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Endogenous Productivity Dynamics in a Two-Sector Business Cycle Model

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Abstract

We develop a stylized two-sector business cycle model with endogenous firm dynamics in the investment goods sector. The positive correlation between firms profitability and the relative price of investment goods generates an endogenous persistence mechanism in productivity dynamics which drives the model response to shocks. A white noise permanent shock to the productivity of new entrants causes endogenous exit and subsequent rounds of productivity increases, due to the competitive pressure generated by falling relative prices of investment goods. The model internal propagation mechanism generates persistent dynamics and a large "multiplier effect" on the initial shock. Neutral productivity shocks affect long run firms productivity in the Investment-goods sector through their effect on relative prices. Firms productivity is also endogenous to shocks to the marginal efficiency of investment. The DSGE version of the model apparently survives the Barro-King curse.
JEL classification: E13, E21, E22, E30, E32

keywords: Productivity shocks, Investment shocks, relative price of investment, DSGE model, Firms entry, Firms exit

1 Introduction

We develop a stylized two-sector model of creative destruction arising from endogenous firms entry/exit in the investment-goods sector (I-sector). Our main goal is to identify the competitive forces that drive the endogenous persistence of firms productivity, in contrast with many business-cycle models which have weak internal propagation mechanisms and rely on systemic and persistent exogenous productivity shocks (Cogley and Nason, 1995). We then investigate the business cycle implications of this mechanism.

The spectacular increase of capital intensity in the production of final goods over the last 50 years has brought broad consensus to the view that investment-specific technological (IST hereafter) change is a fundamental driver of US growth (Greenwood et al., 1997). As a result, the empirical literature on business cycles has closely scrutinized the different effects of IST and neutral productivity shocks (Greenwood et al., 2000; Fisher, 2006). Justiniano et al. (2011) (JPT hereafter) incorporate sectoral technology shocks in an otherwise standard empirical DSGE model of the US, drawing a distinction between IST shocks and temporary shocks that affect the production of installed capital from investment goods, *i.e.* marginal efficiency of investment (MEI henceforth) shocks.

The literature on IST shocks typically assumes that the relative price of investment is equal to the inverse of the investment-specific productivity shifter. In this framework, cyclical variations in demand for investment goods do not affect their relative price, and for this reason they are bound to trigger a strong response of I-goods supply. This point is acknowledged in Moura (2018), whose empirical model embeds price stickiness in the I-sector. He finds that nominal rigidities in the I-sector are important to capture the effects of sector-specific technology shocks, notably contractionary investment supply shocks, and to improve replication of business cycle dynamics. However, in his model the average duration of Calvo contracts in the I-sector takes the implausibly large value of about 14 quarters. Our intuition therefore is that standard sectoral business cycle models should be amended to incorporate some rigidity in the supply function of I-goods, but exclusive reliance on nominal rigidities hides a more complex structure of the supply side of the I-sector which has been neglected so far.

Relative to previous contributions, in the paper we therefore take a radically different approach. Instead of relying on nominal rigidities to model the I-sector supply, we incorporate endogenous firm entry and exit, driven by idiosyncratic productivity and fixed costs. By doing this, we uncover an endogenous persistence mechanism in productivity dynamics which is essentially explained by the positive correlation between firms profitability and the relative price of investment goods. This is the key result of the paper, and we are able to show it drives model dynamics in response to both technology and demand shocks.

Consider for instance the effects of a IST shock, that we model as an increase in the number of new entrants (*NE*s) whose idiosyncratic productivity is sufficiently large to guarantee profitability. The larger inflow of relatively more productive new entrants shifts to the right the supply schedule for investment goods. Lower I-goods prices trigger a "creative destruction" event where the least productive incumbents are driven out of the market.

The exit of less productive firms limits the initial fall in the I-goods prices. For this reason, in subsequent periods the inflow of *NE*s remains above its steady state value until the gradual reduction in the relative price of investment goods becomes sufficiently large. Even if we rule out any autoregressive pattern in the initial shock, numerical simulations show that transition to the new steady state is very persistent, and the cumulative reduction in the relative price of I-goods is associated to a substantial amplification of the initial shock. The endogenous interaction between entry-exit flows and the relative price of I-goods produces significant effects on aggregate variables only at low frequencies. Therefore our model provides a stronger microfoundation for the original JPT result that IST shocks have at best limited relevance at business cycle frequencies.

Incorporating endogenous firm dynamics bears other important implications for business cycle analysis. In fact, both MEI and consumption-sector (C-sector) permanent productivity shocks raise the relative price of I-goods. This, in turn, increases (reduces) entry (exit) flows of I-firms. As a result, the average productivity of the I-sector is also endogenous to such shocks, and in this case behaves countercyclically. Our model is therefore capable of characterizing situations where, depending on the nature of the shocks, an increase in the number of *NE*s exhibits either negative or positive correlation with the average productivity of I-firms. Furthermore, the endogeneity of the relative price of I-goods to permanent C-sector productivity shocks implies that empirical research should find alternatives to the widespread practice of identifying IST shocks from restrictions to the long-run dynamics of the relative price of I-goods (Christiano et al., 2016).

Finally, we obtain intriguing additional results in the DSGE version of our model, that incorporates price stickiness in the consumption goods sector. We find that IST shocks generate initial contractions, a substantial amount of inertia in the relative price of I-goods, and relatively large productivity increases in the long-run. This essentially replicates the results obtained in Moura (2018) even if we do not impose price stickiness in the I-sector. Most importantly, incorporating endogenous firm dynamics seems to protect our model against the Barro-King curse that typically applies to standard DSGE models. First, it prevents the counter-cyclical initial response of consumption to MEI shocks. Second, it increases the positive unconditional correlation between consumption and investment growth. Third, it allows to match the strong positive correlation between consumption growth and output growth. Fourth, it improves the cross-correlations between consumption growth and hours worked. Overall, the improvements are sizable considering the whole cross-correlogram of relevant macroeconomic variables.

To the best of our knowledge we are the first to feature a business cycle model where productivity growth in the I-sector is driven by endogenous firm dynamics. This choice has empirical foundation. Using R&D intensities, the OECD ranks equipment and durable goods production in high/medium-high technology industries (see OECD., 2011). Aghion et al. (2009) find that, in industries close to the technology frontier, incumbents productivity growth is triggered by entry flows of new firms.

Our characterization of the I-sector endogenous evolution is loosely based on Asturias et al. (2017) who develop a deterministic growth model where firms entry and exit affect productivity through competitive pressures in the economy, but our focus is on business cycle analysis. Clementi and Palazzo (2016) investigate the role that entry and exit dynamics play in the propagation of aggregate shocks, but neglect sectorial productivity dynamics and the role of entry in driving productivity growth.

This paper, together with Pinchetti (2017) and Cozzi et al. (2017), adds to a new literature introducing creative destruction as source of endogenous productivity growth in DSGE models, but we focus on sectorial dynamics.

We contribute to a rapidly expanding literature on endogenous firm dynamics in DSGE models. In their seminal work Bilbiie et al. (2012) focus on firm entry as propagation mechanism for TFP shocks. Other works explore the importance of endogenous entry for firms markups and for the optimal monetary policy (see Etro and Colciago, 2010; Etro and Rossi, 2015a; Etro and Rossi, 2015b). However, none of them treats endogenous firm entry as the true engine of innovation, productivity growth and endogenous exit. Rossi (2019) introduces endogenous exit, but focuses on cyclical interactions between endogenous firm dynamics and bank lending.

The remainder of the paper is organized as follows. Sections 2 describes the model economy. Section 3 is devoted to the interpretation of our results. Section 4 investigates the business cycle properties our model. Section 5 concludes. Technical details are left to the Appendix.

2 The model economy

The key players in the economy are I- and C-firms, respectively producing investment and consumption goods. Consumption goods producers are characterized by a standard CRS technology and by a permanent labor-augmenting stochastic technology shifter that is affected by permanent labor augmenting technology shocks (LAT henceforth).

The I-sector is made of a measure η_t of active firms distributed between new entrants, NE_t, and incumbents, INC_t, who survived out of the η_{t-1} firms active at time $t-1$

$$\eta_t = NE_t + INC_t \tag{1}$$

I-firms are endowed with a decreasing returns to scale technology, are characterized by idiosyncratic efficiency levels and face variable and fixed production costs. Fixed production costs and the relative price of I-goods are crucial to identify the productivity threshold that determines entry and exit decisions.

New Entrants draw their idiosyncratic efficiency levels from a new and more productive technology distribution, which embeds a stochastic trend subject to IST shocks. The η_{t-1} firms are subject to idiosyncratic productivity shocks. In combination with the competitive pressures from potential NE firms, such shocks affect the relative price of I-goods, determining actual entry and exit flows. Capital accumulation is subject to standard MEI shocks.

The sequence of events is as follows (Figure 1). At time $t-1$, C-firms rent from households the labor and capital services necessary to produce goods which are then sold to households and to I-firms. Households consume and buy from I-firms the investment goods necessary to accumulate physical capital. At the beginning of period t, LAT, MEI and IST shocks occur, potential NEs observe their individual productivity. Then, entry and exit decisions in the I-sector are made and the markets clear.

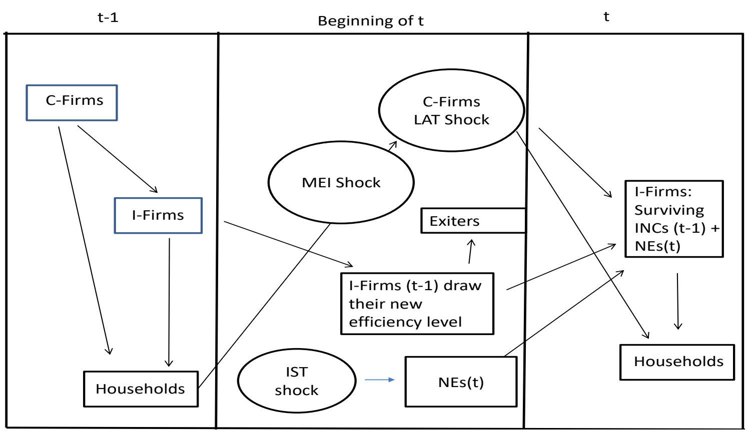

Figure 1: Sequence of events.

2.1 Consumption good Producers

C-firms are perfectly competitive, hire labor from households and exploit capital rented at the end of period $t-1$. Their production function is

$$Y_t = (z_t N_t)^{\chi} (K_{t-1})^{1-\chi} \tag{2}$$

where N defines worked hours, K is the capital stock, z is a permanent LAT shifter, such that $z_t = z_{t-1} g_{z,t}$ where

$$\ln(g_{z,t}) = (1-\rho_z)\ln(g_*) + \rho_z \ln(g_{z,t-1}) + \varepsilon_t^z \tag{3}$$

and $\varepsilon_t^z \sim N(0, \sigma^z)$. The LAT shifter embeds a deterministic trend component, g_*.

The real marginal costs are:

$$mc_t = \left(\frac{r_{k,t}}{1-\chi}\right)^{1-\chi} \left(\frac{w_t}{z_t \chi}\right)^{\chi} \tag{4}$$

2.2 The I-sector

Within a perfectly competitive environment, the I-sector firm j is characterized by the following production function

$$I_{j,t} = A_{j,t} S_{j,t}^{\alpha} \tag{5}$$

where $S_{j,t}$ defines the input of C-goods; $\alpha < 1$ implies that production occurs under decreasing returns to scale. $A_{j,t}$ is the idiosyncratic efficiency level, better identified as firm-specific "knowledge capital".[1]

In terms of consumption goods, profits are

$$\Pi_{j,t} = P_t^I I_{j,t} - S_{j,t} - f_t \tag{6}$$

where P_t^I is the relative price of I-goods and f_t is a fixed production cost which grows at the deterministic gross rate g_* and does not depend on the scale of production as in

[1] Our results would carry over to the case where I firms directly hire labor and capital inputs $I_{j,t} = A_{j,t}\left[(N_{j,t})^{\chi^I} (K_{j,t-1})^{1-\chi^I}\right]^{\alpha}$. Guerrieri et al. (2014) adopt relatively complex input-output interdependencies between the two sectors. Their focus is different, as they wish to enphasize the different implications of MEI and IST shocks in a setting where both shocks have permanent effects and firms dynamics are neglected. In our framework MEI shocks are temporary and, in the spirit of JPT, are better interpreted as shocks proxy for the financial sector effectiveness in channelling savings into productive capital. We opt for the simple production function 2.2 in order to sharpen our focus on endogenous firm dynamics. Needless to say, in our model MEI and IST shocks cause strongly different dynamics.

Comin and Gertler (2006).[2] Differently from Bilbiie et al. (2012), where only new entrants are subject to a sunk entry-cost and each incumbent faces an exogenous exit probability in the next production period, this assumption allows to endogenize also exit decisions.

From profit maximization, the optimal demand for $S_{j,t}$ is

$$S_{j,t} = \left(P_t^I \alpha A_{j,t}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

implying that j_{th} firm's dividends in period t can be characterized as

$$d_{j,t} = \left(\alpha P_t^I A_{j,t}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha} - f_t \tag{8}$$

Intertemporal profit maximization drives entry/exit decision. The firm's value can be written recursively as

$$V_t(A_{j,t}) = d_{j,t} + E_t\left[\bar{\Lambda}_{t+1} V_{t+1}(A_{j,t+1})\right] \tag{9}$$

where the characterization of $E_t\left[\bar{\Lambda}_{t+1} V_{t+1}(A_{j,t+1})\right]$ will be discussed below. Entry/exit decisions are identified by the productivity cut-off level $\hat{A}_t$, such that

$$V_t(\hat{A}_t) = 0 \tag{10}$$

2.2.1 New Entrants

A unit probability mass of potential NEs draw their individual $A_{j,t}$ every period from a Pareto distribution

$$\int_{\overline{e}_t}^{+\infty} \frac{\gamma \overline{e}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) = 1;\ A_{j,t} \geq \overline{e}_t \tag{11}$$

where γ is the tail index describing the distribution skewness and and $\overline{e}_t = \overline{e}_{t-1} g_{e,t}$, then

$$\ln(g_{e,t}) = (1-\rho_e)\ln(g_e) + \rho_e \ln(g_{e,t-1}) + \varepsilon_t^e;\ \varepsilon_t^e \sim N(0, \sigma^e) \tag{12}$$

In this framework the technology shock consists of a sudden and unexpected shift to the right of the potential NEs' pfd due to ε_t^e. For any given market cutoff, this causes an inflow of a higher mass of more productive NEs in the market, stiffening competition faced by incumbent I-firms.

The mean of (11), *i.e.* the average expected productivity of potential NEs, is

$$\mu(A_{j,t}) = \frac{\gamma}{\gamma-1}\overline{e}_t \tag{13}$$

[2] Right from the outset, it is worth noting that by imposing a common, deterministic fixed production cost to all firms greatly simplifies model tractability. This comes at some cost. For instance, by renouncing the relatively complex stochastic characterization adopted in Clementi and Palazzo (2016) we cannot replicate the empirical correlation between firms age and productivity.

and is driven by the lower bound of the support, which grows at the gross rate g_e in the deterministic steady state. This is our re-interpretation of Investment-Specific Technology shock popularized in the RBC-DSGE literature.

The mass of entering NE firms is obtained by cutting the pfd in (11) at $\hat{A}_t$:

$$NE_t = \int_{\hat{A}_t}^{+\infty} \frac{\gamma \bar{e}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) = \left(\frac{\bar{e}_t}{\hat{A}_t}\right)^{\gamma} ; \; \hat{A}_t \geq \bar{e}_t \tag{14}$$

2.2.2 Incumbents

To identify $\hat{A}_t$, note that $(t+1)$-period incumbents survive out of the η_t firms, conditional to the new profitability conditions occurring in $t+1$. We assume that firm j expects to draw its next-period productivity from a probability distribution which is

$$\int_{\hat{A}_t g_e (1-\delta^I)}^{+\infty} \frac{\gamma \left[\hat{A}_t g_e (1-\delta^I)\right]^{\gamma}}{A_{j,t+1}^{\gamma+1}} d(A_{j,t+1}) \tag{15}$$

where $g_e(1-\delta^I) < 1$. The lower support of (15) implies that, on average, incumbent firms deplete their knowledge capital.

The actual mass of INC_{t+1} firms will be

$$\eta_t \int_{\hat{A}_{t+1}}^{+\infty} \frac{\gamma \left[\hat{A}_t g_e (1-\delta^I)\right]^{\gamma}}{A_{j,t+1}^{\gamma+1}} d(A_{j,t+1}) = \eta_t H_{t+1} \equiv \eta_t \left[\frac{\hat{A}_t g_e (1-\delta^I)}{\hat{A}_{t+1}}\right]^{\gamma}$$

where $\hat{A}_{t+1} > \hat{A}_t g_e (1-\delta^I)$ because the fraction of η_t firms characterized by $A_{j,t+1} < \hat{A}_{t+1}$ will exit the market, and $H_{t+1} = \left[\frac{\hat{A}_t g_e (1-\delta^I)}{\hat{A}_{t+1}}\right]^{\gamma}$ characterizes the endogenous survival probability.[3]

Identifying the mass of t-period incumbents is now straightforward

$$INC_t = \eta_{t-1} H_t \tag{16}$$

and the law of motion for the mass of active firms is

$$\eta_t = \left(\frac{\bar{e}_t}{\hat{A}_t}\right)^{\gamma} + \eta_{t-1} H_t \tag{17}$$

Then, using (9) and (10)

$$\left(\alpha P_t^I \hat{A}_t\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha} = f_t - E_t \left\{\bar{\Lambda}_{t+1} H_{t+1} V_{t+1}^{av}\right\} \tag{18}$$

where V_{t+1}^{av} defines the average value of INC_{t+1} firms.

[3]By definition $INC_t \leq \eta_{t-1}$, *i.e.* $\left[\frac{\hat{A}_{t-1} g_e (1-\delta^I)}{\hat{A}_t}\right]^{\gamma} \leq 1$. This condition is always satisfied. in the deterministic steady state, when $\frac{P_t^I}{P_{t-1}^I} = 1$. In the stochastic model one must either impose an upper bound on shocks that trigger an increase in P_t^I or solve the model under the occasionally binding constraint $INC_t = \eta_{t-1}$. In this paper we opt for the first choice. Needless to say, the assumption $\delta^I > 0$ renders the stochastic constraint $INC_t = \eta_{t-1}$ less stringent.

From (18) it can be easily seen how the I-sector threshold depends negatively on the dynamics of the relative price of investment, P^I, and on the future expected profitability which evolves according to[4]

$$V_{t+1}^{av} = \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\alpha P_{t+1}^{I}\hat{A}_{t+1}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha} - f_{t+1} + E_{t+1}\left\{\bar{\Lambda}_{t+2}H_{t+2}V_{t+2}^{av}\right\} \tag{19}$$

As a result, dynamics of the relative price of investment goods is key to determine endogeneity of I-firms dynamics.

Figure 2 provides a graphical representation of how the I-sector evolves over time. Panels a and b depict the Pareto distributions of NEs and INCs at the end of period $t-1$, when all surviving firms are characterized by idiosyncratic efficiency $A_{j,t-1} \geq \hat{A}_{t-1}$. At the beginning of t the η_{t-1} firms are subject to idiosyncratic efficiency shocks, and their "knowledge capital" depreciates on average, implying a leftward translation of their idiosyncratic efficiency distribution (Panel c). Finally, as production in t takes place, panel d identifies the distribution of the η_{t-1} firms between exiters and surviving INC_t firms.

Finally, given that NE_t and INC_t firms share the same identical support, η_t firms productivity distribution can be characterized as [5]

$$\int_{\hat{A}_t}^{+\infty}\frac{\gamma\hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}}\,d(A_{j,t}) \tag{20}$$

and their average productivity is

$$\mu_t = \frac{\gamma}{\gamma-1}\hat{A}_t\ . \tag{21}$$

2.2.3 I-firms production and the process of creative destruction

The aggregate (inverse) supply function of I-firms is easily computed.[6]

$$\begin{aligned}
I_t &= \eta_t\int_{\hat{A}_t}^{+\infty}A_{j,t}\cdot\left(P_t^I\alpha A_{j,t}\right)^{\frac{\alpha}{1-\alpha}}d\mathbf{F}(A_{j,t}) \\
&= \eta_t\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\hat{A}_t^{\frac{1}{1-\alpha}}\left(P_t^I\alpha\right)^{\frac{\alpha}{1-\alpha}} \\
&= \left\{\bar{e}_t^{\gamma} + \eta_{t-1}\left[\hat{A}_{t-1}g_e(1-\delta^I)\right]^{\gamma}\right\}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\times \\
&\times\left[\alpha\frac{f_t-\bar{\Lambda}_{t+1}H_{t+1}V_{t+1}^{av}}{(1-\alpha)}\right]^{(1-\alpha)\left(\frac{1}{1-\alpha}-\gamma\right)}\left(P_t^I\right)^{\gamma-1}(\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{22}$$

[4]see section A.1 for additional details.

[5]See section A.2.

[6]See section A.3 for the details of the derivation.

