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Differentiated goods in a dynamic Cournot duopoly with emission charges on outputs

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Abstract

We extend the dynamic Cournot duopoly framework with emission charges on outputs by Mamada and Perrings (2020), which encompassed homogeneous products in its original formulation, to the more general case of differentiated goods, in order to highlight the richness in its static and dynamic outcomes. In the model each firm is taxed proportionally to its own emission only and charge functions are quadratic. Moreover, due to an adjustment capacity constraint, firms partially modify their output level toward the best response. Like it happened in Mamada and Perrings (2020), the only model steady state coincides with the Nash equilibrium. We find that the full efficacy of the environmental policy, which applies to an equilibrium that is globally asymptotically stable anytime it is admissible, is achieved in the case of independent goods, as well as with a low interdependence degree between goods in absolute value, independently of being substitutes or complements. On the other hand, when goods are substitutes and their interdependence degree is high, the considered environmental policy is still able to reduce pollution at the equilibrium, but the latter is stable just when the policy intensity degree is high enough. When instead goods are complements and their interdependence degree is high in absolute value, the considered environmental policy produces detrimental effects on the pollution level and the unique equilibrium is always unstable, when admissible. This highlights that, from a static viewpoint, even in the absence of free riding possibilities, the choice of the mechanism to implement has to be carefully pondered, according to the features of the considered economy.

Keywords: dynamic Cournot duopoly, differentiated products, emission charges, pollution control, comparative statics, stability analysis.

JEL classification: C62, D43, Q51, Q58

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1 Introduction

In the past decades, environmental policies to control pollution have been proposed in several works, such as Segerson (1988), Katsoulacos and Xepapadeas (1995) and Suter et al. (2008). In particular, some of them are based on both charges and incentives, according to a comparison between the aggregate emission level and the ambient standard, mainly in the case of non-point source (NPS) pollution, like for instance in Ganguli and Raju (2012), Matsumoto and Szidarovszky (2021) and Matsumoto et al. (2018), while other ones are based just on charges, as it happens e.g. in Mamada and Perrings (2020). Indeed, Mamada and Perrings (2020) propose a Cournot duopoly with homogeneous products where the environmental policy is characterized by a tax, for each firm, proportional to its own emission only and charge functions are quadratic. Moreover, due to an adjustment capacity constraint, firms partially modify their output level toward the best response. Following the suggestion contained in the concluding remarks by Mamada and Perrings (2020), in the present contribution we investigate the static and dynamic effects produced by the introduction of differentiated goods in the framework proposed therein, finding a much richer variety of results than dealing with homogeneous products. Namely, for their setting Mamada and Perrings (2020) showed that a unique steady state exists, coinciding with the Nash equilibrium, which is stable when the policy intensity degree is high enough, i.e., for instance when marginal emission charges are increasing in emissions, while it is unstable when the policy is too soft. According to Mamada and Perrings (2020), the latter case, characterized by the system instability, possibly allows for the transition from a duopolistic to a monopolistic framework. Furthermore, in the homogeneous good context it holds that the policy is effective, from a static viewpoint, in reducing pollution. On the other hand, a comparative statics result is economically grounded if it concerns an equilibrium which is asymptotically stable and thus orbits converge towards it after a transient period. As recalled above, this is not the case with homogeneous goods when the policy intensity degree is low.
Although also in the differentiated product framework there exists a unique steady state, which coincides with the Nash equilibrium, different scenarios may occur in regard to the efficacy of the considered environmental policy and the stability of the steady state, according to the fact that goods are complements or substitutes, as well as according to the strength of the interdependence degree between the two goods and of the environmental policy. In more detail, we find that when goods are substitutes the considered environmental policy is effective, from a static viewpoint, in reducing pollution and the steady state may be always stable, when admissible, or the stabilizing scenario can occur, with the steady state being stable just when the policy intensity level is sufficiently high. In particular, when products are independent the former possibility arises, with a consequent full efficacy of the environmental policy, while, within the framework of substitutes, the case of

