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A test of time reversibility based on L-moments with an application to the business cycles of the G7 economies

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Abstract. We study the performance of tests of distributional symmetry based on the coefficient of skewness and on L-moments and present a bootstrap implementation of such tests that is suitable in time series applications. We show with Monte Carlo simulations that both tests are correctly sized – provided that their null distribution is approximated with the bootstrap – and that the procedure based on L-moments has more power than that based on the conventional coefficient of skewness. An empirical application analyses the symmetry of business cycles for the G7 countries implementing tests of symmetry as tools to investigate time reversibility.

Key Words: business cycle; L-moments; symmetry; skewness; time reversibility.

JEL Codes: C22, C46, C52, C55, E32.

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1 Introduction

Tests of normality are routinely implemented for model-building purposes as well as to answer substantive questions in economics and finance (see e.g. Affleck-Graves and McDonald, 1989; Cecchetti et al., 1990; Hutson et al., 2008). A case in point is the analysis of time-reversibility (TR, henceforth) that hinges on the distributional properties of time series. Broadly speaking, TR holds if the statistical properties of a time series are not affected when it is observed in reverse time. Tests of TR have found several applications, including the analysis of business cycle symmetry (see e.g. Ramsey and Rothman, 1996; DeLong and Summers, 1986), the existence of Edgeworth price cycles in retail gasoline markets (Beare and Seo, 2014; McCausland, 2007) and tests of the mixture-of-distribution hypothesis in financial markets (Fong, 2003). Violations of TR arise for three main reasons: nonnormality of the distributional form, nonlinearity of the regression function and nonconstancy of the innovation variance (Cox, 1981). A time series that is not TR is called time-irreversible or directional.

TR can be assessed relying on the coefficient of skewness and build a test of symmetry of the r-th order difference of a time series (DeLong and Summers, 1986; Sichel, 1993). Conventional tests of skewness have a long history in statistics and form the basis of omnibus tests of normality (see e.g. D'Agostino and Pearson, 1973). The Jarque and Bera (1980, 1981) procedure (JB, henceforth) is largely the most widely used test of Normality. There are at least four issues related with the usage of the standard JB test, that extend also to tests of symmetry based on the conventional coefficient of skewness. First, even for independently and identically distributed (iid) random variables, the JB test is incorrectly sized and has low power in small samples (see e.g. Jarque and Bera, 1987; Dufour et al., 1998; White and MacDonald, 1980; Poitras, 2006). Second, it is based on the method-of-moments estimation of the coefficients of skewness and kurtosis that is not robust to outliers (see e.g. Bastianin, 2020; Bonato, 2011; Kim and White, 2004; Brys et al., 2004; Thomas, 2009). Third, when applied to serially correlated data, the sampling distribution does not coincide with that derived for iid observations and correct implementation of the test involves either resorting to a consistent estimator of the long-run covariance matrix (Bao, 2013; Bai and Ng, 2005;

Bontemps and Meddahi, 2005; Lobato and Velasco, 2004; Richardson and Smith, 1993) or relying on appropriate bootstrap or simulation methods (Dufour et al., 2003; Kilian and Demiroglu, 2000; Psaradakis and Vávra, 2018). Fourth, the JB test requires the existence of the eighth moment,[1] which is not satisfied by commonly used distributions and might be problematic in macroeconomic and financial applications, where there is often an issue of moment failure (Chen et al., 2000; De Lima, 1997; Mittnik and Rachev, 1993; Loretan and Phillips, 1994).

We consider a test of symmetry based on L-moments and its application to time series data. Hosking (1990) showed that the shape of distributions can be described relying on linear functions of expectations of order statistics, known as L-moments, that have a number of advantages over standard moments. In fact, L-skewness and L-kurtosis are more robust to outliers than conventional moments that raise the difference from the mean to the third or fourth power. Moreover, L-skewness and L-kurtosis identify deviations from Normality better than conventional moment-based measures. Lastly, L-moments uniquely characterize a set of distributions that is larger than that for which conventional moments can be applied. In fact, any distribution with finite mean is uniquely characterized by its L-moments, even when conventional moments do not exist.

We make four contributions to the literature on tests of symmetry for dependent data. First, we introduce the test based on L-moments (L-test, henceforth) due to Harri and Coble (2011) in the time series econometrics literature.[2] Second, we study the comparative performance of tests of symmetry based on the coefficient of skewness and on L-moments in an extensive set Monte Carlo experiments with sample size and data generating processes aimed at mimicking the dataset typically analyzed in macroeconomics and finance. Third, we introduce a bootstrap version of the L-test that is suitable for time series applications. Fourth, in the empirical application we analyse the symmetry of business cycles for the G7 countries.

[1] For a test of skewness the requirement is that the sixth moment exists.

[2] Two exceptions are Darolles et al. (2009) who constructed measures of fund performance based on L-moments and Bastianin (2020) who analyzed the performance of several robust measures of skewness and kurtosis – including those based on L-moments – and applied them to a large monthly database (i.e. the FRED-MD of McCracken and Ng, 2016). Notice however that none of these papers focus on testing symmetry.

Several studies are closely related to this paper. Kim and White (2004) and Bonato (2011) deal with the estimation of skewness and kurtosis in the presence of outlying observations. Although these authors do consider robust measures of skewness and kurtosis, they do not rely on L-moments. Thomas (2009) assesses the small sample behaviour of measures of symmetry based on quantiles and L-moments with a Monte Carlo analysis, but does not focus on time series data. Similarly, Harri and Coble (2011) introduce Normality tests based on L-moments, but focus exclusively on iid samples. Kilian and Demiroglu (2000) study the performance of the JB test for innovations in Vector Autoregressive and Vector Error-Correction models. We follow Psaradakis (2003, 2016), who implemented a symmetrized version of the sieve bootstrap of Bühlmann (1997) to improve the small sample accuracy of symmetry tests. While this author considers a wide class of symmetry tests, he does not investigate the performance of the L-test.

The rest of the paper is organized as follows. Section 2 discusses the two tests of symmetry considered in our contribution and their bootstrap implementation. Section 3 presents the simulation study. Section 4 is devoted to the empirical application and Section 5 concludes. An Appendix completes the paper.

