No 482

SEPTEMBER 2021

CefES

Center for European Studies

PAPER SERIES

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DIPARTIMENTO DI ECONOMIA, METODI QUANTITATIVI E STRATEGIA DI IMPRESA

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

Business Dynamism, Sectoral Reallocation and Productivity in a Pandemic*

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September 2021

Abstract

Asymmetric effects across sectors are the distinctive features of the Covid-19 shock. Business Formation Statistics in the United States show a reallocation of entry and exit opportunities across sectors in the initial phase of the pandemic. To explain these facts, we propose an Epidemiological-Industry Dynamic model with heterogeneous firms and endogenous firms dynamics. Our analysis suggests that the cleansing effect on business dynamism of the Covid-19 crisis, which typically characterizes recessions, is sector-specific. The framework can rationalize the dynamics of aggregate productivity during the crisis. Monetary policy and sticky wages are central ingredients to capture reallocation effects. Social distancing, by smoothing out cleansing in the social sector, slows down the reallocation process and prolongs the recession, but saves lives.

Keywords: Covid-19; Productivity; Entry; Reallocation.
JEL classification: E3, L16, I3.

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*The views expressed here are those of the authors, and do not necessarily reflect those of De Nederlandsche Bank. We wish to thank Marta Giagheddu, Agnieszka Markiewicz, Makoto Nakajima, Mathias Trabandt, our discussants Bjoern Bruegemann, and seminar participants at De Nederlandsche Bank, the 11th RCEA Money, Macro and Finance Conference, the Erasmus School of Economics and Tinbergen Institute Jamboree for useful comments and suggestions.

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1 Introduction

The crisis caused by the COVID-19 pandemic is like no other. It stands out for being a mixture of both demand and supply shocks and for its asymmetric impact across sectors and countries, as recently stressed by IMF World Economic Outlook (April, 2021). Moreover, the IMF flagship publication highlights the effects of the pandemic on sectoral and aggregate productivity (see also, e.g., Bloom et al., 2020). Theory and evidence suggest that a substantial fraction of aggregate productivity growth is accounted for by the reallocation of resources from lower-productivity to higher-productivity firms. Firm entry and exit are an especially critical component of productivity dynamics induced through reallocation (see, e.g., Foster et al., 2001). The strong asymmetric feature of the Pandemic shock induces a reallocation of business opportunities from less profitable industries to more profitable ones, that plays a critical role for allocative efficiency and productivity.

This paper provides a quantitative framework to study the role played by business dynamism, within and across sectors, in shaping the dynamics of sectoral and aggregate productivity during the first wave of the pandemic, and how that affected macroeconomic aggregates.

The Business Formation Statistics (BFS) data show a clear reallocation pattern since the outburst of the Pandemic. These data provide monthly measures of new business applications and formations in the United States.[1] We consider statistics concerning business applications with planned wages in the period 2019:1-2021:3. These are high-propensity business applications, that is applications with a much higher likelihood of becoming employer businesses with respect to the typical business application, given the intention to pay wages. Data are available for 2-Digit NAICS sectors. We assign industries to either the socially-intensive sector, indexed

[1] The BFS is based on applications for Employer Identification Numbers (EINs). Businesses that hire employees need an EIN for payroll tax purposes. The Monthly BFS data cover the period starting from July 2004 (2004q3) at a monthly frequency. Monthly BFS data are released approximately 11-12 days after the end of the observed month.

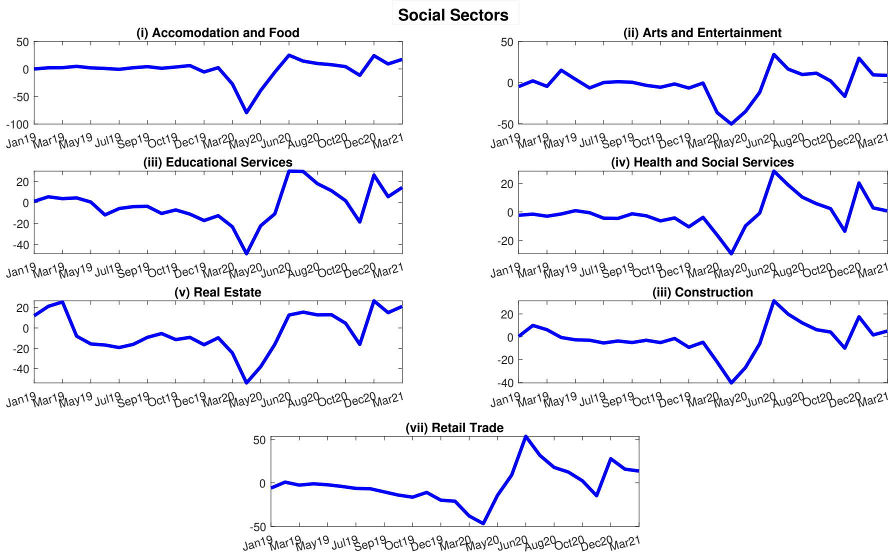

Figure 1: Business Applications in the Social Sectors

The figure displays percentage deviations from trend of business applications with planned wages in the sectors characterised by a high exposure to the virus, i.e. the social sectors. Observations come from the BFS at a monthly frequency. The trend has been computed with an HP filter with parameter 14400*, applied to monthly data from 2004.*

by (s), or to the non-social sector, (ns), following the partition of industries proposed by Kaplan et al. (2020). Figures 1 and 2 display percentage deviations from trend of business applications with planned wages in the (s) sector and the (ns) sector, respectively.

Two facts emerge from a visual observation of the Figures. The first one is that in the early phase of the crisis, that is March-April 2020, there was a drop, with respect to trend, in business applications in the (s) sector, followed by a slight rebound in the summer of the same year. The second one is that, after a mild impact response, a surge in business applications in the (ns) sector can be observed in the same early phase of the crisis.[2] The data indicate that the pandemic represents a large and temporary shock to the (s) sector, that shifted entry opportunities from the social sector to the non-social one.

The effect of the pandemic on business dynamism could affect productivity

[2]The only exception is the construction sectors that interestingly, and contrary to Kaplan et al. (2020), the IMF World Economic Outlook (April, 2021) labels as high-contact sector.

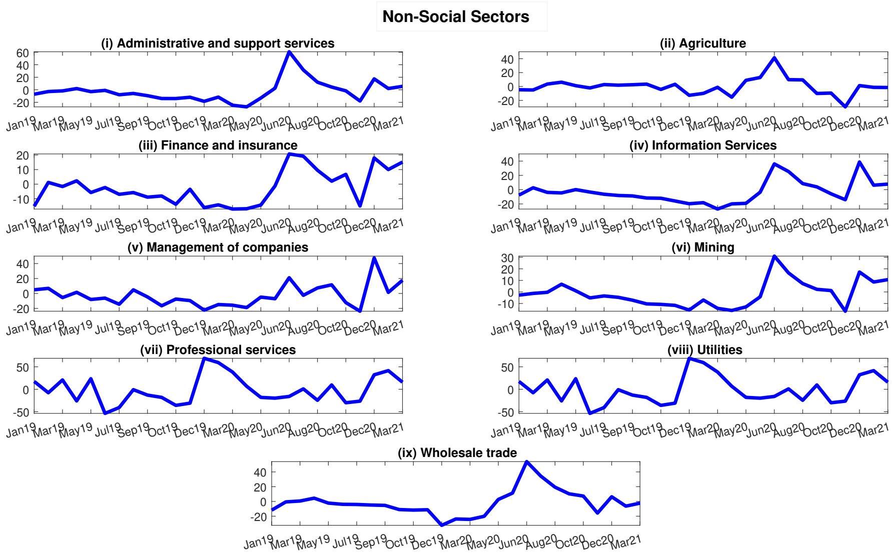

Figure 2: Business Applications in the Non-Social Sectors

The figure displays percentage deviations from trend of business applications with planned wages in the sectors characterised by a low degree of social interaction, i.e. the non-social sectors. Observations come from the BFS at a monthly frequency. The trend has been computed with an HP filter with parameter 14400*, applied to monthly data from 2004.*

through at least three channels: cleansing, reallocation, and production networks. First, the pandemic shock could actually improve sectoral productivity via a standard cleansing effect during recessions, namely by inducing exit of the less productive businesses. Second, reallocation of activity from social towards non-social sectors affects the relative number of firms in those sectors through entry and exit with an, a priori, unclear effect on aggregate productivity. Thus, business dynamism determines a composition effect, within and across sectors, that is important in shaping the dynamics of sectoral and aggregate productivity. Finally, sectoral spillovers and production network could act as an important amplification mechanism of the previous effects.

To capture these effects, we build an Epidemiological-Industry Dynamic model with the following features. First, the economic block of the DSGE model builds on the heterogeneous-firms literature with endogenous firm dynamics á la Melitz (2003), augmented for nominal rigidities as in Bilbiie et al. (2007) and Colciago and Silvestrini (2020). Firms face initial uncertainty concerning their future productivity when making an investment decision to enter the market. Following Bilbiie et al. (2012), firm entry is subject to sunk product development costs, which investors pay in expectation of future profits. Firms join the market up the point where the expected value of their newly created product equals its sunk cost. After entry, firms' production depends on their productivity levels. Firms face fixed production costs. As a result, given aggregate conditions, firms with idiosyncratic productivity levels below a specific threshold will be forced to discontinue production and stay inactive until production becomes profitable again.

Second, we add to our DSGE model an epidemiological block consisting of a SIR model á la Kermack and McKendrick (1927), augmented by a feedback from the economic behavior to the dynamics of the pandemic, which makes the transmission rate of the virus affected endogenously by individual work and consumption choices.

Third, consistently with the sector classification in Figure 1 and 2, we model a two-sector economy with a social and a non-social sector, both populated by an endogenous mass of heterogeneous firms that produce differentiated goods, which are aggregated into sectoral goods. The defining feature of the social goods is that their consumption requires social interaction with other individuals. High social interaction translates into faster transmission of the disease. Examples of social consumption include dining in a restaurant, going to a movie, and traveling by air. Indeed, during the pandemic, individuals voluntarily substitute away from the consumption of the social goods, inducing an endogenous reduction in the demand of those goods (see, e.g., Yan et al., 2021).

Fourth, to analyze the potential role of monetary policy during the pandemic, our framework assumes nominal rigidities in the form of sticky wages. Fifth, to consider the role of production network and sectoral spillovers, the model incorporates a roundabout production structure wherein firms use the outputs of other firms as a factor of production.[3]

[3]The literature (see, e.g., Christiano et al., 2011; Ascari et al., 2018) showed that a roundabout production structure - a feature which Christiano (2016) refers to as "firms networking" after

In response to the COVID-19 shock, our model economy features reallocation effects within and across sectors that are absent in a standard one-sector, homogeneous-firms framework. Specifically, since the disease is transmitted through demand interactions in the social sector, in response to the outburst of the crisis households' expenditure shifts towards the non-social sector, through a substitution channel. As a result, we observe cleansing in the social sector together with a reallocation of entry opportunities to the temporarily more profitable non-social sector. The reallocation process featured in our frameworks allows to successfully reproduce the pattern of business dynamism displayed in Figures 1 and 2 in response to the COVID-19 shock. The reason is as follows. Due to the drop in revenues, break even in the (s) sector requires higher idiosyncratic productivity. This affects both the entry and exit margins. Indeed, only firms with higher productivity will find convenient to enter in the (s) sector, resulting in a drop in the number of potential entrants. Turning to the exit margin, there is cleansing of low-productivity firms, which become temporary idle, causing an increase in the effective exit rate and in average sectoral productivity.[4] Opposite dynamics with respect to those just described characterized the (ns) sector. The reallocation of demand to the non-social sector implies that firms with lower idiosyncratic productivity will now break-even on their costs. As a result, we will observe a higher number of potential entrants, a lower effective exit, and a drop in average sectoral productivity. Our analysis suggests that one distinctive feature of the Covid-19 crisis is that the cleansing effect on business dynamism that typically characterizes recessions is sector-specific. Cleansing in the social sector, together with reallocation across sectors, are the key dimensions to consider in order to explain the empirical dynamics of aggregate labor productivity during the Pandemic. Indeed, our analysis indicates that the latter results from two driving forces. The first one results from the change in average productivity within each of the

Acemoglu et al. (2015) - can act as an amplification source for real shocks, especially in presence of nominal rigidities (see, e.g., Basu, 1995; Huang et al., 2004; Nakamura and Steinsson, 2010).

[4] The effective exit considers both idle firms that stop producing and firms that exogenously exit the market.

two sectors in response to the demand reallocation. The second one derives from a composition effects due to the change in the relative size of sectors. The interaction between these two forces determines the dynamics of aggregate productivity.

A further notable result regards the role of monetary policy. The analysis indicates that nominal wage rigidities and monetary policy, described via a standard Taylor rule, are crucial ingredients to replicate the differing patterns of business creation across sectors observed during the pandemic. The persistent decline of the real interest rate in response to the crisis, that our model features in the presence of nominal wage rigidities, supports consumption, and favors the reallocation of business opportunities to the non-social sector. Absent the endogenous response of the real rate, we would simply observe business destruction in both sectors.

Consistently with the evidence proposed in the IMF World Economic Outlook (April, 2021), our model features a positive relationship between the size of the social sector and the severity of the recession. Additionally, we find that social distancing, by smoothing out fluctuations in aggregate productivity, leads to a slower reallocation across sectors, and to a prolonged recession with respect to the case where no measures are taken.

The literature studying the effects of the COVID-19 pandemic through the lenses of macroeconomic models is already vast and rapidly expanding. Eichenbaum et al. (2020a) and Eichenbaum et al. (2020b) integrate the SIR model in a general equilibrium setting, in order to study the interaction between the economic system and the pandemic. Baqaee and Farhi (2020), and Guerrieri et al. (2020) study the relative importance of demand and supply shocks. Various authors, such as Alfaro et al. (2020), Toxvaerd (2020), Moser and Yared (2020), Alvarez et al. (2020) and Jones et al. (2020) identify externalities in individual distancing decisions, and study optimal lockdown and social distancing policies. Considering a two sectors economy, Guerrieri et al. (2021) design optimal monetary policy in response to asymmetric shocks that shift demand from a sector to the other. Much less vast, but closely related to our paper, is the literature studying the role of microeconomic heterogeneity at shaping the interaction between the epidemic and the economy. Considering heterogeneity across households, Kaplan et al. (2020) integrate the SIR model into a framework with income and wealth inequality, as well as occupational and sectoral heterogeneity, to study the distributional and welfare effects associated with the US policy response to the pandemic. The health and economic policies they consider entail large and heterogeneous welfare costs across households. Hur (2020) integrates a SIR model into an heterogeneous agent-life cycle economy. He designs Pareto-improving mitigation policies, and shows that the latter can reduce deaths by nearly 60 percent relative to a no mitigation scenario. Our paper is complementary to these analysis, since it considers heterogeneity in the supply side of the economy. We integrate the SIR epidemiological model into a New Keynesian Industry Dynamic framework with two sectors, heterogeneous firms and endogenous entry and exit dynamics in order to study the effects of the pandemic and social distancing measures on productivity and business dynamism. Firm level heterogeneity is modeled similarly to Clementi and Palazzo (2016), Hamano and Zanetti (2017), and Rossi (2019).

The remainder of the paper is as follows. Section 2 introduces the theoretical model. Section 3 discusses the calibration. Section 4 presents the results of the benchmark simulation, and Section 5 shows the implications for the response of aggregate productivity. Section 6 analyses the effects of monetary policy. Section 7 presents the results from two further experiments, one introducing social distancing measures, and the other assuming different relative sizes of the social and non-social sectors. Section 8 concludes.

2 The Model

The economy features two sectors indexed by (q), where $(q) = (s)$ identifies the social sector that produces a good whose consumption require social interactions, while and $(q) = (ns)$ identifies the non-social sector. Each sector is characterized by ex-ante heterogeneous firms, which produce a good in different varieties and compete monopolistically. The length of the mass of firms in each sector is determined endogenously by firms' entry and exit, which are modeled at the sectoral level.

The economy features a unitary continuum of homogeneous households or families, who use the final good for consumption and investment purposes. Each family is populated by a unitary continuum of ex-ante homogeneous individuals. Individuals' ex-post heterogeneity comes from their contagion status. The evolution of the disease is governed by a standard SIR model. The transmission rate of the virus depends endogenously on individual working decisions, and on consumption of the social good. In what follows, we describe the main features of the model and leave details and derivations to Appendix A.

2.1 SIR and Contagion

The epidemiological block of our framework is based on the SIR model by Kermack and McKendrick (1927). In each time period t, individuals can be in one of four epidemiological states. The total number of people susceptible to the disease is $\mathbb{S}_t$. $\mathbb{I}_t$ represents the aggregate number of infected individuals. Let $\mathbb{D}_t$ denote the cumulative mass of dead people, then the total number of recovered individuals at time t is $1 - \mathbb{I}_t - \mathbb{D}_t - \mathbb{S}_t$. Since the population is initially normalized to one, these terms represent also the fractions over the initial population.[5]

We follow the approach by Jones et al. (2020), where households' members are not aware of their epidemiological state. As in Eichenbaum et al. (2020a) and

[5]For clarification, in any period t the number of infected individuals $\mathbb{I}_t$ also represents the fraction of infected individuals with respect to the initial unitary population, while the fraction of infected over the current population is $\mathbb{I}_t/(1 - \mathbb{D}_t)$, and the same holds for $\mathbb{S}_t$ and $\mathbb{R}_t$.

