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A Preference-Based Model of Competition among Platforms

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Abstract

We propose a novel approach to study competition among platforms. In particular, we consider representative consumer's preferences over the services platforms provide and the commodities they intermediate. Platforms are assumed to be large, intermediating a variety of commodities offered by sellers under monopolistic competition and free entry, and competing *à la* Cournot. We use a duopoly setting to discuss the welfare implications of platform exchange commissions, which are typically significant in real-world cases. Our preliminary finding is that positive commission actually worsens consumer welfare by reducing the platform price-adjusted quality indexes.

JEL Classification: D11, L13, L41, L51

Keywords: platform competition, two-sided markets, market intermediation

1 Introduction

In this paper we propose a novel approach to study competition among *platforms*. With the latter term we refer to intermediaries which allow buyers and sellers to interact (i.e., a so-called two-sided market), providing a number of complementary services (one of them being market creation, since the exchanges would not take place without those services): see e.g. Belleflamme and Peitz (2015: Part IX). We think of platform operators, or marketplaces, as directly selling their services to the consumers, and also affecting the trade they intermediated, being able to charge transaction fees or commissions on it. We focus on competition among platforms, and are interested in the resulting price structure and allocation, and in its welfare consequences.

The by-now canonical approach to platforms competition (Armstrong, 2006) assumes that they are horizontally differentiated and that consumers obtain an

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indirect utility which depends on the set of intermediated sellers. This framework can be used to account for the variety provided by the sellers, and for their price structure. For example, in his recent contribution Etro (2021) employs an additive consumer surplus function to model sellers' contribution to the value created by platforms. We suggest a somehow more direct approach, namely to exploit utility theory to model consumer preferences over the set of goods and services jointly provided by each platform. We present a duopoly example of the suggested approach, and use it to investigate the role of commission fees. While we think that in principle our approach could be applied to platforms as Amazon Marketplace and App Store by Apple, or more generally to pay televisions, game platforms, credit cards and software operating systems, in practise no single set of preference assumptions can hope to fit all the previous settings, and other examples should be explored in future investigations to assess the robustness of our preliminary results (we briefly discuss a rather special alternative in the Appendix). In addition, we do not deal with the relevant (hybrid) case in which platforms are also allowed to sell their own products, which would raise additional competition concerns.

In particular, we assume that a representative consumer with homothetic preferences has to decide how to spread her expenditure across platforms. In our main example the services directly provided by each platform are related to the goods it intermediates by a simple Cobb-Douglas structure, and sellers are differentiated according to CES preferences (*à la* Dixit and Stiglitz, 1977). From the representative consumer' point of view platforms provide perfect substitutes, and she only cares for the quality supplied by each of them. From the supply side, sellers are interested in the expenditure level captured by each platform because this, jointly with the commission level, ultimately determines their profit level. Our approach delivers a fully-fledged microeconomic foundation to the measurement of consumer welfare, able to account for price changes and gains from variety, and that would be easily generalized to the case of many platforms.

Our approach, and the preferences we employ in our main example are presented and discussed in section 2. Competition is analyzed in section 3: platforms compete *à la* Cournot, while goods are supplied under monopolistic competition and free entry. We study the duopoly symmetric equilibrium with or without the ability of platforms to commit in advance to their commission levels. We also discuss the welfare implications of the exchange commissions adopted by the platforms which, as a matter of fact, are usually significant (for example, Amazon's commissions are typically in the 15%-30% range). In our setting without commissions platforms could not appropriate of the surplus created by the intermediated trade, due to the assumptions of Cobb-Douglas preferences and of platform perfect substitutability (any "quality improvement" would be competed away in a symmetric equilibrium). Thus in the market equilibrium platform commissions are indeed positive. Their impact is to raise sellers' prices and to reduce their variety, thus decreasing platform quality, even though they also reduce the prices required by platforms for their services. Eventually, positive commissions reduce consumer welfare with respect to the benchmark case of zero commissions. These results are partially in contrast to those of Etro (2021), who in a setting with deviced-funded platforms finds that their incentives in determining commissions are largely aligned with those of consumers. However, the alternative, special example that we briefly discuss in the Appendix shows that commissions actually need not affect welfare.

