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Paolo Bertoletti

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Department of Economics, Management and Statistics
University of Milano – Bicocca
Piazza Ateneo Nuovo 1 – 2016 Milan, Italy
http://dems.unimib.it/

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Paolo Bertoletti*
University of Milan-Bicocca

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Abstract

We study platform competition by modelling the preferences of a "representative buyer" over the services platforms provide and the commodities they intermediate. This captures an intensive margin of buyers' participation which is neglected by the canonical setting, and delivers a welfare measure of platform quality. Assuming that sellers offer a large variety of commodities under monopolistic competition and free entry, in contrast to previous results we find that in a duopoly setting strategically chosen commissions (whose value depends on sellers' expenditure share and demand elasticity) actually worsen buyers' welfare, which improves if platforms set commissions in advance of sellers' entry.

JEL Classification: D11, L13, L41, L51

Keywords: platform competition, market intermediation, exchange commissions

1 Introduction

There is by now a large literature on platform economics, where with platform operators (or marketplaces) we refer to intermediaries which allow buyers and sellers to interact, providing a number of complementary services (see e.g. Belleflamme and Peitz, 2015: Part IX, for a general introduction). One of these services is market creation, since it is usually the case that the exchanges would not take place without those services: see for instance Masden and Vellodi (2021), who report that most products which are intermediated by online marketplaces are not sold in any physical store. However, in practise many platforms also

*I am grateful to Federico Etro, Matthew Mitchell and Julian Wright for very useful comments on previous versions of this paper (the usual disclaimer applies). Correspondence: Department of Economics, Management and Statistics, Piazza dell'Ateneo Nuovo, 1 - 20126 Milan (Italy). E-mail: paolo.bertoletti@unimib.it

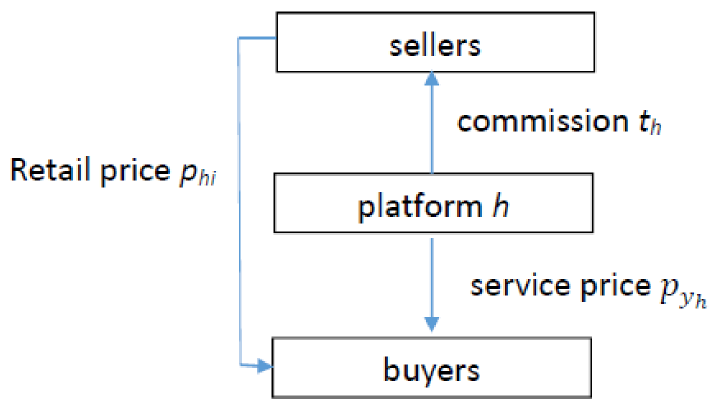

Figure 1: **Platform h business model**

provide and sell additional[1] services, which are possibly an important component of each platform supply: e.g., Amazon offers a Prime option providing fast deliveries and streaming of movies and music. In a similar fashion, payment card systems typically offer multiple credit services, TV networks produce and sell infotainment and providers of operating systems usually develop and sell additional software. Accordingly, in this paper we model platforms as directly selling their services to buyers, and affecting the trade they intermediate, being able to charge commissions on it.

Figure 1 (adapted from Belleflamme and Peitz, 2015: p. 651, Fig. 22.1) illustrates the business model we have in mind: platform h offers its services, whose quantity is measured by a scalar y_h, to buyers at unit price p_{y_h}, and ask an *ad valorem* exchange commission (a "transaction fee") t_h to the sellers, whose quantities x_{hi}, sold at retail prices p_{hi}, $i = 1, .., n_h$, is intermediating (the arrows point to price-taking parties). It suggests that the commodities intermediated by platforms and the services they directly provide to buyers jointly determine the surplus the latter enjoy. The by-now canonical model of platform competition (see e.g. Armstrong, 2006 and Belleflamme and Peitz, 2015: section 22.3) assumes that (usually identical) buyers obtain through the platform an indirect utility which depends on the set and prices of intermediated sellers. It has been microfounded by assuming that buyers have quasi-linear preferences and platforms are horizontally[2] differentiated, and used to account

[1] We do not deal here with the "hybrid" case in which platforms are allowed to sell their own products in competition with third party sellers, nor with the case in which buyers are not also consumers.

[2] See Etro (2021a) for a setting in which platforms can also be vertically differentiated.

for the variety provided by the sellers, and for their price structure: see e.g. Hagiu (2009).

However, the canonical model also assumes that buyers have unit demands for the services provided by platforms: as an implication, the participation of buyers to platforms has only an "extensive margin", and the prices the latter directly receive from buyers have the nature of "access fees". Thus, not only these fees do not affect buyers demand for the intermediated commodities, but possibly allow platforms to monetize the surplus the latter generate, thus internalizing the welfare impact of its changes (see Etro, 2021a and b). This setting has actually led to a number of interesting results, but it remains unclear how much they rely on the previous assumptions, which may not fit all the business environments. In particular, while they may well capture the case of device-funded platforms, in which a single device provided by the platform (think of a mobile phone) is possibly combined with a set of related services (applications), it is not obvious that they are suitable to study cases where an "intensive margin" of buyers' participation actually exists.

With the aim of investigating the possible role of this intensive margin, and more generally of exploring an alternative to the canonical setting, in this paper we take the novel (as far as we know) approach to introduce a "representative buyer" with homothetic preferences over the set of all goods jointly provided by platforms, who has to decide how to spread her expenditure E across them.[3] Formally, we assume that she has preferences over the set of goods provided by m competing platforms that can be represented by the direct utility function:

$$U(\mathbf{y}, \mathbf{x}) = F(U_1(y_1, \mathbf{x}_1), .., U_m(y_m, \mathbf{x}_m,)), \tag{1}$$

where the vector $\mathbf{x}_h = [x_{h1}, .., x_{hn_h}]$ denotes the quantities of commodities sold by n_h sellers through platform h $(j = 1, .., m)$, $\mathbf{y} = [y_1, .., y_m]$ and $\mathbf{x} = [\mathbf{x}_1, ..., \mathbf{x}_m]$ are respectively the vectors of all platforms' services and commodities, and F is increasing in its arguments. In this way we can fully exploit utility theory to account for all the goods offered across platforms, and exactly measure the welfare implications of their business organization, without constraining y_h to capture only the extensive margin of buyers' participation to platform h. Notice that in (1), we make the assumption that the goods provided by platform h are "separable" from those provided by other platforms, implying the existence of fully-fledged sub-utility functions $U_h(y_h, \mathbf{x}_h)$, $j = 1, .., m$, representing the utility contribution of each platform.

In addition, in this paper we also adopt the simplifying assumption that platforms intermediate a large number of commodities, and that accordingly their sellers interacts non-strategically, namely, in a monopolistically competitive fashion under free entry (*à la* Dixit and Stiglitz, 1977): see Etro (2021b)

[3] The assumption that the expenditure E is given can be formally justified by assuming that the representative buyer's preferences have a Cobb-Douglas structure, but it is also consistent with a large behavioural literature which suggests that consumers engage in "multiple budgets" planning procedures: see e.g. Thaler (1985). Alternatively, we might have endogenized E by assuming that preferences were quasi-linear and F (see below) a concave transformation of its arguments.

for a similar assumption. On the contrary, platforms are assumed to compete strategically by choosing the levels of their commissions, and setting the quantities of their services to attract the expenditure E_h under the budget constraint $\sum_{j=1}^{m} E_h = E$. We are interested in the impact of platform competition on the resulting price structure and allocation, and in its welfare consequences.

