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Who killed business dynamism in the U.S.?

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Abstract

We offer a new interpretation of the long-term dynamics in the U.S. firm entry rate. Its decline was the consequence of a persistent combination of adverse(favorable) productivity shocks to potential entrants(incumbents), while the long-term increase in price markups did not play a significant role. In spite of the "Schumpeterian" structure of our model, not all recessions had a "cleansing" effect, because the combination of shocks associated to the specific episodes had markedly different effects on the dispersion of firms' efficiency. Finally, the extensive margin allows to rationalize the procyclical pattern of TFP growth and its long-term decline.

Keywords: firm entry rate, endogenous firm dynamics, productivity, business cycle, Bayesian estimation, DSGE.
JEL codes: C11, E20, E30, E32.

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1 Introduction.

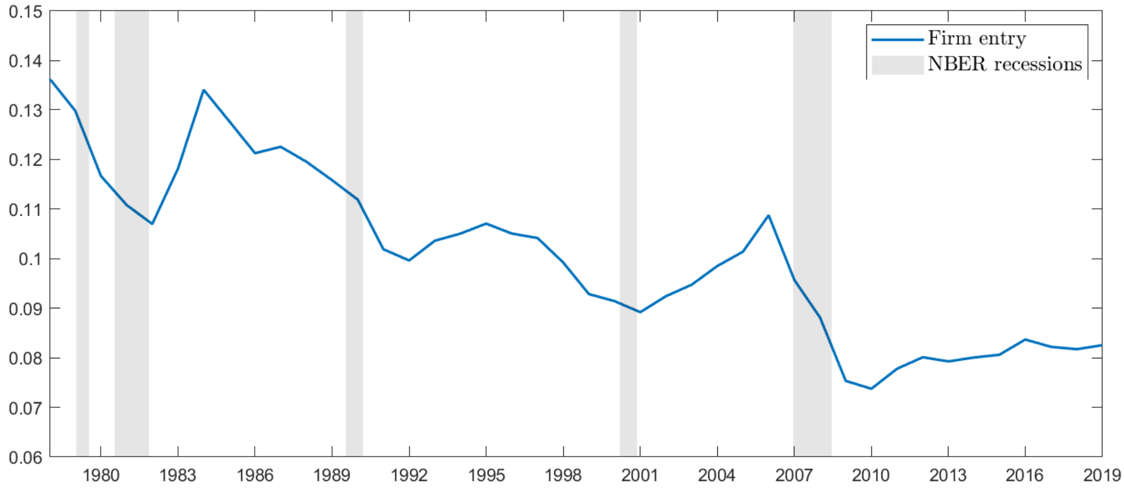
We offer a new interpretation of the long-term dynamics in the U.S. firm entry rate, documented in Figure 1: it exhibits a procyclical pattern and gradually falls up to 2010. Since then, it is essentially flat. In fact, a consistent reduction in the number of young firms (Decker et al., 2014) and a slowdown in productivity growth (Fernald, 2014; Boppart and Li, 2021) characterized the U.S. economy over the last decades.

In this regard, the literature on endogenous firm dynamics sees new businesses as the principal source of innovation in the economy (Asturias et al., 2017; Alon et al., 2018), and treats the falling entry rate as the key factor behind the productivity slowdown. For instance, Gourio et al. (2016) document that entry shocks cause a 1-1.5 percent increase in GDP, lasting over ten years. Increasing concentration and greater market power, reported in studies such as Autor et al. (2020) and Grullon et al. (2019), are identified as the main culprits, leading to the rise in markups documented in De Loecker et al. (2020), Gutiérrez and Philippon (2016), and Eggertsson et al. (2021).

By contrast, other studies emphasize the important contribution of older firms to innovation and productivity growth. According to Hsieh and Klenow (2018), incumbents account for the lion's share of innovation through improvements on their own products, whereas at most one quarter of U.S. productivity growth is ascribable to creative destruction and inputs reallocation towards relatively new firms. Fort et al. (2018) document the limited effect of firm entry and exit on the overall decline in U.S. manufacturing employment between 1977 and 2012, and conclude that incumbents might have been successful in raising their productivity relative to new entrants. Garcia-Macia et al. (2019) find that most growth comes from incumbents' contribution, with the role of entrants and creative destruction fading over the latest decades. Similarly, Klenow and Li (2021) show how fluctuations in U.S. productivity growth have been mostly driven by variations in incumbents' ability to innovate.

