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Uncertainty measures from partially rounded probabilistic forecast surveys

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Abstract

Although survey-based point predictions have been found to outperform successful forecasting models, corresponding variance forecasts are frequently diagnosed as heavily distorted. Forecasters who report inconspicuously low ex-ante variances often produce squared forecast errors that are much larger on average. In this paper, we document the novel stylized fact that this variance misalignment is related to the rounding behavior of survey participants. Rounding may reflect the fact that some survey participants employ a rather judgmental approach to forecasting as opposed to using a formal model. We use the distinct numerical accuracies of panelists' reported probabilities as a means to propose several alternative and easily implementable corrections that i) can be carried out in real time, i.e., before outcomes are observed, and ii) deliver a significantly improved match between ex-ante and ex-post forecast uncertainty. According to our estimates, uncertainty about inflation, output growth and unemployment in the U.S. and the Euro area is higher after correcting for the rounding effect. The increase in the share of non-rounded responses in recent years also helps to understand the trajectory of survey-based average uncertainty during the years since the financial and sovereign debt crisis.

JEL classification: C32, C52, C53, C83

Keywords: Survey data, probabilistic forecasting, rounding, macroeconomic uncertainty.

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1 Introduction

Forecasts that dispense with uncertainty bands are increasingly regarded as incomplete. It has been argued that to express how strongly a point prediction is expected to deviate from the ex-post observed outcome, point forecasts should be complemented by a quantification of ex-ante uncertainty (Dawid, 1984; Bruine de Bruin et al., 2010). While it has been documented that survey forecasts for inflation, GDP growth or unemployment outperform model-based forecasts (cf. Ang et al., 2007; Faust and Wright, 2009), the informative content of survey predictions for the conditional variance has been recently contested, e.g., by Clements (2016). In case of the Survey of Professional Forecasters (SPF) that is conducted by the Federal Reserve Bank of Philadelphia (FED) and the European Central Bank (ECB), point forecasts are elicited along with probabilistic forecasts in the form of histograms. This allows to derive a measure of ex-ante uncertainty by computing the variance of the reported histograms. Several desirable properties of this index have been documented. For example, Lahiri and Sheng (2010) show that the cross-sectional average variance increases with the forecast horizon. However, it has been found that the ex-ante variance (in our terms, 'uncertainty') deviates considerably from the average squared ex-post forecast error. This finding is sometimes interpreted as evidence for 'over- or underconfidence' (Kenny et al., 2014, 2015; Clements, 2014). The term 'overconfidence' might in this context either be understood as a reflection of forecasters' inherent characteristics or rather as a mere description of an ex-ante variance that is small compared to a predefined benchmark such as the ex-post squared forecast error. Regardless of the interpretation, this finding suggests that the average variance of the SPF histograms as proposed by Zarnowitz and Lambros (1987) has to be interpreted with caution.

In this paper, we ask under which conditions the second moments from the SPF data are relatively well aligned with the variability of prediction errors. The derivation of an ex-ante measure of forecast uncertainty that takes potential distortions into account is difficult since the survey data does not contain any covariates that might help to understand forecasters' behavior.[1] Thus, hypotheses about the dependence of individuals' reported ex-ante uncertainty on misperceptions of their own capability to forecast cannot be easily examined empirically. Against the background of these difficulties, we propose to relate the ex-ante variance of forecasters to the properties of the predictions themselves which are observed prior to the outcome.

A misalignment of ex-ante and ex-post forecast variances has been documented empirically by Giordani and Söderlind (2003, 2006), Kenny et al. (2014, 2015) and Clements (2014). Our main finding is that these deviations of survey participants' forecast uncertainty prior to and after the outcome can partially be ascribed to the response pattern

[1] One notable exception is a categorical variable in the FED-SPF data that reports whether forecasters are employed in the financial services industry, a research institute or any other employer.

of a large group of forecasters that provide their histogram predictions in a particular form. A striking feature of this group is that their forecasts are conveyed in a rather coarse form, with apparently strongly rounded numbers and a relatively low number of probability categories that are assigned nonzero numbers. An example of this is depicted in Figure 1.

[FIGURE 1 HERE]

The subfigures show histogram forecasts for the annual inflation rate in 2016 reported by two participants in the 2016Q4 survey wave of the ECB-SPF. Two differences are apparent. First, the forecasted probabilities in Example A are multiples of 10%, whereas those in Example B do not seem to have a common divisor. Second, the number of outcome intervals that contain nonzero probability numbers is considerably smaller in the left graph. In other words, the right histogram exhibits larger variance. Moreover, Figure 2 summarizes the share of probabilities that contain between one and ten decimal numbers out of all reported probability numbers in the SPF data, pooled across forecasters, time periods and forecast horizons. To improve readability, the '0'-category is omitted from the graph.

[FIGURE 2 HERE]

The left panel of Figure 2 shows that the ECB-SPF contains two clearly separated groups of forecasters that are distinguished both in terms of the number of bins for which they fill in nonzero numbers and the number of digits in their numerical values. The right part of the figure shows the counterpart for the case of the FED-SPF. As it is suggested in Figure 1, separating the two groups, we find that the ex-ante variances of those forecasters who report more strongly rounded numbers are substantially smaller than those of survey participants who appear to round less or not at all. Moreover, the ex-ante and ex-post uncertainties of the non-rounding group of forecasters are clearly more in line with each other than in the case of the group which reports strongly rounded histogram probabilities. This holds for both the ECB- and the FED-SPF. However, the number of responses that entail a large number of digits is substantially larger in the former than in the latter.

There are several potential explanations for this finding. First, the degree of coarseness of the forecasted probability numbers might be related to individuals's choices whether to employ a formal model to derive the histogram forecasts or to rely primarily on less formal considerations. In the latter case, one might speak of 'judgmental forecasting'. This possibility is discussed in Section 5, where we examine the results from two special questionnaires of the ECB-SPF that indicate that two groups of survey participants can be separated based on the degree to which they rely on formal models to arrive at forecasts. Interestingly, the size of the group which relies on models as opposed to judgment roughly equals the size of the non-rounders, respectively rounders. Although different degrees of formalization in forecasters' conceptual frameworks might be the most intuitive explanation, other reasons for the observed data patterns may also play a certain role. In particular, a second explanation is that the coarseness of the reported numbers may reflect specifics of the survey design. The FED-SPF is elicited in a rather traditional way by asking respondents to fill in the questionnaire by means of paper-and-pencil. In contrast, the ECB-SPF questions can be answered on the computer via an Excel spreadsheet. Hence, it is conceivable that respondents find it substantially easier to report numbers with many digits in the case of the ECB-SPF. Third, rounding itself could to an extent reduce the number of bins with nonzero numbers if survey participants would round small tail probabilities towards zero.

As mentioned above, it would be difficult to test such hypotheses empirically due to the absence of explanatory variables in the SPF data. Instead, this paper aims at establishing a reliable means to adjust a measure of aggregate uncertainty for the marked influence of coarse histogram forecasts. We show that there exists a pervasive correlation between the rounding behavior of individuals and their respective variance misalignment. Thus, rounded probability numbers are a pertinent indicator for the subgroup of histogram forecasts that show the sort of coarseness which gives rise to suspiciously low ex-ante variances at certain forecast horizons. This provides a reliable way to single out these forecasts since the classification is essentially unaffected by distinct ways to define rounding.

Our findings have three implications that are important for users of histogram-based uncertainty measures. First, the distortion of an index of overall uncertainty that is computed as the average across the individual variances (Lahiri and Sheng, 2010; Lahiri et al., 2015) can be reduced ex-ante by focusing on forecasts that are non-rounded. Second, the trajectory of average uncertainty during recent years is at least partly affected by the overall increase in the share of forecasters who do not report strongly rounded numbers. Third, an improvement in the identification of rounders vs. non-rounders would be possible if survey participants were given the opportunity to state if their responses were rounded or not by asking them to comment on this issue in the questionnaire as it has been suggested by Manski and Molinari (2010).

We conclude that while uncertainty is likely higher than what is reflected by the average forecast variance due to the presence of considerable rounding, the increase in uncertainty during the years after the financial crisis is likely overstated due to changes in the composition of the SPF panel.

The remainder of this paper is structured as follows. After briefly reviewing the related literature in Section 2, the data are introduced in Section 3. We discuss the categorizations that are used to classify survey participants as rounders or non-rounders in Section 4. In Section 5, we analyze the size of both groups in the SPF data and examine the potential connection between rounding and judgmental forecasting. Next, the findings regarding the performance of the histogram forecasts are presented. Based on our results we discuss potential deficiencies of aggregate forecast uncertainty as it is measured with the SPF data and highlight a way to derive a more meaningful uncertainty measure. Finally, a comparison of our results with those from a related study by Binder (2017) is provided. Section 6 summarizes and concludes.

2 Rounding and the information content of histogram forecasts in the related literature

Though surveys like the SPF have become a popular data source to quantify forecast uncertainty, it is not well understood to what extent numerical inaccuracies such as rounded numbers may distort the variance of histogram forecasts. Heitjan and Rubin (1991) discuss the implications of rounding and similar forms of incomplete survey responses on the likelihood of parameter estimates that are based on survey data. Similarly, Tay and Wallis (2002) note that the communication of uncertainty from survey-based density forecasts faces several distinct problems.[2] Some of the crucial steps like the design of the survey questionnaire, the timing of the elicitation process, the production and reporting of forecasts by survey participants as well as the interpretation and evaluation by users of the survey may introduce distortions in the conveyed information.

The question we address in this paper is how rounding may affect ex-ante and ex-post measures of forecast uncertainty. We are particularly interested in the implications of the observation that forecasters who provide strongly rounded responses also show a tendency to provide narrow histograms with only a small number of outcomes to which they attach nonzero probabilities. It has been previously noted that such response behavior may affect conditional second moment statistics from survey data. For example, Boero et al. (2015) interpret the decision of forecasters to round the probabilities of surveys histograms as an expression of what they call 'uncertain uncertainty'. Other studies such as Manski and Molinari (2010) also highlight the importance of rounding choices on the outcomes of histogram forecasts as they are provided by the SPF.

A distinct approach is taken by Binder (2017), who derives an index of inflation uncertainty based on rounding outcomes in a survey of consumer expectations. The construction of the index in Binder (2017) is based on the assumption that rounding can be seen as an expression of uncertainty. This is also reflected in Bruine de Bruin and Carman (2012) or Ruud et al. (2014). These hypotheses regarding the link between rounding and uncertainty connect to the more general literature which discusses rounding and other forms of *data coarsening* (Heitjan and Rubin, 1991; Ruud et al., 2014). We do not employ rounding as the single source of information regarding uncertainty, but derive a direct measure of uncertainty based on the SPF histograms. This enables us to discuss potential distortions from rounding in the computation of the resulting uncertainty index.

[2] We use the terms 'density forecast' and 'histogram forecast' synonymously throughout.

In a recent paper, Clements (2016) examines the informative content of density forecasts in terms of their capability to deliver variance forecasts and concludes that the SPF data provided by the ECB contains little reliable information beyond the forecast for the conditional mean. In the current study, we draw upon such findings and examine to what extent the misalignment between ex-ante uncertainty and ex-post forecast performance can be linked to the tendency to concentrate the entire probability mass in a small share of the outcome intervals from the survey questionnaire. In a related study, Clements (2011) documents that the mismatch between the reported probabilities of a decline in output growth and corresponding probabilities derived from the histogram forecasts can be partially explained by the rounding choices of the forecasters in the FED-SPF. Since more than 75% of the SPF participants' responses appear to be rounded to some extent, it is crucially important to investigate the implications of this particular data feature for the assessment of macroeconomic uncertainty.

3 Data

In this section, the data used to quantify ex-ante and ex-post uncertainty in both the Euro area and the U.S. are described.

The survey data are provided by the SPF of the ECB and the U.S.-FED. Both surveys elicit point and density forecasts of future inflation, real GDP growth and unemployment rates in the Eurozone and the U.S. at the quarterly frequency. For inflation and output growth, the outcome variable x_t refers to year-on-year growth rates, i.e.,

$$x_t = 100 \times \left(\frac{X_t}{X_{t-1}} - 1 \right), \tag{1}$$

where X_t denotes the annual average of either the respective price index or real GDP in year $t = 1, \dots, T$.[3] In the case of the unemployment rate, x_t is calculated as the annual average over the civilian unemployment rates that are observed at the monthly frequency, i.e., $x_t = X_t$. Data on the realizations for the Euro area and the U.S. are drawn from the Statistical Data Warehouse of the ECB and the Real-Time Data Set for Macroeconomists of the Federal Reserve Bank of Philadelphia, respectively.[4,5] Both databases provide data vintages for all outcome variables. For each vintage, we calculate X_t in all cases where consecutive observations for each month (Harmonized Index of Consumer Prices, unemployment rate) or quarter (GDP price index, real GDP) of year

[3] The ECB-SPF inflation forecasts refer to the monthly Harmonized Index of Consumer prices. For the FED-SPF we use the quarterly chain-weighted GDP price index. We prefer GDP inflation over CPI inflation because density forecasts for the latter are only available since 2007 in the FED-SPF, whereas predictions for the former are available for the entire sample period. For the computation of output growth, we use quarterly real GDP.

[4] `http://sdw.ecb.europa.eu/`

[5] `https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data`

t are available and compute x_t. In the empirical analysis, we employ the first-releases of x_t, which are most closely related to the information available to forecasters when they produce their predictions. Moreover, Jo and Sekkel (2018) show that ex-post forecast variances based on the most recent data vintage tend to be underestimated.

The survey data from the SPF consist of so-called 'fixed-event' density forecasts, which are characterized by a fixed target year t and a quarterly forecast horizon h. The nature of these forecasts implies that h diminishes in each consecutive quarter in which the survey is conducted until the arrival of the realization in t. We consider both the predictions for the current and the next year. This obtains a sequence of individual h-step-ahead density forecasts with forecast horizons $h \in \{8, 7, \ldots, 1\}$ as depicted in Table 1.

[TABLE 1 HERE]

In the case of the inflation rate and output growth, forecasters in our sample target the years 2000 to 2017. This means that the time period when forecasts are made and collected ranges from 1999Q1 to 2017Q4.[6] Density forecasts for the unemployment rate in the FED-SPF are available only since 2009Q2, whereas the responses in the ECB-SPF are available for the entire sample period. For the U.S., we thus focus on the unemployment rates in the years 2011 to 2017, for which h-step-ahead predictions are available for each forecast horizon.

In the questionnaire, survey participants $i = 1, \ldots, N$ are requested to assign probabilities to a prespecified number of outcome intervals, the so-called 'bins'. Let $p_{i,k,t,h} \in [0, 100]$ for $k = 1, \ldots, K$ denote the probability number assigned to the k-th bin. The bins have a width of 0.4 percentage points in case of the ECB-SPF as can be seen in Figure 1. In the FED-SPF, the bin width is 0.9 percentage points except in a few cases.[7] As in Abel et al. (2016), the gaps between the interior bins are closed by extending the lower and upper bound of each bin by 0.05 percentage points. This seems to be in line with how most of the survey participants interpret their reporting task, as it is documented in a special survey conducted by the ECB, where 76% of the SPF participants stated that they interpret an interval like $[1.5, 1.9]$ to actually indicate a range as given by $[1.45, 1.95]$ (ECB, 2009). The bins at the lower and upper end of the support are assumed to have twice the width of the interior intervals, i.e., one or two percentage points depending on the survey and variable. The bounds of the individual histograms are fixed at the leftmost

[6]Forecasts for inflation and output growth in the U.S. are available since 1968Q4. However, we prefer to focus on a common sample period for both the ECB- and FED-SPF and exclude these earlier predictions. This also helps to avoid various methodological changes in the FED-SPF such as the switch from gross national product to gross domestic product. Since no five- to eight-step-ahead forecasts for the year 1999 are available in the ECB-SPF data, we exclude the current year predictions from the surveys conducted between 1999Q1 and 1999Q4. Similarly, no one- to four-step-ahead forecasts for the year 2018 are available in both surveys.