Eichenbaum et al. (2020b), we assume that susceptible people can become infected in three ways: by purchasing social goods, working, and through random interactions unrelated to economic activities. The evolution of the disease is internalized by the household. Thus, in the following, we use the calligraphic italics letters to describe the contagion types within the households, i.e. $\mathcal{I}_t$ vs. $\mathbb{I}_t$ and so on. The number of newly infected household's members at time t, $\mathcal{T}_t$, is given by:

$$\mathcal{T}_t = \mathcal{S}_t \mathbb{I}_t \pi_1 c_t\left(s\right) C_t\left(s\right) + \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d + \pi_3 \mathcal{S}_t \mathbb{I}_t. \tag{1}$$

Following Jones et al. (2020), we assume one source of externality that the household does not internalize. When considering exposure, household scales it to $\mathbb{I}_t$ and not to $\mathcal{I}_t$, thus neglecting the effects of individual choices on the aggregate pool of infected. As a result, their mitigation efforts are lower than what would be socially optimal.

As mentioned above, newly infected individuals comes from three types of interactions. The term $\mathcal{S}_t \mathbb{I}_t \pi_1 c_t\left(s\right) C_t\left(s\right)$ yields the number of newly infected household's members due to shopping activities, where $\mathcal{S}_t$ denotes the number of susceptible members within the household, π_1 is a multiplier that scales the probability of becoming infected as a result of consumption activities, $c_t(s)$ is the individual consumption, i.e. of each living household's member, of the social good and $C_t(s)$ is aggregate consumption of the social good.[6]

The term $\mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d$ yields the number of newly infected household's members from supplying labor, where π_2 reflects the probability of becoming infected as a result of work interactions, l_t^s denotes individual labor supply and L_t^d represents aggregate labor demand. Note that, differently from consumption, labor in both the social and the non-social sector yield the same contagion risk.

Finally, the exogenous component of the transition equation (1) comes from the

[6]The only thing that we need to generate reallocation between sectors is a higher contagion risk in the social sector, and this is exactly how these industries are defined in the data. For this reason, we normalize to zero the contagion risk from consumption of the non-social good without loss of generality.

number of random pairings between susceptible household's members and infected people. These meetings result in $\pi_3 \mathcal{S}_t \mathbb{I}_t$ newly infected household's members. Equation (1), that determines the number of newly infected individual, is central in our framework as it affects the behavioral response of the households during the pandemic, which is responsible for the reallocation of demand and supply toward the non-social sector.

Given $\mathcal{T}_t$, the number of household's members who are susceptible ($\mathcal{S}_t$), infected ($\mathcal{I}_t$), recovered ($\mathcal{R}_t$) and the cumulative number of dead members ($\mathcal{D}_t$) evolves as:

$$\mathcal{S}_{t+1} = \mathcal{S}_t - \mathcal{T}_t, \tag{2}$$

$$\mathcal{I}_{t+1} = \mathcal{I}_t + \mathcal{T}_t - (\pi_r + \pi_d)\,\mathcal{I}_t, \tag{3}$$

$$\mathcal{R}_{t+1} = \mathcal{R}_t + \pi_r \mathcal{I}_t, \tag{4}$$

$$\mathcal{D}_{t+1} = \mathcal{D}_t + \pi_d \mathcal{I}_t, \tag{5}$$

where π_r and π_d are the probability of infected to recover and to die, respectively.

2.2 Households

The representative family is initially of size 1, while the mass of the living population within the household at time t is $1 - \mathcal{D}_t$. The time-t utility of the representative household is:

$$(1 - \mathcal{D}_t)\, log\,(c_t) - (1 - \mathcal{D}_t)\,\nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi} \right) - u_d \mathcal{D}_t, \tag{6}$$

where c_t is the individual consumption of the final good and u_d is the disutility from death, which includes the flow value of the psychological costs of death on surviving members. The final good is a composite good defined as a CES (Constant Elasticity of Substitution) function over the household's consumption levels of the social, $c_t(s)$, and the non-social good, $c_t(ns)$, as: $c_t = \left[\chi^{\frac{1}{\eta}} c_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}}\, c_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$. Both

$c_t(s)$ and $c_t(ns)$ are aggregators of goods produced in the social and non-social sector, respectively. The parameter χ captures the relative importance of the social good in the consumption basket and determines the steady state size of the sector, while the parameter $\eta > 1$ measures the elasticity of substitution between the social and the non-social goods.

In each time period t, agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1}/\pi_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t, and π_{t+1} denotes the inflation rate over the same period. Households choose consumption, hours of work, and how much to invest in state-contingent assets and in risky stocks $b_{t+1}(q)$. Stock ownership ensures to households a flow of dividend distributed by operative firms. The timing of investment in the stock market is as in Bilbiie et al. (2012) and Chugh and Ghironi (2011). At the beginning of period t, the household owns $b_t(q)$ shares of a sector mutual fund that represents the ownership of the $N_t(q)$ incumbents in sector (q) in period t, with $(q) = \{(s), (ns)\}$.

The period-t asset value of the portfolio of firms held in sector (q) is the total firms' value in sector (q), given by the product between the average value of a firm $\tilde{v}_t(q)$ and the existing mass of firms $N_t(q)$ in the same sector. To obtain the total value of the portfolio held by households, one needs to sum over the two sectoral funds. During period t, the household purchases $b_{t+1}(q)$ shares in new sectoral funds to be carried to period $t+1$. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants, $N_t^e(q)$, although at the very end of period t a fraction of these firms disappears. The value of total stock market purchases is thus $\sum_{q=s,ns} \tilde{v}_t(q) \left(N_t(q) + N_t^e(q)\right) b_{t+1}(q)$.

Households derive income from three sources: labor, dividend, and from interests on loans to firms. We assume a continuum of differentiated labor inputs indexed by $j \in [0,1]$. Wages are set by labor type specific unions, indexed by $j \in [0,1]$. Given the nominal wage, W_t^j, set by union j, agents stand ready to supply as many hours to labor market j, L_t^j , as required by firms, that is:

$$L_t^j = (W_t^j / W_t)^{-\theta_w} L_t^d, \tag{7}$$

where θ_w is the elasticity of substitution between labor types, W_t is an aggregate nominal wage index, and L_t^d is aggregate labor demand. The latter can be obtained by integrating firms' individual labor demand over the distribution of idiosyncratic productivities. Agents are distributed uniformly across unions, hence aggregate demand for labor type j is spread uniformly across households. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by:

$$\int_0^1 (w_t^j L_t^j) dj = L_t^d \int_0^1 w_t^j (w_t^j / w_t)^{-\theta_w} dj. \tag{8}$$

Stock ownership entitles households to dividend income. Operative firms distribute dividends, following the production and sales of varieties in the imperfectly competitive goods markets. Operative firms in sector (q), that we denote as $N_{o,t}(q)$ and formally define below, are the firms that are actively producing in each sector at time t. As shown in Appendix A.10, total dividends received by a household in a sector can be written as $N_{o,t}(q)\tilde{e}_t(q)$, where $\tilde{e}_t(q)$ denotes average sectoral dividends, that is the amount of dividends distributed by the firm with average sectoral productivity. Finally, a fraction of the resources of households is deposited to financial intermediaries that provide loans to firms. Firms use one-period loans to finance a fraction $\alpha_w \in [0, 1]$ of the wage bill in advance of production. In equilibrium, a real amount equal to $\alpha_w w_t L_t^d$ must be gathered for this purpose. The deposit yields a gross interest rate R_t. Interests on deposits are distributed to households at the end of each period t in a lump sum fashion.

We can then write the flow budget constraint of the representative household:

$$
\begin{aligned}
&(1-\mathcal{D}_t)\sum_{q=s,ns}\rho_t\left(q\right)c_t\left(q\right)+E_t r_{t,t+1}a_{t+1}+ \\
&\sum_{q=s,ns}\tilde{v}_t(q)\left(N_t(q)+N_t^e(q)\right)b_{t+1}(q)=L_t^d\int_0^1 w_t^j\left(\frac{w_t^j}{w_t}\right)^{-\theta_w}dj+\frac{a_t}{\pi_t} \\
&\qquad+\sum_{q=s,ns}\left(N_t(q)\tilde{v}_t(q)+N_{o,t}(q)\tilde{e}_t(q)\right)b_t(q)+\left(R_t-1\right)\alpha_W w_t L_t^d, \quad (9)
\end{aligned}
$$

where w_t denotes real wages and $\rho_t\left(q\right)$ is the price of the good produced in sector (q) expressed in real terms, that we define in Appendix A.3.

Appendix A.1 shows the solution of the infinitely-lived representative household's problem of maximising the present discounted value of (6) - where β is the subjective discount factor - subject to the flow of budget constraints (9) and the constraints coming from the SIR block.

The demand functions of the sectoral goods are key objects in our analysis, since they generate sectoral reallocation, by internalizing the asymmetric contagion risk that leads to the behavioral response of the household. These demands can be found from either the first order conditions of the households (Appendix A.1) or by postulating the existence of a fictitious final good bundler (Appendix A.7), and they are taken as given in the firms' problem we present below. The demand for the production of the social good $Y_t(s)$ reads as:

$$
\left(\frac{Y_t(s)}{Y_t}\right)=\chi\left[\lambda_t\rho_t\left(s\right)+\lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)\right]^{-\eta}\left(\frac{C_t}{\left(1-\mathcal{D}_t\right)}\right)^{-\eta}, \qquad (10)
$$

where Y_t is aggregate production, λ_t is the Lagrange multiplier of the budget constraint, $\lambda_{\mathcal{T},t}$ represents the shadow cost of a new infected, $C_t\left(s\right)$ is aggregate consumption of the social good and C_t is aggregate consumption. As mentioned above, this demand departs from the standard CES demand since it internalizes the exposure to contagion.[7] *Ceteris paribus*, a higher contagion risk, e.g. coming from a higher number of infected or from a larger consumption of the social good, depresses the demand for $Y_t(s)$.

2.3 Firms

Each sector (q) is populated by a mass $N_t(q)$ of atomistic firms. Upon entry, firms draw a time invariant idiosyncratic productivity level, denoted by z, from a known distribution function, $g(z)$, which is identical across sectors and has a positive support. Within their sector of operation, the only source of heterogeneity across firms is the idiosyncratic productivity level, so that we can can index firms within a sector with z. Firms compete monopolistically within the sector and are subject to entry and exit. Each firm produces an imperfectly substitutable good $y_{z,t}(q)$, using the following constant return to scale production function with roundabout:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha}, \tag{11}$$

where the variable Z_t is an exogenous level of productivity, common to all firms. The two inputs are labor, $l_{z,t}(q)$, and an intermediate input, $X_{z,t}(q)$. The former is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0, 1]$, defined as:

$$l_{z,t} = \left(\int_0^1 (l^j_{z,t})^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}, \tag{12}$$

where $\theta_w > 1$ is the degree of substitution between labor inputs. The latter is a composite of all the goods in the economy, combined through the same CES function as consumption. The goods $y_{z,t}(q)$ are input to the production of a sectoral bundle, $Y_t(q)$, by a sectoral good producer that operates in perfect competition. The latter

[7] The demand function of the non-social good, that is good (ns), is:

$$\left(\frac{Y_t(ns)}{Y_t} \right) = (1 - \chi) \left[\lambda_t \rho_t (ns) \right]^{-\eta} \left(\frac{C_t}{(1 - \mathcal{D}_t)} \right)^{-\eta}.$$

adopts a CES production function defined as:

$$Y_t(q) = \left(\int_0^{\infty} N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}}, \tag{13}$$

where $\theta > 1$ is the degree of substitution between goods within a specific sector.

We assume that firms finance a fraction $0 \leq \alpha_W \leq 1$ of their wage bill resorting to loans from financial intermediaries. Loans are reimbursed at the end of the period at the gross risk-free interest rate R_t. Additionally, firms face fixed costs of production $f_{x,t}$, defined in terms of the final good. Appendices A.4 and A.5 provides the technical derivations concerning both the cost minimization and the profit maximization problem of firm z. The equilibrium optimal real price $\rho_{z,t}(q)$ is:

$$\rho_{z,t}(q) = \frac{\theta}{\theta-1} \frac{1}{Z_t z} \left(\frac{(\alpha_W R_t + 1 - \alpha_W) w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha}, \tag{14}$$

where the ratio $\frac{\theta}{\theta-1}$ represents the markup over real marginal costs under monopolistic competition with atomistic firms. Optimal pricing delivers real profits:

$$e_{z,t}(q) = \frac{1}{\theta} \rho_{z,t}(q)^{1-\theta} \rho_t(q)^{\theta} Y_t(q) - f_{x,t}. \tag{15}$$

2.4 Entry and Exit

Potential entrants must pay a sunk entry cost, $f_{e,t}(q)$, measured in units of the final good, to draw their individual productivity level, z, from a p.d.f. $g(z)$ common to both sectors. We assume that the entry costs take the form: $f_{e,t}(q) = \psi_0 + \psi_1 \left(N_t^e(q) \right)^{\gamma}$. Entry costs are composed of a constant term, ψ_0, and of a term which increases with the mass of potential entrants, $\psi_1 \left(N_t^e(q) \right)^{\gamma}$. The variable term could be motivated by various factors, among which we can list congestion externalities, as in Jaef and Lopez (2014) and Casares et al. (2018), and diminishing quality in managerial ability, as in Bergin et al. (2018). Notice that when $\gamma = 0$ the entry cost is a constant. As γ increases, the entry rates will respond less to any given exogenous shock. Firms enter the market up to the point where the sunk cost of entry is equal, in expectation, to the value of discounted future profits. Since the idiosyncratic productivity z is unknown ex-ante, the expected value of discounted profits is evaluated using the value of the average firm in period t, $\tilde{v}_t(q)$.[8] Thus, the free entry condition is $f_{e,t}(q) = \tilde{v}_t(q)$. Due to the fixed costs of production, not all $N_t(q)$ firms have non negative profits, but just those with idiosyncratic productivity, z, above a certain minimum cut-off productivity level $z_t^c(q)$. Appendix A.6 shows that $z_t^c(s)$ is given by:

$$z_t^c(q) = \Omega_t^c \left(\frac{1}{\rho_t \left(q\right)^{\theta} Y_t(q)} \right)^{\frac{1}{\theta-1}}, \tag{16}$$

where:

$$\Omega_t^c = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1} \frac{1}{Z_t} \left(\frac{\left(\alpha_W R_t + 1 - \alpha_W\right) w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \left(f_{x,t}\right)^{\frac{1}{\theta-1}}$$

is common across sectors.[9] Firms with idiosyncratic productivity lower than $z_t^c(q)$ become idle, as in Ghironi and Kim (2019) and Colciago and Silvestrini (2020). Idle firms discontinue production, but stand ready to join again the mass of operative firms when their idiosyncratic productivity becomes again larger than $z_t^c(q)$. This could be interpreted as an endogenous voluntary lockdown of not profitable firms. On top of the endogenous inactivity margin, firms permanently exit the market when hit by an exogenous exit shock. The latter wipes out a fraction δ of existing firms in each period t, no matter if new entrants or incumbent, or if active or inactive. The exit rate, δ, is common across the two sectors. In this framework, the fraction of firms that becomes idle in period t represents the endogenous component of the exit rate. Indeed, in order to be actively operating, firms must be endowed with an

[8]Although in the following we define the notion of an active firm, i.e. an incumbent engaged in production with non-negative profits, note that the expected firm's value is computed considering every incumbent in the market and not just active firms. This is so since an inactive firm with zero current profits could become active in the future, conditional on surviving.

[9]The latter is affected by households' preferences, through θ. Additionally, it increases in the magnitude of both fixed cost, $f_{x,t}$, and in the common, across sectors, components of marginal costs of production, i.e. the real wage and the gross interest rate.

idiosyncratic productivity level above the cut-off. We denote the mass of operative firms at time t as $N_{o,t}(q)$. The latter is formally defined as:

$$N_{o,t}(q) = N_t(q) Pr\left[z > z_t^c(q)\right] = \left[1 - G\left(z_t^c(q)\right)\right] N_t(q) \quad \text{for} \quad (q) = \{(s), (ns)\},$$

where $G(z)$ is the cumulative distribution function associated to $g(z)$: $G(z) = \int_0^z g(x)dx$. Importantly for our purposes, cut-off productivities levels, and thus the size of the mass of operative firms, is directly affected by the pandemic. The reason is the following. In response to the outburst of the pandemic, agents curtail consumption of the social good, partially substituting it with that of the non social good. As a result, output increases in the non-social sector and decreases in the social one. The demand reallocation translates in a higher cut-off productivity threshold, $z_t^c(s)$, in the social sector and in a lower cut-off productivity in the non-social one. As we show in the remained of the analysis, changes in cut-off productivities due to the reallocation of demand ultimately affect both aggregate productivity and business dynamism.

To conclude the description of the entry and exit processes we assume, as in Bilbiie et al. (2012) and many other studies in the literature, a one time period to build, i.e. a one period lag between the decision to enter the market and the beginning of production. This period represents the amount of time required to set up production facilities. As a result, the number of firms in each sector evolves according to:

$$N_t(q) = (1 - \delta)\left(N_{t-1}(q) + N_{t-1}^e(q)\right) \quad \text{for} \quad (q) = \{(s), (ns)\}, \tag{17}$$

where $N_{t-1}(q)$ is the mass of firms in sector (q) in period $t-1$ and $N_{t-1}^e(q)$ denotes the mass of potential entrants between periods $t-1$ and t in the same sector.