We build a new model that encompasses these alternative views. We incorporate endoge-

Figure 1: Firm entry rate, U.S., 1978-2019



Note: Annual data (Business Dynamics Statistics)

nous firm dynamics, as in Hopenhayn (1992), Asturias et al. (2017), and Piersanti and Tirelli (2020), into a stochastic growth model where technology improvements are determined by the different shocks that hit potential new entrants and incumbent firms. Long-term stochastic growth and business cycle dynamics are interpreted on the grounds of a set of shocks that hit the economy at low and high frequencies. In our model, the entry rate falls either if the internal productivity growth of potential entrants is subject to an adverse shock, or if favorable productivity shocks hit incumbents, or if price markups increase. Further, the entry rate falls if adverse demand shocks lower the value of the entry decision.

Our results are summarized as follows. The estimated model predicts a declining entry rate even if we exclude entry data from the set of observables. This is a very important preliminary result, suggesting that we are not forcing the model to rationalize long-run entry data "artificially" included in the set of observables. A persistent combination of adverse(favorable) productivity shocks to potential entrants(incumbents) causes the long-run decline in the model-predicted entry rate. This pattern is fully confirmed when we add the entry rate to the set of observed variables. The model predicts a long-term increase in price markups that is consistent with documented evidence, but we cannot find a persistent effect of markup shocks on the entry rate decline. In fact, markup shocks only have temporary effects.

Our model allows to estimate the dynamics of both TFP and average firm efficiency. The growth of firm efficiency is relatively stable and acyclical. By contrast, TFP growth is subject to a gradual slowdown and has an unambiguous procyclical pattern. Our original contribution is that adjustments in the extensive margin, *i.e.* the mass of firms, drive these findings. Non-technology shocks explain the cyclical pattern of TFP growth, via their effect on the extensive margin. We also find that recessions induced by adverse demand shocks do not have a cleansing effect, *i.e.* they do not reduce the dispersion of firms' idiosyncratic efficiency.

We contribute to a growing literature on business cycle models of endogenous firm dynamics. Lewis and Poilly (2012) and Lewis and Stevens (2015) estimate models in the tradition of Bilbiie et al. (2007) and Bilbiie et al. (2012), but their focus is different as they neglect the analysis of long-term entry dynamics and the implications for long-run growth. Just like us, Clementi and Palazzo (2016) build upon Hopenhayn (1992)'s model, but they treat entry and exit as an exogenous amplification mechanism of productivity shocks that symmetrically hit all firms. To the best of our knowledge, this is the first contribution that incorporates the effect of asymmetric productivity shocks on the entry rate.

We also contribute to the literature that investigates the procyclical pattern of TFP and the sluggish recoveries following major crises. Anzoategui et al. (2019) focus on the role of R&D; Qiu and Ríos-Rull (2022) link firms' TFP to the number of varieties each firm is able to sell. Other studies obtain procyclical TFP either in consequence of sectoral productivity changes (Swanson, 2006) or through a combination of increasing returns and increased utilization of the production factors (Gottfries et al., 2021). In our framework, instead, the extensive margin of goods production drives TFP dynamics.

Finally, the paper provides an interpretation for entry rate dynamics that is complementary to contributions that emphasize the importance of demographic factors, such as

Hopenhayn et al. (2018), Pugsley and Şahin (2018), Karahan et al. (2019) and Peters and Walsh (2021).

The paper is organized as follows: section 2 describes the model, section 3 provides information on the estimation procedure, results are presented in section 4, section 5 concludes.

2 The Model.[1]

Households demand a bundle of differentiated retail goods

$$C_t = \left(\int_0^1 C_t(r)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} dr \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}, \tag{1}$$

supply capital services, k_t, to firms in the intermediate-goods producing sector (INT henceforth), and sell the services of a differentiated labor type ι to the competitive labor packers who assemble the labor bundle

$$l_t = \left(\int_0^1 l_t(\iota)^{\frac{\epsilon^w - 1}{\epsilon^w}} dh \right)^{\frac{\epsilon^w}{\epsilon^w - 1}} \tag{2}$$

that enters the production of INT-goods. INT-goods are sold to retailers.