2 Symmetry tests and their bootstrap implementation

2.1 Symmetry test based on the coefficient of skewness

Let $\{X_t\}_{t=1}^{T}$ be a time series with mean μ and r-th central moment $\mu_r = E\left[(x-\mu)^r\right]$.The coefficient of skewness is defined as:

$$SK = \frac{\mu_3}{\sigma^3} = \frac{E\left[(x-\mu)^3\right]}{E\left[(x-\mu)^2\right]^{3/2}} \tag{1}$$

For symmetric distributions $\mu_3 = 0$ and $SK = 0$. Moreover if X_t is iid normally distributed, a test of symmetry is based on the squares of the sample skewness coefficient, $\widehat{SK}$:

$$\hat{\tau}_3 = T\frac{\widehat{SK}^2}{6} \xrightarrow{d} \chi_1^2 \tag{2}$$

The null hypothesis of the test, $H_0 : SK = 0$, is rejected whenever $\hat{\tau}_3$ is greater than the upper critical value of a χ^2_1.

2.2 Symmetry test based on L-moments

The first four L-moments of a random variable are (Hosking, 1990):

$$\ell_1 = \int_0^1 Q(u)\mathrm{d}u \tag{3}$$

$$\ell_2 = \int_0^1 Q(u)(2u - 1)\mathrm{d}u \tag{4}$$

$$\ell_3 = \int_0^1 Q(u)(6u^2 - 6u + 1)\mathrm{d}u \tag{5}$$

$$\ell_4 = \int_0^1 Q(u)(20u^3 - 30u^2 + 12u - 1)\mathrm{d}u \tag{6}$$

where $Q(\alpha)$ be the quantile function. Much like conventional moments, L-moments uniquely characterize statistical distributions. Thus, ℓ_1 and ℓ_2 can be regarded as measures of location and scale. Population L-skewness (SK_L, for $r = 3$) and L-kurtosis (KR_L, for $r = 4$) are defined as ratios of L-moments, ℓ_r/ℓ_2 for $r = 3, 4$. Table 1 shows that for a standard Normal variate $SK_L = 0$ and $KR_L = 0.1226$. Since $|\ell_r/\ell_2| < 1$ for $r \geq 3$, SK_L and KR_L are bounded on the unit interval. This property makes their interpretation somehow easier than conventional skewness and kurtosis that can take arbitrarily large values.

A test of the null of symmetry $H_0 : SK_L = 0$ relies on the squares of $\widehat{SK}_L$ (Harri and Coble, 2011; Hosking, 1990):

$$\hat{\tau}_{3,L} = \frac{\widehat{SK}_L^2}{(0.1866T^{-1} + 0.8000T^{-2})} \xrightarrow{d} \chi^2_1 \tag{7}$$

2.3 Bootstrapping tests of symmetry

The asymptotic distributions of $\hat{\tau}_3$ and $\hat{\tau}_{3,L}$ – shown in Equation (2) and (7), respectively – depend crucially on the assumption that the underlying data are iid. To accommodate a wider class of data generating processes and in particular serially correlated and persistent variables, we propose to rely on a bootstrap approximation of the null sampling distribution of $\hat{\tau}_3$ and $\hat{\tau}_{3,L}$.

Following Psaradakis (2003, 2016) we rely on a symmetrized version of the sieve bootstrap of Bühlmann (1997) to estimate the null sampling distribution of our symmetry tests. The idea underlying the sieve bootstrap is to approximate the unknown data generating process with an autoregressive model of order p, AR(p). Once the model parameters have been estimated, they are used to generate residual-based replicates of the observed data. Since we are interested in an approximation of the null sampling distribution of the symmetry tests, we rely on a variant of the bootstrap algorithm that resamples from the symmetrized empirical distribution function of residuals from fitting the AR(p) model to the data. Details about the implementation of the the bootstrap procedure are presented in Section A of the Appendix.

3 Simulation study

3.1 Experimental design

Given the prominence of Vector Autoregressive models in macroeconometrics, we focus on autoregressive models of order 1, AR(1), with different degrees of persistence[3]:

$$y_t = \rho y_{t-1} + \varepsilon_t \quad \text{for } \rho = 0.0, 0.5, 0.9 \tag{8}$$

To investigate the size of different tests we rely on $\varepsilon_t \overset{iid}{\sim} N(0,1)$ as well as on different symmetric parametrizations of the Generalized Lambda Family (GLF). Table 1 highlights that the GLF encompasses symmetric (S1-S3) and asymmetric (A1-A4) distributions (Ramberg and Schmeiser, 1974). The distribution is defined by its quantile function: $Q(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$. We select seven members of the GLF, that are considered also by Bai and Ng (2005) and Psaradakis and Vávra (2018). Symmetric distributions S1-S3 – used to investigate the size of tests – all have kurtosis in excess of the Normal distribution. Similarly, distributions A1-A4 have increasing degree of skewness and excess kurtosis. More-

[3]The time needed to absorb half of a unit shock, or "half-life" (HL), is a widely used measure of persistence. For an AR(1) model $HL = \log(2)/\log|\rho|$, therefore given $\rho = 0.5, 0.9$ we have $HL = 1, 6.6$ time periods.

over, recall that a test of symmetry based on the standard coefficient of skewness requires to have six finite moments, while S3, A2 and A4 possess less than six moments.

All results are based on errors standardized to have zero mean and unit variance. We consider three different sample sizes – 40, 160 and 480 – that would correspond to 40 years of yearly, quarterly or monthly data. We use 100 burn-in observations to minimize dependence of the AR process on initial conditions and rely on 199 bootstrap samples.

Table 1: Error distributions

	λ_1	λ_2	λ_3	λ_4	SK	KR	SK_L	KR_L	Moments
N(0,1)	—	—	—	—	0.00	3.00	0.00	0.12	∞
S1	0.0000	-1.000000	-0.0800	-0.0800	0.00	5.99	0.00	0.20	12
S2	0.0000	-0.397912	-0.1600	-0.1600	0.00	11.61	0.00	0.23	6
S3	0.0000	-1.000000	-0.2400	-0.2400	0.00	126.90	0.00	0.27	4
A1	0.0000	-1.000000	-0.0075	-0.0300	1.52	7.46	0.21	0.18	33
A2	0.0000	-1.000000	-0.1009	-0.1802	2.00	21.11	0.16	0.23	5
A3	0.0000	-1.000000	-0.0010	-0.1300	3.16	23.75	0.39	0.22	7
A4	0.0000	-1.000000	-0.0001	-0.1700	3.88	40.73	0.41	0.23	5

Notes: *Notes*: N denotes the standard Normal distribution, while S1-A4 are members of the Generalized Lamba Familiy (GLF) with parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. SK = skewness, KR = kurtosis, SK_L = L-skewness, KR_L = L-kurtosis. "Moments" indicates the number of finite moments of each distribution. GLF have been standardized to have zero mean and unit variance.