2.5 Labor Unions, Monetary Policy and Aggregation

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $(1-\alpha^*)$ of reoptimizing the wage. The optimal nominal wage in sector j set at time t, that we denote with W_t^*, is chosen to maximize agents' lifetime utilities (see Appendix A.8 for details). Due to symmetry, the newly reset wage is identical across labor markets.

The Central Bank sets the nominal interest rate, R_t, according to the following Taylor rule with smoothing:

$$\left(\frac{R_t}{R}\right) = \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi}\left(\frac{Y_t}{Y}\right)^{\varphi_Y}\right]^{1-\varphi_R}\left(\frac{R_{t-1}}{R}\right)^{\varphi_R}, \tag{18}$$

where variables without time subscript denote steady state values. For simplicity, we assume that the steady state gross inflation rate equals one.

In equilibrium, the representative household holds the entire portfolio of firms and the trade of state-contingent asset trade is nil. As a result, $b_{t+1}(q) = b_t(q) = 1$, and $a_{t+1} = a_t = 0$, so that:

$$C_t + N_t^e(s)\tilde{v}_t(s) + N_t^e(ns)\tilde{v}_t(ns) = (\alpha_W R_t + 1 - \alpha_W)\, w_t L_t^d + N_{o,t}(s)\tilde{e}_t(s) + N_{o,t}(ns)\tilde{e}_t(ns). \tag{19}$$

Y_t is either consumed, used as intermediate input in the production process or used to cover fixed costs of production and entry costs, thus:

$$Y_t = C_t + X_t + (N_{o,t}(s) + N_{o,t}(ns))\, f_{x,t} + N_t^e(s) f_{e,t}(s) + N_t^e(ns) f_{e,t}(ns). \tag{20}$$

Finally, the representative household assumption implies $\mathbb{I}_t = \mathcal{I}_t$, $\mathbb{S}_t = \mathcal{S}_t$, $\mathbb{D}_t = \mathcal{D}_t$ and $\mathbb{R}_t = \mathcal{R}_t$.

To obtain tractable results, a Pareto distribution is assumed for the p.d.f. $g(z)$ with minimum $zmin$ and tail parameter κ. This assumption simplifies considerably several equilibrium conditions and allows us to compute analytical solutions.

Following Melitz (2003), a special average productivity is defined over operating firms. In our case, however, the special average productivity is sector-specfic and it is defined as $\tilde{z}_t(q)$. The special average productivity allows to represent each sector as one populated by a mass of homogeneous firms $N_{o,t}(q)$, each of which endowed with idiosyncratic productivity $\tilde{z}_t(q)$, as we show in Appendix A.10. Thanks to the properties of the Pareto distribution, we can write $\tilde{z}_t(q)$ as a function of the cut-off productivity, $z_t^c(q)$, as follows:

$$\tilde{z}_t(q) = \left[\frac{1}{1 - G\left(z_t^c(q)\right)} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \Gamma z_t^c(q), \tag{21}$$

where $\Gamma = \left[\frac{\kappa}{\kappa-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $1 - G\left(z_t^c(q)\right) = \left(\frac{zmin}{z_t^c(q)}\right)^{\kappa}$. The latter illustrates that changes in the cut-off productivity levels, due either to the pandemic or to other exogenous disturbances, lead to changes in average sectoral productivities.

3 Calibration

The time period is a week. The discount factor β equals $0.98^{1/52}$, as in Eichenbaum et al. (2020a). The coefficient measuring the disutility of labor, ν, is set to 1, while the inverse Frisch elasticity of labor supply equals 4, as in many other studies of the business cycle. We set $u_d = 10$ such that the value of a statistical life (VSL) is included between 1 and 50 millions dollars, depending on the definition of income in our model, in all the experiments we consider.[10]

In order to match the US empirical level of 10% job destruction rate, the weekly exit probability δ is calibrated to a value of 0.00211. The aggregate productivity level Z_t is normalized to 1. In the benchmark calibration $\chi = 1/2$, such that the two sectors have the same size. The elasticity of substitution between the social and

[10] To calibrate the psychological cost of death, u_d, we consider the VSL which measures how much the average US citizen is willing to pay for a reduction in mortality rates equivalent to saving one life on average. Greenstone and Nigam (2020) estimate the VSL to be 11.5 million dollars.

the non-social good, η, is 1.5, as estimated by Edmond et al. (2015). On the other hand, the elasticity of substitution between goods belonging to the same sector is $\theta = 3.8$, following Bernard et al. (2003), who calibrated the value of θ to fit US plant and macro trade data. The selected value of θ entails a price markup equal to 35%, within the range estimated by De Loecker and Eeckhout (2018). The elasticity of substitution across labor types, θ_w, equals 4, which implies a steady state wage markup equal to 33%.

Turning to the parameters that determine entry frictions, ψ_0 is normalized to 1, as in Bilbiie et al. (2012). We set the elasticity of entry cost equal to $\gamma = 1.5$, in line with the estimate of Gutierrez Gallardo et al. (2019), who exploit the comovement between industry-level entry rates and stock prices to pin down this parameter. The parameter ψ_1 is such that the steady state ratio between investment and GDP, $\frac{f_e(s)N_e(s)+f_e(ns)N_e(ns)}{Y-X}$, is approximately 15%. As in Ghironi and Kim (2019), given our equilibrium entry costs, we calibrate the entry costs to fixed costs of production $\frac{f_e(s)+f_e(ns)}{2f_x}$ to 4.5, as in Collard-Wexler (2013).

The parameterization of the productivity distribution is as follows. We normalize $zmin$ to 1 with no loss of generality. In the spirit of Gabaix (2011) and Di Giovanni and Levchenko (2012), our sectors can be defined as granular when $1 < \frac{\kappa}{\theta-1} < 2$. Given the value of θ, we set $\kappa = 6$. In this case, as discussed by Colciago and Silvestrini (2020), our sectors are just short of being granular, but the Herfindal-Hirschman Index (HHI) of concentration is well defined.

The calibration of the SIR model follows Eichenbaum et al. (2020a), which is based on data on the infection from South Korea. Assuming that the average duration of the disease is 14 days, the recovery probability π_d and the death probability π_r are calibrated such that $\pi_r + \pi_d = 7/14$. Based on the evidence, the death probability is fixed at 0.2%, i.e. $\pi_d = 7(0.002/14)$. The parameters of the transition equation which govern, respectively, the risk of infection from consumption π_1, from work π_2 and from the mere interaction between susceptible and infected within the household π_3, are calibrated such that the initial contagion is due for 1/6 to consumption activities, for 1/6 to working activities and for 2/3 to random interactions. Finally, the parameters are calibrated such that the exogenous SIR framework converges to the so-called Merkel scenario, where 60% of the population is either recovered or dead. The 60% threshold is regarded to deliver heard immunity.

The Calvo parameter α^* is set to 0.98 in order to observe, on average, one wage change per year. The parameters of the Taylor Rule are $\vartheta_\pi = 1.5$, $\vartheta_Y = 0.5/52$, and $\vartheta_R = 0.8$, as in Eichenbaum et al. (2020a). In the benchmark calibration firms don't borrow to pay wages, i.e. α_w is set to 0, and the Cobb-Douglas parameter α is 1/3. We begin our simulation from an initial infection seed of 0.1%, i.e. $\mathbb{I}_0 = 1e^{-3}$. For reader's convenience, Table 1 reports the values of the calibrated parameters.

4 The Asymmetric Transmission of an Epidemic

In this section, we discuss the impact of an epidemic in our two-sector model. Figure 3 displays percentage deviations from the steady state of key macro variables in response to the pandemic shock.[11]

Consumption and output drop substantially. As described by Eichenbaum et al. (2020a), the pandemic entails both negative demand and supply shocks. The demand shock arises because agents optimally curtail their consumption to limit their exposure to the virus. For the same reason, a negative supply shock results from the reduction in households' desire to work.

In the presence of endogenous business dynamism, heterogeneous firms, and sectors characterized by different degree of social contact, the pandemic shock spurs additional effects. Specifically, since the disease is transmitted when consuming the s-goods, spending shifts towards the ns-goods. The same holds for entry opportunities. The reason is that, due to the drop in revenues, break-even in the s-sector

[11] Unless otherwise stated, the Figures that follows report percentage deviations of variables from their respective steady state.

Table 1: **Calibration of exogenous parameters**

Parameter	Calibration	Target
		SIR
π_r	0.499	Duration infection 14 days, Eichenbaum et al. (2020a)
π_d	0.001	Mortality rate of 0.2%, Eichenbaum et al. (2020a)
π_1	9.254	1/6 contagion from consumption, Eichenbaum et al. (2020a)
π_2	0.143	1/6 contagion from labor, Eichenbaum et al. (2020a)
π_3	0.5	2/3 contagion from interaction, Eichenbaum et al. (2020a)
$\mathbb{I}_0$	0.001	Initial seed of infected 0.1%, Eichenbaum et al. (2020a)
$\mathbb{R}_{end} + \mathbb{D}_{end}$	0.6	60% Merkel scenario, Eichenbaum et al. (2020a)
u_d	10	VSL $\approx$ 10 millions, Greenstone and Nigam (2020)
		Nominal Stickiness
α^*	0.98	Wages change once a year, Eichenbaum et al. (2020a)
ϑ_π	1.5	Standard for weekly frequency, Eichenbaum et al. (2020a)
ϑ_Y	0.5/52	Standard for weekly frequency, Eichenbaum et al. (2020a)
ϑ_R	0.8	Standard smoothing Taylor Rule, Christiano et al. (2005)
		Firms
α	1/3	Standard for Cobb-Douglas production
ψ_0	1	Normalization, Bilbiie et al. (2012)
ψ_1	1000	Investment to GDP Ratio $\approx$ 15%
γ	1.5	Co-mov. entry and stocks, Gutierrez Gallardo et al. (2019)
f_x	0.47	Ratio $f_e/f_x \approx 4.5$, Collard-Wexler (2013)
α_w	0	No working capital constraint
δ	0.00211	Yearly job destruction rate $\approx$ 10%, Colciago (2016)
Z	1	Normalization
θ_w	4	Wage markup of 33%
$zmin$	1	Normalization
κ	6	Almost granular economy, Colciago and Silvestrini (2020)
		Households
χ	0.5	Ex-ante homogeneous sectors
η	1.5	Inter-sectoral substitutability, Edmond et al. (2015)
θ	3.8	US plant data in Bernard et al. (2003)
ν	1	Normalization
ϕ	4	Frisch elasticity as in King and Rebelo (1999)
β	$0.98^{1/52}$	$\approx$ 4% yearly interest rate, Eichenbaum et al. (2020a)

Notes: The table summarizes the calibration of the exogenous parameters. The second column describes the value assigned to the parameters. The third column describes the targets of the calibration and their sources.

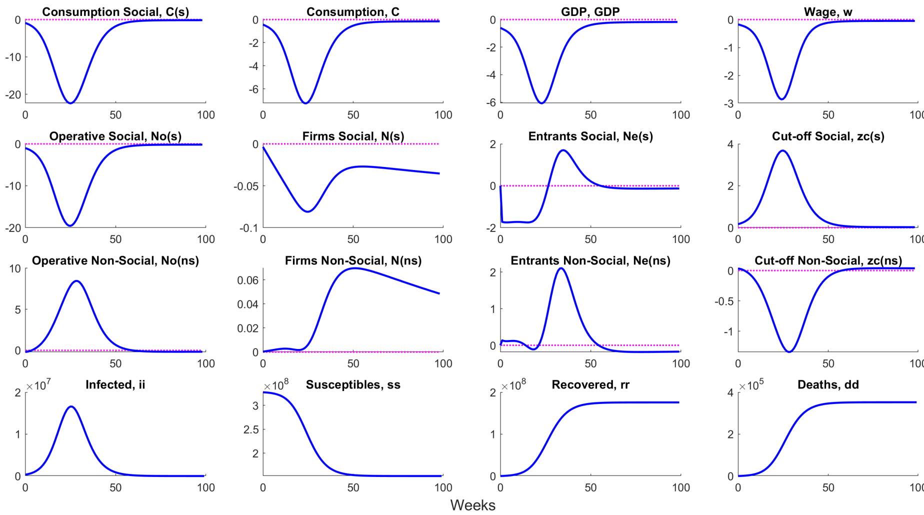

Figure 3: IRFs of the main macroeconomic variables to the pandemic shock.

requires higher idiosyncratic productivity. This, in turn, has two effects. The first one is an increase in the effective exit rate coming from the inactivity margin, the second one is a drop in the number of potential entrants. Indeed, only firms with higher productivity will find convenient to continue production or to newly enter in the s-sector. Both effects increase average productivity in the social sector. Opposite forces, with respect to those just described, characterized the ns-sector: firms with lower idiosyncratic productivity will now break-even on their costs. Thus, the reallocation of demand to the non-social sector results in a lower effective exit, a higher number of potential entrants, and a drop in average sectoral productivity.

Our analysis suggests that one distinctive feature of the Covid-19 crisis is that the cleansing effect on business dynamism that typically characterizes recessions is sector-specific. Entry opportunities are shifted away from temporary less profitable sectors and concentrated in more profitable ones. This pattern qualitatively reflect the dynamics of US data on business applications reported in Figures 1 and 2. Indeed, the data show a drop in business applications in social sectors. On the contrary, business applications in non-social sectors are characterize by a rise, after an initially flat response. The dynamics just described occur in the early phases of the pandemic, prior to the introduction of lockdown measures. The timing of events suggests that the behavioural response of agents to the spread of the disease had a key role in the reallocation process. Finally, the dynamics characterizing the epidemics are those typically observed in the data, where the response of the number of infected people to the diffusion of the disease displays an inverted U shape. After the peak in the number of infected, the economy converges to a steady state where aggregate output, consumption, and entry shrink permanently due the decline in population. [12]

5 Aggregate Productivity

In this section, we discuss the empirical pattern displayed by aggregate labor productivity during the COVID-19 pandemic. Then, we show that our model qualitatively replicates the observed dynamics. Specifically, we consider a simplified version of our framework to show analytically that aggregate productivity is a size-weighted harmonic mean of sectoral average productivities, corrected for the sectoral shares of operating firms. Using this measure as a proxy for aggregate labor productivity in the benchmark model, we argue that the sector-specific cleansing effect and the reallocation of demand and of operative firms across sectors are key to explain the empirical dynamics of aggregate labor productivity.

The top left panel of Figure 4 displays the empirical series of aggregate labor productivity, measured as output per hour. Data are from 2020Q2 to 2021Q1.[13]

[12]Note that while the fluctuations in entry in the model qualitatively reflect the dynamics on business applications, the former should not be quantitatively compared to the latter for two main reasons. First, the magnitude of the fluctuations in business applications were not reflected in the Q2 and Q3 entry data from the Business Employment Dynamics, as noticed by Bilbiie and Melitz (2020). While these latter data confirm the reallocation of entry opportunities across social and non-social sectors, they feature fluctuations in establishment openings of an amplitude ranging from one quarter (construction) to half (accommodation services) of those characterizing business application data. Second, our model studies the implications of the endogenous behavioral response of households to the shock, and thus abstract from government measures to contain the spread of the virus - such as lockdown measures - that obviously impacted on actual fluctuations in entry and other macroeconomic variables. Section 7.1 discusses the impact of the introduction of social distancing measures in our model.

[13]The source of the data is the FRED-MD dataset. Note that, in order to align all panels to the

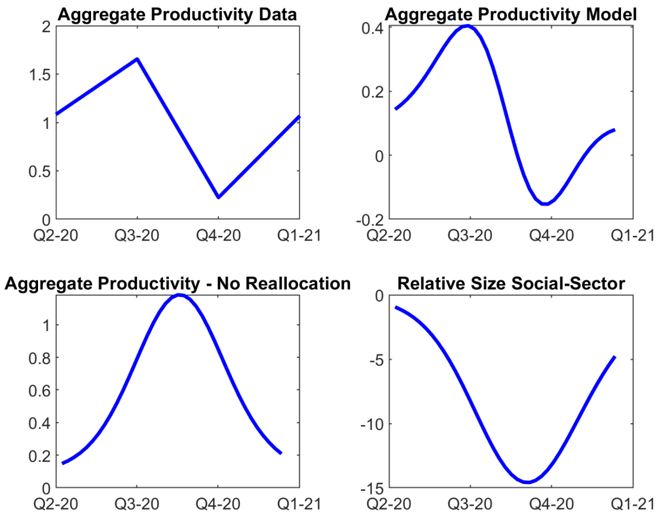

Figure 4: Top Panels: aggregate productivity in the model and in the data during the pandemic. Bottom Panels: productivity decomposition.