Our approach provides a workable and flexible setting that in principle can be applied to online marketplaces as Amazon or application stores as App Store by Apple, and to other cases as credit cards and game platforms. We find natural to start by assuming that from the representative buyer's point of view platforms overall provide perfect substitutes, i.e., by specializing (1) to:

$$U(\mathbf{y}, \mathbf{x}) = \sum_{j=1}^{m} U_h(y_j, \mathbf{x}_j), \tag{2}$$

where U_h is linear homogenous and identical across platform. In this case the representative buyer only cares for the "quality" supplied by each platform, which is increasing in the number of the intermediated sellers and decreasing with respect to their prices and the price of platform's services. However, other assumptions could also be investigated, introducing for instance some asymmetry across platforms. From the supply side, sellers are interested in the expenditure level E_h captured by each platform because this, jointly with the commission level, ultimately determines their profit levels. Accordingly, our setting delivers the kind of "indirect network effects" which are the hallmark of platforms serving groups of customers who value each other's participation/level of transaction (see Belleflamme and Peitz, 2015: p. 577-9). Finally, while most of the literature has focused on the interplay between buyers and sellers, and on how this interaction affects the pricing decisions by the competing platforms, in this paper we explore the welfare implications of their business organization.

In particular, as a first application of the suggested approach, in a duopoly example (i.e., $m = 2$) we focus on the role of the exchange commissions adopted by platforms which, as a matter of fact, are usually significant (for example, Amazon's commissions are typically in the 15%-30% range). They have been recently investigated by Etro (2021a, b), who has argued that their level tends to be neutral on buyers' welfare in a canonical setting, because competing platforms redistribute all the corresponding revenues through lower access prices.[4] In our example the services directly provided by each platform are related to the commodities it intermediates by a simple Cobb-Douglas structure (i.e., they are independent for a given expenditure E_h), and sellers are differentiated according to Constant Elasticity of Substitution (CES) preferences *à la* Dixit and Stiglitz (1977). As anticipated above, our setting provides a fully-fledged microeconomic foundation to the measurement of buyers' welfare, able to account for

[4] There is an interesting ongoing debate on the possibility that a hybrid marketplace would on the contrary systematically increase commissions on third party sellers to favor its own sales: see Hagiu *et al.* (2020), Anderson and Bedre-Defolie (2021) and Zennyo (2021).

price changes, income effects and gains from variety, and that would be easily generalized to the case of $m > 2$.

Given platform perfect substitutability, they are assumed to set the quantities of their services by competing *à la* Cournot: we study the duopoly symmetric equilibrium with or without the ability of platforms to commit to their commission levels in advance with respect to sellers' entry. In our example, without commissions platforms could not appropriate of the surplus created by the intermediated trade. Thus, in the market equilibrium platform commissions are generally positive. Their impact is to raise sellers' prices, thus decreasing platform quality, even though they also reduce the equilibrium prices required by platforms for their services. In addition, by reducing sellers' net profitability, under free entry they also decrease sellers' variety, further reducing platform quality. Eventually, positive commissions reduce buyers' welfare with respect to the "benchmark" case of zero commissions: in contrast to Etro (2021a, b), platform incentives in determining commissions are not generally aligned with those of buyers, who are not fully compensated by corresponding smaller prices of platform services. Moreover, buyers are better off when platforms compete by setting their commission in advance to attract sellers: this enhances platform competition, and may lead commissions to zero. Finally, the equilibrium level of commissions set in advance of sellers' entry increases with respect to the substitutability of their products, and decreases with respect to their collective expenditure share (two implications which are in principle empirically testable).

Our preliminary results say that, as it should be expected, the assumptions concerning the services provided by the platforms, and the way they are monetized, which are somehow overlooked by the canonical approach, do matter. In fact, the case in which those services are perfect complement with respect to sellers' commodities that we illustrate in Appendix B suggests that it is only in such a special case, that mimics the canonical setting, that the level of commissions is neutral on the market outcomes, and their strategic setting not a welfare issue.

Our approach and the preferences we employ in the main example are illustrated and discussed in section 2. Platform competition is analyzed in section 3. Section 4 concludes, while Appendix A contains the main computations and Appendix B develops the case of platform goods which are perfect complements.

2 Preferences over platforms' goods

Suppose that a *representative* buyer[5] has preferences over the set of commodities and services provided by a limited number of competing platforms. Each platform h is intermediating a large number n_h of commodities which are complementary with the services provided by the platform itself (whose amount is given by the scalar y_h). Overall, platforms are offering goods which are perfect

[5] This should be interpreted as covering both the case in which different buyers use different platforms and the case of multihoming buyers.

substitutes. Let the representative buyer's preferences over the goods provided by platform h be given by the utility function:

$$U_h\left(y_h, \mathbf{x}_h\right) = \left[y_h u_h\left(\mathbf{x}_h\right)^{\beta}\right]^{\frac{1}{1+\beta}}, \tag{3}$$

where $u_h\left(\mathbf{x}_h\right) = \left(\sum_{i=1}^{n_h} x_{hi}^{\rho}\right)^{\frac{1}{\rho}}$ is the familiar CES quantity index, $0 < \rho < 1$ and $\beta > 0$. U_h has a Cobb-Douglas upper-tier structure (with expenditure shares $\frac{1}{1+\beta}$ and $\frac{\beta}{1+\beta}$) and it is linear homogeneous (and strictly quasi-concave for $[y_h, \mathbf{x}_h] > \mathbf{0}$, where $\mathbf{0}$ is the relevant null vector): goods whose quantities are given by y_h and $\mathbf{x}_h$ are complements in the sense that they must be consumed together. Sellers supply commodities that are differentiated à la Dixit and Stiglitz (1977), namely, accordingly to CES preferences: in particular, $\sigma = 1/(1-\rho) > 1$ is the Constant Elasticity of Substitution among the n_h commodities.

Demands are provided by the FOCs for utility maximization:[6]

$$y_h\left(p_{y_h}, E_h\right) = \frac{E_h}{(1+\beta)\, p_{y_h}}, \quad x_{hj}\left(\mathbf{p}_h, E_h\right) = \frac{\beta p_{hj}^{-\sigma} E_h}{(1+\beta)\, P_h\left(\mathbf{p}_h\right)^{1-\sigma}}, \tag{4}$$

where p_{y_h} is the price that platform h asks for its services, p_{hj} is the retail price of seller hj $(j = 1, .., n_h)$, $\mathbf{p}_h = [p_{h1}, .., p_{hn_h}]$ is the vector of sellers' prices at platform h, $P_h\left(\mathbf{p}_h\right) = \left(\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the corresponding CES price index and E_h is the overall expenditure on this platform (notice that $p_{y_h} y_h\left(p_{y_h}, E_h\right) + P_h\left(\mathbf{p}_h\right) u_h\left(\mathbf{x}_h\left(\mathbf{p}_h, E_h\right)\right) = E_h$). Accordingly, the indirect utility function which corresponds to (3) is given by:

$$V_h\left(p_{y_h}, \mathbf{p}_h, E_h\right) = \frac{\beta^{\frac{\beta}{1+\beta}} E_h}{1+\beta}\left[\frac{s_h\left(\mathbf{p}_h\right)}{p_{y_h}}\right]^{\frac{1}{1+\beta}},$$

where $s_h\left(\mathbf{p}_h\right) = P_h\left(\mathbf{p}_h\right)^{-\beta}$ can be interpreted as a "quality index" for platform h, which positively depends on the number of sellers (an instance of the so-called buyers' "love for variety": see e.g. Benassy, 2006) and negatively on their prices.