homogeneous goods, analyzed in Mamada and Perrings (2020), turns out to be the less favorable to assess the efficacy of the considered environmental policy, due to the reduced equilibrium stability interval. On the other hand, with complements much worse situations can occur when the interdependence degree between goods is high in absolute value. Indeed, when goods are complements two different frameworks may take place, according to the strength of the interdependence degree between the two products. When the interdependence degree is low in absolute value, the considered environmental policy is effective, from a static viewpoint, in reducing pollution and the equilibrium is always stable, when admissible, like in the case of independent products. This means that the full efficacy of the environmental policy, which applies to an equilibrium that is globally asymptotically stable anytime it is admissible, is achieved in the case of independent goods, as well as with a low interdependence degree between goods in absolute value, independently of being substitutes or complements. When instead goods are complements and the interdependence degree between them is high in absolute value, the considered environmental policy produces a negative effect on the equilibrium pollution level, which increases, and the unique steady state is always unstable, when admissible. Such comparative statics finding highlights that, even in the absence of free riding possibilities, the choice of the mechanism to implement has to be carefully pondered, according to the features of the considered economy. Namely, even if it would seem natural to expect that higher charges lower the emission volume, sometimes counterintuitive outcomes may be observed, especially in frameworks in which the aggregate pollution level is taken into account. A similar situation occurs for instance in Ganguli and Raju (2012), where a higher emission volume may come as a consequence of increased charges in a Bertrand duopoly setting. Indeed, the framework in Ganguli and Raju (2012) encompasses strategic interactions between firms both on the demand side in the market and in regard to emissions in the environmental sphere. The latter aspect leads to a public good game setting, which may give rise to free riding possibilities. On the contrary, we here prove that a similar detrimental effect occurs in the framework by Mamada and Perrings (2020), in the absence of strategic interactions, when goods are complements and the interdependence degree between them is high in absolute value. We stress that the relevance of the latter result is limited by the instability of the Nash equilibrium in the corresponding scenario, in the presence of the mechanism of gradual adjustment towards the best response considered in Mamada and Perrings (2020). On the other hand, the significance of the steady state should be further assessed by dealing with other adjustment mechanisms that would still make the model dynamic in nature, such as the local monopolistic approximation introduced in Bischi et al. (2007) or the gradient rule discussed in Bischi et al. (2010).

The remainder of the paper is organized as follows. In Section 2 we present the model with differentiated products, comparing it with the homogeneous product framework in Mamada and Perrings (2020). In Section 3 we perform the model analysis, focusing on substitutes in Subsection 3.1 and on complements in Subsection 3.2. In Section 4 we conclude.

2 The model

Following the duopoly formulation in Mamada and Perrings (2020), but supposing that the goods produced by the two firms are differentiated, we assume that in each time period t firm $i \in \{1, 2\}$ maximizes the profit function

$$\pi_{i,t} = (p - \beta q_{i,t} - \gamma q_{j,t})q_{i,t} - cq_{i,t}^2 - C_{i,t}^e \tag{2.1}$$

where $q_{i,t}$ and $q_{j,t}$ are the output levels by firms i and j, respectively, with $i \neq j \in \{1, 2\}$, and for the chock price p and the production costs c we suppose that they are positive like in Mamada and Perrings (2020). In regard to β and γ, as usual in the case of differentiated goods, we assume that $|\gamma| < \beta$: if $\gamma > 0$ (resp. $\gamma < 0$) the two goods are substitutes (resp. complements), while they are independent if $\gamma = 0$. We recall that the framework with homogeneous products is obtained as limit case with $\gamma = \beta = k$, where k is the price-depressing effect of oligopoly. Cf. Singh and Vives (1984) and Motta (2004) for further details.

Denoting by $\varepsilon > 0$ emissions per unit output, so that $u_{i,t} = \varepsilon q_{i,t}$ are emissions by firm $i \in \{1, 2\}$ at time t, Mamada and Perrings (2020) propose the following quadratic formulation for emission charges

$$C_{i,t}^e = bu_{i,t} + \frac{1}{2}du_{i,t}^2, \tag{2.2}$$

with $b > 0$ and $d \in \mathbb{R}$, that we will consider, too. Since the marginal emission charge is given by $\frac{dC_{i,t}^e}{du_{i,t}} = b + du_{i,t}$, it may be increasing or decreasing according to the sign of d. In particular, if d is negative, the condition

$$0 < q_{i,t} < \frac{-b}{\varepsilon d} \tag{2.3}$$

is needed to guarantee the positivity of the marginal emission charge.

