \right)^{-\eta} c_i^t$$

where

$$P_t = \left( \int_0^1 p(z)^{1-\eta} \, dz \right)^{1/\eta}$$

is the aggregate price consumption index and $P(z)_t$ defines the price set by the firm producing good $z$.

5.3 Ricardian Households

The Ricardian household’s period budget constraint in real terms reads as:

$$c_o^t + i_o^t + m_o^t = r^K_t K_o^t +$$

$$+ w_t h^o_t + \frac{d^o_t}{1-\Omega} + \frac{R_{t-1}}{\pi_t} m^o_{t-1}$$

where $i_o^t$ denotes the real purchases of investment goods at time $t$. Ricardian households accumulate physical capital $K_o^t$ and rent it out to firms at a real rental rate $r^K_t$. $d^o_t$ defines individual holdings of firms dividends, $m^o_t$ defines individual money holdings, which are used to finance firms’ wage bills at the nominal rate $R_t$.  

28
The capital stock evolves according to the following law of motion:

\[ K_{t+1}^o = (1 - \delta)K_t^o + i_t^o \]

where \( \delta \) is the capital depreciation rate.

Following SGU (2005), the Ricardian household’s first order conditions with respect to \( c_t^o \), \( m_t^o \), \( K_t^o \), respectively are:

\[ \frac{1}{c_t^o} = \lambda_t^o \]

\[ \lambda_t^o = \beta R_t \frac{\lambda_{t+1}^o}{\pi_{t+1}} \]

\[ \lambda_t^o = \beta \lambda_{t+1}^o (1 - \delta + r_k^{t+1}) \]

5.4 Rule of Thumb Households

Non-Ricardian entirely consume their income in each period:

\[ c_t^{rt} = h_t^{d} \int_0^1 w_t^{j} \left( \frac{w_t^{J}}{w_t} \right)^{-\eta_w} \]

Their marginal utility of consumption is:

\[ \frac{1}{c_t^{rt}} = \lambda_t^{rt} \]

5.5 Firms

Firms compete monopolistically by producing good \( z \) according to the following technology:

\[ y_t (z) = (K_t (z))^{\vartheta} (h_t (z))^{(1-\vartheta)} \]

Firms are subject to a cash in advance constraint on the wage bill:

\[ m_{zt} = \nu w_t h_{zt} \]

where \( m_{zt} \) denotes the real money balances obtained by firm \( z \) and \( \nu \) is the fraction of labor costs which is payed in advance. Firms financial needs are supplied by Ricardian households at the gross nominal interest rate.

Following SGU (2005) real marginal costs and factors demands are:

\[ mc_t = \left( \frac{r_k^{t+1}}{\vartheta} \right)^\vartheta \left( \frac{w_t \left[ 1 + \nu \left( 1 - \frac{1}{\pi_t} \right) \right]}{1 - \vartheta} \right)^{1-\vartheta} \]
\[ r_t^k = m_c t \vartheta \left( \frac{h_t}{K_t} \right)^{1-\vartheta} \]

\[ w_t \left[ 1 + \nu \left( 1 - \frac{1}{K_t} \right) \right] = m_c t \left( 1 - \vartheta \right) \left( \frac{K_t}{h_t} \right)^\vartheta \]

### 5.5.1 Price Setting

**Calvo**  According to the Calvo (1983) framework, each period a firm faces a constant probability \((1 - \alpha)\) of being able to reoptimize prices. In other words, \(\alpha\) denotes the degree of price stickiness.

The optimal price \(P_t^*\) is chosen in order to maximize the discounted value of expected future profits. Moreover, it’s important to remind here that only ricardian households own firms. Hence, the firms’ maximization problem is:

\[
\max_{P_t^*} \sum_{s=0}^{\infty} (\beta \alpha)^s \frac{X^s_{t+s}}{X_t^s} (P_t^* - P_{t+s} m c_{t+s}) y_{t,t+s} (z) \\
\text{subject to:} \\
y_{t,t+s} (z) = \left( \frac{P_t^*}{P_{t+s}} \right)^{(-\eta)} y_{t+s}^d
\]

where \(y_d^d\) is the aggregate demand and \(\frac{\beta^s X^s_{t+s}}{X_t^s}\) denotes the stochastic discount factor of Ricardian households.