Labor productivity had a peculiar pattern during the pandemic. It increased up to summer/fall of 2020, to drop sharply afterwards, and to start growing again at the beginning of 2021. To provide economic intuition about the response of labor productivity to the pandemic shock, we simplify our model to consider the case where there is no contagion from consumption, that is where $\pi_1 = 0$, and where labor is the only factor of production. As a result of the latter assumption, the model features no network effects. In this simplified case, as detailed in Appendix A.12, the quantity of final output produced in the economy can be written as:

$$Y_t = N_{o,t}^{\frac{1}{\theta-1}} Z_t \tilde{Z}_t L_t^d, \tag{22}$$

where L_t^d are aggregate hours of work, $N_{o,t}$ denotes the total number of operating firms in the economy, and $\tilde{Z}_t$ defines the endogenous component of aggregate productivity. The latter is given by:

$$\tilde{Z}_t = \left(\chi \rho_t(s)^{-\eta} \omega_t(s)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_t(s))^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)} \right)^{-1}, \tag{23}$$

first of April 2020, we discarded the first month of observations from the simulated model, as we assume that the behavioral response started in March.

where $\omega_t(q) = N_{o,t}(q)/N_{o,t}$. In the restricted model, the endogenous component of aggregate productivity, $\tilde{Z}_t$, is a weighted harmonic average of the average sectoral productivities, $\tilde{z}_t(s)$ and $\tilde{z}_t(ns)$, corrected for a measure of the sectoral fractions of operative firms, that is $\omega_t(s)^{\frac{1}{1-\theta}}$. The weights are represented by the relative size of sectors, $\chi\rho_t(s)^{-\eta}$ and $(1-\chi)\rho_t(ns)^{-\eta}$. We next use equation (23) to construct a measure of aggregate productivity in the fully-fledged model.[14] The latter is displayed in the top right panel of Figure 4.

When using equation (23) to build labor productivity, our model delivers a sinusoidal-shaped series that mirrors closely that in the data, at least from a qualitative standpoint. The reason for this relative success, is that our model accounts for both sector-specific cleansing in the social sector, and for the reallocation of demand across sectors that characterized the pandemic. To see this, consider the two bottom panels of the figure. The bottom left panel plots again labor productivity as in (23), but shutting down the reallocation channel. Specifically, we assume that $\omega_t(q)$ and the sectors' relative size in equation (23) remain constant at their pre-pandemic steady steady values. The bottom right panel, instead, plots the change of the relative weight of the social sector in response to the pandemic shock.

A joint reading of two lower panels suggests that the response of aggregate productivity to the pandemic shock is the result of two driving forces. The first one results from the change in sectoral average productivity in the two sectors. The second force is a composition effect due to the reallocation of demand across sectors, that alters their relative size. The interaction between these two forces determines the overall productivity dynamics. In the initial part of the sample, the cleansing in the social sector dominates, because the relative size of the social sector displayed a

[14] As argued in Ghironi and Melitz (2005) and Bilbiie et al. (2012) when using the model for empirical statements, one has to recognize that empirically relevant variables, as opposed to welfare-consistent concepts, net out the effect of changes in the range of available varieties. Unfortunately, in our rich benchmark framework, we cannot analytically pin-down the relationship between welfare consistent and empirically relevant variables. On the contrary, we can identify that relationship in the simplified framework, and for that reason we use the endogenous measure of productivity obtained in the latter case as a proxy for aggregate productivity in the benchmark model.

limited decline. As a result overall productivity rises (even if much less than without reallocation, lower left panel). In the central part of the sample, instead, the decline in the relative size of the social sector becomes quantitatively sizeable, thus the composition effect dominates, leading to a sharp decline in aggregate productivity (much stronger - and earlier - than without reallocation). As a result, aggregate productivity overshoots and then approaches the long-run level of productivity from below at the end of sample, as the relative size of the social sector converges to steady state, pushing up again productivity. In short, our analysis shows that the composition effect due to reallocation is key to generate a sinusoidal shape of the response of aggregate productivity. Thus, neglecting the reallocation of demand across sectors that characterized the pandemic, as we do in the bottom left panel of Figure 4, entails counterfactual dynamics in labor productivity.

6 Monetary Policy in an Epidemic

This Section studies the role of monetary policy during the pandemic. Monetary policy is very powerful in influencing the dynamics of output and the reallocation of activity across sectors. Figure 5 compares the dynamics of our benchmark specification with the ones under flexible wages. Dynamics of output and consumption are similar across specifications, although the recession is stronger in the flexible prices case. Importantly, there is a substantial difference in business dynamism between the two cases. Under flexible wages, we do not observe the reallocation of entry opportunities that characterize US data. Indeed, few periods after the shock, entry diminishes sharply in both sectors. Despite the sizeable reduction in the productivity cutoff, the recession is so severe to induce a contraction in the number of entrants also in the non-social sector. While powerless in the flexible wage case, in presence of nominal rigidities monetary policy affects the real interest rate.[15] Specifically, in

[15]While the real interest rate surges slightly on impact to stay flat for one year before starting decreasing under flexible wages, with nominal rigidities the real interest rate drops on impact and

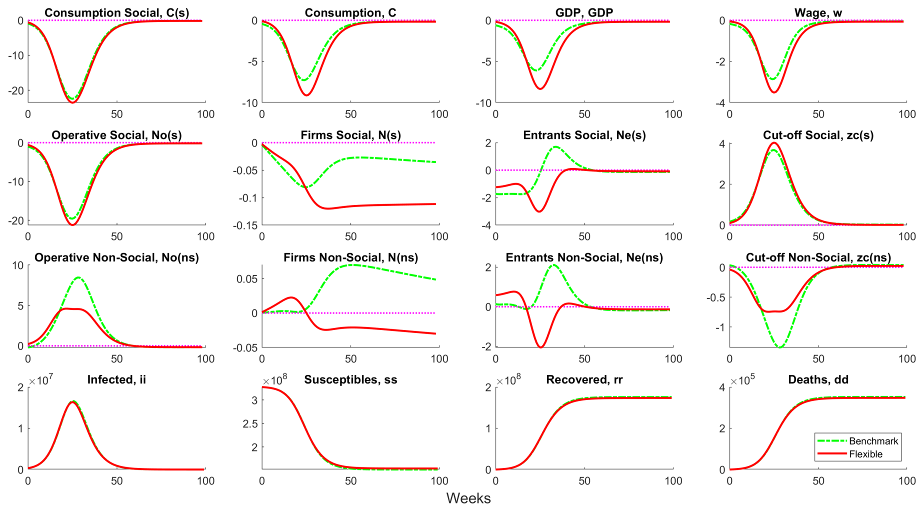

Figure 5: IRFs of the main macroeconomic variables to the pandemic shock. Sticky Wages Vs. Flexible Wages.

our benchmark case, the monetary response is such that the real rate decreases persistently. This supports consumption during the crisis, leading to a milder recession with respect to the flexible wages scenario. Due to fear of contagion, the additional consumption flows to the non-social sector. Hence, by sustaining aggregate demand through a blunt - and not tailored - instrument as the real interest rate, monetary policy induces a larger reallocation between the two sectors. While studying the optimality of this measure goes beyond the scope of this paper, we point out that the dynamics of the real interest rate is a key ingredient to replicate the sectoral business dynamism observed in US data.

To further investigate this point, Figure 6 shows model dynamics under alternative parameterizations of the output gap coefficient response in the interest rate rule. A larger weight on output in the Taylor rule leads to a stronger reduction in the real rate in response to the crisis and, thorough the demand channel, to a milder recession and hence a stronger reallocation. The larger is ϑ_y, the lower is the cut-off in the non-social sector at the outset of the pandemic. Hence, entry and the number of firms respond strongly and are always above steady state. The social sector also

keep reducing for around one year and half before reverting to steady state.

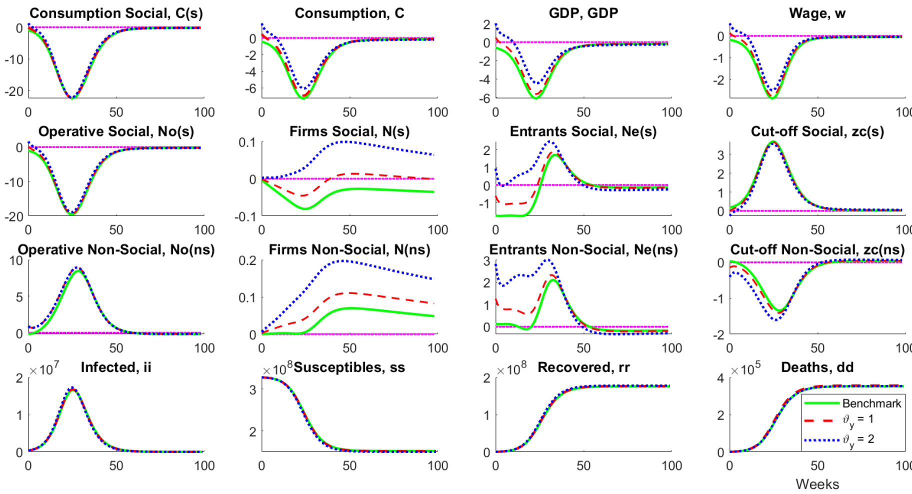

Figure 6: IRFs of the main macroeconomic variables to the pandemic shock under different Values of the output response coefficient in the interest rate rule.

benefits from a lower recession, because the dynamics of both entry and the number of firms raises with ϑ_y almost uniformly - in the sense that they exhibit a similar shape, but just shifted upwards.

The next Section studies the effects of the pandemic shock when public authorities impose social distancing measures. Additionally, we study the role played by the relative size of the social sector for the propagation of the epidemic.

7 Further Results

This Section discusses the role of social distancing measures, and of the economic structure of a country, in terms of the relative size of the social sector, for the propagation of the pandemic. We do not discuss the effect of roundabout production in the main text since our analysis suggests that the network linkages - as far as modelled as standard and symmetric roundabout - did not have strong economic effects during the pandemic.

7.1 Social Distancing

The Social distancing (SD, henceforth) measures implemented by various government around the globe, were aimed at reducing the diffusion of the disease along the three possible contagion channels that we consider: consumption, work and random encounters between susceptible and infected agents. In other words, they aimed at reducing the probabilities π_1, π_2 and π_3. We compare the benchmark simulation without SD to three alternative scenarios, that differ only in the duration of containment measures. Under all scenarios characterized by SD, we assume a generalized 20% cut in social contacts with respect to the benchmark case. Figure 7 displays transition dynamics under the cases we consider. Green dashed lines refer to the benchmark transition with no containment measures, blue solid lines to the case in which containment measures are imposed for 6 months, yellow dotted lines to the case in which social distancing applies for 1 year, and finally red dotted-dashed lines refer to the case in which SD is imposed permanently, i.e. for the whole transition.

Independently of their length, SD measures dampen the peak effect of the recession on consumption, output and other macroeconomic variables with respect to the benchmark no-SD scenario. Nevertheless, they extend the duration of the recession, and the more so, the longer their duration. The reason is that SD smooths out the reallocation process across sectors with respect to the benchmark case. Indeed, fluctuations in sectoral productivities are dampened and last longer with respect to what observed in the baseline scenario. Turning to epidemiological effects, containment measures are effective at diminishing the number of infected people, but their effects on the number of dead individuals depend on their duration. Specifically, when measures are imposed for half a year the effects are simply delayed. Indeed, with respect to our no-SD baseline scenario the difference in the number of dead individuals at the end of the transition is minor. However, we do not consider the possibility that the temporary strategy would prove effective in buying time to authorities for a reorganization of health care facilities, and for the development of a

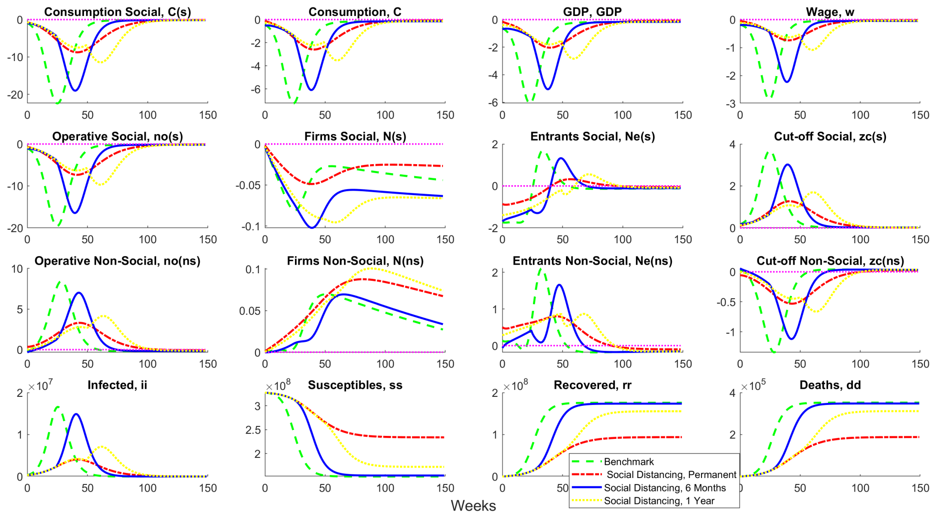

Figure 7: Simulation Benchmark vs. Social Distancing

vaccine or a cure of the disease. In the case of a more extended SD policy, where measures are imposed for an year, we observe a substantial reduction in the loss of lives. Lifting containment measures after a year, however, leads to a second wave of infection as suggested by the dynamics of the number of infected.

7.2 Economic Structure

The IMF World Economic Outlook (April, 2021) argues that output losses have been particularly severe for countries with a large relative size of high-contact sectors. For this reason, we compare the response of our model economy to the pandemic under two alternative scenarios concerning the relative size of the social sector. The first one is meant to mimic the US. Kaplan et al. (2020) report that the US value added share of social sectors is approximately 0.26. We set $\chi_{US} = 0.34$ to match this target. The second scenario is meant to represent the case of an economy with a larger social sector. To do so we set $\chi = 1 - \chi_{US} = 0.66$. The fact that the relative sizes of social sectors across scenarios sum to 1 allows to hold the aggregate size of the economy unchanged across the two cases, so that the epidemiological block of the model is comparable across the two scenarios.

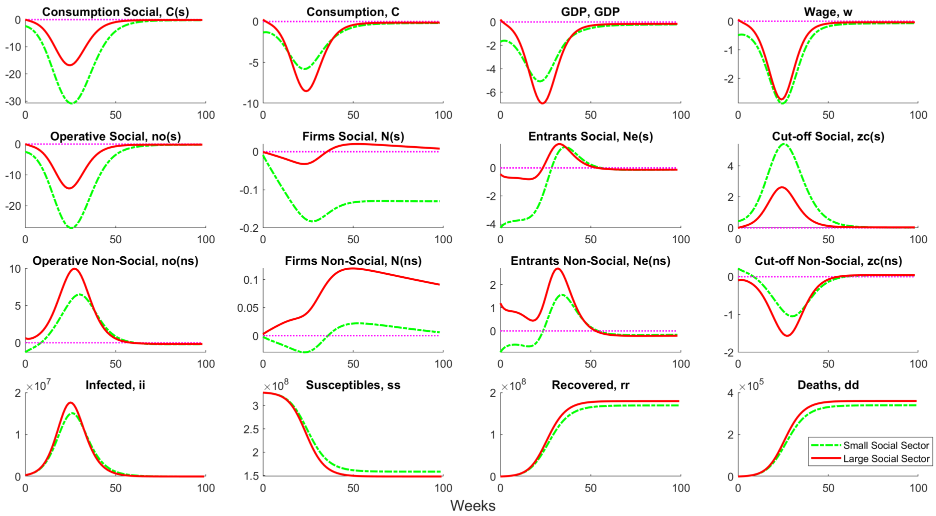

Figure 8: Alternative Sizes of the Social Sector

Red solid lines in Figure 8 refer to the case of an economy characterized by a large social sector, while green dotted-dashed ones to the case of a smaller social sector. Consistently with the evidence uncovered by the IMF, output and consumption losses are larger in the former case. The deeper recession observed in the case of a large social sector is accompanied by an abrupt reallocation across sectors. Indeed, the number of operative firms in the social sectors collapses as well as firm entry. Notice that, when the size of the social sector is large, percentage deviations from the initial equilibrium in consumption and operative firms in the social sector are smaller with respect to those observed in the case of a smaller social sector. Nevertheless, since the absolute size of the social sector is significantly larger when $\chi = 0.66$, the absolute magnitude of the reallocation across sectors is more sizeable in the former case. The relative size of sectors is reflected also in the epidemiological block of the model. When the size of the social sector is larger, there is a higher number of infected both at the peak and during the transition, which ultimately result in a higher death toll.

8 Conclusions

A key dimension to consider in order to understand firms economic exposure to the COVID-19 pandemic is their sector of operation. Considering US statistics on Business formation, we argue that during the pandemic business startups reallocated from sectors characterized by social interaction to those characterized by low social contact. To rationalize this fact, we build an Epidemiological-Industry Dynamic model characterized by firms with heterogeneous productivity and endogenous business dynamism. The epidemiological block of the model consists of a SIR model. We showed that in response to the outburst of the pandemic, our framework reproduces the reallocation of entry opportunities across sectors observed in US data. The latter entails a response in sectoral and aggregate productivity. At the sector level, there is cleansing of low-productivity firms in the social sector, and the opposite in the non-social one. Accounting for productivity movement at the sectoral level does not, however, suffice to explain the empirical pattern of aggregate labor productivity during the pandemic. We showed that a paramount ingredient to replicate the latter is to capture the reallocation of demand and of operative firms across sectors that we observed during the pandemic. Monetary policy, through its effect on the real interest rate, affected business dynamism during the Covid-19 crisis. Indeed, the latter cannot be replicated in a the flexible-wage counterpart of our model. Finally, our framework could naturally rationalize the different quantitative effects of the Covid-19 shock observed in countries caracterized by different relative sizes of the social vs. non-social sectors. Our analysis has not considered the lockdown measures governments have taken in response to the COVID-19 pandemic, but emphasized the role of the behavioural response of households for the propagation of the pandemic to the economy. On the basis of our results, we expect that lockdown measures would quantitatively amplify, but not qualitatively alter, the reallocation process across sectors that we described in the paper. Exploring the effects of lockdown measures on productivity represents an interesting avenue for future research.

