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On the detrimental effects of concave emission charges in a dynamic Cournot duopoly model

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Abstract

We reconsider the dynamic Cournot duopoly framework with homogeneous goods by Mamada and Perrings (2020), in order to highlight the richness in its outcomes. In the model each firm is taxed proportionally to its own emission only and charge functions are quadratic. While Mamada and Perrings (2020) focus on the case of convex, and partially on the case of concave, charge functions, we show that completing the analysis for concave functions it may happen that, with the raise in the emission charges, the equilibrium production levels for the two firms, which are directly proportional to their emissions, increase, both with homogeneous and with differentiated products. This highlights that, even in the absence of free riding possibilities, too soft environmental policies can produce detrimental effects on the pollution level, and thus the choice of the mechanism to implement has to be carefully pondered.

Keywords: dynamic Cournot duopoly, differentiated products, emission charges, pollution control, comparative statics, stability analysis.

JEL classification: C62, D43, Q51, Q58

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1 Introduction

In the past decades, environmental policies to control pollution have been proposed in several works, such as Segerson (1988), Katsoulacos and Xepapadeas (1995) and Suter et al. (2008). In particular, some of them are based on both charges and incentives, according to a comparison between the aggregate emission level and the ambient standard, mainly in the case of non-point source (NPS) pollution, like for instance in Ganguli and Raju (2012), Matsumoto and Szidarovszky (2021) and Matsumoto et al. (2018), while other ones are based just on charges, as it happens e.g. in Mamada and Perrings (2020). Even if it would seem natural to expect that higher charges lower the emission volume, sometimes counterintuitive outcomes may be observed, especially in frameworks in which the aggregate pollution level is taken into account. This occurs for instance in Ganguli and Raju (2012), where a higher emission volume may come as a consequence of increased charges in a Bertrand duopoly setting. Indeed, the framework in Ganguli and Raju (2012) encompasses strategic interactions between firms both on the demand side in the market and in regard to emissions in the environmental sphere. The latter aspect leads to a public good game setting, which may give rise to free riding possibilities. On the contrary, in the present work we prove that a similar detrimental effect occurs in the framework by Mamada and Perrings (2020), who propose a Cournot duopoly with homogeneous products where the decisional mechanism is based on a gradual adjustment towards the best response, and in which each firm is taxed proportionally to its own emission only. In more detail, Mamada and Perrings (2020) consider quadratic charge functions, focusing on the case of convex, and partially on the case of concave, charges. On the other hand, completing the analysis for concave functions performed therein, we discovered a larger richness in the outcomes of the model, since with the raise in the emission charges the equilibrium production levels for the two firms, which are directly proportional to their emissions, may increase when the environmental policy is too soft, both with homogeneous and with differentiated products. We recall that the extension to the case of differentiated products has been suggested in their concluding remarks by Mamada and Perrings (2020).
The remainder of the paper is organized as follows. In Section 2 we present the model with differentiated products, comparing it with the homogeneous product framework in Mamada and Perrings (2020). In Section 3 we perfom the model analysis. In Section 4 we conclude.

2 The model

Following the duopoly formulation in Mamada and Perrings (2020), but supposing that the goods produced by the two firms are differentiated, we assume that in each time period t firm $i \in \{1, 2\}$ maximizes the profit function

$$\pi_{i,t} = (p - \beta q_{i,t} - \gamma q_{j,t})q_{i,t} - cq_{i,t}^2 - C_{i,t}^e \tag{2.1}$$

where $q_{i,t}$ and $q_{j,t}$ are the output levels by firms i and j, respectively, with $i \neq j \in \{1, 2\}$, and for the chock price p and the production costs c we suppose that they are positive like in Mamada and Perrings (2020). In regard to β and γ, as usual in the case of differentiated goods, we assume that $|\gamma| < \beta$: if $\gamma > 0$ (resp. $\gamma < 0$) the two goods are substitutes (resp. complements), while they are independent if $\gamma = 0$. We recall that the framework with homogeneous products is obtained as limit case with $\gamma = \beta = k$, where k is the price-depressing effect of oligopoly. Cf. Singh and Vives (1984) and Motta (2004) for further details.
Denoting by $\varepsilon > 0$ emissions per unit output, so that $u_{i,t} = \varepsilon q_{i,t}$ are emissions by firm $i \in \{1, 2\}$ at time t, Mamada and Perrings (2020) propose the following quadratic formulation for emission charges

$$C_{i,t}^e = bu_{i,t} + \frac{1}{2}du_{i,t}^2, \tag{2.2}$$

with $b > 0$ and $d \in \mathbb{R}$, that we will consider, too. Since the marginal emission charge is given by $\frac{dC_{i,t}^e}{du_{i,t}} = b + du_{i,t}$, it may be increasing or decreasing according to the sign of d. In particular, if d is negative, the condition