The perfectly competitive INT-firms have mass η_t, distributed between new entrants, NE_t, and incumbents, INC_t, who survived out of the η_{t-1} firms active at time $t-1$:

$$\eta_t = NE_t + INC_t. \tag{3}$$

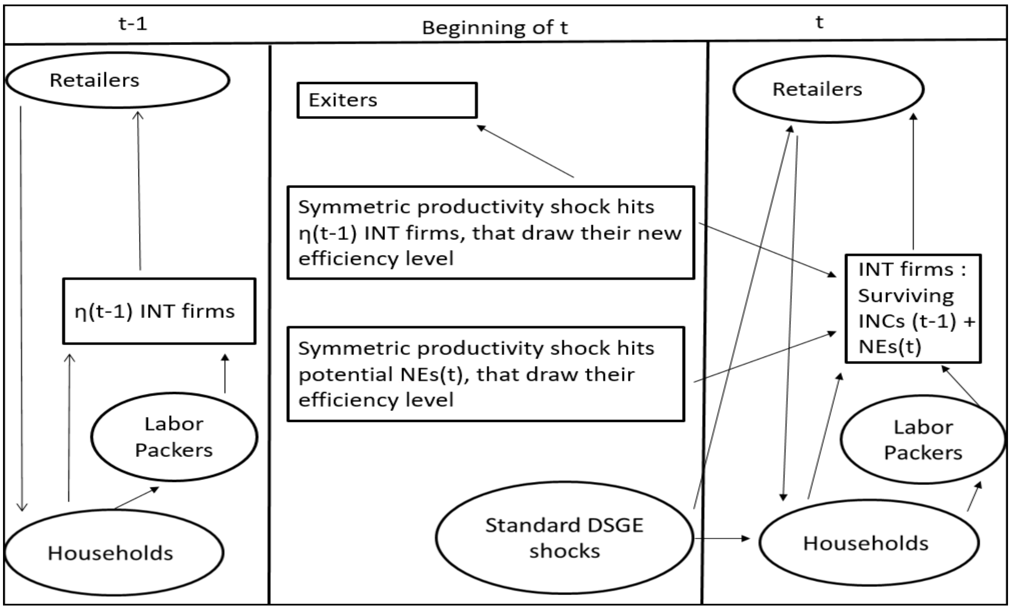
Both NEs and $INCs$ group heterogeneous firms that are subject to idiosyncratic productivity shocks.

At the beginning of each period, two sets of shocks hit the economy. The first one is a set of demand and supply shocks that characterize standard DSGE models, *i.e.* marginal

[1] See Appendix C for the full set of equilibrium conditions and for the derivation of key equations.

efficiency of investment, retail price markups and labor supply, monetary and fiscal policy. The second one includes two independent productivity shocks that symmetrically affect the idiosyncratic efficiency distribution of NEs and $INCs$ respectively. The sequence of events unfolds as in Figure 2.

Figure 2: Model sequence of events



2.1 INT-sector.

The production function of a generic firm f is:

$$y_t^{f,j} = A_t^{f,j} \left(Z_t^{f,j} \right)^{\gamma}, \tag{4}$$

$$Z_t^{f,j} = \left[(k_t^{f,j})^{\alpha} (l_t^{f,j})^{(1-\alpha)} \right], \tag{5}$$

where $j = NE, INC$. $A_t^{f,j}$ defines the firm-specific level of productivity, $\gamma < 1$ is the degree of decreasing return to scale, $Z_t^{f,j}$ is a Cobb-Douglas bundle of factor inputs. Firm dividends are

$$d_t^{f,j} = p_t y_t^{f,j} - r_t^k k_t^{f,j} - w_t l_t^{f,j} - w_t \phi^j \ , \tag{6}$$

where p_t is the consumption price of INT-goods, r_t^k is the real rental rate of capital, w_t is the consumption real wage and ϕ^j is the exogenous fixed production cost defined in labor units. Factor demands are:

$$k_t^{f,j} = \alpha\gamma \frac{p_t y_t^{f,j}}{r_t^k} \tag{7}$$

$$l_t^{f,j} = (1-\alpha)\gamma \frac{p_t y_t^{f,j}}{w_t} \tag{8}$$

and

$$p_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)} \tag{9}$$

is the consumption price of Z_t. Note that the capital intensity of the input bundle $Z_t^{f,j}$ does not vary across firms, but its scale obviously grows with firm efficiency.