Now suppose that there are just 2 platforms (i.e., $m = 2$), with representative buyer's preferences given by:

$$U\left(\mathbf{y}, \mathbf{x}\right) = \left[y_1 u_1\left(\mathbf{x}_1\right)^{\beta}\right]^{\frac{1}{1+\beta}} + \left[y_2 u_2\left(\mathbf{x}_2\right)^{\beta}\right]^{\frac{1}{1+\beta}},$$

where $\mathbf{y} = [y_1, y_2]$ and $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]$. Notice that platforms offer *perfect* substitutes: preferences are overall homothetic and additively separable in 2 groups, corresponding to platforms, and there is symmetry between platforms (except possibly for the number of sellers).[7] Clearly, the representative buyer's expenditure $E = E_1 + E_2$ could be spread over platforms only if both offered the

[6] By the separability of (1), a version of two-stage budgeting (see e.g. Deaton and Muellbauer, 1980: sections 5.1- 5.2) applies and demands of platform h's goods only depends on platform expenditure E_h.

[7] Exploting additivity across platforms, it would be easy to generalize this setting to the case of $m > 2$ competiting platforms and to introduce some additional asymmetries among platforms.

maximum "price-adjusted quality index" $s_h(\mathbf{p}_h)/p_{y_h}$, delivering indirect utility:

$$V(\mathbf{p}_y, \mathbf{p}, E) = \frac{\beta^{\frac{\beta}{1+\beta}} E}{1+\beta} \left[\max\left\{\frac{s_1(\mathbf{p}_1)}{p_{y_1}}, \frac{s_2(\mathbf{p}_2)}{p_{y_2}}\right\}\right]^{\frac{1}{1+\beta}},$$

where $\mathbf{p}_y = [p_{y_1}, p_{y_2}]$ and $\mathbf{p} = [\mathbf{p}_1, \mathbf{p}_2]$.

3 Competition

Under the assumption that their equilibrium numbers n_1 and n_2 are large, we use monopolistic competition *à la* Dixit and Stiglitz (1977) to model the non-strategic behavior of (identical) sellers whose products are intermediated by the platforms. To establish the market equilibrium between platforms, we consider their strategic choice of quantities y_h (Cournot competition)[8] and commissions t_h. We discuss two scenarios. We start with a three-stage setting in which: 1) first, sellers enters on each platform, paying a setup cost $F > 0$;[9] 2) then platforms simultaneously set the quantities of their services and the ad-valorem commissions $1 > t_h \geq 0$ ($t_h = 0$ in the benchmark case) that sellers will have to pay,[10] $h = 1, 2$; 3) finally, sellers set their own prices, and exchanges take place. In this setting platforms take as given the number of their sellers, and their choices (together with the prices chosen by the sellers) determine the distribution of the representative buyer's expenditure across platforms. We study the symmetric equilibrium in which the number of sellers is determined by free entry.

We then move to a four-stage setting in which: 1) platforms commit to their own commission levels, strategically affecting entry decisions by sellers; 2) sellers enters on each platform and pay the set-up cost; 3) platforms simultaneously set the quantities of their services; 4) sellers set their own prices, and exchanges take place. Again we study the symmetric equilibrium with free entry, and discuss the welfare implications of commissions in both settings.

3.1 Monopolistic competition among sellers

Let us assume that seller hi at the third stage has a (common) constant marginal cost $c > 0$, and that the number of sellers n_h is large and the expenditure shares $b_{hi} = \frac{p_{hi} x_{hi}}{\sum_{j=1}^{n_h} p_{hj} x_{hj}}$ ($h = 1, 2,\ i = 1, .., n_h$) are small. The latter assumption implies that each seller faces an individual demand whose elasticity (see (4))

[8] We briefly discuss the case of Bertrand competition in section 3.5.

[9] In our setting with monopolistic competition and identical sellers these may be thought as being either singlehoming or multihoming, but in the latter case they are assumed to pay a set-up cost for each of the platforms that intermediate their product. Accordingly, a seller's decision to join platform h does not affect his decision to join the other platform. This is consistent with the fact, reported by Duch-Brown (2017) for online marketplaces, that multi-homing is reasonably difficult for small sellers (in particular, marketing investments in reputation are often impossible to transfer).

[10] Qualitatively similar results would arise assuming that the exchange fee t_h were paid by buyers, or shared with them.

$|\partial \ln x_{hi} / \partial \ln p_{hi}| = \sigma + (1-\sigma) b_{hi}$ is approximately equal to the elasticity of substitution σ:[11] $\mu = 1/(\sigma - 1)$ is thus an approximately profit-maximizing markup. Accordingly, in the unique monopolistic competition equilibrium (see Dixit and Stiglitz, 1977)[12] sellers use the pricing rule:

$$p_{hi}(t_h) = \frac{c}{\rho(1-t_h)}. \tag{5}$$

It follows that the quality of platform h, s_h, depends on the number of its sellers n_h and on its commission level t_h according to:

$$s_h(t_h, n_h) = \left[\frac{(1-t_h)\rho}{c}\right]^{\beta} n_h^{\frac{\beta}{\sigma-1}}. \tag{6}$$

The useful simplification of using monopolistic competition is that the pricing rule (5) does not depend on the behavior of competing sellers. However, it does depend on the commission levels t_h, $h = 1, 2$ (chosen by platform at the second stage). The impact of a positive commission t_h is to increase the price of the intermediated commodities, thus reducing their consumption (i.e., seller size at the platform) for given platform expenditure and number of sellers. Also note that the quality index s_h is increasing with respect to n_h and decreasing with respect to t_h. In particular:

$$\frac{\partial s_h(t_h, n_h)}{\partial t_h} = -\beta \frac{s_h(t_h, n_h)}{(1-t_h)} < 0, \quad \frac{\partial^2 s_h(t_h, n_h)}{(\partial t_h)^2} = \beta(\beta - 1)\frac{s_h(t_h, n_h)}{(1-t_h)^2},$$

which stresses that the sensitivity of s_h with respect to t_h depends on the sellers' collective market share β.