[7]Since 2014Q1, the bin width for inflation is 0.4 percentage points. Similarly, the interior bins for the unemployment rate have a width of 0.4 percentage points throughout the sample period.

and rightmost bin with nonzero probability mass. Moreover, the maximum range covered by the bins differs across surveys, outcome variables and time instances.[8]

We exclude observations from the sample whenever the sum over the reported probabilities deviates by at least 0.9 percentage points from the required 100% overall probability in absolute terms.[9] Moreover, there is a small group of forecasters that assign 100% to a single bin.[10] To find out if this affects our conclusions, we conducted the empirical analysis with and without these histograms and found the difference in results to be negligible in most cases. Thus, we present our findings based on the full sample of observations unless stated otherwise.[11]

The participants in both surveys include employees of research institutes and the financial services industry.[12] The occupation of the anonymous survey participants is provided in the case of the FED-SPF. Depending on the survey period under consideration, 22-50% of the participants of the FED-SPF are classified as 'financial service providers' and 39-70% as 'non-financial service providers'. A third category of unclassified 'others' is also included, which amounts to 0-15% of the cross-section. In the case of the ECB-SPF, this information is not provided. An identification number allows to track the anonymous individual forecasters. We observe a relatively large number of entries and exits of SPF participants in each survey round. In order to analyze whether participation varies systematically across forecast horizons, we define the participation indicator variable $D^{\mathcal{P}}_{i,t,h}$, which is equal to unity if forecaster i issues an h-step-ahead density forecast for x_t, and zero else. For each forecast horizon $h \in \{8, 7, \ldots, 1\}$, Table 2 displays the number of density forecasts reported in both versions of the SPF, i.e., $\sum_{i=1}^{N} \sum_{t=1}^{T} D^{\mathcal{P}}_{i,t,h}$.

[TABLE 2 HERE]

The sample size is roughly constant across variables and forecast horizons in both surveys with the obvious exception of the unemployment rate in the FED-SPF. This suggests that the cross-section of forecasters is relatively similar. Although the total number of participants is higher in the FED- than in the ECB-SPF (116 versus 104), the sample size for both inflation and real GDP growth is considerably larger in the latter case. In other

[8] A summary of the definitions and ranges of the bins in the ECB-SPF is available at https://www.ecb.europa.eu/stats/prices/indic/forecast/shared/files/SPF_dataset_description.pdf?e62bee48524d7af2dac96bbf8d72a201. A similar description for the FED-SPF is provided at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

[9] We permit small deviations in order to keep the non-rounded histograms in the sample. In such cases the probabilities may not add up to exactly 100%.

[10] Approximately 1% of the histograms submitted to the ECB-SPF and around 2% in the FED-SPF.

[11] Results based on a sample that excludes all single-bin histograms will be provided upon request.

[12] A partial list of the participants in the FED-SPF is available at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/2012/spfq412.pdf. For the ECB-SPF, see https://www.ecb.europa.eu/stats/ecb_surveys/survey_of_professional_forecasters/html/index.en.html → 'About the survey' → 'Institutions which have contributed to the SPF'

words, average participation is lower in the FED-SPF. Between 1999Q1 and 2017Q4, the number of forecasters who contributed to the ECB-SPF has declined from 63 to 50. Over the same time period, the number of participants in the FED-SPF is relatively constant. In particular, 28 forecasters have submitted predictions in 1999Q1 compared to 34 in 2017Q4.

In order to compute first and second moments of the histograms, it is common to assume that the entire probability mass within each bin is located at the midpoint (Lahiri et al., 1988; Kenny et al., 2015). Alternatively, one may consider uniformly distributed distributions within bins or compute the moments of a smoothed density function as it is done in Engelberg et al. (2009) or Glas and Hartmann (2016). However, this choice has little impact on either the first or second moments. Thus, we focus on the 'mass-at-midpoint' assumption below.[13] Based on this approach, the mean of forecaster i's histogram is given by

$$\mu_{i,t,h} = \frac{1}{100} \sum_{k=1}^{K} p_{i,k,t,h} \times m_k, \tag{2}$$

with m_k denoting the midpoint of the k-th bin. The h-step-ahead 'consensus' forecast is calculated as the equally-weighted average over the individual histogram means, that is,

$$\bar{\mu}_{t,h} = \frac{1}{N} \sum_{i=1}^{N} \mu_{i,t,h}. \tag{3}$$

In order to analyze which data release is predicted by the SPF participants, Figure 3 depicts the realizations of each outcome variable in the Euro area and the U.S. using observations from both the first release (solid line) and the most recent data vintage (dashed line). Moreover, each plot includes the consensus forecasts, i.e., $\bar{\mu}_{t,h}$ from Eqn. (3), for horizons $h \in \{8, 7, \ldots, 1\}$. The one- and eight-step-ahead predictions are highlighted distinctly from the other forecast horizons.

[FIGURE 3 HERE]

The evidence from Figure 3 shows that the accuracy of the average forecast improves as the target period approaches. In other words, forecast errors decline with h. In particular, the deviation between x_t and $\bar{\mu}_{t,1}$ is smaller than the difference between x_t and $\bar{\mu}_{t,8}$ in almost all cases. Moreover, in cases where the first and last data releases deviate substantially, $\bar{\mu}_{t,1}$ is more closely associated with the former. This finding suggests that participants of the SPF predict the first release of the respective outcome variable. This supports our choice of focusing on this particular data release in the empirical analysis. However, using last-release data has little impact on the empirical findings.

[13] Results based on other distributional assumptions are provided upon request.

To compare the mismatch between ex-ante and ex-post uncertainty, we need a quantification of the variances of the reported histograms that enables us to retain the information regarding the rounding choices of forecasters. Based on the means from Eqn. (2), we calculate the individual variance as

$$\sigma^2_{i,t,h} = \frac{1}{100} \sum_{k=1}^{K} p_{i,k,t,h} \times (m_k - \mu_{i,t,h})^2. \tag{4}$$

This variable serves as a measure of forecaster i's ex-ante uncertainty. To obtain an indicator of aggregate uncertainty, we follow Lahiri and Sheng (2010) and compute the cross-sectional average of the h-step-ahead variances from Eqn. (4),

$$\overline{\sigma^2_{t,h}} = \frac{1}{N} \sum_{i=1}^{N} \sigma^2_{i,t,h}. \tag{5}$$

Analogously to Figure 3, Figure 4 depicts the time series of the h-step-ahead average forecast variances.

[FIGURE 4 HERE]

Average ex-ante uncertainty declines with the forecast horizon, i.e., the average forecaster becomes increasingly more confident as the target period approaches and more information about the realization is available. Moreover, an increase in average uncertainty is visible in most cases after the outbreak of the financial crisis in 2008. Owing to an adjustment of the bin definitions in 2014Q1, a break in the time series of ex-ante uncertainty is visible for the predictions of the inflation rate in case of the FED-SPF. However, there is almost no effect on our results if the data for the years 2014-2017 is discarded. Thus, these observations remain in the sample.

So far, we have described the characteristics of the entire cross-section of SPF participants in both the U.S. and the Euro area. However, it may be that panelists differ systematically with respect to the coarseness of their predictions. In the next step, we aim to isolate two distinct groups of forecasters based on the way that individual survey participants decide to round (or not to round) the reported probability numbers.

4 Classification of survey participants

In this section, we discuss alternative classification schemes that serve as a means to distinguish non-rounders from rounders based on their reporting behavior.

Though rounding is one of the most striking characteristics of the histogram forecasts in the SPF, an unambiguous classification into rounders and non-rounders is not possible. Since the coarseness of the responses appears to vary across individual forecasters, we propose several distinct categorization schemes in order to assess the robustness of our findings. Due to the anonymous nature of participation in the SPF, reputational concerns should not play an important role in the decision whether or not to round a prediction. In most empirical research on rounding of survey-based forecasts, the participants are classified as rounders based on whether the point forecast is a multiple of a particular integer number (e.g., Binder, 2017). In contrast, we analyze the histograms reported in the SPF. Thus, the employed rounding schemes are based on multiple reported numbers for each individual, instead of just a single one. Moreover, we consider two distinct types of categorizations that differ in terms of what constitutes a rounded probability.

4.1 Decimal-based categorization

The first type of categorization is based on the number of decimals of each probability number, $p_{i,k,t,h}$, which is denoted as $d_{i,k,t,h}$. For notational convenience, we suppress all subscripts except for i and k in the following subsections.[14] Let $K_i \in \{1, \ldots, K\}$ denote the number of bins to which forecaster i assigns nonzero probability, i.e., cases where $p_{i,k} > 0$. Similarly, $K_i^\star \in \{0, \ldots, K_i\}$ indicates the number of bins with nonzero probability that contain decimals numbers, i.e., cases where both $p_{i,k} > 0$ and $d_{i,k} > 0$. The share of probabilities in forecaster i's histogram that contain nonzero decimals numbers is thus given by

$$\rho_i = \frac{K_i^\star}{K_i}. \tag{6}$$

Based on ρ_i, we define distinct classification schemes that are introduced here in terms of how strictly we delineate the definition of a non-rounder. That is, each of the rules that are successively introduced below is less likely to classify a forecaster as a non-rounder than the previous one. The first approach is to treat a forecaster as a non-rounder if *any* of the individually reported probability numbers are stated by means of using decimals, that is,

$$D_i^{\text{any}} = \begin{cases} 1 & \text{if } \rho_i > 0 \text{ and} \\ 0 & \text{else.} \end{cases} \tag{7}$$

It is likely that this indicator will classify some forecasters as non-rounders even though the majority of reported numbers entail a rather strong degree of rounding. Consider an example where five bins are available, i.e., $K = 5$, and a survey participant reports probabilities $(p_{i,1}, \ldots, p_{i,5})' = (0.5\%, 30\%, 39\%, 30\%, 0.5\%)'$, such that $K_i = 5$ and $K_i^\star = 2$. Despite the fact that only the probabilities in the tails include decimals, such a forecaster is considered as a non-rounder based on D_i^{any} since $\rho_i = 0.4$. A more restrictive rule to

[14] This does not mean that we assume that variation across time or forecast horizons plays no role. We analyze the importance of these factors in Section 5.

single out non-rounders is obtained if a survey participant is regarded as a non-rounder if *most* of the probabilities are reported with nonzero decimal numbers, i.e.,

$$D_i^{\text{most}} = \begin{cases} 1 & \text{if } \rho_i > 0.5 \text{ and} \\ 0 & \text{else.} \end{cases} \tag{8}$$

This approach categorizes forecasters as non-rounders if more than 50% of the probabilities reported in a given histogram contain decimal numbers. Note that if K_i is even and half of the probabilities contain decimals while the other half do not, i.e., if $K_i^\star = K_i/2$, the scheme in Eqn. (8) classifies a survey participant as a rounder. Based on this categorization, the forecaster from the example above is considered to be a rounder because only 40% of the probabilities contain decimal numbers. The most restrictive approach is to classify a forecaster as a non-rounder if *all* probabilities are stated by means of nonzero decimal numbers, that is,

$$D_i^{\text{all}} = \begin{cases} 1 & \text{if } \rho_i = 1 \text{ and} \\ 0 & \text{else.} \end{cases} \tag{9}$$

In this case, forecasters are only considered to be non-rounders if each probability number is stated with nonzero decimal numbers, i.e., cases where $K_i^\star = K_i$. Based on the scheme in Eqn. (9), the forecaster from the example above is considered as a rounder because three out of five probabilities do not contain decimal numbers.

To summarize, the categorizations described in Eqns. (7)-(9) classify survey participants as rounders if any, most, or all of the probabilities are stated with nonzero decimal numbers. It thus follows that $\sum_{i=1}^N D_i^{\text{any}} \geq \sum_{i=1}^N D_i^{\text{most}} \geq \sum_{i=1}^N D_i^{\text{all}}$.

4.2 Integer-based categorization

In the first part of her empirical analysis, Binder (2017) classifies consumers as rounders based on whether their point forecast is a multiple of five. Similarly, Manski (2004) notes that probabilistic forecasts are frequently multiples of an integer number. For example, D'Amico and Orphanides (2006), Engelberg et al. (2009) or Clements (2011) observe that the probabilities reported in the FED-SPF tend to be multiples of five or ten. Boero et al. (2015) documents similar evidence for the predictions from the Survey of External Forecasters. A similar integer-based approach is considered here, which contrasts with the previous categorization that classifies survey participants based on whether the reported probabilities contain decimal numbers. In order to analyze whether the decimal- and integer-based approaches yield comparable results in isolating rounders and non-rounders, we analyze whether the probability in the k-th bin of forecaster i's histogram is a multiple of integer $\tau \in \mathbb{N}$ by defining

$$\tilde{D}^{\mathrm{m}\tau}_{i,k} = \begin{cases} 1 & \text{if } \tau \cdot \left\lfloor \frac{p_{i,k}}{\tau} \right\rfloor = p_{i,k} \text{ and} \\ 0 & \text{else,} \end{cases} \tag{10}$$

where $\lfloor p_{i,k}/\tau \rfloor$ is the integer part of $p_{i,k}/\tau$. Based on the bin-specific indicator variables from Eqn. (10), forecasters are classified as rounders according to the following rule:

$$\tilde{D}^{\mathrm{m}\tau}_{i} = \begin{cases} 1 & \text{if } \mathrm{mode}(\tilde{D}^{\mathrm{m}\tau}_{i,1}, \ldots, \tilde{D}^{\mathrm{m}\tau}_{i,K}) = 1 \text{ and} \\ 0 & \text{else.} \end{cases} \tag{11}$$

Thus, a survey participant is treated as a rounder if the majority of the probabilities are multiples of τ. If the modal value in Eqn. (11) is not uniquely defined, we set $\tilde{D}^{\mathrm{m}\tau}_{i}$ to zero. Thus, if half of the probabilities are multiples of τ, but the other half are not, the corresponding forecaster is considered a non-rounder. Note that $\tilde{D}^{\mathrm{m}\tau}_{i}$ is used to isolate rounders, whereas the decimal-based categorizations in Eqns. (7)-(9) isolate non-rounders. In order to faciliate the comparison between both approaches, we use

$$D^{\mathrm{m}\tau}_{i} = 1 - \tilde{D}^{\mathrm{m}\tau}_{i} \tag{12}$$

in most cases instead of $\tilde{D}^{\mathrm{m}\tau}_{i}$. Thus, forecasters are considered to be non-rounders if most of the probabilities are *not* multiples of τ. In reference to the evidence documented in Boero et al. (2015) that many of the probabilities submitted to the SPF are multiples of five or ten, the forecaster considered in the example from the previous subsection is classified as a non-rounder based on both $D^{\mathrm{m}5}_{i}$ and $D^{\mathrm{m}10}_{i}$ since only two out of the five probabilities are multiples of either five or ten.

5 Rounding patterns and variance misalignment

In this section, we characterize and distinguish the groups of rounders and non-rounders in the SPF based on the methodology from Section 4. We document that this categorization helps to understand the finding of a mismatch between the ex-ante and ex-post uncertainties of individual forecasters. In order to test if variance misalignment and rounding choices are systematically related, inferential results regarding the differences in the histogram characteristics of rounders and non-rounders are reported.

5.1 Rounders and non-rounders in the SPF data

To investigate which reasons can be considered as viable explanations for the observation that survey responses are coarse in distinct ways, we first employ the cross-sectional dimension to examine how pervasive the habit of reporting rounded probability numbers is among the SPF panelists. Comparing the relative size of the rounding versus non-rounding categories, as well as conditioning on time periods or forecast horizons, provides tentative explanations for the observed response patterns. Moreover, special survey questions that are provided in the ECB-SPF regarding the use of formal models versus judgment can be related to the relative size of the two groups of forecasters. For the sake of brevity, we choose to focus on one of the decimal-based categorizations and consider the integer-based approach for one particular value of τ in the following subsections. However, the results from the empirical analysis are robust to the choice of the considered categorization.

Figure 2 shows that relatively few participants in the FED-SPF state their probabilities in terms of decimal numbers. In contrast, the share of probabilities that contain decimal numbers is considerably larger in the ECB-SPF. Moreover, the participants in the FED-SPF use a relatively narrow range of at most four decimals, whereas the panelists in the ECB-SPF use up to ten. This may be due to systematic differences in either the cross-section or the structure of both surveys such as the differences in the bin width. Based on the small number of probabilities with $d_{i,k,t,h} > 0$ in case of the FED-SPF, we choose to focus on $D^{\text{any}}_{i,t,h}$ as the preferred decimal-based classification scheme. This is recommendable since the explanatory power of the distinction between rounders and non-rounders may be reduced due to the smaller number of forecasters that are classified as non-rounders based on $D^{\text{most}}_{i,t,h}$ and $D^{\text{all}}_{i,t,h}$.