$$0 < q_{i,t} < \frac{-b}{\varepsilon d} \tag{2.3}$$

is needed to guarantee the positivity of the marginal emission charge.
Like in Mamada and Perrings (2020), we assume that, due to an adjustment capacity constraint, firms adjust the output level according to (the size and the extent of) the difference between their best response and their current output level with a reactivity parameter $\lambda \in (0, 1)$, so that

$$q_{i,t+1} = q_{i,t} + \lambda(R_i(q_{j,t}) - q_{i,t}) \tag{2.4}$$

where $R_i(q_j)$ is the best response function of firm i to the output q_j produced by firm j. Notice that there is no adjustment when $\lambda = 0$, while adjustment

is complete and instantaneous when $\lambda = 1$. See Bischi et al. (2010) for dynamical aspects connected with oligopoly models. Although overadjustment, given by $\lambda > 1$, is possible, Mamada and Perrings (2020) disregard such eventuality and we will stick to their choice, too.
From the FOC coming from the maximization of (2.1) we find

$$R_i(q_{j,t}) = \frac{p - \gamma q_{j,t} - b\varepsilon}{2(\beta + c) + d\varepsilon^2} \tag{2.5}$$

as best response function for $i \neq j \in \{1, 2\}$, which is well defined when $2(\beta+c)+d\varepsilon^2 \neq 0$. We will maintain such assumption along all the manuscript, even when not explicitly mentioned, so that the unique (symmetric) Nash equilibrium is given by

$$(q_1^*, q_2^*) = \left(\frac{p - b\varepsilon}{2(\beta + c) + d\varepsilon^2 + \gamma}, \frac{p - b\varepsilon}{2(\beta + c) + d\varepsilon^2 + \gamma} \right). \tag{2.6}$$

In the case of homogeneous goods, the Nash equilibrium becomes

$$(\bar{q_1}^*, \bar{q_2}^*) = \left(\frac{p - b\varepsilon}{3k + 2c + d\varepsilon^2}, \frac{p - b\varepsilon}{3k + 2c + d\varepsilon^2} \right). \tag{2.7}$$

In order to ensure the positivity for (2.7), in Mamada and Perrings (2020) it is supposed that $p > b\varepsilon$ and that $3k + 2c + d\varepsilon^2 > 0$. Nonetheless, also the framework with $p < b\varepsilon$ and $3k + 2c + d\varepsilon^2 < 0$ is admissible. We will obtain the results for the latter case in Subsection 3.2, as corollaries of the findings for the corresponding heterogeneous good framework with $p < b\varepsilon$ and $2(\beta + c) + d\varepsilon^2 + \gamma < 0$, while in Subsection 3.1 we will focus on the case with $p > b\varepsilon$ and $2(\beta + c) + d\varepsilon^2 + \gamma > 0$, which generalizes the framework analyzed in Mamada and Perrings (2020). We stress that all such conditions have to be considered jointly with the constraints coming from (2.3) at the Nash equilibrium. Focusing for instance on the condition, considered by Mamada and Perrings (2020), for the positivity of the denominator of (2.7), i.e., on $d > -\frac{3k+2c}{\varepsilon^2}$, it is satisfied for every $d > 0$, in which case C_i^e in (2.2) is convex, for $i \in \{1, 2\}$, as well as for $d \in \left(-\frac{3k+2c}{\varepsilon^2}, 0\right)$, that is, when C_i^e is concave, but production variations lead to emission charge variations close to those that we would have in the linear case, corresponding to $d = 0$. Notice also that the Nash equilibrium in (2.7) fulfills the right constraint in (2.3) if $d \in \left(-\frac{b(3k+2c)}{\varepsilon p}, 0\right)$ and the latter condition is stricter than $d \in \left(-\frac{3k+2c}{\varepsilon^2}, 0\right)$ under the therein maintained assumption $p > b\varepsilon$. Hence, in such case, (2.3) is satisfied by the Nash equilibrium for the homogeneous

good setting if $d \in \left(-\frac{b(3k+2c)}{\varepsilon p}, 0\right)$. In Section 3, taking into account (2.3), we will derive similar conditions for the heterogeneous product case under the various assumptions on the parameters, considering then what happens in the limit case in which products are homogeneous.
Supposing that firms partially adjust their output level toward the best response according to (2.4) with $\lambda \in (0,1)$, when inserting (2.5) therein we obtain the dynamical system

$$\begin{cases} q_{1,t+1} = q_{1,t} + \lambda(R_1(q_{2,t}) - q_{1,t}) = q_{1,t} + \lambda\left(\frac{p-\gamma q_{2,t}-b\varepsilon}{2(\beta+c)+d\varepsilon^2} - q_{1,t}\right) \\ q_{2,t+1} = q_{2,t} + \lambda(R_2(q_{1,t}) - q_{2,t}) = q_{2,t} + \lambda\left(\frac{p-\gamma q_{1,t}-b\varepsilon}{2(\beta+c)+d\varepsilon^2} - q_{2,t}\right) \end{cases} \tag{2.8}$$

whose only steady state is given by the Nash equilibrium in (2.6), and which coincides with the model (2) on page 373 in Mamada and Perrings (2020) for $\gamma = \beta = k$.