2 Preferences over platforms' commodities

Suppose that a *representative* consumer has preferences over the set of commodities and services provided by a limited number of competing platforms. The intuition is that each platform h is intermediating a large number n_h of commodities (whose quantities are given by the vector $\mathbf{x}_h$) which are complementary with the service provided by the platform itself (whose amount is captured by the scalar y_h). Overall, platforms are offering goods which are substitutes. Let preferences over the commodities provided by platform h be represented by the utility function:

$$U_h\left(y_h, \mathbf{x}_h\right) = y_h \sum_{i=1}^{n_h} x_{hi}^{\rho}, \tag{1}$$

with $0 < \rho < 1$. U_h has a Cobb-Douglas upper-tier structure: preferences are homothetic, and goods y_h and $\mathbf{x}_h$ are complementary in the sense that they must be consumed together.[1] Sellers supply commodities that are differentiated à la Dixit and Stiglitz (1977).

Demands are provided by the FOCs for utility maximization:

$$y_h\left(p_{y_h}, E_h\right) = \frac{E_h}{\left(\rho+1\right) p_{y_h}}, \quad x_{hj}\left(\mathbf{p}_h, E_h\right) = \frac{\rho E_h p_{hj}^{-\sigma}}{\left(\rho+1\right)\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}},$$

where $\sigma = 1/(1-\rho)$ is the elasticity of substitution among the n_h commodities, and E_h is the overall expenditure on platform h. Notice that ρ also affects the expenditure shares. Accordingly, the indirect utility function is given by:

$$\begin{aligned} V_h\left(p_{y_h}, \mathbf{p}_h, E_h\right) &= \frac{\rho^{\rho}}{\left(\rho+1\right)^{1+\rho}} \frac{E_h^{1+\rho}\left[\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}\right]^{\frac{1}{\sigma}}}{p_{y_h}} \\ &= \frac{\rho^{\rho} E_h^{1+\rho}}{\left(\rho+1\right)^{1+\rho}} \frac{s_h\left(\mathbf{p}_h, n_h\right)}{p_{y_h}}, \end{aligned}$$

[1] U_h is strictly quasi-concave (for $[y_h, \mathbf{x}_h] > \mathbf{0}$):

$$D^2 U_h(y_h, \mathbf{x}_h) = \begin{bmatrix} 0 & \nabla g\left(\mathbf{x}_h\right) \\ \nabla g\left(\mathbf{x}_h\right)' & y_h D^2 g\left(\mathbf{x}_h\right) \end{bmatrix},$$

where $g\left(\mathbf{x}_h\right) = \sum_i x_{hi}^{\rho}$, with

$$\mathbf{z}' D^2 U_h(y_h, \mathbf{x}_h)\mathbf{z} = -2\frac{z_1^2 g\left(\mathbf{x}_h\right)}{y_h} + y_h \mathbf{z}_{-1}' D^2 g\left(\mathbf{x}_h\right)\mathbf{z}_{-1} < 0,$$

for all $\mathbf{z}$ such that $\mathbf{z}' D U_h(y_h, \mathbf{x}_h) = 0$.

where $s_h(\mathbf{p}_h) = \left[\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}\right]^{\frac{1}{\sigma}}$ can be interpreted as a "quality index" for platform h.

Now suppose that there are just 2 platforms, with:

$$U(\mathbf{y}, \mathbf{x}) = y_1 \sum_{i=1}^{n_1} x_{1i}^{\rho} + y_2 \sum_{j=1}^{n_2} x_{2j}^{\rho}.$$

Notice that platforms offer *perfect* substitutes: preferences are still homothetic and additively separable in 2 groups, corresponding to the platforms, and there is symmetry between platforms (except possibly for the number of sellers).[2] Clearly, consumer expenditure could be spread over platforms only if both offer the maximum "price-adjusted quality index" $s_h(\mathbf{p}_h)/p_{y_h}$, delivering indirect utility:

$$V(\mathbf{p}_y, \mathbf{p}, E) = \frac{\rho^{\rho} E^{\rho+1}}{(\rho+1)^{1+\rho}} \max\left\{\frac{s_1(\mathbf{p}_1)}{p_{y_1}}, \frac{s_2(\mathbf{p}_2)}{p_{y_2}}\right\}.$$

3 Competition

We use monopolistic competition *à la* Dixit and Stiglitz (1977) to model the behavior of sellers selling through platforms, and Cournot competition to establish the equilibrium between platforms.[3] We start by considering as given the number of sellers of each platform, and assuming that they pay to it an ad-valorem commission $1 > t_h \geq 0$ ($t_h = 0$ in the benchmark case). We then characterize the symmetric equilibrium in which the number of sellers is determined by free entry under a setup cost $F > 0$.[4] Finally, we move to a two-stage setting in which commission are set in a first-stage, strategically affecting entry decisions by sellers, and again we characterize the symmetric equilibrium. We also briefly discuss the welfare implications of commissions in both settings.