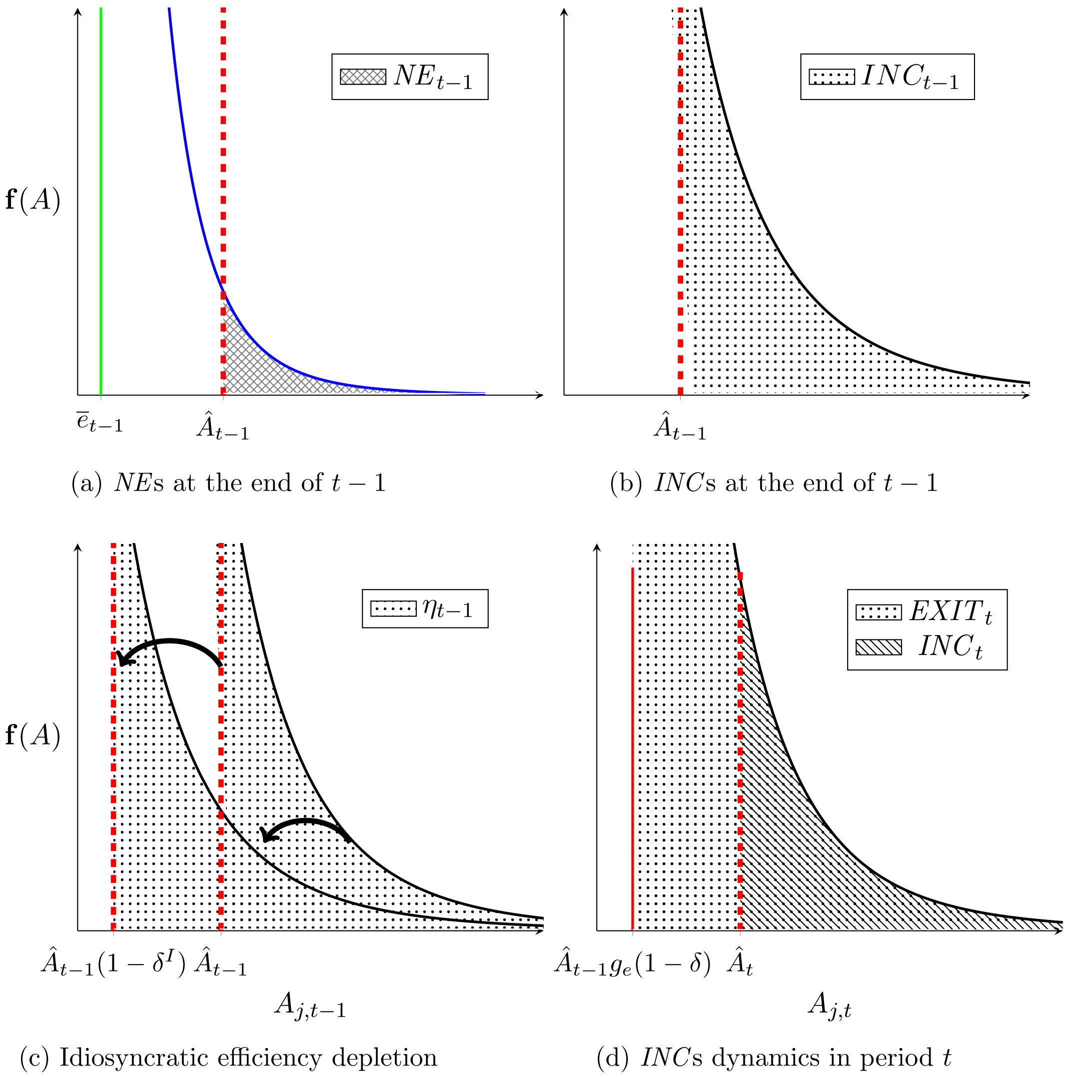

Figure 2: I-firms dynamics

From (14) it is easy to see that a shock to $\overline{e}_t$ shifts to the right the NE_t supply of investment goods. For any given demand for investment goods (31) this puts downward pressure on in the relative price P_t^I. As a result, given (18), from (16) it can be seen that both the mass and the supply of surviving incumbents shrink. This is the essence of the "creative destruction" process triggered by IST shocks.

2.2.4 Comparison with the standard (No-Entry) characterization of the I sector

To benchmark our results we sketch here a representative firm, "No-Entry" (*NoE* henceforth) version of our model where entry/exit flows have been removed, and the I-sector representative firm is characterized by the following production function

$$I_t^{NoE} = A_t^{NoE} S_t^{NoE}$$
$$A_t^{NoE} = g_{A,t}^{NoE} A_{t-1}^{NoE}$$

$$\ln\left(g_{A,t}^{NoE}\right) = (1-\rho_A)\ln\left(g_*^{NoE}\right) + \rho_A \ln\left(g_{A,t-1}^{NoE}\right) + \varepsilon_{A,t}^{NoE}$$

Profit maximization implies

$$P_t^{I,NoE} = \frac{1}{A_t^{NoE}} \tag{23}$$

A_t^{NoE} follows a random walk, therefore a IST shock has symmetrical opposite effects on the stochastic trends driving $P_t^{I,NoE}$ and A_t^{NoE}. Differently from (22), condition (23) implies that the supply of I-goods is infinitely elastic to non-IST shocks.

In our "Entry" (E henceforth) model instead, from condition (7) for a generic firm j the following condition must hold

$$P_t^I = \frac{S_{j,t}^{1-\alpha}}{\alpha A_{j,t}} \tag{24}$$

where each firm chooses to produce I-goods up to the point where the marginal cost equals P_t^I. Due to decreasing returns to scale, an exogenous increase in the demand for I-goods can therefore trigger an increase in I_t only if P_t^I also increases. This, in turn, limits exit flows and raises the number of *NE*s.

The supply of I-goods is therefore less elastic than in the *NoE* model. Indeed, condition (22) ties the price elasticity of I_t directly to γ, the parameter that shapes the Pareto distribution. Finally, one crucial implication of the E model is that P^I is endogenous to changes in the demand for I-goods, independently on the nature of the shock hitting the economy. As shown in section 4.3 below, this will significantly improve the dynamic performance of the model even when IST shocks play a minor role in explaining output volatility.

2.3 The Representative Household

We assume a standard characterization of households preferences,[7]

$$U_t(C,N) = \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \ln(C_{t+s}) - \Phi \frac{N_{t+s}^{1+\theta}}{1+\theta} \right\} \tag{25}$$

the flow budget constraint in real terms is

$$C_t + P_t^I I_t = r_{k,t} K_{t-1} + w_t N_t + D_t \tag{26}$$

where D_t are aggregate dividends payd by I-firms. The law of motion of capital is

$$K_t = (1-\delta)K_{t-1} + \mu_t \left[1 - \mathcal{J}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{27}$$

[7] Households preferences are logarithmic in consumption to guarantee the existence of a balanced growth path.

where δ is the capital depreciation rate, $\mathcal{J}\left(\frac{I_t}{I_{t-1}}\right)=\frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2$ defines investment adjustment costs, μ_t is a shock to the marginal efficiency of investment (*i.e.*, MEI) as in JPT:

$$\ln\left(\mu_t\right)=\rho_\mu \ln\left(\mu_{t-1}\right)+\varepsilon_t^\mu \tag{28}$$

and $\varepsilon_t^\mu \sim N\left(0,\sigma^\mu\right)$ is an i.i.d. innovation term.

The F.O.C.s are

$$\lambda_t=\left(C_t\right)^{-1} \tag{29}$$

$$\frac{\Phi N_t^\theta}{\lambda_t}=w_t \tag{30}$$

$$P_t^I=\left\{\begin{array}{c} Q_t\mu_t\left[1-\left(\mathcal{J}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}+\mathcal{J}\left(\frac{I_t}{I_{t-1}}\right)\right)\right]+ \\ +\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}\mu_{t+1}\mathcal{J}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array}\right\} \tag{31}$$

$$\lambda_t=\beta E_t\left\{\lambda_{t+1}\left[\frac{r_{k,t+1}}{Q_t}+\frac{Q_{t+1}}{Q_t}(1-\delta)\right]\right\} \tag{32}$$

Where $Q_t=\frac{\phi_t}{\lambda_t}$ is the shadow value of the capital stock in units of consumption goods, ϕ_t is the Lagrange multiplier of the law of motion of capital and $\mathcal{J}'(\cdot)\equiv\gamma_I\left(\frac{I_{t+s}}{I_{t+s-1}}-1\right)$.

2.4 Market clearing

Recalling that

$$S_t=\eta_t\int_{\hat{A}_t}^{+\infty}S\left(A_{j,t}\right)d\mathbf{F}(A_{j,t}) \tag{33}$$

is the amount of C-good inputs necessary for I-goods production, the C- and I-goods market clearing conditions respectively are

$$Y_t=C_t+S_t+\eta_t f_t \tag{34}$$

$$Y_t=\left(z_tN_t\right)^\chi\left(K_{t-1}\right)^{1-\chi} \tag{35}$$

3 Results

To support intuition in the presentation of our simulation results, it is convenient to apply to each endogenous variable the following partition

$$X_t^\iota=x_t^\iota\overline{X}_t^\iota$$

where $\iota = E$, NoE identifies the (Entry, No-Entry) features of the model, x_t^ι and $\overline{X}_t^\iota$ respectively define the stationary (cyclical) and trend components of variable X_t^ι.[8] Identification of $\overline{X}_t^\iota$ allows to identify the long run effects of permanent sectoral productivity shocks.

We obtain the following characterizations for $\overline{X}_t^\iota$ [9]

$$\overline{Y}_t^E = \overline{S}_t^E = \overline{C}_t^E = \overline{W}_t^E = \Gamma_t;\ \text{with } \Gamma_t = \frac{e_t^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} z_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{t(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{36}$$

$$\overline{Y}_t^{NoE} = \overline{C}_t^{NoE} = \overline{W}_t^{NoE} = z_t \left(A_t^{NoE}\right)^{\frac{1-\chi}{\chi}} \tag{37}$$

The stochastic trends governing K and I are

$$\overline{K}_t^E = \overline{I}_t^E = \Lambda_t;\ \text{with } \Lambda_t = \frac{e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} z_t^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{t[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}}$$

$$\overline{K}_t^{NoE} = \overline{I}_t^{NoE} = z_t \left(A_t^{NoE}\right)^{\frac{1}{\chi}}.$$

P_t^I, Q_t and $r_{k,t}$ are driven by

$$\overline{P}_t^{I,E} = \overline{Q}_t^E = \overline{r}_{k,t}^E = \frac{\Gamma_t}{\Lambda_t};\ \text{with } \frac{\Gamma_t}{\Lambda_t} = \frac{z_t^{\frac{1}{1+\chi(\gamma-1)}} g_*^{\frac{t\chi[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}}{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}} \tag{38}$$

$$\overline{P}_t^{I,NoE} = \overline{Q}_t^{NoE} = \overline{r}_{k,t}^{NoE} = \left(A_t^{NoE}\right)^{-1} \tag{39}$$

Note that in the NoE model A_t^{NoE} is the only driver of $\overline{P}_t^{I,NoE}$, $\overline{Q}_t^{NoE}$ and $\overline{r}_{k,t}^{NoE}$. By contrast, in the E model the LAT shifter z_t has a positive effect on $\overline{P}_t^{I,E}$, $\overline{Q}_t^E$ and $\overline{r}_{k,t}^E$. Our result obtains because LAT shocks raise the demand for I goods and this, in turn, determines an increase in their relative price.

The stochastic trend governining NE, INC and η is

$$\Xi_t = \frac{e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} z_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{t\gamma\left\{(1-\alpha)-\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}\right\}}} \tag{40}$$

whilst the one governing I-firms cutoff, $\hat{A}_t$, is[10]

$$\Theta_t = \frac{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{z_t^{\frac{\chi}{1+\chi(\gamma-1)}} g_*^{t\left\{\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}-(1-\alpha)\right\}}} \tag{41}$$

Finally, with reference to average firm productivity in the I sector

[8] The details of stochastic trend identification and removal are left in section B.1.

[9] Note that $\bar{e}_t = e \cdot e_t$, where e is the initial condition characterizing $\bar{e}_t$, see section B.2.3.

[10] The I-firms fixed costs trend is by assumption deterministic and equals the BGP growth rate, g_*^t.

$$A_t^{av} = \frac{\gamma}{\gamma - 1}\hat{A}_t$$

we obtain

$$\overline{A}_t^{av} = \Theta_t \tag{42}$$

because the trend component of average productivity in the I sector is the same as sectorial thresholds. Note that permanent shocks to z_t impact on $\overline{A}_t^{av}$, in sharp contrast with the No-Entry model. A LAT shock unambiguously raises the relative price of I-goods. This, in turn, relaxes competitive pressures on endogenous entry-exit flows, and expands the tail of relatively less productive I firms.

3.1 Calibration

For sake of comparison, we strictly follow JPT estimates of common parameters and exogeous process parameters. We calibrate Φ at a value such that $N^{ss} = \exp(0.192)$ and set the inverse of the Frisch elasticity, θ, at 4.492.[11] The investment adjustment cost, $\gamma_I = 3.142$, and the capital depreciation rate, $\delta = 0.025$, are standard. The the capital income share $1 - \chi = 0.167$ replicates the value estimated by JPT.[12]

Some deviations from JPT are inevitably due to our modeling choices concerning the I-sector. For instance, we assume that I-sector production occurs under decreasing returns to scale. To maintain the rental rate of capital and the great ratios at the JPT values, we must therefore choose slightly different calibrations for β, which is now equal to 0.994 rather than 0.9985, and for the quarterly growth rate of the economy g_*, now 1.004 instead of 1.003.

Turning to the innovative features of our model, we postulate a business destruction rate $\frac{NE^{ss}}{\eta^{ss}}$ equal to 10% on annual basis, equal to the US business destruction rate reported in Bilbiie et al. (2012). From condition (42) we have that $\overline{A}_t^{av} = \Theta_t$, therefore in SS the fraction of productivity increases generated by *NE*s is 10% *per annum*, matching the findings in Garcia-Macia et al. (2019), who have that firms of age < 1 contribute to 9% of TFP growth. The fixed costs in the I-sector amount to 1% of total GDP. The I-goods market clears when $P^I = 1$ and the I-firms efficiency depreciation is implicitly $\delta^I = 0.0046$.

I-firms returns to scale, $\alpha = 0.8$, are set at the lower bound of Basu and Fernald (1997) estimates, and the tail index of the Pareto distribution, $\gamma = 5.5$, is set fairly close to Asturias et al. (2017).[13] Finally, the MEI shock calibration is $\rho_\mu = 0.813$ and $\sigma^\mu = 5.786\%$, while the calibration of IST and LAT shocks is properly discussed in sections (3.2.1) and (3.2.3) below.

[11] From (30) it is easy to see that worked hours are stationary.

[12] We also experimented with $\chi = 1 - 0.33$ and could not detect any significant difference in our results.

[13] We perform a sensitivity analysis of our key results in Section D.1.

Parameters calibration is summarized in Table 1.

Table 1: **Parameters**

Households		
g_*	1.004	Gross BGP rate
β	0.994	Discount factor
δ	0.025	Capital depreciation
γ_I	3.142	Investment adjustment costs
θ	4.492	Inverse Frisch elasticity of labor supply
N^{ss}	exp(0.192)	SS labor
C-Producers		
χ	0.833	Labor share of income
I-firms		
g_e	$g_*^{1-\alpha}$	Technology frontier BGP rate
α	0.8	I-firms returns to scale
H	0.975	I-firms survival rate
γ	5.5	Tail index of I-firms distributions
f	0.01*Y	Fixed Cost initial condition
δ^I	$1 - H^{1/\gamma} g_*^{1-\alpha}$	I-firms efficiency depreciation
Exogenous Processes		
ρ_{μ^i}	0.813	MEI shock persistence
ρ_e	0	IST shock growth persistence, Entry model
ρ_A	0.163	IST shock growth persistence, No Entry model
ρ_z	0.287	LAT shock growth persistence
σ^A	0.629%	IST shock sd, No-Entry model
$\sigma^{z,NoE}$	0.943%	LAT shock sd, No-Entry model
σ^{μ}	5.786%	MEI shock sd
σ^e	$\frac{(1-\chi)}{\chi}\frac{1+\chi(\gamma-1)}{(1-\chi)\gamma}\frac{\sigma^A}{1-\rho_A}$	IST shock sd, Entry model
σ^z	$\frac{1+\chi(\gamma-1)}{\chi\gamma}\sigma^{z,NoE}$	LAT shock sd, Entry model

3.2 Permanent shocks: long run effects and transitional dynamics

We investigate here the impact of permanent IST and LAT shocks. The model is solved by means of a first order perturbation method.

3.2.1 IST shock

The IST shock in our model is normalized to generate the same long-term shift to C-goods production obtained in the *NoE* under the JPT calibration (Table 2). In JPT the intial IST shock is equal to $\sigma^A = 0.629\%$, and the long-run productivity shift $\frac{\sigma^A}{1-\rho_A} = 0.752\%$ is matched by an identical fall in P^I. The long run shift in consumption goods production is equal to $\frac{1-\chi}{\chi}\frac{\sigma^A}{1-\rho_A}$ (see condition 37). In our model we impose $\rho^e = 0$ and according to condition (36) the long run shift in the C-sector amounts to $\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}\sigma^e$, and imposing that $\sigma^e = \frac{(1-\chi)}{\chi}\frac{1+\chi(\gamma-1)}{(1-\chi)\gamma}\frac{\sigma^A}{1-\rho_A}$ ensures the same long run shift for real variables. Given our calibration for χ and γ, normalization requires that $\sigma^e \equiv 0.779\%$. The bulk of the difference between σ^e and σ^A is explained by the exogenous persistence parameter ρ_A, but why do we need $\sigma^e > \frac{\sigma^A}{1-\rho_A}$ to match the long-run increase in F-goods production?

To answer this question, note that to obtain the same C-goods increase in the two models we need the same increase in the capital-labor ratio. This, in turn requires an identical increase in the long-run production of I-goods. Moreover, in both models, the new steady state is characterized by an unchanged allocation of C-goods to consumption and to production of I-goods.[14] As a result, since $\Delta\overline{P}_t^{I,\iota} + \Delta\overline{I}_t^{\iota} = \Delta\overline{S}_t^{\iota}$, we obtain that $\Delta\overline{P}_t^{I,E} = \Delta\overline{P}_t^{I,NoE}$. In addition, the two models by assumption generate an identical increase in average firm productivity, respectively A in the NoE model and $\frac{A}{\eta}$ in the E model.

One key difference between the two sets of long-run adjustments is that in the E model we obtain an increase in both the aggregate I-sector productivity shifter of A and in the mass of firms η. Understanding why this happens allows to rationalize the required calibration for σ^e. In our model, decreasing returns imply that, given the same variations in $\frac{A}{\eta}$ and P^I, I-firms choose a smaller size of production than in the NoE model, characterized by constant returns to scale. As a result, the E model can induce the same increase in production only if the number of firms increases.

From conditions (14), (16), (17) it is easy to see that condition[15]

$$\left\{P^{I,ss}\alpha^{\alpha}\overline{e}_{t-1}g_{e,t}\left[\frac{1-\alpha}{fg_*^t}\frac{\gamma(1-\alpha)-1+\beta H}{\gamma(1-\alpha)-1}\right]^{1-\alpha}\right\}^{\gamma} = \eta^{ss}\left[1-\left(1-\delta^I\right)^{\gamma}\right] \tag{43}$$

implies that

$$\frac{\Delta\eta^{ss}}{\eta^{ss}} = \gamma\left\{\frac{\Delta g_{e,t}}{g_{e,t}} + \frac{\Delta P^{I,ss}}{P^{I,ss}}\right\} = \gamma\left\{\sigma^e - \frac{\sigma^A}{1-\rho_A}\right\}$$

must hold in the new steady state, confirming that the number of firms can increase only if $\sigma^e > \frac{\sigma^A}{1-\rho_A}$.[16] Note also that for constant returns to scale, $\alpha = 1$, and a constant mass of I-firms, $\eta\left[1-\left(1-\delta^I\right)^{\gamma}\right] = 1$, (43) boils down to the standard $P^I = \frac{1}{A^{NoE}}$ condition.[17]

Figure 3 plots IRFs to the IST shock. From condition (14) we know that the IST shock implies an inflow of NEs which shifts the supply schedule to the right. In the periods following the initial shock, NEs dynamics are driven by two opposite effects (condition 14). The first one, positive, is the permanent increase in the $\bar{e}_t$ shifter after the shock. The second one, negative, is determined by the cumulative fall in P^I whose adjustment to

[14]Long run constant $\frac{c}{s}$ is needed to preserve the balanced growth path.

[15]See section B.2.1 for the threshold computation in the deterministic steady state.

[16]It would be straightforward to show that the increase in η drives the variation in A^E.

[17]Note that in this case it must be that $e_t = A_t^{NoE}$.

Table 2: **Variables $\Delta\%$ from old SS following a permanent IST shock**

Variable	ENTRY	NO-ENTRY
Y	0.1507	0.1507
C	0.1507	0.1507
S	0.1507	0.1507
I	0.9022	0.9022
N	0	0
P^I	-0.7515	-0.7515
A	0.9022	0.7515
η	0.1507	-

its new steady state value gradually tightens the entry threshold, causing the reduction in *NE*s flows towards their new steady-state value.

The fall in the relative price of investment goods lowers profitability of incumbents, raising the mass of exiting firms well above the contemporaneous *NE*s inflow. In fact, it takes about 20 periods before *INC*s mass returns to the initial steady state level. In a sense, creative destruction limits the fall inP^I relatively to what would happen in the *NoE* model, where all firms benefit from the productivity increase and there are no endogenous exits. The increase in exits allows the persistence of the *NE*s inflow above its new steady-state value.

In spite of the immediate fall in P^I, which calls for greater demand, the expectation of further reduction in their relative price and in the shadow price of capital (see condition 31) induces households to postpone investment, which remains below the initial steady state value for 10 quarters. Consumption is characterized by a moderate increase for the first 20 periods and then begins to gain momentum. Weak investment and the depreciation of P^I imply that output remains stationary during the first 20 periods. This pattern, in turn, drives the evolution of employment.

