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## Cheap Talk with Multiple Experts and Uncertain Biases

Gùlen Karakoç

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Department of Economics, Management and Statistics
University of Milano – Bicocca
Piazza Ateneo Nuovo 1 – 2016 Milan, Italy
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# Cheap Talk with Multiple Experts and Uncertain Biases*

Gülen Karakoç†

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**Abstract**

A decision maker solicits information from two partially informed experts and then makes a choice under uncertainty. The experts can be either moderately or extremely biased relative to the decision maker, which is their private information. I investigate the incentives of the experts to share their private information with the decision maker and analyze the resulting effects on information transmission. I show that it may be optimal to consult a single expert rather than two experts if the decision maker is sufficiently concerned about taking advice from extremely biased experts. In contrast to what may be expected, this result suggests that getting a second opinion may not always be helpful for decision making.

JEL Classification: C72, D82, D83.

Keywords: Cheap Talk, Multiple Experts, Asymmetric Information.

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†*University of Naples Federico II*, Department of Economics. E-mail: gulenkarakoc@gmail.com.

# 1 Introduction

Conventional wisdom suggests that getting a second opinion is helpful for decision making and it is common in many real-life situations, for example: in healthcare markets, patients often seek a second opinion to find the right diagnosis; universities often ask more than one recommendation letter before making tenure decisions; and customers often talk to several salespeople to find the product that better fits their needs. These examples suggest that decision maker may wish to consult more than one expert to make sound decisions.

However, experts often have different preferences vis-à-vis the decision maker, and this makes communication difficult.[1] In particular, when the talk is cheap and hence, unverifiable, biased experts may have incentives to strategically alter their advice to push the decision makers in a certain direction, leading them to take a wrong decision. Moreover, as an outsider, the decision maker may not always know the actual preferences of the experts. This, in turn, makes it even harder for the decision maker to make inferences from the experts' opinions.[2] Hence, getting advice from multiple experts creates an opportunity to extract more information but, at the same time, creates a new challenge for the decision maker to resolve potentially conflicting opinions.

Many existing models explain why, and under which conditions, an uninformed decision maker benefits from consulting multiple experts before making a decision (See, e.g., Sobel, 2013, for a survey). However, most of these models assume that the experts' biases are known, whereas little is known about the communication when the bias of the expert is private information. Do experts have incentives to share their private information with the decision maker? What is the effect of this information asymmetry on the decision maker's behavior? And, is it better to consult two experts or just one?

I address these issues by analyzing a simple cheap talk model adapted from Austen-Smith (1993). I consider an environment in which an uninformed decision maker seeks advice from either one or two partially informed experts before taking a payoff relevant action. Each expert receives a private binary signal about the state of the world and then provides information to the decision maker through simultaneous cheap talk. The decision maker and the experts have different preferences (e.g., biases) over actions.[3] The decision maker's preferences are common knowledge across players, while each expert is privately informed about his bias. The experts may differ in terms of how biased they are relative to the decision maker, which is a natural assumption in many real-life situations. For instance, one could think of political leaders relying upon the recommendations of economic and military advisers before conducting a military intervention abroad: both types of advisers wish to influence the leader but possibly to differing degrees. To formalize this idea, I introduce two types of experts: an expert is either moderately biased (hereafter moderate expert), whose bias is

[1]For instance, a salesman may promote a specific product to get a higher commission or a financial advisor may earn extra compensation by pushing his/her clients to invest heavily in a particular product (see, for example, Wonsuk and Harbaugh 2018 for the former, and Piccolo *et* al. 2016 for the latter).

[2]There are ample reasons why this should be a real concern for decision making. For instance, the incentives of mortgage brokers to distort advice towards higher-commission (see, for example, Gambacorta *et* al. 2017), of money managers to push their clients to invest in more expensive products (see, for example, Piccolo *et al.* 2016), of academics to write inflated recommendation letters to help their students to gain admission to competitive graduate schools (see, for example, Rosovsky and Hartley 2002), and of doctors to recommend expensive treatments (see, for example, Evans 1974) depends on the experts' preferences.

[3]For ease of discussion, hereafter, I refer to the decision maker as "she," and each expert "he."

small; or extremely biased (hereafter extreme expert), whose bias is large in absolute terms. Because an expert's bias measures how distant his preferences are relative to those of the decision maker, a moderate expert is assumed to be less biased than an extreme expert.

Building on this insight, I focus on two informative equilibria in which the decision maker can learn some information from the experts' messages. As a benchmark, I consider a fully-revealing equilibrium in which experts of either type truthfully reveal their privately observed signals about the state of the world and the decision maker believes them. Then, I consider a semi-revealing equilibrium in which a moderate expert is willing to send informative messages to the decision maker depending on his privately observed signal, while an extreme expert reports the same message independent of his private information so that no information can be inferred from his message.

I first examine the effect of uncertain biases on the decision maker's action and the experts' truth-telling incentives. I show that in a fully-revealing equilibrium, the conditions for the existence of such equilibrium are not different from those that one would obtain if the biases were known. By contrast, in a semi-revealing equilibrium, the fact that the decision maker is uncertain about the experts' biases affects the incentives to disclose truthful information. In particular, the interval that supports truth-telling as equilibrium is small compared to that of the fully-revealing equilibrium. This happens because, other things being equal, in a semi-revealing equilibrium, the decision maker knows that with some probability that each expert reports a message that does not necessarily reflect the privately observed signal and the decision maker updates her beliefs accordingly. This provides an incentive to lie and, hence, makes the truth-telling condition tighter.

Next, I show that in both fully-revealing and semi-revealing equilibria information transmission is more difficult with two experts. When the expert is consulted alone, the decision maker's action is conditioned only on his message and, hence, restraining the expert's incentive to misreport his private signals due to the *overshooting effect* that was highlighted in Morgan and Stocken (2008). With multiple experts, instead, each expert knows that his report is less relevant in affecting the decision maker's final action. Consequently, the presence of another expert provides a strong incentive to lie via the overshooting effect compared to communication with only one expert.

After characterizing the conditions for the existence of fully-revealing and semi-revealing equilibria, I next develop a welfare analysis using the ex-ante expected utility of the decision maker as a welfare measure. Interestingly, I show that the fully-revealing equilibrium with one expert may be informationally superior to the semi-revealing equilibrium with two experts. Specifically, uncertainty over biases is detrimental to the decision maker because it allows experts to distort information relatively more often as compared to fully-revealing case, thereby reducing the information content of the messages. However, with two experts, the decision maker has a higher chance to get truthful information from one of the experts, which may provide more information than the one-expert communication does. The net effect on the decision maker's expected profit depends on the relative likelihood of messages being distorted by the extreme type. This happens because if the probability that the decision maker believes the expert to be moderate is sufficiently low, then consulting two experts with uncertain biases increases the likelihood of receiving distorted information from the extreme experts. In this case, the decision maker prefers to consult only one expert because even if he is an extreme expert, then he reveals the true signal in equilibrium via overshooting effect.

In sum, the analysis unveils a novel effect arising because of the presence of information asymmetries in a canonical multi-expert cheap talk framework. The results suggest that talking to multiple experts may not always be optimal for a decision maker who deals with privately informed biased experts. Therefore, when the decision-maker consults a single expert, asymmetric information tends to put additional pressure on him to communicate truthfully via the overshooting effect. This insight may help to explain why, in reality, some doctoral programmes have softened their requirement for "at least two letters of recommendation" policy when they make an application decision.[4] Although I develop the arguments in a decision maker – expert framework, the scope of the analysis is broader. The results can easily be adapted to many situations that involve simultaneous communication between an uninformed party and informed parties, such as management consulting, and medical, political, and financial advice.

The rest of the paper is organized as follows. After discussing the related literature, Section 2 describes the baseline model. Section 3.1 characterizes the conditions under which a fully-revealing equilibrium exists. In Section 3.2, I characterize the conditions under which a semi-revealing equilibrium exists. Welfare is discussed in Section 4. The last section concludes. All proofs are in the Appendix.

**Related Literature.** I build on and contribute to two strands of literature. First, this paper relates to the literature on cheap talk with multiple experts. Gilligan and Krehbiel (1989) first characterized the cheap talk model with two perfectly informed experts in a one-dimensional environment. Krishna and Morgan (2001) consider a cheap talk model with two perfectly informed experts to show that when the decision maker sequentially consults two experts who are biased in the same direction, then the most informative equilibrium is obtained by consulting the less biased expert alone. Gick (2006) studies a cheap talk model in which an uninformed decision maker seeks advice from two perfectly informed experts. He shows that having a second expert, even if he/she is more biased than the first one, improves the information structure when the communication is simultaneous.[5] The analysis in this paper is related to that in Austen-Smith (1993), who considers a uniform state space, and assumes that the experts are partially informed about the underlying state, as this paper does. However, I allow the decision maker to be uncertain about the experts' biases. Specifically, Austen-Smith (1993) shows that simultaneously consulting two experts leads to higher welfare than consulting only one expert, while in this paper I find that there are some circumstances under which two-expert communication is not necessarily superior to one-expert communication.

Second, this paper is related to cheap talk literature with uncertain individual preferences. There is a growing literature that considers experts' reputational/career concerns as a source of uncertainty. For instance, Sobel (1985), Bénabou and Laroque (1992), Morris (2001), Gentzkow and Shapiro (2006), Ottaviani and Sørensen (2006) consider uncertainty about

[4] Bocconi university, for instance, requires *up to two* reference letters for admission to their doctoral programmes.