3.1 Monopolistic competition among sellers

Let us assume that good x_{hi} is produced by seller hi with a (common) constant marginal cost $c > 0$: by the CES structure under monopolistic competition its price is given by:

$$p_{hi}(t_h) = \frac{c}{\rho(1-t_h)},$$

with

$$x_{hi}(t_h, E_h, n_h) = \frac{(1-t_h)\rho^2 E_h}{(\rho+1) n_h c}, \quad p_{hi} x_{hi} = \frac{\rho E_h}{(\rho+1) n_h}, \quad n_n p_{hi} x_{hi} = \frac{\rho E_h}{(\rho+1)},$$

[2] Exploting additivity across platforms, it would be easy to generalize this model to the case of $N > 2$ competiting platforms and to introduce some additional asymmetries among platforms.

[3] We briefly discuss the case of Bertrand competition in section 3.5.

[4] In our setting sellers are identical and may be multihoming, but in the latter case they pay a set-up cost for each of the platforms that intermediate their product.

and

$$s_h(t_h, n_h) = [(1 - t_h)\rho]^{\frac{\sigma-1}{\sigma}} n_h^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}}. \quad (2)$$

The impact of a positive commission is to increase the price of the intermediated commodities, thus reducing their consumption (i.e., seller size) for given platform expenditure and number of sellers. Also note that the quality index s_h is increasing with respect to n_h and decreasing with respect to t_h. In particular:

$$\frac{\partial s_h(t_h, n_h)}{\partial t_h} = -\rho^2 \frac{s_h(t_h, n_h)}{[(1-t_h)\rho]} < 0, \quad \frac{\partial^2 s_h(t_h, n_h)}{\partial t_h \partial t_h} = -\frac{\rho^3}{\sigma} \frac{s_h(t_h, n_h)}{[(1-t_h)\rho]^2} < 0.$$

The variable profit of each seller is given by:

$$\pi_{hi}(t_h, E_h, n_h) = \frac{(1-t_h)\rho E_h}{(2\sigma - 1) n_h}.$$

Note that profit π_{hi} is not monotonic with respect to ρ (for given t_h, E_h and n_h an increase of ρ raises π_{hi} if and only if $\rho \geq \sqrt{2} - 1$), due to the double role played by this parameter.[5]

3.2 Cournot competition among platforms

To model competition among platform we use Cournot competition:[6] inverse demand

$$p_{y_h}(\mathbf{y}, \mathbf{t}, \mathbf{n}) = \frac{s_h(n_h, t_h) E / (\rho + 1)}{s_1(n_1, t_1) y_1 + s_2(n_2, t_2) y_2}$$

is well defined, while revenue E_h spent by consumer on platform h is given by:

$$E_h(\mathbf{y}, \mathbf{t}, \mathbf{n}) = (\rho + 1) p_{y_h}(\mathbf{y}, \mathbf{t}, \mathbf{n}) y_h.$$

Profit of platform h with constant unit cost d is thus given by:

$$\begin{aligned} \Pi_h(\mathbf{y}, \mathbf{t}, \mathbf{n}) &= (p_{y_h}(\mathbf{y}, \mathbf{t}, \mathbf{n}) - d) y_h + \frac{t_h \rho E_h(\mathbf{y}, \mathbf{t}, \mathbf{n})}{(\rho + 1)} \\ &= \kappa(t_h) \frac{s_h(n_h, t_h) y_h}{s_1(n_1, t_1) y_1 + s_2(n_2, t_2) y_2} - d y_h, \end{aligned}$$

where $\kappa(t_h) = \frac{E}{\rho+1}(1 + t_h \rho)$ accounts for the additional revenue platform h obtains through the commission t_h. Note that $\kappa(t_h) = \frac{E}{\rho+1}$ if $t_h = 0$ and $\kappa''(t_h) = 0$, and that all the results at the platform level depend on the relative quality offered, $s_h(n_h, t_h) / s_{-h}(n_{-h}, t_{-h})$.

Direct differentiation shows that $\frac{\partial^2 \Pi_h}{(\partial y_h)^2}, \frac{\partial^2 \Pi_h}{(\partial t_h)^2} < 0$: the determinant of the Hessian $D^2_{y_h, t_h} \Pi_h$ is rather involved, but one can show that it is certainly

[5] The two roles of parameter ρ could be easily distinguished by generalizing (1) to $U_h(y_h, \mathbf{x}_h) = y_h \left[\left(\sum_{i=1}^{n_h} x_{hi}^{\rho} \right)^{\frac{1}{\rho}} \right]^{\beta}$, with $1 > \beta > 0$.