The choice of τ for the integer-based categorization is guided by the evidence from Figure 5, which depicts the share of rounded histograms in the SPF data based on Eqn. (11) for a pooled sample of observations across all time periods and forecast horizons in the case of inflation (first row), output growth (second row) and unemployment rates (third row). This share is calculated as 100 times the number of rounded histograms that are classified by means of $\tilde{D}^{\text{m}\tau}_{i,t,h}$ for $\tau \in \{1, \ldots, 10\}$ divided by the total number of reported predictions, i.e.,

$$\tilde{S}^{\text{m}\tau} = 100 \times \frac{\sum_i \sum_t \sum_h \tilde{D}^{\text{m}\tau}_{i,t,h}}{\sum_i \sum_t \sum_h D^{\mathcal{P}}_{i,t,h}}. \tag{13}$$

[FIGURE 5 HERE]

The results are remarkably similar across all outcome variables and both versions of the SPF. The majority of survey participants are classified as rounders if we set τ to unity, i.e., most histogram forecasts consist of probabilities that almost exclusively do not contain decimal numbers. This is not surprising given that Figure 2 shows that only a small fraction of the SPF participants reports probabilities with decimal numbers. There are two notable spikes in the cases where τ is set to either five or ten. This squares with the evidence documented in Engelberg et al. (2009) and Boero et al. (2015), who show that many of the probabilities reported in surveys of macroeconomic expectations are multiples of five or ten. In particular, 74-79% of all histograms in the SPF data consist of probabilities that are for the most part multiples of five. Similar numbers are reported in Clements (2011). Thus, we isolate non-rounders by setting τ to five and use $D^{\mathrm{m5}}_{i,t,h}$ in the following analysis due to the fact that the share of rounded histograms is particularly large in this case.

We observe a considerable overlap in the groups of rounders and non-rounders classified by the decimal- and integer-based approaches. Depending on the outcome variable, 86-87% (81-84%) of all histograms in the ECB-SPF (FED-SPF) are unanimously classified as either rounded or non-rounded by $D^{\mathrm{any}}_{i,t,h}$ and $D^{\mathrm{m5}}_{i,t,h}$. Thus, the choice of the employed categorization has little impact on the status of individual survey participants.

The classification of forecasters is consistent across outcome variables. For the ECB-SPF, the share of predictions that are classified as either rounded or non-rounded across all three outcome variables is 94% for $D^{\mathrm{any}}_{i,t,h}$ and 88% for $D^{\mathrm{m5}}_{i,t,h}$. The corresponding numbers for the FED-SPF are 95% and 79%, respectively. This finding suggests that variable-specific considerations do not play an important role in the rounding choices of the panelists.

One explanation for the decision to round a forecast may be the amount of information that is available to all forecasters at the time a prediction is made rather than systematic differences between certain groups of panelists. In a fixed-event setting, the information set of a survey participant increases as h declines. In order to analyze the size of the groups of rounders and non-rounders, Table 3 summarizes the share of non-rounded observations in the SPF data for each forecast horizon, that is,

$$S^{\mathcal{R}}_h = 100 \times \frac{\sum_i \sum_t D^{\mathcal{R}}_{i,t,h}}{\sum_i \sum_t D^{\mathcal{P}}_{i,t,h}}, \tag{14}$$

where $D^{\mathcal{R}}_{i,t,h} \in \{D^{\mathrm{any}}_{i,t,h}, D^{\mathrm{m5}}_{i,t,h}\}$ denotes either the preferred decimal- or integer-based rounding scheme described in Eqns. (7) and (12), respectively.

[TABLE 3 HERE]

Table 3 shows that the share of non-rounded observations indicated by $D^{\mathrm{any}}_{i,t,h}$ is relatively small in both surveys. Between 11-14% (ECB-SPF) and 4-10% (FED-SPF) of all histograms consist of probabilities that are stated with decimal numbers and are thus classified as being non-rounded. As discussed before, the larger value of S^{any}_h in the case of the ECB–SPF may be related to the reporting practices in both surveys. The share of non-rounded histograms based on $D^{\mathrm{m5}}_{i,t,h}$ is considerably larger and relatively similar in both versions of the SPF. In particular, S^{any}_h lies between 19-25% (ECB-SPF) and 18-28% (FED-SPF).[15] Notably, the share of non-rounders is stable across forecast horizons. This finding suggests that the decision to round is not merely the result of more information being available as the target period approaches.

[15] Naturally, if the probabilities are stated with decimal numbers, they cannot be multiples of an integer number. Conversely, if the probabilities are not multiples of a particular integer, they do not necessarily contain decimal numbers. Thus, the share of non-rounders isolated via $D^{\mathrm{any}}_{i,t,h}$ is a subset of the share classified by $D^{\mathrm{m5}}_{i,t,h}$.

Although the group of non-rounders is considerably smaller than the group of rounders in both surveys, the evidence documented in Table 3 shows that the share of non-rounders is relatively similar across outcome variables and forecast horizons. As a means to analyze the fluctuations in the status of active forecasters, Figure 6 depicts the time variation in the share of non-rounders for each variable across the predictions for both the current and the next year (defined analoguously to Eqn. (14)). As before, non-rounders are classified in terms of either $D^{\text{any}}_{i,t,h}$ (first row) or $D^{\text{m5}}_{i,t,h}$ (second row).

[FIGURE 6 HERE]

For each variable, the share of non-rounders in the ECB-SPF has considerably increased from approximately 5-15% of the cross-section during the initial years to 30-45% in recent survey periods. Over the same time period, the share of non-rounders in the FED-SPF has also increased, although it rarely exceeds 15% in the case of the categorization via $D^{\text{any}}_{i,t,h}$. In contrast, the share based on $D^{\text{m5}}_{i,t,h}$ is relatively similar in both versions of the SPF. This is in line with the previously documented evidence from Figures 2 and 5, which shows that participants of the FED-SPF rarely state probabilities in terms of decimal numbers, but more frequently not as multiples of five.

Overall, Figure 6 documents an increase in the share of non-rounded histogram forecasts during more recent years. This is partly the result of an increasing number of new entrants to both surveys who are classified as non-rounders.[16] However, incumbent participants' transitions from the rounding to the non-rounding group are also more frequent than transitions in the other direction. In general, such changes in the response pattern for a given identification number might be either due to changes in personnel or reorganizations of the forecasting process within the institutions that participate in the SPF. In particular, it may be the case that rounding choices reflect the fact that some survey participants use formal models to arrive at their forecasts, whereas others rely more on judgment and intuition.

In order to shed light on the reporting practices of its participants, the ECB-SPF conducted two special surveys (ECB, 2009, 2014). Among other questions, respondents were asked if their probability distributions are based on a model, judgment or a mixture of the two. In the first special survey, 79% of the survey participants answered that their reported probabilities are judgment-based, whereas the remaining panelists replied that they are derived from a formal model or a functional form. Interestingly, the fraction of forecasters who stated that they rely entirely on judgment is very close to the relative frequency of rounded observations classified by means of D^{m5} (see Table 3). In the second survey, the share of forecasters who indicated that their reported probabilities are based on judgment varies between 68% for the medium-term inflation and GDP growth forecasts and 79% for the short-term unemployment rate forecasts. On average, the predominance

[16]The participation and status of the individual SPF participants in each survey round is depicted in Figures A.1 and A.2 in the Appendix.

of forecasters who rely on judgment has slightly declined compared to the first special survey. This squares with the increase in the share of non-rounders in recent survey periods depicted in Figure 6.[17] Notably, the share of forecasters who replied that they compute their probabilites only for the SPF (79%), as opposed to producing them for purposes related to their regular work, is the same as the fraction of forecasters who stated that they rely on judgment. Consequently, it is also very similar to the share of rounders as measured by $\tilde{S}^{\mathrm{m5}}$.

It is tempting to examine the link between the results from the special surveys and the rounding choices in the quarterly SPF questionnaires. The questions in the special surveys refer to fixed-horizon forecasts, i.e., predictions with a constant forecast horizon. Thus, we consider the share of non-rounders for the fixed-horizon forecasts reported in the surveys that correspond to the dates when the special surveys were sent out, i.e., 2008Q3 and 2013Q2. Note, however, that the number of forecasters in the 2013Q2 survey (39) does not always match the number of responses from the second special survey in all cases (35-40). The share of non-rounders in 2008Q3 based on D^{m5} (17-30% depending on the variable and horizon; 21% on average) closely mirrors the share of forecasters who reported that they use some sort of model when they report their probabilities (21%).[18] The share of non-rounders classified by means of D^{m5} in 2013Q2 (34-47%; 38% on average) is relatively similar to the fraction of forecasters who replied that they use either a model or a combination of model and judgment in the second special survey (26-33%). Thus, it appears that there is a close association between our distinction of rounders and non-rounders on the one hand and the non-judgment versus judgment-based forecast grouping documented in the special survey of the ECB on the other hand.[19] A more thorough analysis is not possible because the individual responses from the special surveys are not publicly available.

[17]The share of cases where judgment is applied is considerably smaller for the point predictions and rarely exceeds 50% in the first special survey. In the second special survey, the fraction of point predictions based on judgment has further declined. In particular, the share of forecasters who replied that their point forecasts are essentially judgment-based is 35% or less for the forecast horizons of at most three years ahead. The share is considerably larger for the long-term predictions, but remains below 50%. Out of the remaining panelists, 14-28% indicated that their point predictions are model-based, while the remaining 25-60% replied that they use a mixture of judgment and models.

[18]We consider both the category 'econometric model' and what is referred by the ECB as a 'functional form' as cases where forecasters employ some generic form of model.

[19]The FED-SPF also conducted a special survey on the forecasting techniques of its participants in 2009Q4. 80% of the respondents reported that they use a mixture of a model-based approach and judgment. The summary does not specify whether the panelists were asked about the point forecasts, the histograms or their predictions in general. A summary of the results is available at `https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-special-survey-on-forecast-methods.pdf?la=en`.

5.2 Counterfactual rounding experiment

In this subsection, we investigate in which aspects the reported histograms of the non-rounders differ from those of the rounders. Although rounding appears to be related to the reported level of ex-ante uncertainty, rounding is unlikely the only determinant. In order to disentangle the effect of rounding from any additional unobserved influences, we examine how the ex-ante variance of responses that are classified as non-rounded changes after the corresponding probabilities are artifically rounded.

We focus on two features of the histograms which are related to the histogram width and thus ex-ante uncertainty. First, the number of bins to which a forecaster assigns a nonzero probability as represented by the count statistic $K_{i,t,h}$. For both rounders and non-rounders, we calculate the average number of bins used by the individuals in each group,

$$\bar{K} = \frac{\sum_i \sum_t \sum_h K_{i,t,h} \times D^{\mathcal{R}}_{i,t,h}}{\sum_i \sum_t \sum_h D^{\mathcal{R}}_{i,t,h}} \tag{15}$$

and

$$\overline{\tilde{K}} = \frac{\sum_i \sum_t \sum_h K_{i,t,h} \times \tilde{D}^{\mathcal{R}}_{i,t,h}}{\sum_i \sum_t \sum_h \tilde{D}^{\mathcal{R}}_{i,t,h}}, \tag{16}$$

where $\tilde{D}^{\mathcal{R}}_{i,t,h} = 1 - D^{\mathcal{R}}_{i,t,h}$. Second, we consider the variance of the individual histograms, i.e., $\sigma^2_{i,t,h}$ from Eqn. (4). We compute the average variance of each group, i.e.,

$$\overline{\sigma^2} = \frac{\sum_i \sum_t \sum_h \sigma^2_{i,t,h} \times D^{\mathcal{R}}_{i,t,h}}{\sum_i \sum_t \sum_h D^{\mathcal{R}}_{i,t,h}} \tag{17}$$

and

$$\overline{\tilde{\sigma}^2} = \frac{\sum_i \sum_t \sum_h \sigma^2_{i,t,h} \times \tilde{D}^{\mathcal{R}}_{i,t,h}}{\sum_i \sum_t \sum_h \tilde{D}^{\mathcal{R}}_{i,t,h}}. \tag{18}$$

Note that it is unclear from an ex-ante point of view whether rounders or non-rounders report histograms with a higher dispersion. The results based on the decimal- and integer-based categorizations are depicted in Figures 7 and 8 for the average number of bins and variances, respectively.

[FIGURES 7 AND 8 HERE]

As shown in Figure 7, the rounders in both the ECB- and FED-SPF assign nonzero probabilities to four bins on average, whereas the non-rounders use twice as many in most cases. Similarly, the variances of the non-rounders are, on average, approximately twice as large as those of the rounders. These findings are remarkably robust across outcome variables and the employed categorization. However, there is substantial heterogeneity in the level of ex-ante uncertainty across surveys and outcome variables. This squares with the evidence from Figure 4.

The evidence from Figures 7 and 8 suggests that rounding is related to the width of the reported histograms. For example, this may be the case if forecasters round small probabilities in the tails of the histogram to zero. To illustrate the connection between rounding and ex-ante uncertainty, consider the two histograms depicted in Figure 9.

[FIGURE 9 HERE]

The histogram depicted in the left plot corresponds to Example B from Figure 1. This forecast is classified as non-rounded by both the $D^{\text{any}}_{i,t,h}$ and the $D^{\text{m5}}_{i,t,h}$ scheme. Moreover, nonzero probabilities are assigned to each bin, such that $K_{i,t,h} = 12$. Based on Eqn. (4), the variance of this histogram is given by $\sigma^2_{i,t,h} = 0.72$. The right plot depicts the histogram that obtains if the probabilities in the reported histogram are artificially rounded to the nearest multiple of five. As a result, all of the probabilities in the right tail are rounded to zero, such that $K_{i,t,h}$ reduces to seven. In addition, the variance of the histogram reduces to $\sigma^2_{i,t,h} = 0.67$. This is a reduction by 7%. Thus, rounding has a considerable impact on the histogram width as measured by both $K_{i,t,h}$ and $\sigma^2_{i,t,h}$ in this particular case.

Next, an analysis based on all non-rounders helps to clarify if this effect is present for the entire SPF data. In particular, this serves as a means to disentangle the effect of rounding on the ex-ante variance from any other influence like the (unobserved) individual characteristics of the anonymous survey participants, we repeat the artificial rounding exercise from Figure 9 for all non-rounders in the SPF data. For each histogram with $D^{\text{m5}}_{i,t,h} = 1$ we round the reported probabilities to multiples of five. After excluding observations where the artificially rounded probabilities do not sum to 100%, we find that for the remaining cross-section the average variance from Eqn. (17) reduces by 7-10% (ECB-SPF) and 10-11% (FED-SPF), depending on the outcome variable.[20] The average variance based on the artificially rounded histograms remains higher than the one of the rounders from Eqn. (18), which suggests that other factors besides rounding explain part of the differences in the reported level of uncertainty. Since K, the number of available bins, is considerably larger in the ECB-SPF across all variables, the similarities in the results across both versions of the SPF suggest that these findings are not just an inherent consequence of the different survey designs.

To summarize, we find that the non-rounders in the SPF report higher ex-ante uncertainty than the rounders. Thus, the degree of the variance misalignment may be related to the rounding behavior of individual forecasters. We examine these issues in detail in the next subsection.

[20]For brevity, these results are not reported in detail here, but are available upon request.

5.3 Analysis of variance misalignment

The evidence reported in the previous subsection suggests a relationship between the dispersion of the reported histograms and the rounding choices of individual forecasters. Based on this observation, we compare the ex-ante and ex-post uncertainties of the SPF participants while accounting for the fact that there may be differences in the degree of the variance misalignment between rounders and non-rounders. In the case of survey-based fixed-event forecasts, the survey participants should become better informed as the forecast horizon shrinks during successive survey rounds. If this is the case, the differences in the variance misalignment may be related to the forecast horizon. This hypothesis is examined next. We measure the ex-ante uncertainty of forecaster i at forecast horizon h by means of the individual-specific average variance, which is defined as

$$\overline{\sigma^2_{i,h}} = \frac{1}{T_{i,h}} \sum_{t=1}^{T_{i,h}} \sigma^2_{i,t,h}, \tag{19}$$

where $T_{i,h} = \sum_{t=1}^{T} D^{\mathcal{P}}_{i,t,h}$ indicates the number of times forecaster i has reported h-step-ahead histogram forecasts and $\sigma^2_{i,t,h}$ denotes the variance from Eqn. (4). In order to analyze the degree of the variance misalignment in the SPF, the ex-ante uncertainty from Eqn. (19) is compared to the mean squared error (MSE), as given by

$$\text{MSE}_{i,h} = \frac{1}{T_{i,h}} \sum_{t=1}^{T_{i,h}} e^2_{i,t,h}, \tag{20}$$

which serves as a quantification of ex-post uncertainty. The MSE in Eqn. (20) is based on the individual forecast errors,

$$e_{i,t,h} = x_t - \mu_{i,t,h}, \tag{21}$$

with x_t denoting the realization of the outcome variable and $\mu_{i,t,h}$ indicating the mean of forecaster i's histogram as defined in Eqn. (2). To compare ex-post and ex-ante uncertainty across all survey participants, we compute the average misalignment ratio,

$$m_h = \frac{1}{N_h} \sum_{i=1}^{N_h} \frac{\text{MSE}_{i,h}}{\overline{\sigma^2_{i,h}}}, \tag{22}$$

for each forecast horizon, where N_h denotes the number of survey participants who report h-step-ahead histogram forecasts for outcome variable x_t. If forecasters provide an accurate ex-ante quantification of the average size of their forecast errors, the value of the statistic in Eqn. (22) equals unity.[21] Values above unity are typically interpreted

[21] Note that the statistic in Eqn. (22) differs from the one employed in Clements (2014) where the root MSE and the standard deviations are used to compute a similar ratio. Due to the nonlinearity of this

as evidence of 'overconfidence', i.e., cases where ex-ante uncertainty is, on average, too small compared to ex-post uncertainty. We compute the m_h-series across all forecasters as well as separate ratios for the rounders and non-rounders. The corresponding series do not include histograms with 100% probability in a single bin to avoid excessively large ratios of ex-post to ex-ante uncertainty. In extreme cases where forecasters set *all* their h-step-ahead variances equal to zero, the denominator of Eqn. (22) is zero. Thus, it seems advisable to exclude these observations from the calculation of the m_h-series.[22] The results based on $D^{\text{any}}_{i,t,h}$ and $D^{\text{m5}}_{i,t,h}$ are depicted in Figures 10 and 11, respectively.