3 Analysis

We split the analysis of the model described by (2.8) according to the sign of the numerator N and of the denominator D of the Nash equilibrium in (2.6): in particular, in Subsection 3.1 we consider what occurs when they are both positive (Case $N > 0$, $D > 0$), as done in Mamada and Perrings (2020) for the setting with homogeneous products, while in Subsection 3.2 we focus on the scenario in which both of them are negative (Case $N < 0$, $D < 0$).

3.1 Case $N > 0\,,\ D > 0$

In the present subsection, we focus on the case

$$p - b\varepsilon > 0, \qquad 2(\beta + c) + d\varepsilon^2 + \gamma > 0. \tag{3.1}$$

With reference to (2.6), since for $i \in \{1,2\}$ it holds that

$$\frac{\partial q_i{}^*}{\partial b} = \frac{-\varepsilon}{2(\beta+c)+d\varepsilon^2+\gamma}, \qquad \frac{\partial q_i{}^*}{\partial d} = \frac{-(p-b\varepsilon)\varepsilon^2}{(2(\beta+c)+d\varepsilon^2+\gamma)^2}, \tag{3.2}$$

the next result about comparative statics immediately follows:

Proposition 3.1 *Under* (3.1), *in regard to* (q_1^*, q_2^*) *in* (2.6) *it holds that, for* $i \in \{1,2\}$, q_i^* *decreases when* b *or* d *increase.*

This result, highlighting the efficacy of the environmental policy described by the emission charges in (2.2) under (3.1), holds true also in the homogeneous product framework in Mamada and Perrings (2020), due to the assumption made therein about the sign of the numerator and of the denominator of the Nash equilibrium in (2.7).
On the other hand, under different conditions on the parameters (cf. (3.4)), we will find in Subsection 3.2 that opposite outcomes arise in regard to b and d. The same is true also with respect to parameter ε, connected with the produced pollution level, too, as an increase in it describes the transition for firms to more polluting technologies. The proof is omitted for brevity's sake.
In relation to the model dynamic outcomes, we have the following result about the stability of the Nash equilibrium:

Proposition 3.2 *Under* (3.1)*,* (q_1^*, q_2^*) *is admissible according to* (2.3) *for* $d > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$*. If this is the case, it is globally asymptotically stable for System* (2.8) *when* $d > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$.

Proof. We investigate the system stability by using the well-known Jury conditions

$$(i)\ \det(J) < 1, \quad (ii)\ 1 + \mathrm{tr}(J) + \det(J) > 0, \quad (iii)\ 1 - \mathrm{tr}(J) + \det(J) > 0,$$

where

$$J = \begin{bmatrix} 1-\lambda & \frac{-\lambda\gamma}{2(\beta+c)+d\varepsilon^2} \\ \frac{-\lambda\gamma}{2(\beta+c)+d\varepsilon^2} & 1-\lambda \end{bmatrix}$$

is the Jacobian matrix for (2.8), and $\det(J)$, $\mathrm{tr}(J)$ denote its determinant and trace, respectively.[1] Thus, we have $\det(J) = 1 - 2\lambda + \lambda^2\left(1 - \frac{\gamma^2}{(2(\beta+c)+d\varepsilon^2)^2}\right)$ and $\mathrm{tr}(J) = 2 - 2\lambda$. Hence, (iii) reads as

$$1 - \frac{\gamma^2}{(2(\beta+c)+d\varepsilon^2)^2} > 0. \tag{3.3}$$

Since $\lambda \in (0,1)$, conditions (i) and (ii) are then always fulfilled. Condition (3.3) can be rewritten as $(2(\beta+c)+d\varepsilon^2-\gamma)(2(\beta+c)+d\varepsilon^2+\gamma) > 0$, which implies that $2(\beta+c)+d\varepsilon^2-\gamma > 0$, since we are supposing that $2(\beta+c)+d\varepsilon^2+\gamma > 0$. Observing that, under (3.1), the conditions in (2.3) lead to $d > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p} > -\frac{2\beta+2c+\gamma}{\varepsilon^2}$, the desired conclusion follows. □

[1]Notice that, differently from d, parameter e plays no role on the stability of the Nash equilibrium, being not present in J.