The *NoE* model generates quite different dynamics. The IST shock is quickly incorporated into a productivity increase, the relative price of investment goods and the shadow price of capital fall abruptly to the new steady state level. The strong fall in P^I and Q triggers a surge in demand for investment goods which is stronger than the increase in I-firms productivity. As a result, since P^I is constrained by the productivity shifter dynamics (see eq. 23), a persistent fall in consumption is necessary to reallocate resources to the production of investment goods.

Summing up, even if shock calibrations imply identical long run adjustments in the C-sector, in consumption and in production of investment goods, predicted transitional dynamics are quite different. The No-Entry model is associated to quick and strong responses in productivity and investment, and to a sharp and fall in the relative price of I-goods. Transition is much slower in the endogenous-entry model. The exit of less productive firms is crucial in determining the gradual fall in P^I while preventing output from increasing in the medium-short run. Thus the creative destruction effect is at the root of the endogenous persistence mechanism. In consequence of different supply responses to the shock, the two models imply quite different consumption and labor supply patterns.

3.2.2 The role of endogenous exit in the IST shock transmission

The investigate the specific role played by endogenous exit, we benchmark our a model against a simplified version where exit is exogenous. In this alternative framework, I-firms are still subject to idiosyncratic productivity shocks, but they are randomly hit by an exogenous exit shock so that $INC_{t+1} = \eta_t H$. Figure 4 reports selected impulse response functions. First, note that exogenous exit is associated with a sudden and deeper fall in the relative price of investment. On impact, this translates into a reduction in the I-firms mass. Then, as the flows of new entrants adjust, the mass of I-firms grows steadily determining a similar pattern for investment and output. As a result, creative destruction does not materialize, the production of investment never falls and the transition to the new steady state is much faster. Finally, note that what determines the persistence of the relative price of investment is endogenous entry rather than exit.

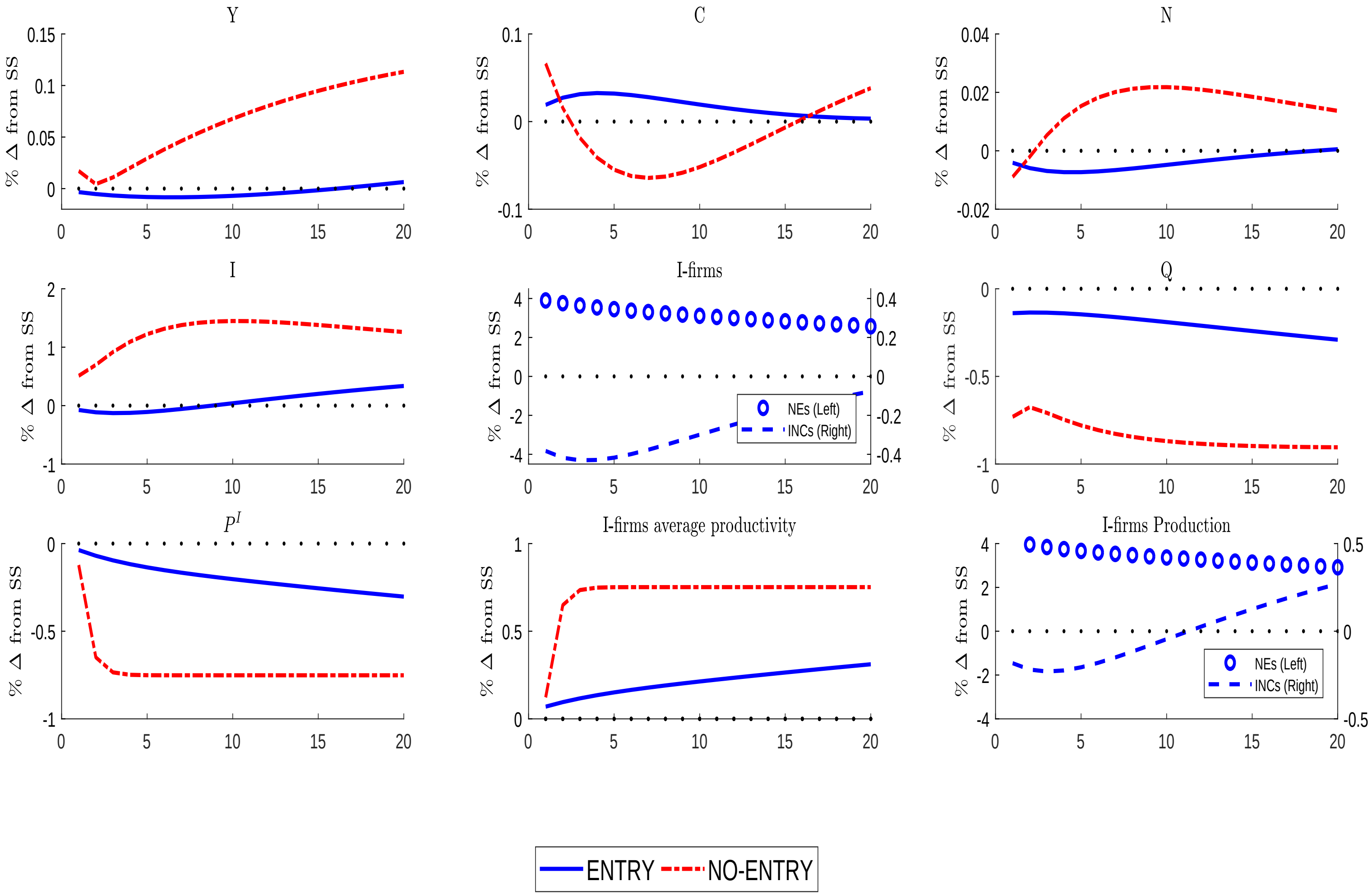

Figure 3: Impulse response functions to a permanent IST shock.

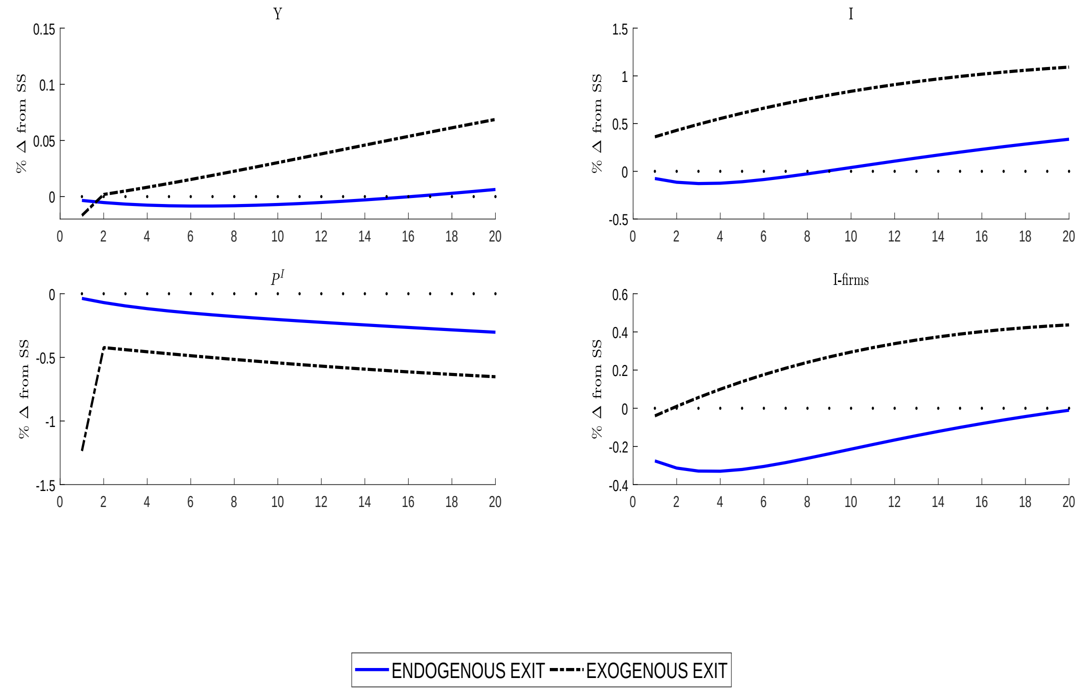

Figure 4: Selected impulse response functions to a permanent IST shock.

3.2.3 LAT shock

In JPT the LAT shock is equal to 0.943, and ρ^z is 0.287. We choose an indentical value for ρ^z and set $\sigma^z = \frac{1+\chi(\gamma-1)}{\chi\gamma}0.943\% \equiv 0.9774\%$. This increase in σ^z is due to the endogeneity of P^I (and I-firms entry/exit) to the LAT shock and is explained by the same mechanism that drives long run adjustments to the IST shock.

Table 3: **Variables Δ% from old SS following a permanent LAT shock**

Variable	ENTRY	NO-ENTRY
Y	1.3226	1.3226
C	1.3226	1.3226
S	1.3226	1.3226
I	1.0821	1.3226
N	0	0
P^I	0.2405	0
A	1.0821	0
η	1.3226	-

Table 3 displays steady state percentage variations of key variables. In the long run all variables comove positively. In the endogenous entry model the greater demand for

I-goods is associated to an increase in the sectoral relative price. This, in turn, raises the entry rate and reduces firms exits. As a consequence average productivity of I-firms, $\frac{A}{\eta}$, falls. Condition (24) is once more crucial to rationalize our results.

Impulse responses to a LAT shock are shown in Figure 5. Both models predict expansionary effects in response to the shock and differences are essentially explained by the I-sector adjustments. The LAT shock acts as a demand shock for the I-sector. In the endogenous entry model the shock triggers an increase in the relative price of investment goods, facilitating both the entry of new firms and the survival of incumbents. Our calculations show that average productivity falls. In the *NoE* model P^I and I-firms productivity remain constant. This explains why in the *NoE* model we obtain that the supply of I-goods grows more vigorously whereas the shadow price of capital is less sensitive to the shock.

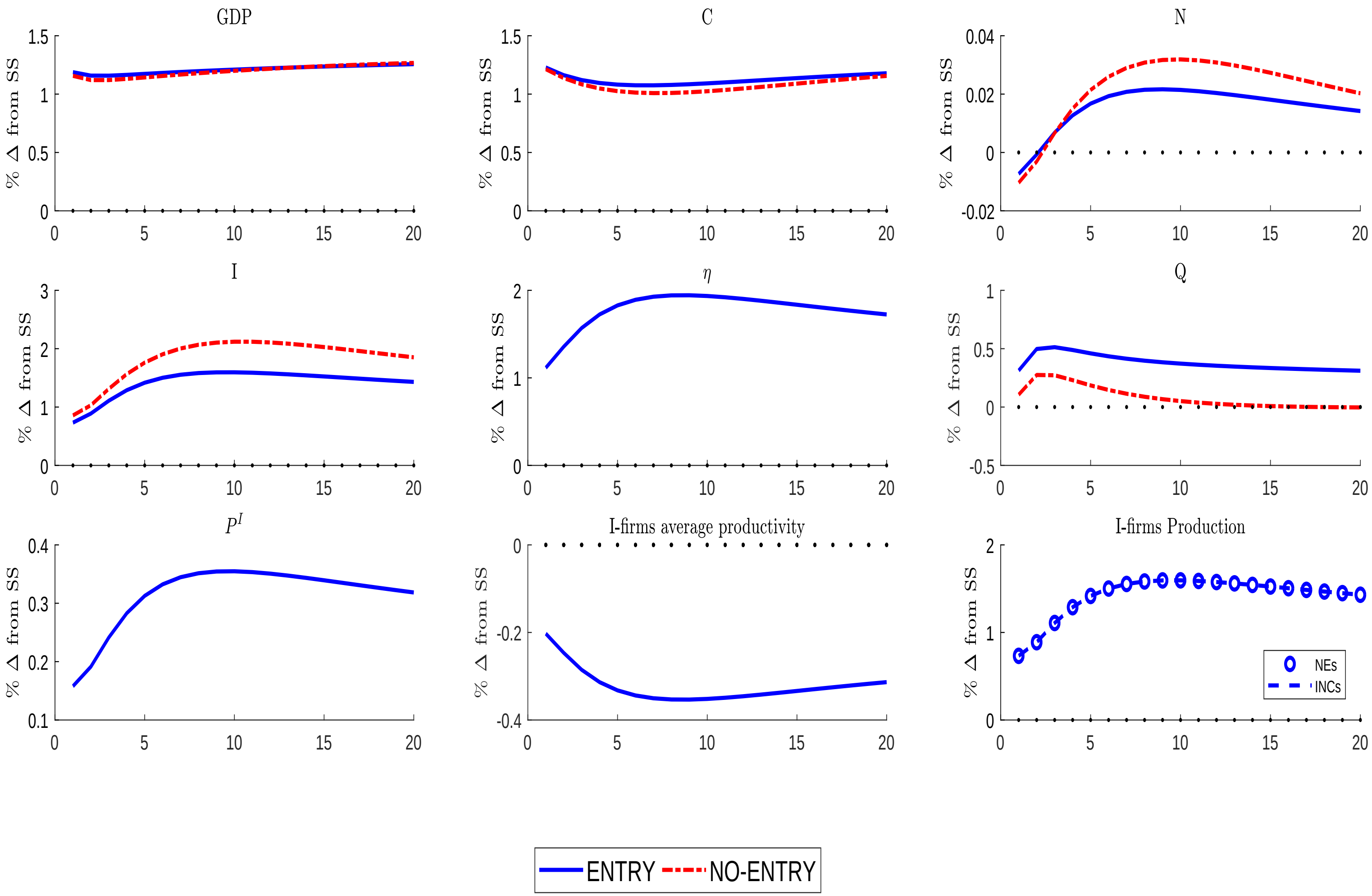

Figure 5: Impulse response functions to a permanent LAT shock.

4 Extension: endogenous firm dynamics in a New Keynesian DSGE model

By incorporating a limited number of frictions which are standard in the DSGE literature, *i.e.* nominal rigidities and consumption habits, this section facilitates comparison with previous contributions such as JPT and Moura (2018). A detailed description of the DSGE version of the model is left to section C and the additional parameters calibration is reported in Table 4.[18]

Ascari et al. (2019) point out that standard DSGE models are subject to the Barro and King (1984) curse, *i.e.* they fail to replicate: i) the positive unconditional correlation between the growth rates of consumption and investment; ii) the strongly positive unconditional correlation between consumption growth and output growth; iii) the cross correlations between consumption growth and the level of hours.[19]

Table 4: **Additional DSGE Parameters**

Households		
a	0.858	Habit in Consumption
Retailers		
ν_p	6.55	C-goods elasticity of substitution
λ_p	0.787	Calvo prices
γ_p	0.131	Prices indexation
Ω	0.2446	Fixed production cost
Labor Packagers		
ν_w	7.94	Wages elasticity of substitution
λ_w	0.777	Calvo wages
γ_w	0.092	Wages indexation
Government		
ρ^R	0.86	Interest rate smoothing
κ_π	1.688	Taylor rule inflation
κ_x	0.046	Taylor rule outputgap
$\kappa_{\Delta x}$	0.211	Taylor rule outputgap growth
G^{ss}	0.25	public expenditure to GDP ratio
Shocks		
σ^R	0.21%	Monetary policy shock

When evaluating the performance of DSGE models, one key issue concerns the choice

[18] Parameters and exogenous variables calibration strictly follow JPT. For sake of simplicity we abstract from capital utilization costs.

[19] To overcome these anomalies they incorporate firms networking in an otherwise standard DSGE model.

of shocks. Medium-scale models typically rely on a relatively large number of shocks. This widespread practice is open to criticism because several of such shocks lack a clear economic interpretation (Chari et al., 2009). We take this criticism here, and restrict the number of shocks to four: LAT, monetary policy (MP hereafter), IST and MEI. Following JPT, the MEI shock affects the transformation of savings into capital input. Relative to the calibration discussed in section 3.1, additional parameters and shocks are borrowed from JPT (see Table 4).

4.1 The recessionary impact of IST shocks

Figure 6 depicts the impact of a permanent IST shock in the DSGE version of our model in comparison with the canonical formulation of JPT.[20] Relative to the the RBC case discussed above, nominal frictions in the C-sector cause a recession on impact. Such a recession is short lived in JPT, whilst it is much more prolonged in our model. In addition, our model allows for a procyclical comovement of real variables on impact, something that is completely missed by JPT. More interestingly, our results essentially replicate the findings of Moura (2018)[21] who relies on an implausibly long duration of Calvo contracts in the I-sector (around 14 quarters). This suggests that the exclusive reliance on nominal rigidities hides structural factors that determine the rigidity of the I-sector supply function.

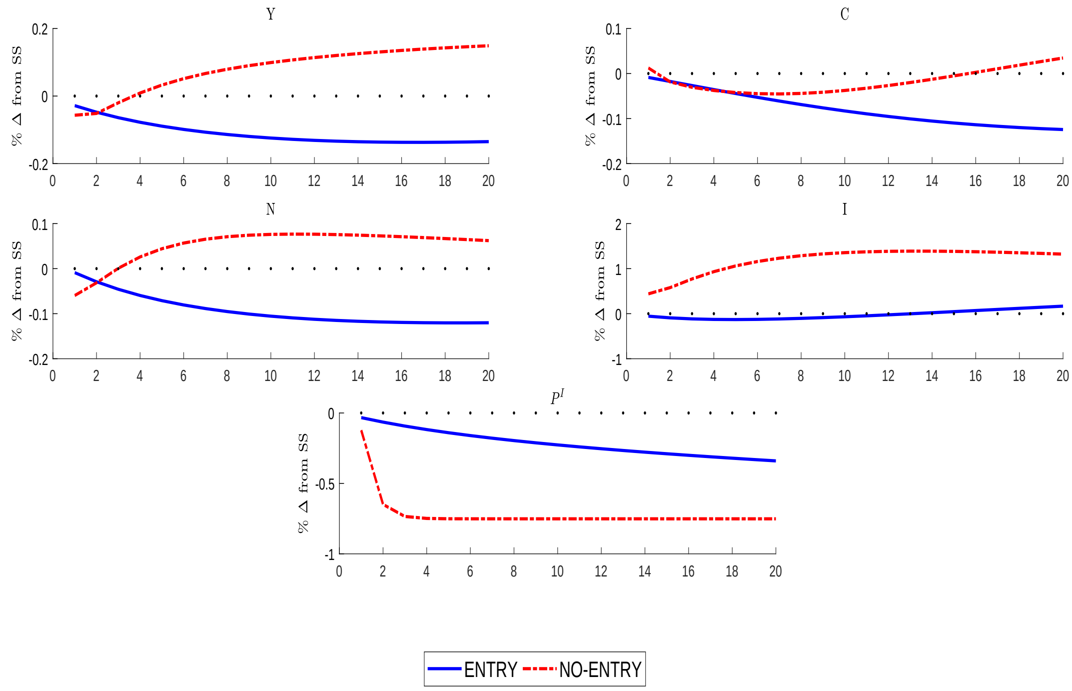

Figure 6: Selected impulse response functions to a permanent IST shock, DSGE model.

[20] A full display of DSGE model impulse responses to a IST, LAT and monetary policy shock is left to section C.3.1 in the Appendix.

[21] See Figure 3, page 57 in Moura (2018).

4.2 The impact of MEI shocks

Figure 7 displays impulse responses of E and NoE models to a MEI shock, which is the main business cycle driver according to JPT and Moura (2018). In the NoE model P^I is exogenous and the MEI shock entirely falls upon the production of I-goods, this displays a crowding out effect on consumption which falls on impact even though output increases. By contrast, in our model any increase in the production of I-goods is inevitably associated to higher production costs and therefore causes a surge in P^I that, in turn, dampens the response of I-goods and prevents consumption from falling.[22] As a results, our model also displays a smaller fall in the shadow price of capital, Q, due to a relatively weaker capital accumulation in response to the shock. The limited response of capital accumulation inevitably dampens C-firms production and demand for labor.

With reference to the Barro-King curse, our model shows i) a stronger correlation between the growth rates of consumption and investment; ii) a stronger correlation between consumption growth and output growth; iii) improved cross correlations between consumption growth and the level of hours.

[22]The cumulative fall of consumption, in percentage deviations from the ss, in the first two periods amounts to -0.025% in the No-Entry model. By contrast, in the Entry model the cumulative increase of consumption in the same period is equal to 0.045%.