[5] See also Li (2008) for a cheap talk model with multiple experts and sequential communication.

expert types and focus on the reputational incentives which this paper does not address. Few papers focus on the informativeness of the communication with uncertain biases. In particular, Morgan and Stocken (2003) and Dimitrakas and Sarafidis (2005) show that the revelation of the expert's bias weakens the communication when the magnitude of the bias is uncertain. Interestingly, Li (2004) and Li and Madarász (2008) characterize cheap talk equilibria with uncertain (and exogenous) biases in one expert mechanism.[6] Both these papers consider uniform state space and allow two values of the bias as this paper does. However, they posit that the expert can perfectly observe the state. Specifically, they show that the revelation of the bias always weakens the communication when there is uncertainty on the direction of the bias. In contrast, in this paper I show that transparency of biases enlarges the truth-telling interval and hence improves the incentives to communicate with the decision maker truthfully.

## 2 The Model

**Players and Environment.** Consider a decision maker (female), $D$, who seeks advice from two (male) experts, $A_1$ and $A_2$. The decision maker takes an action $y \in \mathbb{R}$ that affects the payoffs of all players. The state of the world, $\theta$, is a random variable and uniformly distributed on $[0, 1]$, with density $f(\theta) = 1$. The decision maker has no further information about $\theta$, while each expert privately observes an informative signal about the state. Specifically, each expert, say $A_i$, observes a binary signal $s_i \in \mathcal{S} \triangleq \{0, 1\}$ such that each signal is equally likely $\Pr[s_i] = \frac{1}{2}, s_i \in \mathcal{S}, i = 1, 2$.

Following Austen-Smith (1993), I assume that the signals are conditionally independent across experts given the underlying state $\theta$. Specifically, I assume that each signal $s_i$ has the following conditional probability

$$\Pr[s_i|\theta] = \theta^{s_i} (1-\theta)^{1-s_i}, \qquad s_i \in \mathcal{S}. \tag{1}$$

Conditional on the state $\theta$; therefore, the joint probability distribution of the signals is such that

$$\Pr[s_i, s_j|\theta] = \theta^{s_i+s_j} (1-\theta)^{2-s_i-s_j}, \qquad s_i, s_j \in \mathcal{S}. \tag{2}$$

Based upon the realized signal, each expert simultaneously reports a message to the decision maker. Let $m_i$ be $A_i$'s message, and, for simplicity, I consider a binary message space such that $m_i \in \mathcal{M} \triangleq \{0, 1\}$.[7]

Based upon the received messages, the decision maker takes an action $y(m_i, m_j)$ that affects the payoffs of all players.

All players have quadratic loss utility functions. Specifically, $D$'s utility is

$$\mathcal{U}_D(y, \theta, b_D) \triangleq -(y - \theta - b_D)^2,$$

[6] Hence, they do not provide welfare comparison between one and two-experts mechanisms.

[7] The use of binary messages is without loss of insight because the state is uniformly distributed on the unit interval and the signal space is assumed to be binary. Hence, the decision maker's uncertainty is just relative to these binary signals about the state so that a binary message space has enough elements to transmit any information available for the experts — see, e.g., Kawamura (2011).

and $A_i$'s utility is

$$\mathcal{U}_i(y,\theta,b_i) \triangleq -(y-\theta-b_i)^2, \quad i=1,2.$$

The quadratic loss utility function is commonly used in the cheap talk literature (e.g., Crawford and Sobel, 1982; Austen-Smith, 1993; Morgan and Stocken, 2008; among many others) because it allows us to obtain (tractable) closed form solutions. The quadratic loss utility function has an important implication because it guarantees the concavity of $D$'s objective function and hence, the uniqueness of the optimal action. Hence, given quadratic loss specification, in state $\theta$, the decision maker's most preferred action is $\theta + b_D$ and $A_i$'s most preferred action is $\theta + b_i$.

The parameter $b_D \geqslant 0$ represents the decision maker's bias and is common knowledge across players.[8] The parameter $b_i \in \mathcal{B} \triangleq \{b_M, b_E\}, i = 1,2$, instead, represents $A_i$'s bias and measures how distant his preferences are relative to those of the decision maker. Specifically, if $b_i = b_M$, then the bias is moderate and $A_i$ is said to be a moderate expert. Meanwhile, if $b_i = b_E$, then the bias is extreme and $A_i$ is said to be an extreme expert where a moderate expert is assumed to be less biased than an extreme expert, that is $|b_M - b_D| < |b_E - b_D|$.

More importantly, $A_i$'s bias is his private information and is drawn from the following distribution

$$\Pr[b_i = b_M] \triangleq \nu \triangleq 1 - \Pr[b_i = b_E], \qquad i = 1,2.$$

Hence, $A_i$ knows his own bias, while $D$ and $A_j$ have only a prior about that.[9] Finally, all players are expected utility maximizers.

**Timing.** The timing is as follows.

- Nature randomly chooses $\theta$ according to a uniform distribution on $[0,1]$.
- Nature independently chooses the types of the experts and privately informs them.
- Each expert privately observes $s_i$.
- Each expert simultaneously sends $m_i$ to the decision maker.
- Based upon the received messages, $D$ takes an action $y \in \mathbb{R}$.

**Equilibrium.** The solution concept is Perfect Bayesian Equilibrium (PBE). For simplicity, I consider only pure strategies for the experts (see, e.g., Austen-Smith, 1993; Li, 2004 among many others).[10]

As is common in cheap talk models, multiple equilibria exist. In particular, a babbling equilibrium always exists, in which the messages do not depend on the experts' private information about the underlying state. Indeed, given such a strategy, it is optimal for the decision maker to ignore the messages but then babbling is actually the best response for the experts. However, I focus on two informative equilibria: (i) fully-revealing equilibrium

[8] The assumption is used to formalize the idea that, in reality, the Internet search is widely used to collect some information before talking to the experts.

[9] For a similar approach, see also Morris (2001), Morgan and Stocken (2003) and Li (2004).

[10] Notice that all messages fall on the equilibrium path. Hence, no off-equilibrium path beliefs are required.

in which experts of either type truthfully report their signals about the underlying state and the decision maker believes them; (ii) semi-revealing equilibrium in which a moderate expert truthfully reports his private signal while an extreme expert reports the same message regardless of his private information about the state.

Without loss of generality, in the analysis that follows, I assume that the extreme expert is rightward biased; that is $b_D < b_E$. Assuming that a rightward biased extreme expert is with no loss of generality because experts' payoffs are symmetric and the message space is binary.[11] As it will be clear shortly, in a semi-revealing equilibrium, a rightward biased extreme expert always reports, with a slight abuse of notation, $m_E = 1$ independent of his signal; that is such that he wants as high action as possible relative to the decision maker. When he observes a signal equal to one, he wants to report $m_E = 1$ instead of zero because, by doing so, he can shift the decision maker's action rightward. Moreover, I do not impose any restrictions on the direction of moderate bias because a moderate expert, in equilibrium, is willing to send both messages (both 0 and 1) depending on his privately observed signal. Therefore, he wants as high (resp. low) action as possible if $b_D < b_M$ (resp. $b_D > b_M$).

# 3 Equilibrium Analysis

I now characterize the decision maker's optimal action after receiving any messages. I will then analyze experts' incentives to communicate in fully-revealing and semi-revealing equilibria with one and two experts.[12]

## 3.1 Fully-Revealing Equilibrium

To gain intuition about the central result of the paper, I first analyze a simple case in which the experts simultaneously and truthfully report their private signals — i.e., such that $m_i = s_i$ and $m_j = s_j$ in equilibrium — and the decision maker believes them. Because the experts' messages reflect the true realizations of the signals, $D$'s best response to such a strategy is

$$\begin{aligned} y^F(s_i, s_j) &= \arg\max_{y \in \mathbb{R}} \int_\theta -(y - \theta - b_D)^2 f(\theta | s_i, s_j)\, d\theta, \\ &= b_D + \mathrm{E}[\theta | s_i, s_j] \qquad \forall (s_i, s_j) \in \mathcal{S}^2, \end{aligned} \tag{3}$$

where, abusing slightly notation, I define $y^F_{s_i, s_j} \triangleq y^F(s_i, s_j)$ and the superscript $F$ denotes the optimal action taken by the decision maker after being truthfully informed about the signals. The expression in (3) reflects that when $D$ receives truthful messages from the experts, her optimal action is just the conditional expectation of the state shifted by her bias $b_D$.

The following lemma characterizes the decision maker's optimal action after being truthfully informed by one or two experts.

[11] Hence, the equilibrium in which the extreme expert is leftward biased expert is just the mirror image of the equilibrium with the rightward biased extreme expert.

[12] It is worth pointing out that the model with one expert is similar to the model with two experts. A detailed equilibrium analysis with one expert can be found in the Appendix.