[FIGURES 10 AND 11 HERE]

The evidence for the entire cross-section shows that the variance misalignment can be diagnosed in both versions of the SPF. The values of the m_h-ratio in both surveys tend to be substantially larger than unity at forecast horizons of one year or more, i.e., ex-post and ex-ante uncertainty are better aligned as the target period approaches. In particular, there is a notable drop in m_h as the forecast horizon diminishes from five to four quarters ahead. As discussed in Lahiri and Sheng (2008), this may be related to the availability of first releases of data for x_t for the respective year or alternative sources of information about the outcome. At the shortest forecast horizons, the ex-ante variances are frequently larger than the MSE statistics. In these cases, forecasters overstate their ex-ante uncertainty compared to the squared forecast errors and should, on average, reduce the variance of their histogram close to the target. These findings square with similar evidence documented in Giordani and Söderlind (2003, 2006) and Clements (2014, 2016) for the FED-SPF and also in Kenny et al. (2014) and Krüger (2017) for the ECB-SPF. In particular, we confirm the result of Clements (2014) that the ex-ante uncertainty of forecasters in the SPF exceeds ex-post uncertainty at short forecast horizons. The observed pattern is remarkably consistent across variables. The inflation rate forecasts in the FED-SPF are an exception since they are relatively well aligned even at long forecast horizons. Moreover, in most cases the degree of the variance misalignment is larger in the ECB-SPF than in the FED-SPF.

Empirical studies on the variance misalignment in surveys of macroeconomic expectations typically evaluate the entire cross-section of forecasters. By isolating rounders and non-rounders in the SPF by means of either $D^{\text{any}}_{i,t,h}$ or $D^{\text{m5}}_{i,t,h}$, we find that the average ratios of the non-rounders are much closer to unity at forecast horizons of one year or more, which are particularly those horizons for which the studies listed above tend to

transformation, the two statistics cannot be directly compared. Lahiri et al. (2015) discuss the distinct interpretations that arise due to the ordering by means of which aggregation and the root-transformation are applied. To avoid this type of ambiguity, we opt for employing the variance and the MSE instead.

[22] We also exclude the forecaster with identification number 563 from the FED-SPF sample for the analysis in this section. This survey participant is classified as a non-rounder and reports relatively small one-quarter-ahead ex-ante uncertainty compared to his/her one-quarter-ahead ex-post uncertainty, which disproportionately affects the magnitude of our findings for this particular forecast horizon. However, including this forecaster does not affect the qualitative conclusions of our analysis.

find the most substantial evidence of 'overconfidence'. In contrast, the average ratios of the rounders and non-rounders are relatively similar as the target period approaches. In sum, the results indicate that the ex-ante and ex-post uncertainties of the non-rounders are better aligned than those of the rounders at forecast horizons of one year or more. Thus, it appears the variance misalignment is at least partially explained by the rounding choices of the SPF participants. Rounding may affect both the numerator and the denominator of the statistic in Eqn. (22). On the one hand, the histogram mean can be affected. This has an impact on the size of the prediction errors. On the other hand, rounding may be related to the ex-ante uncertainty as measured by the variance of the histogram. This is analyzed in the next subsection.

5.4 Differences in histogram characteristics

The improved alignment of the ex-ante and ex-post uncertainties of the non-rounders documented in the previous subsection may be due to a higher dispersion of the reported histograms and/or be the consequence of smaller forecast errors. The evidence from Figures 7 and 8 shows that, on average, ex-ante uncertainty reported by the non-rounders is considerably larger, which means that the denominator of the ratio in Eqn. (22) is larger for this particular group. To shed light on the potential reasons for the misalignment of variances, we analyze the forecast performance and histogram characteristics of rounders and non-rounders below. To evaluate the impact of non-rounding, we estimate horizon-specific regressions of the form

$$y_{i,t,h} = \alpha_h + \beta_h D^{\mathcal{R}}_{i,t,h} + \gamma_{2,h} D2_{t,h} + \ldots + \gamma_{T,h} DT_{t,h} + \varepsilon_{i,t,h}, \tag{23}$$

where $y_{i,t,h} \in \{K_{i,t,h}, \sigma^2_{i,t,h}, |e_{i,t,h}|, e^2_{i,t,h}\}$ denotes distinct histogram characteristics, variation measures and loss functions, respectively, $D^{\mathcal{R}}_{i,t,h} \in \{D^{\text{any}}_{i,t,h}, D^{\text{m5}}_{i,t,h}\}$ indicates the employed categorization for (non-)rounding and $\varepsilon_{i,t,h}$ is the error term. The first group of histogram characteristics consists of variables that capture the histogram width, i.e., the number of bins used by forecasters, $K_{i,t,h}$, and the individual variance defined in Eqn. (4). These variables are observable ex-ante and affect the denominator of Eqn. (22). The second group captures the individual ex-post forecast performance based on the realizations and the histogram means. In particular, we consider the absolute forecast errors, $|e_{i,t,h}| = |x_t - \mu_{i,t,h}|$, as well as the squared forecast errors, $e^2_{i,t,h} = (x_t - \mu_{i,t,h})^2$. Both are related to the numerator of the ratio in Eqn. (22). In order to capture unobserved time variation, the specification in (23) includes time-fixed effects $D2_{t,h}, \ldots, DT_{t,h}$. In particular, the unobserved sources of heterogeneity include changes in the design of the survey questionnaire such as adjustments in the bin definitions.

In Eqn. (23), each candidate variable for $y_{i,t,h}$ is regressed on $D^{\mathcal{R}}_{i,t,h}$, i.e., the indicator for non-rounding. The slope coefficients $\beta_8, \ldots, \beta_1$ capture the differences in the histogram characteristics of non-rounders and rounders for distinct forecast horizons

$h \in \{8, 7, \ldots, 1\}$. The parameter vector $(\alpha_h, \beta_h, \gamma_{2,h}, \ldots, \gamma_{T,h})'$ is estimated via ordinary least squares (OLS). The sample size used in the estimation for each h is reported in Table 2. Since the data used in each regression are observed at the annual frequency, the error terms in Eqn. (23) are correlated across time periods due to the overlapping forecast horizons in cases where $h > 4$. In order to account for the autocorrelation patterns in the data, we apply the variance-covariance estimator by Newey and West (1987).

Figures 12-15 display the estimates of β_h over h for each outcome variable. The significant and insignificant estimates are highlighted differently. In particular, a diamond '$\diamond$' indicates that the estimate is significantly different from zero at the 5% critical level against a two-sided alternative. Generally, the results are robust to the choice of the classification scheme.[23] Note that the estimates for the FED-SPF are more strongly affected by individual observations due to the smaller share of non-rounders in this survey (see Table 3).

5.4.1 Differences in individual histogram ranges and variances

The evidence from Figures 7 and 8 shows that the non-rounders use more of the available bins and report higher variances than the rounders for a pooled sample of observations based on all forecast horizons. Yet it is not clear whether these differences vary with h. This may be the case if rounders and non-rounders update their information sets at different frequencies, e.g., due to heterogeneity in the level of information stickiness or differences in the horizons forecasters are concerned with as part of their principal occupation. In order to analyze the importance of the forecast horizon, Figures 12 and 13 depict the estimates of β_h that result when either the employed number of bins, $K_{i,t,h}$, or the individual ex-ante variance, $\sigma^2_{i,t,h}$, are used as the dependent variable in the model from Eqn. (23). Forecasters are classified as non-rounders based on either $D^{\text{any}}_{i,t,h}$ (first row) or $D^{\text{m5}}_{i,t,h}$ (second row).

[FIGURES 12 AND 13 HERE]

The results for $K_{i,t,h}$ confirm the evidence from Figure 7. The non-rounders in both surveys use significantly more bins than the rounders. The finding that non-rounders fill in a larger number of bins is also found for all particular forecast horizons. On average, the difference is approximately equal to four bins. However, in most cases the differences become less pronounced as the forecast horizon diminishes. Thus, the larger variances of the non-rounders are revised downwards more strongly as the target is approached during the forecasting process. This pattern is particularly apparent for the estimates based on the inflation and unemployment rate forecasts in the ECB-SPF. The values of the adjusted R^2-statistics (not shown) are lower in the FED-SPF than in the ECB-SPF. In the former case, the models explain 10-43% of the variation in $K_{i,t,h}$, whereas 22-55% are explained in

[23]The results for the other categorizations are reported in Figures A.3-A.6 in the Appendix.

the latter case. It could be that differences in the survey methodology are the reason for the improved goodness of fit. The reporting practices permitted in the case of the ECB-SPF may allow the employed rounding classification to isolate uninformed from informed survey participants. In contrast, the categorizations might be less precise in the case of the FED-SPF due to the fact that the panelists are required to fill in the questionnaire manually. The larger bin width in some cases may further mask the difference between rounders and non-rounders in the FED-SPF.

Overall, the results from Figure 12 suggest that the non-rounders use significantly more bins than the rounders and that this difference frequently becomes smaller as the horizon is dimninishing. The latter finding is in line with the evolution of the average misalignment ratios from Figures 10 and 11. Moreover, our results are not strongly affected by the choice of the employed classification scheme.

The evidence that is obtained when $\sigma^2_{i,t,h}$ is used as the dependent variable is in line with Figure 8 in the sense that non-rounders report significantly wider histograms. For the decimal-based categorization, the decreasing pattern of the estimated slope coefficients from Figure 7 is visible here as well. The goodness of fit in the case of $\sigma^2_{i,t,h}$ is 1-23% and 10-40% in the FED- and ECB-SPF, respectively. In both surveys, the difference in the average variances tends to decline in both magnitude and significance as the target approaches. The estimates for the FED-SPF are driven by a smaller number of individual observations than in the case of the ECB-SPF.

In sum, the results for $K_{i,t,h}$ and $\sigma^2_{i,t,h}$ confirm that non-rounders in the SPF use more bins and report larger variance forecasts than the rounders. In most cases, this implies that the denominator of the m_h-statistic from Eqn. (22) is larger for the non-rounders. The differences become smaller as the target approaches, which provides a potential explanation for the similar alignment of the ex-post and ex-ante uncertainties reported by rounders and non-rounders at the shortest forecast horizons (see Figures 10 and 11). The results are robust to the choice of the categorization if the share of non-rounders in the cross-section is sufficiently large. This can be observed for both surveys, but it is particularly visible in the ECB-SPF, which contains a larger number of individuals that we classify as non-rounders than the FED-SPF.

5.4.2 Differences in forecast errors

The results from Figures 12 and 13 reveal that the histograms reported by the non-rounders are more dispersed than those of the rounders. This is particularly the case for forecast horizons of one year or more. These horizons correspond to those for which the difference in the variance misalignment between both groups is particularly large (see Figures 10 and 11). Apart from the denominator of Eqn. (22), the numerator can also be the reason for the variance misalignment. The size of the numerator depends on the individual forecast errors. As seen in Figure 3, average prediction errors decline as the target approaches. However, it may be that the average forecast errors of rounders and non-rounders decline at different rates. To analyze the impact of non-rounding on the predictive accuracy of the histogram means, Figures 14 and 15 depict the estimated slope coefficients when either absolute or squared forecast errors, $|e_{i,t,h}|$ and $e_{i,t,h}^2$, are considered as the dependent variable in Eqn. (23).

[FIGURES 14 AND 15 HERE]

We find no significant differences in the ex-post forecast performance of rounders and non-rounders in terms of either absolute or squared forecast errors. In the case of the ECB-SPF, the estimates of β_h are very close to zero for both types of prediction errors and both categorizations. The results for the FED-SPF are more erratic. Nonetheless, the null hypothesis that β_h equals zero is not rejected in almost all cases. Overall, the results suggest that the histogram mean is relatively robust to the rounding choices of the survey participants. This is in line with the evidence of Engelberg et al. (2009), who show that rounding has little impact on the mean of a forecaster's subjective distribution. Similarly, Binder (2017) decomposes disagreement, defined as the cross-sectional dispersion of the point forecasts, into the contributions of the rounding and non-rounding group and shows that almost all of the cross-sectional variability can be ascribed to variation within the respective groups, i.e., rounders and non-rounders, meaning that the group-specific means do not differ substantially.

To summarize, our results suggest that the ex-ante and ex-post uncertainties of SPF participants deviate substantially at forecast horizons of one year or more. This misalignment can be at least partially explained by the rounding choices of the panelists. In particular, we show that the uncertainties of the non-rounders are better aligned due to the fact that this group of forecasters reports larger ex-ante variances but does not substantially differ from the rounders in terms of ex-post prediction errors. Thus, rounding choices affect the denominator of the misalignment ratio in Eqn. (22), but not the numerator.[24] The implication of this finding is that a better calibrated quantification of ex-ante uncertainty can be obtained by focusing on the non-rounders. The share of non-rounded responses has been increasing recently as seen in Figure 6. However, the large difference in the share of non-rounders in the ECB- and FED-SPF based on the decimal-based categorization, i.e., $D_{i,t,h}^{\text{any}}$ (see Table 3) suggests that reporting techniques may play a role in the decision of a forecaster to report rounded numbers. Participants in the ECB-SPF can process and submit their responses online, whereas forecasters in the FED-SPF are required to print out the questionnaire and report their forecasts in a hand-written form. This may appear tedious to some non-rounders and induce them to report rounded prob-

[24] We have also analyzed whether the *degree* of (non-)rounding contains information about certain histogram characteristics by replacing $D_{i,t,h}^{\mathcal{R}}$ in Eqn. (23) with the average number of decimals per histogram forecast. The results are remarkably similar to our main results, i.e., each additional decimal numbers is associated with a significantly wider histogram in terms of both the number of bins and the ex-ante variance. In contrast, the average number of decimals has no predictive power for either absolute or squared forecast errors.

abilities instead. If this is the case, surveys of macroeconomic expectations should be designed in such a way that its participants can submit their forecasts with as little effort as possible. Nonetheless, the ECB-SPF sample contains the responses of a considerable number of rounders. This suggests that additional factors such as information deficiencies or ambiguity may play a role.

In additional regressions that are reported in Figures A.7 and A.8 in the Appendix, we analyze whether rounders and non-rounders differ in terms of the forecast performance of the entire histogram as measured by the quadratic probability score (QPS) and the ranked probability score (RPS) as discussed in Boero et al. (2011). The evidence suggests that the histograms of the non-rounders tend to outperform those of the rounders at long forecast horizons. However, the results vary both across versions of the SPF and outcome variables. We do not focus on these findings because they are not directly related to the analysis of the variance misalignment.[25]

5.5 Expert versus consumer surveys

In a related study, Binder (2017) investigates the relationship between ex-ante uncertainty and rounding in two surveys of consumer expectations. In a preliminary analysis, she finds that the average histogram width of the rounders in the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York is approximately twice as large as that of the non-rounders (see her Table 1). In contrast, we find that in the SPF data the histograms of the *non-rounders* are more dispersed. However, as will be discussed below, there are important distinctions between both analyses. Moreover, we show that our categorizations and the one used by Binder (2017) isolate distinct groups of survey participants.