[6] As in the quality-augmented model by Sutton (1991).

positive, and then Π_h is locally (strictly) concave with respect to (y_h, t_h), if $s_1(n_1, t_1)\, y_1$ and $s_2(n_2, t_2)\, y_2$ are sufficiently close. The FOC $\frac{\partial \Pi_h}{\partial y_h} = 0$ gives:

$$\frac{\kappa(t_h)\, s_h}{s_1 y_1 + s_2 y_2} - \frac{\kappa(t_h)\, s_h^2 y_h}{(s_1 y_1 + s_2 y_2)^2} = d,$$

i.e.,

$$(s_1 y_1 + s_2 y_2) - s_h y_h = \frac{d}{\kappa(t_h)\, s_h} (s_1 y_1 + s_2 y_2)^2 .$$

Adding up across platforms,

$$s_1 y_1 + s_2 y_2 = d\,(s_1 y_1 + s_2 y_2)^2 \left(\frac{1}{\kappa(t_1)\, s_1} + \frac{1}{\kappa(t_2)\, s_2} \right),$$

and

$$s_1 y_1 + s_2 y_2 = \frac{1}{d\left(\frac{1}{\kappa(t_1) s_1} + \frac{1}{\kappa(t_2) s_2} \right)}.$$

Thus

$$\begin{aligned} \widetilde{p}_{y_h}(\mathbf{t}, \mathbf{n}) &= \frac{s_h(n_h, t_h)\, Ed \left(\frac{1}{\kappa(t_1) s_1(n_1, t_1)} + \frac{1}{\kappa(t_2) s_2(n_2, t_2)} \right)}{(\rho + 1)} \\ &= d \left(\frac{1}{k(t_h)} + \frac{s_h(n_h, t_h)}{k(t_{-h})\, s_{-h}(n_{-h}, t_{-h})} \right), \end{aligned}$$

where $k(t_h) = (1 + t_h \rho)$. Note that $k(t_h) = 1$ if $t_h = 0$, and that an increase of t_h reduces the profit maximizing price of the services of platform h (for a given behavior of the competing platform and a given number of sellers n_h) both by increasing $k(t_h)$ and reducing s_h.

In addition, manipulating the previous FOC

$$s_h y_h = \frac{k(t_h)^2\, s_h(n_h, t_h)^2\, \kappa(t_{-h})\, s_{-h}(n_{-h}, t_{-h})}{d\,[k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)]^2},$$

thus

$$\widetilde{y}_h(\mathbf{t}, \mathbf{n}) = \frac{k(t_h)^2\, s_h(n_h, t_h)\, \kappa(t_{-h})\, s_{-h}(n_{-h}, t_{-h})}{d\,[k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)]^2},$$

$$\widetilde{E}_h(\mathbf{t}, \mathbf{n}) = (\rho + 1)\, p_{y_h} y_h = \frac{E k(t_h)\, s_h(n_h, t_h)}{k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)}, \qquad (3)$$

$$\widetilde{\Pi}_h(\mathbf{t}, \mathbf{n}) = \frac{\kappa(t_h)\, s_h(n_h, t_h)^2\, k(t_h)^2}{[k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)]^2}, \qquad (4)$$

and

$$\widetilde{\pi}_{hi}(\mathbf{t}, \mathbf{n}) = \frac{(1 - t_h)\, \rho E}{(2\sigma - 1)\, n_h} \frac{k(t_h)\, s_h(n_h, t_h)}{k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)}. \qquad (5)$$

Notice that the ratio $k(t_h) s_h(n_h, t_h) / [k(t_{-h}) s_{-h}(n_{-h}, t_{-h})]$, which depends on $(\mathbf{t}, \mathbf{n})$, affects all these results.

Consider now the profit-maximizing choice of commission t_h: an increase of t_h raises platform revenue but decreases its quality index. From the FOC $\frac{\partial \Pi_h}{\partial t_h} = 0$, t_h must satisfy

$$y_h \frac{\left[\kappa'(t_h) s_h + \kappa(t_h) \frac{\partial s_h}{\partial t_h}\right][s_1 y_1 + s_2 y_2] - [\kappa(t_h) s_h] \frac{\partial s_h}{\partial t_h} y_h}{(s_1 y_1 + s_2 y_2)^2} = 0,$$

which becomes after some manipulation

$$\frac{1 - t_h}{1 + t_h \rho} = \frac{s_{-h}}{s_1 y_1 + s_2 y_2} y_{-h} = \frac{E_{-h}}{E}.$$

Accordingly, the profit-maximizing value of the commission depends on the expenditure distribution across platforms, with $t_h \in (0, 1)$ for $E_{-h}/E \in (0, 1)$.