**Lemma 1** *In a fully-revealing equilibrium, when D consults only one expert, her optimal actions are*

$$y_0^F = b_D + \frac{1}{3}, \qquad y_1^F = b_D + \frac{2}{3},$$

*while when D simultaneously consults two experts, her optimal actions are*

$$y_{0,0}^F = b_D + \frac{1}{4}, \qquad y_{0,1}^F = y_{1,0}^F = b_D + \frac{1}{2}, \qquad y_{1,1}^F = b_D + \frac{3}{4}.$$

Hence, in a fully-revealing equilibrium (both with one and two experts), uncertainty about the expert's types has no consequence on the optimal actions because $D$ believes that experts truthfully report their private signals regardless of their type. Moreover, the optimal actions are such that $y_{0,0}^F < y_{0,1}^F < y_{1,1}^F$. The reason is simple: when $D$ receives two different signals, she takes action based on her prior beliefs about the state. Instead, when $D$ receives two identical signals from the experts, she has a more precise idea regarding the state because both experts report their signals truthfully. Consequently, this shifts the decision maker's action rightward when she receives $(s_i, s_j) = (1,1)$, and shifts it leftward when she receives $(s_i, s_j) = (0,0)$ from the experts. A similar logic applies when $D$ consults one expert.

Consider now the experts' incentives to reveal the observed signals. Without loss of generality, I focus on the truth-telling incentives of $A_i$ because the experts are ex-ante symmetric. Notice that from $A_i$'s perspective $A_j$ truthfully reports his signal; that is, in equilibrium $m_j = s_j$. Hence, there exists a fully-revealing equilibrium if there is an incentive for $A_i$ to reveal $m_i = s_i$ instead of sending false message $1 - s_i$ along the equilibrium path. This condition is

$$\sum_{s_j \in \mathcal{S}} \int_\theta - \left( y_{s_i,s_j}^F - \theta - b_i \right)^2 f\left(s_j, \theta | s_i\right) d\theta \geqslant \\ \geqslant \sum_{s_j \in \mathcal{S}} \int_\theta - \left( y_{1-s_i,s_j}^F - \theta - b_i \right)^2 f\left(s_j, \theta | s_i\right) d\theta, \qquad b_i \in B. \tag{4}$$

Let

$$\Delta y^F\left(s_i, s_j\right) \triangleq y_{1-s_i,s_j}^F - y_{s_i,s_j}^F,$$

be the difference between $D$'s action after receiving false and correct signal from $A_i$ given that $A_j$ reports his signal truthfully in equilibrium. Taking into account $D$'s optimal action after hearing the truthful messages (3), by integrating and rearranging terms the above constraint simplifies to

$$\left(b_i - b_D\right) \sum_{s_j \in \mathcal{S}} \Pr\left[s_j | s_i\right] \underbrace{\Delta y^F\left(s_i, s_j\right)}_{\text{Overshooting Effect}} \leqslant \sum_{s_j \in \mathcal{S}} \Pr\left[s_j | s_i\right] \frac{\Delta y^F\left(s_i, s_j\right)^2}{2}. \tag{5}$$

Condition (5) reflects that $A_i$'s incentive to report his private signal is shaped by $D$'s reaction to receiving false information from $A_i$ — i.e., the *overshooting effect* (highlighted in Morgan and Stocken, 2008): a deviation from a truthful message may shift the decision maker's action too far from the expert's ideal action. More specifically, an expert with rightward bias (resp. leftward bias) may prefer a higher (resp. lower) action than the decision maker, although the displacement in decision maker's action caused by an undetectable lie may be too large relative to the case of truth-telling, which is not desirable for either the expert or the decision maker. As I shall explain later on, this makes truth-telling an optimal strategy for an expert who has preferences close to those of the decision maker. Other things being equal, the sign of the overshooting effect depends on $A_i$'s privately observed signal. More precisely, if $\Delta y^F(s_i, s_j) > 0$, then the overshooting effect is positive and an expert with leftward bias $b_i < b_D$ has no incentive to misreport because sending false message shifts the decision maker's optimal action rightward. Similarly, if $\Delta y^F(s_i, s_j) < 0$, then the overshooting effect is negative and an expert with rightward bias has no incentive lie because, in this case, reporting a false signal to the decision maker cannot be incentive compatible.

The following proposition characterizes a fully-revealing equilibrium with one and two experts.

**Proposition 2** *There exists a fully-revealing equilibrium with the following properties:*

*(i) When $D$ consults only one expert and the expert of either type truthfully reports his signal if and only if*

$$|b_1 - b_D| \leqslant \frac{1}{6}, \qquad b_1 \in \mathcal{B}.$$

*(ii) When $D$ simultaneously consults two experts and the experts of either type truthfully report their signals if and only if*

$$|b_i - b_D| \leqslant \frac{1}{8}, \qquad b_i \in \mathcal{B}, i = 1, 2.$$

There are two key aspects to note about Proposition 2. First, the maximal distance in preferences (both with one expert and two experts) compatible with full information revelation does not depend on the parameter $\nu$ because $D$ believes that an expert of either type truthfully reports his private signal. Accordingly, the impact of each message on $D$'s optimal action is very high. This, in turn, makes truth-telling an optimal strategy for an expert who has preferences close to the those of the decision maker because he cannot do better than report his true signal due to the *overshooting effect*. Because the information asymmetry has no impact on the equilibrium, the conditions for its existence are not different from those that would be obtained if the biases were known.

Second, when $D$ consults one expert, the magnitude of the overshooting effect is $\left|\Delta y^F(s_1)\right| \triangleq \frac{1}{3}$, while when $D$ consults two experts, it is $\left|\Delta y^F(s_i, s_j)\right| \triangleq \frac{1}{4}$. This suggests that, when $D$ consults one expert, the displacement in decision maker's action caused by an undetectable lie is large compared to the case with two experts. This, in turn, increases $A_i$'s incentives to misreport. Hence, $A_i$'s preferences should be even closer to those of the decision maker's (as compared to one expert) to reveal his private information. As a result, having multiple experts makes the truth-telling conditions tighter relative to the case where $D$ consults only one expert.

## 3.2 Semi-Revealing Equilibrium

Consider now a semi-revealing equilibrium, in which the moderate expert truthfully reports his signal, while the extreme expert (rightward biased) reports $m_E = 1$ independent of his private signal. The structure of $D$'s maximization problem is similar to that solved in a fully-revealing equilibrium with the difference that she must now form beliefs about the signals $(s_i, s_j)$ given the message pair $(m_i, m_j)$ because the messages may not necessarily reflect the privately observed signals. The Bayes rule then implies the following posterior:

$$\Pr[s_i, s_j|m_i, m_j] \triangleq \frac{\Pr[m_i, m_j|s_i, s_j]\Pr[s_i, s_j]}{\sum_{(s_i,s_j)\in\mathcal{S}^2}\Pr[m_i, m_j|s_i, s_j]\Pr[s_i, s_j]},$$

where $\Pr[s_i, s_j]$ is the joint probability of the signals.

To understand the updating process, notice that when $D$ receives $(m_i, m_j) = (0, 0)$ from the experts, then she will be sure that these messages come from two moderate experts who tell the truth. As a result, the messages convey full information about the signals; that is,

$$\Pr[s_i = 0, s_j = 0|0, 0] = 1 \quad \text{and} \quad \Pr[s_i = 1, s_j = 1|0, 0] = 0. \tag{6}$$

When the decision maker receives $(m_i, m_j) = (1, 1)$ from the experts instead, then she is uncertain about the types/biases of the experts. Consequently, she must update beliefs discounting the possibility of receiving uninformative messages. In this case, by Bayes' rule $D$'s posterior beliefs are

$$\Pr[s_i = 0, s_j = 0|1, 1] = \frac{(1-\nu)^2}{\nu^2 - 3\nu + 3}, \qquad \Pr[s_i = 1, s_j = 1|1, 1] = \frac{1}{\nu^2 - 3\nu + 3}. \tag{7}$$

Notice that

$$\frac{d\Pr[s_i = 0, s_j = 0|1, 1]}{d\nu} < 0, \qquad \text{and} \qquad \frac{d\Pr[s_i = 1, s_j = 1|1, 1]}{d\nu} > 0.$$

Hence, when $D$ receives $(m_i, m_j) = (1, 1)$, an increase of the probability of being moderate makes her more confident that the signals are $(s_i, s_j) = (1, 1)$, and *vice versa.* A similar reasoning applies (See the Appendix) for the mixed messages and signals.

Hence, $D$'s problem is

$$y^S(m_i, m_j) = \arg\max_{y\in\mathbb{R}} \int_\theta -(y - \theta - b_D)^2 f(\theta|m_i, m_j)\, d\theta,$$

whose solution yields,

$$y^S(m_i, m_j) = b_D + \underbrace{\sum_{(s_i,s_j)\in\mathcal{S}^2}\Pr[s_i, s_j|m_i, m_j]\,\mathrm{E}[\theta|s_i, s_j]}_{\triangleq \mathrm{E}_\nu[\theta|m_i,m_j]}, \tag{8}$$

where, slightly abusing the notation, I define $y^S_{m_i,m_j} \triangleq y^S(m_i, m_j)$ and the superscript $S$ denotes the optimal actions taken by the decision maker in a semi-revealing equilibrium.

The following lemma describes the decision maker's optimal actions in a semi-revealing equilibrium.