3.2 Size

The size of symmetry tests based on the standard coefficient of skewness and on L-skewness are investigated comparing the Monte Carlo rejection frequency against the nominal size of the test set to 5%. Table 2 shows results based on the asymptotic distribution of tests when these are applied to simulated raw data. For the Normal distribution we see that both tests have empirical size close to the nominal level only when the AR parameter does not exceed 0.5. In the remaining cases both tests are highly oversized and their performance deteriorates as the degree of excess kurtosis (i.e. moving from S1 to S3) and/or the serial correlation increases. All in all, Table 2 highlights that implementing the two tests of symmetry based on their asymptotic distribution is not advisable when data feature serial correlation or excess kurtosis.

In Table 3 we investigate the size of tests when their null sampling distribution is approximated with the bootstrap procedure highlighted in Section 2.3. Overall, we see that

Table 2: Size: empirical rejection frequency of tests of symmetry – Asymptotic distribution

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	N(0,1)	S1	S2	S3	N(0,1)	S1	S2	S3
40	0.0	0.0370	0.2382	0.3744	0.4865	0.0380	0.1502	0.2442	0.3083
	0.5	0.0460	0.1942	0.3013	0.3764	0.0691	0.1712	0.2492	0.2993
	0.9	0.0971	0.1081	0.1512	0.2072	0.1842	0.2252	0.2492	0.3123
160	0.0	0.0551	0.3904	0.5756	0.7017	0.0541	0.1612	0.2683	0.4024
	0.5	0.0741	0.3193	0.4855	0.6306	0.0691	0.1832	0.2963	0.3894
	0.9	0.2412	0.3363	0.3664	0.4374	0.3283	0.3994	0.4154	0.4665
480	0.0	0.0450	0.5125	0.6567	0.7928	0.0521	0.1882	0.2943	0.3934
	0.5	0.0771	0.4505	0.5986	0.7447	0.0701	0.2092	0.3073	0.4254
	0.9	0.3303	0.4535	0.5345	0.6116	0.3864	0.4595	0.5115	0.5516

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1. A well sized test should have empirical rejection frequency close to its nominal size that in this case is 0.05. Results based on 999 simulations.

the symmetrized sieve bootstrap dramatically improves the performance of tests that now feature empirical rejection frequencies close to the 5% nominal level. Much like for the results based on the asymptotic distributions, both tests tend to be slightly oversized as the degree of excess kurtosis and or the serial correlation increases. Moreover, the empirical rejection frequency gets closer to the 5% nominal level as the sample size increases.

Table 3: Size: empirical rejection frequency of tests of symmetry – Sieve Bootstrap

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	N(0,1)	S1	S2	S3	N(0,1)	S1	S2	S3
40	0.0	0.0290	0.0671	0.0581	0.0721	0.0390	0.0691	0.0621	0.0761
	0.5	0.0480	0.0601	0.0521	0.0581	0.0531	0.0551	0.0631	0.0571
	0.9	0.0791	0.0601	0.0490	0.0541	0.0791	0.0791	0.0751	0.0761
160	0.0	0.0561	0.0400	0.0480	0.0440	0.0551	0.0420	0.0480	0.0761
	0.5	0.0661	0.0430	0.0430	0.0661	0.0681	0.0450	0.0450	0.0681
	0.9	0.0731	0.0450	0.0531	0.0631	0.0681	0.0561	0.0621	0.0661
480	0.0	0.0591	0.0511	0.0420	0.0370	0.0571	0.0651	0.0621	0.0450
	0.5	0.0591	0.0470	0.0480	0.0450	0.0571	0.0581	0.0521	0.0511
	0.9	0.0541	0.0460	0.0591	0.0581	0.0581	0.0470	0.0531	0.0511

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1. A well sized test should have empirical rejection frequency close to its nominal size that in this case is 0.05. Results based on 999 simulations and 199 bootstrap samples.

In Section C of the Appendix we repeat the analysis focusing on residuals of $AR(p)$ models, with p selected relying on the Akaike Information Criterion. In this case, the performance of the tests is not sensitive to the degree of persistence of the data, in that they are filtered with the $AR(p)$ model. Besides that, results mimic what we have seen for the raw data. When the error distribution features excess kurtosis (S1-S3), the implementation of the symmetry tests using asymptotic critical values leads to empirical rejection frequencies well above the 5% nominal level. When the null distributions are approximated with the symmetrized sieve bootstrap both tests are well sized.

To sum up, the analysis carried out in this section highlights that symmetry tests based on L-moments are appropriate both for serially correlated time series and autoregressive residuals if an appropriate bootstrap procedure is used to approximate their distributions under the null.

3.3 Power

To study the power of tests we rely on four asymmetric distributions. As shown in Table 1, distributions A1 to A4 have increasing degree of asymmetry and excess kurtosis. As documented in the previous section, both tests are oversized when relying on their asymptotic distributions, therefore we present only results based on bootstrap critical values.

The power analysis is summarized in Table 4 and Figure 1. The test based on L-skewness has always more power than the test based on the conventional skeweness coefficient. Power gains from L-moments are highest in smaller samples and increase as the degree of excess kurtosis and asymmetry rise. Section C of the Appendix suggests that these conclusions carry over to AR(p) residuals.