$$Z_t^{f,j} = \left[\frac{p_t}{p_t^z} A_t^{f,j} \gamma\right]^{\frac{1}{1-\gamma}} . \tag{10}$$

The firm supply function therefore is

$$y_t^{f,j} = \left(A_t^{f,j}\right)^{\frac{1}{1-\gamma}} \left[\gamma \frac{p_t}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} . \tag{11}$$

From (6) and (11), the firm's value can be written recursively as

$$V_t\left(A_t^{f,j}\right) = (1-\gamma)\left[A_t^{f,j} \frac{p_t \gamma^{\gamma}}{(p_t^z)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_t \phi^j + E_t\left\{\Lambda_{t+1} V_{t+1}\left(A_{t+1}^{f,j}\right)\right\} \ , \tag{12}$$

where ϕ^j allows to identify the cutoff values $\hat{A}_t^j$ that define the entry and exit productivity thresholds

$$V_t\left(\hat{A}_t^j\right) = 0. \tag{13}$$

Right from the outset, note that these thresholds react to current economic conditions, *i.e.* an increase in p_t unambiguously raises the firm value and lowers the idiosyncratic efficiency level that meets the profitability condition, whereas an increase in the price of inputs or in the fixed cost would work in the opposite direction. Future valuation of the firm also matters, and firms may operate under temporarily negative profitability.

2.1.1 New entrants.

At the beginning of period t, potential NEs draw their productivity level $A_t^{f,NE}$ from the Pareto distribution,

$$f_t\left(A_t^{NE}\right) = \int_{z_t}^{+\infty} \frac{\xi\left(z_t\right)^{\xi}}{\left(A_t^{f,NE}\right)^{\xi+1}} d\left(A_t^{f,NE}\right) = 1, \tag{14}$$

where

$$z_t = z_{t-1} g_t^z \tag{15}$$

defines the technology frontier, g_t^z is the stochastic exogenous firm productivity driver in the long run

$$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z \ln(g_{t-1}^z) + \varepsilon_t^z;\ \varepsilon_t^z \sim N\left(0, \sigma^z\right) \tag{16}$$

and g^z defines the deterministic productivity growth trend. The mass of new entrants is:

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi\left(z_t\right)^{\xi}}{\left(A_t^{f,NE}\right)^{\xi+1}} d\left(A_t^{f,NE}\right) = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi}, \tag{17}$$

where $\hat{A}_t^{NE}$ defines the productivity threshold such that $V_t\left(\hat{A}_t^{NE}\right) = 0$.

2.1.2 Incumbents.

At the beginning of period t, the η_{t-1} firms draw their idiosyncratic productivity from the following distribution

$$f_t(\hat{A}_t^{INC}) = \int_{\hat{A}_{t-1}^{INC} g^z (1-\delta^{INC})\Psi_t}^{+\infty} \frac{\xi \left[\hat{A}_{t-1}^{INC} g^z \left(1-\delta^{INC}\right) \Psi_t\right]^{\xi}}{\left(A_t^{f,INC}\right)^{\xi+1}} d(A_t^{f,INC}) \; , \tag{18}$$

where $\hat{A}_{t-1}^{INC}$, $V_t\left(\hat{A}_{t-1}^{INC}\right) = 0$, defines the productivity threshold that characterized the distribution of INC_{t-1} firms; by setting $g^z\left(1-\delta^{INC}\right) < 1$ we assume that, on average, the η_{t-1} firms deplete their knowledge capital.[2] Finally,