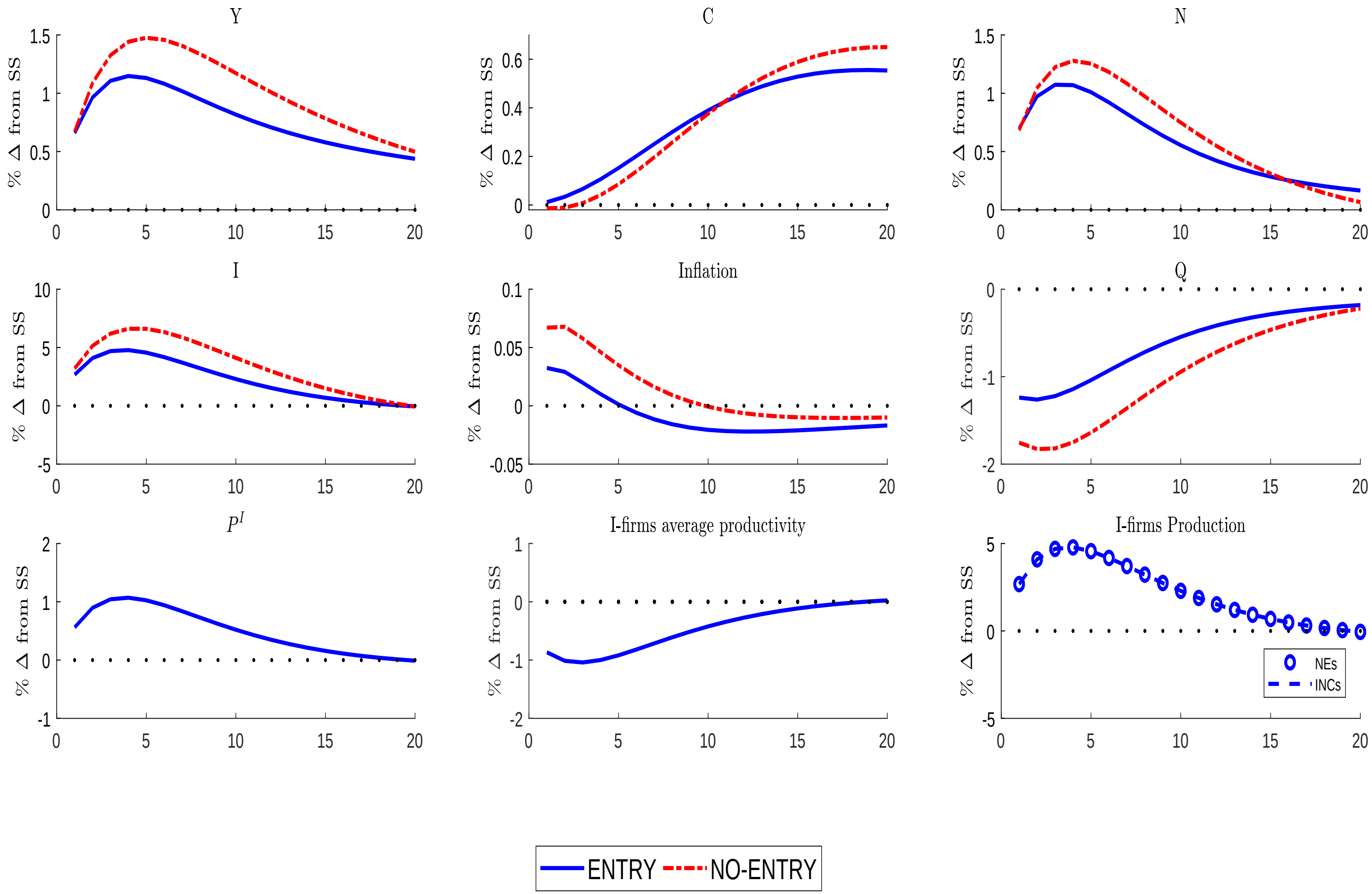

Figure 7: Impulse response functions to a transitory MEI shock, DSGE model.

4.3 Business cycle analysis

In Table 5 we show how our model performs relative to JPT conditional on LAT, MP, MEI and IST shocks.

In contrast with JPT, our model predicts a correlation between output and consumption which is stronger than in JPT and much closer to the data-generated moment. Importantly, this does not cause the model to inflate the comovement between output and investment. Our model also does a relatively better job in replicating the correlation between investment and consumption, which is another weak spot of traditional medium-scale DSGE models. We also obtain a striking improvement in the correlations of both output and consumption with worked hours. These results are not restricted to the contemporaneous correlations but occur throughout the cross-correlogram, and are particularly striking if one looks at cross-correlations of consumption with output, investment and worked hours (Figure 8).

Finally, Table 6 reports the variance decomposition for the two models. In the E model the MEI shock still explains more than sixty percent of output volatility but its contribution to the the volatility of consumption growth is almost halved when compared to the NoE model. This result is fully in line with our discussion of model dynamics in response to a MEI shock. Interestingly, the IST shock is much less relevant for business cycle dynamics. Essentially this happens because it takes many periods before consumption and investment react to the shock.

Table 5: **Moments in the Benchmark (Entry) and JPT (No Entry) model**

	$\sigma(\Delta Y)$	$\sigma(\Delta C)$	$\sigma(\Delta \widetilde{I})$	$\sigma(N)$	$\rho(\Delta Y, \Delta C)$
JPT Data	(0.97)	(0.48)	(3.58)	(3.68)	(0.58)
Entry	0.97	0.44	4.37	3.35	0.55
No Entry	1.08	0.45	4.62	3.97	0.42
	$\rho(\Delta Y, \Delta \widetilde{I})$	$\rho(\Delta \widetilde{I}, \Delta C)$	$\rho(\Delta Y, N)$	$\rho(\Delta C, N)$	$\rho(\Delta \widetilde{I}, N)$
JPT Data	(0.89)	(0.36)	(0.05)	(0.13)	(0.02)
Entry	0.93	0.22	0.08	0.16	0.04
No Entry	0.95	0.11	0.17	0.34	0.07

Note: in the first row data obtained from the dataset used in JPT (1954Q3 to 2009Q1) are shown. In the second and third rows we report selected theoretical business cycle moments implied by our model (Entry) and by JPT's (No Entry).
Investment is defined in real terms as $\widetilde{I} = P^I I$ to comply with national accounting standards.

Table 6: **Variance decomposition**

Entry	IST	LAT	MP	MEI
ΔY	0.29%	32.61%	4.90%	62.20%
ΔC	0.79%	80.69%	4.77%	13.75%
$\Delta\widetilde{I}$	0.12%	9.44%	2.76%	87.69%
N	5.48%	7.74%	9.89%	76.89%

No Entry	IST	LAT	MP	MEI
ΔY	0.96%	28.12%	4.20%	66.72%
ΔC	0.77%	74.50%	3.62%	21.11%
$\Delta\widetilde{I}$	1.16%	8.74%	2.61%	87.49%
N	0.74%	11.24%	7.62%	80.40%

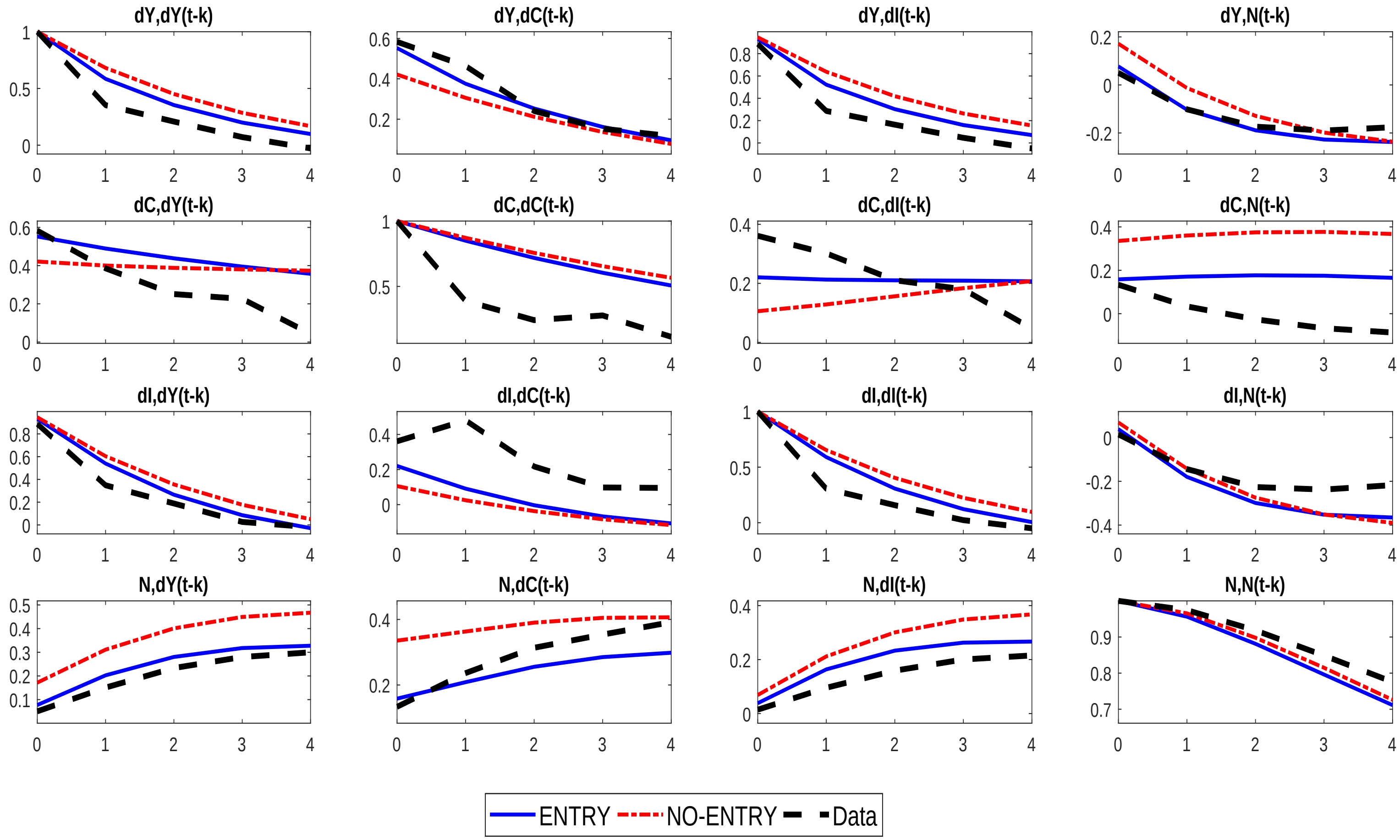

Figure 8: Cross-correlogram for key macroeconomic variables.

5 Conclusions

We constructed a novel two-sector model where firm entry/exit flows and firm-productivity dynamics in the investment-goods sector are endogenous. In this regard we characterize an investment supply shock as a sudden inflow of new more productive firms. Conditional on such shock, the combination of these two features generates strongly persistent dynamics in the relative price of investment that impact real variables only at low frequencies. In addition, we are able to show that productivity shocks in the consumption goods sector have permanent (negative) spillover effects on the investment goods sector productivity. This happens because higher productivity in the consumption goods sector raises the relative price of investment goods, allowing the survival of less productive firms in this sector.

We also find that the DSGE version of our model apparently escapes the Barro-King curse. In this regard, the key difference between the Entry and the No-Entry models seems to be response of the I-sector supply to changes in the relative price of investment goods induced by investment demand shocks. In the No-Entry model such response is infinitely elastic, whereas the Entry model is characterized by an upward-sloping supply function.

On the grounds of these results, we see two directions of future research. The first one is essentially empirical, and concerns the identification of investment-specific technology shocks. Sectorial VAR models typically infer such shocks from innovations to the relative price of investment goods, and DSGE models that incorporate price stickiness in the investment sector, such as Moura (2018), impose that long-run changes in the price of investment goods identify permanent sectorial productivity changes. In fact neither assumption survives in our model. The second direction of future research is theoretical, and concerns the design of optimal monetary policy when sectoral productivity is endogenous to cyclical entry flows.

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A Derivations and Proofs

A.1 Computation of I-firm's expected value

To define firms cutoff (18) and (19) we need to identify $H_{t+1}V_{t+1}^{av}$. To this end, note that

$$E_t\left\{V_{t+1}\left(A_{j,t+1}\right)\right\} = \int_{\hat{A}_{t+1}}^{+\infty} V_{t+1}\left(A_{j,t+1}\right)\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right)$$
$$= \int_{\hat{A}_{t+1}}^{+\infty}\left[\left(\alpha P_{t+1}^I A_{j,t+1}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha} - f_{t+1} + \beta E_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}V_{t+2}\left(A_{j,t+2}\right)\right\}\right]\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right) \tag{44}$$

where

$$\int_{\hat{A}_{t+1}}^{+\infty}\left[-f_{t+1} + \beta E_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}V_{t+2}\left(A_{j,t+2}\right)\right\}\right]\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right) =$$
$$= \left[-f_{t+1} + \beta E_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}V_{t+2}\left(A_{j,t+2}\right)\right\}\right]H_{t+1}$$

because $\mathbf{f}\left(A_{j,t+1}\right) = \frac{\gamma\left[\hat{A}_t g_e(1-\delta^I)\right]^\gamma}{A_{j,t+1}^{\gamma+1}}$, and $\int_{\hat{A}_{t+1}}^{+\infty}\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right) = H_{t+1}$ is the endogenous expected survival probability. Furthermore

$$\int_{\hat{A}_{t+1}}^{+\infty}\left(\alpha P_{t+1}^I A_{j,t+1}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha}\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right) =$$
$$= H_{t+1}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\alpha P_{t+1}^I\hat{A}_{t+1}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha}$$

This implies that (44) can be rewritten as

$$E_t\left\{V_{t+1}\left(A_{j,t+1}\right)\right\} \equiv \int_{\hat{A}_{t+1}}^{+\infty} V_{t+1}\left(A_{j,t+1}\right)\mathbf{f}\left(A_{j,t+1}\right)d\left(A_{j,t+1}\right) = H_{t+1}V_{t+1}^{av} \tag{45}$$

where

$$V_{t+1}^{av} = \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\alpha P_{t+1}^I\hat{A}_{t+1}\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha} - f_{t+1} + \beta E_{t+1}\left\{H_{t+2}\frac{\lambda_{t+2}}{\lambda_{t+1}}V_{t+2}^{av}\right\} \tag{46}$$

Now we can identify the cutoff (18) where $V_t\left(\hat{A}_t\right) = 0$.

$$0 = \left[\left(\alpha P_t^I\hat{A}_t\right)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{\alpha} - f_t\right] + \beta E_t\left\{H_{t+1}\frac{\lambda_{t+1}}{\lambda_t}V_{t+1}^{av}\right\} \tag{47}$$

Conditions (46) and (47) drive the dynamics of the I-sector threshold.

A.2 I sector productivity distribution

We claim that productivity distribution of the η_t I-firms is

$$\int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \tag{48}$$

This can be easily shown by the fact that for NEs

$$\begin{aligned} NE_t &= \int_{\hat{A}_t}^{+\infty} \frac{\gamma \bar{e}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \\ &= \left(\frac{\bar{e}_t}{\hat{A}_t}\right)^{\gamma} \int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \\ &= NE_t \int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \end{aligned} \tag{49}$$

where we exploited the fact that $NE_t = \left(\frac{\bar{e}_t}{\hat{A}_t}\right)^{\gamma}$. In addition, given the pdf definition

$$\int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \equiv 1 \tag{50}$$

Then the lhs and the rhs of (49) are equivalent.

For INCs instead

$$\begin{aligned} INC_t &= \eta_{t-1} \int_{\hat{A}_t}^{+\infty} \frac{\gamma \left[\hat{A}_{t-1} g_e (1-\delta^I)\right]^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \\ &= \eta_{t-1} \left[\frac{\hat{A}_{t-1} g_e (1-\delta^I)}{\hat{A}_t}\right]^{\gamma} \int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \\ &= INC_t \int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \end{aligned} \tag{51}$$

where we exploited the fact that $\eta_{t-1} \left[\frac{\hat{A}_{t-1} g_e (1-\delta^I)}{\hat{A}_t}\right]^{\gamma} = INC_t$ and (50).

Then, since $\eta_t = NE_t + INC_t$ and given (50), it must be that

$$\eta_t \int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t}) \equiv \eta_t$$

This implies that

$$\int_{\hat{A}_t}^{+\infty} \frac{\gamma \hat{A}_t^{\gamma}}{A_{j,t}^{\gamma+1}} d(A_{j,t})$$

describes the η_t I-firms productivity distribution.

A.3 I-sector Production

Here we derive overall production in the I-sector.

A.3.1 Derivation of I-firms total production

New Entrants

Let us start from new entrants. We know that the production function for the generic NE firm can be expressed as

$$I_{j,t}^{NE} = A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \tag{52}$$

Then, by exploiting the transformation theorem we can compute the expected value of NEs production

$$\begin{aligned}
I_t^{NE} &= \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \mathbf{dF}(A_{j,t}) \\
\Rightarrow I_t^{NE} &= \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \mathbf{f}(A_{j,t}) d(A_{j,t}) \\
\Rightarrow I_t^{NE} &= \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \gamma \bar{e}_t^{\gamma} \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}-\gamma-1} d(A_{j,t}) \\
\Rightarrow I_t^{NE} &= \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \gamma \bar{e}_t^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} A_{j,t}^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t}^{+\infty} \\
\Rightarrow I_t^{NE} &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{A}_t^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{53}$$

Where we exploited the fact that $NE_t = \left(\frac{\hat{A}_t}{\bar{e}_t}\right)^{-\gamma}$ and by assumption it must hold true that $\gamma(1-\alpha)-1>0$.

Incumbents

Let us repeat the same computation for incumbents. The production function for the generic incumbent firm is

$$I_{j,t}^{INC} = A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \tag{54}$$

Then, as before we have

$$\begin{aligned}
I_t^{INC} &= \eta_{t-1} \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \mathbf{dF}(A_{j,t}) \\
\Rightarrow I_t^{INC} &= \eta_{t-1} \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \mathbf{f}(A_{j,t})\, d(A_{j,t}) \\
\Rightarrow I_t^{INC} &= \eta_{t-1} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \gamma \left[\hat{A}_{t-1} g_e (1-\delta^I)\right]^{\gamma} \int_{\hat{A}_t}^{+\infty} A_{j,t}^{\frac{1}{1-\alpha}-\gamma-1} d(A_{j,t}) \\
\Rightarrow I_t^{INC} &= \eta_{t-1} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \gamma \left[\hat{A}_{t-1} g_e (1-\delta^I)\right]^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_{j,t}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t}^{+\infty} \\
\Rightarrow I_t^{INC} &= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{A}_t^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{55}$$

Where we have exploited the fact that $\eta_{t-1}\left[\frac{\hat{A}_{t-1}(1-\delta^I)}{\hat{A}_t}\right]^\gamma = INC_t$.

All I-firms

Then

$$
\begin{aligned}
I_t &= I_t^{NE} + I_t^{INC} \\
&= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{A}_t^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} + \\
&+ INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{A}_t^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}} \\
&= \eta_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{A}_t^{\frac{1}{1-\alpha}} \left(P_t^I \alpha\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{56}
$$

which comes from the fact that $\eta_t = NE_t + INC_t$.

A.3.2 I-firms dividends and production input

Similarly to the supply of investment, it can be easily shown that

$$S_t = \eta_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha P_t^I \hat{A}_t\right)^{\frac{1}{1-\alpha}} \tag{57}$$

Therefore, profits are

$$\begin{aligned} D_t &= P_t^I I_t - S_t - \eta_t f_t \\ &= \eta_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(P_t^I \hat{A}_t\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - \eta_t f_t \end{aligned} \tag{58}$$

Simply stating that the total value of investment goods (in real terms) produced in the I-sector must be equal to the sum of dividends, the input share of production and the total amount of fixed costs.

B Stochastic Trends Identification and the Deterministic Steady State

B.1 Stationary Representation of the Model

In this section we provide the details of the model's stochastic trend extraction and identification. For sake of generality, we set out the case of the New-Keynesian DSGE version of our model (*i.e.* with both sticky prices and wages) of which the RBC formulation is a special case.[23]

Let us assume Γ_t is the stochastic trend governing Y_t, C_t, w_t and S_t, from which follows that, for instance, $Y_t = \Gamma_t y_t$, where smaller case characters are meant to be detrended variables if not differently specified. Moreover, assume that Λ_t is the stochastic trend governing K_t and I_t, thus $K_t = \Lambda_t k_t$. We claim that both Γ_t and Λ_t are convolutions of the labor augmenting and the *NE*s permanent technology shifters, z_t and e_t.

B.1.1 Final production

Without any loss of generality we can rewrite final production as

$$y_t \Gamma_t = (z_t N_t)^\chi \left(\frac{\Lambda_t k_{t-1}}{\bar{g}_t} \right)^{1-\chi} - \Omega \Gamma_t \tag{59}$$

where $\bar{g}_t = \frac{\Lambda_t}{\Lambda_{t-1}}$. Then we define

$$\Gamma_t = z_t^\chi \Lambda_t^{1-\chi} \tag{60}$$

Which can be interpreted as the non stationary stochastic evolution of TFP in our model. Thus, dividing (59) by (60) we obtain

$$y_t = N_t^\chi \left(\frac{k_{t-1}}{\bar{g}_t} \right)^{1-\chi} - \Omega \tag{61}$$

In a similar fashion we can work out the detrended law of motion of capital by dividing both sides by Λ_t, that is

$$k_t = (1-\delta)\frac{k_{t-1}}{\bar{g}_t} + \mu_t^i \left[1 - \frac{\gamma_I}{2} \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right)^2 \right] i_t \tag{62}$$

Again, without any loss of generality, the demand of capital can be rewritten as

$$r_{k,t} = mc_t (1-\chi) \left(\frac{z_t}{\Lambda_t} \frac{\bar{g}_t N_t}{k_{t-1}} \right)^\chi \tag{63}$$

Then, the stochastic trend leading $r_{k,t}$ is, manipulating (60), $\left(\frac{z_t}{\Lambda_t}\right)^\chi = \frac{\Gamma_t}{\Lambda_t}$, from which follows that the detrended rental rate of capital is

$$r_{k,t}^* = mc_t (1-\chi) \left(\frac{\bar{g}_t N_t}{k_{t-1}} \right)^\chi \tag{64}$$

[23] The details of the DSGE model derivation are left to section C.

where $r^*_{k,t} = r_{k,t}\frac{\Lambda_t}{\Gamma_t}$, is the stochastically detrended rental rate of capital.