3.3 A symmetric equilibrium with free entry

Under free entry with a setup cost F it must be the case that[7]

$$n_h = \frac{(1 - t_h) \rho E_h}{(2\sigma - 1) F}. \tag{6}$$

Note that n_h depends on the behavior of the competing platform through E_h.

In a symmetric equilibrium $E_h = E/2$ and then $t_h = (\rho + 2)^{-1}$, with

$$p_{hi} = \frac{(\rho + 2) c}{(\rho + 1) \rho}, \; \pi_{hi} = \frac{\rho E}{2(\rho + 2) \sigma n_h}.$$

Thus

$$n_h = \frac{\rho E}{2(\rho + 2) \sigma F}, \quad x_{hi} = \frac{(1 - t_h) \rho^2 E_h}{(\rho + 1) n_h c} = \frac{(\sigma - 1) F}{c}.$$

A positive commission increases the price of the commodities sold through the platforms, and reduces their equilibrium number of sellers. However, due to the CES structure, seller size is unaffected by the commission.

In addition,

$$s_h = \frac{\rho (\rho + 1)^\rho}{(\rho + 2)} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}},$$

$$p_{y_h} = \frac{(\rho + 2) d}{1 + \rho}, \; y_h = \frac{E}{2d(\rho + 2)}, \; \Pi_h = \frac{E}{2(\rho + 2)}.$$

[7] As usual in the literature, for the sake of simplicity we treat the equilibrium number of firms as a continuous variable.

3.3.1 Discussion

In our setting the use of commissions by platforms increases their profits (from an equilibrium value without commissions of $\Pi_h^0 = \frac{E}{4(\rho+1)}$) by allowing then to appropriate some of the surplus created by the commodities they are intermediating. This worsens the quality index of the platforms, both by raising the price of the products intermediated and by decreasing sellers' variety. However, it also decreases the price of its services from $p_{y_h}^0 = 2d$, raising their consumption from $y_h^0 = \frac{E}{4(1+\rho)d}$.

Overall, since without any commission free-entry would deliver (in a symmetric equilibrium):

$$n_h^0 = \frac{\rho E}{2(2\sigma - 1)F},$$

and then

$$s_h^0 = \frac{\rho}{(\rho+1)^{1-\rho}} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}},$$

we get a worsening of the *price-adjusted quality index*

$$\frac{s_h}{p_{y_h}} = \frac{\rho(\rho+1)^{1+\rho}}{d(\rho+2)^2} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}} < \frac{\rho}{2d(\rho+1)^{1-\rho}} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}} = \frac{s_h^0}{p_{y_h}^0},$$

and thus of consumer welfare with respect to the benchmark (second-best) case of no commissions.

3.4 Commitment

Suppose now that, instead of determining them simultaneously to the prices of their services, platforms can set commissions in advance. Then commissions can be used strategically also to affect sellers' entry. In particular, by using the second-stage equilibrium value of sellers' profit (5) and the quality index definition (2), free entry implies:

$$\begin{aligned} n_h &= \frac{(1-t_h)\rho}{(2\sigma-1)F} \frac{Ek(t_h)s_h(n_h,t_h)}{[k(t_2)s_2(n_2,t_2)+k(t_1)s_1(n_1,t_1)]} \qquad (7)\\ &= \frac{\rho E}{(2\sigma-1)F} \frac{1-t_h}{1+\frac{1+t_{-h}\rho}{1+t_h\rho}\left(\frac{1-t_{-h}}{1-t_h}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma}}} \qquad (8) \end{aligned}$$

for $h = 1, 2$, and

$$\frac{n_h}{n_{-h}} = \left(\frac{1-t_h}{1-t_{-h}}\right)^{\frac{2\sigma-1}{\sigma-1}} \left(\frac{1+t_h\rho}{1+t_{-h}\rho}\right)^{\frac{\sigma}{\sigma-1}}.$$

Substituting last expression into (8) we obtain the second-stage equilibrium number of sellers:

$$\widehat{n}_h(\mathbf{t}) = \frac{\rho E}{(2\sigma - 1) F} \frac{1 - t_h}{1 + \left[\frac{(1-t_{-h})(1+t_{-h}\rho)}{(1-t_h)(1+t_h\rho)}\right]^{\frac{\sigma}{\sigma-1}}}, \tag{9}$$

showing that it is decreasing with respect to t_h and increasing with respect to t_{-h}, as one should expect.