Like in Mamada and Perrings (2020), we assume that, due to an adjustment capacity constraint, firms modify the output level according to (the size and the extent of) the difference between their best response and their current output level with a reactivity parameter $\lambda \in (0, 1)$, so that

$$q_{i,t+1} = q_{i,t} + \lambda(R_i(q_{j,t}) - q_{i,t}) \tag{2.4}$$

where $R_i(q_j)$ is the best response function of firm i to the output q_j produced by firm j. Notice that there is no adjustment when $\lambda = 0$, while adjustment is complete and instantaneous when $\lambda = 1$. Although overadjustment, given by $\lambda > 1$, is possible, Mamada and Perrings (2020) disregard such eventuality

and we will stick to their choice, too. We also stress that the mechanism in (2.4) is just one of the possible adjustment rules. See Bischi et al. (2010) for dynamical aspects connected with oligopoly models.
From (2.1) it follows that

$$\frac{\partial \pi_{i,t}}{\partial q_{i,t}} = p - b\varepsilon - (2(\beta + c) + d\varepsilon^2)q_{i,t} - \gamma q_{j,t} \tag{2.5}$$

for $i \neq j \in \{1, 2\}$, and thus profits are strictly concave when

$$2(\beta + c) + d\varepsilon^2 > 0. \tag{2.6}$$

This condition is satisfied for every $d > 0$, in which case C_i^e in (2.2) is convex, for $i \in \{1, 2\}$, as well as for $d \in \left(-\frac{2(\beta+c)}{\varepsilon^2}, 0\right)$, that is, when C_i^e is concave, but production variations lead to emission charge variations close to those that we would have in the linear case, corresponding to $d = 0$. We will maintain assumption (2.6) along the manuscript, even when not explicitly mentioned. Moreover, setting $\partial \pi_{i,t}/\partial q_{i,t} = 0$, from (2.5) we find

$$R_i(q_{j,t}) = \frac{p - \gamma q_{j,t} - b\varepsilon}{2(\beta + c) + d\varepsilon^2} \tag{2.7}$$

as best response function for $i \neq j \in \{1, 2\}$, which is well defined under (2.6), so that the unique (symmetric) Nash equilibrium is given by

$$(q_1^*, q_2^*) = \left(\frac{p - b\varepsilon}{2(\beta + c) + d\varepsilon^2 + \gamma}, \frac{p - b\varepsilon}{2(\beta + c) + d\varepsilon^2 + \gamma}\right). \tag{2.8}$$

In the case of homogeneous goods, the Nash equilibrium becomes

$$(\bar{q_1}^*, \bar{q_2}^*) = \left(\frac{p - b\varepsilon}{3k + 2c + d\varepsilon^2}, \frac{p - b\varepsilon}{3k + 2c + d\varepsilon^2}\right). \tag{2.9}$$

In order to ensure the positivity for (2.9), in Mamada and Perrings (2020) it is supposed that $p > b\varepsilon$ and that $3k + 2c + d\varepsilon^2 > 0$, since this is the only case compatible with (2.6) when goods are homogeneous. More generally, we will see in Subsection 3.1 that the only case which may occur with substitutes is $p > b\varepsilon$ and $2(\beta + c) + d\varepsilon^2 + \gamma > 0$, which extends the framework analyzed in Mamada and Perrings (2020). On the other hand, in Subsection 3.2 we will understand that with complements also the case $p < b\varepsilon$ and $2(\beta + c) + d\varepsilon^2 + \gamma < 0$ has to be taken into account. We stress that all such conditions have to be considered jointly with (2.6), as well as with the constraints coming from (2.3) at the Nash equilibrium. For instance, notice that the Nash equilibrium in (2.9), for the homogeneous good setting, fulfills the right constraint in (2.3) if $d \in \left(-\frac{b(3k+2c)}{\varepsilon p}, 0\right)$. In Section 3, taking into account (2.3) and (2.6), we will derive similar conditions for the differentiated product case under the various assumptions on the parameters, analyzing