Finally, we claimed that w_t shares the same stochastic trend as Y_t, therefore

$$w^*_t \Gamma_t = mc_t \chi z^\chi_t \Lambda^{1-\chi}_t \left(\frac{k_{t-1}}{\bar{g}_t N_t}\right)^{1-\chi}$$

implying that

$$w^*_t = mc_t \chi \left(\frac{k_{t-1}}{\bar{g}_t N_t}\right)^{1-\chi} \tag{65}$$

Where $w = w^*\Gamma_t$ and the marginal cost, mc_t, is stationary by itself since $\Gamma_t = z^\chi_t \Lambda^{1-\chi}_t$,

$$mc_t = \left(\frac{w^*_t \Gamma_t}{\chi z_t}\right)^\chi \left[\frac{r^*_{k,t}\Gamma_t}{(1-\chi)\Lambda_t}\right]^{1-\chi} \equiv \left(\frac{w^*_t}{\chi}\right)^\chi \left[\frac{r^*_{k,t}}{(1-\chi)}\right]^{1-\chi}$$

Let us consider equations (125) and (126). For what concerns these two terms, they do have, by construction, the same stochastic trend as Y. Therefore their detrended version is

$$d^p_t = \pi^*_t y_t + \beta \left[\lambda_p \frac{\lambda^*_{t+1}}{\lambda^*_t}\frac{\pi^*_t}{\pi^*_{t+1}}\left(\frac{\pi_{t+1}}{\pi^{\gamma_p}_t}\right)^{\nu_p - 1} d^p_{t+1}\right] \tag{66}$$

And

$$f^p_t = mc_t y_t + \beta \left[\lambda_p \frac{\lambda^*_{t+1}}{\lambda^*_t}\left(\frac{\pi_{t+1}}{\pi^{\gamma_p}_t}\right)^{\nu_p} f^p_{t+1}\right] \tag{67}$$

Thus, the dynamics of inflation follow

$$d^p_t = \frac{\nu_p}{\nu_p - 1} f^p_t \tag{68}$$

Finally, price dispersion and price evolution are unchanged.

B.1.2 Households

From before, we implicitly assumed $C_t = c_t \Gamma_t$, where we also define $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$. At this point we also have that $\lambda_t = \frac{\lambda^*_t}{\Gamma_t}$ where λ^*_t is the stochastically detrended marginal utility of consumption (MUC).

Then, MUC can be rewritten as[24]

$$\frac{\lambda^*_t}{\Gamma_t} = \frac{1}{\Gamma_t c_t - a\Gamma_{t-1}c_{t-1}} - \beta a \frac{1}{\Gamma_{t+1}c_{t+1} - a\Gamma_t c_t}$$

Then multiplying on both sides by Γ_t and rearranging we have

$$\lambda^*_t = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \tag{69}$$

[24] We present here the utility function formulation including habits. The case without habits is a particular case of equation (69) with $a = 0$.

therefore from the bond-Euler

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \frac{R_{n,t}}{\pi_{t+1}} \right\} \tag{70}$$

Before moving to the capital-Euler, we remark that the shadow price of capital in consumption units is $Q_t = \frac{\phi_t^k}{\lambda_t}$, where we know that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ and it must also hold true that $\phi_t^k = \frac{\phi_t^{*,k}}{\Lambda_t}$ since Λ_t is the stochastic trend governing capital. This implies that $Q_t = \frac{\phi_t^{*,k}/\Lambda_t}{\lambda_t^*/\Gamma_t} \equiv q_t^* \frac{\Gamma_t}{\Lambda_t}$. Plugging the latter into the capital-Euler yields

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{q_t^*} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} + \frac{q_{t+1}^*}{q_t^*} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} (1-\delta) \right] \right\}$$

which boils down to

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\bar{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{q_t^*} + \frac{q_{t+1}^*}{q_t^*} (1-\delta) \right] \right\} \tag{71}$$

Further, the optimal investment condition implies that the stochastic trend of Q is the same leading P^I, implying $P_t^I = p_t^I \frac{\Gamma_t}{\Lambda_t}$. Then, dividing both sides of (31) by $\frac{\Gamma_t}{\Lambda_t}$ and rearranging, we have[25]

$$\begin{aligned} p_t^I = q_t^* \mu_t \left\{ 1 - \left[\gamma_I \left(\frac{i_t \bar{g}_t}{i_{t-1}} - g^* \right) \frac{i_t \bar{g}_t}{i_{t-1}} + \frac{\gamma_I}{2} \left(\frac{i_t \bar{g}_t}{i_{t-1}} - g^* \right)^2 \right] \right\} + \\ + \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{q_{t+1}^*}{\bar{g}_{t+1}} \mu_{t+1} \gamma_I \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} - g^* \right) \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} \right)^2 \right\} \end{aligned} \tag{72}$$

B.1.3 Optimal wage households choice

First of all, let us observe that $w = w^* \Gamma_t \Rightarrow w^\# = w^{\#,*} \Gamma_t$, that is the optimal (real) wage households would set should wages be flexible, has the same stochastic trend as the (real) wage effectively earned.

From equation (136), to guarantee the existence of a balanced growth path, it must be that the ratio between equations (137) and (138) yields a certain stochastic trend, *i.e.* $\Gamma_t^{1+\nu_w \theta}$, which can be easily verified. This implies that we have

$$\left(w_t^{\#,*} \right)^{1+\nu_w \theta} = \frac{\nu_w}{\nu_w - 1} \frac{f_t^w}{d_t^w} \tag{73}$$

where

$$f_t^w = \Phi \left(w_t^* \right)^{(1+\theta)\nu_w} N_t^{1+\theta} + \beta \lambda_w \left[\frac{\pi_{t+1}}{(\pi_t \widetilde{g}_t)^{\gamma_w} g_*^{1-\gamma_w}} \right]^{(1+\theta)\nu_w} f_{t+1}^w \widetilde{g}_{t+1}^{(1+\theta)\nu_w} \tag{74}$$

and

[25] Note that in the deterministic setady state we have that $\widetilde{g} = \bar{g} = g^*$ (see section B.1.8)

$$d_t^w = \lambda_t^* \left(w_t^*\right)^{\nu_w} N_t + \beta\lambda_w \left[\frac{\pi_{t+1}}{(\pi_t \widetilde{g}_t)^{\gamma_w} g_*^{1-\gamma_w}}\right]^{\nu_w - 1} d_{t+1}^w \widetilde{g}_{t+1}^{\nu_w-1} \tag{75}$$

The wage evolution implicitly holds as

$$\left(w_t^*\right)^{1-\nu_w} = (1-\lambda_w)\left(w_t^{\#,*}\right)^{1-\nu_w} + \lambda_w \left[w_{t-1}^* \frac{(\pi_{t-1}\widetilde{g}_{t-1})^{\gamma_w}(\pi g_*)^{1-\gamma_w}}{\pi_t \widetilde{g}_t}\right]^{1-\nu_w} \tag{76}$$

As usual for $\nu_w \to \infty$ and $\lambda_w = 0$ we have that $w_t^{\#,*} = w_t^*$ and equation (73) boils down to the leisure-consumption relationship governing the labor supply in the RBC version of the model

$$w_t^* = \Phi \frac{N_t^\theta}{\lambda_t^*} \tag{77}$$

B.1.4 I-firms

Let us define

$$\hat{A}_t = \Theta_t \hat{a}_t$$

where Θ_t is the stochastic trend driving the I-firms threshold.

We remark that by assumption $f_t = g_*^t f$, *i.e.* the trend leading fixed costs is purely deterministic, and that $\frac{1}{\widetilde{g}_t}$ is the stochastic growth rate associated to λ_t, where in the deterministic steady state $\widetilde{g}_t = g_*$. Note also that $H_{t+i} = \left[\frac{\hat{A}_{t+i-1} g_e (1-\delta^I)}{\hat{A}_{t+i}}\right]^\gamma$ is implicitly stationary as the stochastic growth rate associated to $\hat{A}_t$ is such that in the steady state $\frac{\Theta_t}{\Theta_{t-1}} = g_{\Theta,t} = g_e$.[26]

Given the fixed cost deterministic trend, this implies that all the addends in condition (18) must grow at the same deterministic rate. Therefore, (18) can be rewritten as

$$\left(\alpha \frac{\Gamma_t}{\Lambda_t} p_t^I \Theta_t \hat{a}_t\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha} = f g_*^t - \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^* \widetilde{g}_{t+1}} \left[\frac{\hat{a}_t g_e (1-\delta^I)}{\hat{a}_{t+1} g_{\Theta,t+1}}\right]^\gamma v_{t+1}^{av} g_*^{t+1} \right\}$$

To guarantee the existence of a balanced growth path it must be that $\Theta_t = \frac{\Lambda_t}{\Gamma_t} g_*^{t(1-\alpha)}$, and dividing the above equation by g_*^t on both sides it becomes

$$\left(\alpha p_t^I \hat{a}_t\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha} = f - \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^* \widetilde{g}_{t+1}} \left[\frac{\hat{a}_t g_e (1-\delta^I)}{\hat{a}_{t+1} g_{\Theta,t+1}}\right]^\gamma v_{t+1}^{av} g_* \right\} \tag{78}$$

This implies that (19) can be rewritten as

$$v_{t+1}^{av} = \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha p_{t+1}^I \hat{a}_{t+1}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha} - f + \beta E_t \left\{ \frac{\lambda_{t+2}^*}{\lambda_{t+1}^* \widetilde{g}_{t+2}} \left[\frac{\hat{a}_{t+1} g_e (1-\delta^I)}{\hat{a}_{t+2} g_{\Theta,t+2}}\right]^\gamma v_{t+2}^{av} g_* \right\} \tag{79}$$

[26] The claims about $\widetilde{g}_t = g_*$ and $g_{\Theta,t} = g_e$ can be easily verified in section B.1.8.

Then, exploiting the fact that $\frac{\Lambda_t}{\Gamma_t} = \left(\frac{\Lambda_t}{z_t}\right)^{\chi}$ we have that

$$\hat{A}_t = \left(\frac{\Lambda_t}{z_t}\right)^{\chi} g_*^{t(1-\alpha)} \hat{a}_t \tag{80}$$

At this point, we impose that $\overline{e}_t = e \cdot e_t$, and we can easily rewrite the mass of active NEs as

$$NE_t = \left(\frac{\overline{e}_t z_t^{\chi}}{\hat{a}_t \Lambda_t^{\chi} g_*^{t(1-\alpha)}}\right)^{\gamma} = \left(\frac{e}{\hat{a}_t}\right)^{\gamma} \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$$

Implying that $NE_t = ne_t \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and so

$$ne_t = \left(\frac{e}{\hat{a}_t}\right)^{\gamma} \tag{81}$$

By BGP conditions we know that also η_t and INC_t share the same stochastic trend as NE_t, this implies

$$\eta_t^* = ne_t + inc_t \tag{82}$$

Then, substituting for (80) into (16), it turns out that

$$\begin{aligned} inc_t &= \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left[\frac{\hat{a}_{t-1}}{\hat{a}_t}\left(\frac{g_{z,t}}{\bar{g}_t}\right)^{\chi}(1-\delta)^I\right]^{\gamma} \\ &= \eta_{t-1}^* \frac{g_*^{\gamma(1-\alpha)}}{g_{e,t}^{\gamma}} \left[\frac{\hat{a}_{t-1}(1-\delta)^I}{\hat{a}_t}\right]^{\gamma} \end{aligned} \tag{83}$$

Where of course we exploited the fact that $INC_t = inc_t \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and $\eta_t = \eta_t^* \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and therefore η_{t-1}^* in (83) must be expressed accordingly.

B.1.5 Stochastic Trends Identification

Notice that from the aggregate resource constraint in (34) it turns out that the stochastic trend leading $\eta_t f$ must be, by construction, the same leading Y_t, C_t and S_t, *i.e.* Γ_t. Thus, given that $\eta_t f g_*^t \equiv \eta_t^* f \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} g_*^t$, we can easily work out

$$\Gamma_t = \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}} \tag{84}$$

Then, plugging the relationship $\Gamma_t = z_t^{\chi} \Lambda_t^{1-\chi}$ into (84) (and solving for Λ_t) allows for the identification of the stochastic trend leading both K_t and I_t, that is

$$\Lambda_t = \frac{e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} z_t^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{t[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{85}$$

Then, plugging (85) into (84) we have

$$\Gamma_t = \frac{e_t^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} z_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{t(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{86}$$

Which is the stochastic trend driving all aggregate variables but K_t and I_t. Thus, the stochastic trend governing aggregate variables is a Cobb-Douglas of the the permanent shifters governing the NEs technology shifter and consumption goods production, respectively. [27]

At this point we can also identify the stochastic trend driving I-firms cutoff. For instance, substituing for (85) into the long run component of (80) we obtain that the corresponding stochastic trend is

$$\Theta_t = \frac{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{z_t^{\frac{\chi}{1+\chi(\gamma-1)}} g_*^{t\left\{\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}-(1-\alpha)\right\}}} \tag{87}$$

Finally, plugging (85) into the long run component of (81), (82) and (83) it turns out that the stochastic trend steering the I-firms industry composition is

$$\Xi_t = \frac{e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} z_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{t\gamma\left\{(1-\alpha)-\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}\right\}}} \tag{88}$$

B.1.6 I-firms production

At this point, since we claimed that K_t and I_t are governed by the same stochastic trend, *i.e.* Λ_t, this implies that $I_t = i_t\Lambda_t$. Then

$$i_t\Lambda_t = \eta_t^* \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\frac{\hat{a}_t \Lambda_t^{\chi} g_*^{t(1-\alpha)}}{z_t^{\chi}}\right)^{\frac{1}{1-\alpha}} \left[\alpha p_t^I \left(\frac{z_t^n}{\Lambda_t}\right)^{\chi}\right]^{\frac{\alpha}{1-\alpha}}$$

From which rearranging

$$i_t\Lambda_t = \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \eta_t^* \hat{a}_t^{\frac{1}{1-\alpha}} \left(\alpha p_t^I\right)^{\frac{\alpha}{1-\alpha}} \frac{e_t^{\gamma} z_t^{\chi(\gamma-1)}}{\Lambda_t^{\chi(\gamma-1)} g_*^{t[\gamma(1-\alpha)-1]}}$$

But from (85) we know that $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma} z_t^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$ which therefore implies

$$i_t = \eta_t^* \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \hat{a}_t^{\frac{1}{1-\alpha}} \left(\alpha p_t^I\right)^{\frac{\alpha}{1-\alpha}} \tag{89}$$

[27] According to our parametrization $\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)} < 1$.

B.1.7 Aggregate Resources Constraint

There is only one variable to be detrended yet. By construction it must be $S_t = s_t\Gamma_t$.

Then it is sufficient to show that

$$s_t\Gamma_t = \eta_t^* \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha p_t^I \left(\frac{z_t}{\Lambda_t}\right)^{\chi} \hat{a}_t \left(\frac{\Lambda_t}{z_t}\right)^{\chi} g_*^{t(1-\alpha)}\right]^{\frac{1}{1-\alpha}}$$

can be rewritten as

$$s_t\Gamma_t = \eta_t^* \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha p_t^I \hat{a}_t\right]^{\frac{1}{1-\alpha}} \frac{z_t^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}}$$

and again, since $\Gamma_t = z_t^{\chi}\Lambda^{(1-\chi)}$ and $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma} z_t^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$, it must be that

$$s_t = \eta_t^* \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha p_t^I \hat{a}_t\right]^{\frac{1}{1-\alpha}} \tag{90}$$

Moreover, given condition (139) it is clear that public expenditure follows the same trend as Y_t, implying that $G_t = g_t^*\Gamma_t$

Thus, we have proven that

$$y_t = c_t + s_t + g_t^* + \eta^* f_t \tag{91}$$

holds true.

B.1.8 Stochastic Growth Rates Identification

First of all, note that according to (12) and (119), in a deterministic setup $g_{e,t} = g_e$ and $g_{z,t} = g_*$.

Further, we claimed that $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$, then exploiting (86) it turns out that

$$\widetilde{g}_t = \frac{g_{e,t}^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} g_{z,t}^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{92}$$

meaning that the stochastic BGP growth rate is a convolution of the stochastic growth rate of e_t and z_t. Similarly for $\frac{\Lambda_t}{\Lambda_{t-1}} = \bar{g}_t$ it follows that

$$\bar{g}_t = \frac{g_{e,t}^{\frac{\gamma}{1+\chi(\gamma-1)}} g_{z,t}^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{\gamma(1-\alpha)-1}{1+\chi(\gamma-1)}}} \tag{93}$$

In the deterministic steady state we have that $g_e = g_*^{1-\alpha}$ (see Section B.2). Moreover, also $\bar{g} = \widetilde{g} = g_*$ must hold true, *i.e.* the deterministic BGP is the same for all aggregated variables, which is verified plugging $g_e = g_*^{1-\alpha}$ into the deterministic formulation of (92) and (93).

Finally, it can be easily verified that: i) the growth rate of (87), *i.e.* the one governing the I-firms cutoffs, is equal to $g_*^{1-\alpha}$; ii) the growth rate of (88), *i.e.* the one governing the I-firms mass, is equal to 1 in a deterministic setup which proves that the mass of I-firms is stationary in the steady state.

B.2 Deterministic Steady State and the Balanced Growth Path

In the deterministic steady state

$$\frac{z_t}{z_{t-1}} = g_* \tag{94}$$

Without any loss of generality we assume that output, capital, investment, consumption and the real wage all grow at g_* gross rate. From (6) it is straightforward to show that the fixed costs f_t also grows at the BGP rate g_*, whereas the relative price of investment is constant.

From condition (31) the shadow price of capital is equal to the price of investment.

$$Q^{ss} = P^{I,ss} \tag{95}$$

The steady state rental rate of capital stems from (71) and is

$$\frac{g_*}{\beta} - 1 + \delta = \frac{r_k^{ss}}{P^{I,ss}} \tag{96}$$

The constant labor supply, $N^{ss} = \bar{N}^{ss}$, is pinned down by the preference parameter Φ in (136). The marginal cost is defined in terms of consumption goods units $mc^{ss} = \frac{\nu_p - 1}{\nu_p} < 1$, and this allows to obtain the detrended capital stock value $K^{ss} = \frac{K_t^{ss}}{g_*^t}$:

$$K^{ss} = \left\{ \left[\frac{(\nu_p - 1)}{\nu_p} \frac{(1-\chi)}{r_k^{ss}} \right]^{\frac{1}{\chi}} g_* \bar{N}^{ss} \right\} \tag{97}$$

As a result the detrended value of consumption goods output is

$$Y^{ss} = \left(\bar{N}^{ss}\right)^{\chi} \left(\frac{K^{ss}}{g_*} \right)^{1-\chi}$$

From the capital accumulation condition, investment is

$$I^{ss} = \left(1 - \frac{1-\delta}{g_*} \right) K^{ss} \tag{98}$$

And the real wage is obtained solving

$$\frac{\nu_p - 1}{\nu_p} = \left(\frac{r_k^{ss}}{1-\chi} \right)^{1-\chi} \left(\frac{W^{ss}}{\chi} \right)^{\chi} \tag{99}$$

B.2.1 I-firms Operating Threshold

Keeping in mind that in steady state $\beta\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta}{g_*} \equiv \frac{1}{(1+r)}$; $\beta^2\frac{\lambda_{t+2}}{\lambda_t} = \frac{\beta^2}{g_*^2} \equiv \frac{1}{(1+r)^2}$;...$\beta^i\frac{\lambda_{t+i}}{\lambda_t} = \frac{\beta^i}{g_*^i} \equiv \frac{1}{(1+r)^i}$, that $P^{I,ss} = P_t^I = P_{t+i}^I$, $H = H_t = H_{t+i}$, $\hat{A}_{t+i}^{ss} = \hat{A}_t^{ss} g_e^i$ and that $g_e^{\frac{1}{1-\alpha}} = g_*$ and $f_t = f g_*^t$. Plugging (46) into (47) and iterating forward we can rewrite (47) as

$$\left(\alpha P^I \hat{A}_t^{ss}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}\left[1 + \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\beta H + \beta^2 H^2 + ... + \beta^i H^i\right)\right] = f_t + f_t\beta H + f_t\beta^2 H^2 + ... + f_t\beta^i H^i$$

or

$$\left(\alpha P^{I,ss} \hat{A}_t^{ss}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}\left[1 + \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\sum_{i=0}^{+\infty}(\beta H)^i - \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\right] = f_t\sum_{i=0}^{+\infty}(\beta H)^i \tag{100}$$

given that $\beta, H < 1$, taking the limit of (100) as $i \to \infty$ yields

$$\left(\alpha P^{I,ss} \hat{A}_t^{ss}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}\frac{\gamma(1-\alpha) - (1-\beta H)}{[\gamma(1-\alpha)-1](1-\beta H)} = f_t\frac{1}{1-\beta H}$$

from which the threshold is

$$\hat{A}_t^{ss} = \left[\frac{f_t}{1-\alpha}\frac{\gamma(1-\alpha)-1}{\gamma(1-\alpha)-1+\beta H}\right]^{1-\alpha}\frac{1}{\alpha^\alpha P^{I,ss}} \tag{101}$$

Since $\beta, H > 0$, then $\frac{\gamma(1-\alpha)-1}{\gamma(1-\alpha)-1+\beta H} < 1$ implying that

$$\hat{A}_t^{ss} < \left(\frac{f_t}{1-\alpha}\right)^{1-\alpha}\frac{1}{\alpha^\alpha P^{I,ss}} \tag{102}$$

that is, in the deterministic steady state some firms decide to stay in the market even if the current level of their productivity generates negative profits. This obviously happens because the current loss is compensated by the expectation of future profits.

In fact, by plugging (101) into (47) we get:

$$V_{t+1}^{av,ss} = f_t\left[1 - \frac{\gamma(1-\alpha)-1}{\gamma(1-\alpha)-1+\beta H}\frac{1}{\alpha}\right]\frac{1}{\beta H} \tag{103}$$

where $V_{t+1}^{av,ss} > 0$ because $\frac{\gamma(1-\alpha)-1}{\gamma(1-\alpha)-1+\beta H}\frac{1}{\alpha} < 1$.

B.2.2 Stationary mass of I-firms

To fully characterize the incumbents distribution in the deterministic steady state, let us define H as the survival rate, *i.e.* $H \in (0, 1)$ so that the law of motion of I-firms now reads $\eta_t = NE_t + \eta_{t-1}H$, from which it is clear that $INC_t = \eta_{t-1}H$.