Finally, notice that (7) and (4) imply that we can write the second-stage equilibrium profit of platform h as:

$$\Pi_h = \frac{(2\sigma - 1)\sigma F^2}{\rho^2 E} \frac{(1 + t_h\rho) n_h^2}{(1 - t_h)^2}.$$

It follows by using (9) that the first-stage, reduced-form for the profit of platform h, which is increasing with respect to t_{-h}, is given by

$$\widehat{\Pi}_h(\mathbf{t}) = \frac{E}{\rho + 1} \frac{1 + t_h\rho}{\left\{1 + \left[\frac{(1-t_{-h})(1+t_{-h}\rho)}{(1-t_h)(1+t_h\rho)}\right]^{\frac{\sigma}{\sigma-1}}\right\}^2}, \tag{10}$$

with

$$\frac{\partial \widehat{\Pi}_h}{\partial t_h} = \frac{\rho E}{\rho + 1} \frac{1 + \left[\frac{(1-t_{-h})(1+t_{-h}\rho)}{(1-t_h)(1+t_h\rho)}\right]^{\frac{\sigma}{\sigma-1}} \left[1 - 2\frac{1-\rho+2t_h\rho}{1-t_h}\right]}{\left\{1 + \left[\frac{(1-t_{-h})(1+t_{-h}\rho)}{(1-t_h)(1+t_h\rho)}\right]^{\frac{\sigma}{\sigma-1}}\right\}^3}.$$

Since computation shows that the SOC $\frac{\partial^2 \widehat{\Pi}_h}{\partial t_h^2} < 0$ is always satisfied, in a symmetric sub-game perfect Nash equilibrium we get:

$$t_h = \frac{\rho}{1 + 2\rho},\ n_h = \frac{\rho E}{2\sigma(1 + 2\rho) F},\ p_{hi} = \frac{(1 + 2\rho) c}{\rho(1 + \rho)},$$

$$s_h = \frac{\rho(1 + \rho)^\rho}{(1 + 2\rho)} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}},$$

$$p_{y_h} = \frac{2(1 + 2\rho) d}{(1 + \rho)^2},\ y_h = \frac{(1 + \rho) E}{4d(1 + 2\rho)},\ \Pi_h = \frac{(1 + \rho) E}{4(1 + 2\rho)}.$$

3.4.1 Discussion

Competition through commissions to attract sellers decreases their equilibrium value and then prices, raising the number of sellers under free entry. This raises the quality index of platforms, but it also raises the equilibrium price they ask for their services, and decreases their supply. All in all, platform profits decrease, but since

$$\frac{s_h}{p_{y_h}} = \frac{\rho(1 + \rho)^{2+\rho}}{2d(1 + 2\rho)^2} \left[\frac{E}{2\sigma F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}}$$

their *price-adjusted quality index* increases under commitment, and so consumer welfare. However, it would be still better for the representative consumer that the platforms could not ask for commissions.

3.5 Bertrand competition among platforms

It is worth discussing briefly the case in which within our framework platforms engage in Bertrand competition. Remember that $s_h(n_h, t_h)$ is a decreasing function with respect to t_h. In a two-stage setting, at the second stage price competition would lead the platform offering the best quality index, say $s_h = \max\{s_1(n_1, t_1), s_2(n_2, t_2)\}$, to use a price equal to:

$$p_{y_h} = \frac{s_h(n_h, t_h)}{s_{-h}(n_{-h}, t_{-h})} d.$$

Unless $s_1(n_1, t_1) = s_2(n_2, t_2)$ and $p_{y_h} = d$ too, platform $-h$ would then command no expenditure (and would attract no sellers under free entry). Accordingly, both platform would set $t_h = 0$ in the first stage, obtaining $\Pi_h = 0$ and delivering the symmetric allocation:

$$n_h = \frac{\rho E}{(2\sigma - 1) 2F},\ p_{hi} = \frac{c}{\rho},\ y_h = \frac{E}{2(\rho + 1) d},$$

with

$$\frac{s_h}{p_{y_h}} = \frac{\rho}{d(1+\rho)^{1-\rho}} \left[\frac{E}{\sigma 2F}\right]^{\frac{1}{\sigma}} c^{\frac{1-\sigma}{\sigma}} = 2\frac{s_h^0}{p_{y_h}^0}.$$

The same (optimal) allocation would arise under simultaneous price-commission choices and free entry.