also, when suitable, what happens in the limit case in which products are homogeneous.
Supposing that firms partially adjust their output level toward the best response according to (2.4) with $\lambda \in (0,1)$, when inserting (2.7) therein we obtain the dynamical system

$$\begin{cases} q_{1,t+1} = q_{1,t} + \lambda(R_1(q_{2,t}) - q_{1,t}) = q_{1,t} + \lambda\left(\frac{p-\gamma q_{2,t}-b\varepsilon}{2(\beta+c)+d\varepsilon^2} - q_{1,t}\right) \\ q_{2,t+1} = q_{2,t} + \lambda(R_2(q_{1,t}) - q_{2,t}) = q_{2,t} + \lambda\left(\frac{p-\gamma q_{1,t}-b\varepsilon}{2(\beta+c)+d\varepsilon^2} - q_{2,t}\right) \end{cases} \tag{2.10}$$

whose only steady state is given by the Nash equilibrium in (2.8), and which coincides with the model (2) on page 373 in Mamada and Perrings (2020) for $\gamma = \beta = k$.

3 Analysis

We split the analysis of the model described by (2.10) according to the sign of the parameter γ, measuring the interdependence degree between goods, since the focus of the paper is on the static and dynamic effects produced by the extension of the setting in Mamada and Perrings (2020) to the case of differentiated goods. In particular, in Subsection 3.1 we consider what occurs when γ is non-negative and thus goods are substitutes or independent, while in Subsection 3.2 we focus on the scenario in which γ is negative and hence goods are complements. We stress that the homogeneous goods case is encompassed as limit case in the framework investigated in Subsection 3.1.

3.1 The case of substitutes

In the present subsection, we focus on the case $\gamma \geq 0$. Then, recalling (2.6), it holds that $2(\beta+c)+d\varepsilon^2+\gamma > 0$, and thus, in order to ensure the positivity of the Nash equilibrium in (2.8), we need that

$$p - b\varepsilon > 0. \tag{3.1}$$

Under such conditions, we present our results about comparative statics (cf. Proposition 3.1) and about the model dynamics (see Proposition 3.2).

Proposition 3.1 *When $\gamma \geq 0$, under (2.6) and (3.1), in regard to (q_1^*, q_2^*) in (2.8) it holds that, for $i \in \{1,2\}$, q_i^* decreases when b or d increase.*

Proof. The desired conclusions immediately follow by observing that, with reference to (2.8), for $i \in \{1,2\}$ it holds that

$$\frac{\partial {q_i}^*}{\partial b} = \frac{-\varepsilon}{2(\beta+c)+d\varepsilon^2+\gamma}, \qquad \frac{\partial {q_i}^*}{\partial d} = \frac{-(p-b\varepsilon)\varepsilon^2}{(2(\beta+c)+d\varepsilon^2+\gamma)^2}. \tag{3.2}$$

$\square$

In particular, this result, highlighting the efficacy from a static viewpoint, with substitutes and independent goods, of the environmental policy described by the emission charges in (2.2) under (2.6) and (3.1), holds true also in the homogeneous product framework in Mamada and Perrings (2020).
On the other hand, when dealing with complements, under different conditions on the parameters (cf. (3.5)), we will find in Subsection 3.2 that opposite comparative statics outcomes may arise in regard to b and d (see Proposition 3.5). Noticing that

$$\frac{\partial q_i{}^*}{\partial \varepsilon} = \frac{-b(2(\beta+c)+d\varepsilon^2+\gamma)-(p-b\varepsilon)2d\varepsilon}{(2(\beta+c)+d\varepsilon^2+\gamma)^2},$$

the same is true also with respect to parameter ε, connected with the produced pollution level, too, as an increase in it describes the transition for firms to more polluting technologies.
In relation to the model dynamic behavior, we have the following result about the stability of the Nash equilibrium:

Proposition 3.2 *When* $\gamma \geq 0$, *under* (2.6) *and* (3.1), (q_1^*, q_2^*) *is admissible according to* (2.3) *for* $d > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$. *If this is the case, it is globally asymptotically stable for System* (2.10) *when* $d > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$.