It follows that, in the third stage equilibrium, by using (4):

$$x_{hi}(t_h, E_h, n_h) = \frac{(1-t_h)\rho\beta E_h}{(1+\beta) n_h c}, \quad p_{hi}x_{hi} = \frac{\beta E_h}{(1+\beta) n_h}, \quad n_n p_{hi} x_{hi} = \frac{\beta E_h}{(1+\beta)},$$

while the variable profit of each seller is given by:

$$\pi_{hi}(t_h, E_h, n_h) = [(1-t_h) p_{hi}(t_h) - c] x_{hi}(t_h, E_h, n_h) = \frac{(1-t_h)\beta E_h}{\sigma(1+\beta) n_h} : \tag{7}$$

notice that profit π_{hi} increases with respect to β and decreases with respect to σ (for given t_h, n_h and E_h).

[11] Notice that under symmetric sellers' prices from (4) one gets $|\partial \ln x_{hj} / \partial \ln p_{hi}| = \sigma + (1-\sigma)/n_h$. The relevant demand elasticity is here a constant due to the CES assumption: in the case of a symmetric equilibrium with non-CES (but still homothetic) preferences it could depend on the number of sellers: see Bertoletti and Etro (2016).

[12] The monopolistic competition equilibrium approximates its oligopolistic (Bertrand and Cournot) counterparts when market shares are indeed negligible: see Bertoletti and Etro (2021).

3.2 Strategic competition between platforms

At the second stage platforms compete by choosing the supply of their services and the level of their commissions, given the numbers of sellers and anticipating their third-stage equilibrium prices. Given perfect substitutability between platforms' goods, for values of the quality indexes given by (6) it must be the case that the inverse demand system $p_{y_h}(\mathbf{y},\mathbf{t},\mathbf{n})$ (where $\mathbf{t}=[t_1,t_2]$ and $\mathbf{n}=[n_1,n_2]$ are respectively the vectors of platform commissions and numbers of sellers), $h=1,2$, satisfies the condition $\frac{s_1(t_1,n_1)}{p_{y_1}}=\frac{s_2(t_2,n_2)}{p_{y_2}}$, as in the quality-augmented model of Cournot competition by Sutton (1991) (also see Belleflamme and Peitz, 2015: section 4.3.1),[13] and the adding-up condition (see (4))

$$p_{y_1}y_1+p_{y_2}y_2=\frac{E}{(1+\beta)}.$$

Accordingly, each platform h chooses the quantity of its services y_h, facing an inverse demand given by $(h=1,2)$

$$p_{y_h}(\mathbf{y},\mathbf{t},\mathbf{n})=\frac{s_h(n_h,t_h)\,E/(1+\beta)}{s_1(n_1,t_1)\,y_1+s_2(n_2,t_2)\,y_2} \tag{8}$$

(which is well defined for $\mathbf{y}>\mathbf{0}$), while from (4) revenue E_h is given by:

$$E_h(\mathbf{y},\mathbf{t},\mathbf{n})=(1+\beta)\,p_{y_h}(\mathbf{y},\mathbf{t},\mathbf{n})\,y_h.$$

To be able to sell more of its services y_h platform h has to accept to receive a smaller price p_{y_h}, which increases in the quality it provides (and in buyers' expenditure) and decreases with respect to the quality provided by the competing platform and the quantity of the latter services.

At the second stage, profit of platform h with a constant (common) unit cost d for its services is thus given by:

$$\begin{aligned}\Pi_h(\mathbf{y},\mathbf{t},\mathbf{n}) &= \left(p_{y_h}(\mathbf{y},\mathbf{t},\mathbf{n})-d\right)y_h+\frac{t_h\beta E_h(\mathbf{y},\mathbf{t},\mathbf{n})}{(1+\beta)}\\ &= \kappa(t_h)\,\frac{s_h(n_h,t_h)\,y_h}{s_1(n_1,t_1)\,y_1+s_2(n_2,t_2)\,y_2}-dy_h,\end{aligned}$$

where $\kappa(t_h)=\frac{E}{1+\beta}(1+t_h\beta)$ accounts also for the revenue platform h obtains through the commission t_h. Note that an increase of β reduces $\kappa(t_h)$ for a given, positive t_h (so that each platform overall revenue would be smaller in a symmetric equilibrium), and that the value of all variables at the platform level depend on its relative quality, $s_h(n_h,t_h)/s_{-h}(n_{-h},t_{-h})$ $(h,-h=1,2,\,h\neq -h)$.

Direct differentiation shows that $\frac{\partial^2\Pi_h}{(\partial y_h)^2}<0$, and that $\frac{\partial^2\Pi_h}{(\partial t_h)^2}<0$ under the sufficient condition that $1\geq\beta t_h$, which is satisfied in the equilibrium (see

[13] See Hagiu (2009) and Correia-da-Silva *et al.* (2018) for examples of platforms competing by choosing "quantities" (i.e., numbers of identical sellers or buyers) rather than prices, as usually assumed.

below), and *a fortiori* if $\beta \leq 1$. The determinant of the Hessian matrix $D^2_{y_h,t_h}\Pi_h$ is rather involved, but one can show that it is certainly positive, and then Π_h is locally (strictly) concave with respect to (y_h, t_h), if $s_1(n_1, t_1)\, y_1$ and $s_2(n_2, t_2)\, y_2$ are sufficiently close. In Appendix A we prove that in a sub-game perfect Nash-Cournot equilibrium of the second stage it must be the case that:

$$\widetilde{p}_{y_h}(\mathbf{t}, \mathbf{n}) = d\left(\frac{1}{k(t_h)} + \frac{s_h(n_h, t_h)}{k(t_{-h})\, s_{-h}(n_{-h}, t_{-h})}\right), \tag{9}$$

where $k(t_h) = (1 + t_h\beta)$. Note that an increase of t_h reduces the profit maximizing price of the services of platform h (for a given behavior of the competing platform and a given number of sellers n_h) both by increasing $k(t_h)$ and reducing s_h, and that also an increase of β would reduce it for any positive commission level in any symmetric equilibrium. In addition, a rise of t_{-h} would increase $\widetilde{p}_{y_h}$, which increases with respect to n_h and decreases with respect to n_{-h} by their impacts on the relative quality of the competing platforms.

Manipulating the FOC $\frac{\partial \Pi_h}{\partial y_h} = 0$ we also get:

$$\widetilde{y}_h(\mathbf{t}, \mathbf{n}) = \frac{k(t_h)^2\, s_h(n_h, t_h)\, \kappa(t_{-h})\, s_{-h}(n_{-h}, t_{-h})}{d\left[k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)\right]^2},$$

$$\widetilde{E}_h(\mathbf{t}, \mathbf{n}) = \frac{E k(t_h)\, s_h(n_h, t_h)}{k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)}, \tag{10}$$

$$\widetilde{\Pi}_h(\mathbf{t}, \mathbf{n}) = \frac{\kappa(t_h)\, s_h(n_h, t_h)^2\, k(t_h)^2}{\left[k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)\right]^2}, \tag{11}$$

and

$$\widetilde{\pi}_{hi}(\mathbf{t}, \mathbf{n}) = \frac{(1 - t_h)\,\beta E}{\sigma(1 + \beta)\, n_h} \frac{k(t_h)\, s_h(n_h, t_h)}{k(t_2)\, s_2(n_2, t_2) + k(t_1)\, s_1(n_1, t_1)}. \tag{12}$$

Notice that the ratio $k(t_h)\, s_h(n_h, t_h) / \left[k(t_{-h})\, s_{-h}(n_{-h}, t_{-h})\right]$, which depends on $\mathbf{t}$ and $\mathbf{n}$, affects all these results.