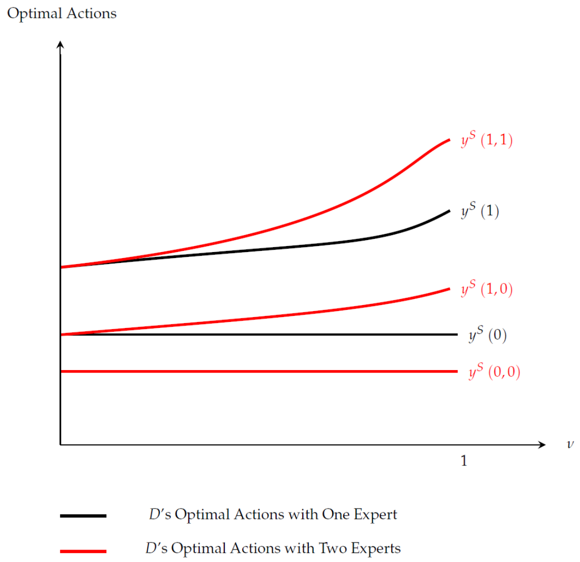


Figure 1: $D$'s optimal actions in a semi-revealing equilibrium.

**Lemma 3** *In a semi-revealing equilibrium, when $D$ consults only one expert, her optimal actions are*

$$y_0^S = b_D + \frac{1}{3}, \qquad y_1^S = b_D + \frac{3-\nu}{3\,(2-\nu)},$$

*while when $D$ consults two experts, her optimal actions are*

$$y_{0,0}^S = b_D + \frac{1}{4}, \qquad y_{0,1}^S = y_{1,0}^S = b_D + \frac{2-\nu}{2\,(3-2\nu)}, \qquad y_{1,1}^S = b_D + \frac{\nu^2-4\nu+6}{4\,(\nu^2-3\nu+3)}.$$

The decision maker's optimal actions in a semi-revealing equilibrium are depicted in Figure 1. Hence, even with uncertain biases, the optimal actions are such that $y_{0,0}^S < y_{0,1}^S < y_{1,1}^S$. Clearly, when $D$ receives $(m_i, m_j) = (0,0)$ from the experts, then she will be sure that the messages are sent by two moderate experts who tell the truth because the extreme expert is rightward biased and has no incentive to report zero. In this case, the decision maker's optimal action in a semi-revealing equilibrium coincides with her optimal action in a fully-revealing equilibrium — i.e., $y_{0,0}^S = y_{0,0}^F$ — as expected. By contrast, when $D$ receives any other messages that contain at least one message equal to one, then she discounts the possibility of receiving false information and hence the experts' messages have a lower impact on the action taken by the decision maker. This implies that $D$'s optimal action in a semi-revealing equilibrium is lower than the one in a fully-revealing equilibrium — i.e., $y_{1,0}^F > y_{1,0}^S$ and $y_{1,1}^F > y_{1,1}^S$ for all $\nu \in (0,1)$. Notice also that the higher are the chances of being moderate, the more 'accurate' the inference that $D$ can make on the messages given the signals. Hence, the optimal actions converge to those found in a fully-revealing equilibrium as $\nu$ tends to one.

A similar reasoning applies when $D$ consults a single expert. More precisely, when $D$ receives $m_1 = 0$ from the expert, then she will be sure that the expert is moderate and is reporting truthfully. Hence, the decision maker will assign probability one to $b_1 = b_M$. In this case, not surprisingly, $D$'s optimal action in a semi-revealing equilibrium with one expert coincides with her optimal action in a fully-revealing equilibrium — i.e., $y_0^F = y_0^S$. By contrast, when $D$ receives message $m_1 = 1$, she discounts the possibility that the expert is extreme (in which case the message reveals no information), and hence, the expert's message has a lower impact on the final decision than the one in a fully-revealing equilibrium. An important point here is to note that the optimal actions are $y_1^S < y_{1,1}^S$ and $y_{1,0}^S < y_1^S$ for all $\nu \in (0,1)$. The first inequality follows from the fact that when $D$ consults two experts, then she has a higher chance to get truthful information from one of the experts. The second inequality follows from observing that, when $D$ receives any messages that contain at least one message equal to zero, she can infer with certainty that a moderate expert sends the message. Hence, the decision maker's optimal action is lower when she receives $(m_i, m_j) = (1,0)$ than receiving only one message $m_1 = 1$.

Consider now the experts' incentives to reveal their private signals. As before, I focus on the truth-telling incentives of $A_i$ since experts are ex-ante symmetric. Suppose that $A_i$ is moderate; that is, such that $b_i = b_M$. Given that $A_j$'s bias is his private information, from $A_i$'s perspective $A_j$ is either moderate with probability $\nu$ or extreme with probability $1-\nu$. Hence, $A_i$'s expected utility when reporting $m_i = s_i$ is higher than his expected utility when reporting a false message $m_i = 1 - s_i$ if

$$\sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \mathcal{S}} \int_\theta -\left(y_{m_i,m_j}^S - \theta - b_M\right)^2 f(s_j, \theta | s_i)\, d\theta \geqslant$$
$$\geqslant \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \mathcal{S}} \int_\theta -\left(y_{1-m_i,m_j}^S - \theta - b_M\right)^2 f(s_j, \theta | s_i)\, d\theta. \tag{9}$$

For any $m_j$, let

$$\Delta y^S(m_i, m_j) \triangleq y_{1-m_i,m_j}^S - y_{m_i,m_j}^S,$$

be the difference between $D$'s action after receiving false and correct messages from $A_i$. Taking into account $D$'s optimal action after hearing the signals (8), integrating and rearranging terms, (9) simplifies to

$$(b_M - b_D) \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \mathcal{S}} \Pr(s_j | s_i) \underbrace{\Delta y^S(m_i, m_j)}_{\text{Overshooting Effect}} \leqslant$$
$$\leqslant \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \mathcal{S}} \Pr(s_j | s_i) \left\{ \frac{\Delta y^S(m_i, m_j)^2}{2} + \Delta y^S(m_i, m_j) \underbrace{\left(\mathrm{E}_\nu[\theta | m_i, m_j] - \mathrm{E}[\theta | s_i, s_j]\right)}_{\text{goes to 0 as } \nu \longrightarrow 1} \right\}. \tag{10}$$

In contrast to the case of fully-revealing, the overshooting effect now depends on the parameter $\nu$. Because $A_j$'s bias is his private information and the other players have only a prior about that, $A_j$'s report plays an important role on $A_i$'s incentive to truthfully report his private information. Specifically, the overshooting effect is stronger when $A_j$ reports $m_j = 1$ than when he reports $m_j = 0$ — i.e., $\left|\Delta y^S(m_i, 0)\right| < \left|\Delta y^S(m_i, 1)\right|$.[13] The reason is that when $D$ receives $m_j = 1$, she anticipates the risk that it is an uninformative message, and

[13] In fact, for $\nu \in (0,1)$, one gets

$$\frac{1}{4(3-2\nu)} \triangleq \left|\Delta y^S(m_i, 0)\right| < \left|\Delta y^S(m_i, 1)\right| \triangleq \frac{\nu^2 - 6\nu + 6}{4(3-2\nu)(\nu^2 - 3\nu + 3)}.$$

discounts accordingly $A_j$'s message. This, in turn, puts more weight on $A_i$'s message so that a lie from $A_i$ has a stronger impact on the decision maker's action.

To complete the characterization of the semi-revealing equilibrium, suppose that $A_i$ is extreme — i.e., such that $b_i = b_E$. I then check that $A_i$ has no incentive to report $m_i = 0$ when his private signal is $s_i = 0$. Hence, $A_i$'s expected utility from reporting $m_i = 1$ is higher than his utility from reporting truthfully $m_i = 0$ if

$$\sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \int_\theta -\left(y^S_{0,m_j} - \theta - b_E\right)^2 f(s_j, \theta | s_i = 0)\, d\theta <$$
$$< \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \int_\theta -\left(y^S_{1,m_j} - \theta - b_E\right)^2 f(s_j, \theta | s_i = 0)\, d\theta, \qquad (11)$$

which, by integrating and rearranging terms, simplifies to

$$(b_E - b_D) \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0)\, \Delta y^S(0, m_j) >$$
$$> \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0) \left\{ \frac{\Delta y^S(1, m_j)^2}{2} + \Delta y^S(1, m_j)\left(\mathrm{E}_\nu[\theta|0, m_j] - \mathrm{E}[\theta|0, s_j]\right) \right\}. \qquad (12)$$

Clearly, when $s_i = 1$, a rightward biased extreme expert has an incentive to report $m_E = 1$. The following proposition characterizes a semi-revealing equilibrium with one and two experts.

**Proposition 4** *There exists a semi-revealing equilibrium with the following properties:*

*(i) When $D$ consults only one expert, there exist two thresholds $\alpha_1(\nu)$ and $\beta_1(\nu)$, with $0 < \alpha_1(\nu) < \beta_1(\nu)$, such that the moderate expert truthfully reports his signal, while the extreme expert reports $m_E = 1$ if and only if*

$$-\beta_1(\nu) \leqslant b_M - b_D \leqslant \alpha_1(\nu) \qquad \text{and} \qquad b_E - b_D > \alpha_1(\nu).$$

*(ii) When $D$ simultaneously consults two experts, there exist two thresholds $\alpha_2(\nu)$ and $\beta_2(\nu)$, with $\alpha_2(\nu) < \alpha_1(\nu)$ and $\beta_2(\nu) < \beta_1(\nu)$, such that the moderate expert truthfully reports his signal, while the extreme expert reports $m_E = 1$ if and only if*

$$-\beta_2(\nu) \leqslant b_M - b_D \leqslant \alpha_2(\nu) \qquad \text{and} \qquad b_E - b_D > \alpha_2(\nu).$$

*Moreover, $\alpha_1(\nu)$ and $\alpha_2(\nu)$ are increasing in $\nu$, while $\beta_1(\nu)$ and $\beta_2(\nu)$ are decreasing in $\nu$.*

Figure 2 graphically summarizes the results of Proposition 4, for which semi-revealing equilibrium with one and two experts exist.[14] Uncertainty about the experts' biases has two effects on information transmission. First, the interval that supports truth-telling as an equilibrium shrinks as the probability of being moderate tends to zero. The intuition is straightforward: in a semi-revealing equilibrium, $D$ knows that with some probability, each

[14] For the sake of clarity, hereafter, I restrict attention to the case where both types of experts are biased in the same direction relative to the decision maker — i.e., $b_D < b_M < b_E$.