Proof. We investigate the system stability by using the well-known Jury conditions

$$(i)\ \det(J) < 1, \quad (ii)\ 1+\operatorname{tr}(J)+\det(J) > 0, \quad (iii)\ 1-\operatorname{tr}(J)+\det(J) > 0,$$

where

$$J = \begin{bmatrix} 1-\lambda & \frac{-\lambda\gamma}{2(\beta+c)+d\varepsilon^2} \\ \frac{-\lambda\gamma}{2(\beta+c)+d\varepsilon^2} & 1-\lambda \end{bmatrix}$$

is the Jacobian matrix for (2.10), and $\det(J)$, $\operatorname{tr}(J)$ denote its determinant and trace, respectively.[1] Thus, we have $\det(J) = 1-2\lambda+\lambda^2\Big(1-\frac{\gamma^2}{(2(\beta+c)+d\varepsilon^2)^2}\Big)$ and $\operatorname{tr}(J) = 2-2\lambda$. Hence, (iii) reads as

$$1-\frac{\gamma^2}{(2(\beta+c)+d\varepsilon^2)^2} > 0. \tag{3.3}$$

Since $\lambda \in (0,1)$, conditions (i) and (ii) are then always fulfilled. Condition (3.3) can be rewritten as $(2(\beta+c)+d\varepsilon^2-\gamma)(2(\beta+c)+d\varepsilon^2+\gamma) > 0$, which implies that $2(\beta+c)+d\varepsilon^2-\gamma > 0$, since $2(\beta+c)+d\varepsilon^2+\gamma > 0$ under the maintained assumptions. Observing that in the considered case the conditions in (2.3) lead to $d > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$, the desired conclusion follows. $\square$

[1]Notice that, differently from d, parameter e plays no role on the stability of the Nash equilibrium, being not present in J.

We stress that we have analyzed the system stability because a comparative statics result is economically grounded if it concerns an equilibrium which is asymptotically stable and thus orbits converge towards it after a transient period.
Considering the homogeneous product setting, the stability condition $d > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$, found in Proposition 3.2, becomes $d > -\frac{k+2c}{\varepsilon^2}$ in agreement with the result on page 374 in Mamada and Perrings (2020), which highlights the stabilizing role of d on $(\bar{q_1}^*, \bar{q_2}^*)$ in (2.9). Together with the constraints coming from (2.3) discussed in Section 2, we can conclude that $(\bar{q_1}^*, \bar{q_2}^*)$ is stable and admissible for $d > \max\{-\frac{b(3k+2c)}{\varepsilon p}, -\frac{k+2c}{\varepsilon^2}\}$. Notice that $-\frac{b(3k+2c)}{\varepsilon p} > -\frac{k+2c}{\varepsilon^2}$ for $b\varepsilon < p\left(\frac{k+2c}{3k+2c}\right)$, in which case the Nash equilibrium is always stable when it is admissible according to (2.3). More generally, in the case of differentiated products, it holds that the stability threshold found in Proposition 3.2 is not admissible according to (2.3), i.e., $-\frac{b(2\beta+2c+\gamma)}{\varepsilon p} > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$, when $b\varepsilon < p\left(\frac{2\beta+2c-\gamma}{2\beta+2c+\gamma}\right)$. Under (3.1), the latter condition could be fulfilled when the two products are substitutes, while it is granted in the case of independent goods. In such positive eventualities, the comparative statics results reported in Proposition 3.1, which show that the environmental policy described by the emission charges in (2.2) is effective in reducing pollution under (3.1), are robustly grounded from an economic viewpoint, since the Nash equilibrium is a global attractor.
Summarizing, we can say that in the case of substitutes the considered environmental policy is always effective from a static viewpoint. From a dynamic perspective, Proposition 3.2, which supports the comparative statics results obtained in Proposition 3.1, shows that when d is large enough, i.e., when the policy intensity degree is sufficiently high, orbits converge towards the steady state after a transient period. When the convergence is reached, raising d further leads to a decrease in emissions and to a full efficacy of the environmental policy. In particular, since for $b\varepsilon < p\left(\frac{2\beta+2c-\gamma}{2\beta+2c+\gamma}\right)$ it holds that the Nash equilibrium is always stable anytime it is admissible according to (2.3), we notice that such condition is more easily verified for small non-negative values of the interdependence degree between goods,[2] as the term on the right-hand side is decreasing in γ. Indeed, it is granted in the case of independent goods under (3.1). Hence, within the framework of substitutes, the case of homogeneous goods, analyzed in Mamada and Perrings (2020), turns out to be the less favorable to assess the efficacy of the considered environmental policy, due to the reduced equilibrium stability interval. On the other hand, as we shall see in the next subsection, with complements much worse situations can occur when the interdependence degree between goods is high in absolute value (cf. Propositions 3.5 and 3.6).