Consider now the profit-maximizing choice of commission t_h (for a given $\mathbf{n}$): an increase of t_h raises platform revenue *ceteris paribus* but decreases its quality index. From the FOC $\frac{\partial \Pi_h}{\partial t_h} = 0$, t_h must satisfy

$$\frac{1 - t_h}{1 + t_h\beta} = \frac{s_{-h} y_{-h}}{s_1 y_1 + s_2 y_2} = \frac{E_{-h}}{E}.$$

Accordingly, the profit-maximizing value of the commission depends on the expenditure distribution across platforms and on β, with $t_h \in (0, 1)$ for $E_{-h}/E \in (0, 1)$. Notice that in a symmetric equilibrium the quality indexes would cancel out.

3.3 A symmetric equilibrium with free entry

Under free entry with a setup cost F it must be the case that $\pi_{hi} = F$, i.e., using (7),[14] the equilibrium number of sellers must satisfy:

$$n_h = \frac{(1 - t_h)\,\beta E_h}{\sigma\,(1 + \beta)\,F}. \tag{13}$$

n_h increases with respect to buyers' expenditure over platform h, and decreases with respect to its commission level, the substitutability among the commodities provided by the sellers (which increases demand elasticity and reduces their profitability), and the entry fixed cost. Note that n_h depends on the behavior of the competing platform through E_h.

Moreover, in a symmetric equilibrium $E_h = E/2$ and then $t_h = (2 + \beta)^{-1}$: intuitively, a rise of t_h *ceteris paribus* increases its revenue but it also raises sellers' prices and decreases the quality index of the platform. In a symmetric equilibrium an increase of β raises both the relevant terms of this trade off but it has a larger impact on the second, and the equilibrium value of t_h must decrease. In contrast, for a given $\mathbf{n}$, in a symmetric equuilibrium σ does not affect the marginal profitability of increasing t_h and thus has no impact on its equilibrium value.[15]

Thus, in a symmetric, sub-game perfect Nash equilibrium with free entry of sellers:

$$p_{hi} = \frac{(2+\beta)\,c}{\rho\,(1+\beta)},\ \pi_{hi} = \frac{\beta E}{2\sigma\,(2+\beta)\,n_h},\ n_h = \frac{\beta E}{2\sigma\,(2+\beta)\,F},$$

$$x_{hi} = \frac{(1-t_h)\,\rho\beta E_h}{(1+\beta)\,n_h c} = \frac{(\sigma-1)\,F}{c}.$$

A positive commission increases the price of the commodities sold through the platforms, and reduces their number of sellers in a free entry equilibrium. However, due to the CES structure, seller size (at each platform) is unaffected by the commission. Notice that an increase in the sellers' collective share β raises their number less than proportionally, because it also reduces their prices by decreasing the equilibrium commission.

In addition,

$$p_{y_h} = \frac{(2+\beta)\,d}{1+\beta},\ y_h = \frac{E}{2d\,(2+\beta)} = \frac{\Pi_h}{d}.$$

[14] As usual in the literature, for the sake of simplicity we treat the equilibrium number of firms as a continuous variable.

[15] In a symmetric equilibrium:

$$\frac{\partial \Pi_h}{\partial t_h} = \frac{\kappa'(t_h)}{2} - \frac{\kappa(t_h)\,\beta}{4\,(1-t_h)} = \frac{\beta E}{2\,(1+\beta)} - \frac{\beta E\,(1+t_h\beta)}{4\,(1+\beta)\,(1-t_h)}.$$

3.3.1 Discussion

In the first scenario of our setting the larger is the collective expenditure share β of sellers the worse is platform profitability: a larger β increases the share of buyers' expenditure that platforms do not directly serve and enhances their competition. On the contrary, the elasticity of substitution σ among sellers' commodities (which determines both the representative buyer's love for variety and her demand elasticity: see Benassy, 1996), has an impact on their performance (the larger σ the smaller their equilibrium price and number, and the larger their size) but does not affect platform profitability.

Turning to the role of exchange commissions, their use by platforms increases their profits from an equilibrium value without commissions of $\Pi_h^0 = \frac{E}{4(1+\beta)}$ (from now onwards we use the suffix ^{0} to denote the values that variables assume in the equilibrium case with no commission) by allowing then to appropriate some of the surplus created by the commodities they are intermediating. This worsens the quality index of the platforms, both by raising the price of the products intermediated and by decreasing sellers' variety. In particular, since with no commissions free entry would deliver (in a symmetric equilibrium):

$$n_h^0 = \frac{\beta E}{2\sigma(1+\beta)F},$$

then, from (6),

$$s_h^0 = \left[\frac{\rho}{c}\right]^\beta \left[\frac{\beta E}{2\sigma(1+\beta)F}\right]^{\frac{\beta}{\sigma-1}} > \left[\frac{(1+\beta)\rho}{(2+\beta)^{\frac{\sigma}{\sigma-1}}c}\right]^\beta \left[\frac{\beta E}{2\sigma F}\right]^{\frac{\beta}{\sigma-1}} = s_h.$$

However, by decreasing the quality provided, positive commissions also decrease the price of platform services from $p_{y_h}^0 = 2d$, raising their consumption from $y_h^0 = \frac{E}{4(1+\beta)d}$. Overall, since

$$\frac{p_{y_h}^0}{p_{y_h}} = \frac{2(1+\beta)}{(2+\beta)} < \left[\frac{2+\beta}{1+\beta}\right]^{\frac{\sigma\beta}{\sigma-1}} = \frac{s_h^0}{s_h}$$

is equivalent to

$$g_1(\beta,\sigma) = 2^{\frac{1}{\frac{\sigma\beta}{\sigma-1}+1}} < \frac{2+\beta}{1+\beta} = g_2(\beta), \tag{14}$$

which is always satisfied for $\beta > 0$ and $\sigma > 1$, we get a worsening of the *price-adjusted quality index*, and thus of buyers' welfare with respect to the equilibrium with no commissions.[16]

[16] Since $\sup_\sigma g_1(\beta,\sigma) = 2^{\frac{1}{\beta+1}}$, a sufficient condition for (14) to hold is

$$2^{\frac{1}{1+\beta}} \leq \frac{2+\beta}{1+\beta}, \text{ i.e., } \ln 2 \leq (1+\beta)\ln\frac{2+\beta}{1+\beta} = h(\beta),$$

which is always satisfied since $h(0) = \ln 2$, $h'(0) > 0 > h''(\beta)$ and $\lim_{\beta\to\infty} h'(\beta) = 0$.