First, we consider professional forecasters, whereas Binder (2017) focuses on consumers. There may be systematic differences in the way that each group computes their predictions. As discussed in Section 5.1, survey participants may rely on either formal models or judgment in the forecasting process. It seems likely that the relative importance of judgmental forecasting is higher for consumers than it is for experts. Second, we classify the SPF participants as rounders or non-rounders based on their histogram forecasts. Binder (2017) focuses on the point forecasts instead. For consumer surveys, this may be advantageous since consumers who are not expert forecasters may focus their attention on approximating the first moment and put less effort into a sophisticated quantification of higher moments. The categorizations employed in our study have the advantage that they are based on more than just one number due to the fact that almost all SPF participants

[25]In unreported regressions we have also considered higher moments and found no clear evidence for substantial deviations in the skewness of the histograms reported by both groups. On average, the SPF histograms tend to be relatively symmetric. Following Andrade et al. (2015), we have found that the histograms of the non-rounders exhibit a higher kurtosis than those of the rounders.

assign nonzero probabilities to multiple bins. Thus, the two approaches can be considered as complementary to each other. However, it is possible that survey participants who report rounded point forecasts differ from respondents who round the probabilities. We show that this is the case below. Third, the employed survey data differ in other important aspects. The SCE sample used by Binder (2017) to obtain the estimates in her Table 1 only covers a short period from January 2013 to September 2015, whereas we examine the SPF data for the period 1999Q1-2017Q4. Moreover, the bins in the SCE have a width of two percentage points and are thus much wider than those in the SPF. Furthermore, generalized beta distributions are fitted to the histograms of the SCE. Binder (2017) uses the interquartile range of the individual beta distributions in order to measure the dispersion. We follow Zarnowitz and Lambros (1987) and examine the individual variance as a measure of ex-ante uncertainty. Finally, the SCE differs from the SPF in terms of the sampling scheme by means of which surveyed individuals are selected. In particular, the SCE constitutes a rotating panel, whereas most of the SPF forecasters have a fairly long history of survey participation. The accumulated experience of some forecasters may also be related to their rounding choice.

In order to analyze whether the distinct approaches based on point and histogram forecasts isolate the same SPF participants, we first consider the correlations between the decimal- and integer-based categorizations for the reported probabilities based on a pooled sample of observations across all forecast horizons. Here, we only consider the case of the ECB-SPF. We have documented in the previous section that both approaches work well in isolating two distinct groups of forecasters who appear to rely on either judgment or models to compute their probabilities. If this is the case, the correlations between $D^{\text{any}}_{i,t,h}$ and $D^{\text{m5}}_{i,t,h}$ are expected to be positive and large.

In the second step, we follow Binder (2017) and categorize rounders based on whether the point prediction, $\mu^{\star}_{i,t,h}$, is a multiple of 0.5, i.e.,

$$\tilde{D}^{\text{m0.5}}_{i,t,h} = \begin{cases} 1 & \text{if } 0.5 \cdot \left\lfloor \frac{\mu^{\star}_{i,t,h}}{0.5} \right\rfloor = \mu^{\star}_{i,t,h} \text{ and} \\ 0 & \text{else.} \end{cases} \tag{24}$$

Note that Binder (2017) classifies consumers as rounders if the point forecast is a multiple of five, not 0.5. This is due to the fact that the range of point forecasts for inflation reported in the SCE is considerably larger than in the SPF. As in the case of the integer-based categorizations, we consider

$$D^{\text{m0.5}}_{i,t,h} = 1 - \tilde{D}^{\text{m0.5}}_{i,t,h} \tag{25}$$

in order to focus on non-rounders. If the categorizations based on point and histogram forecasts perform equally well, the correlations between $D^{\text{m0.5}}_{i,t,h}$ and either $D^{\text{any}}_{i,t,h}$ or $D^{\text{m5}}_{i,t,h}$ should also be positive and large. Table 4 summarizes the correlations based on a pooled sample of observations across all survey participants, time instances and forecast horizons.

[TABLE 4 HERE]

The correlation statistics between $D^{\text{any}}_{i,t,h}$ and $D^{\text{m5}}_{i,t,h}$ have the expected sign and amount to 0.57, 0.57 and 0.59 for inflation, real GDP growth and unemployment, respectively. This shows again that there is a large overlap in the groups of survey participants that are classified as non-rounders by both approaches (see Section 5.1). In contrast, the corresponding correlations between $D^{\text{any}}_{i,t,h}$ and $D^{\text{m0.5}}_{i,t,h}$ are considerably smaller and close to zero. In other words, these categorizations isolate distinct groups of forecasters. It may be the case that the weak association is due to methodological differences between the decimal-based approach and $D^{\text{m0.5}}_{i,t,h}$. If this were the only explanation, it may be expected that the categorizations from Eqns. (10) and (24) are more closely related, such that the association between $D^{\text{m5}}_{i,t,h}$ and $D^{\text{m0.5}}_{i,t,h}$ should be stronger. However, the corresponding correlation statistics are again close to zero, which suggests that categorizations based on point and histogram forecasts isolate distinct groups of forecasters.

6 Conclusion

We analyze the misalignment between ex-ante and ex-post uncertainty that is frequently observed in surveys of macroeconomic expectations. In the analysis of the Survey of Professional Forecasters for the Euro area and the U.S., we employ a variety of distinct categorizations to isolate two distinct types of forecasters based on their reporting behavior. We find that the variance misalignment is considerably smaller for survey participants who report non-rounded histogram forecasts. This is a consequence of the fact that this group reports significantly larger ex-ante variances. In contrast, the forecast errors of rounders and non-rounders do not seem to differ in a systematic way. Thus, rounding has little impact on the first-moment dynamics but has a substantial effect on the second moments. Our results have important implications for the evaluation of the cross-section of survey participants. In particular, measures of aggregate ex-ante uncertainty that are more aligned with ex-post squared forecast errors can be derived by focusing on the non-rounders and discarding the remaining responses. Due to the relatively small share of non-rounded histograms, this would result in a substantial loss of information. However, the share of non-rounders has increased substantially over time. This suggests that the quality of the SPF predictions has improved in recent years and increases the feasibility of focusing on the non-rounders. Designers of surveys of macroeconomic expectations should improve their questionnaires in such a way that reporting less strongly rounded probabilities is further encouraged. To facilitate the distinction between survey participants that provide rounded numbers and ones who do not, inquiring about participants' respective intentions in the survey questionnaire might be helpful.

Our results also have implications for the usefulness of using disagreement as a proxy for forecast uncertainty. Since we do not find evidence of substantial differences in the means of the histograms reported by rounders and non-rounders, measures of forecaster disagreement for both groups are likely to be relatively similar (see Binder, 2017). However, measures of aggregate uncertainty, e.g., the cross-sectional average variance, are strongly affected by the rounding choices of the panelists due the higher dispersion of the histograms reported by non-rounders. This suggests that one potential explanation for the increase in the difference between uncertainty and disagreement documented by, among others, Lahiri and Sheng (2010) and Glas and Hartmann (2016), is the growing share of non-rounders in recent survey periods.

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Tables and Figures

Table 1: Horizon structure of fixed-event forecasts

	Forecast horizon h	
	'Current year'	'Next year'
Q1	4	8
Q2	3	7
Q3	2	6
Q4	1	5

Notes: This table depicts the structure of the forecast horizons associated with the predictions for the current and the next calendar year from the SPF data.

Table 2: Number of density forecasts provided by SPF participants

		Forecast horizon h								
SPF	Variable	8	7	6	5	4	3	2	1	$\sum_h$
ECB	Inflation	942	955	863	967	967	961	877	966	7498
	GDP growth	948	956	867	973	972	963	878	972	7529
	Unemployment	908	916	825	914	925	919	830	910	7147
FED	Inflation	626	654	635	663	654	662	637	657	5188
	GDP growth	652	676	654	685	677	685	659	680	5368
	Unemployment	263	255	257	264	258	251	254	254	2056

Notes: For each outcome variable, this table displays the number of reported histograms per forecast horizon, i.e., $\sum_i \sum_t D^{\mathcal{P}}_{i,t,h}$, as well as the total number of observations across all horizons. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Table 3: Share of non-rounded observations

SPF	Variable	Scheme	Forecast horizon h 8	7	6	5	4	3	2	1
ECB	Inflation	D^{any}	14.12	13.82	14.14	12.93	13.24	12.80	12.77	11.28
		D^{m5}	23.46	23.14	22.83	22.34	22.96	22.89	22.69	20.29
	GDP growth	D^{any}	13.82	14.33	14.42	13.77	12.96	12.56	12.30	11.63
		D^{m5}	23.84	24.69	22.95	23.12	24.38	24.20	22.10	19.75
	Unemployment	D^{any}	12.67	13.21	12.85	12.47	12.76	12.51	13.49	11.65
		D^{m5}	22.80	21.94	21.82	23.74	21.84	21.87	22.77	19.34
FED	Inflation	D^{any}	4.15	4.13	4.72	4.07	4.28	4.53	5.02	4.11
		D^{m5}	21.57	22.94	21.42	20.21	18.35	20.39	21.19	18.11
	GDP growth	D^{any}	6.13	6.80	6.42	7.30	6.50	6.57	6.53	5.00
		D^{m5}	24.69	24.70	23.55	23.50	23.63	23.80	22.91	25.00
	Unemployment	D^{any}	9.51	7.84	8.17	7.95	8.53	7.17	7.87	5.12
		D^{m5}	23.19	24.71	26.85	25.38	28.29	25.10	28.35	24.41

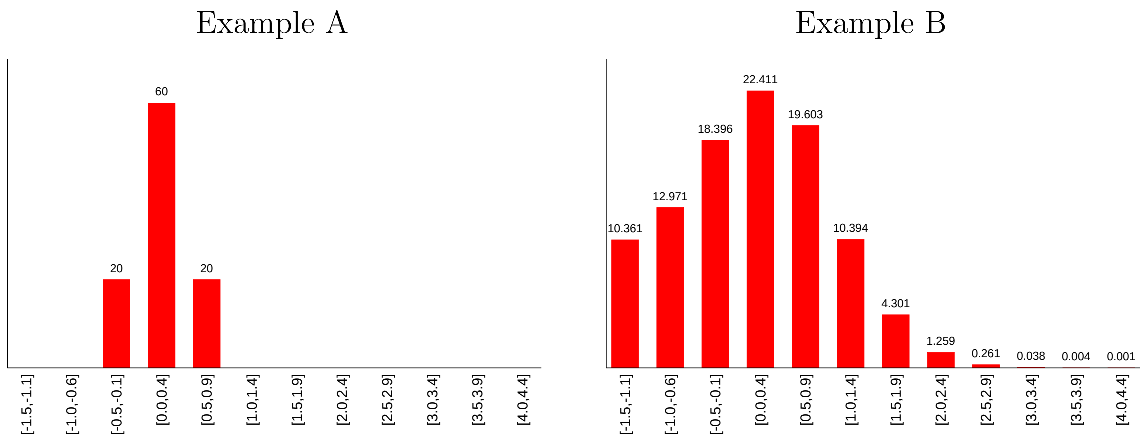
Notes: For each forecast horizon, this table displays the share of non-rounded observations in the sample, i.e., $S_h^{\mathcal{R}} = 100 \times (\sum_i \sum_t D_{i,t,h}^{\mathcal{R}})/(\sum_i \sum_t D_{i,t,h}^{\mathcal{P}})$ for the preferred decimal- and integer-based classification schemes $D_{i,t,h}^{\mathcal{R}} \in \{D_{i,t,h}^{\text{any}}, D_{i,t,h}^{\text{m5}}\}$ from Eqns. (7) and (12). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Table 4: Correlations across categorizations in the ECB-SPF

	Inflation	GDP growth	Unemployment
$\widehat{\mathbf{Corr}}[D^{\text{any}}, D^{\text{m5}}]$	0.57	0.57	0.59
$\widehat{\mathbf{Corr}}[D^{\text{any}}, D^{\text{m0.5}}]$	-0.07	-0.07	-0.08
$\widehat{\mathbf{Corr}}[D^{\text{m5}}, D^{\text{m0.5}}]$	-0.06	-0.07	-0.08

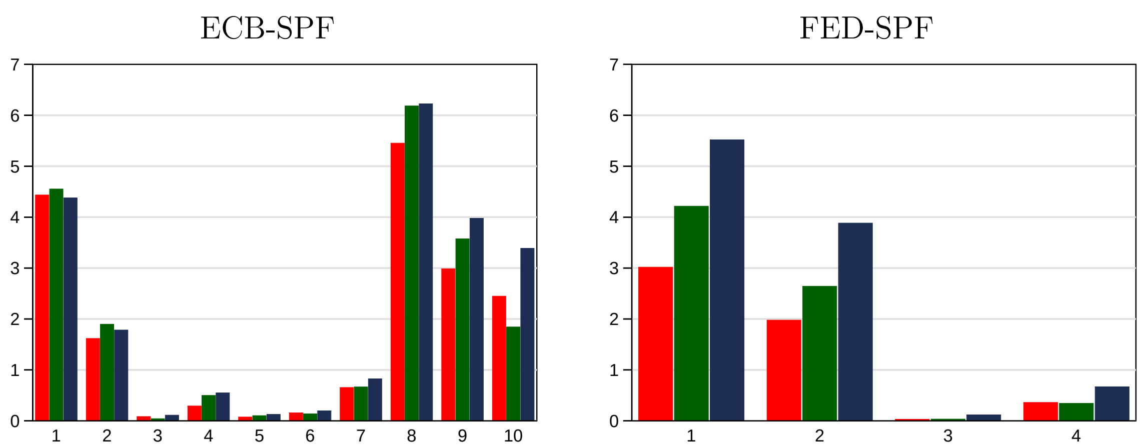
Notes: For each outcome variable, this table displays the bivariate correlations between distinct categorizations for non-rounders in the ECB-SPF for a pooled sample of observations across all survey participants, time instances and forecast horizons. The sample period is 1999Q1-2017Q4.

Figure 1: Two examples of histogram forecasts for the inflation rate from the ECB-SPF



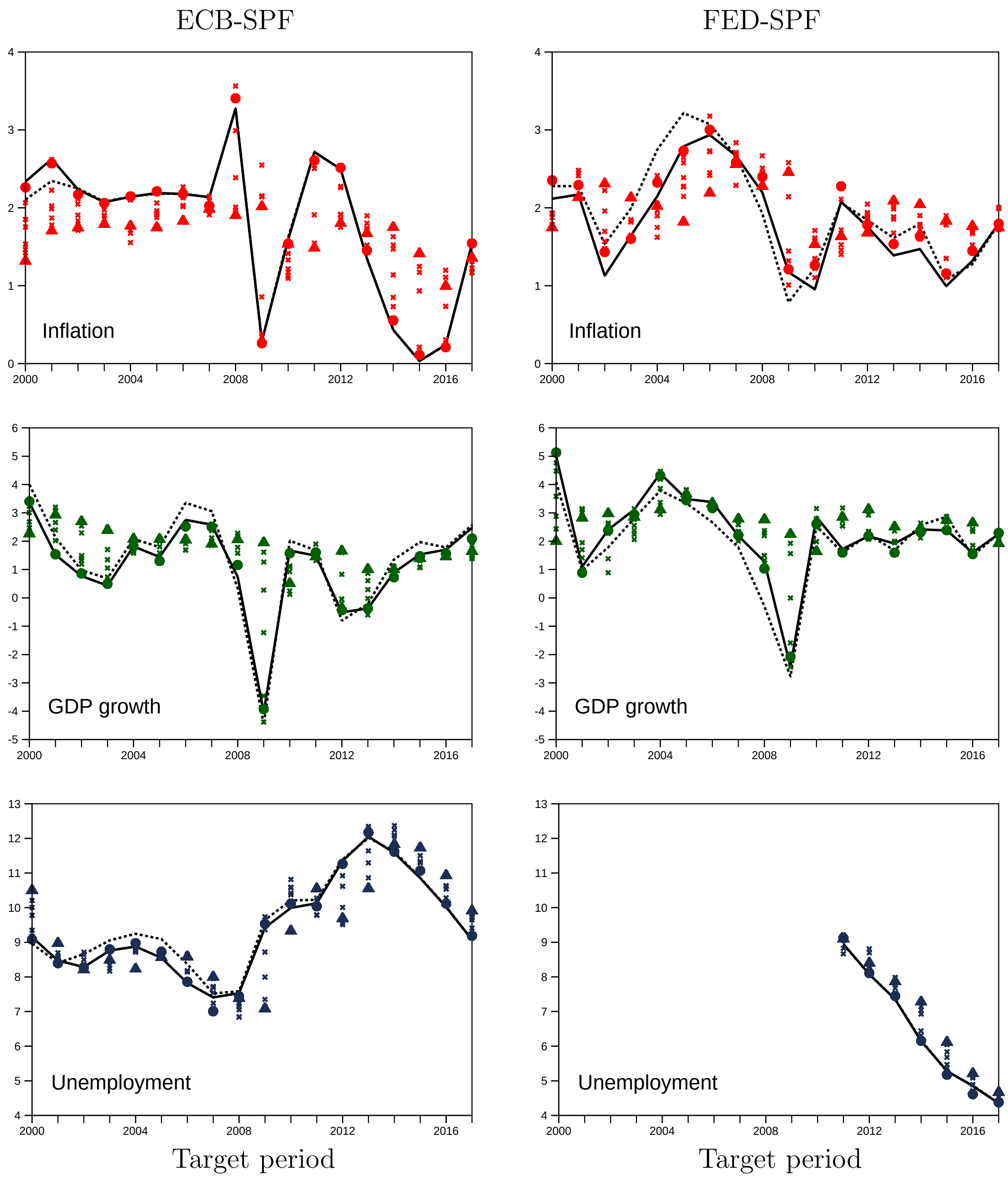
Notes: The graphs depict two examples of one-quarter-ahead histogram forecasts for the inflation rate from the ECB-SPF. Both predictions are taken from the 2016Q4 survey. The left plot depicts the histogram of forecaster 102. The right subfigure displays the probabilities reported by forecaster 95. In the latter case, the decimal numbers attached to the probabilities are cut off at the third decimal, i.e., the original histogram in the SPF data contains additional decimal numbers.