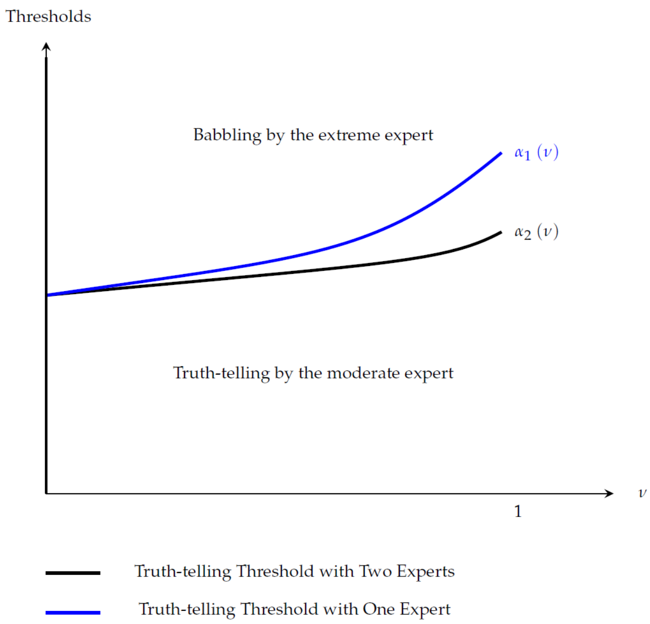


Figure 2: Truth-telling thresholds for semi revealing equilibrium.

expert reports a message which does not necessarily reflect the privately observed signal. Hence, $D$ updates her beliefs discounting the possibility of receiving uninformative messages. This implies that each message has a lower impact on $D$'s action in equilibrium. This, in turn, makes the incentives to lie stronger when an expert observes a signal that would shift the decision maker's action in an undesired direction if reported truthfully. Moreover, observe that the thresholds $\alpha_2(\nu)$ is increasing in $\nu$ and $\beta_2(\nu)$ is decreasing in $\nu$. That is, when the probability of being moderate increases, the truth-telling interval in a semi-revealing equilibrium enlarges, and it eventually coincides with the truth-telling interval in a fully-revealing equilibrium.[15]

Second, the conditions for truth-telling are tighter when the decision maker consults two experts rather than just one. To understand why, recall that an expert, say $A_i$, has only a prior about the type of the other expert. When $A_i$ is consulted alone, $D$'s optimal action is conditioned only on his report and this makes him relatively sure of the consequence of the message that he sends to $D$. In contrast, when there are two experts, $A_i$ is unsure about the weight of his message because $D$'s optimal action depends on $A_j$'s report too. As a result, the presence of another expert with an unknown bias makes incentives to lie stronger relative to the communication with one expert.

## 4 Welfare Analysis

To study the welfare effects, I now compare the decision maker's ex-ante expected utility among the types of equilibria defined in Proposition 2 and 4. To begin with, I compare $D$'s ex-ante expected utility with one and two experts within each equilibria.

[15] In the Appendix I derive a closed form solution for the thresholds $\alpha_i(\nu)$ and $\beta_i(\nu)$, $i = 1, 2$, as a function of the bias parameter $\nu$. The model is based on the quadratic-uniform setting, and this permits to obtain closed form solutions for the threshold equilibria. Hence, closed form solutions deliver additional comparative statics to those mentioned above.

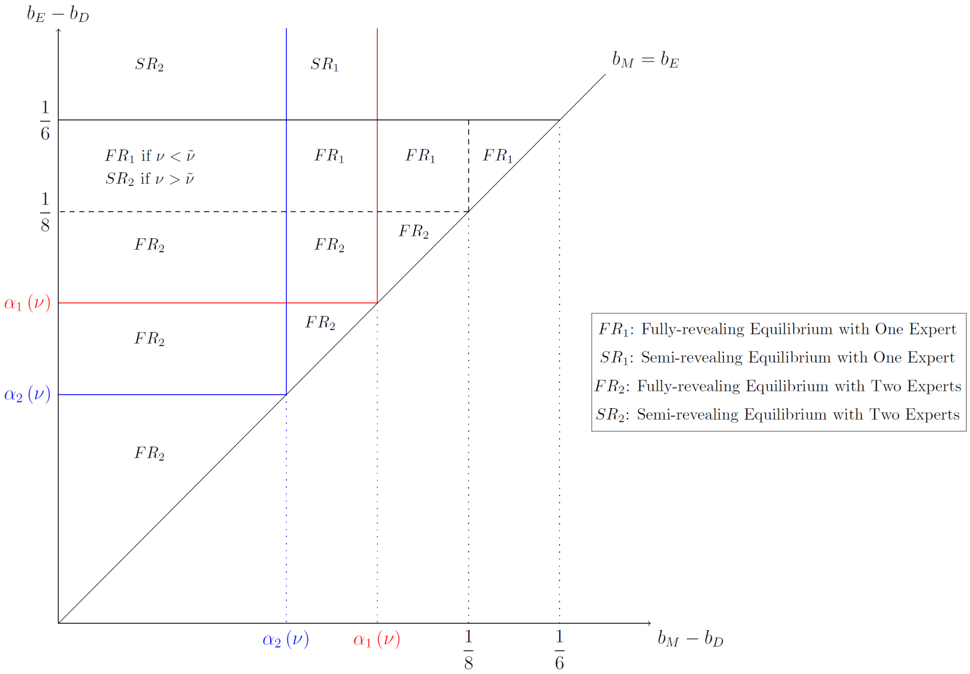


Figure 3: Welfare Maximizing Equilibria.

**Proposition 5** *In both fully-revealing and semi-revealing equilibria, consulting two experts is informationally superior to consulting just one.*

Not surprisingly, in a fully-revealing equilibrium, two experts provide more information to the decision maker than a single expert. In other words, although two-expert communication reduces the size of the interval, which supports truth-telling as an equilibrium, it induces $D$ to take action as a combination of two truthful messages. By doing so, $D$ can have a more precise idea about the underlying state, allowing her to take more precise action. Therefore, the fully-revealing equilibrium with two experts is informationally superior to the fully-revealing equilibrium with one expert.

The same conclusion holds even when the experts report noisy information to the decision maker. Although the magnitude of the overshooting effect is attenuated in a semi-revealing equilibrium relative to a fully-revealing equilibrium, the improvement in information transmission in two-expert communication happens because $D$ has a higher chance to get truthful information from one of the experts.

Figure 3 plots the welfare maximizing equilibrium within each interval defined in Propositions 2 and 4. For the sake of clarity, hereafter I focus on the situation where both types of experts are biased in the same direction relative to the decision maker — i.e., $b_D < b_M < b_E$.

By comparing $D$'s expected utility in a fully-revealing equilibrium with one expert and a semi-revealing equilibrium with two experts, I can establish the following result:

**Proposition 6** *There exists a threshold $\widetilde{\nu}$ such that fully-revealing with one expert is informationally superior to semi-revealing equilibrium with two experts if $\nu \leqslant \widetilde{\nu}$.*

Surprisingly, when the probability of being moderate is sufficiently low, the decision maker prefers to consult a single expert. To understand why, consider first the region of parameters where the two equilibria exist — i.e., a fully-revealing revealing equilibrium with one expert and a semi-revealing equilibrium with two experts (See Figure 3). It is easy to see that these two equilibria obtain when

- A moderate type has preferences close enough to those of the decision maker to induce him to report his signal truthfully regardless of the strategy of the other expert — i.e., $b_M - b_D < \alpha_2(\nu)$, and
- An extreme expert has distant enough preferences that induce him not to report truthfully if the other expert does so *but* close enough that, if consulted alone, he would report his signal truthfully — i.e., such that $\frac{1}{8} < b_E - b_D < \frac{1}{6}$.

Now, in the region of parameters mentioned above, if $\nu$ is low, the truth-telling interval in a semi-revealing equilibrium is small too because the threshold $\alpha_2(\nu)$ is increasing in $\nu$. Hence, for low values of $\nu$, the moderate expert has lower incentives to report his signal truthfully, and, hence, the information content of his message decreases. In this case, consulting two experts with uncertain biases increases the likelihood of receiving false information from the experts, which, in turn, lowers the ex-ante expected profit of the decision maker. Therefore, when $\nu$ is sufficiently low, the decision maker prefers to consult a single expert who reports truthfully his signal. In contrast, when $\nu > \widetilde{\nu}$, the decision maker prefers to consult two experts with uncertain biases rather than a single expert because the experts distort information less when $\nu$ is high. In this case, $D$ has a higher chance to get truthful information from the experts, who provide more information than a single expert.