3.4 Pre-Commitment to commission levels

Let us now consider the second scenario in which, instead of determining them simultaneously to the quantities of their services, taking as given the number of sellers, platforms can set commissions in advance of sellers' entry.[17] Then commissions can be used strategically to affect entry at each platform. At the fourth stage sellers' pricing rules are still given by (5), while at the third stage (Cournot equilibrium) the relevant profit expressions respectively for the platforms and for sellers are provided by (11) and (12). In Appendix A we obtain the second-stage equilibrium number of sellers:

$$\widehat{n}_h(\mathbf{t}) = \frac{\beta E}{\sigma(1+\beta)F}\frac{1-t_h}{1+f(\mathbf{t})}, \tag{15}$$

where

$$f(\mathbf{t}) = \left(\frac{1+t_{-h}\beta}{1+t_h\beta}\right)^{\frac{\sigma-1}{\sigma-1-\beta}}\left(\frac{1-t_{-h}}{1-t_h}\right)^{\frac{\beta\sigma}{\sigma-1-\beta}}, \tag{16}$$

and the first-stage, reduced-form for the profit of platform h, which is increasing with respect to t_{-h}:

$$\widehat{\Pi}_h(\mathbf{t}) = \frac{E}{(1+\beta)}\frac{1+t_h\beta}{[1+f(\mathbf{t})]^2}. \tag{17}$$

Taking logs of (16) it is easy to see that $\partial f(\mathbf{t})/\partial t_h > 0 > \partial f(\mathbf{t})/\partial t_{-h}$ if and only $\sigma - 1 > \beta$. Since this implies that, as one should expect, $\widehat{n}_h(\mathbf{t})$ is decreasing with respect to t_h and increasing with respect to t_{-h}, in the following we make the assumption that this condition is satisfied. Taking logs of (17) the FOC for profit maximization can be written as:

$$\frac{\partial \ln \widehat{\Pi}_h}{\partial t_h} = \frac{\beta}{1+t_h\beta} - 2\frac{\frac{\partial f(\mathbf{t})}{\partial t_h}}{1+f(\mathbf{t})} = 0, \tag{18}$$

and computation shows that the SOC is globally satisfied (assuming $\sigma - 1 > \beta$) since $\frac{\partial \ln\left\{\frac{\partial f(\mathbf{t})}{\partial t_h}\right\}}{\partial t_h} > \frac{\partial \ln f(\mathbf{t})}{\partial t_h}$.

Since

$$\frac{\partial f(\mathbf{t})}{\partial t_h} = f(\mathbf{t})\left[-\frac{\sigma-1}{\sigma-1-\beta}\beta(1+t_h\beta)^{-1} + \frac{\beta\sigma}{\sigma-1-\beta}(1-t_h)^{-1}\right]$$

and $f(t\boldsymbol{\iota}) = 1$, where ι is the relevant unit vector and t a common commission value, (18) delivers (from now onwards we use the suffix ^{c} to denote the values variables assume in the symmetric, sub-game perfect Nash equilibrium of the commitment case):[18]

$$t_h^c = \max\left\{\frac{\sigma-2-\beta}{(\sigma-1)(2+\beta)}, 0\right\}.$$

[17] This scenario somehow captures a sort of a long-run equilibrium with respect to the case in which both p_{y_h} and t_h are simultaneously determined for given numbers of sellers.

[18] Notice that (18) might be satisfied by a negative commission value, suggesting that platform competition could lead to subsidize sellers. However, this seems hardly realistic and we do not consider this case here.

This shows that in a symmetric equilibrium commissions are smaller under this scenario than in the case of simultaneous platform choices, and actually zero whenever $(\sigma - 1 >)\ \beta \geq \sigma - 2$. Notice that t_h^c still (weakly) decreases with respect to β, but in this scenario it also (weakly) increases with respect to σ. Intuitively, a larger σ reduces the commission "pass-through", as measured by $\frac{\partial p_{hi}}{\partial t_h}$ (see (5)), and the marginal profitability of an increase of t_h rises.[19]

For the case of positive commissions ($\beta < \sigma - 2$) we get the following subgame perfect, Nash equilibrium values:

$$p_{hi}^c = \frac{(2+\beta)\,c}{(1+\beta)},\ n_h^c = \frac{\beta E}{2(\sigma-1)(2+\beta)F},\ s_h^c = \left[\frac{(1+\beta)}{\rho^{\frac{1}{\sigma-1}}(2+\beta)^{\frac{\sigma}{\sigma-1}}c}\right]^{\beta}\left[\frac{\beta E}{2\sigma F}\right]^{\frac{\beta}{\sigma-1}},$$

$$p_{y_h}^c = \frac{2d(\sigma-1)(2+\beta)}{(\sigma-1)(2+\beta)+\beta(\sigma-2-\beta)},\ \ y_h^c = \frac{E}{4d(1+\beta)}\left(1 + \frac{\sigma-2-\beta}{(\sigma-1)(2+\beta)}\beta\right) = \frac{\Pi_h^c}{d}.$$

3.4.1 Discussion

Competition through commissions to attract sellers decreases their (symmetric) equilibrium value and then prices, raising the number of sellers under free entry. This increases the quality index of platforms, but it also raises the equilibrium price they ask for their services, and decreases their supply. All in all, platforms' profits decrease by an "enhanced-competition effect", but their *price-adjusted quality index* must improve under commitment, and so buyers' welfare. To see the latter result, notice that in the case of a common commission t, by using (6), (8) and (15) we obtain:

$$\frac{s_h}{p_{y_h}} = \left[\frac{(1-t)\rho}{c}\right]^{\beta}\frac{(1+\beta t)\left[\frac{\beta E}{\sigma(1+\beta)F}\frac{1-t}{1+f(t\iota)}\right]^{\frac{\beta}{\sigma-1}}}{2d},$$

which is everywhere decreasing with respect to t. However, unless $\sigma - 1 > \beta \geq \sigma - 2$ and thus $t_h^c = 0$, it would be still better for the representative buyer that the platforms could not ask for commissions. This is perhaps a natural outcome, but in contrast to the results obtained by Etro (2021a and b) in a canonical setting.

3.5 Price competition between platforms

It is worth discussing briefly the case in which within our example platforms engage in Bertrand competition over the prices p_{y_h}, $h = 1, 2$. Remember that $s_h(n_h, t_h)$ is a decreasing function with respect to t_h. In a four-stage setting, at the third stage price competition would lead the platform offering the best

[19] Formally, $\frac{\partial^2 f}{\partial t_h \partial \sigma} < 0$ in a symmetric equilibrium.

quality index, say h (that is, $s_h = \max\{s_1(n_1,t_1), s_2(n_2,t_2)\}$), to use a price equal to:

$$p_{y_h} = \frac{s_h(n_h,t_h)}{s_{-h}(n_{-h},t_{-h})}d.$$

Unless $s_1(n_1,t_1) = s_2(n_2,t_2)$ and $p_{y_h} = d$ too, platform $-h$ would then command no expenditure (and would attract no sellers under free entry). Accordingly, both platform would set $t_h^B = 0$ in the first stage, obtaining $\Pi_h^B = 0$ and delivering the symmetric allocation (we use the suffix ^{B} to denote the values variables assume in the sub-game perfect Nash-Bertrand equilibrium):

$$p_{hi}^B = \frac{c}{\rho},\ n_h^B = \frac{\beta E}{2\sigma(1+\beta)F},\ p_{y_h}^B = d,\ y_h^B = \frac{E}{2(1+\beta)d},$$

with

$$\frac{s_h^B}{p_{y_h}^B} = \left[\frac{\rho}{cd^{\frac{1}{\beta}}}\right]^{\beta}\left[\frac{\beta E}{2\sigma(1+\beta)F}\right]^{\frac{\beta}{\sigma-1}} = 2\frac{s_h^0}{p_{y_h}^0}.$$

The same allocation would arise under a scenario with simultaneous price-commission choices and free entry. Notice that this allocation would not be (even second-best) optimal because concentrating all sellers within the same platform would raise the corresponding quality index and then buyers' welfare.