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A Appendix

A.1 Households

The economy features a continuum of homogeneous households or family of mass one, and markets are complete. For these reasons we consider a representative household from the outset. Each household is populated by a continuum of individuals, who differ in their contagion type. The family is initially of unitary length, while at time t has size $(1 - \mathcal{D}_t)$, where $\mathcal{D}_t$ denoted the cumulative number of dead members of the household. The time-t utility of the representative household is:

$$(1 - \mathcal{D}_t)\, log\,(c_t) - (1 - \mathcal{D}_t)\, \nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi} \right) - u_d \mathcal{D}_t \tag{A1}$$

where c_t is the individual, i.e. of each living household's member, consumption of the final good, l_t^s denotes individual labor supply and u_d is the flow disutility from death, which includes the psychological costs of death on surviving members. The final good is a composite good defined as: $c_t = \left[\chi^{\frac{s}{\eta}} c_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} c_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$, where $c_t(s)$ and $c_t(ns)$ are the individual consumption levels of the social and the non-social good, respectively. Both $c_t(s)$ and $c_t(ns)$ are defined as aggregators of goods produced in the social and non-social sector, respectively. The parameter χ captures the relative importance of the social good in the consumption basket, while the parameter $\eta > 1$ measures the elasticity of substitution between the social and the non-social goods.

A.1.1 Income and Investment

In each time period t, agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t + 1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1}/\pi_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period t+1 and t, and π_{t+1} denotes the inflation rate over the same period. Households choose consumption, hours of work, and how much to invest in state-contingent assets and in risky stocks $b_{t+1}(q)$.

Stock ownership ensures to households a flow of dividend distributed by operative firms. We assume a continuum of differentiated labor inputs indexed by $j \in [0,1]$. Wages are set by labor type specific unions, indexed by $j \in [0,1]$. Given the nominal wage, W_t^j, set by union j, agents stand ready to supply as many hours to labor market j, L_t^j , as required by firms, that is

$$L_t^j = (W_t^j / W_t)^{-\theta_w} L_t^d, \tag{A2}$$

where W_t is an aggregate wage index, and L_t^d is aggregate labor demand. The latter can be obtained by integrating firms' individual labor demand over the distribution of idiosyncratic productivities. Formal definitions of the labor demand and of the wage index can be found in the sections devoted to firms. Agents are distributed uniformly across unions, hence aggregate demand for labor type j is spread uniformly across households. Total hours must satisfy the time resource constraint $L_t^s = \int_0^1 L_t^j \, dj$. Combining the latter with equation (A2) we obtain

$$L_t^s = L_t^d \int_0^1 (w_t^j / w_t)^{-\theta_w} \, dj$$

where lower case letters denote wages in real terms. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by

$$\int_0^1 (w_t^j L_t^j) = L_t^d \int_0^1 w_t^j (w_t^j / w_t)^{-\theta_w} \, dj.$$

Besides labor income, households enjoy dividend income through stock ownership. The timing of investment in the stock market is as in Bilbiie et al. (2012) and Chugh and Ghironi (2011). At the beginning of period t, the household owns $b_t(q)$ shares of a sector mutual fund that represents the ownership of the $N_t(q)$ incumbents in sector (q) in period t, with $(q) = \{(s), (ns)\}$.

The period-t asset value of the portfolio of firms held in sector (q) can be expressed as the sum of two components. First, the total firms' value in sector (q), which is the product between the average value of a firm $\tilde{v}_t(q)$ and the existing mass of firms $N_t(q)$ in the same sector. Second, following the production and sales of varieties in the imperfectly competitive goods markets, total firms' dividends, distributed only by operative firms. Operative firms in sector (q), that we denote as $N_{o,t}(q)$ and formally define below, are the set of firms that are actively producing in each sector at time t. As shown in Appendix A.10, total dividends received by a household in a sector can be written as $N_{o,t}(q)\tilde{e}_t(q)$, where $\tilde{e}_t(q)$ denotes average sectoral dividends, that is the amount of dividends distributed by the firm with average sectoral productivity. To obtain the total value of the portfolio held by households, one needs to sum both components over the two sectoral funds.

During period t, the household purchases $b_{t+1}(q)$ shares in new sectoral funds to be carried to period $t+1$. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants, $N_t^e(q)$, although at the very end of period t a fraction of these firms disappears. The value of total stock market purchases is thus $\sum_{q=s,ns} \tilde{v}_t(q) \left(N_t(q) + N_t^e(q)\right) b_{t+1}(q)$.

A fraction of the resources of household is deposited to financial intermediaries that provide loans to firms. Firms use one-period loans to finance a fraction $\alpha_w \in [0,1]$ of the wage bill in advance of production. In equilibrium, a real amount equal to $\alpha_w w_t L_t^d$ must be gathered for this purpose. The deposit yields a gross interest rate R_t. Interests on deposits are distributed to households at the end of each period t in a lump sum fashion. We can finally write the flow budget constraint of the representative household as:

$$(1-\mathcal{D}_t) \sum_{q=s,ns} \rho_t\left(q\right) c_t\left(q\right) + E_t r_{t,t+1} a_{t+1} + \sum_{q=s,ns} \tilde{v}_t(q) \left(N_t(q) + N_t^e(q)\right) b_{t+1}(q) =$$

$$= L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\theta_w} dj + \frac{a_t}{\pi_t} + \sum_{q=s,ns} \left(N_t(q)\tilde{v}_t(q) + N_{o,t}(q)\tilde{e}_t(q)\right) b_t(q) + \left(R_t - 1\right) \alpha_W w_t L_t^d$$

where $\rho_t\,(q)$ is the price of the good produced in sector (q) expressed in real terms, that we define in the section devoted to firms.

A.1.2 Utility Maximization

Denoting with V_t the household's value at time t, utility can be written in recursive form as:

$$V_t(\cdot) = (1 - \mathcal{D}_t)\, log\,(c_t) - (1 - \mathcal{D}_t)\, \nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi}\right) - u_d \mathcal{D}_t + \beta E_t V_{t+1}(\cdot) \qquad \text{(A3)}$$

The household maximizes (A3) with respect to $c_t\,(s)$, $c_t\,(ns)$, l_t^s, $\mathcal{D}_{t+1}$, $\mathcal{I}_{t+1}, \mathcal{S}_{t+1}$, $\mathcal{R}_{t+1}$ (dropped from the maximization as $\mathcal{R}_t = 1 - \mathcal{D}_t - \mathcal{I}_t - \mathcal{S}_t$), a_{t+1}, $b_{t+1}\,(s)$ and $b_{t+1}\,(ns)$ at any t. Constraints to the problem are the household's budget constraint presented above, the time resource constraint $L_t^s = L_t^d \int_0^1 (w_t^j/w_t)^{-\theta_w}\, dj$ and the equations defining contagion, presented in the main text. The recursive utility maximization problem reads as:

$$V_t(\mathcal{S}_t, \mathcal{I}_t, \mathcal{D}_t, a_t, b_t(s), b_t(ns)) = (1-\mathcal{D}_t)\, log\,(c_t) - (1-\mathcal{D}_t)\,\nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi} \right) - u_d \mathcal{D}_t +$$

$$+\beta E_t V_{t+1}(\mathcal{S}_{t+1}, \mathcal{I}_{t+1}, \mathcal{D}_{t+1}, a_{t+1}, b_{t+1}(s), b_{t+1}(ns))$$

$$+\lambda_t \left[L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj + \frac{a_t}{\pi_t} + \sum_{q=s,ns} \left(N_t(q)\tilde{v}_t(q) + N_{o,t}(q)\tilde{e}_t(q) \right) b_t(q) \right.$$

$$+ (R_t - 1)\,\alpha_W w_t L_t^d - (1-\mathcal{D}_t) \sum_{q=s,ns} \rho_t\,(q)\, c_t\,(q) - E_t r_{t,t+1} a_{t+1} +$$

$$\left. - \sum_{q=s,ns} \tilde{v}_t(q)\, (N_t(q) + N_t^e(q))\, b_{t+1}(q) \right] +$$

$$+ \frac{\lambda_t w_t}{\tilde{\mu}_t} \left[(1-\mathcal{D}_t)\, l_t^s - L_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj \right] +$$

$$+ \lambda_{\mathcal{T},t} \left[\mathcal{T}_t - \mathcal{S}_t \mathbb{I}_t \pi_1 c_t\,(s)\, C_t\,(s) - \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d - \pi_3 \mathcal{S}_t \mathbb{I}_t \right] +$$

$$+ \lambda_{\mathcal{I},t} \left[\mathcal{S}_{t+1} - \mathcal{T}_t - \mathcal{I}_t + (\pi_r + \pi_d)\, \mathcal{I}_t \right] +$$

$$+ \lambda_{\mathcal{S},t} \left[\mathcal{S}_{t+1} - \mathcal{S}_t + \mathcal{T}_t \right] +$$

$$+ \lambda_{\mathcal{D},t} \left[\mathcal{D}_{t+1} - \mathcal{D}_t - \pi_d \mathcal{I}_t \right]$$

where $\rho_t\,(q) = \frac{P_t(q)}{P_t}$ and $c_t = \left[\chi^{\frac{1}{\eta}} c_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}}\, c_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$. The First Order Conditions (FOCs) are the following:[16]

$$c_t\,(s) : \chi^{\frac{1}{\eta}} \left(\frac{c_t(s)}{c_t} \right)^{\frac{-1}{\eta}} = \lambda_t \rho_t\,(s)\, c_t + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t} \pi_1 C_t\,(s)\, c_t$$

[16]Rearranged from

$$c_t\,(s) : (1-\mathcal{D}_t) \left[\frac{\eta}{\eta-1} c_t^{\frac{1}{\eta}-1} \chi^{\frac{1}{\eta}} \frac{\eta-1}{\eta} c_t(s)^{\frac{-1}{\eta}} \right] - (1-\mathcal{D}_t)\, \lambda_t \rho_t\,(s) - \lambda_{\mathcal{T},t} \mathcal{S}_t \mathbb{I}_t \pi_1 C_t\,(s) = 0$$

$$c_t\,(ns) : (1-\mathcal{D}_t) \left[\frac{\eta}{\eta-1} c_t^{\frac{1}{\eta}-1} (1-\chi)^{\frac{1}{\eta}} \frac{\eta-1}{\eta} c_t(ns)^{\frac{-1}{\eta}} \right] = (1-\mathcal{D}_t)\, \lambda_t \rho_t\,(ns)$$

$$l_t^s : -v\,(1-\mathcal{D}_t)\,(l_t^s)^{\phi} + \frac{\lambda_t w_t}{\tilde{\mu}_t} (1-\mathcal{D}_t) - \lambda_{\mathcal{T},t} \mathcal{S}_t \mathbb{I}_t \pi_2 L_t^d = 0$$

$$c_t\,(ns) : (1-\chi)^{\frac{1}{\eta}} \left(\frac{c_t(ns)}{c_t}\right)^{\frac{-1}{\eta}} = \lambda_t \rho_t\,(ns)\,c_t$$

$$l_t^s : \nu\,(l_t^s)^{\phi} = \frac{\lambda_t w_t}{\tilde{\mu}_t} - \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t} \pi_2 L_t^d$$

$$\mathcal{T}_t : \lambda_{\mathcal{T},t} + \lambda_{\mathcal{S},t} - \lambda_{\mathcal{I},t} = 0$$

$$\mathcal{I}_{t+1} : \beta E_t V_{\mathcal{I},t+1} + \lambda_{\mathcal{I},t} = 0$$

$$\mathcal{S}_{t+1} : \beta E_t V_{\mathcal{S},t+1} + \lambda_{\mathcal{S},t} = 0$$

$$\mathcal{D}_{t+1} : \beta E_t V_{\mathcal{D},t+1} + \lambda_{\mathcal{D},t} = 0$$

$$a_{t+1} : \beta E_t V_{a,t+1} - \lambda_t E_t r_{t,t+1} = 0$$

$$b_{t+1}(s) : \beta E_t V_{b(s),t+1} - \lambda_t \tilde{v}_t(s)\,(N_t(s) + N_t^e(s)) = 0$$

$$b_{t+1}(ns) : \beta E_t V_{b(ns),t+1} - \lambda_t \tilde{v}_t(ns)\,(N_t(ns) + N_t^e(ns)) = 0$$

Finally, the envelope conditions are:

$$V_{\mathcal{I},t} = -\lambda_{\mathcal{I},t}\,(1-\pi_r) + \pi_d\,(\lambda_{\mathcal{I},t} - \lambda_{\mathcal{D},t})$$

$$V_{\mathcal{S},t} = \lambda_{\mathcal{T},t} \left[-\mathbb{I}_t \pi_1 c_t\,(s)\,C_t\,(s) - \mathbb{I}_t \pi_2 l_t^s L_t^d - \pi_3 \mathbb{I}_t\right] - \lambda_{\mathcal{S},t}$$

$$V_{\mathcal{D},t} = -log(c_t) + \nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi}\right) - u_d + \lambda_t\left[\rho_t\,(s)\,c_t\,(s) + \rho_t\,(ns)\,c_t\,(ns)\right] - \frac{\lambda_t w_t}{\tilde{\mu}_t} l_t^s - \lambda_{\mathcal{D},t}$$

$$V_{a,t} = \lambda_t \frac{1}{\pi_t}$$

$$V_{b(s),t} = \lambda_t \left[N_t(s)\tilde{v}_t(s) + N_{o,t}(s)\tilde{e}_t(s)\right]$$

$$V_{b(ns),t} = \lambda_t \left[N_t(ns)\tilde{v}_t(ns) + N_{o,t}(ns)\tilde{e}_t(ns) \right]$$

Jones et al. (2020) point out that there is one externality not internalized by households. When considering current exposure risk in the newly infected equation, households scales it to $\mathbb{I}_t$ and not to $\mathcal{I}_t$. For this reason it neglects the risk of infecting others more tomorrow due to current choices. As a result, their mitigation efforts are lower than what would be socially optimal. Given the available workforce, it follows that aggregate labor supply is $L_t^s = (1 - \mathcal{D}_t)\, l_t^s$. The no arbitrage condition, such that the expected returns on different assets classes are equalized, is $E_t r_{t,t+1} = 1/R_t$.

A.2 Consumption of Individual Goods and Price Indexes

Let $c_{z,t}(q)$ be the consumption of the good produced by the firm with productivity z in sector (q), or, in short, the consumption of good z in sector (q). Consider minimizing expenditure when buying goods from the social sector, i.e. the sector which entails exposure externalities. The expenditure minimization problem of the household reads as follows. We assume that the levels of consumption involved here are individual, i.e. relative to one member of the household. The problem is:

$$\min_{c_{z,t}(s),} \int_0^\infty N_t(s) p_{z,t}(s)\, c_{z,t}(s)\, g(z) dz$$

such that:

$$c_t(s) = \left(\int_0^\infty N_t(s) c_{z,t}(s)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}}$$

The Lagrangian is:

$$\mathcal{L} = \int_0^\infty N_t(s) p_{z,t}(s)\, c_{z,t}(s)\, g(z) dz + \Theta_t(s) \left[c_t(s) - \left(\int_0^\infty N_t(s) c_{z,t}(s)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}} \right]$$

where $c_{z,t}(s)$ is the consumption of good z in the bundle $c_t(s)$. The F.O.C. is:

$$N_t(s)p_{z,t}(s) - \Theta_t \frac{\theta}{\theta-1}\left(\int_0^\infty N_t(s)c_{z,t}(s)^{\frac{\theta-1}{\theta}} g(z)dz\right)^{\frac{1}{\theta-1}} N_t(s)\frac{\theta-1}{\theta} c_{z,t}(s)^{\frac{-1}{\theta}} = 0$$

or

$$p_{z,t}(s) = \Theta_t(s)c_t(s)^{\frac{1}{\theta}} c_{z,t}(s)^{-\frac{1}{\theta}}$$

Raise to the power of $1-\theta$, multiply by $N_t(s)$ and integrate over the distribution of the idiosyncratic productivitities

$$\int_0^\infty N_t(s)p_{z,t}(s)^{1-\theta} g(z)dz = \Theta_t(s)^{1-\theta}c_t(s)^{\frac{1-\theta}{\theta}} \int_0^\infty N_t(s)c_{z,t}(s)^{\frac{\theta-1}{\theta}} g(z)dz$$

raise to $\frac{1}{1-\theta}$ to get

$$\left(\int_0^\infty N_t(s)p_{z,t}(s)^{1-\theta} g(z)dz\right)^{\frac{1}{1-\theta}} = \Theta_t(s)c_t(s)^{\frac{1}{\theta}} \left(\int_0^\infty N_t(s)c_{z,t}(1)^{\frac{\theta-1}{\theta}} g(z)dz\right)^{\frac{1}{1-\theta}}$$

Since the term in round brackets in the RHS equals $c_t(s)^{-\frac{1}{\theta}}$ it follows that

$$\left(\int_0^\infty N_t(s)p_{z,t}(s)^{1-\theta} g(z)dz\right)^{\frac{1}{1-\theta}} = \Theta_t(s)$$

$\Theta_t(s)$ represents the cost of relaxing the constraint when the objective is minimized, thus it amounts to the minimum cost of acquiring a unit of $\mathrm{c}_t(s)$, and it can be given the interpretation of the price index in sector s, that is $P_t(s)$, which can thus be written as:

$$P_t(s) = \left(\int_0^\infty N_t(s)p_{z,t}(s)^{1-\theta} g(z)dz\right)^{\frac{1}{1-\theta}}.$$

Substituting the latter into the initial FOC we obtain

$$\frac{c_{z,t}(s)}{c_t(s)} = \left(\frac{p_{z,t}(s)}{P_t(s)}\right)^{-\theta}$$

Given all member of the surviving members of the households have the same consumption, it follows that the latter can be rewritten as

$$\frac{C_{z,t}(s)}{C_t(s)} = \left(\frac{p_{z,t}(s)}{P_t(s)}\right)^{-\theta}$$

where recall that capital letters denote aggregate variables, thus $C_{z,t}(s)$ is the aggregate consumption of good z in sector s, and $C_t(s)$ is the aggregate consumption of bundle s. From the household problem that the individual demand of bundle s is given by:

$$\chi^{\frac{1}{\eta}}\left(\frac{c_t(s)}{c_t}\right)^{\frac{-1}{\eta}} = \left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right] c_t$$

or

$$\left(\frac{c_t(s)}{c_t}\right) = \chi\left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} c_t^{-\eta}$$

The latter represents the demand of good s by an individual member of the household. Aggregate consumption levels are:

$$C_t(s) = (1-\mathcal{D}_t)\, c_t(s)$$

and

$$C_t = (1-\mathcal{D}_t)\, c_t$$

As a result, the aggregate consumption of bundle s is

$$\left(\frac{C_t(s)}{C_t}\right) = \chi\left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta}$$

or

$$C_t(s) = (1-\mathcal{D}_t)^{\eta}\chi\left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta}(C_t)^{1-\eta}$$

Turning to the consumption of bundle ns, from the household problem we get:

$$(1-\chi)^{\frac{1}{\eta}}\left(\frac{C(ns)}{C_t}\right)^{\frac{-1}{\eta}} = \lambda_t\rho_t(ns)\frac{C_t}{(1-\mathcal{D}_t)}$$

or

$$C_t(ns) = (1-\chi)(1-\mathcal{D}_t)^{\eta}\left[\lambda_t\rho_t(ns)\right]^{-\eta}C_t^{1-\eta}$$

Note that both demand functions simplify to standard CES demands if there is no pandemic.