Figure 2: Relative frequencies of the number of decimals reported in the SPF



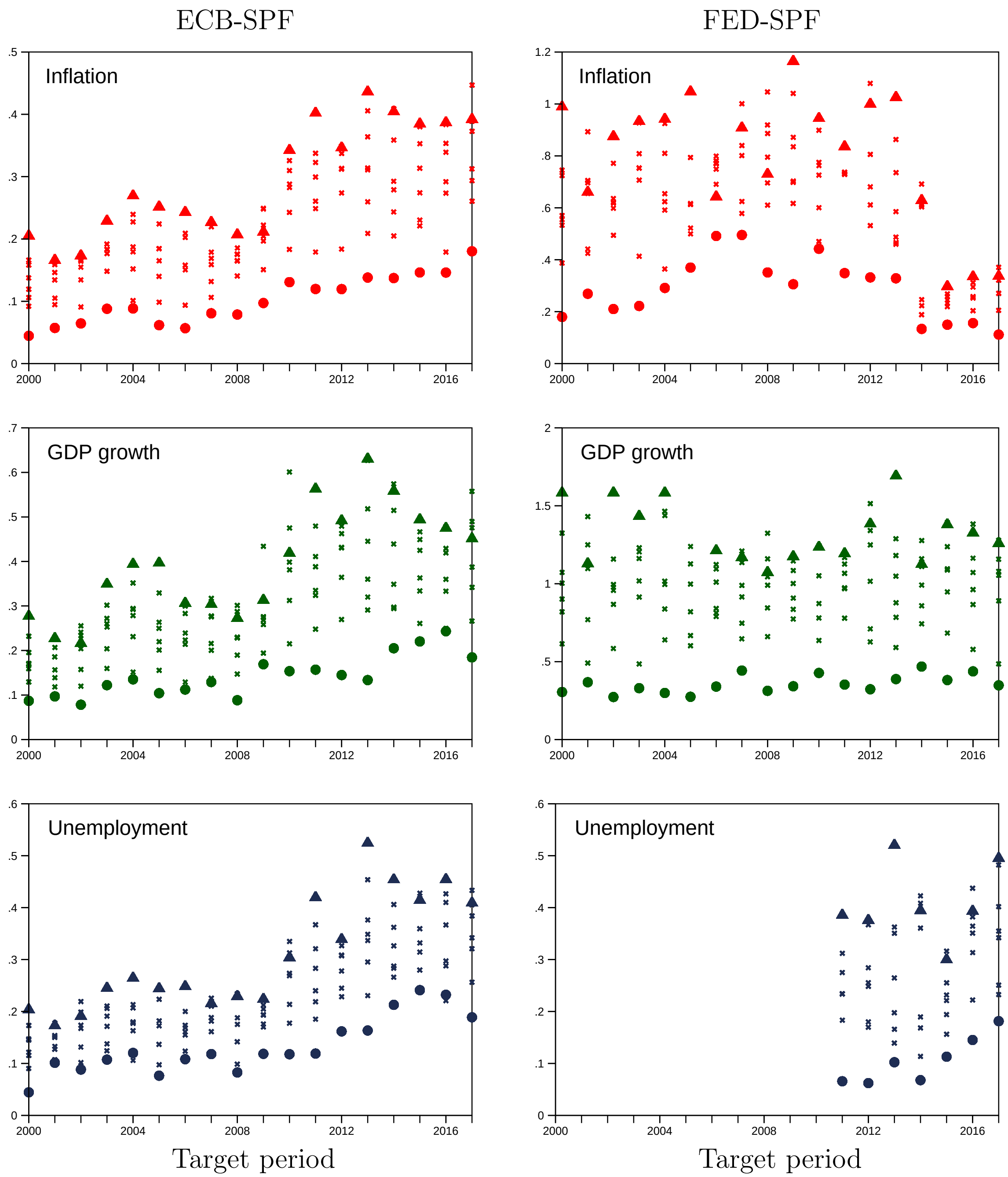
Notes: The graphs depict the share of probabilities that contain $d \in \{1, \ldots, 10\}$ decimal numbers out of all reported probabilities in the SPF data for inflation, output growth and unemployment. The '0'-category is omitted to improve the readability. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 3: Realizations and consensus forecasts from the SPF



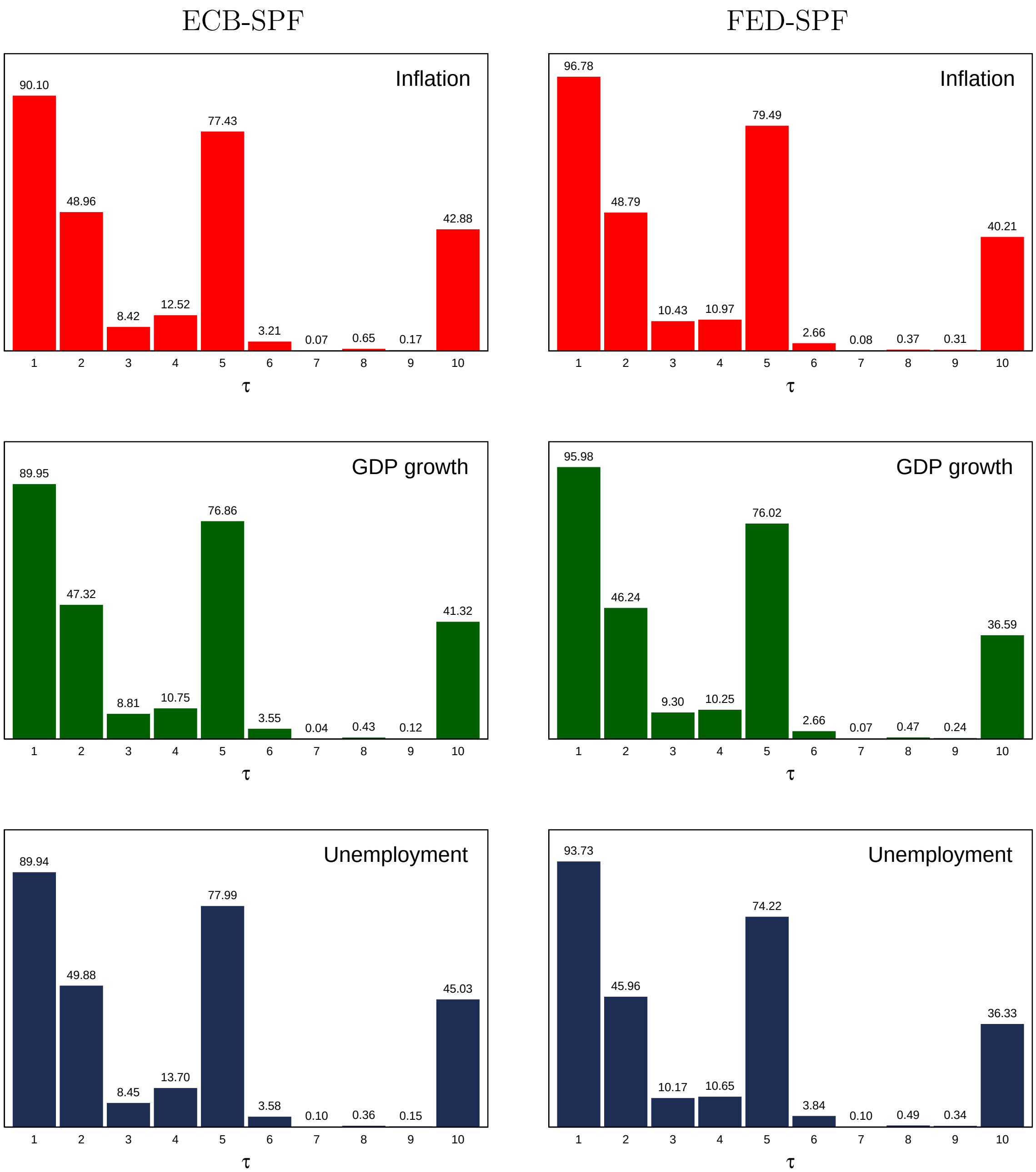
Notes: The graphs depict the time series of the annual realizations x_t for inflation (first row), output growth (second row) and unemployment (third row) in the Eurozone and the U.S. based on first-release (solid black lines) and last-release (dashed black lines) data vintages. In addition, each plot displays the cross-sectional average across the means of the individual h-step-ahead histogram forecasts, i.e., $\bar{\mu}_{t,h}$ from Eqn. (3). Triangles '$\triangle$' and bullets '$\bullet$' indicate the eight- and one-step-ahead consensus forecasts, respectively. Crosses '$\times$' indicate the predictions for the intermediate forecast horizons. The horizontal axis depicts the target year. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 4: Average ex-ante uncertainty



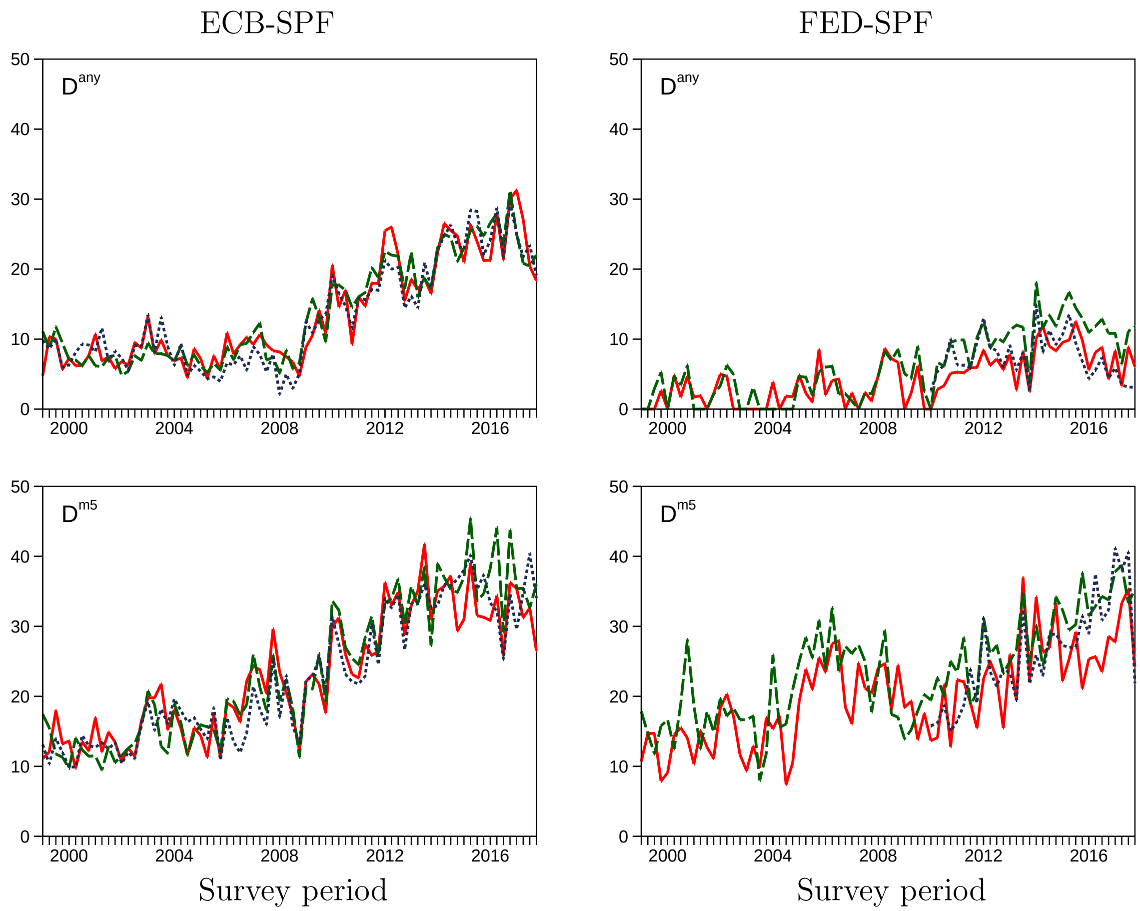
Notes: The graphs depict the time series of the cross-sectional average across the h-step-ahead variances from the individual histograms for inflation (first row), output growth (second row) and unemployment (third row) in the Eurozone and the U.S., i.e., $\overline{\sigma^2_{t,h}}$ from Eqn. (5). Triangles '△' and bullets '•' indicate the eight- and one-step-ahead average variances, respectively. Crosses '×' indicate the average variances for the intermediate forecast horizons. The horizontal axis depicts the target year. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 5: Share of rounded histograms (integer-based categorization)



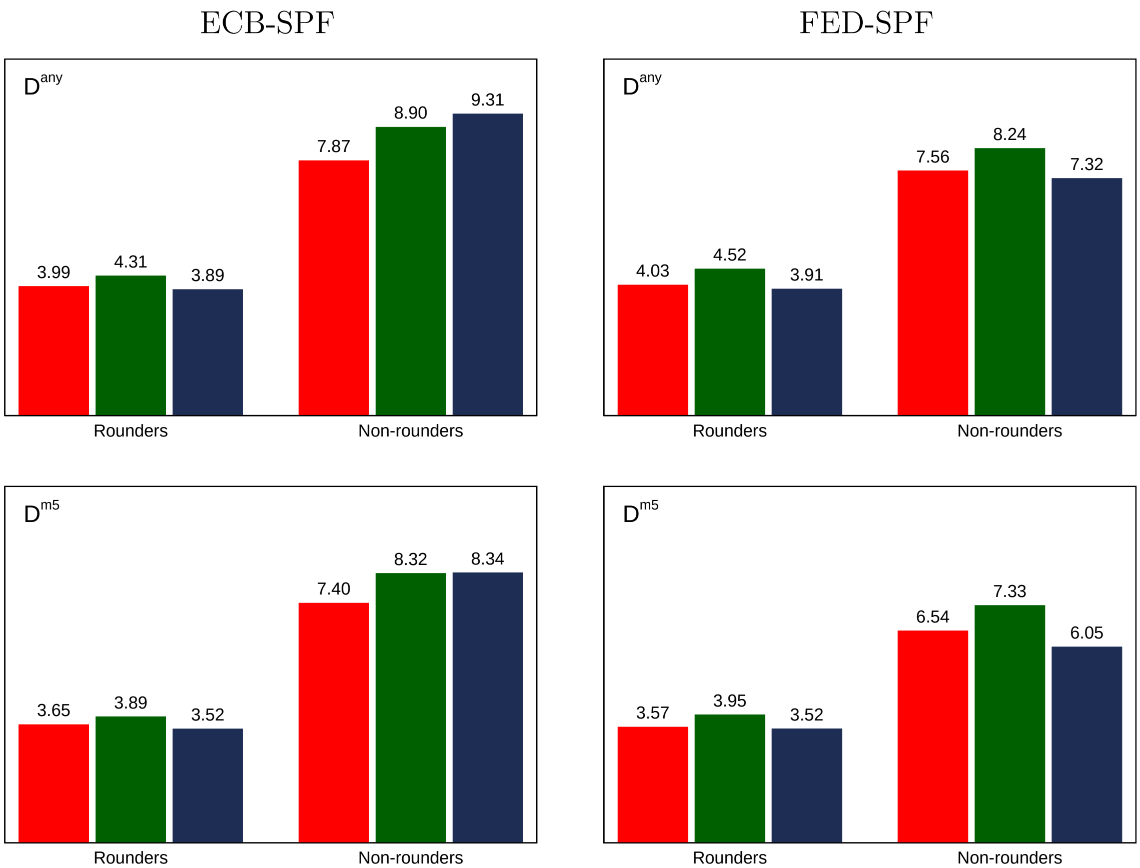
Notes: The graphs depict the share of rounded histogram forecasts classified via the integer-based categorization from Eqn. (12), i.e., $\tilde{S}^{\mathrm{m}\tau} = 100 \times (\sum_i \sum_t \sum_h \tilde{D}^{\mathrm{m}\tau}_{i,t,h})/(\sum_i \sum_t \sum_h D^{\mathcal{P}}_{i,t,h})$ for $\tau \in \{1, \ldots, 10\}$, based on a pooled sample of observations across all forecast horizons for inflation (first row), output growth (second row) and unemployment (third row). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 6: Time-variation in the share of non-rounders