4 Conclusions

In this paper we have proposed to use preferences by a representative buyer to investigate platform competition. This allows us to investigate the implications of a possible intensive margin in the participation of buyers, which is overlooked by the canonical setting. We have put our approach at work by studying a duopoly example with platforms offering perfect substitutes and competing by setting the quantities of their services and ad-valorem exchange commissions on the commodities they intermediate, provided by a large number of sellers (who pay the commissions) under monopolistic competition. We have characterized the symmetric equilibrium under free entry of sellers and simultaneous platform choices, showing that the equilibrium level of commissions decreases with respect to sellers' expenditure share (an empirically testable implication), and that the adoption of positive commissions worsens buyers' welfare by increasing prices and reducing good variety. In the case in which platforms can commit to their commission levels in advance with respect to sellers' entry we find that the equilibrium level of commissions increases with respect to the substitutability of sellers' products (again, in principle a testable implication) but it is reduced by the competition for attracting them, which improves the platform quality index (even though it also raises the price each platform requires for its services), and so buyers' welfare. Anyway, in most cases the representative buyer would still be better off in the equilibrium case with no commissions.

While an additional advantage of our approach is that it can be easily generalized to the case of many competing platforms and to some asymmetry among them, our results concerning the welfare implications of exchange commissions should be compared to those recently obtained by Etro (2021a and b), who finds on the contrary an important alignment of platform incentives to those of buyers. Namely, in his model, taking into account the free entry of monopolistically competitive sellers, platforms set commissions to the level which maximizes buyers' welfare. This is due to the fact that in a canonical setting any change in the surplus created by sellers is (in a symmetric equilibrium) shifted back to buyers by an adjustment of prices which leaves platforms with a profit level that only depends on their intrinsic differentiation. In turn, this property depends on the fact that the participation of buyers to a platform only provides an extensive margin, implying that the prices they directly paid to it have the nature of access fees.

We interpret our different results as saying that the possible presence of an intensive margin does matter. In addition, in Appendix B we sketch the analysis of an alternative preference structure for platform goods which exhibits a "super-neutrality" of commission levels, due to perfect complementarity between the service provided by each platform and its sellers' commodities (still differentiated *à la* Dixit and Stiglitz, 1977). In summary, in such a setting platforms must allow the representative buyer enough purchasing power to buy their sellers' commodities, and thus their prices are used to exactly offset the impacts of commissions on sellers' prices, with no equilibrium effects on buyers' welfare and platform profits. This suggests that it is only in such a special case, which mimics the canonical setting, that the level of commissions is neutral on market outcomes.

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Appendix A

Proof of expression (9) The FOC $\frac{\partial \Pi_h}{\partial y_h}=0$ (for a given $\mathbf{n}$) gives:

$$\frac{\kappa\left(t_h\right) s_h\left(s_1 y_1+s_2 y_2\right)-\kappa\left(t_h\right) s_h^2 y_h}{\left(s_1 y_1+s_2 y_2\right)^2}=d,$$

i.e.,

$$\left(s_1 y_1+s_2 y_2\right)-s_h y_h=\frac{d}{\kappa\left(t_h\right) s_h}\left(s_1 y_1+s_2 y_2\right)^2 .$$

Adding up across platforms we also get

$$s_1 y_1+s_2 y_2=d\left(s_1 y_1+s_2 y_2\right)^2\left(\frac{1}{\kappa\left(t_1\right) s_1}+\frac{1}{\kappa\left(t_2\right) s_2}\right),$$

and

$$s_1 y_1+s_2 y_2=\frac{1}{d\left(\frac{1}{\kappa\left(t_1\right) s_1}+\frac{1}{\kappa\left(t_2\right) s_2}\right)} .$$

Thus, in a Cournot equilibrium of the second stage it must be the case that:

$$\widetilde{p}_{y_h}(\mathbf{t}, \mathbf{n})=\frac{s_h\left(n_h, t_h\right) E d\left(\frac{1}{\kappa\left(t_1\right) s_1\left(n_1, t_1\right)}+\frac{1}{\kappa\left(t_2\right) s_2\left(n_2, t_2\right)}\right)}{(1+\beta)},$$

that provides equation (9).

Proof of expressions (15) and (17) By using the third-stage equilibrium value of sellers' profit (12) and the quality index definition (6), free entry implies:

$$n_h = \frac{(1-t_h)\beta E}{\sigma(1+\beta)F}\frac{k(t_h)s_h(n_h,t_h)}{[k(t_2)s_2(n_2,t_2)+k(t_1)s_1(n_1,t_1)]} \tag{19}$$

$$= \frac{\beta E}{\sigma(1+\beta)F}\frac{1-t_h}{1+\frac{1+t_{-h}\beta}{1+t_h\beta}\left(\frac{1-t_{-h}}{1-t_h}\right)^{\beta}\left(\frac{n_{-h}}{n_h}\right)^{\frac{\beta}{\sigma-1}}} \tag{20}$$

for $h = 1, 2$, and then

$$\frac{n_h}{n_{-h}} = \frac{(1-t_h)k(t_h)s_h(n_h,t_h)}{(1-t_{-h})k(t_{-h})s_{-h}(n_{-h},t_{-h})} = \left[\frac{1-t_h}{1-t_{-h}}\right]^{\frac{(\sigma-1)(\beta+1)}{\sigma-1-\beta}}\left[\frac{1+t_h\beta}{1+t_{-h}\beta}\right]^{\frac{\sigma-1}{\sigma-1-\beta}}.$$

Substituting last expression into (20) we obtain the second-stage equilibrium number of sellers (15). Finally, notice that by (19) and (11) free entry implies that we can write the third-stage equilibrium profit of platform h as:

$$\Pi_h = \frac{\sigma^2(1+\beta)F^2}{\beta^2 E}\frac{(1+t_h\beta)n_h^2}{(1-t_h)^2}.$$

It follows by using (15) that the first-stage, reduced-form for the profit of platform h is given by (17).