4 Empirical application: the symmetry of business cycles of the G7 economies

One way to implement a test of TR is to focus on the symmetry of the distribution of the r-difference of a time series (DeLong and Summers, 1986; Sichel, 1993). This form of TR is known as lag reversibility (see e.g. Tsay, 1992; Paparoditis and Politis, 2002) and implies that the joint distributions of (Y_t, Y_{t-r}) and (Y_{t-r}, Y_t) are equal for all t and all $r = 1, 2, \ldots$ In the presence of lag reversibility, $\Delta^r Y_t = Y_t - Y_{t-r}$ has a symmetric distribution and hence

Table 4: Power: empirical rejection frequency of tests of symmetry – Sieve Bootstrap

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	A1	A2	A3	A4	A1	A2	A3	A4
40	0.0	0.4815	0.1772	0.7818	0.7508	0.6346	0.2803	0.9880	0.9910
	0.5	0.2022	0.0901	0.2993	0.2683	0.3253	0.1602	0.7157	0.7157
	0.9	0.0581	0.0541	0.0440	0.0621	0.0781	0.0741	0.0791	0.0901
160	0.0	0.9419	0.5205	0.9099	0.8759	0.9990	0.7538	1.0000	1.0000
	0.5	0.7207	0.3253	0.7568	0.7197	0.9139	0.5215	0.9990	0.9980
	0.9	0.1091	0.1061	0.1221	0.1461	0.1261	0.1181	0.1662	0.2182
480	0.0	0.9960	0.8118	0.9610	0.9239	1.0000	0.9990	1.0000	1.0000
	0.5	0.9790	0.7247	0.9399	0.9019	1.0000	0.9640	1.0000	1.0000
	0.9	0.2112	0.1572	0.3473	0.3303	0.2723	0.2242	0.5395	0.5826

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1. Higher rejection frequencies indicate higher power. Results based on 999 simulations and 199 bootstrap samples.

$P(\Delta^r Y_t > 0) = P(\Delta^r Y_t < 0) = \frac{1}{2}$.

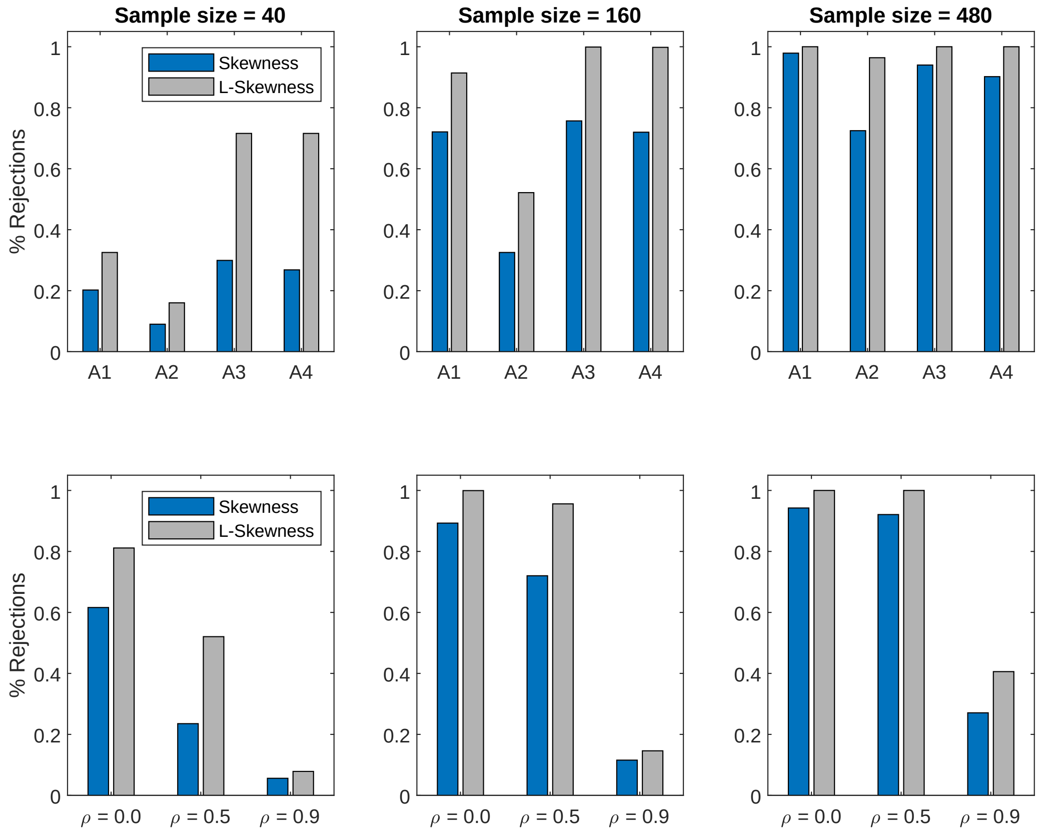
The concept of TR has been widely used in the analysis of business cycles to investigate the so-called Mitchell–Keynes hypothesis, which posits that expansions are more gradual than recessions (see e.g. Neftçi, 1984; DeLong and Summers, 1986; Ramsey and Rothman, 1996; Sichel, 1993). We focus on quarterly real GDP for the G7 economies over the period 1970:Q1-2019:Q4. We analyse whether $\Delta^r y_t = \ln(GDP_t/GDP_{t-r})$ for $r = 1, \ldots, 4$ is TR.[4]

Table 5 shows that, consistently with the Mitchell–Keynes hypothesis, the sample skewness of real GDP growth is negative for all countries except UK. Interestingly, if we omit the single largest observation in absolute value, $\widehat{SK}$ turns negative also for UK, which is expected, given that the low degree of resistance to outliers is a feature of conventional moments (see e.g. Brys et al., 2004). On the contrary, $\widehat{SK}_L$ for UK preserves its sign even when omitting the largest observation in absolute value.

In Table 5 we also present the p-values of tests of symmetry based on $\widehat{SK}$ and $\widehat{SK}_L$ for Δy_t as well as the Holm-Bonferroni p-values for the joint null hypothesis that the distributions of $\Delta^r Y_t$ for $r = 1, \ldots, 4$ are symmetric. We have consistent evidence against symmetry only for Japan. On the other hand, we can reject the null of symmetry for Italy when using the test based on $\widehat{SK}$, but not when relying on $\widehat{SK}_L$. A robustness analysis using monthly Industrial Production data – shown in Section C of the Appendix – provides

[4]Data sources and further results are presented in Sections B and C of the Appendix.

Figure 1: Power: median empirical rejection frequency of symmetry tests – Sieve bootstrap



Notes: the three plots in the upper panel show the median empirical rejection frequency of symmetry tests for each sample size and distribution. The three plots in the lower panel show the median empirical rejection frequency of symmetry tests for each sample size and autoregressive parameter, ρ. Blue bars denote the test based on the standard coefficient of skewness ($\hat{\tau}_3$), while gray bars identify the test based on L-skewness ($\hat{\tau}_{3,L}$). Higher rejection frequencies indicate higher power. See notes to Table 4.

further evidence against the symmetry of the Japanese business cycle, moreover it also provides evidence against TR for the US. All in all, the Mitchell–Keynes hypothesis seems to be strongly supported only for Japan.