As shown in SGU (2005) the first order condition with respect to \(P_t^*\) is:

\[
\sum_{s=0}^{\infty} (\beta \alpha)^s \frac{E_t}{X_t^s} \left( \frac{P_t^*}{E_t (P_{t+s} m c_{t+s})} \right)^{(-\eta)} y_{t+s}^d^* = 0
\]

where \(\frac{n}{\eta-1}\) is the markup which would obtain in absence of price stickiness. The price level is a weighted average of the prices set by optimizing and non-optimizing firms:

\[
P_t = \left[ (1 - \alpha) P_{t-1}^{1-\eta} + \alpha \left( P_{t-1} \right)^{1-\eta} \right]^{1-\eta}
\]

Straightforward manipulations allow to obtain the average price markup over marginal costs, \(\mu_{t,\text{Calvo}}\):

\[
\mu_{t,\text{Calvo}} = \frac{1}{m_c t} \left[ (1 - \alpha) \left( \frac{P_t^*}{P_t} \right)^{1-\eta} + \alpha \left( \frac{1}{P_t} \right)^{1-\eta} \right]^{1-\eta}
\]
**Rotemberg**  Under Rotemberg the firm maximizes discounted profits:

$$\max_{P_t(z)} E_t \left[ \sum_{s=0}^{\infty} \beta^s \frac{\lambda^s_{t+s}}{\lambda^s_t} \left( \frac{P_{t+s}(z)}{P_{t+s}} - mc_{t+s} \right) y_{t+s}(z) \right]$$

subject to

$$y_{t+s}(z) = \left( \frac{P_{t+s}(z)}{P_{t+s}} \right)^{-\eta} y_{t+s}$$

and to a quadratic price adjustment cost:

$$\frac{\xi_p}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 y_t$$

where $\xi_p > 0$ measures the degree of nominal price rigidity.

In the symmetrical equilibrium, where price dispersion is absent by assumption, the FOC to the problem is:

$$mc_t = \left( \eta - 1 \right) + \frac{\xi_p}{\eta} (\pi_t - 1) \pi_t - \frac{\xi_p}{\eta} E_t \frac{\lambda^0_{t+1}}{\lambda^0_t} \left( \tilde{E}_t \{\pi_{t+1}\} - 1 \right) \tilde{E}_t \{\pi_{t+1}\} \frac{E_t y_{t+1}}{y_t}$$

where the real markup is

$$\mu^p_{t,Rotemberg} = \frac{1}{mc_t}$$

### 5.6 Wage Setting

#### 5.6.1 Calvo

In each period a labor union faces a constant probability $(1 - \alpha_w)$ of being able to reoptimize wages. In other words, $\alpha_w$ denotes the degree of wage stickiness.

Each optimizing union sets $W^*_t$ to maximise a weighted average of the two household types utility functions, conditional to the probability that the wage cannot be reoptimized in the future.

$$L^u = E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s \left\{ [(1 - \theta) U^c(c^s_{t+s}) + \theta U^r(c^s_{t+s})] - U(h_{t+s}) \right\}$$

$L^u$ is maximized subjecto to the firms demand constraint

$$h^*_t = \left( \frac{W^*_t}{W_t} \right)^{-\eta_w} h^d_t$$

The first order condition is:

$$E_t \sum_{s=0}^{\infty} (\beta \alpha_w)^s \lambda^s_{t+s} h^d_{t+s} \left( \frac{w^*_t}{w_{t+s}} \right)^{-\eta_w} \prod_{k=1}^{s} \left( \frac{\pi_{t+k}}{\pi^w_{t+k-1}} \right)^{(\eta_w)}$$
\[
\left( \frac{\eta_w - 1}{\eta_w} \right) \frac{w_t^*}{\prod_{k=1}^{s} \left( \frac{\pi_{t+k}}{\pi_{t+k-1}} \right)} - mrs_{t+s} = 0
\]

where

\[\lambda_{t+s} = \left[ (1 - \Omega) \lambda_{t+s}^\sigma + \Omega \lambda_{t+s}^\nu \right]\]

is the average marginal utility of consumption, \(mrs_{t+s} = -\frac{U_{h_{t+s}}}{\lambda_{t+s}}\) defines the average marginal rate of substitution and \(\frac{\eta_w - 1}{\eta_w}\) is the markup that would prevail under flexible nominal wages and \(\chi_w\) denotes wage indexation to past inflation. The aggregate real wage is a weighted average of the real wages set by optimizing and non-optimizing unions:

\[w_t^{(1-\eta_w)} = (1 - \alpha_w) w_t^*^{(1-\eta_w)} + \alpha_w \left( \frac{\chi_w}{\pi_{t-1}} w_{t-1} \right)^{(1-\eta_w)}\]

5.6.2 Rotemberg

In each period all unions maximise

\[L = \sum_{s=0}^{\infty} (\beta)^s \left\{ [(1 - \theta) U^o(h_{t+s}) + \theta U^r(r_{t+s})] - U(h_{t+s}) \right\}\]

subject to firms labor demand

\[h_t^d = \left( \frac{W_j^d}{W_t} \right)^{-\eta_w} h_t^d\]

and to a quadratic adjustment cost:

\[\frac{\xi_w}{2} \left( \frac{W_j^d}{(\pi_w) W_t^d} - 1 \right)^2 h_t.\]