4 Conclusions

In this paper we propose to use preferences by a representative consumer to investigate platform competition. In our suggested duopoly example, platforms offer perfect substitutes and compete *à la* Cournot, setting ad-valorem commissions on the goods they intermediate, provided under monopolistic competition by a large number of sellers. We have characterized the symmetric equilibrium in a setting with free entry of sellers and simultaneous platform choices, showing that the adoption of positive commissions worsen consumer welfare by increasing prices and reducing good variety. In the case in which platforms can commit in advance to their commission levels, we find that competition for attracting sellers improves the platform quality index (even though it also raises the price each platform requires for its services), and so consumer welfare. Since the model can be easily generalized to the case of many platforms (and possibly to some asymmetry among them) and/or considering different preferences over the services they provide and the commodities they intermediated, future research should investigate the robustness of our preliminary results. In particular, in the Appendix we sketch the analysis of an alternative setting, which exhibits a (perhaps surprising) neutrality of commission levels, due to perfect complementarity between the service provided by each platform and sellers' commodities (still differentiated *à la* Dixit and Stiglitz, 1977).[8]

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Appendix In this Appendix we sketch the analysis of an alternative utility tree, for which a set of sellers' commodities must be consumed together with a unit of the single service/good provided by each platform. In particular, suppose that

$$U_h\left(y_h, \mathbf{x}_h\right) = \min\left\{y_h, u\left(\mathbf{x}_h\right)\right\}, \tag{11}$$

where $u\left(\mathbf{x}_h\right) = \left[\sum_{i=1}^{n_h} x_{hi}^{\rho}\right]^{\frac{1}{\rho}}$ and $0 < \rho < 1$. In this case U_h has a "perfect complements" upper-tier structure and $u\left(\mathbf{x}\right)$ is the familiar CES quantity index: preferences are still homothetic, and demands are given by:

$$y_h\left(p_{y_h}, \mathbf{p}_h, E_h\right) = \frac{E_h}{T_h\left(p_{y_h}, \mathbf{p}_h\right)} = u\left(\mathbf{x}_h\left(p_{y_h}, \mathbf{p}_h, E_h\right)\right),$$

$$x_{hj}\left(p_{y_h}, \mathbf{p}_h, E_h\right) = \frac{P_h\left(\mathbf{p}_h\right) E_h}{T_h\left(p_{y_h}, \mathbf{p}_h\right)} \frac{p_{hj}^{-\sigma}}{\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}},$$

where $T_h\left(p_{y_h}, \mathbf{p}_h\right) = p_{y_h} + P_h\left(\mathbf{p}_h\right)$ can be interpreted as the overall "tariff" of platform h, and $P_h\left(\mathbf{p}_h\right) = \left[\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the price index which corresponds to $u\left(\mathbf{x}_h\right)$. The indirect utility function corresponding to (11) is given by:

$$V_h\left(p_{y_h}, \mathbf{p}_h, E_h\right) = \frac{E_h}{T_h\left(p_{y_h}, \mathbf{p}_h\right)}.$$

[8] This special setting may somehow capture the case of device-funded platforms, in which a single device provided by the platform can be combined with a set of related services (applications): see Etro (2021).

With 2 platforms, we get the overall utility functions:

$$
\begin{aligned}
U(\mathbf{y},\mathbf{x}) &= \min\left\{y_1, \left[\sum_{i=1}^{n1} x_{1i}^{\rho}\right]^{\frac{1}{\rho}}\right\} + \min\left\{y_2, \left[\sum_{i=1}^{n_2} x_{2i}^{\rho}\right]^{\frac{1}{\rho}}\right\}, \\
V(\mathbf{p}_y, \mathbf{p}, E) &= E \max\left\{\frac{1}{T_1(p_{y_1}, \mathbf{p}_1)}, \frac{1}{T_2(p_{y_2}, \mathbf{p}_2)}\right\}.
\end{aligned}
$$

Notice platforms are still providing perfect substitutes.

