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Ramsey pricing: a simple example of a subordinate commodity*

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Abstract

We present preferences exhibiting a so-called subordinate good, namely a commodity that receives a negative price-cost margin according to Ramsey pricing. We also show that they deliver Ramsey quantities proportional to the efficient ones.

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1 Introduction

It is well known that in the case of a multiproduct firm Ramsey pricing (of which monopolistic pricing is an example) may involve (some) negative price-cost margin, and that this requires some complementarity among goods: see e.g. Tirole (1988: section 1.1.2) and Belleflamme and Peitz (2015: section 2.2.2). The optimal pricing literature has longly provided an explanation of this possibility based on the, rather involved, so-called "superelasticities" of demand: see e.g. Brown and Sibley (1986: chapter 3). However, Armstrong and Vickers (2018) have recently showed that the condition of having a commodity with a negative Ramsey margin boils down to consumer surplus being (locally) decreasing with respect to the quantity of that good. Moreover, Bertoletti (2018) has argued that this is equivalent to that commodity having a negative (inverse) "outside substitutability", the latter being measured by (minus) the scale elasticity of its inverse demand, and has suggested to classify similar goods as "subordinates". In the case of two goods (in addition to the outside commodity), Bertoletti (2018) has also showed that a subordinate commodity has a relatively poor substitutability, a relatively small budget share and it is a luxury (in terms of

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the preferences over inside commodities). However, we are not aware of any example of preferences delivering such a commodity.[1] The aim of this note is to provide such an example, exploiting a simple linear demand system with two goods. In addition, we show that the monopoly price of the other commodity is larger than the corresponding duopoly price, a result due to strategic substitutability. Finally, it turns out that Ramsey quantities are proportional to efficient ones: in fact, preferences belong to the class studied by Armstrong and Vickers (2018).

2 A simple model

Consider the following (indirect utility) function:

$$S(\mathbf{p}) = \max\left\{0, \frac{(a-p_1-p_2)^2 - p_2^2}{2}\right\}, \tag{1}$$

where $a > 0$: notice that S is decreasing and (strictly, whenever strictly decreasing) convex, and then it is a legitimate consumer surplus measure. By Hotelling's Lemma we get the following (direct) linear demand system (from now onwards we restrict our attention to internal solutions, meaning $\mathbf{x}$, $\mathbf{p} > \mathbf{0}$):

$$x_1(\mathbf{p}) = a - p_1 - p_2, \tag{2}$$
$$x_2(\mathbf{p}) = a - p_1 - 2p_2. \tag{3}$$

Note that commodities are complements (i.e., $\frac{\partial x_i}{\partial p_j} < 0$ for $x_i > 0$, i, $j = 1$, 2, $i \neq j$) and $x_1 > x_2$ for $x_1 > 0$: indeed, they are somehow close to the case of perfect complements.[2]

Suppose that commodities 1 and 2 are produced under constant returns to scale, with constant unit costs $c_1 \geq 0$ and $c_2 > 0$. The corresponding profit functions, given by $\pi_i(\mathbf{p}) = (p_i - c_i)\, x_i(\mathbf{p})$, $i = 1$, 2, are concave, and overall profit is $\Pi = \pi_1 + \pi_2$. In what follows we assume that a is sufficiently large to make feasible all the market allocations considered below.[3]

2.1 Ramsey pricing

Ramsey prices (see e.g. Bertoletti, 2018) maximize $W(\mathbf{p}) = \Pi(\mathbf{p}) + \alpha S(\mathbf{p})$ for $1 \geq \alpha \geq 0$. Notice that $W(\mathbf{p})$ is concave. FOCs can be written as:

[1] It is well known that examples of negative price-cost magins can be constructed in terms of access pricing and within the literature on the so-called two-sided markets: see e.g. Belleflamme and Peitz (2015: chapter 22).

[2] With perfect complements Ramsey prices would not be uniquely defined: see e.g. Tirole (1988: p. 71, Exercise 1.5).

[3] A *sufficient* condition is $a > c_1 + 4c_2$.

$$
\begin{aligned}
(p_1 - c_1)\frac{\partial x_1(\mathbf{p})}{\partial p_1} + (p_2 - c_2)\frac{\partial x_2(\mathbf{p})}{\partial p_1} &= -(1-\alpha)\, x_1(\mathbf{p}), \\
(p_1 - c_1)\frac{\partial x_1(\mathbf{p})}{\partial p_2} + (p_2 - c_2)\frac{\partial x_2(\mathbf{p})}{\partial p_2} &= -(1-\alpha)\, x_2(\mathbf{p}),
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
(p_1 - c_1) + (p_2 - c_2) &= (1-\alpha)\,(a - p_1 - p_2)\,, \\
(p_1 - c_1) + 2\,(p_2 - c_2) &= (1-\alpha)\,(a - p_1 - 2p_2)\,.
\end{aligned}
$$