5 Conclusions

Our Monte Carlo simulations show that symmetry tests based on L-moments have more power than tests based on the conventional coefficient of skewness. We also highlight that tests of symmetry can be applied to serially correlated and persistent time series, provided that an appropriate bootstrap algorithm is implemented to simulate their distributions under the null hypothesis. In fact, asymptotic results for iid data cannot applied to time series

Table 5: Symmetry test - real GDP

	Skewness			L-Skewness		
Country	$\widehat{SK}$	$r=1$	$r=1,\ldots,4$	$\widehat{SK}_L$	$r=1$	$r=1,\ldots,4$
Canada	-0.2182	0.4645	0.1041	-0.0286	0.4975	0.3203
France	-0.4897	0.1982	0.4324	-0.0016	0.9710	1.0000
Germany	-0.6645	0.3914	0.0841	-0.0121	0.8068	1.0000
Italy	-0.7051	0.0541	0.0000	-0.0199	0.7157	0.1522
Japan	-1.6380	0.0581	0.0000	-0.1421	0.0851	0.0080
UK	0.1448	0.7838	0.0721	-0.0372	0.5155	0.1001
US	-0.2985	0.4925	0.3844	-0.0425	0.3934	0.4765

Notes: the table shows the coefficient of skewness ($\widehat{SK}$) and L-Skewness ($\widehat{SK}_L$) for Δy_t and the p-values of tests of symmetry. The null hypothesis of the test is that the distribution of $\Delta^r y_t = \ln(Y_t/Y_{t-r})$ is symmetric, where Y_t is real GDP. We report the Bonferroni p-value for the joint null hypothesis that the distributions of $\Delta^r y_t$ for $r = 1, 2, \ldots$ are symmetric.

data in that they yield badly sized tests. A byproduct of our paper is to extend the results of Psaradakis (2003, 2016) showing that the symmetrized version of the sieve bootstrap he envisaged works well also for tests based on L-moments.

In the empirical application symmetry tests are a tools to investigate Mitchell–Keynes business-cycle hypothesis for the G7 countries. We have made a link between the analysis of the symmetry of business cycles and tests of TR in the form of lag reversibility. While in our application we have used the Holm-Bonferroni correction as a way to control for the family-wise error rate in multiple comparisons, a more detailed analysis of these is left for future research. Similarly, we believe that the investigation of an omnibus test of normality based on L-moments, as well as the application of our methods to test for the presence of Edgeworth price cycles in retail gasoline markets might deserve some attention.

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Appendix to
"A test of time reversibility based on L-moments with an application to the business cycles of the G7 economies"

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A Symmetry tests and bootstrap implementation: further details

A.1 Estimation of conventional skewness

Given a sample of size T, $\{x\}_{t=1}^{T}$, SK is estimated as follows:

$$\widehat{SK}_1 = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{x_t - \hat{\mu}}{\hat{\sigma}}\right)^3 \tag{1}$$

where $\hat{\mu} = T^{-1}\sum_{t=1}^{T} x_t$ and $\hat{\sigma} = \sqrt{T^{-1}\sum_{t=1}^{T}(x_t - \hat{\mu})^2}$.

If X_t is iid normally distributed, then:

$$\sqrt{T}\ \widehat{SK} \xrightarrow{d} N(0,6) \tag{2}$$

Tests of symmetry are often based on the squares of $\widehat{SK}$:

$$\hat{\tau}_3 = T\frac{\widehat{SK}^2}{6} \xrightarrow{d} \chi_1^2 \tag{3}$$

The null hypothesis of the test,$H_0 : SK = 0$ is rejected whenever $\hat{\tau}_3$ is greater than the upper critical value of a χ_1^2 (i.e. $\chi^2_{1(0.99)} = 6.6349$, $\chi^2_{1(0.95)} = 3.8415$ and $\chi^2_{1(0.90)} = 2.7055$).

A.2 Estimation of L-moments

L-moments can be generally defined as:

$$\ell_r = \frac{1}{r}\sum_{j=0}^{r-1}(-1)^j\binom{r-1}{j}E\left(X_{r-j:r}\right) \quad \text{for } r = 1,2,\ldots \tag{4}$$

The expectation of an order statistics, $E\left(X_{j:r}\right)$, can be written as (David and Nagaraja, 2017):

$$\begin{aligned} E\left(X_{j:r}\right) &= \frac{r!}{(j-1)!(r-j)!}\int_{-\infty}^{\infty} xF^{j-1}(x)\left[1-F(x)\right]^{r-j}f(x)\mathrm{d}x \\ &= \frac{r!}{(j-1)!(r-j)!}\int_{0}^{1} u^{j-1}\left[1-u\right]^{r-j}Q(u)\mathrm{d}u \end{aligned} \tag{5}$$

Substituting (5) in (4) and rearranging, we get:

$$\ell_r = \int_0^1 P^*_{r-1}(u)Q(u)\mathrm{d}u \tag{6}$$

where $P_r^*(u) = \sum_{j=0}^{r} p_{r,j}^* u^j$ and $p_{r,j}^* = (-1)^{r-j}\binom{r}{j}\binom{r+j}{j}$. Notice that $P_r^*(u)$ is the r-th shifted Legendre polynomial, related to the Legendre polynomial by $P_r^*(u) = P_r(2u-1)$. L-moments can be estimated by sample L-moments. These are defined as:

$$\hat{\ell}_r = \sum_{j=0}^{r-1} p_{r-1,j}^* b_j \qquad \text{where} \quad b_j = \frac{1}{n}\sum_{i=1}^{n} \frac{(i-1)(i-2)\dots(i-j)}{(n-1)(n-2)\dots(n-j)} x_{i:n} \tag{7}$$

An estimator of L-skewness is thus given by: $\widehat{SK}_L = \hat{\ell}_3/\hat{\ell}_2$.