Monopolistic competition among sellers gives, by the CES structure:

$$
p_{hi}(t_h) = \frac{c}{\rho(1-t_h)}, \quad P_h(t_h, n_h) = \frac{c}{\rho(1-t_h) n_h^{\frac{1}{\sigma-1}}},
$$

and then

$$
x_{hi}(p_{y_h}, t_h, E_h, n_h) = \frac{E_h}{n_h^{\frac{1}{\rho}} T_h(p_{y_h}, t_h, n_h)},
$$

$$
\pi_{hi}(p_{y_h}, t_h, E_h, n_h) = \frac{c}{\sigma-1} \frac{E_h}{n_h^{\frac{1}{\rho}} T_h(p_{y_h}, t_h, n_h)}. \tag{12}
$$

The price index P_h increases with respect to t_h and decreases with respect to n_h. For given expenditure on the platform, E_h, and given price, p_{y_h}, both the sellers' sizes and profits are decreasing with respect to the number of sellers n_h and the commission level t_h.

Let us consider once again Cournot competition among platforms. Inverse demand must satisfy $T_1 = T_2$ and the budget constraint $T_1 y_1 + T_2 y_2 = E$, and thus it is provided by $(h = 1, 2)$:

$$
p_{y_h}(\mathbf{y}, t_h, n_h) = \frac{E}{y_1 + y_2} - P_h(t_h, n_h),
$$

with

$$
E_h(\mathbf{y}) = \frac{y_h E}{y_1 + y_2}, \quad T_h(\mathbf{y}) = \frac{E}{y_1 + y_2}.
$$

Notice that the platform price p_{y_h} must allow the representative consumer enough purchasing power to buy the (perfectly complementary) sellers commodities, and that the commission level t_h does directly affect neither T_h nor the platform expenditure E_h.

Profit of platform h (assuming for the sake of simplicity a null unit cost) is thus given by:

$$
\begin{aligned}
\Pi_h(\mathbf{y}, n_h) &= \left(p_{y_h}(\mathbf{y}, t_h, n_h) + t_h P_h(t_h, n_h)\right) y_h \\
&= E_h(\mathbf{y}) - \frac{c y_h}{\rho n_h^{\frac{1}{\sigma-1}}},
\end{aligned}
$$

and does not either directly depend on t_h (this commission neutrality is of course also due to the complete cost pass-through that characterizes the CES case). Moreover, since the FOC $\frac{\partial \Pi_h}{\partial y_h} = 0$ provides:

$$\frac{y_{-h} E}{(y_1 + y_2)^2} = \frac{c}{\rho n_h^{\frac{1}{\sigma-1}}},$$

adding up across platforms we get

$$\begin{aligned}
\widetilde{T}_h(\mathbf{n}) &= \frac{E}{y_1 + y_2} = \frac{c}{\rho}\left[n_h^{\frac{1}{1-\sigma}} + n_{-h}^{\frac{1}{1-\sigma}}\right], \quad y_1 + y_2 = \frac{\rho E}{c\left[n_h^{\frac{1}{1-\sigma}} + n_{-h}^{\frac{1}{1-\sigma}}\right]}, \\
\widetilde{y}_h(\mathbf{n}) &= \frac{n_{-h}^{\frac{1}{\sigma-1}} \rho E}{c\left[\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}} + 1\right]^2}, \quad \widetilde{p}_{y_h}(\mathbf{n}, t_h) = \frac{c}{\rho n_{-h}^{\frac{1}{\sigma-1}}}\left[1 - \frac{t_h \left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}}{(1-t_h)}\right], \\
\widetilde{E}_h(\mathbf{n}) &= \frac{E}{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}} + 1}, \quad \widetilde{\Pi}_h(\mathbf{n}) = \frac{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{1-\sigma}} E}{2 + \left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}} + \left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{1-\sigma}}},
\end{aligned}$$

and, from (12),

$$\widetilde{\pi}_{hi}(\mathbf{n}) = \frac{E}{\sigma} \frac{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}}{n_h\left[\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}} + 1\right]^2}.$$

These equilibrium results (which hold if platforms are not too asymmetric) show that in this setting there is a sort of superneutrality of the commission levels, that affect neither the overall platform tariffs (and then consumers) nor the profit levels. Commissions only changes the platform prices to offset their impacts on sellers' prices. Setting $t_h = 0$ we get, in a symmetric equilibrium of free entry (with set-up cost $F > 0$):

$$y_h = \frac{\rho E n_h^{\frac{1}{\sigma-1}}}{4c}, \quad p_{y_h} = \frac{c}{\rho n_h^{\frac{1}{\sigma-1}}} = P_h = \frac{T_h}{2}, \quad \Pi_h = \frac{E}{4},$$

$$p_{hi} = \frac{c}{\rho}, \quad n_h = \frac{E}{4\sigma F}, \quad x_{hi} = \frac{(\sigma-1)F}{c}.$$