Taken together, this result suggests that it may be optimal to consult a single expert rather than two experts whenever the biases of the experts are not too similar, the extreme expert is not too extreme, and the probability of getting undistorted information is sufficiently low.

# 5 Conclusion

It is commonly believed that seeking advice from multiple sources improves the information transmission between the uninformed party and the informed parties. This presumption may be incorrect, especially when there is uncertainty about the experts' biases. Interestingly, contrary to what conventional wisdom suggests, I have shown that the decision maker may prefer to consult a single expert rather than two experts when the decision maker is sufficiently concerned about taking advice from extreme experts because, in this case, the extreme expert does not communicate truthfully with the decision maker in the presence of another expert, while he communicates truthfully if he is consulted alone. This suggests that even though the decision maker is uncertain about the experts' biases, consulting a single expert can be used as a tool to prevent opportunistic behavior by the experts in the first place. Hence, talking to multiple experts to elicit information from them about the true state is not always ex-ante efficient.

# 6 Appendix

**Proof of Lemma 1.** *(i) One Expert.* Suppose that $D$ consults one expert who truthfully reports his signal. Since the utility function is concave in $y$, the (expected) utility maximizing action of the decision maker after receiving $m_1 = s_1$ can be defined as follows

$$y_{s_1}^F = \arg\max_{y\in\mathbb{R}} \int_\theta -(y-\theta-b_D)^2 f(\theta|s_1)\,d\theta,$$
$$= b_D + \underbrace{\int_\theta \theta f(\theta|s_1)\,d\theta}_{\triangleq \mathrm{E}[\theta|s_1]}, \qquad \forall s_1 \in \mathcal{S}, \tag{A1}$$

where the conditional density of $\theta$ given the signal $s_1$ is

$$f(\theta|s_1) = \frac{\Pr[s_1|\theta]\,f(\theta)}{\int_\theta \Pr[s_1|\theta]\,f(\theta)\,d\theta}.$$

Using the conditional probability distribution of the signal from (1), I obtain

$$f(\theta|s_1=0) = 2(1-\theta), \qquad f(\theta|s_1=1) = 2\theta. \tag{A2}$$

Substituting (A2) into (A1), it is immediate to verify that

$$y_0^F = b_D + \underbrace{\frac{1}{3}}_{\mathrm{E}[\theta|s_1=0]}, \qquad y_1^F = b_D + \underbrace{\frac{2}{3}}_{\mathrm{E}[\theta|s_1=0]}, \tag{A3}$$

as claimed.

*(ii) Two Experts.* From (3) I know that $D$'s optimal action after receiving $m_i = s_i$ and $m_j = s_j$ is

$$y_{s_i,s_j}^F = b_D + \underbrace{\int_\theta \theta f(\theta|s_i,s_j)\,d\theta}_{\triangleq \mathrm{E}[\theta|s_i,s_j]}, \tag{A4}$$

where the conditional density of $\theta$ given the signals $s_i$ and $s_j$ is

$$f(\theta|s_i,s_j) = \frac{\Pr[s_i,s_j|\theta]\,f(\theta)}{\int_\theta \Pr[s_i,s_j|\theta]\,f(\theta)\,d\theta},$$

Using the conditional probability distribution of the signals from (2), I obtain

$$f(\theta|s_i=0,s_j=0) = 3(1-\theta)^2, \qquad f(\theta|s_i=1,s_j=1) = 3\theta^2, \tag{A5}$$
$$f(\theta|s_i=0,s_j=1) = f(\theta|s_i=1,s_j=0) = 6\theta(1-\theta). \tag{A6}$$

Substituting (A5) and (A6) into (A4) yields the decision maker's optimal actions

$$y_{0,0}^F = b_D + \underbrace{\frac{1}{4}}_{\mathrm{E}[\theta|0,0]}, \qquad y_{0,1}^F = y_{1,0}^F = b_D + \underbrace{\frac{1}{2}}_{\mathrm{E}[\theta|0,1]}, \qquad y_{1,1}^F = b_D + \underbrace{\frac{3}{4}}_{\mathrm{E}[\theta|1,1]}, \tag{A7}$$

as claimed.■

**Proof of Proposition 2.** *(i) One Expert.* Consider $A_1$'s incentive to report truthfully his private signal. $A_1$'s expected utility from reporting $m_1 = s_1$ is higher than his expected utility from reporting $m_1 = 1 - s_1$ if and only if

$$\int_\theta -\left(y_{s_1}^F - \theta - b_1\right)^2 f(s_1|\theta)\,d\theta \geqslant \int_\theta -\left(y_{1-s_1}^F - \theta - b_1\right)^2 f(s_1|\theta)\,d\theta, \qquad \forall s_1 \in \mathcal{S}, b_1 \in \mathcal{B},$$

which substituting $f(s_1|\theta) = f(\theta|s_1)\Pr[s_1]$ by Bayes' rule and integrating yields

$$-\left(y_{s_1}^F - \mathrm{E}[\theta|s_1] - b_1\right)^2 \Pr[s_1] \geqslant -\left(y_{1-s_1}^F - \mathrm{E}[\theta|s_1] - b_1\right)^2 \Pr[s_1].$$

Using $D$'s best response from (A1) and rearranging terms, I obtain

$$(b_D - b_1)^2 \Pr[s_1] \leqslant (b_D + \underbrace{\mathrm{E}[\theta|1-s_1] - \mathrm{E}[\theta|s_1]}_{\triangleq \Delta y^F(s_1)} - b_1)^2 \Pr[s_1], \tag{A8}$$

Expanding squares and rearranging terms, (A8) further simplifies to

$$(b_1 - b_D)\,\Delta y^F(s_1) \leqslant \frac{\Delta y^F(s_1)^2}{2}, \tag{A9}$$

where I have used the fact that $\Pr[s_1] = \frac{1}{2}, \forall s_1 \in \mathcal{S}$. Solving (A9) jointly with $D$'s optimal actions from Lemma 1, it is immediate to verify that when $s_1 = 0$ truth-telling by $A_1$ requires

$$b_1 - b_D \leqslant \frac{1}{6},$$

while, when he observes $s_1 = 1$, truth-telling condition is

$$b_1 - b_D \geqslant -\frac{1}{6},$$

where $b_1 \in \mathcal{B}$. The result follows immediately.

*(ii) Two experts.* Without loss of generality, I focus on $A_i$'s incentive to report truthfully his signal, because experts are ex-ante symmetric. $A_i$'s expected utility from reporting $m_i = s_i$ is higher than his expected utility from reporting a false message $m_i = 1 - s_i$ if and only if

$$\sum_{s_j \in \mathcal{S}} \int_\theta -\left(y^F_{s_i,s_j} - \theta - b_i\right)^2 f(s_j,\theta|s_i)\,d\theta \geqslant \sum_{s_j \in \mathcal{S}} \int_\theta -\left(y^F_{1-s_i,s_j} - \theta - b_i\right)^2 f(s_j,\theta|s_i)\,d\theta, \tag{A10}$$

which, substituting $f(s_j,\theta|s_i) = f(\theta|s_i,s_j)\Pr[s_j|s_i]$ by Bayes' rule and following the same steps as I did above, simplifies to

$$(b_i - b_D)\sum_{s_j \in \mathcal{S}} \Pr[s_j|s_i]\,\Delta y^F(s_i,s_j) \leqslant \sum_{s_j \in \mathcal{S}} \Pr[s_j|s_i]\,\frac{\Delta y^F(s_i,s_j)^2}{2}. \tag{A11}$$

In order to compute $\Pr[s_j|s_i]$, notice first that conditional probability distribution of the signals can be written as follows

$$\Pr(s_i,s_j|\theta) = \frac{f(s_i,s_j,\theta)}{f(\theta)}. \tag{A12}$$

Then, using (A12) together with the fact that $f(\theta) = 1$, I obtain

$$\Pr[s_j|s_i] = \int_\theta f(s_j,\theta|s_i)\,d\theta = \int_\theta \frac{f(s_i,s_j,\theta)}{\Pr(s_i)}d\theta = \Pr[s_i]\int_\theta \Pr(s_i,s_j|\theta)\,d\theta. \tag{A13}$$

Using (2) together with $\Pr[s_i] = \frac{1}{2}, s_i \in \mathcal{S}$, it can be easily verified that

$$\Pr[s_j = 0|s_i = 0] = \Pr[s_j = 1|s_i = 1] = \frac{2}{3}, \tag{A14}$$

$$\Pr[s_j = 1|s_i = 0] = \Pr[s_j = 0|s_i = 1] = \frac{1}{3}. \tag{A15}$$

Finally, substituting (A14), (A15) into (A11) and using $D$'s optimal actions from Lemma **1**, when $s_i = 0$, truth-telling by $A_i$ requires

$$b_i - b_D \leqslant \frac{1}{8},$$

while, when $s_i = 1$, truth-telling by $A_i$ requires

$$b_i - b_D \geqslant -\frac{1}{8}.$$

where $b_i \in \mathcal{B}, i = 1, 2$. The result follows immediately. ■

**Proof of Lemma 3.** *(i) One Expert.* In a semi-revealing equilibrium, $D$'s maximization problem after receiving $m_1 \in \mathcal{M}$ is

$$\begin{aligned} y^S_{m_1} &= \arg\max_{y \in \mathbb{R}} \int_\theta -(y - \theta - b_D)^2 f(\theta|m_1)\,d\theta, \\ &= b_D + \underbrace{\sum_{s_1 \in \mathcal{S}} \Pr[s_1|m_1]\,\mathrm{E}[\theta|s_1]}_{\triangleq \mathrm{E}_\nu[\theta|m_1]}, \qquad \forall m_1 \in \mathcal{M}. \end{aligned} \tag{A16}$$