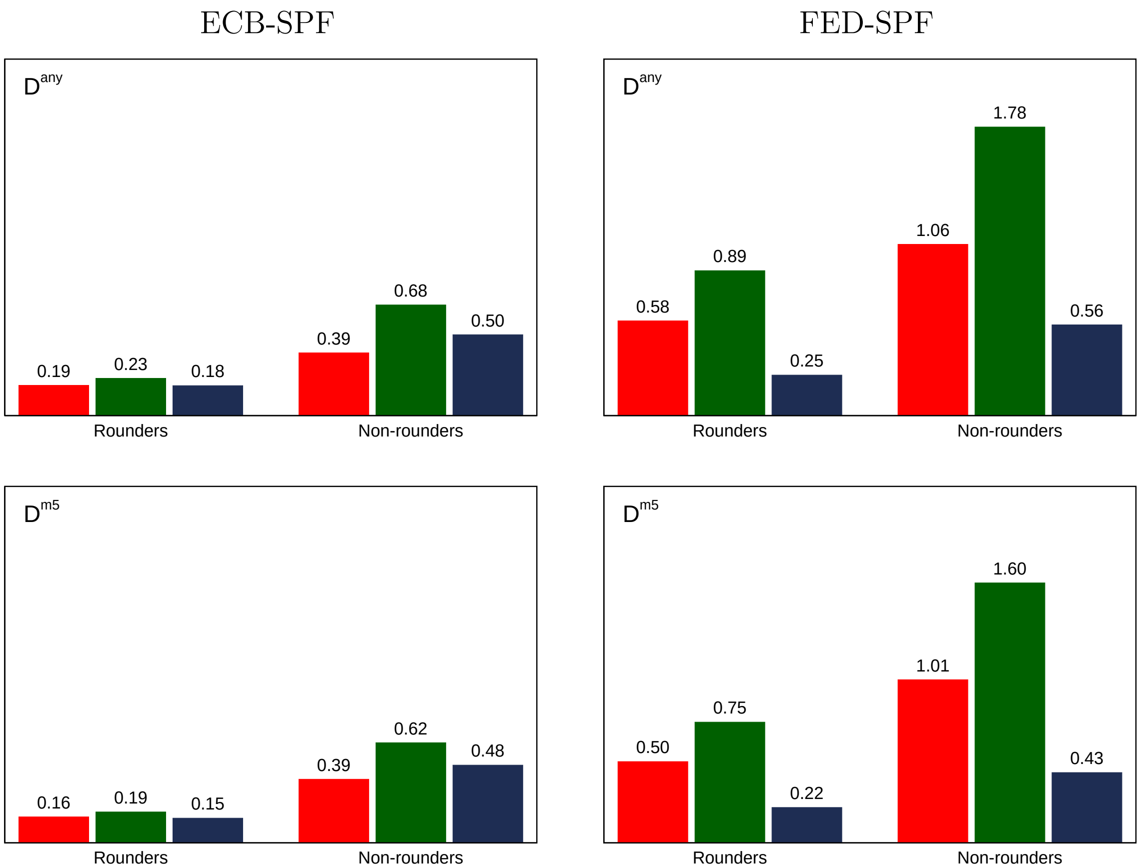
Notes: The graphs depict the share of non-rounded histogram forecasts for inflation (solid), output growth (dashed) and unemployment (dotted) based on $D^{\text{any}}_{i,t,h}$ (first row) and $D^{\text{m5}}_{i,t,h}$ (second row) for a pooled sample of observations across the predictions for the current ($h \leq 4$) and the next year ($h \geq 5$). The horizontal axis depicts the quarter during which predictions are reported. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 7: Average number of bins used by rounders and non-rounders



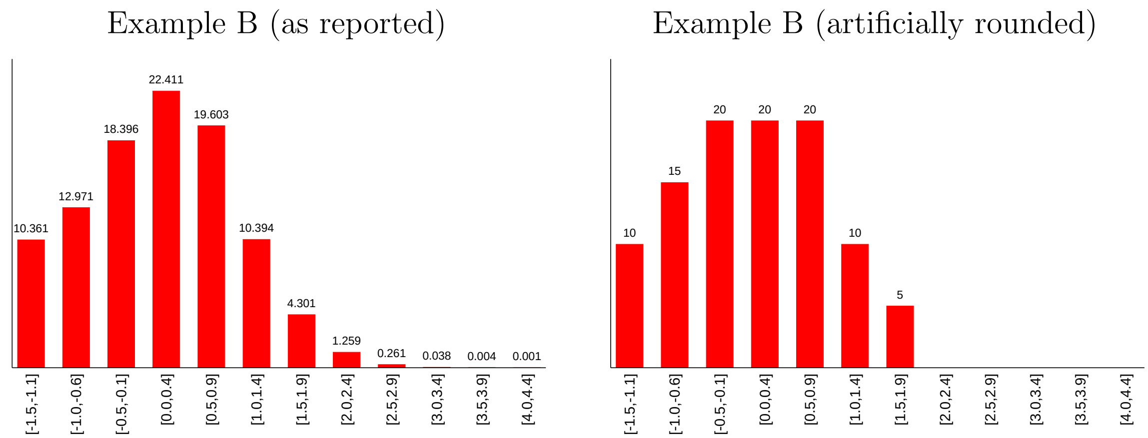
Notes: The graphs depict the average number of bins used by rounders and non-rounders based on the predictions for inflation, output growth and unemployment for a pooled sample of observations across forecasters, time periods and forecast horizons. Non-rounders are classified by means of $D^{\text{any}}_{i,t,h}$ (first row) or $D^{\text{m5}}_{i,t,h}$ (second row). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 8: Average ex-ante variances reported by rounders and non-rounders



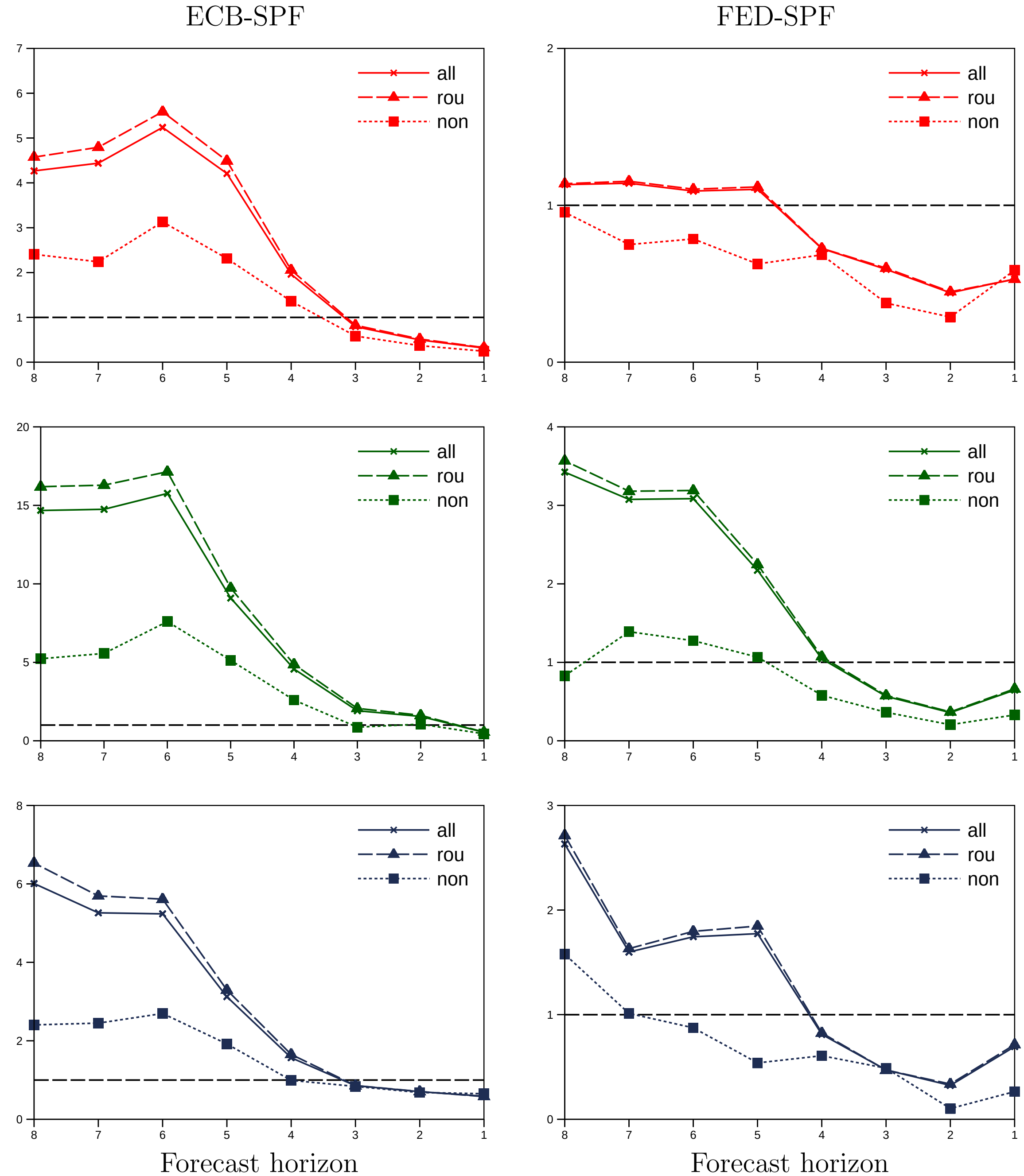
Notes: The graphs depict the average across the ex-ante variances reported by rounders and non-rounders based on the predictions for inflation, output growth and unemployment for a pooled sample of observations across forecasters, time periods and forecast horizons. Non-rounders are classified by means of $D^{\text{any}}_{i,t,h}$ (first row) or $D^{\text{m5}}_{i,t,h}$ (second row). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 9: Artificial rounding exercise



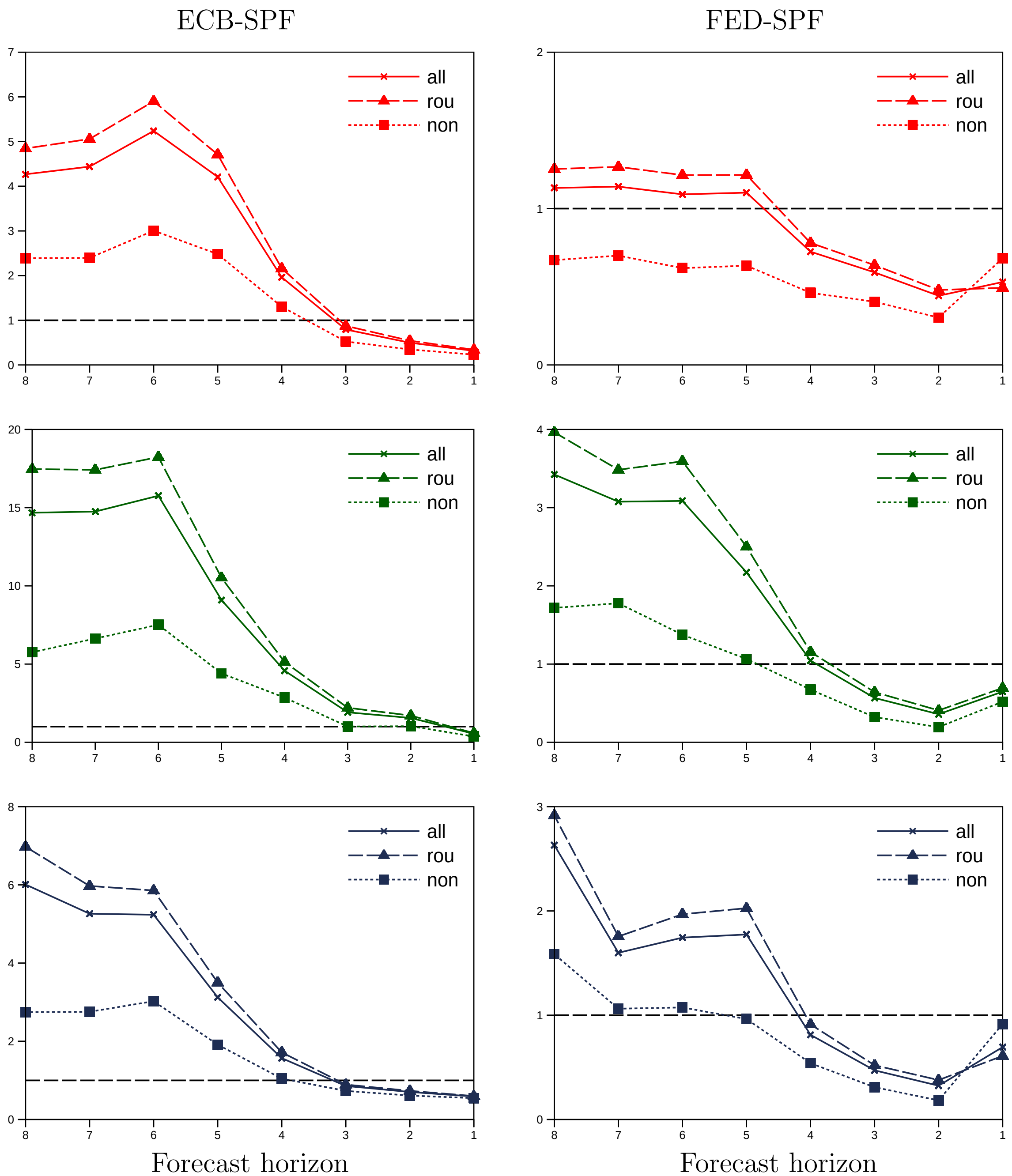
Notes: The left plot depicts the one-quarter-ahead histogram forecast for the inflation rate reported by forecaster 102 in the 2016Q4 survey round of the ECB-SPF (see Figure 1). The right subfigure displays the result of artificially rounding the originally reported probabilities to multiples of five.