Since in ss it must be that $NE_t = NE_{t-1} = ... = NE^{ss}$, and $\eta_t = \eta_{t-1} = \eta^{ss}$, solving backward we have that

$$\eta^{ss} = NE^{ss} \sum_{i=t-n}^{t} H^i + \eta^{ss} H^n \tag{104}$$

Then, taking the $lim_{n\to\infty}$ for $t \geq n$, and since $H \in (0,1)$, it turns out that in the deterministic steady state η_t must be constant *i.e.*,

$$\eta^{ss} = NE^{ss} \frac{1}{1-H} \Rightarrow \frac{NE^{ss}}{\eta^{ss}} = 1 - H \tag{105}$$

From which follows that

$$INC^{ss} = \eta^{ss} - NE^{ss} \equiv NE^{ss} \frac{H}{1-H} \Rightarrow \frac{INC^{ss}}{\eta^{ss}} = H \tag{106}$$

That is, the survival rate defines the share of incumbent firms over the total.

B.2.3 I-sector production and market clearing

The NEs technology frontier embeds a stochastic efficiency trend: $\overline{e}_t = e \cdot e_{t-1} g_{e,t}$, where e is the initial condition characterizing the distribution support so that $\overline{e}_t = e \cdot e_t$ and $e_t = e_{t-1} g_{e,t}$.[28] Therefore in the steady state $\overline{e}_t \equiv e g_e^t$ and $f_t \equiv f g_*^t$, from (14) we obtain

$$\begin{aligned} NE^{ss} &= \left(\frac{e g_e^t}{\hat{A}^{ss}} \right)^{\gamma} \\ &= \left\{ P^{I,ss} \alpha^{\alpha} e g_e^t \left[\frac{1-\alpha}{f g_*^t} \frac{\gamma(1-\alpha) - 1 + \beta H}{\gamma(1-\alpha) - 1} \right]^{1-\alpha} \right\}^{\gamma} \end{aligned} \tag{107}$$

Thus, in order to have a constant non zero and non diverging mass of NEs, it turns out that $g_e = g_*^{1-\alpha}$ must necessarily hold true. Then, given that $H = \left[\frac{\hat{A}_{t-1} g_e (1-\delta^I)}{\hat{A}_t} \right]^{\gamma} = (1-\delta^I)^{\gamma}$, from (16) and (17) we obtain

$$INC^{ss} = \eta^{ss} \left[\frac{\hat{A}_{t-1}^{ss} g_e (1-\delta^I)}{\hat{A}_t^{ss}} \right]^{\gamma} = \eta^{ss} \left[\frac{\hat{A}_t^{ss} (1-\delta^I)}{\hat{A}_t^{ss}} \right]^{\gamma} = \eta^{ss} (1-\delta^I)^{\gamma}$$

Exploiting (105)

$$\eta^{ss} = \frac{NE^{ss}}{1-(1-\delta^I)^{\gamma}} \tag{108}$$

$$INC^{ss} = \frac{NE^{ss}(1-\delta^I)^{\gamma}}{1-(1-\delta^I)^{\gamma}} \tag{109}$$

We can now solve for steady state investments. From condition (22) we get[29]

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[28] The initial condition e allows for a free calibration of I-firms fixed cost initial condition, f.

[29] It is easy to see that $\gamma(1-\alpha) > 1$ is necessary to obtain positive invetment levels. This requirement is easily met under standard calibrations of I-firms returns to scale and tail index in the Pareto distribution (11).

$$I^{ss} = \left\{ e^{\gamma} + \eta^{ss} \left[\frac{\hat{A}^{ss}}{g_e} g_e (1-\delta^I) \right]^{\gamma} \right\} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}^{ss} \right)^{\frac{1}{1-\alpha}-\gamma} \left(P^{I,ss} \alpha \right)^{\frac{\alpha}{1-\alpha}} \tag{110}$$

using (107), (108), (110), (98), (101), (97) and (95) we obtain the market clearing condition for the I-goods sector:

$$\left(1 - \frac{1-\delta}{g_*}\right) \left\{ \left[\frac{(\nu_p - 1)}{\nu_p} \frac{(1-\chi)}{r_k^{ss}} \right]^{\frac{1}{\chi}} g_* \bar{N}^{ss} \right\} = e^{\gamma} \left[\frac{1-\alpha}{f} \frac{\gamma(1-\alpha) - 1 + \beta \left(1-\delta^I\right)^{\gamma}}{\gamma(1-\alpha)-1} \right]^{\gamma(1-\alpha)-1} \times$$
$$\times \frac{1}{1-(1-\delta^I)^{\gamma}} \frac{\gamma(1-\alpha)\alpha^{\gamma\alpha}}{\gamma(1-\alpha)-1} \left(P^{I,ss}\right)^{\gamma-1} \tag{111}$$

At this stage, for given values of g_*, δ, $\bar{N}^{ss}$, r_k^{ss}, ν_p, χ, f, δ^I, γ, α and β, the appropriate choice of e ensures that condition (111) holds when

$$P^{I,ss} = 1$$

Finally, the model is closed by the following conditions

$$P^{I,ss} I^{ss} = D^{ss} + S^{ss} \tag{112}$$

$$C^{ss} = Y^{ss} - S^{ss} - G^{ss} - \eta^{ss} f \tag{113}$$

$$S^{ss} = \eta^{ss} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha P^{I,ss} \hat{A}^{ss} \right)^{\frac{1}{1-\alpha}} \tag{114}$$

$$D^{ss} = \eta^{ss} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(P^{I,ss} \hat{A}^{ss} \right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - \eta^{ss} f \tag{115}$$

C New Keynesian DSGE Formulation of the model

C.1 Additional features

In this section we introduce all the necessary features to characterize the DSGE version of our model embedding the same frictions as in JPT. In this regard, the fixed cost of production of intermediate C-goods producers, Ω, is now calibrated as estimated in JPT (see Table 4) rather than 0, as implicitly assumed for the RBC model.

C.1.1 C-good retailers

Retail firms assemble the consumption good bundle Y_t using a continuum of intermediate inputs Y_t^h.[30] The representative firm profit maximization problem is:

$$\max_{Y_t, Y_t^h} P_t Y_t - \int_0^1 P_t^h Y_t^h dh$$

$$\text{s.t. } Y_t = \left[\int_0^1 \left(Y_t^h \right)^{\frac{\nu_p - 1}{\nu_p}} dh \right]^{\frac{\nu_p}{\nu_p - 1}}$$

where $\nu_p > 1$ is the elasticity of substitution among differentiated goods. From the first order conditions, we obtain:

$$Y_t^h = \left(\frac{P_t^h}{P_t} \right)^{-\nu_p} Y_t \tag{116}$$

$$P_t = \left[\int_0^1 \left(P_t^h \right)^{1-\nu_p} dh \right]^{\frac{1}{1-\nu_p}} \tag{117}$$

C.1.2 Intermediate good Producers

C-firms are monopolistically competitive, hire labor from households and exploit capital rented at the end of period $t-1$. Their production function is

$$Y_t^h = (z_t N_t)^{\chi} (K_{t-1})^{1-\chi} - \Omega \Gamma_t \tag{118}$$

where N defines worked hours, K is the capital stock, z is a permanent labor augmenting technology shifter (LAT hereafter), such that $z_t = z_{t-1} g_{z,t}$ where

$$\ln(g_{z,t}) = (1 - \rho_z) \ln(g_*) + \rho_z \ln(g_{z,t-1}) + \varepsilon_t^z \tag{119}$$

and $\varepsilon_t^z \sim N(0, \sigma^z)$. The LAT shifter embeds a deterministic trend component, g_*. Ω is a fixed cost of production indexed at the stochastic trend governing the economy, Γ_t, to ensure the existence of a balanced growth path.

[30] Intermediate goods production is described in section C.1.2.

where w_t and $r_{k,t}$ respectively define the real wage and the rental rate of capital defined in consumption goods. The real marginal costs are:

$$mc_t = \left(\frac{r_{k,t}}{1-\chi}\right)^{1-\chi}\left(\frac{w_t}{z_t\chi}\right)^{\chi} \tag{120}$$

Price stickiness is based on the Calvo mechanism. In each period intermediate firms face a probability $1-\lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t^h = \pi_{t-1}^{\gamma_p} p_{t-1}^h \tag{121}$$

where $\gamma_p \in [0,1]$ represents the degree of price indexation.

All the $1-\lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$, the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t+s$ will read as $\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p}$ where $\Pi_{t,t+s-1} = \pi_t ... \pi_{t+s-1} = \frac{P_{t+s-1}}{P_{t-1}}$. $\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \bar{\Lambda}_{t+s}\left(\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p} - P_{t+s}mc_{t+s}\right)Y_{t+s}^h$$

subject to:

$$Y_{t+s}^h = Y_{t+s}\left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}}\right)^{-\nu_p} \tag{122}$$

where Y_t is aggregate demand and $\bar{\Lambda}_t$ is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \bar{\Lambda}_{t+s}Y_{t+s}\left[\begin{array}{c} (1-\nu_p)\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{1-\nu_p}\widetilde{P}_t^{-\nu}(P_{t+s})^{\nu_p} + \\ +\nu_p\widetilde{P}_t^{-\nu_p-1}P_{t+s}^{\nu_p+1}mc_{t+s}\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{-\nu_p}\end{array}\right] = 0 \tag{123}$$

After standard manipulations condition (123) can be rewritten as

$$D_t^p = \frac{\nu_p}{\nu_p-1}F_t^p \tag{124}$$

where

$$D_t^p = \pi_t^{\#}Y_t + \beta E_t\left\{\lambda_p\frac{\lambda_{t+1}}{\lambda_t}\frac{\pi_t^{\#}}{\pi_{t+1}^{\#}}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu_p-1}D_{t+1}^p\right\} \tag{125}$$

and

$$F_t^p = mc_tY_t + \beta E_t\left\{\lambda_p\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu_p}F_{t+1}^p\right\} \tag{126}$$

where $\pi_t^{\#} = \frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}$ is the inflation rate chosen by re-optimizing firms. Finally, (127) describes price evolution.

$$1 = (1 - \lambda_p) \left(\pi_t^{\#}\right)^{1-\nu^p} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{1-\nu^p} \tag{127}$$

C.2 Households and Wage Setting

We assume a standard characterization of households preferences,

$$U_t(C, N) = \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \ln(C_{t+s} - aC_{t+s-1}) - \Phi \frac{\left(N_{t+s}^l\right)^{1+\theta}}{1+\theta} \right\} \tag{128}$$

Parameter a defines internal consumption habits. The flow budget constraint in real terms is

$$C_t + P_t^I I_t + \frac{B_t}{P_t} = R_{n,t-1} \frac{B_{t-1}}{P_t} + r_{k,t} K_{t-1} + w_t^l N_t^l + \Pi_t^{C,I} \tag{129}$$

where B is a nominally riskless bond of one-period maturity with gross nominal remuneration R_n and $\Pi^{C,I} = \Pi_t^C + D_t$ are aggregate dividends paid by C- and I-firms. The law of motion of capital and any sort of things related to the investment sector are unchanged with respect to section 2.3 and so are not repeated. Among the other things, maximizing with respect to the riskless bond, B, and consumption gives rise to the F.O.C.s :

$$\lambda_t = (C_t - aC_{t-1})^{-1} - \beta a (C_{t+1} - aC_t)^{-1} \tag{130}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{R_{n,t}}{\pi_{t+1}} \right\} \tag{131}$$

accounting for the presence of habits in consumption and the riskless bond smoothing consumption equation.

C.2.1 Labor packagers

Households supply a differentiated type of labor $l \in (0, 1)$ to the labor packagers which bundle and sell it to intermediate firms, according to

$$N_t = \left[\int_0^1 \left(N_t^l\right)^{\frac{\nu_w - 1}{\nu_w}} dl\right]^{\frac{\nu_w}{\nu_w - 1}}$$

Where $\nu_w > 1$ is the elasticity of substitution among different labor types. The maximization problem of the competitive labor packager is

$$\max_{n_t^l} W_t \left[\int_0^1 \left(N_t^l\right)^{\frac{\nu_w - 1}{\nu_w}} dl\right]^{\frac{\nu_w}{\nu_w - 1}} - \int_0^1 W_t^l N_t^l dl$$

from which the following demand function is obtained

$$N_t^l = \left(\frac{W_t^l}{W_t} \right)^{-\nu_w} N_t \tag{132}$$

where W_t^l is the remuneration for the labor supplied by household l, while the remuneration payd by the intermediate firm to the labor union for the homogeneous labor input provided is

$$W_t = \left[\int_0^1 \left(W_t^l \right)^{1-\nu_w} dl \right]^{\frac{1}{1-\nu_w}}$$

In terms of wage setting, we assume that in each period a fraction λ_w of households cannot freely set its wage, but the wage setting is conducted according to the following indexation rule

$$W_t^l = W_{t-1}^l \left(\pi_{t-1} \widetilde{g}_{t-1} \right)^{\gamma_w} \left(\pi g_* \right)^{1-\gamma_w} \tag{133}$$

in order to preserve balance growth in the model. γ_w is the degree of wage dynamic indexation and π is the steady state inflation.

C.2.2 Optimal wage choice

Similarly as before, the probability that the nominal wage chosen in t will be still operative in $t+s$ is $(\lambda_w)^s$. The real wage a given household charges in period $t+s$ if the nominal wage is still stuck at period's t choice is

$$w_{t+s}^l = \frac{W_t^l}{P_t} \frac{P_t}{P_{t+s}}$$

where as before $\Pi_{t,t+s} = \frac{P_{t+s}}{P_t}$ is the gross inflation rate between between $t+s$ and t, therefore it should be that

$$w_{t+s}^l = w_t^l \Pi_{t,t+s}^{-1}$$

But given the indexation rule (133) defined above, we have that

$$w_{t+s}^l = w_t^l \Pi_{t,t+s}^{-1+\gamma_w} \left(\pi g_* \right)^{(1-\gamma_w)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{\gamma_w} \tag{134}$$

Considering just the parts of the Lagrangian related to the choice of labor, the household maximization problem reads

$\max_{w_t^l} \mathcal{L} =$

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_w)^s \left\{ \begin{array}{c} -\Phi \dfrac{\left(\frac{w_t^l \Pi_{t,t+s}^{-1+\gamma_w} (\pi g_*)^{(1-\gamma_w)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{\gamma_w}}{w_{t+s}} \right)^{-\nu_w(1+\theta)} N_{t+s}^{1+\theta}}{1+\theta} + \lambda_{t+s} P_{t+s} \times \\ \times \left[w_t^l \Pi_{t,t+s}^{-1+\gamma_w} (\pi g_*)^{(1-\gamma_w)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{\gamma_w} \left(\frac{w_t^l \Pi_{t,t+s}^{-1+\gamma_w} (\pi g_*)^{(1-\gamma_w)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{\gamma_w}}{w_{t+s}} \right)^{-\nu_w} N_{t+s} \right] \end{array} \right\}$$

the F.O.C. of this problem is

$$\left(w_t^{\#}\right)^{1+\nu_w\theta} = \frac{\nu_w}{\nu_w - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\lambda_w)^s \Phi w_{t+s}^{\nu_w(1+\theta)} \left[\Pi_{t,t+s}^{1-\gamma_w} (\pi g_*)^{(\gamma_w - 1)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{-\gamma_w}\right]^{(1-\theta)\nu_w} N_{t+s}^{1+\theta}}{E_t \sum_{s=0}^{\infty} (\beta\lambda_w)^s \lambda_{t+s} P_{t+s} w_{t+s}^{\nu_w} \left[\Pi_{t,t+s}^{1-\gamma_w} (\pi g_*)^{(\gamma_w - 1)s} \prod_{i=0}^{s} \widetilde{g}_{t+i}^{-\gamma_w}\right]^{\nu_w - 1} N_{t+s}} \tag{135}$$

Since all households are identical, they will update to the same wage and therefore $w_t^l \equiv w_t^{\#}$. Note that without any loss of generality for $\nu_w \to \infty$ and $\lambda_w = 0$ we have that $w_t^{\#} = w_t$ and condition (135) boils down to (30).

Similarly to retailers, equation (135) can be rearranged as

$$\left(w_t^{\#}\right)^{1+\nu_w\theta} = \frac{\nu_w}{\nu_w - 1} \frac{F_t^w}{D_t^w} \tag{136}$$

where

$$F_t^w = \Phi w_t^{(1+\theta)\nu_w} N_t^{1+\theta} + \beta\lambda_w \left[\frac{\pi_{t+1}}{(\pi_t \widetilde{g}_t)^{\gamma_w} g_*^{1-\gamma_w}}\right]^{(1+\theta)\nu_w} F_{t+1}^w \tag{137}$$

and

$$D_t^w = \lambda_t w^{\nu_w} N_t + \beta\lambda_w \left[\frac{\pi_{t+1}}{(\pi_t \widetilde{g}_t)^{\gamma_w} g_*^{1-\gamma_w}}\right]^{\nu_w - 1} D_{t+1}^w \tag{138}$$

C.3 Government

Fiscal policy is Ricardian and the government finances it budget deficit through short term bonds. Public expenditure follows an exogenous path as a time varying fraction of GDP

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t \tag{139}$$

where the government spending shock follows the usual stochastic process

$$\ln(g_t) = (1 - \rho_g)\ln(\bar{G}) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \; ; \; \varepsilon_t^g \sim N(0,1) \tag{140}$$

and $\bar{G} = \frac{1}{1 - G^{ss}/Y^{ss}}$.

The monetary authority sets the following monetary policy rule

$$\frac{R_{n,t}}{R_n} = \left(\frac{R_{n,t-1}}{R_n}\right)^{\rho^R} \left[\left(\frac{\pi_t}{\pi}\right)^{\kappa_p} X_t^{\kappa_y}\right]^{1-\rho^R} (\Delta X_t^{\kappa_{\Delta X}})\, \varepsilon_t^R \tag{141}$$

where $\varepsilon_t^R \sim N(0, \sigma^R)$ is the monetary policy shock. ρ^R is the interest rate smoothing parameter and $X_t = \frac{Y_t}{\widetilde{Y}_t}$ defines the efficient output gap and $\widetilde{Y}_t$ is the GDP holding in the flexible prices world.[31]

[31] Making use of GDP instead of Y leaves results virtually unchanged.

Finally, the market clears

$$Y_t = C_t + S_t + G_t + \eta_t f_t \tag{142}$$

C.3.1 DSGE Model impulse responses

IST shock Figure 9 shows the impulse responses to a positive IST shock in the DSGE model as described in section C.1. The persistence of the shock is nil and the developments in the I-sector resemble the ones described in section 3.2.1.The differences with respect to Figure 3 are entirely due to the presence of nominal rigidities and to the calibration of intermediate producers fixed production cost, Ω, which is now greater than 0. These features are consistent with the original formulation of JPT.

Overall, the cyclical impact of the IST shock is more pronounced given the inclusion of both price and wage stickiness in the model. In spite of this, the benchmark model response to the shock is still relatively sluggish as compared to JPT. Wage stickiness prevents wages and therefore consumption from increasing on impact in both models. Also the presence of fixed cost in the intermediate goods production acts in this direction.

In the benchmark model, creative destruction materializes in the I-sector displaying twofold effects in the economy: i) it prevents investment from exerting any downward pressure on consumption; ii) it implicitly reduces the I-sector demand of consumption goods, keeping inflation relatively flat, so that hours worked do not abruptly fall on impact. Moreover, expectations of a future reductions in P^I avoid a subsequent deeper fall in consumption. Then as investment keeps moderately falling, so does consumption and hours worked implying a slight but persistent recession.

In the no entry model investment suddenly increases crowding out consumption. The presence of price stickiness impedes a fall in the final good price which would avoid an abrupt fall in the demand of labor and thus in production. However, since the fall of the relative price of investment is relatively fast, so is the adjustment of consumption and hours worked. The former falls persistently and the latter quickly increase.

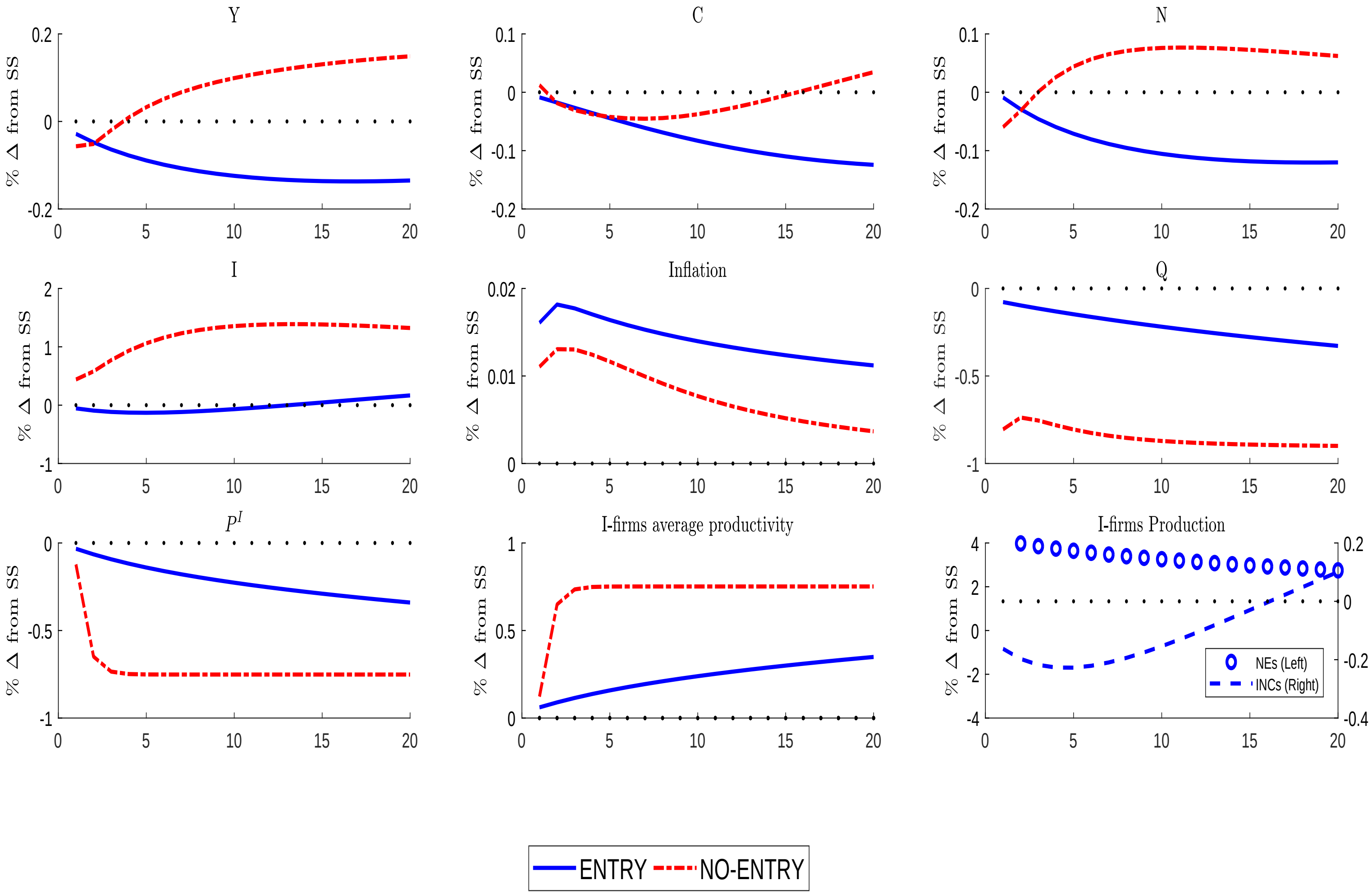


Figure 9: Impulse response functions to a permanent IST shock.