Bayes rule implies that $D$'s posterior beliefs about $s_1$ can be written as follows

$$\Pr[s_1|m_1] = \frac{\Pr[m_1|s_1]\Pr[s_1]}{\sum_{s_1\in\mathcal{S}}\Pr[m_1|s_1]\Pr[s_1]}. \tag{A17}$$

When $D$ receives $m_1 = 1$, her posteriors beliefs are

$$\Pr[s_1 = 1|m_1 = 1] = \frac{1}{2-\nu}, \qquad \Pr[s_1 = 0|m_1 = 1] = \frac{1-\nu}{2-\nu}, \tag{A18}$$

while when she receives $m_1 = 0$, her posterior beliefs are

$$\Pr[s_1 = 1|m_1 = 0] = 0, \qquad \Pr[s_1 = 0|m_1 = 0] = 1. \tag{A19}$$

Substituting the posterior beliefs (A18) and (A19) into (A16), and using the conditional expectations $\mathrm{E}[\theta|s_i, s_j]$ from the proof of Lemma 2, I have

$$y_0^S = b_D + \underbrace{\frac{1}{3}}_{\mathrm{E}_\nu[\theta|m_1=0]}, \qquad y_0^S = b_D + \underbrace{\frac{3-\nu}{3(2-\nu)}}_{\mathrm{E}_\nu[\theta|m_1=1]},$$

as claimed.

*(ii) Two Experts.* From (8) I know that $D$'s optimal action after receiving $m_i$ and $m_j$ is

$$y_{m_i,m_j}^S = b_D + \underbrace{\sum_{(s_i,s_j)\in\mathcal{S}^2}\mathrm{E}[\theta|s_i,s_j]\Pr[s_i,s_j|m_i,m_j]}_{\mathrm{E}_\nu[\theta|m_i,m_j]}, \qquad \forall(m_i,m_j)\in\mathcal{M}^2. \tag{A20}$$

Bayes' rule implies that $D$'s posterior beliefs about the signals can be written as follows

$$\Pr[s_i,s_j|m_i,m_j] = \frac{\Pr[m_i,m_j|s_i,s_j]\Pr[s_i,s_j]}{\sum_{(s_i,s_j)\in\mathcal{S}^2}\Pr[m_i,m_j|s_i,s_j]\Pr[s_i,s_j]}. \tag{A21}$$

Given that the extreme expert's babbling strategy is to report $m_E = 1$, when $D$ receives $(m_i, m_j) = (1, 1)$, her posterior beliefs about $(s_i, s_j)$ are

$$\Pr[s_i = 1, s_j = 1|1,1] = \frac{1}{\nu^2 - 3\nu + 3}, \qquad \Pr[s_i = 0, s_j = 0|1,1] = \frac{(1-\nu)^2}{\nu^2 - 3\nu + 3}, \tag{A22}$$

and

$$\Pr[s_i = 0, s_j = 1|1,1] = \Pr[s_i = 1, s_j = 0|1,1] = \frac{1-\nu}{2(\nu^2 - 3\nu + 3)}. \tag{A23}$$

By the same token, when $D$ receives $(m_i, m_j) = (0, 1)$, the posteriors are

$$\Pr[s_i = 0, s_j = 0|0,1] = \frac{2(1-\nu)}{3-2\nu}, \qquad \Pr[s_i = 0, s_j = 1|0,1] = \frac{1}{3-2\nu}, \tag{A24}$$

and zero, otherwise. Since the message space is binary, symmetric argument applies to the case where decision maker receives $(m_i, m_j) = (1, 0)$. Finally, when $D$ receives $(m_i, m_j) = (0, 1)$ the posteriors are

$$\Pr[s_i = 0, s_j = 0|0,0] = 1, \tag{A25}$$

and zero, otherwise. Next, I need to compute the joint probability of the signals. Notice that, Bayes rule implies that $\Pr[s_i, s_j]$ can be written as follows:

$$\Pr[s_i,s_j] = \Pr[s_j|s_i]\Pr[s_i]. \tag{A26}$$

Then substituting $\Pr[s_j|s_i]$ from equations (A14) and (A15) into (A26), it follows that

$$\Pr[s_i = 1, s_j = 1] = \Pr[s_i = 0, s_j = 0] = \frac{1}{3}, \tag{A27}$$

$$\Pr[s_i = 0, s_j = 1] = \Pr[s_i = 1, s_j = 0] = \frac{1}{6}. \tag{A28}$$

Finally, substituting $\mathrm{E}[\theta|s_1, s_2]$ from the proof of Lemma 1 and the joint probability of the signals (A27) and (A28) into (A20), it is immediate to verify that

$$y^S_{0,0}= b_D+ \underbrace{\frac{1}{4}}_{\mathrm{E}_\nu[\theta|0,0]} \qquad y^S_{0,1}= y^S_{1,0}= b_D+\underbrace{\frac{2-\nu}{2\,(3-2\nu)}}_{\mathrm{E}_\nu[\theta|0,1]} \qquad y^S_{1,1}= b_D+\underbrace{\frac{\nu^2-4\nu+6}{4\nu^2-12\nu+12}}_{\mathrm{E}_\nu[\theta|1,1]},$$

as desired. ■

**Proof of Proposition 4.** *(i) One Expert.* Suppose that $A_1$ is moderate — i.e., such that $b_1 = b_M$. Then $A_1$ has an incentive to report truthfully if and only if

$$\int_\theta -\left(y^S_{m_1} - \theta - b_M\right)^2 f\left(s_1|\theta\right) d\theta \geqslant \int_\theta -\left(y^S_{1-m_1} - \theta - b_M\right)^2 f\left(s_1|\theta\right) d\theta,$$

which following the same steps as I did in the proof of Proposition 2 and rearranging terms, simplifies to

$$\Pr\left[s_1\right]\left(b_M - b_D\right)\Delta y^S\left(m_1\right) \leqslant$$
$$\leqslant \Pr\left[s_1\right]\left\{\frac{\Delta y^S\left(m_1\right)^2}{2} + \Delta y^S\left(m_1\right)\left(\mathrm{E}_\nu\left[\theta|m_1\right] - \mathrm{E}\left[\theta|s_1\right]\right)\right\}, \qquad \text{(A29)}$$

where $\Delta y^S\left(m_1\right) \triangleq y^S_{1-m_1} - y^S_{m_1}$. Now substituting the optimal actions from Lemma 1 and Lemma 3 into (A29), it follows that whenever $s_1 = 0$, truth-telling by the moderate expert requires

$$b_M - b_D \leqslant \alpha_1\left(\nu\right) \triangleq \frac{1}{2\left(6-3\nu\right)}.$$

Similarly, when $s_1 = 1$ is observed, truth-telling by the moderate expert requires

$$b_M - b_D \geqslant -\beta_1\left(\nu\right),$$

where

$$\beta_1\left(\nu\right) \triangleq \frac{3-2\nu}{2\left(6-3\nu\right)}.$$

Moreover, $\alpha_1\left(\nu\right)$ is increasing in $\nu$ and $\beta_1\left(\nu\right)$ is decreasing in $\nu$ — i.e.,

$$\frac{d}{d\nu}\left[\alpha_1\left(\nu\right)\right] = \frac{1}{6\left(2-\nu\right)^2} > 0, \qquad \frac{d}{d\nu}\left[\beta_1\left(\nu\right)\right] = -\frac{1}{6\left(2-\nu\right)^2} < 0,$$

as expected. To complete the proof, I need to check that the extreme expert has no incentive to report $m_1 = 0$ when his signal is $s_1 = 0$. Adopting the same logic used above, this required condition is

$$\left(b_E - b_D\right)\Delta y^S\left(0\right) > \frac{\Delta y^S\left(0\right)^2}{2} + \Delta y^S\left(0\right)\left(\mathrm{E}_\nu\left[\theta|0\right] - \mathrm{E}\left[\theta|0\right]\right). \qquad \text{(A30)}$$

Substituting the optimal actions from Lemma 1 and Lemma 3 into (A30), whenever $s_1 = 0$, babbling condition required by the extreme expert is

$$b_E - b_D > \alpha_1\left(\nu\right).$$

Finally, when $s_1 = 1$, a rightward biased extreme expert has a strict incentive to report $m_1 = 1$.