To build their test of normality based on L-moments Harri and Coble (2011) start from the following results in Hosking (1990):

$$\frac{\widehat{SK}_L}{\sqrt{(0.1866T^{-1} + 0.8000T^{-2})}} \xrightarrow{d} N(0,1) \tag{8}$$

A test of the null of symmetry $H_0 : SK_L = 0$ relies on the squares of $\widehat{SK}_L$:

$$\hat{\tau}_{3,L} = \frac{\widehat{SK}_L^2}{(0.1866T^{-1} + 0.8000T^{-2})} \xrightarrow{d} \chi_1^2 \tag{9}$$

A.3 Bootstrapping tests of symmetry

Suppose that we observe a sample of data $\{y_t\}_{t=1}^{T}$ whose data generating process can be written as:

$$y_t - \mu = \sum_{j=1}^{\infty} \phi_j (y_{t-j} - \mu) + \varepsilon_t \tag{10}$$

where $\{\phi_j\}_{j=1}^{\infty}$ is a square-summable sequence, $\mu \equiv E(y_t)$ and ε_t is an iid symmetrically distributed random variable. Notice that Equation (10) encompasses a large set of stochastic processes, including Autoregressive Moving Average (ARMA) models. Moreover, it is worth pointing out that the symmetry of the distribution of y_t is implied by the symmetry of the distribution of the error term ε_t.

The idea underlying the sieve bootstrap is to approximate the unknown data generating process in Equation (10) with a Autoregressive model of order p, AR(p), where the autoregressive order increases slowly with the sample size.

Let $\mathcal{T}$ be one of the symmetry tests we consider, then its asymptotic distribution under the null hypothesis is estimated relying on the following bootstrap algorithm:

1. Select the order p of an $AR(p)$ with the Akaike Information Criterion
2. Get the estimates of the coefficients $\hat{\rho}_1, \dots, \hat{\rho}_p$ of the $AR(p)$ model $y_t - \hat{\mu} = \sum_{j=1}^{p} \rho_j (y_{t-j} - \hat{\mu}) + u_t$ where $\hat{\mu}$ is the sample average of y_t.
3. Construct the residuals $\hat{u}_t = (y_t - \hat{\mu}) - \sum_{j=1}^{p} \hat{\rho}_j (y_{t-j} - \hat{\mu})$ for $t = p+1, \dots, T$

4. Draw a random sample $\left\{u_t^b\right\}_{t=1}^T$ from the empirical distribution function of $\tilde{u}_t$ where:

$$\tilde{u}_t = \begin{cases} \hat{u}_t & \text{if } t = p+1, \ldots, T \\ -\hat{u}_t & \text{if } t = T+1, \ldots, 2T-p \end{cases}$$

5. Generate bootstrap replicates $\left\{y_t^b\right\}_{t=1}^T$ relying on:

$$y_t^b - \mu = \sum_{j=1}^{p} \hat{\phi}_j \left(y_{t-j}^b - \mu\right) + u_t^b$$

6. Construct the bootstrap analog of $\mathcal{T}$, denoted as $\mathcal{T}_b$, applying the test of symmetry to the bootstrap time series $\left\{y_t^b\right\}_{t=1}^T$

7. Repeat steps 4-6 a large number of times to obtain a sample of size B: $\{\mathcal{T}_b\}_{b=1}^B$

The empirical distribution of $\{\mathcal{T}_b\}_{b=1}^B$ is used to as a bootstrap approximation of the null sampling distribution of $\mathcal{T}$. The bootstrap p-values can be computed as $p_b = B^{-1}\sum_i^B \mathbb{I}(|\mathcal{T}_b| > |\hat{\mathcal{T}}|)$ where $\hat{\mathcal{T}}$ is the observed value of $\mathcal{T}$ and $\mathbb{I}(\cdot)$ is the indicator function taking unit value if the condition in brackets is satisfied. The bootstrap test rejects the null of symmetry if $p_b > \alpha$ where $\alpha \in (0,1)$ is the nominal level of the test. Equivalently, we reject the null of symmetry if $\hat{\mathcal{T}}$ exceeds the $(\lceil B(1-\alpha)\rceil)$-th order statistic of $\{\mathcal{T}_b\}_{b=1}^B$.

B Empirical application: data sources

We have sourced real GDP and Industrial Production (IP) for G7 countries (i.e. Canada, France, Germany, Italy, Japan, UK, US) from the Main Economic Indicators database maintained by the OECD. We measure as GDP by Expenditure in Constant Prices (Index 2015=100, Seasonally Adjusted) Canada, France US, UK, Germany for G7 countries. Real GDP series cover different sample periods. For Canada, France, Germany, UK, US data span 1970:Q1-2019:Q4 (200 observations); for Italy 1981:Q1-2019:Q1 (156 observations); for Japan 1994:Q1-2019Q4 (104 observations). Data on IP span January 1961-December 2019 (708 observations). In both cases we focus on $\Delta^r y_t = \ln(Y_t/Y_{t-r})$ where Y_t is either real GDP or IP.

C Additional tables and figures

Table C1: Size: empirical rejection frequency of tests of symmetry for residuals – Asymptotic distribution

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	N(0,1)	S1	S2	S3	N(0,1)	S1	S2	S3
40	0.0	0.0350	0.2302	0.3744	0.4725	0.0450	0.1451	0.2402	0.3063
	0.5	0.0330	0.2282	0.3734	0.4695	0.0470	0.1461	0.2352	0.2953
	0.9	0.0350	0.2262	0.3744	0.4464	0.0430	0.1411	0.2262	0.2923
160	0.0	0.0511	0.3734	0.5796	0.7027	0.0551	0.1562	0.2593	0.3924
	0.5	0.0470	0.3804	0.5766	0.7057	0.0561	0.1562	0.2653	0.3904
	0.9	0.0490	0.3784	0.5746	0.7017	0.0591	0.1592	0.2653	0.3944
480	0.0	0.0430	0.5115	0.6577	0.7898	0.0501	0.1872	0.2943	0.3894
	0.5	0.0430	0.5125	0.6547	0.7888	0.0511	0.1872	0.2923	0.3834
	0.9	0.0470	0.5115	0.6627	0.7868	0.0531	0.1902	0.2883	0.3834

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1 of the paper. A well sized test should have empirical rejection frequency close to its nominal size that in this case is 0.05. Results based on 999 simulations. Tests are applied on the residuals of an $AR(p)$ model where p is selected with the Akaike Information Criterion.