LAT shock Figure 10 shows the impulse to a permanent LAT shock in the DSGE model. The transmission mechanisms are standard and the differences with respect to the RBC formulation are all due to the introduction of the DSGE features into the model.

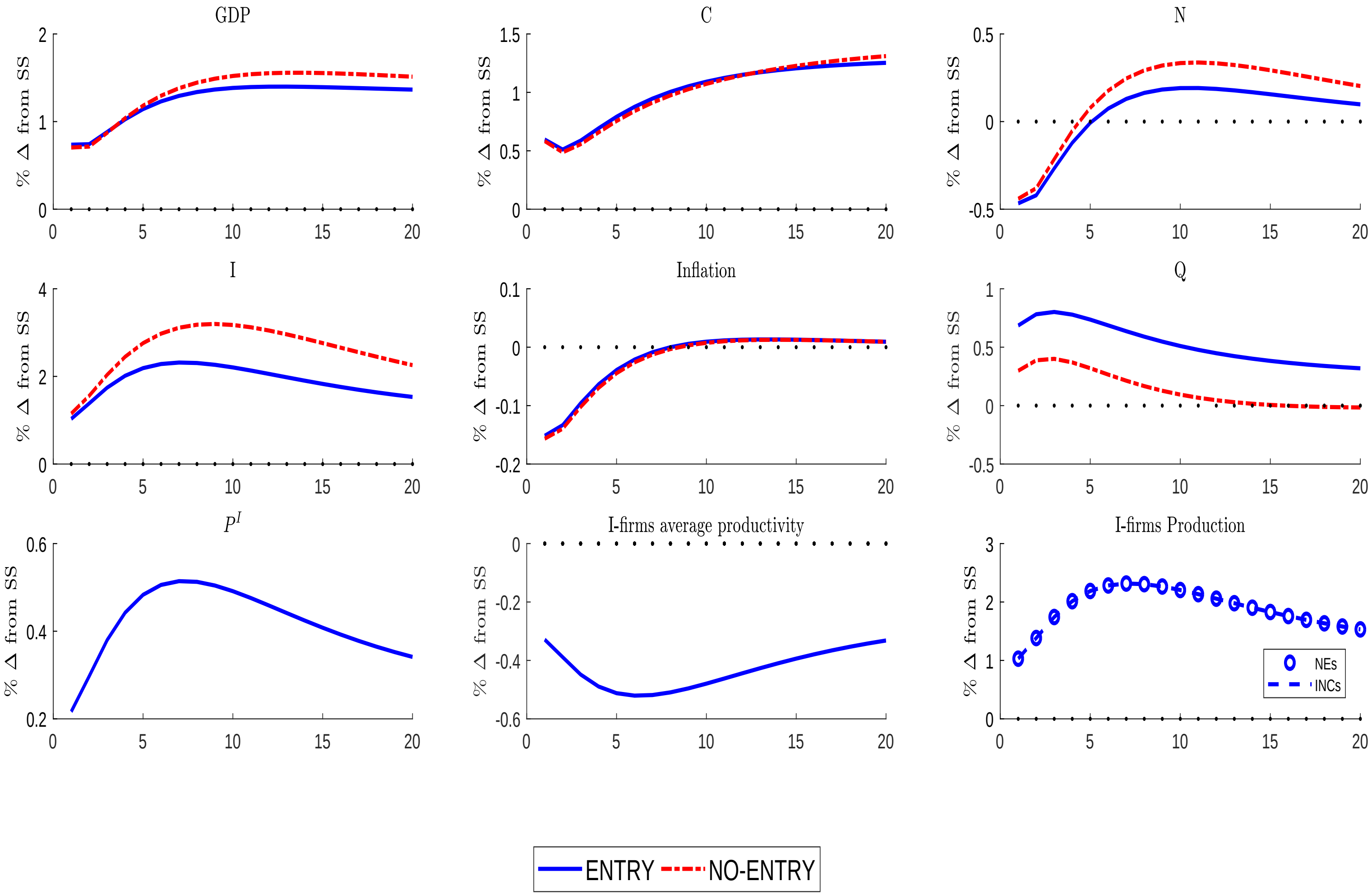


Figure 10: Impulse response functions to a permanent LAT shock.

Monetary Policy Shock Figure 11 shows the impulse to a transitory monetary policy shock. The transmission of the shock is standard as it is not altered by our I-sector formulation. The only difference is that rigidity of the investment supply schedule dampens the transmission of the shock as part of it is absorbed by the endogenous relative price of investment.

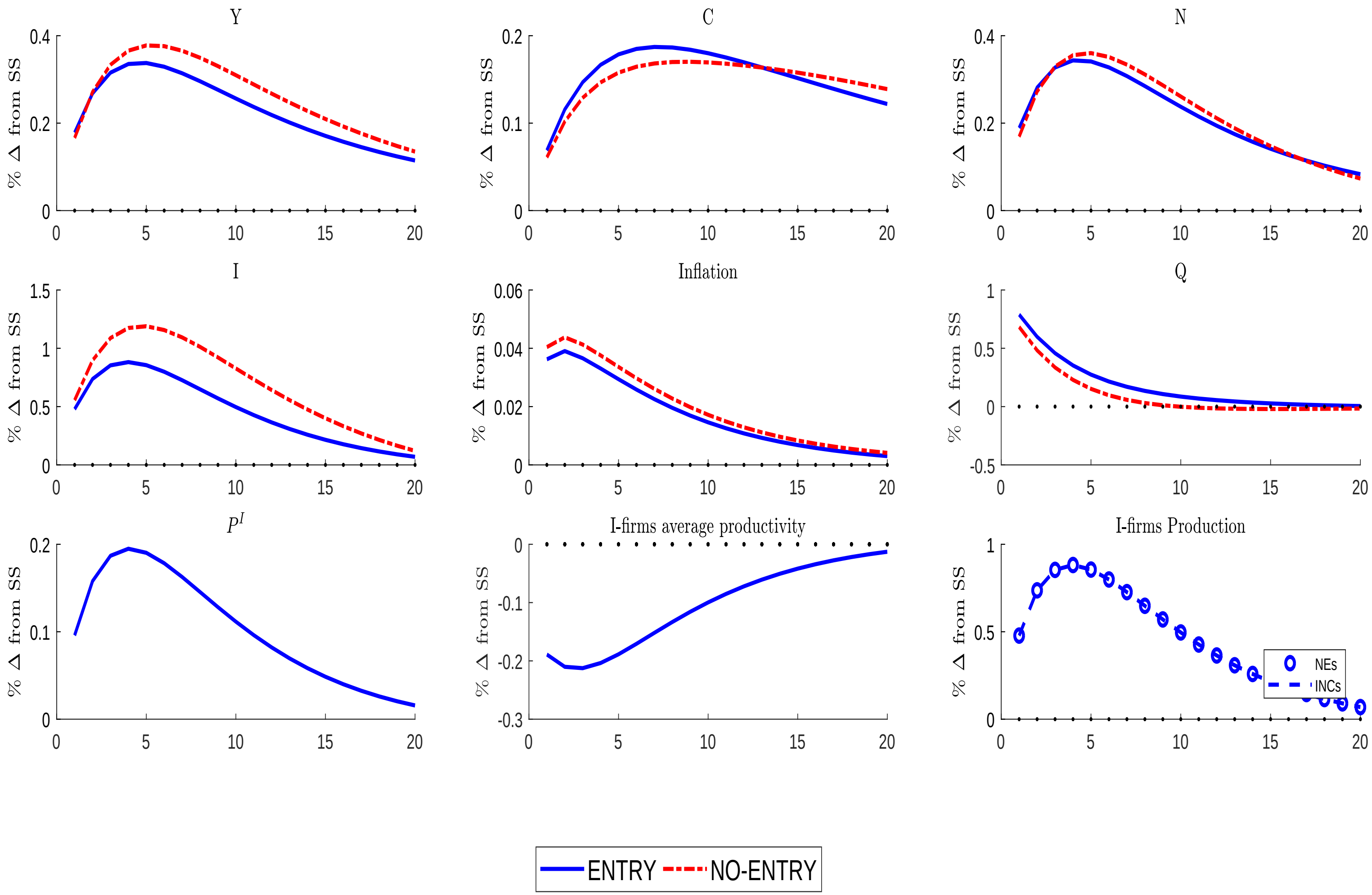


Figure 11: Impulse response functions to a monetary policy shock.

D Sensitivity Analysis

D.1 Different calibration of the I-sector

Here we perform a sensitivity analysis for possibly different parametrizations of our I-sector. The first thing to notice is that condition $\gamma(1-\alpha) > 1$ is binding. This implies that a larger value of α must be coupled with larger values of γ so that the constraint is not violated. From (11) it is easy to see that, as $\gamma \to \infty$, the I-firms distribution collapses to a point mass of I-firms all with identical idiosyncratic productivity. Thus, the larger γ, the smaller the responsiveness of I-firms average productivity to entry/exit flows driven by the different shocks. In fact, γ describes the rate at which the productivity pdf decays, therefore a smaller γ implies that highly productive firms are relatively more likely. It follows that, when an IST shock occurs, the recovery from creative destruction is faster for lower values of γ. Note that γ determines the price-elasticity of I-goods production, $\gamma - 1$.

In a nutshell, an increase in both α and γ boils down to a less rigid supply schedule in the I-sector, hence dampening the P^I response and boosting the reaction of investment production. Values of tail index have been chosen so that $\gamma = \frac{1}{1-\alpha} + 0.01$. The couples of values are ($\gamma = 2.501; \alpha = 0.6$), ($\gamma = 5.01; \alpha = 0.8$) and ($\gamma = 100.01; \alpha = 0.99$). These values cover three different cases: the first one refers to a situation where the investment supply schedule is extremely rigid, the second figures out a standard calibration very close to the one reported in the main text, and the last approaches a situation where the productivity distribution almost entirely collapses to a data point on the lower bound on the support and where I-firms produce almost under constant returns to scale and therefore endogenous firm dynamics is the only departure from the standard *NoE* model.

Figures 12, 13, 14 and 15 depict IRFs to IST, LAT, MEI and monetary policy shocks, obtained from simulations of the DSGE version of our model.

Considering the couple ($\gamma = 100.01; \alpha = 0.99$) for the IST case, most of the persistent dynamics of P^I falls upon endogenous firms dynamics rather than endogenous productivity because are entry flows, and therefore competitive forces, that determine the dynamics of the relative price of investment. Moreover note that as γ increases, creative destruction hits harder and the recovery takes longer: as more firms are located on the fat tail of the productivity distribution, more firms exit the market. By contrast, considering the LAT shock, the relative price of investment is virtually unaffected by the increase in the investment demand. This is because I-firms virtually produce with a CRS technology and therefore do not need to increase the price when production cost increases (see condition 24). The same reasoning applies also to other transitory shocks and therefore impulses are more in line with the standard *NoE* model even if endogenous firm dynamics is still at play.

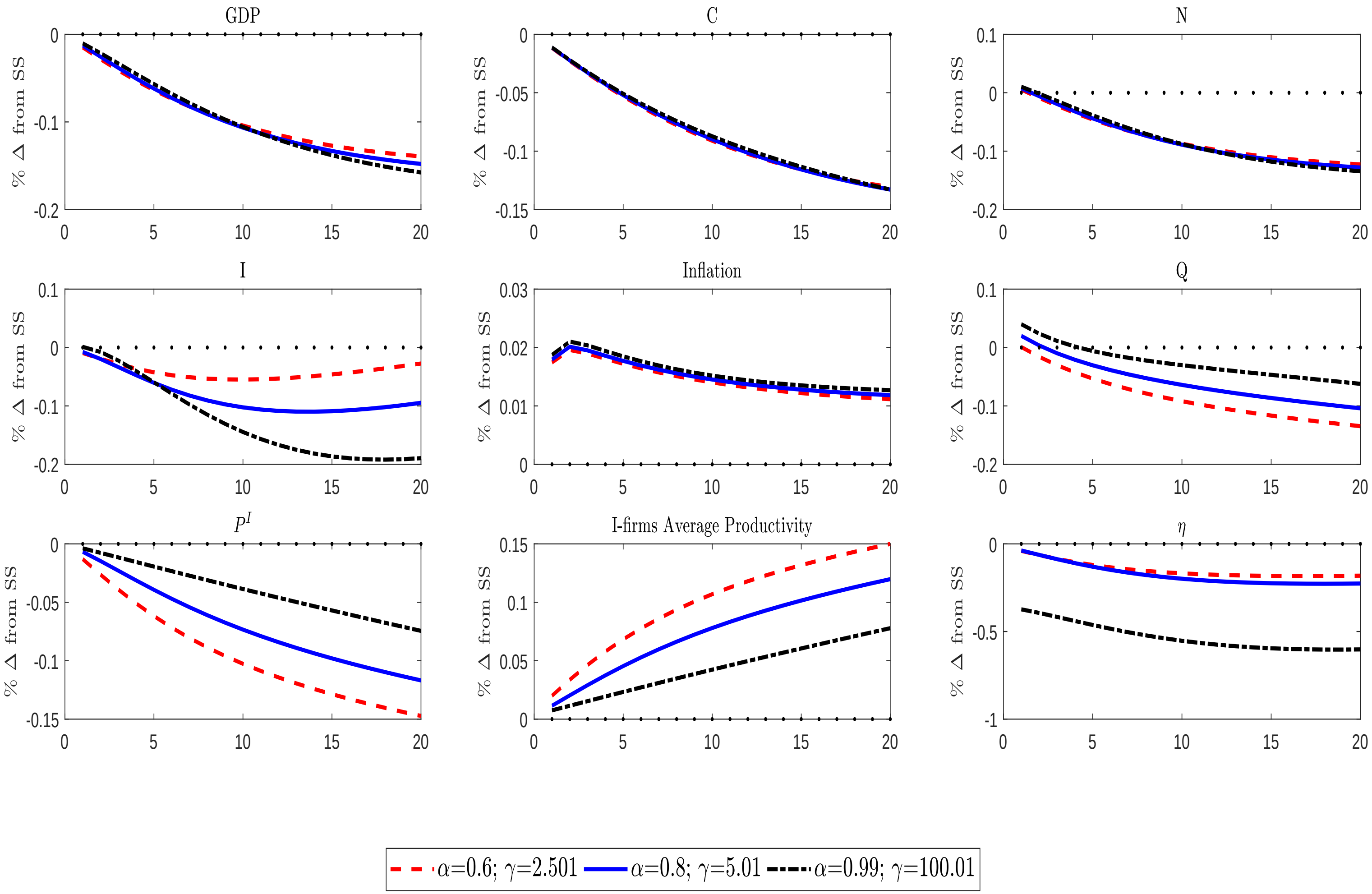


Figure 12: Impulse response functions to a permanent IST shock for different values of α and γ.

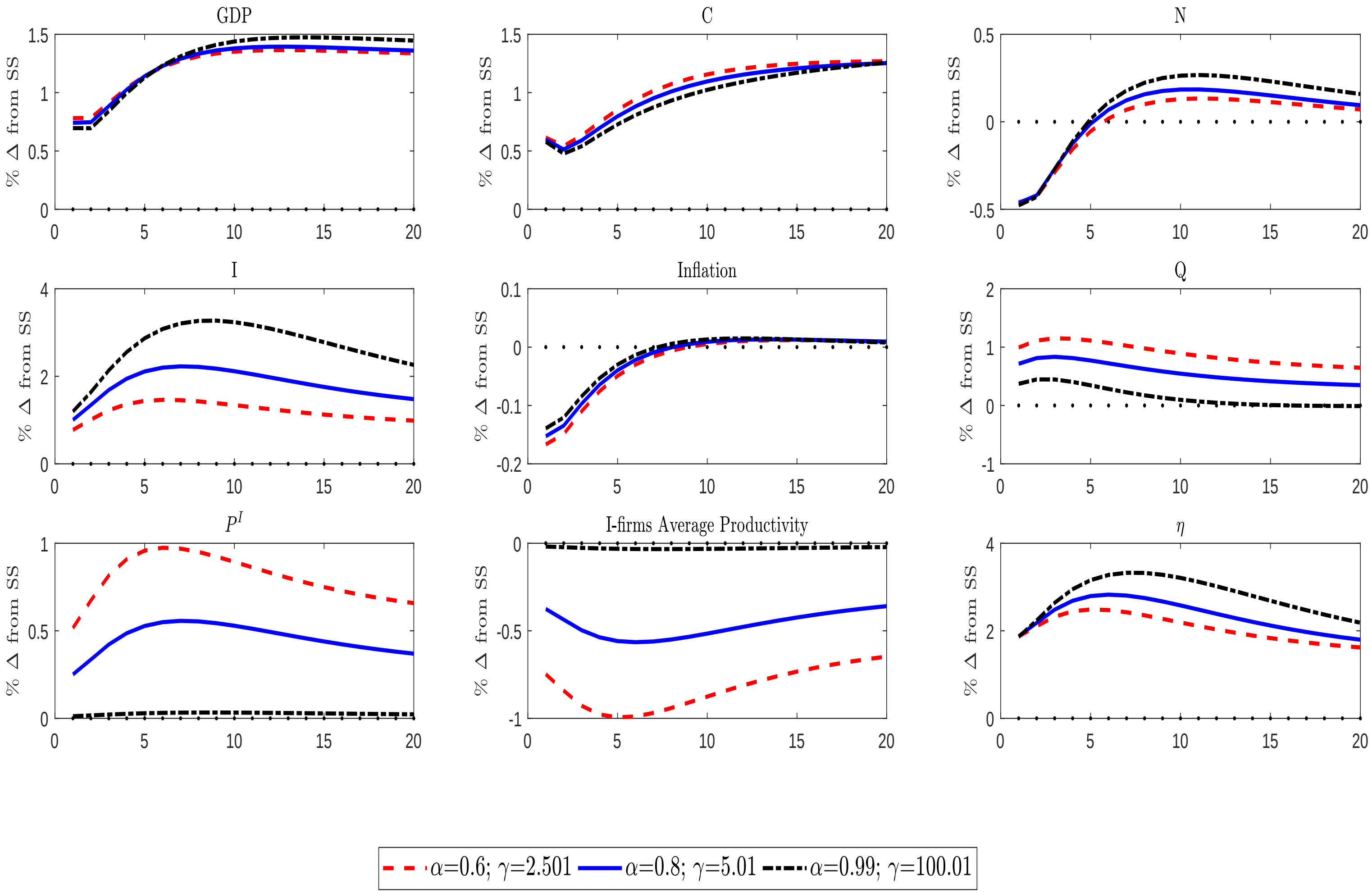


Figure 13: Impulse response functions to a permanent LAT shock for different values of α and γ.