From the first order condition the wage setting (wage markup) equation is:

\[\mu_{w,Rotemberg} = \frac{w_t}{mrs_t} = \frac{\eta_w}{\eta_w - 1} \left\{ 1 - \left\{ \frac{\xi_w}{\eta_w} \left( \frac{w_t}{w_{t-1}} \pi_{t-1} - 1 \right) \frac{w_t}{w_{t-1}} \pi_{t-1} + \frac{w_{t+1}}{w_t} \pi_{t+1} \right\} \right\} \]
5.7 Market clearing

Consider the individual firm demand function:

\[ y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\eta} y^d_t \]

where \( y^d_t = c_t + i_t \)

defines absorption of resources for consumption and capital accumulation. Integrating over all firms yields:

\[ y_t = s^X_t y^d_t \]

where \( s^X_t (X = Calvo, Rotemberg) \) defines the output wedge, i.e. the output costs of inflation under nominal rigidities.

\( s^Calvo_t \) denotes the resource cost determined by relative price dispersion in the Calvo model. As shown in Schmitt-Grohé and Uribe (2005)

\[ s^Calvo_t = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\eta} dz = \alpha (\pi_t)^{\eta} s^Calvo_{t-1} + (1 - \alpha) \left( \frac{P^*_t}{P_t} \right)^{-\eta} \]

Where \( \frac{P^*_t}{P_t} \), given the characterization of the aggregate price index, must satisfy:

\[ \alpha \pi_t^{(\eta-1)} + (1 - \alpha) \left( \frac{P^*_t}{P_t} \right)^{(1-\eta)} = 1 \]

SGU (2005) have show that \( s_t \) has a lower bound at 1 and that it matters up to first order when the deterministic steady state features a non zero inflation rate.

Under Rotemberg the output wedge is determined by the output costs of price and nominal wage adjustments.

\[ s^{Rotemberg}_t = \frac{1}{1 - \frac{\xi_p}{2} (\pi - 1)^2 - \frac{\xi_w}{2} \left( \frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}} - 1 \right)^2} \]

5.7.1 Labour market equilibrium

The equilibrium on the labour market is given by:

\[ h^*_t = s^X_t h^d_t \]

where \( h^d_t = \left( \frac{y_t}{K_t} \right)^{\frac{1}{1-\eta}} \) defines firms labor demand and \( s^X_t \) denotes the labor market wedge. \( s^{Rotemberg}_t \) denotes the labor market wedge.

\[ s^{Rotemberg}_t = \frac{1}{1 - \frac{\xi_p}{2} \left( \frac{\pi_t}{\pi_{t-1}} - 1 \right)^2} \]

\( s^Calvo_t \) is the additional labor effort due to relative wage dispersion in the Calvo model. It evolves according to:
where

\[ w_t^* = \left( \frac{w_{t-1} \left(1 - \eta_w\right) \left(\pi_{t-1} \eta_w \right) \left(1 - \eta_w\right)}{(1 - \alpha_w)} \right)^{\frac{1}{1 - \eta_w}} \]

6 Appendix B. Steady state derivation

In the following we present a recursive derivation of the steady state values for the variables discussed in the main text.

From the Ricadian households first order conditions

\[ R = \frac{\pi}{\beta} \]

\[ \rho^k = \frac{1}{\beta} - 1 + \delta \]

Given that

\[ s_t^{Calvo} = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\eta} dz = \alpha (\pi_t)^{\eta} s_{t-1}^{Calvo} + (1 - \alpha) \left( \frac{P_t^{\star}}{P_t} \right)^{-\eta} \]

\[ \alpha \pi_t^{(\eta-1)} + (1 - \alpha) \left( \frac{P_t^{\star}}{P_t} \right)^{(1-\eta)} = 1 \]

Price dispersion under Calvo is:

\[ s^{Calvo} = \frac{(1 - \alpha)}{1 - \alpha \pi^{\eta}} \left[ 1 - \alpha \pi^{-(1-\eta)} \right]^{\frac{\pi}{1-\eta}} \]

Derivation of markups:

- Under Calvo:

\[ \mu_{p,Calvo} = \eta \frac{(1 - \beta \alpha \pi^{-(1-\eta)})}{\eta - 1} \left( \frac{P_t^{\star}}{P_t} \right) \]

\[ = \eta \frac{(1 - \beta \alpha \pi^{-(1-\eta)})}{\eta - 1} \left( \frac{1 - \beta \alpha \pi^{-(1-\eta)}}{(1 - \alpha)} \right)^{\frac{1}{1-\eta}} \]

- Under Rotemberg:

\[ mc = \left( \frac{\eta - 1}{\eta} \right) + \frac{\xi_p}{\eta} (1 - \beta) (\pi - 1) \pi \]