*(ii) Two Experts.* Without loss of generality, I focus on $A_i$'s incentives to disclose his private information, since experts have symmetric payoffs. Consider first that $A_i$ is a moderate such that $b_i = b_M$. Given that $A_j$'s type is his private information, $A_i$'s incentive compatibility constraints writes as

$$\sum_{b_j\in\mathcal{B}} \Pr\left[b_j\right] \sum_{s_j\in\mathcal{S}} \int_\theta -\left(y^S_{m_i,m_j} - \theta - b_M\right)^2 f\left(s_j,\theta|s_i\right) d\theta \geqslant$$
$$\geqslant \sum_{b_j\in\mathcal{B}} \Pr\left[b_j\right] \sum_{s_j\in\mathcal{S}} \int_\theta -\left(y^S_{1-m_i,m_j} - \theta - b_M\right)^2 f\left(s_j,\theta|s_i\right) d\theta. \qquad \text{(A31)}$$

Following the same steps as I did in the proof of Proposition 2, the above constraint can be rewritten as follows

$$(b_M - b_D) \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|s_i) \Delta y^S(m_i, m_j) \leqslant$$
$$\leqslant \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|s_i) \left\{ \frac{\Delta y^S(m_i, m_j)^2}{2} + \Delta y^S(m_i, m_j)(\mathrm{E}_\nu[\theta|m_i, m_j] - \mathrm{E}[\theta|s_i, s_j]) \right\}. \tag{A32}$$

Using the optimal actions from Lemma 1 and Lemma 3 and $\Pr[s_j|s_i]$ from equations (A14) and (A15) , when $s_i = 0$, truth-telling by the moderate expert requires

$$b_M - b_D \leqslant \alpha_2(\nu) \triangleq \frac{5\nu^4 - 34\nu^3 + 84\nu^2 - 90\nu + 36}{8(3-2\nu)(\nu^2 - 3\nu + 3)(3\nu^2 - 8\nu + 6)}.$$

By the same token, when $s_i = 1$, truth-telling by the moderate expert requires

$$b_M - b_D \geqslant -\beta_2(\nu),$$

where

$$\beta_2(\nu) \triangleq \frac{5\nu^4 - 33\nu^3 + 80\nu^2 - 87\nu + 36}{8(3-2\nu)(\nu^2 - 3\nu + 3)(2-\nu)}.$$

Moreover, it can be shown that

$$\frac{d}{d\nu}[\alpha_2(\nu)] = \frac{30\nu^8 - 408\nu^7 + 2329\nu^6 - 7374\nu^5 + 14\,262\nu^4 - 17\,316\nu^3 + 12\,906\nu^2 - 5400\nu + 972}{8(3-2\nu)^2(\nu^2 - 3\nu + 3)^2(3\nu^2 - 8\nu + 6)^2} \geqslant 0,$$

and

$$\frac{d}{d\nu}[\beta_2(\nu)] = \frac{\nu^6 + 10\nu^5 - 112\nu^4 + 384\nu^3 - 627\nu^2 + 504\nu - 162}{8(2-\nu)^2(3-2\nu)^2(\nu^2 - 3\nu + 3)^2} < 0.$$

To complete the proof, I need to check that extreme expert has no incentive to report $m_i = 0$ when his signal is $s_i = 0$. Adopting the same logic used above, $A_i$'s expected utility from reporting $m_E = 1$ is higher than his expected utility when reporting truthfully $m_i = 0$ if

$$(b_M - b_D) \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0) \Delta y^S(0, m_j) >$$
$$> \sum_{b_j \in \mathcal{B}} \Pr[b_j] \sum_{s_j \in \{0,1\}} \Pr(s_j|0) \frac{\Delta y^S(0, m_j)^2}{2} + \Delta y^S(0, m_j)(\mathrm{E}_\nu[\theta|0, m_j] - \mathrm{E}[\theta|0, s_j]).$$

Using the optimal actions from Lemma 1 and Lemma 3, when $s_1 = 1$, babbling by the rightward biased extreme expert requires

$$b_E - b_D > \alpha_2(\nu),$$

while when $s_1 = 1$, the rightward biased extreme expert has an incentive to report truthfully his signal. ■

**Proofs of Propositions 5 and 6.** I first compare $D$'s expected utility from consulting one and two experts within each equilibrium. Let $\mathrm{E}U_i^F, i = 1, 2$, be the decision maker's ex-ante expected utility in a fully-revealing equilibrium. More precisely, in a fully-revealing equilibrium, $D$'s expected profit from consulting one expert is

$$\mathrm{E}U_1^F \triangleq \int_\theta \sum_{s_1 \in \mathcal{S}} -\left(y_{s_1}^F - \theta - b_D\right) \Pr[s_1|\theta] f(\theta) d\theta, \tag{A33}$$

which using (1) and using the results of Lemma 1 yields

$$\mathrm{E}U_1^F = -\frac{1}{18}. \tag{A34}$$

Similarly, in a fully-revealing equilibrium, $D$'s expected profit from consulting two experts is

$$\mathrm{E}U_2^F \triangleq \int_\theta \sum_{(s_i,s_j)\in\mathcal{S}^2} -\left(y^F_{s_i,s_j} - \theta - b_D\right) \Pr\left[s_i, s_j|\theta\right] f(\theta)\, d\theta. \tag{A35}$$

Using (2) and using the optimal actions from Lemma 1, I have

$$\mathrm{E}U_2^F = -\frac{1}{24}. \tag{A36}$$

Comparing this with (A34),

$$\mathrm{E}U_2^F - \mathrm{E}U_1^F = \frac{1}{72} > 0.$$

Therefore, in a fully-revealing equilibrium, $D$'s ex-ante expected utility is higher with two experts. Now let $\mathrm{E}U_i^S, i = 1, 2$ be the decision maker's ex-ante expected utility in a semi-revealing equilibrium. More precisely, in a semi-revealing equilibrium $D$'s ex-ante expected profit from consulting one expert is

$$\mathrm{E}U_1^S \triangleq \int_\theta \sum_{m_1\in\mathcal{M}} -\left(y^S_{m_1} - \theta - b_D\right) \Pr\left[m_1|\theta\right] f(\theta)\, d\theta, \tag{A37}$$

where

$$\Pr\left[m_1|\theta\right] = \sum_{s_1\in\mathcal{S}} \Pr\left[m_1|s_1\right] \Pr\left[s_1|\theta\right]. \tag{A38}$$

Substituting the conditional probability distribution of $s_1$ from (1) and the corresponding prior beliefs into (A38), I have

$$\Pr\left[m_1 = 1|\theta\right] = \theta + (1-\nu)(1-\theta) \qquad \text{and} \qquad \Pr\left[m_1 = 1|\theta\right] = \nu(1-\theta).$$

Hence,

$$\mathrm{E}U_1^S = -\frac{3-2\nu}{18(2-\nu)}, \tag{A39}$$

where I have used the optimal actions from Lemma 3. Similarly, in a semi-revealing equilibrium $D$'s ex-ante expected profit from consulting two experts is

$$\mathrm{E}U_2^S \triangleq \int_\theta \sum_{(m_i,m_j)\in\mathcal{M}^2} -\left(y^S_{m_i,m_j} - \theta - b_D\right) \Pr\left[m_i, m_j|\theta\right] f(\theta)\, d\theta,$$

where

$$\Pr\left[m_i, m_j|\theta\right] = \sum_{(s_i,s_j)\in\mathcal{S}^2} \Pr\left[m_i, m_j|s_i, s_j\right] \Pr\left[s_i, s_j|\theta\right]. \tag{A40}$$

Substituting the conditional probability distribution of $(s_i, s_j)$ from (2) and the corresponding prior probabilities into (A40), I have

$$\Pr\left[m_i = 1, m_j = 1|\theta\right] = (1-\nu(1-\theta))^2, \qquad \Pr\left[m_i = 0, m_j = 0|\theta\right] = \nu^2(1-\theta)^2$$

$$\Pr\left[m_i = 0, m_j = 0|\theta\right] = \Pr\left[m_i = 1, m_j = 0|\theta\right] = \nu(1-\theta)(1-\nu(1-\theta)),$$

Then using the optimal actions from Lemma 3, I obtain

$$\mathrm{E}U_2^S = -\frac{36(1-\nu)^2 + 13\nu^2(1-\nu) + 2\nu^2}{48(3-2\nu)(\nu^2-3\nu+3)}. \tag{A41}$$

Comparing (A39) and (A41),

$$\mathrm{E}U_2^S - \mathrm{E}U_1^S = \frac{(3-\nu)\left(\nu^3 + 6\nu(2-\nu)(1-\nu)\right)}{144(2-\nu)(3-2\nu)(\nu^2-3\nu+3)},$$

which is positive. Therefore, in a semi–revealing equilibrium, $D$'s ex-ante expected utility is higher with two experts. Finally, I compare $D$'s ex-ante expected utility in a semi-revealing equilibrium with two experts with her ex-ante expected utility when she consults one expert who reports truthfully his signal. Direct comparison of (A34) and (A41) yields

$$\mathrm{E}U_2^S - \mathrm{E}U_1^F = \frac{1}{144}\frac{96\nu - 81\nu^2 + 23\nu^3 - 36}{(3-2\nu)(\nu^2-3\nu+3)}. \tag{A42}$$

Since the denominator is positive, the sign of (A42) depends on the sign of

$$\mu(\nu) \triangleq 96\nu - 81\nu^2 + 23\nu^3 - 36.$$

Notice that

$$\mu(0) = -36 < 0,$$

$$\mu(1) = 2 > 0.$$

Moreover,

$$\frac{d\mu(\nu)}{d\nu} = 3\left(23\nu^2 - 54\nu + 32\right) > 0.$$

Hence, by mean value theorem there exists a unique $\widetilde{\nu} \triangleq 0.74$ such that $\mu(\nu) < 0$ (so that the decision maker's ex-ante expected utility is higher with one accurate expert) if and only if $\nu < \widetilde{\nu}$. ■

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