A.3 Aggregate price index

The aggregate price index must be such that

$$P_tC_t = P_t(s)\,C_t(s) + P_t(ns)\,C_t(ns)$$

substituting for the demand functions:

$$P_tC_t = P_t(s)\,\chi(1-\mathcal{D}_t)^{\eta}\left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1C_t(s)\right]^{-\eta}C_t^{1-\eta}+$$

$$+P_t(ns)(1-\chi)(1-\mathcal{D}_t)^{\eta}\left(\lambda_t\rho_t(ns)\right)^{-\eta}C_t^{1-\eta}$$

or

$$P_t = \left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta}\left\{P_t(s)\,\chi\left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1C_t(s)\right]^{-\eta} + P_t(ns)(1-\chi)\left(\lambda_t\rho_t(ns)\right)^{-\eta}\right\}$$

Notice that when $\mathbb{I}_t = 0$, $\mathcal{D}_t = 0$ and, thus, $\lambda_t = \frac{1}{C_t}$:

$$P_t = \chi P_t(s)^{1-\eta}P_t^{\eta} + (1-\chi)\,P_t(ns)^{1-\eta}P_t^{\eta}$$

$$P_t = \left[\chi P_t(s)^{1-\eta} + (1-\chi)\,P_t(ns)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

which is the traditional price index under CES production function.

A.4 Firms and Cost Minimization

Each sector (q) is populated by a mass $N_t(q)$ of atomistic firms. Once upon entry, firms draw a time invariant idiosyncratic productivity level, denoted by z, from a known distribution function, $g(z)$, which is identical across sectors and has a positive support. Within their sector of operation, the only source of heterogeneity across firms is the idiosyncratic productivity level, so that we can can index firms within a sector with z. Firms compete monopolistically within the sector and are subject to entry and exit. Each firm produces an imperfectly substitutable good $y_{z,t}(q)$, which is an input to the production of a sectoral bundle $Y_t(q)$ by a sectoral good producer. The latter adopts a CES production function defined as:

$$Y_t\left(q\right) = \left(\int_0^{\infty} N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz\right)^{\frac{\theta}{\theta-1}} \tag{A4}$$

where $\theta > 1$ is the degree of substitution between sectoral goods. The production function of individual goods producers is a constant return to scale Cobb-Douglas function, with parameter $0 \leq \alpha \leq 1$. The two inputs are labor, $l_{z,t}(q)$, and an intermediate input, $X_{z,t}(q)$. The latter is a composite of all the goods in the economy, i.e. we define a roundabout in production. The individual production function reads as:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha}$$

where the variable Z_t is an exogenous, and common to all firms, level of productivity. The labor input is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0,1]$, defined as:

$$l_{z,t}(q) = \left(\int_0^{1} (l^j_{z,t}(q))^{\frac{\theta_w-1}{\theta_w}} dj\right)^{\frac{\theta_w}{\theta_w-1}} \tag{A5}$$

where $\theta_w > 1$ is the degree of substitution between labor inputs. The minimization of total labor costs, $\int_0^1 W_t^j l_{z,t}^j(q) dj$, delivers firm z's demand of labor input j and the definition of the aggregate nominal wage index, which are respectively:

$$l_{z,t}^j(q) = \left(\frac{w_t^j}{w_t}\right)^{-\theta_w} l_{z,t}(q) \tag{A6}$$

and

$$w_t = \left(\int_0^1 \left(w_t^j\right)^{1-\theta_w} dj\right)^{\frac{1}{\theta_w - 1}} \tag{A7}$$

where W_t^j (w_t^j) is the nominal (real) wage of labor input j, and $l_{z,t}(q)$ denotes the demand of the labor bundle by firm z.

Our analysis encompasses the case where firms have to partially pay their workers before production takes place. In this case, firms finance a fraction $0 \leq \alpha_W \leq 1$ of their wage bill resorting to loans from financial intermediaries. Loans are reimbursed at the end of the period at the gross risk-free interest rate R_t. When $\alpha_W = 1$ the entire wage bill must be borrowed in advance of production, while when $\alpha_W = 0$ firms do not borrow at all.[17] Additionally, firms face fixed costs of production $f_{x,t}$, defined in terms of the final good.

Before maximizing profits, firms choose the optimal levels of labor and intermediate input to minimize the costs of production for a given level of idiosyncratic output. The minimization of the costs of production for a firm is:

$$\min_{l_{z,t}(q), X_{z,t}(q)} \left(\alpha_W R_t + 1 - \alpha_W\right) W_t l_{z,t}(q) + P_t X_{z,t}(q) + P_t f_{x,t}$$

subject to the definition of the production function:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha} \tag{A8}$$

Note that $f_{x,t} P_t$ are the nominal fixed costs of production (since $f_{x,t}$ are the real

[17] The latter is assumed in the benchmark specification.

fixed production costs). The Lagrangian is:

$$\mathcal{L} = (\alpha_W R_t + 1 - \alpha_W) W_t l_{z,t}(q) + P_t X_{z,t}(q) + P_t f_{x,t} + \lambda_{z,t}(q) \left[y_{z,t}(q) - Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha} \right]$$

The F.O.C. with respect to $l_{z,t}(q)$ is:

$$(\alpha_W R_t + 1 - \alpha_W) W_t = \lambda_{z,t}(q) (1-\alpha) Z_t z l_{z,t}(q)^{-\alpha} X_{z,t}(q)^{\alpha} \tag{A9}$$

while the F.O.C. with respect to $X_{z,t}(q)$ is:

$$P_t = \lambda_{z,t}(q) \alpha Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha-1} \tag{A10}$$

Combining the two F.O.C.s we get the optimal ratio between the two inputs, which does not depend on idiosyncratic variables nor on sectoral quantities:

$$\frac{X_{z,t}(q)}{l_{z,t}(q)} = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t} (\alpha_W R_t + 1 - \alpha_W)$$

because of this reason, this optimal condition holds in both sectors and for all firms.

Moreover, it is easy to show that $\lambda_{z,t}(q)$ is the marginal cost. First, substitute (A9) and (A10) in the cost function

$$\begin{aligned} (\alpha_W R_t + 1 - \alpha_W) W_t l_{z,t}(q) + P_t X_{z,t}(q) + f_{x,t} P_t &= \lambda_{z,t}(q) (1-\alpha) Z_t z l_{z,t}(q)^{-\alpha} X_{z,t}(q)^{\alpha} l_{z,t}(q) + \\ &= + \lambda_{z,t}(q) \alpha Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha-1} X_{z,t}(q) + f_{x,t} P_t = \\ &= \lambda_{z,t}(q) Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^{\alpha} + f_{x,t} P_t = \lambda_{z,t}(q) y_{z,t}(q) + f_{x,t} P_t \end{aligned}$$

Hence the cost function is linear in the output (CRTS) and $\frac{\partial TC_{z,t}(q)}{\partial y_{z,t}(q)} = MC_{z,t}(q) = \lambda_{z,t}(q)$.

Second, note that from (A9) and (A10) we can get the expression for the marginal cost which is given by $\lambda_{z,t}(q)$

$$\left(\alpha_W R_t + 1 - \alpha_W\right) W_t = \lambda_{z,t}(q)\left(1-\alpha\right) Z_t z \left(\frac{X_{z,t}(q)}{l_{z,t}(q)}\right)^{\alpha}$$

or

$$\left(\alpha_W R_t + 1 - \alpha_W\right) W_t = \lambda_{z,t}(q)\left(1-\alpha\right) Z_t z \left(\frac{\lambda_{z,t}(q)\alpha Z_t z}{P_t}\right)^{\frac{\alpha}{1-\alpha}} =$$

$$= \lambda_{z,t}(q)^{\frac{1}{1-\alpha}}\left(1-\alpha\right)\left(Z_t z\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{P_t}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\frac{\left(\alpha_W R_t + 1 - \alpha_W\right) W_t}{1-\alpha}\left(\frac{P_t}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} = \left[\lambda_{z,t}(q)\left(Z_t z\right)\right]^{\frac{1}{1-\alpha}}$$

$$\frac{1}{Z_t z}\left[\frac{\left(\alpha_W R_t + 1 - \alpha_W\right) W_t}{1-\alpha}\right]^{1-\alpha}\left(\frac{P_t}{\alpha}\right)^{\alpha} = \lambda_{z,t}(q) \tag{A11}$$

Thus:

$$MC_{z,t}(q) = MC_{z,t} = \frac{1}{Z_t z}\left[\frac{\left(\alpha_W R_t + 1 - \alpha_W\right) W_t}{1-\alpha}\right]^{1-\alpha}\left(\frac{P_t}{\alpha}\right)^{\alpha}. \tag{A12}$$

Marginal costs are affected by both the idiosyncratic productivity level, z, and by aggregate productivity, Z_t. The aggregate price level, P_t, appears in the definition of marginal costs because it represents the cost of one unit of the intermediate input $X_{z,t}(q)$. Real profits of firm z in sector (q) read as :

$$e_{z,t}(q) = p_{z,t}(q)y_{z,t}(q) - \left(\alpha_W R_t + 1 - \alpha_W\right) W_t l_{z,t}(q) - P_t X_{z,t}(q) - P_t f_{x,t} \tag{A13}$$

Profits maximization is presented in the following Appendix.

A.5 Profits Maximization

Firms maximize their per-period nominal profits by choosing the optimal price $p_{z,t}(q)$. Maximization is, thus:

$$\max_{p_{z,t}(q)} \quad p_{z,t}(q)y_{z,t}(q) - (\alpha_W R_t + 1 - \alpha_W)\, W_t l_{z,t}(q) - P_t X_{z,t}(q) - P_t f_{x,t} \quad q = 1,2$$

Subject to the definition of production (A8), to the F.O.C. from costs minimization, and to the demand constraint:

$$y_{z,t}(q) = \left(\frac{p_{z,t}(q)}{P_t(q)}\right)^{-\theta} Y_t(q)$$

which derives from the costs minimization of the sectoral bundlers. By combining the optimality condition from the costs minimization and the definition of the Cobb-Douglas, nominal profits can be rewritten as:

$$p_{z,t}(q)y_{z,t}(q) - y_{z,t}(q)\frac{1}{Z_t z}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_t}{\alpha}\right)^{\alpha} - f_{x,t}P_t$$

Using the demand function, profits can be rewritten as:

$$p_{z,t}(q)^{1-\theta}\left(\frac{1}{P_t(q)}\right)^{-\theta} Y_t(q) - p_{z,t}(q)^{-\theta}\left(\frac{1}{P_t\,(q)}\right)^{-\theta} Y_t\,(q)\, MC_{z,t} - f_{x,t}P_t$$

Under monopolistic competition, firm z takes sectoral and aggregate variables as given. As a result, the first order condition for profit maximization with respect to $p_{z,t}(q)$ reads as

$$(1-\theta)\, p_{z,t}(q)^{-\theta}\left(\frac{1}{P_t\,(q)}\right)^{-\theta} Y_t\,(q) + \theta p_{z,t}(q)^{-1-\theta}\left(\frac{1}{P_t\,(q)}\right)^{-\theta} Y_t(q) MC_{z,t} = 0$$

Under monopolistic competition, the optimal real price $\rho_{z,t}(q) = p_{z,t}(q)/P_t$ satisfies:

$$\rho_{z,t}(q) = \frac{\theta}{\theta-1}\frac{MC_{z,t}}{P_t} = \frac{\theta}{\theta-1}\frac{1}{Z_t z}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}$$

Note that the firm-z real price is solely determined by the idiosyncratic productivity level z. The term $\theta/(\theta-1)$ represents the standard time and firm invariant constant markup over marginal costs under monopolistic competition.

Individual real profits read as:

$$e_{z,t}(q) = \frac{p_{z,t}(q)}{P_t} y_{z,t}(q) - y_{z,t}(q)\frac{MC_{z,t}}{P_t} - f_{x,t} \tag{A14}$$

$$e_{z,t}(q) = \rho_{z,t}(q) y_{z,t}(q) - \frac{\theta-1}{\theta}\rho_{z,t}(q) y_{z,t}(q) - f_{x,t}$$

$$e_{z,t}(q) = \frac{1}{\theta}\rho_{z,t}(q) y_{z,t}(q) - f_{x,t} = \frac{1}{\theta}\rho_{z,t}(q)\left(\frac{p_{z,t}(q)}{P_t(q)}\right)^{-\theta} Y_t(q) - f_{x,t}$$

The sectoral quantity shares the same aggregator and demand constraint of the sectoral consumption, which, thus, entails the internalization of the exposure to contagion:[18]

$$\left(\frac{Y_t(s)}{Y_t}\right) = \chi\left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta}$$

While the demand function of the non-social good, that is good ns, reads as:

$$\left(\frac{Y_t(ns)}{Y_t}\right) = (1-\chi)\left[\lambda_t \rho_t(ns)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta}$$

Using the demand functions just provided, individual profits is sector $(q) = (s)$ can be rewritten as:

$$e_{z,t}(s) = \frac{1}{\theta}\rho_{z,t}(s)^{1-\theta}\rho_t(s)^{\theta}\chi\left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t - f_{x,t}$$

[18] See Appendix A.7.

while in sector $(q) = (ns)$ they are:

$$e_{z,t}(ns) = \frac{1}{\theta}\rho_{z,t}(ns)^{1-\theta}\rho_t(ns)^{\theta}(1-\chi)\left[\lambda_t\rho_t\left(ns\right)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta}Y_t - f_{x,t}$$

In the next sections we describe the endogenous exit margin and we compute the cut-off productivity levels, used in the main text.

A.6 Productivity Cut-off

Firms turn inactive when, by producing, they would make negative profits. Using this, we can define a cut-off productivity level, one for each sector, below which firms become idle. Setting equilibrium real profits equal to zero we get:

$$f_{x,t} = \frac{1}{\theta}\rho_{zc,t}(q)^{1-\theta}\rho_t(q)^{\theta}Y_t\left(q\right)$$

or:

$$\left(\frac{f_{x,t}}{\rho_t(q)^{\theta}Y_t(q)}\right)^{\frac{1}{1-\theta}}\theta^{\frac{1}{1-\theta}} = \rho_{zc,t}(q)$$

substituting the real price $\rho_{z,t}$, evaluated at the cut-off zc:

$$\frac{\theta}{\theta-1}\frac{1}{Z_t z_t^c(q)}\left(\frac{\left(\alpha_W R_t + 1 - \alpha_W\right)w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} = \left(\frac{f_{x,t}}{\rho_t(q)^{\theta}Y_t(q)}\right)^{\frac{1}{1-\theta}}\theta^{\frac{1}{1-\theta}}$$

Solving for the sectoral cut-off productivity $z_t^c(q)$:

$$z_t^c(q) = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{1}{Z_t}\left(\frac{\left(\alpha_W R_t + 1 - \alpha_W\right)w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{f_{x,t}}{\rho_t(q)^{\theta}Y_t(q)}\right)^{\frac{1}{\theta-1}}$$

which is the formula we use in the main text.