Table C2: Size: empirical rejection frequency of tests of symmetry for residuals – Sieve Bootstrap

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	N(0,1)	S1	S2	S3	N(0,1)	S1	S2	S3
40	0.0	0.0360	0.0410	0.0410	0.0330	0.0380	0.0561	0.0601	0.0621
	0.5	0.0350	0.0501	0.0541	0.0410	0.0410	0.0541	0.0611	0.0651
	0.9	0.0340	0.0460	0.0571	0.0511	0.0410	0.0621	0.0701	0.0591
160	0.0	0.0511	0.0380	0.0430	0.0430	0.0631	0.0410	0.0490	0.0691
	0.5	0.0561	0.0420	0.0450	0.0440	0.0631	0.0470	0.0511	0.0681
	0.9	0.0501	0.0390	0.0521	0.0420	0.0631	0.0440	0.0621	0.0741
480	0.0	0.0651	0.0450	0.0430	0.0320	0.0521	0.0571	0.0601	0.0460
	0.5	0.0591	0.0501	0.0440	0.0320	0.0601	0.0561	0.0611	0.0440
	0.9	0.0551	0.0531	0.0430	0.0310	0.0551	0.0571	0.0661	0.0430

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1 of the paper. A well sized test should have empirical rejection frequency close to its nominal size that in this case is 0.05. Results based on 999 simulations and 199 bootstrap samples. Tests are applied on the residuals of an $AR(p)$ model where p is selected with the Akaike Information Criterion.

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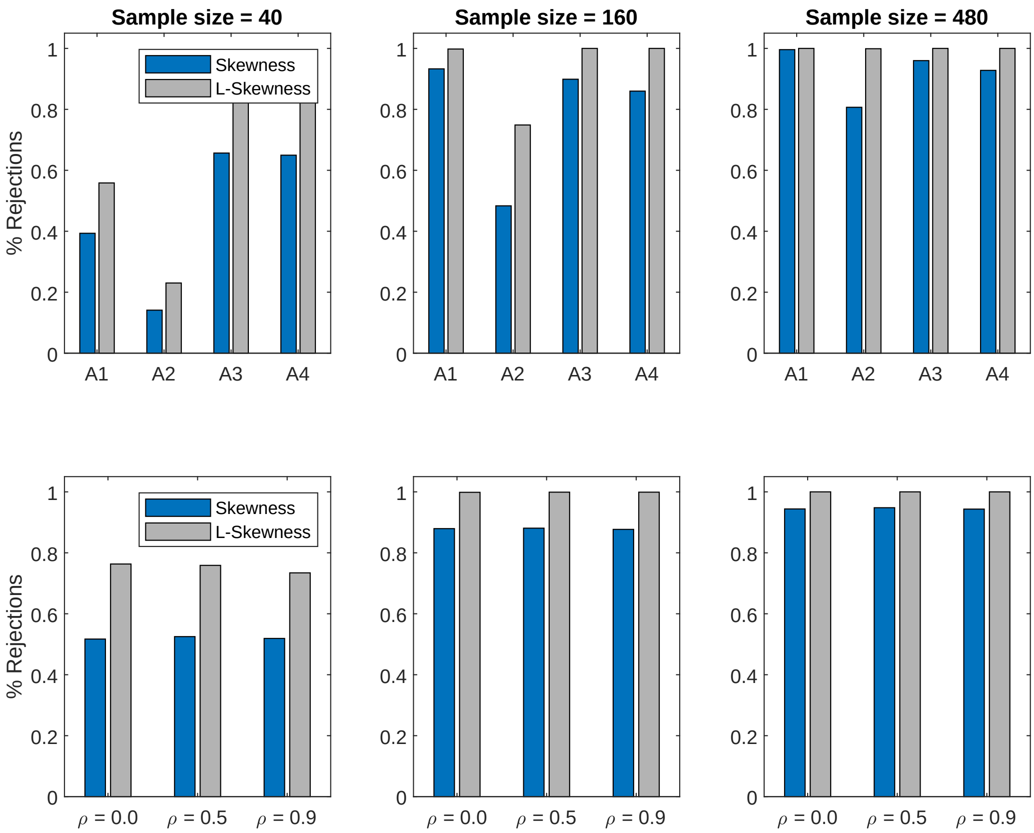
Figure A1: Power: median empirical rejection frequency of symmetry tests for residuals – Sieve bootstrap



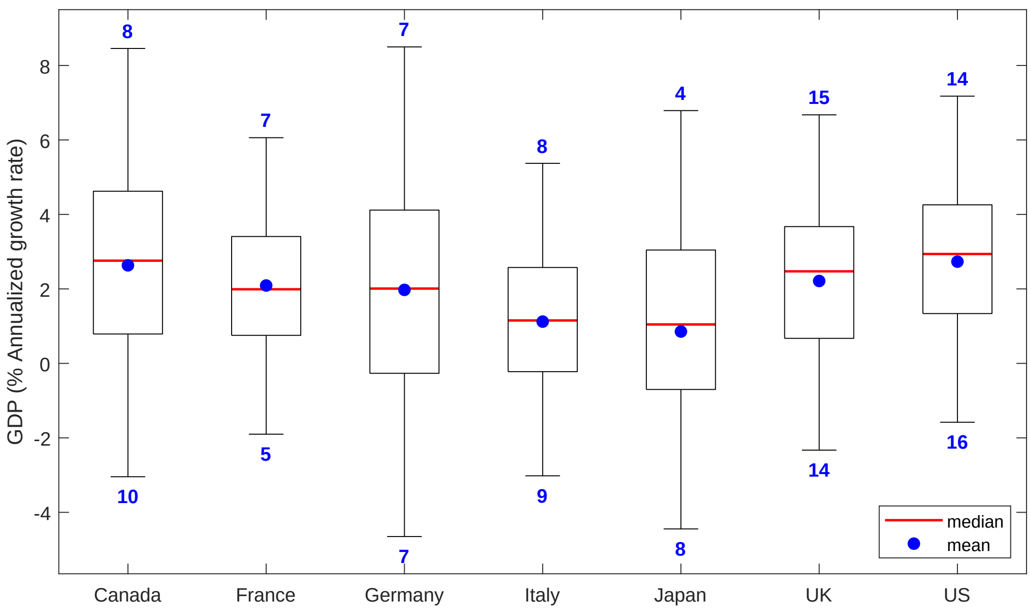
Notes: the three plots in the upper panel show the median empirical rejection frequency of symmetry tests for each sample size and distribution. The three plots in the lower panel show the median empirical rejection frequency of symmetry tests for each sample size and autoregressive parameter, ρ. Blue bars denote the test based on the standard coefficient of skewness ($\hat{\tau}_3$), while gray bars identify the test based on L-skewness ($\hat{\tau}_{3,L}$). Tests are applied on the residuals of an $AR(p)$ model where p is selected with the Akaike Information Criterion.