Appendix B In this Appendix we sketch the case of a utility function over platform goods alternative to (3): in this case an aggregate of sellers' commodities must be consumed in a fixed proportion (one to one) with the services provided by each platform. In particular, suppose that the representative buyer's utility over the goods of platform h is given by

$$U_h(y_h,\mathbf{x}_h) = \min\{y_h, u_h(\mathbf{x}_h)\}, \tag{21}$$

where $u_h(\mathbf{x}_h) = \left(\sum_{i=1}^{n_h} x_{hi}^{\rho}\right)^{\frac{1}{\rho}}$ and $0 < \rho < 1$, as in the main text. In this case the linear-homogeneous U_h has a "perfect complements" upper-tier structure: preferences are still homothetic, and demands are given by:

$$y_h(p_{y_h},\mathbf{p}_h,E_h) = \frac{E_h}{T_h(p_{y_h},\mathbf{p}_h)} = u_h(\mathbf{x}_h(p_{y_h},\mathbf{p}_h,E_h)),$$

$$x_{hj}(p_{y_h},\mathbf{p}_h,E_h) = \frac{p_{hj}^{-\sigma}P_h(\mathbf{p}_h)^{\sigma}E_h}{T_h(p_{y_h},\mathbf{p}_h)},$$

where $P_h(\mathbf{p}_h) = \left(\sum_{i=1}^{n_h} p_{hi}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ and $T_h(p_{y_h},\mathbf{p}_h) = p_{y_h} + P_h(\mathbf{p}_h)$ can be interpreted as the overall "tariff" required by platform h. The indirect utility function corresponding to (21) is given by:

$$V_h(p_{y_h},\mathbf{p}_h,E_h) = \frac{E_h}{T_h(p_{y_h},\mathbf{p}_h)}.$$

With 2 platforms, the overall, homothetic preferences of the representative buyer are given by the utility functions:

$$\begin{aligned} U(\mathbf{y}, \mathbf{x}) &= \min\{y_1, u_1(\mathbf{x}_1)\} + \min\{y_2, u_1(\mathbf{x}_2)\}, \\ V(\mathbf{p}_y, \mathbf{p}, E) &= E \max\left\{\frac{1}{T_1(p_{y_1}, \mathbf{p}_1)}, \frac{1}{T_2(p_{y_2}, \mathbf{p}_2)}\right\}. \end{aligned}$$

Notice that platforms are still providing perfect substitutes.

Monopolistic competition among sellers gives, by the CES structure:[20]

$$p_{hi}(t_h) = \frac{c}{\rho(1-t_h)}, \quad P_h(t_h, n_h) = \frac{c}{\rho(1-t_h) n_h^{\frac{1}{\sigma-1}}},$$

and then at the last stage

$$\begin{aligned} x_{hi}(p_{y_h}, t_h, E_h, n_h) &= \frac{E_h}{n_h^{\frac{1}{\rho}} T_h(p_{y_h}, t_h, n_h)}, \\ \pi_{hi}(p_{y_h}, t_h, E_h, n_h) &= \frac{c}{\sigma-1} \frac{E_h}{n_h^{\frac{1}{\rho}} T_h(p_{y_h}, t_h, n_h)}. \end{aligned} \tag{22}$$

The equilibrium value of the price index P_h increases with respect to t_h and decreases with respect to n_h. For given expenditure on the platform, E_h, and given platform price, p_{y_h}, both the sellers' sizes and profits are decreasing with respect to the number of sellers n_h and the commission level t_h.

Let us consider once again Cournot competition among platforms at the second stage to set the quantities y_h, $j = 1, 2$. Inverse demand must satisfy $T_1 = T_2$ and the budget constraint $T_1 y_1 + T_2 y_2 = E$, and thus it is provided by ($h = 1, 2$):

$$p_{y_h}(\mathbf{y}, t_h, n_h) = \frac{E}{y_1 + y_2} - P_h(t_h, n_h),$$

with

$$E_h(\mathbf{y}) = \frac{y_h E}{y_1 + y_2}, \quad T_h(\mathbf{y}) = \frac{E}{y_1 + y_2}.$$

Notice that the platform price p_{y_h} must allow the representative buyer enough purchasing power to buy the (perfectly complementary) sellers' commodities, and that the commission level t_h does directly affect neither T_h nor the platform expenditure E_h. Accordingly, it also does not directly affect buyers.

Profit of platform h (assuming for the sake of simplicity a null unit cost) is at the second stage thus given by:

$$\begin{aligned} \Pi_h(\mathbf{y}, n_h) &= [p_{y_h}(\mathbf{y}, t_h, n_h) + t_h P_h(t_h, n_h)] y_h \\ &= E_h(\mathbf{y}) - \frac{c y_h}{\rho n_h^{\frac{1}{\sigma-1}}}, \end{aligned}$$

[20] Since $\partial \ln P_h / \partial \ln p_{hi} = b_{hi}$ and $|\partial \ln x_{hi} / \partial \ln p_{hi}| = \sigma - \sigma b_{hi} + \frac{b_{hi}}{1 + p_y y_h / P_h}$, once again sellers' demand elasticity is approximately equal to σ whenever their expenditure shares are small.

and does not either directly depend on t_h (this commission neutrality is of course also due to the complete cost pass-through that characterizes the CES case). Moreover, since the FOC $\frac{\partial \Pi_h}{\partial y_h}=0$ provides:

$$\frac{y_{-h}E}{(y_1+y_2)^2}=\frac{c}{\rho n_h^{\frac{1}{\sigma-1}}},$$

adding up across platforms we get in a sub-game perfect Nash-Cournot equilibrium:

$$\widetilde{T}_h(\mathbf{n}) = \frac{E}{y_1+y_2}=\frac{c}{\rho}\left[n_h^{\frac{1}{1-\sigma}}+n_{-h}^{\frac{1}{1-\sigma}}\right], \quad y_1+y_2=\frac{\rho E}{c\left[n_h^{\frac{1}{1-\sigma}}+n_{-h}^{\frac{1}{1-\sigma}}\right]},$$

$$\widetilde{y}_h(\mathbf{n}) = \frac{n_{-h}^{\frac{1}{\sigma-1}}\rho E}{c\left[\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}+1\right]^2}, \quad \widetilde{p}_{y_h}(\mathbf{n},t_h)=\frac{c}{\rho n_{-h}^{\frac{1}{\sigma-1}}}\left[1-\frac{t_h\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}}{(1-t_h)}\right],$$

$$\widetilde{E}_h(\mathbf{n}) = \frac{E}{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}+1}, \quad \widetilde{\Pi}_h(\mathbf{n})=\frac{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{1-\sigma}}E}{2+\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}+\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{1-\sigma}}},$$

and, from (22),

$$\widetilde{\pi}_{hi}(\mathbf{n})=\frac{E}{\sigma}\frac{\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}}{n_h\left[\left(\frac{n_{-h}}{n_h}\right)^{\frac{1}{\sigma-1}}+1\right]^2}.$$

These equilibrium results (which hold if platforms are not too asymmetric) show that in this setting there is a sort of super-neutrality of the commission levels, that affect neither the overall platform tariffs (and then buyers) nor the platform profit levels. Commissions only changes the platform prices to offset their impacts on sellers' prices. Setting $t_h=0$ we get, in a symmetric, sub-game perfect Nash equilibrium of free entry (with set-up cost $F>0$):

$$p_{hi}=\frac{c}{\rho}, \quad n_h=\frac{E}{4\sigma F}, \quad x_{hi}=\frac{(\sigma-1)F}{c},$$

$$y_h=\frac{(\sigma-1)}{c}\left[\frac{E}{4\sigma F^{\frac{1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}, \quad p_{y_h}=\frac{c}{\rho}\left[\frac{4\sigma F}{E}\right]^{\frac{1}{\sigma-1}}=P_h=\frac{T_h}{2}, \quad \Pi_h=\frac{E}{4}.$$