A.7 Fictitious Bundler of Y_t

The demand functions for the sectoral outputs $Y_t(q)$ can also be obtained from a fictitious bundler that maximizes, in real terms:

$$Y_t - \rho_t(s)Y_t(s) - \rho_t(ns)Y_t(ns)$$

subject to:

$$Y_t = \left[\chi^{\frac{1}{\eta}} Y_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} Y_t(ns)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

and to:

$$\mathcal{T}_t = \mathcal{S}_t \mathbb{I}_t \pi_1 c_t\left(s\right) C_t\left(s\right) + \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d + \pi_3 \mathcal{S}_t \mathbb{I}_t$$

The Lagrangian is:

$$\mathbb{L} = Y_t - \rho_t(s)Y_t(s) - \rho_t(ns)Y_t(ns) +$$
$$+\bar{\lambda}_t \left(\left[\chi^{\frac{1}{\eta}} Y_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} Y_t(ns)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - Y_t\right) +$$
$$+\bar{\lambda}_{t,t}\left(\mathcal{T}_t - \mathcal{S}_t \mathbb{I}_t \pi_1 c_t\left(s\right) C_t\left(s\right) - \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d - \pi_3 \mathcal{S}_t \mathbb{I}_t\right)$$

The first order condition with respect to $Y_t(s)$ is:

$$-\rho_t(s) + \bar{\lambda}_t \frac{\eta}{1-\eta} Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} \frac{\eta-1}{\eta} Y_t(s)^{\frac{-1}{\eta}} - \bar{\lambda}_{t,t} \mathcal{S}_t \mathbb{I}_t \pi_1 c_t\left(s\right) \frac{\delta C_t\left(s\right)}{\delta Y_t(s)} = 0$$

Since $Y_t(s) = C_t(s) + X_t(s) + f_{x,t} N_{o,t}(s) + f_{e,t} N_t^e(s)$ and $c_t(s)(1-\mathcal{D}_t) = C_t(s)$, this can be written as:

$$\bar{\lambda}_t Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} Y_t(s)^{\frac{-1}{\eta}} = \rho_t(s) + \bar{\lambda}_{t,t} \frac{\mathcal{S}_t \mathbb{I}_t}{1-\mathcal{D}_t} \pi_1 C_t\left(s\right)$$

The Lagrange multiplier $\bar{\lambda}_t$ represents the real value in terms of increased revenues of an extra unit of Y_t (indeed, without contagion, the condition would be real marginal cost of $Y_t(s)$, i.e. $\rho_t(s)$, equal to the marginal benefit of $Y_t(s)$, which is the marginal benefit of Y_t times the marginal product of $Y_t(s)$, i.e. $\frac{\delta Y_t}{\delta Y_t(s)}$). The first is equal to 1. For the household, the value of one extra unit of Y_t is hence $1 \cdot \frac{\delta C_t}{\delta Y_t}\frac{\delta c_t}{\delta C_t}\frac{\delta U}{\delta c_t}$, which is $\frac{1-\mathcal{D}_t}{C_t}$.

On the other hand, the Lagrange multiplier $\bar{\lambda}_{t,t}$ represents the costs in terms of newly infected of having one extra unit of $Y_t(s)$. This is equal to $\lambda_{t,t}\frac{\delta C_t(s)}{\delta Y_t(s)} = \lambda_{\mathcal{T},t}$. Finally, since the household owns the bundler, we rescale both multipliers by $1/\lambda_t$ to express everything in terms of utility. Thus:

$$\frac{1-\mathcal{D}_t}{C_t}\frac{1}{\lambda_t}Y_t^{\frac{1}{\eta}}\chi^{\frac{1}{\eta}}Y_t(s)^{\frac{-1}{\eta}} = \rho_t(s) + \frac{\lambda_{\mathcal{T},t}}{\lambda_t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)$$

This gives:

$$Y_t^{\frac{1}{\eta}}\chi^{\frac{1}{\eta}}Y_t(s)^{\frac{-1}{\eta}} = \left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)\right]\frac{C_t}{1-\mathcal{D}_t}$$

Raising to the power of $-\eta$:

$$\frac{Y_t(s)}{Y_t} = \chi\left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathbb{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)\right]^{-\eta}\left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta}$$

With the same reasoning, and knowing that $C_t(ns)$ does not impact the endogenous contagion rate, the first order condition for $Y_t(ns)$ is:

$$\frac{Y_t(ns)}{Y_t} = (1-\chi)\left[\lambda_t\rho_t(ns)\right]^{-\eta}\left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta}$$

Those are the two demand constraints used in the main text.

A.8 Labor Unions

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period, a union faces a constant probability $(1-\alpha^*)$ of re-optimizing the wage. Due to symmetry, we denote the optimal real wage chosen at time t, as w_t^*. This wage is chosen to maximize the relevant part of agents' lifetime utilities. We assume that wages that are not re-optimized are not index to inflation, i.e. $w_{t+s} = \frac{w_t^*}{\prod_{k=1}^{s}\pi_{t+k}}$

Then, the maximization problem of the union can be written as follows:

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s}\pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(w_{t+s}\right)^{\theta_w} \lambda_{t+s} \left\{ \left(w_t^*\right)^{1-\theta_w} \left(\prod_{k=1}^{s}\frac{1}{\pi_{t+k}}\right) - \left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right)\left(w_t^*\right)^{-\theta_w} \right\}$$

The FOCs with respect to w_t^* reads as: -

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s}\pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(w_{t+s}\right)^{\theta_w} \lambda_{t+s} \left[\begin{array}{c} (1-\theta_w)\left(w_t^*\right)^{-\theta_w}\left(\prod_{k=1}^{s}\frac{1}{\pi_{t+k}}\right) + \\ +\theta_w\left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right)\left(w_t^*\right)^{-\theta_w-1} \end{array} \right] = 0$$

or

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s}\pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(\frac{w_t^*}{w_{t+s}}\right)^{-\theta_w} \lambda_{t+s} \left[\begin{array}{c} w_t^*\frac{(\theta_w-1)}{\theta_w}\left(\prod_{k=1}^{s}\frac{1}{\pi_{t+k}}\right) + \\ -\left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right) \end{array} \right] = 0$$

For simplicity, define:

$$\left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right) = \chi_{t+s}^*$$

such that:

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s}\pi_{t+k}\right)^{\theta_w} L_{t+s}^d \left(w_{t+s}\right)^{\theta_w} \lambda_{t+s} \left[w_t^*\frac{(\theta_w-1)}{\theta_w}\left(\prod_{k=1}^{s}\frac{1}{\pi_{t+k}}\right) - \chi_{t+s}^* \right] = 0$$

The latter is equivalent to:

$$\begin{aligned}
& \frac{(\theta_w - 1)}{\theta_w} w_t^* E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\theta_w - 1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \\
= \quad & E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \chi_{t+s}^*
\end{aligned}$$

Define, following Schmitt-Grohé and Uribe (2005):

$$f_t^1 = E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\theta_w - 1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \tag{A15}$$

and

$$f_t^2 = E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \chi_{t+s}^*$$

The first order condition for wage setting is thus:

$$w_t^* = \frac{\theta_w}{(\theta_w - 1)} \frac{f_t^2}{f_t^1}$$

where w_t^* is the newly reset wage in real terms, and f_t^1 and f_t^s are recursively defined as:

$$f_t^1 = L_t^d (w_t)^{\theta_w} \lambda_t + \alpha^* \beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1$$

and

$$f_t^2 = L_t^d (w_t)^{\theta_w} \left(\nu (l_t^s)^{\phi} + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_2 L_t^d \right) + \alpha^* \beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2$$

Since:

$$\lambda_t \chi_t^* = \lambda_t \frac{w_t}{\tilde{\mu}_t} = \nu (l_t^s)^{\phi} + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_2 L_t^d$$

A.9 Aggregation

For the law of large number, in each period t the wage is optimally reset in a fraction $1-\alpha^*$ of the labor markets. Demand of hours in each of those markets is:

$$l_t^* = \left(\frac{W_t^*}{W_t}\right)^{-\theta_w} L_t^d$$

As a result, total demand of hours in market where the wage has been newly reset is : $L_t^* = (1-\alpha^*)\, l_t^*$. In markets where the wage was last reset τ periods ago the demand of hours is:

$$L_{t,\tau} = (1-\alpha^*)\,(\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t}\right)^{-\theta_w} L_t^d$$

Summing across all possible τ we obtain:

$$L_{t,t-\tau} = (1-\alpha^*) \sum_{\tau=1}^{\infty} (\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t}\right)^{-\theta_w} L_t^d$$

Combining these definitions we can write:

$$L_t^s = L_t^* + L_{t,t-\tau} = (1-\alpha^*) \sum_{\tau=0}^{\infty} (\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t}\right)^{-\theta_w} L_t^d = \tau_t^* L_t^d \tag{A16}$$

where τ_t^* measures the resource cost due to wage dispersion. The latter entails an inefficiently large labor supply with respect to the the one that is required for production. The variable τ_t^* can be written recursively as:

$$\tau_t^* = (1-\alpha^*) \left(\frac{w_t^*}{w_t}\right)^{-\theta_w} + \alpha^* \left(\frac{w_{t-1}}{w_t}\right)^{-\theta_w} \pi_t^{\theta_w} \tau_{t-1}^* \tag{A17}$$

Using the wage index $W_t = \left[\int_0^1 \left(W_t^j\right)^{1-\theta_w} dj\right]^{1/(1-\theta_w)}$ one can show that:

$$w_t^{1-\theta_w} = (1-\alpha^*)\,(w_t^*)^{1-\theta_w} + \alpha^* \left(\frac{w_{t-1}}{\pi_t}\right)^{1-\theta_w} \tag{A18}$$

In equilibrium, the representative family holds the entire portfolio of firms and the trade of state-contingent asset trade is nil. As a result, $b_{t+1}(q) = b_t(q) = 1$, and $a_{t+1} = a_t = 0$, and the following resource constraint follows:

$$C_t + N_t^e(s)\tilde{v}_t(s) + N_t^e(ns)\tilde{v}_t(ns) = (\alpha_W R_t + 1 - \alpha_W)\, w_t L_t^d + N_{o,t}(s)\tilde{e}_t(s) + N_{o,t}(ns)\tilde{e}_t(ns) \tag{A19}$$

By definition, Y_t is either consumed, used as intermediate input in the production process or used to cover fixed costs of production and of entry, thus:

$$Y_t = C_t + X_t + (N_{o,t}(s) + N_{o,t}(ns))\, f_{x,t} + N_t^e(s) f_{e,t}(s) + N_t^e(ns) f_{e,t}(ns) \tag{A20}$$

Finally, since the mass of households has unitary measure:

$$\mathbb{I}_t = \mathcal{I}_t, \quad \mathbb{S}_t = \mathcal{S}_t, \quad \mathbb{D}_t = \mathcal{D}_t.$$

A.10 Aggregation Details

Following Melitz (2003), we assume that the distribution function $g(z)$ follows a Pareto distribution with parameters $zmin$ (minimum) and κ (tail). We then define $\tilde{z}_t(q)$ as a special average productivity in each sector (q). This productivity summarizes all the relevant information within a sector, as the industry is isomorphic to one populated by identical $N_{o,t}(q)$ firms endowed with productivity $\tilde{z}_t(q)$, as we show below.

Thanks to the properties of the Pareto distribution, we can write $\tilde{z}_t(q)$ as a function of the cut-off productivity, $z_t^c(q)$, as follows:

$$\tilde{z}_t(q) = \left[\frac{1}{1 - G\left(z_t^c(q)\right)} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \Gamma z_t^c(q) \tag{A21}$$

where $\Gamma = \left[\frac{\kappa}{\kappa-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and, again due to the properties of the Pareto distribution,

$1 - G\left(z_t^c(q)\right) = \left(\frac{zmin}{z_t^c(q)}\right)^{\kappa}$.

In what follows, tilded variables refer to firms characterized by the special average productivity. Given that only some firms are active in each sector, the sectoral price $P_t(q)$ can be written as:

$$P_t(q) = \left[\frac{1}{1 - G\left(z_t^c(q)\right)} \int_{z_t^c(q)}^{\infty} p_{z,t}(q)^{1-\theta} N_{o,t}(q) g(z) dz\right]^{\frac{1}{1-\theta}}$$

Substituting the optimal individual price, $p_{z,t}(q)$, one obtains:

$$P_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}} \left(\frac{W_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{1-\alpha} \left(\frac{P_t}{\alpha}\right)^{\alpha} \frac{\theta}{\theta - 1} \frac{1}{Z_t}$$

$$\left[\frac{1}{1 - G\left(z_t^c(q)\right)} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz\right]^{\frac{1}{1-\theta}}$$

By using the definition of the special average productivity $\tilde{z}_t(q)$ this becomes:

$$P_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}} \left(\frac{W_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{1-\alpha} \left(\frac{P_t}{\alpha}\right)^{\alpha} \frac{\theta}{\theta - 1} \frac{1}{Z_t \tilde{z}_t(q)}$$

or, by using the definition of the optimal individual price, $p_{z,t}(q)$:

$$P_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}} \tilde{p}_t(q)$$

The latter also implies that the ratio $\rho_t(q) = \frac{P_t(q)}{P_t}$ equals

$$\rho_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}} \tilde{\rho}_t\left(q\right)$$

We can use this result to substitute out $\rho_t(q)$ from the equilibrium conditions regarding profits and cut-off productivities.

Moreover, by definition $w_t L_t(q) = \frac{1}{1-G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} w_t l_{t,z}(q) N_{o,t}(q) g(z) dz$ and the same holds for $X_t(q)$ and $\Omega_t(q)$, where $L_t(q)$ is the total labor demanded in sector

(q), $X_t(q)$ is the total intermediate input demanded in sector (q) and $\Omega_t^e(q)$ are the total dividends of sector (q). Following the steps above, namely by substituting for $l_{t,z}(q)$, $X_{t,z}(q)$ and $e_{t,z}(q)$ a function of z only, one can show that:[19]

$$L_t(q) = N_{o,t}(q)\tilde{l}_t(q), \quad X_t(q) = N_{o,t}(q)\tilde{X}_t(q) \quad \text{and} \quad \Omega_t(q) = N_{o,t}(q)\tilde{e}_t(q)$$

These conditions complete the aggregation from the sectors to the whole economy. Indeed, from the market clearing condition we get:

$$L_t^d = L_t(s) + L_t(ns)$$

and

$$X_t = X_t(s) + X_t(ns)$$

and

$$\Omega_t = \Omega_t(s) + \Omega_t(ns)$$

Note that the same results hold also for the sectoral and aggregate firm value as a function of $\tilde{v}_t(q)$. However, in this case we must multiply by $N_t(q)$ instead of $N_{o,t}$, as also inactive firms have a non-zero value due to the possibility of became active in the future upon survival.

From the definition of aggregate price:

$$P_t = \left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta}\left\{P_t(s)\,\chi\left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} + P_t(ns)\,(1-\chi)\,(\lambda_t\rho_t(ns))^{-\eta}\right\}$$

considering that $P_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}}\tilde{p}_t(q)$

$$P_t = \left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta}\{N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{p}_t(s)\chi\left[\lambda_t\rho_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} +$$

$$+N_{o,t}(ns)^{\frac{1}{1-\theta}}\tilde{p}_t(ns)\,(1-\chi)\,(\lambda_t\rho_t(ns))^{-\eta}\}$$

[19] For a deeper derivation see Colciago and Silvestrini (2020).

dividing through by P_t and considering that $\rho_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}}\tilde{\rho}_t(q)$ we get

$$1 = \left(\frac{C_t}{1-\mathcal{D}_t}\right)^{-\eta} \{N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\chi \left[\lambda_t N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{\rho}_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} +$$

$$+N_{o,t}(ns)^{\frac{1}{1-\theta}}\tilde{\rho}_t(ns)(1-\chi)\left(\lambda_t N_{o,t}(ns)^{\frac{1}{1-\theta}}\tilde{\rho}_t(ns)\right)^{-\eta}\}$$

which gives:

$$\left(\frac{C_t}{1-\mathcal{D}_t}\right)^{\eta} = \{N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\chi \left[\lambda_t N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{\rho}_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} +$$

$$+N_{o,t}(ns)^{\frac{1-\eta}{1-\theta}}\tilde{\rho}_t(ns)^{1-\eta}(1-\chi)\lambda_t^{-\eta}\}$$

Notice that when $\mathcal{I}_t = 0$, $\mathcal{D}_t = 0$ and, thus, $\frac{1}{C_t} = \lambda_t$ the condition reduces to:

$$1 = \chi\left[N_{o,t}(s)^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\right]^{1-\eta} + (1-\chi)\left(N_{o,t}(ns)^{\frac{1}{1-\theta}}\tilde{\rho}_t(ns)\right)^{1-\eta}$$

which is the usual price index with two sectors and CES demand.