[2]According to Proposition 3.4, a similar phenomenon holds true in the case of complements with a low interdependence degree in absolute value.

3.2 The case of complements

In the present subsection, we deal with the case $\gamma < 0$. Accordingly, under (2.6), two different scenarios ensure the positivity of the Nash equilibrium in (2.8), i.e.,

$$p - b\varepsilon > 0, \qquad 2(\beta + c) + d\varepsilon^2 + \gamma > 0, \tag{3.4}$$

or

$$p - b\varepsilon < 0, \qquad 2(\beta + c) + d\varepsilon^2 + \gamma < 0. \tag{3.5}$$

The former scenario, in which (2.6) is granted, leads to findings similar to, but in regard to dynamics not coinciding with, those described in Subsection 3.1. Namely, recalling (3.2), the following result about comparative statics holds true.

Proposition 3.3 *When $\gamma < 0$, under (3.4), in regard to (q_1^*, q_2^*) in (2.8) it holds that, for $i \in \{1, 2\}$, q_i^* decreases when b or d increase.*

Hence, like it happened in Proposition 3.1 the environmental policy described by the emission charges in (2.2) is still effective, from a static viewpoint, in reducing pollution when dealing with complements under (3.4), i.e., when the interdependence degree between the two goods is low in absolute value. On the other hand, under the same assumptions, in regard to the system dynamic behavior we have an even better situation than that described in Proposition 3.2, as the following result shows.

Proposition 3.4 *When $\gamma < 0$, under (3.4), (q_1^*, q_2^*) is admissible according to (2.3) for $d > -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$. If this is the case, it is always globally asymptotically stable for System (2.10).*

Proof. Following the same steps in the proof of Proposition 3.2, we find that, for $\gamma < 0$, under (3.4), (q_1^*, q_2^*) is globally asymptotically stable for System (2.10) when $d > -\frac{2\beta+2c-\gamma}{\varepsilon^2}$. However, such condition is granted under (3.4). □

Thus, like in the case of independent goods, the equilibrium is always stable when it is admissible also in the case of complements under (3.4), and the full efficacy of the considered environmental policy still holds true.

Dealing instead with (3.5), i.e., when the interdependence degree between the two goods is high in absolute value, since the partial derivatives in (3.2) are now positive, the considered environmental policy produces detrimental effects. Namely, pollution increases at the Nash equilibrium, as highlighted by the following:

Proposition 3.5 *Under (2.6) and (3.5), in regard to (q_1^*, q_2^*) in (2.8), it holds that, for $i \in \{1, 2\}$, q_i^* increases when b or d increase.*

Recalling that $\gamma \in (-\beta, 0)$ in the case of complements, it holds that (3.5) can be fulfilled just for negative values of d, in which case C_i^e in (2.2) is concave. We can then rephrase Proposition 3.5 by saying that we have proven the inefficacy, from the static viewpoint, of the environmental policy described by the emission charges C_i^e in (2.2), under suitable parameter configurations for which C_i^e is concave and emission charges increase too slowly with production. Also in relation to the model dynamic outcomes, things drastically change when assuming (3.5). Indeed, the following result holds true.

Proposition 3.6 *Under* (2.6) *and* (3.5), (q_1^*, q_2^*) *is admissible according to* (2.3) *for* $d < -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$. *If this is the case, it is always unstable for System* (2.10).