We stress that we have analyzed the system stability because a comparative statics result is economically grounded if the considered equilibrium is asymptotically stable and thus orbits converge towards it after a transient period.
Considering the homogeneous product setting, the stability condition $d > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$ becomes $d > -\frac{k+2c}{\varepsilon^2}$, in agreement with the result on page 374 in Mamada and Perrings (2020), which highlights the stabilizing role of d on $(\bar{q_1}^*, \bar{q_2}^*)$ in (2.7). Together with the constraints coming from (2.3) discussed in Section 2, we can conclude that $(\bar{q_1}^*, \bar{q_2}^*)$ is stable and admissible for $d > \max\{-\frac{b(3k+2c)}{\varepsilon p}, -\frac{k+2c}{\varepsilon^2}\}$. Notice that $-\frac{b(3k+2c)}{\varepsilon p} > -\frac{k+2c}{\varepsilon^2}$ for $b\varepsilon < p\left(\frac{k+2c}{3k+2c}\right)$, in which case the Nash equilibrium is always stable when it is admissible according to (2.3). More generally, in the case of differentiated products, it holds that the stability threshold found in Proposition 3.2 is not admissible according to (2.3), i.e., $-\frac{b(2\beta+2c+\gamma)}{\varepsilon p} > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$, when $b\varepsilon < p\left(\frac{2\beta+2c-\gamma}{2\beta+2c+\gamma}\right)$. Under (3.1), the latter condition could be fulfilled when the two products are substitutes, while it is granted in the case of complements or of independent goods. In such positive eventualities, the comparative statics results reported in Proposition 3.1, which show that the environmental policy described by the emission charges in (2.2) is effective in reducing pollution under (3.1), are robustly grounded from an economic viewpoint, since the Nash equilibrium is a global attractor.

3.2 Case $N < 0\,,\, D < 0$

In the present subsection, we focus on the case

$$p - b\varepsilon < 0, \qquad 2(\beta + c) + d\varepsilon^2 + \gamma < 0, \tag{3.4}$$

in which the following results about comparative statics (see Proposition 3.3) and on the system dynamic behavior (cf. Proposition 3.4) hold true:

Proposition 3.3 *Under* (3.4)*, in regard to* (q_1^*, q_2^*) *in* (2.6)*, it holds that, for* $i \in \{1, 2\}$*,* q_i^* *increases when* b *or* d *increase.*

Proposition 3.4 *Under* (3.4)*,* (q_1^*, q_2^*) *is admissible according to* (2.3) *for* $d < -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$*. If this is the case, it is globally asymptotically stable for System* (2.8) *when* $d < -\frac{2\beta+2c-\gamma}{\varepsilon^2}$.

The result in Proposition 3.3, which is an immediate consequence of the positive sign of the partial derivatives in (3.2), highlights that the environmental policy described by the emission charges in (2.2) is detrimental under

(3.4), since such scheme makes pollution increase in correspondence to the Nash equilibrium. That finding is robustly grounded from an economic viewpoint when the Nash equilibrium is a global attractor and, after a certain transient period, all orbits converge towards it. Accordingly, we studied its stability in Proposition 3.4, whose proof follows similar steps to those used to check Proposition 3.2. In particular, the stability condition $d < -\frac{2\beta+2c-\gamma}{\varepsilon^2}$ comes from (3.3), while the admissibility condition $d < -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$ follows from (2.3). Depending on the relative position of the two thresholds, parameter d may play a destabilizing role on the system stability, or (q_1^*, q_2^*) may be stable for all the admissible values of d. It is easy to show that the former case can occur just when the two products are complements (i.e., $\gamma < 0$), since with substitutes or with independent goods it always holds that $-\frac{2\beta+2c-\gamma}{\varepsilon^2} > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$ under (3.4). Recalling that in the homogeneous good framework it holds that $\gamma = \beta = k > 0$, in such case the system is always stable under (3.4) and thus Proposition 3.3, showing the inefficacy of the considered environmental policy, is economically grounded when dealing with homogeneous products, too.
Noticing that (3.4) can be fulfilled just for negative values of d, in which case C_i^e in (2.2) is concave, we can summarize Propositions 3.3 and 3.4 by saying that, focusing on an equilibrium which attracts all orbits, and which is hence dynamically meaningful, we have proven the inefficacy of the environmental policy described by the emission charges C_i^e in (2.2), under suitable parameter configurations for which C_i^e is concave and emission charges increase too slowly with production.

4 Conclusions

In the present contribution we have shown that, even when each firm is taxed proportionally to its own emission only and thus no free riding possibilities can arise, too soft environmental policies may produce detrimental effects on the pollution level, implying that the choice of the mechanism to implement has to be carefully pondered. In particular, we have obtained the above counterintuitive result completing the analysis performed in Mamada and Perrings (2020) by dealing with parameter configurations not considered therein, in order to let emerge the richness of interesting outcomes hidden in the dynamic Cournot duopoly model with quadratic emission charges proposed in that work.

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