Figure 10: Variance misalignment in the SPF data (decimal-based categorization)



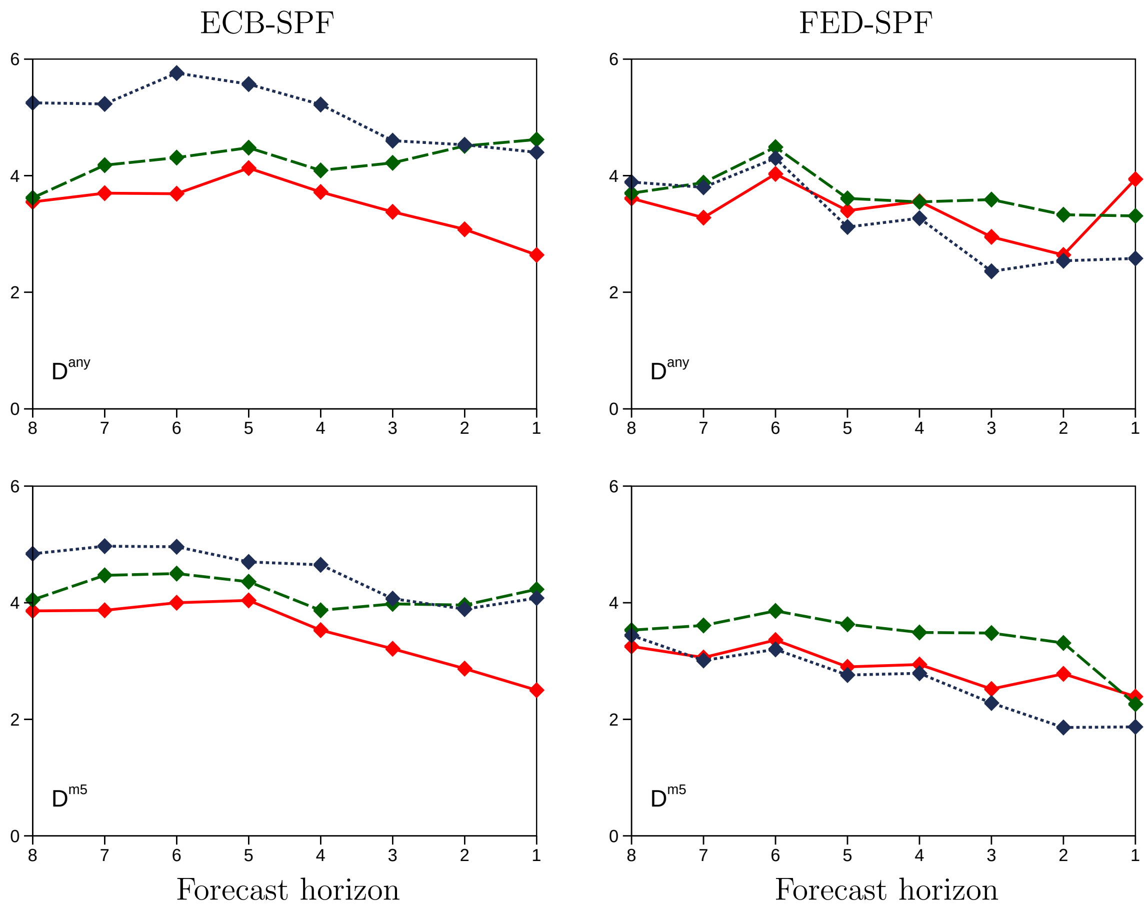
Notes: Each plot depicts the misalignment ratio m_h from Eqn. (22) for inflation (first row), output growth (second row) and unemployment (third row) in the ECB- (first column) and FED-SPF (second column). In addition to the average ratio for the entire cross section (solid line), each plot depicts separate ratios for rounders (dashed line) and non-rounders (dotted line). Non-rounders are classified by means of $D^{\text{any}}_{i,t,h}$. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 11: Variance misalignment in the SPF data (integer-based categorization)



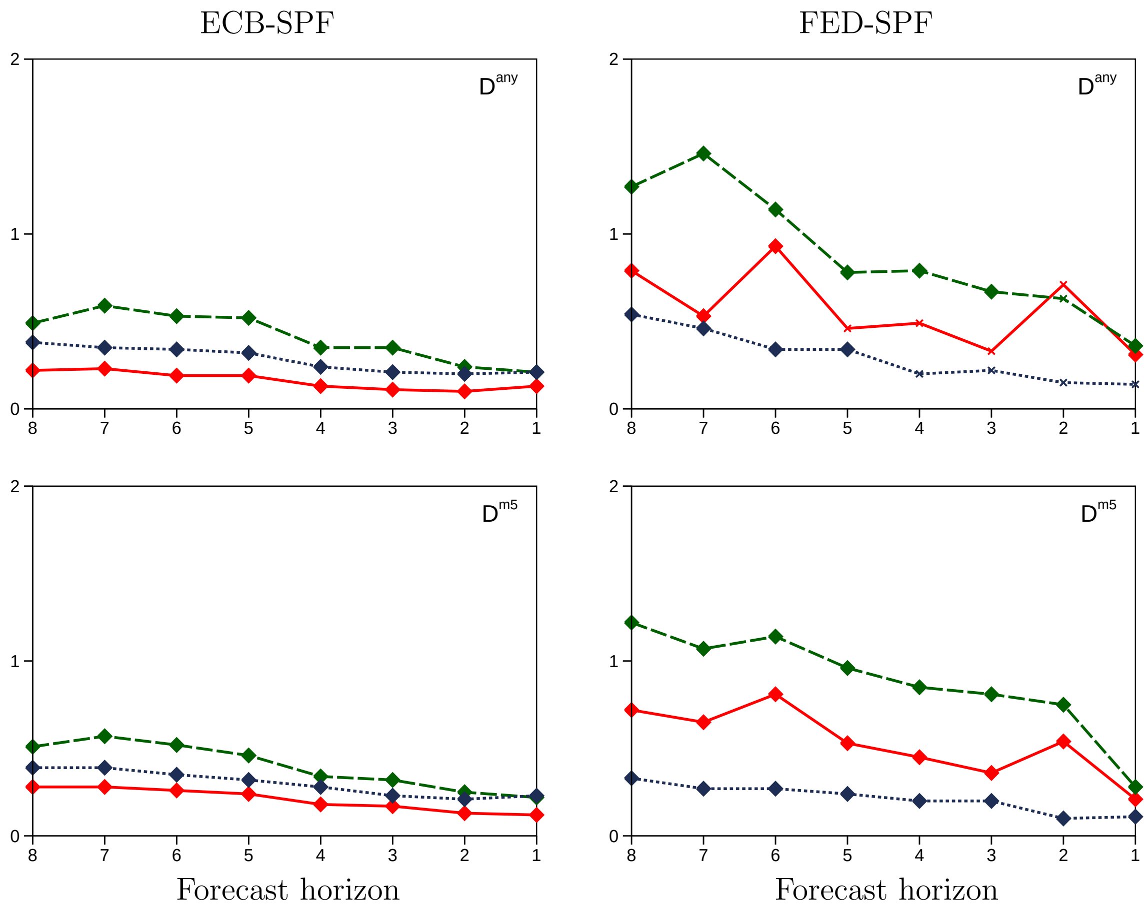
Notes: Each plot depicts the misalignment ratio m_h from Eqn. (22) for inflation (first row), output growth (second row) and unemployment (third row) in the ECB- (first column) and FED-SPF (second column). In addition to the average ratio for the entire cross section (solid line), each plot depicts separate ratios for rounders (dashed line) and non-rounders (dotted line). Non-rounders are classified by means of $D^{\mathrm{m5}}_{i,t,h}$. The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 12: Deviations in the number of bins used by non-rounders and rounders



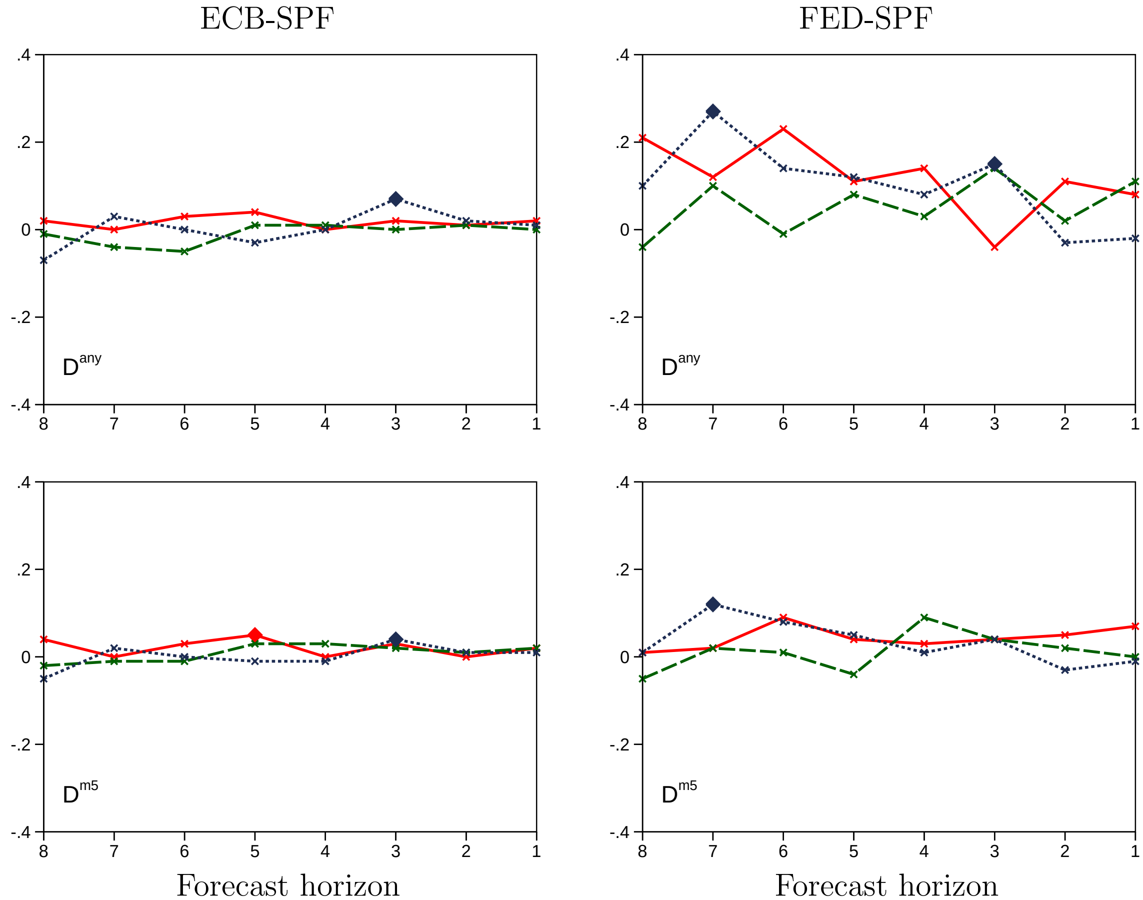
Notes: For each forecast horizon, the graphs depict the difference in the number of bins used by non-rounders and rounders. In particular, each marker denotes an estimate of the slope coefficient, β_h, based on the h-step-ahead predictions for inflation (solid), output growth (dashed) and unemployment (dotted) when $K_{i,t,h}$ is considered as the dependent variable in Eqn. (23). A diamond '$\diamond$' indicates that the number of bins used is distinct among non-rounders and rounders. The significance level is 5%. A cross '$\times$' indicates an insignificant estimate. The explanatory variable $D^{\mathcal{R}}_{i,t,h} \in \{D^{\text{any}}_{i,t,h}, D^{\text{m5}}_{i,t,h}\}$ denotes either the preferred decimal- (first row) or integer-based (second row) categorization from Section 4. Each regression includes time-fixed effects. Coefficients are estimated via OLS. We apply the variance-covariance estimator of Newey and West (1987). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 13: Deviations in the variances reported by non-rounders and rounders



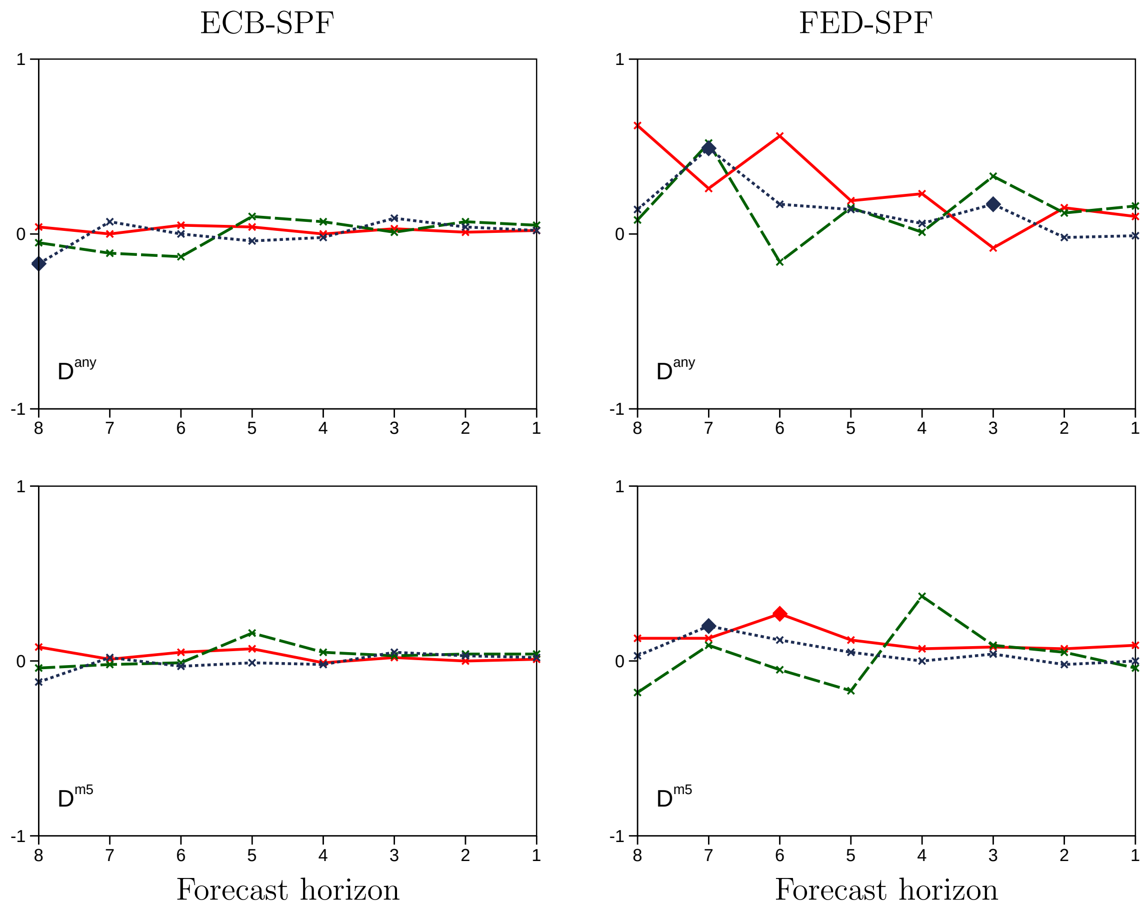
Notes: For each forecast horizon, the graphs depict the difference in the ex-ante variances reported by non-rounders and rounders. In particular, each marker denotes an estimate of the slope coefficient, β_h, based on the h-step-ahead predictions for inflation (solid), output growth (dashed) and unemployment (dotted) when $\sigma^2_{i,t,h}$ is considered as the dependent variable in Eqn. (23). A diamond '$\diamond$' indicates that the reported variance forecasts are distinct among non-rounders and rounders. The significance level is 5%. A cross '$\times$' indicates an insignificant estimate. The explanatory variable $D^{\mathcal{R}}_{i,t,h} \in \{D^{\text{any}}_{i,t,h}, D^{\text{m5}}_{i,t,h}\}$ denotes either the preferred decimal- (first row) or integer-based (second row) categorization from Section 4. Each regression includes time-fixed effects. Coefficients are estimated via OLS. We apply the variance-covariance estimator of Newey and West (1987). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 14: Deviations in the absolute forecast errors of non-rounders and rounders



Notes: For each forecast horizon, the graphs depict the difference in the ex-post absolute forecast errors of non-rounders and rounders. In particular, each marker denotes an estimate of the slope coefficient, β_h, based on the h-step-ahead predictions for inflation (solid), output growth (dashed) and unemployment (dotted) when $|e_{i,t,h}|$ is considered as the dependent variable in Eqn. (23). A diamond '$\diamond$' indicates that the absolute prediction errors are distinct among non-rounders and rounders. The significance level is 5%. A cross '$\times$' indicates an insignificant estimate. The explanatory variable $D^{\mathcal{R}}_{i,t,h} \in \{D^{\text{any}}_{i,t,h}, D^{\text{m5}}_{i,t,h}\}$ denotes either the preferred decimal- (first row) or integer-based (second row) categorization from Section 4. Each regression includes time-fixed effects. Coefficients are estimated via OLS. We apply the variance-covariance estimator of Newey and West (1987). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.

Figure 15: Deviations in the squared forecast errors of non-rounders and rounders



Notes: For each forecast horizon, the graphs depict the difference in the ex-post squared forecast errors of non-rounders and rounders. In particular, each marker denotes an estimate of the slope coefficient, β_h, based on the h-step-ahead predictions for inflation (solid), output growth (dashed) and unemployment (dotted) when $e^2_{i,t,h}$ is considered as the dependent variable in Eqn. (23). A diamond '$\diamond$' indicates that the squared prediction errors are distinct among non-rounders and rounders. The significance level is 5%. A cross '$\times$' indicates an insignificant estimate. The explanatory variable $D^{\mathcal{R}}_{i,t,h} \in \{D^{\text{any}}_{i,t,h}, D^{\text{m5}}_{i,t,h}\}$ denotes either the preferred decimal- (first row) or integer-based (second row) categorization from Section 4. Each regression includes time-fixed effects. Coefficients are estimated via OLS. We apply the variance-covariance estimator of Newey and West (1987). The sample period is 1999Q1-2017Q4, except for the unemployment rate forecasts from the FED-SPF, which are available since 2010Q1 for our purposes.