Proof. The proof follows similar steps to those used to check Proposition 3.2. In particular, from (3.3) we would obtain the stability condition $d < -\frac{2\beta+2c-\gamma}{\varepsilon^2}$, which however contradicts the maintained assumptions.
The admissibility condition $d < -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}$ follows from (2.3). □

Notice that the assumptions in Proposition 3.6 are jointly fulfilled for $d \in \left(-\frac{(2\beta+2c)}{\varepsilon^2}, -\frac{b(2\beta+2c+\gamma)}{\varepsilon p}\right)$, interval that may be empty for each value of $\gamma \in (-\beta, 0)$, in which case the equilibrium is never admissible[3], like it happens e.g. when parameter p, that is an index of the market size, is too low.
To sum up, in the case of complements two opposite situations occur, both in regard to the static efficacy of the environmental policy and the dynamic behavior of the system, according to the interdependence degree between goods. When the latter is low in absolute value, we observe a full efficacy of the considered environmental policy because pollution decreases at the Nash equilibrium, which is stable anytime it is admissible; on the contrary, if the interdependence degree between goods is high in absolute value, pollution increases at the Nash equilibrium, that is however never stable when admissible. Indeed, the relevance of the comparative statics result in Proposition 3.5 is limited by Proposition 3.6, which shows the instability of the Nash equilibrium in the corresponding scenario, in the presence of the mechanism of gradual adjustment towards the best response considered in Mamada and Perrings (2020). On the other hand, the significance of the steady state should be further assessed by dealing with other adjustment mechanisms that would still make the model dynamic in nature, such as the local monopolistic approximation introduced in Bischi et al. (2007) or the gradient rule discussed in Bischi et al. (2010).

[3]We stress that this cannot occur in the frameworks described by Propositions 3.2 and 3.4, since in those cases the conditions on d are all lower bounds.

4 Conclusions

In the present work we have shown the richness in the static and dynamic outcomes arising when replacing homogeneous goods with differentiated products in the dynamic Cournot duopoly framework with emission charges on outputs by Mamada and Perrings (2020), following the suggestion contained in their concluding remarks.

In more detail, we found that the full efficacy of the environmental policy considered therein, according to which each firm is taxed proportionally to its own emission only and charge functions are quadratic, is achieved in the case of independent goods, as well as with a low interdependence degree between products in absolute value, independently of being substitutes or complements. Namely, in such cases pollution decreases at the only model steady state, coinciding with the Nash equilibrium, which is globally asymptotically stable anytime it is admissible. On the other hand, when goods are substitutes and their interdependence degree is high, the considered environmental policy is still able to reduce pollution at the equilibrium, but the latter is stable just for a high enough policy intensity degree. When instead goods are complements and the interdependence degree between them is high in absolute value, the considered environmental policy produces a detrimental effect on the equilibrium pollution level, which increases. This highlights that, from a static viewpoint, even in the absence of free riding possibilities, the choice of the mechanism to implement has to be carefully pondered, according to the features of the considered economy. We stress that the relevance of the latter result is limited by the instability, in the presence of the mechanism of gradual adjustment towards the best response considered in Mamada and Perrings (2020), of the Nash equilibrium in the corresponding scenario. However, instabilities can in some contexts give rise to chaotic dynamics, similar to those observed in the time series of good prices and exchanged quantities in real-world markets, especially in relation to agricultural commodities. In particular, a growing empirical and experimental literature (see e.g. Arango and Moxnes 2012; Chatrath et al. 2002; Gouel 2012; Huffaker et al. 2018) suggests that the therein identified dynamic phenomena may be explained in terms of the endogenous fluctuations generated by the presence of nonlinearities. Accordingly, in order to provide a more accurate description of real-world markets, some forms of nonlinearities should be introduced in the model, so as to guarantee that it is able to generate interesting, non-divergent dynamic outcomes. It is then in such more realistic frameworks that the efficacy of the environmental policy should be assessed. Since comparative statics tools are no more sufficient in that efficacy evaluation in chaotic regimes, the need to introduce alternative, dynamical comparative methods arises, based for instance on the consideration of the behavior of different time series of the cumulative aggregate emission. We will tackle such issues in a future research work.

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