Thus we get the following Ramsey prices:

$$
p_1^R(\alpha) = \frac{c_1 + (1-\alpha)\,a}{2-\alpha} \geq c_1,\; p_2^R(\alpha) = \frac{c_2}{2-\alpha} \leq c_2,
$$

which show that commodity 2 is indeed "subordinate" (see Bertoletti, 2018 for a discussion). Notice that $\frac{dp_2^R}{d\alpha} > 0$ and $\frac{dp_1^R}{d\alpha} < 0$, with $p_i^R(1) = c_i$, $p_1^R(0) = p_1^m = \frac{c_1+a}{2} > c_1$ and $p_2^R(0) = p_2^m = \frac{c_2}{2} < c_2$, where p_i^m denotes the price a multiproduct monopolist would adopt for commodity i. Also note that p_2^R does not depend on the willingness-to-pay parameter a. It is easily computed that:

$$
\begin{aligned}
x_1^R(\alpha) &= \frac{a - c_1 - c_2}{2-\alpha},\; x_2^R(\alpha) = \frac{a - c_1 - 2c_2}{2-\alpha}, \\
\Pi^R(\alpha) &= \frac{1-\alpha}{(2-\alpha)^2}\left[(a - c_1 - c_2)^2 + c_2^2\right], \\
S^R(\alpha) &= \frac{1}{2\,(2-\alpha)^2}\left[(a - c_1)\,(a - c_1 - 4c_2) + 3c_2^2\right].
\end{aligned}
$$

Ramsey quantities $\mathbf{x}^R(\alpha)$ are proportional to the efficient quantities $\mathbf{x}^R(1)$, i.e., x_1^R/x_2^R does not depend on α. In fact, by inverting (2)-(3) we obtain the corresponding inverse demand system:

$$
\begin{aligned}
p_1(\mathbf{x}) &= a - 2x_1 + x_2, \\
p_2(\mathbf{x}) &= x_1 - x_2.
\end{aligned}
$$

The latter integrates into the utility function $u(\mathbf{x}) = ax_1 + x_1x_2 - x_1^2 - x_2^2/2$, which can be written as $h(\mathbf{x}) + g(q(\mathbf{x}))$, where $h(\mathbf{x}) = ax_1$ and $q(\mathbf{x}) = \sqrt{\mathbf{x}'\mathbf{M}\mathbf{x}}$ with $\mathbf{M} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ are linear homogeneous and $g(t) = -t^2/2$ is concave, showing that preferences (1) belong to the class studied by Armstrong and Vickers (2018).[4] Notice that, as a function of quantities, consumer surplus is given by:

$$
\begin{aligned}
s(\mathbf{x}) &= u(\mathbf{x}) - \mathbf{p}(\mathbf{x})'\mathbf{x} \\
&= x_1^2 - x_2x_1 + x_2^2/2,
\end{aligned}
$$

[4] Armstrong and Vickers (2018: p. 1458) mention that this kind of preferences may deliver subordinate commodities, but do not provide an example.

so that $\frac{\partial s}{\partial x_2} = x_2 - x_1 < 0$ (whenever $p_2 > 0$); equivalently, the measure of *outside substitutability* proposed by Bertoletti (2018), $\mu_i(\mathbf{x}) = -\frac{\partial \ln p_i(\rho \mathbf{x})}{\partial \ln \rho}\,|_{\rho=1}$, is actually negative for commodity 2: $\mu_2 = (x_2 - x_1) / (x_1 - x_2) < 0$.

2.2 Price competition

Consider the duopoly equilibrium in which each commodity is produced by an independent firm and firms compete by setting simultaneously their own price. The reaction functions are given by:

$$\begin{aligned} p_1 &= \max\left\{0, \frac{a - p_2 + c_1}{2}\right\}, \\ p_2 &= \max\left\{0, \frac{a - p_1 + 2c_2}{4}\right\}. \end{aligned}$$

In the unique Nash equilibrium:

$$p_1^* = \frac{3a + 2(2c_1 - c_2)}{7} > c_1,\; p_2^* = \frac{a + 4c_2 - c_1}{7} > c_2,$$

$$x_1^* = \frac{3a - 3c_1 - 2c_2}{7} > 0,\; x_2^* = 2\frac{a - c_1 - 3c_2}{7} > 0,$$

$$\pi_1^* = \frac{(3a - 2c_2 - 3c_1)^2}{49},\; \pi_2^* = 2\frac{(a - 3c_2 - c_1)^2}{49},$$

with $p_1^m > p_1^*$, due to strategic substitutability.

3 Conclusions

We have presented a simple example of preferences exhibiting a *subordinate* commodity, namely a good that should be optimally priced below its marginal cost according to Ramsey pricing. It is a complement to another commodity that a multiproduct monopolist would price more than in the corresponding duopoly equilibrium (with competitors producing a single product). Finally, Ramsey quantities enjoy the very convenient property of being proportional to efficient ones: see Armstrong and Vickers (2018) for a discussion.

References

Armstrong, M. and J. Vickers (2018) "Multiproduct Pricing Made Simple", *Journal of Political Economy* 126, 1444-71.

Belleflamme, P. and M. Peitz (2015) *Industrial Organization. Markets and Strategies*, second edition, Cambridge University Press, Cambridge (UK).

Bertoletti, P. (2018) , "A Note on Ramsey Pricing and the Structure of Preferences", *Journal of Mathematical Economics* 76, 45-51.

Brown, S. and D. Sibley (1986) *The Theory of Public Utility Pricing*, Cambridge University Press, Cambridge (UK).

Tirole, J. (1988) *The Theory of Industrial Organization*, MIT Press, Cambridge (USA).