34
where the markup is

$$\mu_{p,Rotemberg} = \frac{1}{mc}$$

The real wage therefore is

$$w = (\mu_{p,X})^{-\frac{\alpha}{1-\eta}} \left( \frac{r^k}{\vartheta} \right)^{\frac{\alpha}{1-\eta}} \frac{1 - \vartheta}{1 + \nu \left(1 - \frac{1}{R} \right)}$$

To derive capital-labor ratio:

$$y_t(z) = (K_t(z))^\vartheta (h_t(z))^{(1-\vartheta)}$$

$$r_t^k = \frac{R_t w_t}{(1 - \alpha) (k_{t-1})^\alpha (h_t)^{-\alpha} (k_{t-1})^{\alpha-1} (h_t)^{1-\alpha}}$$

$$\frac{k}{h} = \left( \frac{\mu_{p,X}}{\vartheta} \right)^{-\frac{1}{\pi-\eta}} \left( \frac{r^k}{\vartheta} \right)^{\frac{\vartheta}{\pi-\eta}} \vartheta r^k$$

$$\frac{wh}{y} = \frac{w}{(\frac{k}{h})^\vartheta} =$$

$$= (\mu_{p,X})^{-\frac{1}{\pi-\eta}} \left( \frac{r^k}{\vartheta} \right)^{\frac{\vartheta}{\pi-\eta}} \frac{1 - \vartheta}{1 + \nu \left(1 - \frac{1}{R} \right)} \left( \frac{\mu_{p,X} r^k}{\vartheta} \right)^{\frac{\vartheta}{\pi-\eta}} =$$

$$= (\mu_{p,X})^{-1} \frac{1 - \vartheta}{1 + \nu \left(1 - \frac{1}{R} \right)}$$

To obtain \( \frac{\epsilon^r}{\epsilon} \) bear in mind that aggregate financial variables and returns are obtained aggregating individual holdings

$$(1 - \Omega) k^\alpha = K$$

$$(1 - \Omega) m^\alpha = \nu wh$$

$$(1 - \Omega) d^\rho = \frac{\left( \mu_{p,X} - 1 - \frac{\varepsilon \pi}{2} (\pi - 1)^2 \right)}{\mu_{p,X}} \frac{y}{s^{(1-\varepsilon)}}$$

where \( \varepsilon = 0,1 \) characterizes the Calvo and Rotemberg cases respectively. Note that the inflation effect on dividends is twofold. On the one hand it affects price markups. On the other hand it generates "inflation adjustment" costs which reduce dividends distributed to households.

Individual consumption levels are

$$c^r = wh = \frac{(1 - \vartheta)}{\mu_{p,X} \left(1 + \nu \left(1 - \frac{1}{R} \right)\right)} y$$

35
\[
c^0 = c^{rt} \left( 1 + \frac{\nu}{(1-\Omega)} \left( 1 - \frac{1}{R} \right) \right) + \left( \frac{\nu}{(1-\Omega)} K \right) + \frac{\nu}{(1-\Omega)} d^o
\]
\[
= c^{rt} \left( 1 + \frac{\nu}{(1-\Omega)} \left( 1 - \frac{1}{R} \right) \right) + \frac{(\nu^k - \delta)}{(1-\Omega)} K + \frac{1}{(1-\Omega)} \left( \frac{\mu p, X - 1}{\mu p, X} \pi y \right) \left( \frac{(\pi - 1)^2}{\mu p, X} \right) y
\]

where \( \frac{K}{y} = \left( \frac{\mu p, X \nu^k}{\sigma} \right)^{-1} \). As a result:

\[
\frac{c^{rt}}{c^0} = \frac{1}{\left[ \frac{\nu}{(1-\Omega)} \left( 1 - \frac{1}{R} \right) \right] + \left( \frac{\nu}{(1-\Omega)} \left( \frac{\mu p, X \nu^k}{\sigma} \right)^{-1} + \frac{1}{(1-\Omega)} \left( \frac{\mu p, X - 1}{\mu p, X} \pi y \right) \left( \frac{(\pi - 1)^2}{\mu p, X} \right) \frac{\mu p, X (1+\nu(1-\pi))}{(1-\pi)} \right]}
\]

\[
\frac{c^{rt}}{c^0} = \frac{1}{\left[ \frac{\nu}{(1-\Omega)} \left( 1 - \frac{1}{R} \right) \right] + \left( \frac{\nu}{(1-\Omega)} \left( \frac{\mu p, X \nu^k}{\sigma} \right)^{-1} + \frac{1}{(1-\Omega)} \left( \frac{\mu p, X - 1}{\mu p, X} \pi y \right) \left( \frac{(\pi - 1)^2}{\mu p, X} \right) \frac{\mu p, X (1+\nu(1-\pi))}{(1-\pi)(1-\theta)} \right]}
\]