$$\ln\left(\Psi_t\right) = \rho^{\Psi} \ln\left(\Psi_{t-1}\right) + \varepsilon_t^{\Psi};\; \varepsilon_t^{\Psi} \sim N\left(0, \sigma^{\Psi}\right) \tag{19}$$

denotes the equivalent of a standard productivity shock. The mass of incumbents is

$$INC_t = \eta_{t-1} H_t \; , \tag{20}$$

where

$$H_t = \int_{\hat{A}_t^{INC}}^{+\infty} \frac{\xi \left[\hat{A}_{t-1}^{INC} g^z \left(1-\delta^{INC}\right) \Psi_t\right]^{\xi}}{\left(A_t^{f,INC}\right)^{\xi+1}} d(A_t^{f,INC}) = \left(\frac{\hat{A}_{t-1}^{INC} g^z \left(1-\delta\right) \Psi_t}{\hat{A}_t^{INC}}\right)^{\xi} \tag{21}$$

is the endogenous survival probability for the η_{t-1} firms. The expected efficiency of the η_{t-1} firms is

$$E_{t-1}\left\{A_t^{\eta_{t-1}}\right\} = \frac{\xi}{\xi-1} E_{t-1}\left\{\hat{A}_{t-1}^{INC} g^z \left(1-\delta\right) \Psi_t\right\} \; . \tag{22}$$

[2] This is akin to Liu et al. (2020) and the literature cited therein.

The mass of exiting firms is

$$EX_t = \eta_{t-1}(1-H_t) = \eta_{t-1}\left(\frac{\hat{A}_t^{INC} - \hat{A}_{t-1}^{INC} g^z (1-\delta)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi}. \quad (23)$$

2.1.3 Thresholds.

We now derive the efficiency thresholds associated to the intertemporal zero profit condition (13). To begin with, from condition (22) notice that firms operative in t are confronted with the same present value of future dividends

$$E_t\left\{V\left(A_{t+1}^{f,j}\right)\right\} = \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} V_{t+1}\left(A_{t+1}^{f,INC}\right) \frac{\xi\left(\widehat{A}_{t+1}^{INC}\right)^{\xi}}{\left(A_{t+1}^{f,INC}\right)^{\xi+1}} d\left(A_{t+1}^{f,INC}\right) = E_t\left\{H_{t+1}V_{t+1}^{av}\right\}, \quad (24)$$

where V_{t+1}^{av} defines the continuation value of the η_t firms conditional to survival in $t+1$. In recursive form,

$$V_{t+1}^{av} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\left[\frac{(1-\gamma)^{1-\gamma}}{\gamma^{\gamma}} \frac{p_{t+1}}{\left(p_{t+1}^z\right)^{\gamma}} \hat{A}_{t+1}^{INC}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi^{INC} + E_{t+1}\left\{\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right\}. \quad (25)$$

Given the shape of the Pareto distribution, condition

$$\xi(1-\gamma) > 1$$

is necessary to ensure that $E_t\left\{V\left(A_{t+1}^{f,j}\right)\right\}$ converges to finite value.