Table C3: Power: empirical rejection frequency of tests of symmetry for residuals – Sieve Bootstrap

		Skewness, $\hat{\tau}_3$				L-Skewness, $\hat{\tau}_{3,L}$			
T	ρ	A1	A2	A3	A4	A1	A2	A3	A4
40	0.0	0.3844	0.1231	0.6647	0.6496	0.5596	0.2302	0.9670	0.9700
	0.5	0.3934	0.1411	0.6567	0.6667	0.5586	0.2322	0.9590	0.9670
	0.9	0.4074	0.1411	0.6306	0.6486	0.5435	0.2292	0.9249	0.9399
160	0.0	0.9329	0.4815	0.8989	0.8599	0.9970	0.7467	1.0000	1.0000
	0.5	0.9349	0.4835	0.8999	0.8619	0.9980	0.7528	1.0000	1.0000
	0.9	0.9329	0.4975	0.8979	0.8559	0.9980	0.7487	1.0000	1.0000
480	0.0	0.9930	0.8048	0.9600	0.9279	1.0000	0.9990	1.0000	1.0000
	0.5	0.9960	0.8068	0.9640	0.9319	1.0000	0.9990	1.0000	1.0000
	0.9	0.9960	0.8088	0.9590	0.9279	1.0000	0.9980	1.0000	1.0000

Notes: T denotes the sample size, while ρ is the autoregressive parameter. N(0,1) denotes the standard Normal distribution, while S1-S4 are the distributions belonging to the Generalized Lambda Family shown in Table 1 of the paper. Higher rejection frequencies indicate higher power. Results based on 999 simulations and 199 bootstrap samples. Tests are applied on the residuals of an $AR(p)$ model where p is selected with the Akaike Information Criterion.

Figure A2: Real GDP (% Annualized growth rate) – boxplot



Notes: for each country the figure represents the sampling distribution of real GDP growth rate with a boxplot. The line in the middle of the box is the median, while the dot is the sample average. The size of the box is proportional to the interquartile range (IQR), namely the distance between the 75-th ($Q_{0.75}$) and 25-th percentile $Q_{0.25}$. The bottom (top) external line, known as whisker, is drawn in correspondence of $Q_{0.25} - IQR$ ($Q_{0.75} + IQR$). Numbers outside the two whiskers represent the number of observations lower (greater) than the bottom (top) whisker.

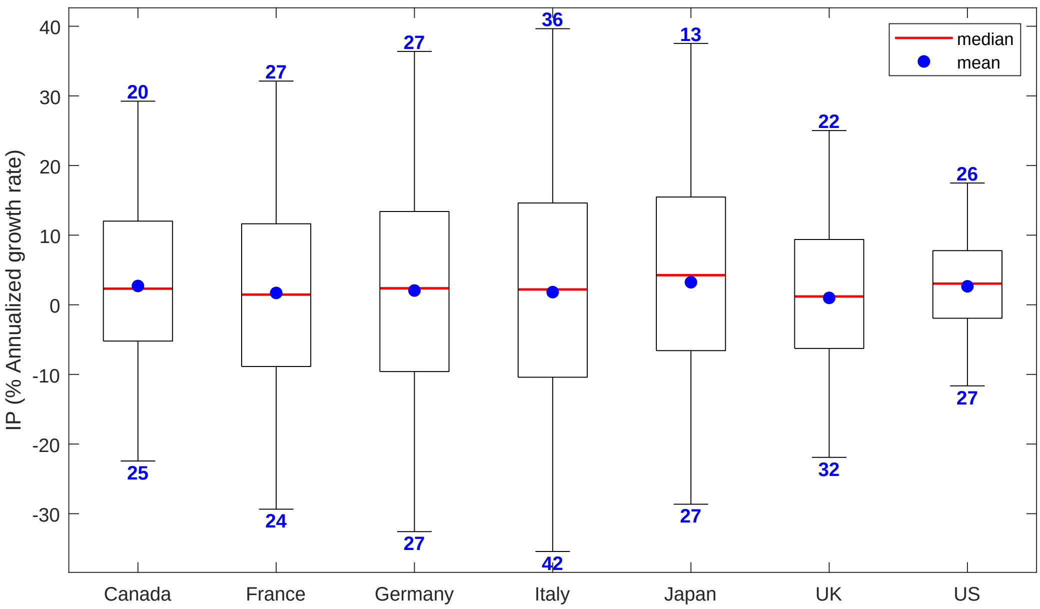
Table C4: Symmetry test - Industrial Production

	Skewness			L-Skewness		
Country	$\widehat{SK}$	$r=1$	$r=1,\ldots,4$	$\widehat{SK}_L$	$r=1$	$r=1,\ldots,4$
Canada	-0.2173	0.0921	0.0150	-0.0275	0.1371	0.0631
France	-3.7781	0.4665	1.0000	0.0028	0.9419	1.0000
Germany	-0.1220	0.6957	0.0000	-0.0150	0.5185	0.0601
Italy	-0.1400	0.7347	1.0000	-0.0045	0.8408	1.0000
Japan	-2.3414	0.0931	0.0000	-0.0910	0.0180	0.0541
UK	-0.2071	0.7207	1.0000	-0.0439	0.1812	0.5435
US	-0.9273	0.0120	0.0000	-0.0595	0.0280	0.0000

Notes: *Notes*: the table shows the coefficient of skewness ($\widehat{SK}$) and L-Skewness ($\widehat{SK}_L$) and the p-values of tests of symmetry. The null hypothesis of the test is that the distribution of $\Delta^r Y_t = \ln(Y_t/Y_{t-r})$ is symmetric, where Y_t is Industrial Production. We report the Bonferroni p-value for the joint null hypotheis that the distributions of $\Delta^r Y_t$ for $r = 1, 2, \ldots$ are symmetric.

Hosking, J. R. (1990). L-moments: analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society. Series B*, pages 105–124.

Figure A3: Industrial production (% Annualized growth rate) – boxplot



Notes: for each country the figure represents the sampling distribution of the growth rate of Industrial Production with a boxplot. The line in the middle of the box is the median, while the dot is the sample average. The size of the box is proportional to the interquartile range (IQR), namely the distance between the 75-th ($Q_{0.75}$) and 25-th percentile $Q_{0.25}$. The bottom (top) external line, known as whisker, is drawn in correspondence of $Q_{0.25} - IQR$ ($Q_{0.75} + IQR$). Numbers outside the two whiskers represent the number of observations lower (greater) than the bottom (top) whisker.