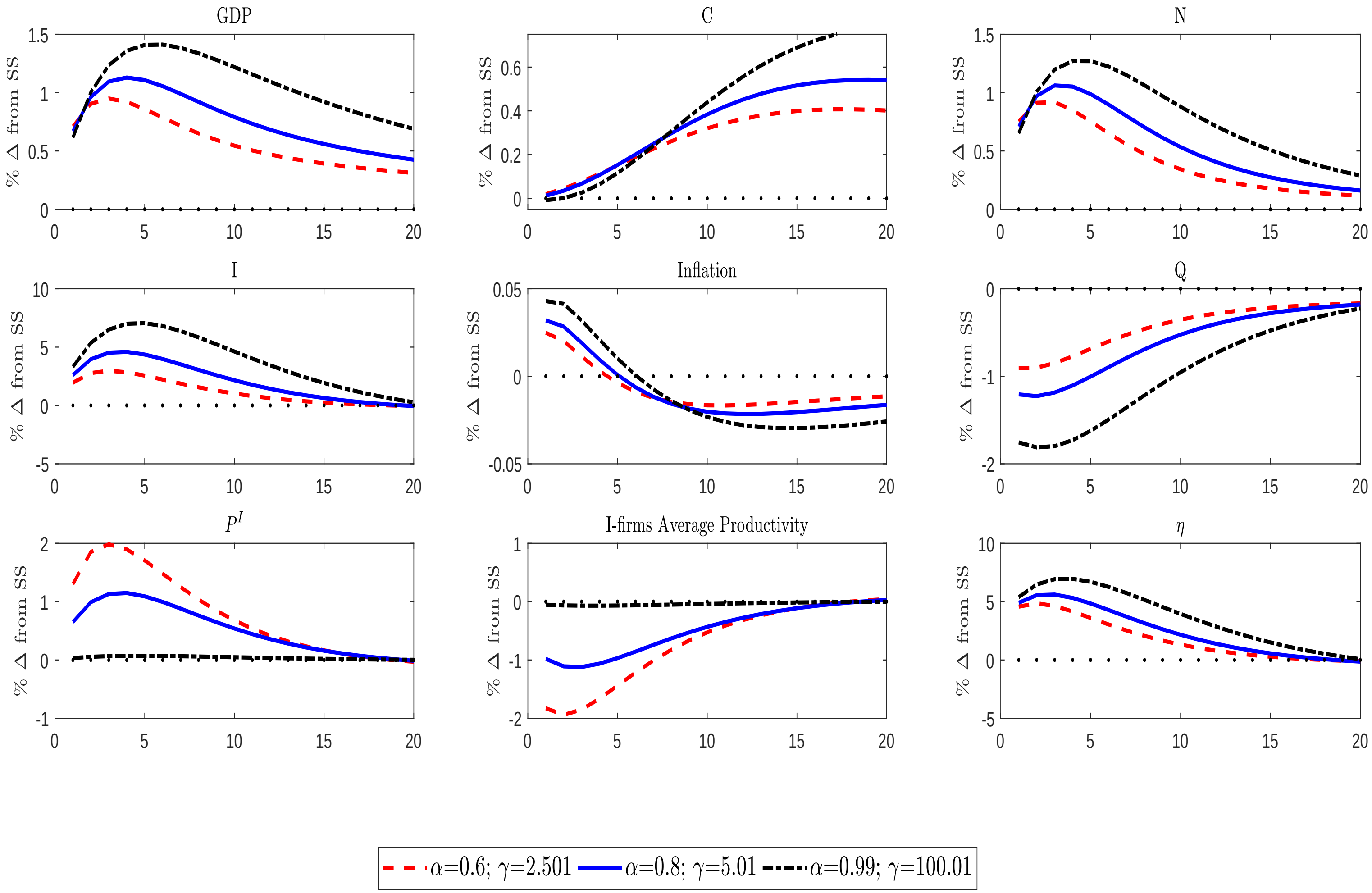


Figure 14: Impulse response functions to a transitory MEI shock for different values of α and γ.

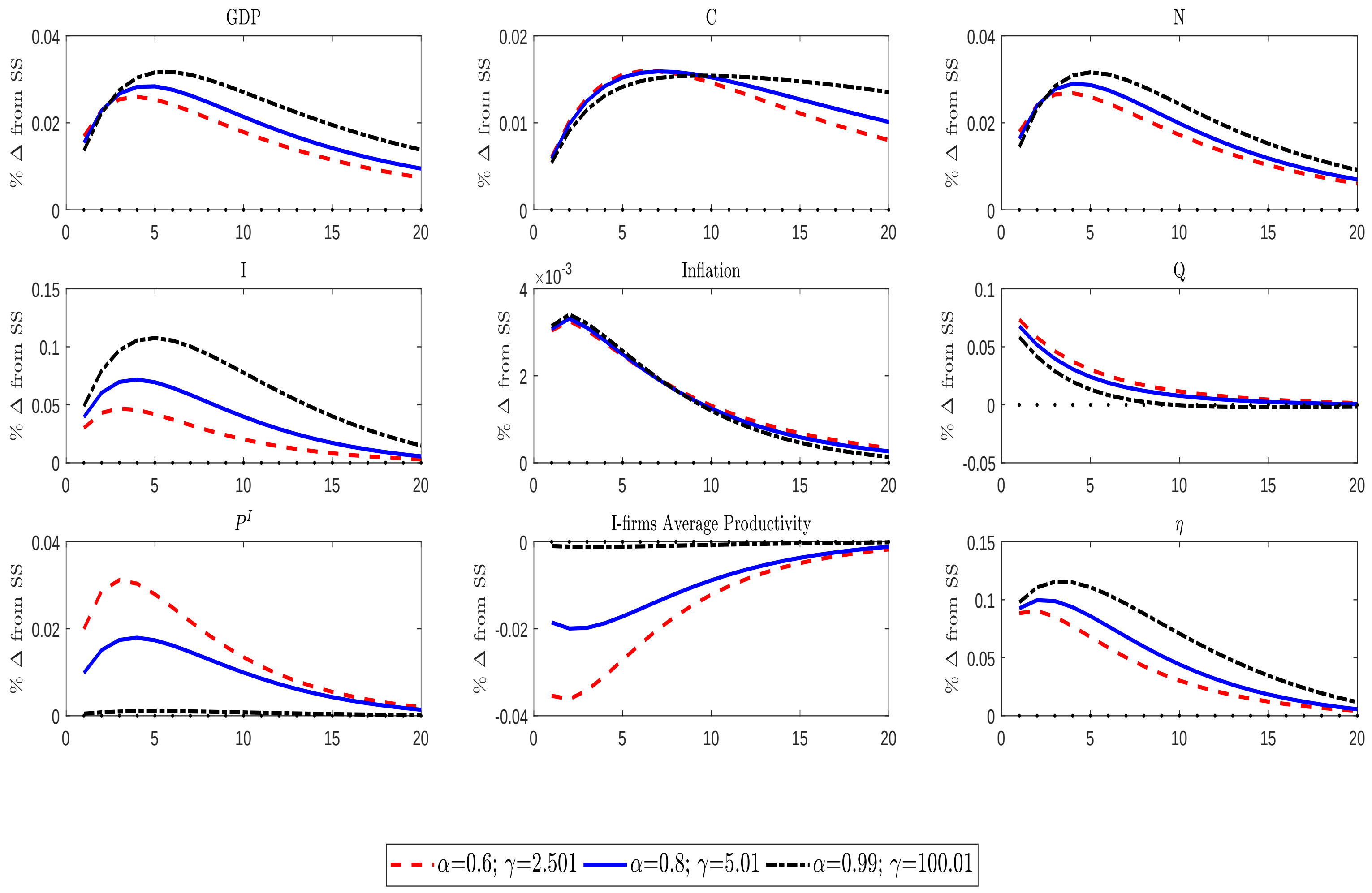


Figure 15: Impulse response functions to a transitory MP shock for different values of α and γ.

D.2 Persistence of MEI shocks

The dynamics triggered by a MEI shock in the no-entry model are strongly affected by the persistence of the shock itself. In particular, higher degrees of persistence reinforce the crowding out effect of investment with respect to consumption. As pointed out by Ascari et al. (2019), the transmission mechanism is relatively easy to explain because persistence strengthens the positive effect of MEI shocks on future investment demand, consumption goods producers internalize this and increase inflation. As a response the central bank raises real rates and therefore curbs consumption. MEI shocks persistence therefore implies a stronger crowding out effect on consumption. In the benchmark entry model the I-sector supply schedule is relatively rigid and this mitigates the fall of consumption.

In Figure 16 we report impulse responses to a positive MEI shock for both entry and no entry models (upper and lower panel respectively) over a grid of different values of the shock persistence ρ_{μ^i} (*i.e.* 0.65, 0.8 and 0.95).

The no entry model predicts a deeper drop in consumption as ρ_μ increases, whereas in the entry model consumption only falls for $\rho_\mu = 0.95$.

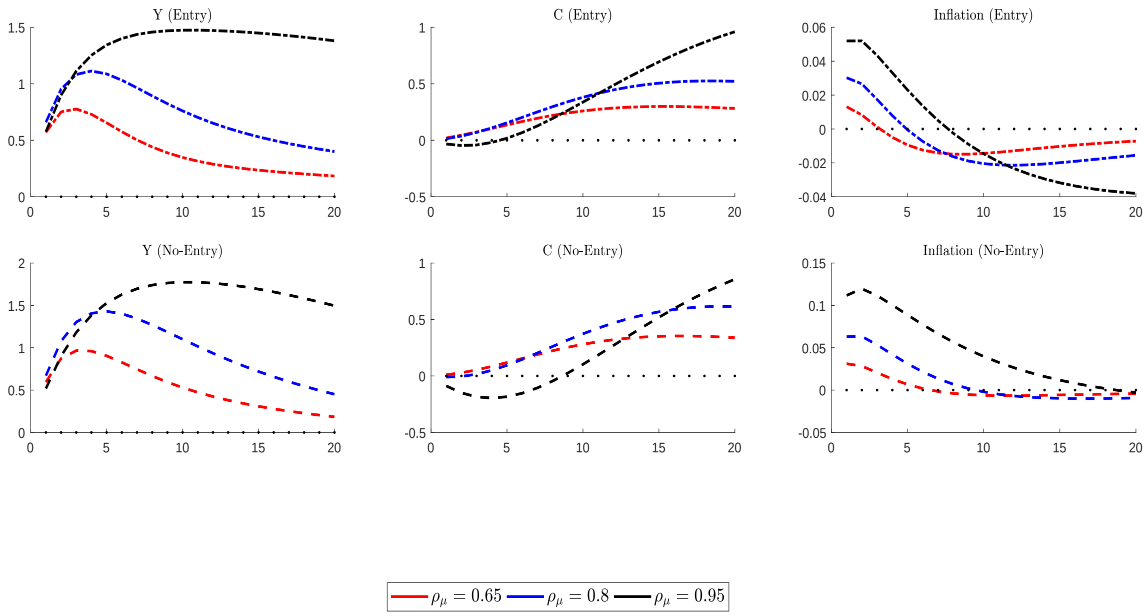


Figure 16: Impulse response functions to a transitory MEI shock for different persistence values.

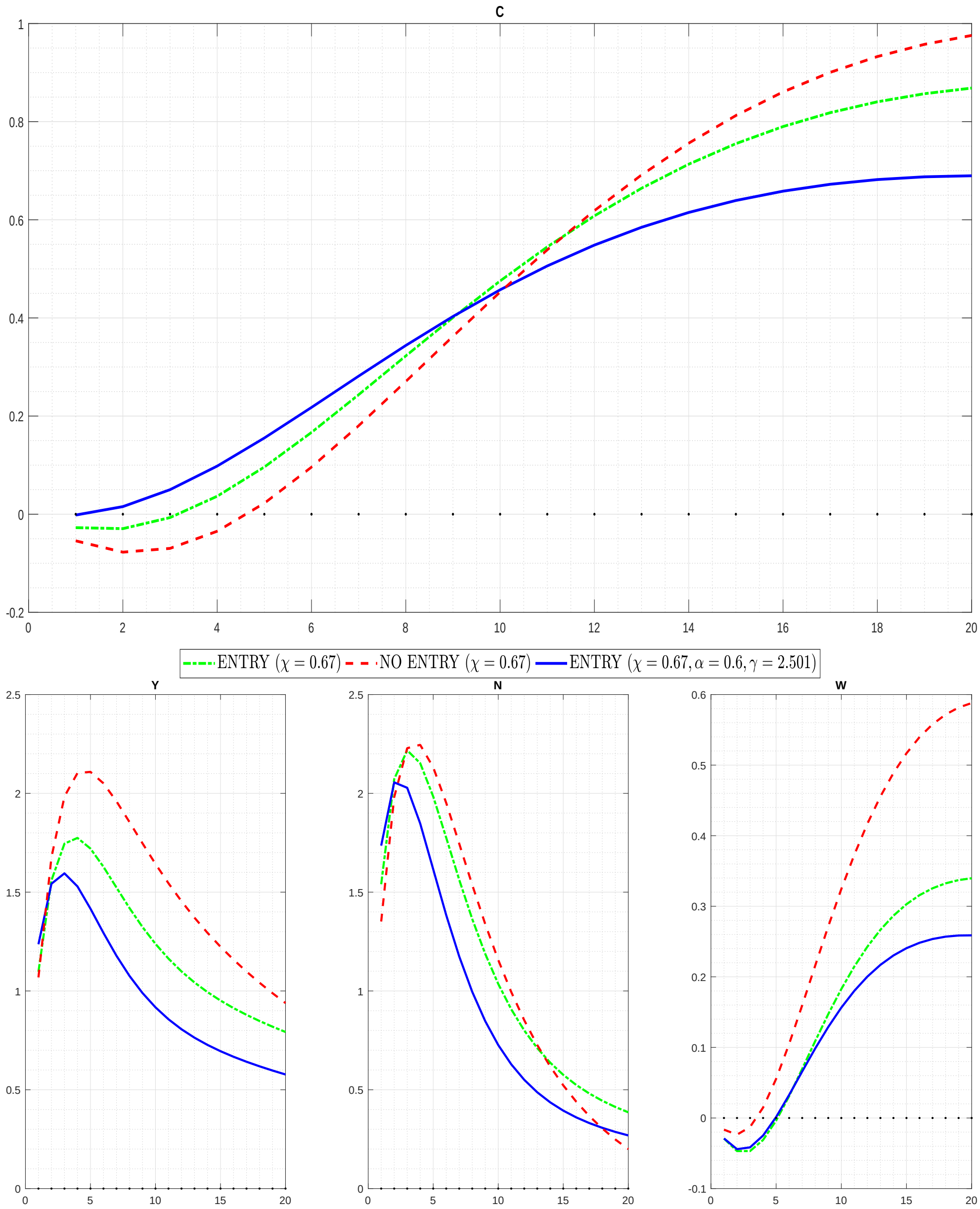
D.3 On the labor share of income

In standard DSGE models the size of the labor income share parameter plays a crucial role in sustaining the predicted contemporaneous correlations of consumption with output and investment. In fact, the larger the value of χ the more rigid is labor demand, and labor market equilibrium requires a stronger correlation between consumption and the real wage. JPT estimate a quite large value of the labor income share parameter, $\chi = 0.83$, that is at odds with the empirical evidence concerning the labor income share (see for instance Karabarbounis and Neiman, 2013).

In this section we conduct some sensitivity analysis to study the dynamics implied by our model according to a different calibration of the labor share of income. In this regard we impose $\chi = 0.67$, as in Ascari et al. (2019), for both E and NoE model. Further, for the E model, we consider a more rigid supply function in the light of the one presented in section D.1, *i.e.* $\alpha = 0.6$ and $\gamma = 2.501$. This is done to show that, when the labor share of income is sensibly lower, a less responsive supply schedule can fix many of the issues.

Impulse response functions are reported in Figure 17. The short run response of consumption in the NoE model is now far more strongly counter-cyclical: it stays abundantly below zero for the first four quarters, then consumption expands much more vigorously after two years. Turning to the E model with $\chi = 0.67$, consumption dynamics are essentially in line with the benchmark NoE model (*i.e.* where $\chi = 0.83$) but the short run response of consumption is still troublesome as it stays negative for the first three quarters. However, a more rigid investment supply schedule, via the mitigation of the crowding out of investment on consumption, more than offsets the relatively more elastic labor demand due to the lower value assigned to χ. As a result, consumption is virtually neutral on impact.

In order to have a more detailed view on the robustness of the business cycle properties of our model, we report in Table 7 some of the most relevant business cycle statistics generated by the two models under the different calibrations. The first thing to notice, is that all the models in place severely overestimate output and hours volatility, this is due to the fact that the models calibration was initially suited to match $\chi = 0.83$ rather than $\chi = 0.67$. However, looking at business cycle correlation we can gain the most interesting insights. In the NoE model, the smaller labor income share parameter yields essentially no correlation between consumption and output, and a counter-cyclical correlation between consumption and investment. This downside effect is much more attenuated for the E model according to which $\rho(\Delta Y, \Delta C)$ is still undoubtedly positive and $\rho(\Delta I, \Delta C)$ is essentially nil. Finally, when the E model emebeds a more rigid supply curve, all the above correlations get closer to the data generated moments. This is testified by Figure 18 where the cross-correlogram spectrum is considered up to lag four.



Note: the upper panel displays impulse responses for consumption to a positive MEI shock; lower panel displays impulses responses for output, hours worked and wage to a positive MEI shock. The MEI shock calibration is reported in Table 1.

Figure 17: Impulse response function to a transitory MEI shock for different calibrations of income labor share.

Table 7: **Moments in the Benchmark (Entry) and JPT (No Entry) model**

toprule	$\sigma(\Delta Y)$	$\sigma(\Delta C)$	$\sigma(\Delta \widetilde{I})$	$\sigma(N)$	$\rho(\Delta Y, \Delta C)$
JPT Data	(0.97)	(0.48)	(3.58)	(3.68)	(0.58)
Entry	1.41	0.40	3.62	5.92	0.15
No Entry	1.45	0.42	3.48	6.43	0.02
Entry ($\alpha = 0.65, \gamma = 2.9$)	1.47	0.41	3.64	5.30	0.26

	$\rho(\Delta Y, \Delta \widetilde{I})$	$\rho(\Delta \widetilde{I}, \Delta C)$	$\rho(\Delta Y, N)$	$\rho(\Delta C, N)$	$\rho(\Delta \widetilde{I}, N)$
JPT Data	(0.89)	(0.36)	(0.05)	(0.13)	(0.02)
Entry	0.98	-0.03	0.23	0.38	0.19
No Entry	0.99	-0.15	0.28	0.52	0.15
Entry ($\alpha = 0.65, \gamma = 2.9$)	0.97	0.11	0.23	0.32	0.19

Note: in the first row data obtained from the dataset used in JPT (1954Q3 to 2009Q1) are shown. In the second and third rows we report selected theoretical business cycle moments implied by our model (Entry) and by JPT's (No Entry). In the forth row the same business cycle moments for the Entry model with $\alpha = 0.65$ and $\gamma = 2.9$ are displayed. The labor share of income calibration is $\chi = 0.67$ for all the models included in the table.
Investment is defined in real terms as $\widetilde{I} = P^I I$ to comply with national accounting standards.

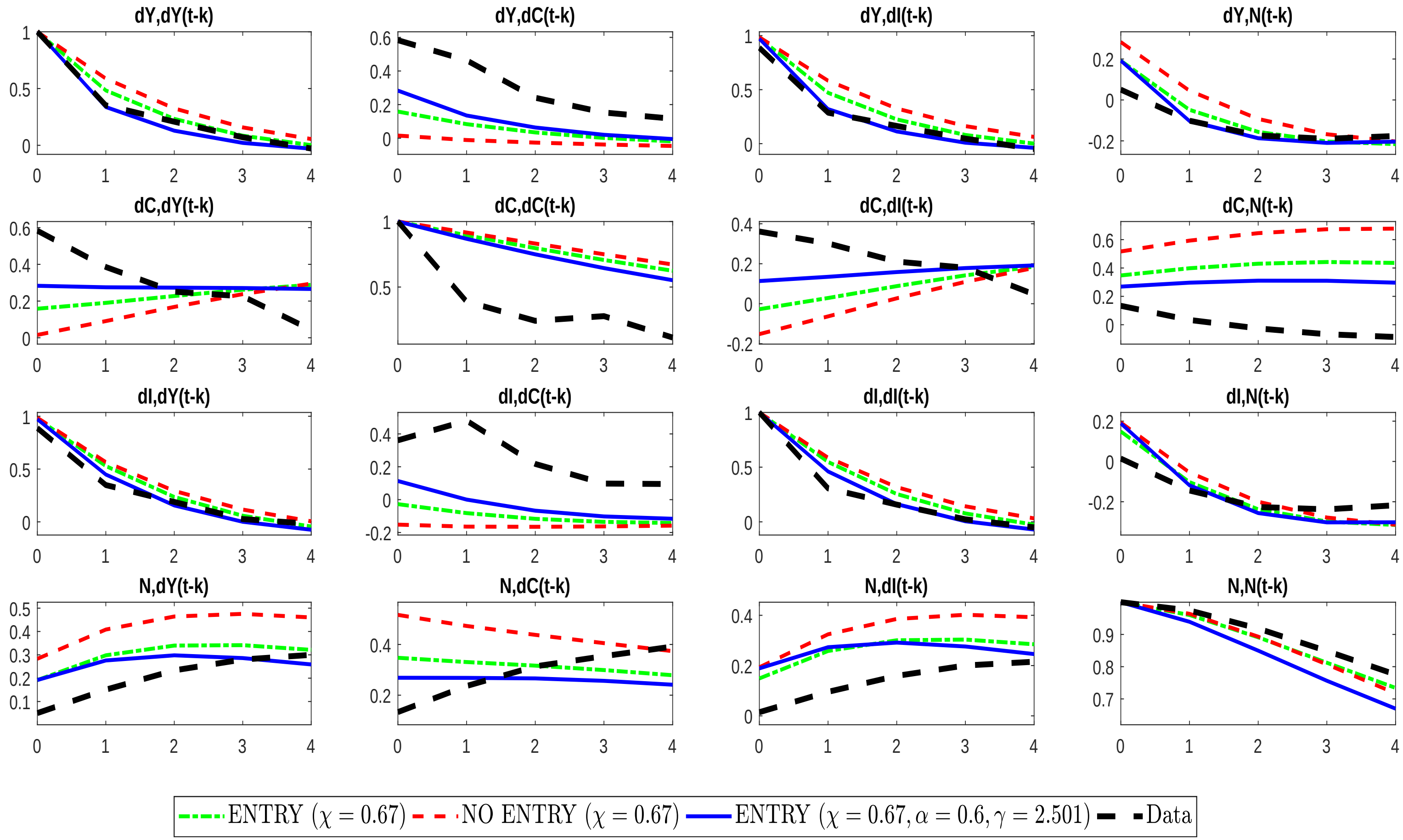


Figure 18: Cross-correlogram for key macroeconomic variables.