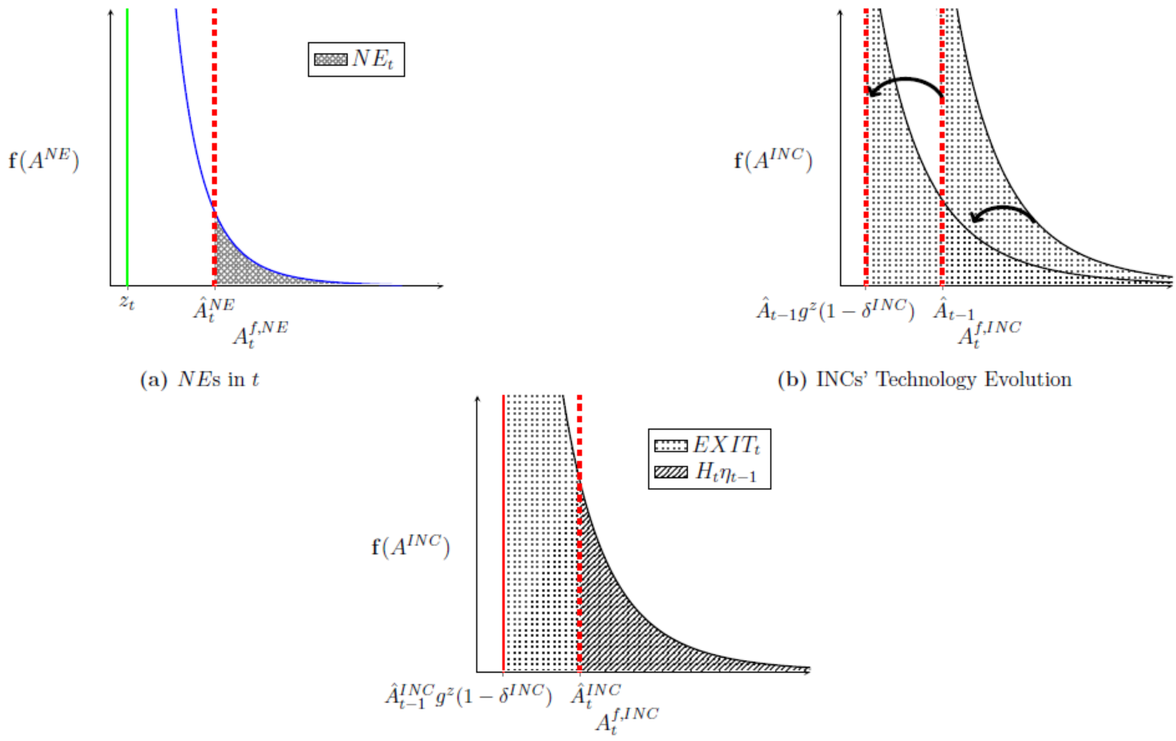
Using (13) and (25), the following condition identifies the thresholds for INC_t and NE_t firms:

$$\hat{A}_t^j = \left[\frac{w_t\phi^j - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma} \frac{\left(p_t^z\right)^{\gamma}}{p_t}. \quad (26)$$

Increases in the participation cost $w_t\phi^j$ and in the price of the input bundle p_t^z raise the productivity threshold, whereas increases in current or discounted future profitability, respectively determined by p_t and $\Lambda_{t+1}H_{t+1}V^{av}_{t+1}$, allow relatively less efficient firms to operate in the market.

Figure 3 provides a graphical representation of how NEs and $INCs$ are distributed. Panel (a) identifies the fraction of potential entrants that choose to operate in t. In Panel (b) we represent the distribution of the depreciated knowledge capital inherited by the η_{t-1} firms. Finally, in Panel (c) $\hat{A}^{INC}_t$ splits the support between exiting and surviving η_{t-1} firms.

Figure 3: Model firm dynamics



(a) NEs in t

(b) INCs' Technology Evolution

(c) Exit in t

The following condition highlights the impact of productivity shocks on firm dynamics:

$$\eta_t = \left(\frac{z_t}{\hat{A}^{NE}_t}\right)^{\xi} + \eta_{t-1}\left(\frac{\hat{A}^{INC}_{t-1}g^z\,(1-\delta)\,\Psi_t}{\hat{A}^{INC}_t}\right)^{\xi}. \tag{27}$$

From (15) it is easy to see that shocks to z_t have permanent effects on the support of the

NEs distribution. Using (26), we get

$$\eta_t = \left[\begin{array}{c} \left(\frac{z_t}{\left[w_t \phi^{NE} - E_t \left\{ \Lambda_{t+1} H_{t+1} V_{t+1}^{av} \right\} \right]^{1-\gamma}} \right)^{\xi} + \\ \eta_{t-1} \left(\frac{\hat{A}_{t-1}^{INC} g^z (1-\delta) \Psi_t}{\left[w_t \phi^{INC} - E_t \left\{ \Lambda_{t+1} H_{t+1} V_{t+1}^{av} \right\} \right]^{1-\gamma}} \right)^{\xi} \end{array} \right] \left(\frac{\gamma^{\gamma} (1-\gamma)^{1-\gamma} p_t}{(p_t^z)^{\gamma}} \right)^{\xi} . \tag{28}$$