In order to close the model, we must write the idiosyncratic $\tilde{X}_t(q)$ as functions of aggregate variables and productivity levels to be able to solve for the equilibrium. In sector (q), from the individual demand, given the relation for the sectoral price obtained above, we get:

$$\tilde{y}_t(q) = N_{o,t}(q)^{\frac{\theta}{1-\theta}}Y_t(q)$$

Recall that the cost minimization problem implies

$$\frac{X_{z,t}(q)}{l_{z,t}(q)} = \frac{\alpha w_t(\alpha_W R_t + 1 - \alpha_W)}{1-\alpha}$$

From the definition of the Cobb-Douglas and the optimality condition from costs minimization we can write:

$$\tilde{y}_t(q) = Z_t\tilde{z}_t(q)\tilde{X}_t(q)^{\alpha}\tilde{l}_t(q)^{1-\alpha} = Z_t\tilde{z}_t(q)\tilde{X}_t(q)\left(\frac{\alpha w_t(\alpha_W R_t + 1-\alpha_W)}{(1-\alpha)}\right)^{\alpha-1}$$

Thus:

$$N_{o,t}(q)^{\frac{\theta}{1-\theta}}Y_t(q) = Z_t\tilde{z}_t(q)\tilde{X}_t(q)\left(\frac{\alpha w_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{\alpha-1}$$

which gives:

$$\tilde{X}_t(q) = \frac{Y_t\left(q\right)N_{o,t}(q)^{\frac{\theta}{1-\theta}}}{Z_t\tilde{z}_t(q)}\left(\frac{\alpha w_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{1-\alpha}$$

Hence, the sectoral variables are:

$$X_t(s) = N_{o,t}(s)\tilde{X}_t(s) \quad \text{and} \quad X_t(ns) = N_{o,t}(ns)\tilde{X}_t(ns)$$

Finally:

$$X_t = X_t(s) + X_t(ns)$$

A.11 List of Equilibrium Conditions

A competitive equilibrium is a set of processes for the following 37 variables $\{\mathcal{T}_t, \mathcal{S}_t, \mathcal{I}_t, \mathcal{D}_t, l_t^s, L_t^d, C_t\left(s\right), C_t(ns), C_t, w_t, \tilde{\mu}_t, \lambda_t, \lambda_{\mathcal{S},t}, \lambda_{\mathcal{I},t}, \lambda_{\mathcal{D},t}, \lambda_{\mathcal{T},t}, z_t^c\left(s\right), N_t\left(s\right),$ $\tilde{\rho}_t\left(s\right), z_t^c\left(ns\right), N_t\left(ns\right), \tilde{\rho}_t\left(ns\right), \tilde{e}_t\left(s\right), \tilde{e}_t\left(ns\right), R_t, \pi_t, w_t^*, f_t^1, f_t^2, X_t, N_t^e\left(s\right), N_t^e\left(ns\right),$ $\tau_t^*, Y_t, Z_t, f_{e,t}(s), f_{e,t}(ns)\}$ that satisfy the 37 equilibrium conditions reported below.

In addition we have 29 parameters $\{\pi_1, \pi_2, \pi_3, \pi_d, \pi_r, \phi, \beta, \eta, \chi, \kappa, \theta, \delta, \theta_w, \alpha^*, \alpha, \alpha_w,$ $\varphi_\pi, \varphi_Y, \varphi_R, \Gamma, f_x, \nu, \psi_0, \psi_1, \gamma, \rho_Z, \varepsilon_Z, zmin, u_d\}$, where Γ is a convolution of other parameters.

Households and SIR

$$1) \quad \mathcal{T}_t = \frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)^2 + \mathcal{S}_t\mathcal{I}_t\pi_2 l_t^s L_t^d + \pi_3\mathcal{S}_t\mathcal{I}_t$$

$$2) \quad \mathcal{I}_{t+1} = \mathcal{T}_t + \mathcal{I}_t - \left(\pi_r + \pi_d\right)\mathcal{I}_t$$

$$3) \quad \mathcal{S}_{t+1} = \mathcal{S}_t - \mathcal{T}_t$$

$$4)\quad \mathcal{D}_{t+1} = \mathcal{D}_t + \pi_d \mathcal{I}_t$$

$$5)\quad \nu \left(l_t^s\right)^{\phi} = \frac{\lambda_t w_t}{\tilde{\mu}_t} - \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathcal{I}_t}{1-\mathcal{D}_t} \pi_2 L_t^d$$

$$6)\quad \lambda_{\mathcal{T},t} = \lambda_{\mathcal{I},t} - \lambda_{\mathcal{S},t}$$

$$7)\quad \lambda_{\mathcal{I},t} = -\beta E_t \left[-\lambda_{\mathcal{I},t+1}\left(1-\pi_r\right) + \pi_d \left(\lambda_{\mathcal{I},t+1} - \lambda_{\mathcal{D},t+1}\right)\right]$$

$$8)\quad \lambda_{\mathcal{S},t} = -\beta E_t \left[\lambda_{\mathcal{T},t+1}\left(-\frac{\mathcal{I}_{t+1}}{1-\mathcal{D}_{t+1}} \pi_1 C_{t+1}\left(s\right)^2 - \mathcal{I}_{t+1}\pi_2 l_{t+1}^s L_{t+1}^d - \pi_3 \mathcal{I}_{t+1}\right) - \lambda_{\mathcal{S},t+1}\right]$$

$$9)\quad \lambda_{\mathcal{D},t} = -\beta E_t \cdot$$

$$\left[-log\left(\frac{C_{t+1}}{1-\mathcal{D}_{t+1}}\right) + \nu\left(\frac{\left(l_{t+1}^s\right)^{1+\phi}}{1+\phi}\right) - u_d + \lambda_{t+1}\frac{C_{t+1}}{1-\mathcal{D}_{t+1}} - \frac{\lambda_{t+1} w_{t+1}}{\tilde{\mu}_{t+1}} l_{t+1}^s - \lambda_{\mathcal{D},t+1}\right]$$

$$10)\quad C_t = C_t\left(s\right)^{\frac{1}{1-\eta}} \left(1-\mathcal{D}_t\right)^{\frac{\eta}{\eta-1}} \chi^{\frac{1}{\eta-1}} \left[\lambda_t \left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}} \tilde{\rho}_t(s) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\left(s\right)\right]^{\frac{\eta}{1-\eta}}$$

$$11)\quad C_t(ns) = \left(1-\chi\right)\left(1-\mathcal{D}_t\right)^{\eta} \left[\lambda_t \left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{1}{1-\theta}} \tilde{\rho}_t(ns)\right]^{-\eta} C_t^{1-\eta}$$

$$12)\quad f_{e,t}(s) = \beta\left(1-\delta\right) E_t\left[\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(f_{e,t+1}(s) + \left(\frac{zmin}{z_{t+1}^c(s)}\right)^{\kappa} \tilde{e}_{t+1}(s)\right)\right]$$

$$13)\quad f_{e,t}(ns) = \beta\left(1-\delta\right) E_t\left[\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(f_{e,t+1}(ns) + \left(\frac{zmin}{z_{t+1}^c(ns)}\right)^{\kappa} \tilde{e}_{t+1}(ns)\right)\right]$$

$$14)\quad f_{e,t}(s) = \psi_0 + \psi_1\left(N_t^e(s)\right)^{\gamma}$$

$$15)\quad f_{e,t}(ns) = \psi_0 + \psi_1\left(N_t^e(ns)\right)^{\gamma}$$

$$16)\quad 1 = \beta E_t\left[\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\frac{R_t}{\pi_{t+1}}\right]$$

Unions

$$17)\quad w_t^* = \frac{\theta_w}{\left(\theta_w - 1\right)}\frac{f_t^2}{f_t^1}$$

$$18)\quad f_t^1 = L_t^d w_t^{\theta_w}\lambda_t + \alpha^*\beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1$$

$$19)\quad f_t^2 = L_t^d w_t^{\theta_w}\left(\nu\left(l_t^s\right)^{\phi} + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_2 L_t^d\right) + \alpha^*\beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2$$

Firms

$$20)\quad \tilde{\rho}_t(s) = \frac{\theta}{\theta-1}\frac{1}{Z_t\Gamma z_t^c(s)}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}$$

$$21)\quad \tilde{\rho}_t(ns) = \frac{\theta}{\theta-1}\frac{1}{Z_t\Gamma z_t^c(ns)}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}$$

$$22)\quad \tilde{e}_t(s) = \frac{1}{\theta}\tilde{\rho}_t(s)\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{\theta}{1-\theta}}\chi\cdot$$

$$\left[\lambda_t\tilde{\rho}_t(s)\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}} + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\,(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t - f_x$$

$$23)\quad \tilde{e}_t(ns) = \frac{1}{\theta}\tilde{\rho}_t(ns)\left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{\theta}{1-\theta}}(1-\chi)\cdot$$

$$\left[\lambda_t\tilde{\rho}_t(ns)\left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{1}{1-\theta}}\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t - f_x$$

Entry and Exit

$$24)\quad N_{t+1}(s) = (1-\delta)\left(N_t(s) + N_t^e(s)\right)$$

$$25)\quad N_{t+1}(ns) = (1-\delta)\left(N_t(ns) + N_t^e(ns)\right)$$

$$26)\quad z_t^c\,(s) = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{1}{Z_t}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\chi^{\frac{1}{1-\theta}}\cdot$$

$$\left(\frac{f_x}{\tilde{\rho}_t(s)^{\theta}\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{\theta}{1-\theta}}\left[\lambda_t\tilde{\rho}_t(s)\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}} + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t\,(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t}\right)^{\frac{1}{\theta-1}}$$

$$27)\quad z_t^c\,(ns) = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{1}{Z_t}\left(\frac{(\alpha_W R_t + 1 - \alpha_W)\, w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(1-\chi)^{\frac{1}{1-\theta}}$$

$$\left(\frac{f_x}{\tilde{\rho}_t(ns)^{\theta-\eta}\left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{\theta-\eta}{1-\theta}}\lambda_t^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t}\right)^{\frac{1}{\theta-1}}$$

Taylor Rule

$$28) \quad \left(\frac{R_t}{R}\right) = \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi}\left(\frac{Y_t}{Y}\right)^{\varphi_Y}\right]^{1-\varphi_R}\left(\frac{R_{t-1}}{R}\right)^{\varphi_R}$$

Aggregation and Market Clearing

$$29) \quad C_t + N_t^e(s) f_{e,t}(s) + N_t^e(ns) f_{e,t}(ns) =$$

$$= \left(\alpha_W R_t + 1 - \alpha_W\right) w_t L_t^d + \left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\tilde{e}_t(s) + \left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\tilde{e}_t(ns)$$

$$30) \quad 1 = (1-\mathcal{D}_t)^{\eta}\left\{\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\chi\left[\lambda_t\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}}\tilde{\rho}_t(s) + \right.\right.$$

$$\left.\left. + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta} + \left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{1-\eta}{1-\theta}}\tilde{\rho}_t(ns)^{1-\eta}(1-\chi)\lambda_t^{-\eta}\right\} C_t^{-\eta}$$

$$31) \quad C_t = \left[\chi^{\frac{1}{\eta}} C_t(s)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} C_t(ns)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$32) \quad Y_t = C_t + X_t + \left(\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s) + \left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right) f_x +$$

$$+ N_t^e(ns) f_{e,t}(ns) + N_t^e(ns) f_{e,t}(ns)$$

$$33) \quad X_t = \frac{\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}}}{Z_t\Gamma z_t^c(s)}\left(\frac{\alpha w_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{1-\alpha}\chi \cdot$$

$$\left[\lambda_t\left(\left[\left(\frac{zmin}{z_t^c(s)}\right)^{\kappa} N_t(s)\right]^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\right) + \lambda_{\mathcal{T},t}\frac{\mathcal{S}_t\mathcal{I}_t}{1-\mathcal{D}_t}\pi_1 C_t(s)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t +$$

$$+ \frac{\left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{1}{1-\theta}}}{Z_t\Gamma z_t^c(ns)}\left(\frac{\alpha w_t\left(\alpha_W R_t + 1 - \alpha_W\right)}{(1-\alpha)}\right)^{1-\alpha}(1-\chi) \cdot$$

$$\left[\lambda_t\left(\left[\left(\frac{zmin}{z_t^c(ns)}\right)^{\kappa} N_t(ns)\right]^{\frac{1}{1-\theta}}\tilde{\rho}_t(ns)\right)\right]^{-\eta}\left(\frac{C_t}{(1-\mathcal{D}_t)}\right)^{-\eta} Y_t$$

$$34) \quad w_t^{1-\theta_w} = (1-\alpha^*)(w_t^*)^{1-\theta_w} + \alpha^*\left(\frac{w_{t-1}}{\pi_t}\right)^{1-\theta_w}$$

$$35) \quad (1-\mathcal{D}_t)\, l_t^s = \tau_t^* L_t^d$$

$$36) \quad \tau_t^* = (1-\alpha^*)\left(\frac{w_t^*}{w_t}\right)^{-\theta_w} + \alpha^*\left(\frac{w_{t-1}}{w_t}\right)^{-\theta_w} \pi_t^{\theta_w}\tau_{t-1}^*$$

Entry and Exit

$$37) \quad Z_t = \rho_z Z_{t-1} + \varepsilon_z$$

A.12 Analytical Derivation of Aggregate Productivity in the Simplified Model

The first step in our derivation is to show that the aggregate price index can be written as $P_t = N_{o,t}^{1/(1-\theta)}\tilde{P}_t$, where $N_{o,t} \equiv N_{o,t}(s) + N_{o,t}(ns)$ and $\tilde{P}_t$ is an average of producers' prices. When $\pi_1 = 0$ the price index equation reduces to:

$$1 = (1-\mathcal{D}_t)^{\eta}\left\{\chi\lambda_t^{-\eta}\left[[N_{o,t}(s)]^{\frac{1}{1-\theta}}\tilde{\rho}_t(s)\right]^{1-\eta} + [N_{o,t}(ns)]^{\frac{1-\eta}{1-\theta}}\tilde{\rho}_t(ns)^{1-\eta}(1-\chi)\lambda_t^{-\eta}\right\}C_t^{-\eta}$$

or, in nominal terms using that $\tilde{\rho}_t(q) = \tilde{p}_t(q)/P_t$:[20]

$$P_t = \left\{\chi\left[N_{o,t}(s)\right]^{\frac{1-\eta}{1-\theta}}\tilde{p}_t(s)^{1-\eta} + (1-\chi)\left[N_{o,t}(ns)\right]^{\frac{1-\eta}{1-\theta}}\tilde{p}_t(ns)^{1-\eta}\right\}^{\frac{1}{1-\eta}}$$

Using the definition of $N_{o,t}$ provided above we get:

$$P_t = N_{o,t}^{\frac{1}{1-\theta}}\left\{\chi\omega_s^{\frac{1-\eta}{1-\theta}}\tilde{p}_t(s)^{1-\eta} + (1-\chi)(1-\omega_s)^{\frac{1-\eta}{1-\theta}}\tilde{p}_t(ns)^{1-\eta}\right\}^{\frac{1}{1-\eta}} = N_{o,t}^{\frac{1}{1-\theta}}\tilde{P}_t$$

where $\omega_s = N_{o,t}(s)/N_{o,t}$. Note that $\tilde{P}_t$ is a form of weighted average of the average producers' prices in the two sectors $\tilde{p}_t(S)$ and $\tilde{P}_t(ns)$. Given the price index, we aggregate the production function to obtain a notion of aggregate labor productivity.

[20]Provided that $(1-\mathcal{D}_t)/(\lambda_t C_t)$ is equal to 1. This can be proven under mild assumptions or it can be obtained by re-scaling the Lagrange multiplier of the household. Thus, in the following we omit this term from the derivations.

From the sectoral demand constraint and the definition of sectoral production we get:

$$Y_t(s) = \chi \rho_t(s)^{-\eta} Y_t = N_{o,t}(s)^{\frac{1}{\theta-1}} Z_t \tilde{z}_t(s) L_t(s)$$

and similar results for the non-social sector. Solving for $L_t(q)$ we get:

$$\chi \rho_t(s)^{-\eta} Y_t N_{o,t}(s)^{\frac{1}{1-\theta}} \frac{1}{Z_t \tilde{z}_t(s)} = L_t(s)$$

and

$$(1-\chi)\rho_t(ns)^{-\eta} Y_t N_{o,t}(ns)^{\frac{1}{1-\theta}} \frac{1}{Z_t \tilde{z}_t(ns)} = L_t(ns)$$

Summing side by side:

$$\frac{Y_t}{Z_t}\left(\chi \rho_t(s)^{-\eta} N_{o,t}(s)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(s)} + (1-\chi)\rho_t(ns)^{-\eta} N_{o,t}(ns)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(ns)}\right) = L_t^d$$

Using again the definition of $N_{o,t}$ and of ω_s this becomes:

$$\frac{Y_t}{Z_t} N_{o,t}^{\frac{1}{1-\theta}} \left(\chi \omega_s^{\frac{1}{1-\theta}} \rho_t(s)^{-\eta} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_s)^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)}\right) = L_t^d$$

Solving for Y_t:

$$Y_t = N_{o,t}^{\frac{1}{\theta-1}} Z_t \left(\chi \omega_s^{\frac{1}{1-\theta}} \rho_t(s)^{-\eta} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_s)^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)}\right)^{-1} L_t^d = N_{o,t}^{\frac{1}{\theta-1}} Z_t \tilde{Z}_t L_t^d$$

The aggregate labor productivity $\tilde{Z}_t$ is a weighted armonic average of the average sectoral productivities $\tilde{z}_t(s)$ and $\tilde{z}_t(ns)$. The statistics we compute and we use for the comparison with the aggregate labor productivity in the data is:

$$\frac{\left(\frac{Y_t P_t}{\tilde{P}_t}\right)}{L_t^d} = Z_t \tilde{Z}_t$$

The reason is the following: in the data, real variables in units of consumption are obtained by deflating the nominal quantities with price deflators as the CPI.

However, these deflators, by being based on averages of producers' prices over a semi-fixed bundle of goods, are conceptually more similar to the average producer price $\tilde{P}_t$ than to the consumer welfare-based price index P_t.[21] For this reason, *real* variables in the data do not correspond to $P_t X_t / P_t = X_t$ in the model but to $P_t X_t / \tilde{P}_t$, and these are the statistics we use. Note that this allows us to *correct* for the presence of love for variety in the model and directly use $\tilde{Z}_t$ and $\tilde{z}_t(q)$ as measures of aggregate and sectoral productivity, respectively.

[21] For a deeper discussion on the topic, see Ghironi and Melitz (2005) and Bilbiie et al. (2012).