Thus, a positive shock to z_t creates a supply congestion effect that lowers p_t and raises the productivity thresholds. Our estimates will show that this is associated to an increase in both entry and exit rates. This mechanism, akin to a Schumpeterian cleansing effect, is enriched by the role of discounted future profitability: ceteris paribus, the larger $\hat{A}_t^{INC}$ also raises the firm survival probability in the next period, H_{t+1}, and causes a persistent downward pressure on the price of INT-goods. The initial ε_t^z shock permanently raises the expected z values in the NEs distribution support. This, combined with the $\hat{A}_t^{INC}$ increase, generates a sequence of falling prices and endogenously increasing firm productivity.[3]

The Ψ_t shock raises the survival probability of the η_{t-1} firms and triggers a fall in p_t. In this case both entry and exit rates fall. Finally, demand shocks also matter for firms' productivity and entry/exit flows. In fact, any change in demand for INT-goods that raises $\frac{p_t}{p_t^z}$ will lower the productivity thresholds, raising both $INCs$ and NEs.

2.1.4 INT-sector aggregation.

Production of INT-goods is

$$Y_t^{INT} = Y_t^{NE} + Y_t^{INC} , \tag{29}$$

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE} \left[\left(k_t^{f,NE} \right)^{\alpha} \left(l_t^{f,NE} \right)^{1-\alpha} \right]^{\gamma} dF \left(A_t^{f,NE} \right) , \tag{30}$$

$$Y_t^{INC} = \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC} \left[\left(k_t^{f,INC} \right)^{\alpha} \left(l_t^{f,INC} \right)^{1-\alpha} \right]^{\gamma} dF \left(A_t^{f,INC} \right) . \tag{31}$$

Straightforward manipulations yield the supply function

[3] See Piersanti and Tirelli (2020) for a detailed discussion.

$$Y_t^{INT} = \frac{\xi\,(1-\gamma)}{\xi(1-\gamma)-1}\left[NE_t\left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}} + INC_t\left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\gamma}}\right]\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}}, \qquad (32)$$

where

$$\frac{\xi\,(1-\gamma)}{\xi(1-\gamma)-1}\left(\hat{A}_t^j\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}}$$

denotes the average production of j-type firms.

Note that an increase in p_t has manifold effects. First, it increases the price/cost margin, $\frac{p_t}{p_t^z}$. Second, it raises the mass of j-firms (see conditions 17, 21 and 26). Third, by loosening the zero-profit condition (26), it reduces the average firm efficiency $\hat{A}_t^j$. The supply elasticity is

$$\frac{\partial Y_t^{INT}}{\partial p_t}\frac{p_t}{Y_t^{INT}} = \xi - 1\ .$$

From conditions (7) and (8), factor-inputs demands are:

$$K_t^{INT} = \alpha\gamma\frac{p_t Y_t^{INT}}{r_t^k}\ , \qquad (33)$$

$$L_t^{INT} = (1-\alpha)\gamma\frac{p_t Y_t^{INT}}{w_t} + \phi^{NE}NE_t + \phi^{INC}INC_t\ . \qquad (34)$$

2.1.5 Retailers.

There is a continuum of monopolistic retailers $r \in (0,1)$, and final output is a CES bundle of differentiated goods:

$$Y_t = \left(\int_0^1 Y_t\,(r)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}}\,dr\right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}, \qquad (35)$$

where

$$\ln(\epsilon_t^p) = (1-\rho^p)\ln(\epsilon^p) + \rho^p\ln(\epsilon_{t-1}^p) + \varepsilon_t^p - \eta^p\varepsilon_{t-1}^p;\ \varepsilon_t^p \sim N\left(0,\sigma^p\right) \qquad (36)$$

allows to identify standard price markup shocks.[4]

Retailers face Calvo rigidities and either re-optimize with probability $1-\Gamma_p$ or follow this simple indexation rule

$$P_t(r) = \left(\pi_{t-1}^{\mu_p} \overline{\pi}_{ss}^{1-\mu_p}\right) P_{t-1} \ . \tag{37}$$

Their price is a combination of steady-state and past inflation indexed by the parameter μ_p.

The solution of the retailers' pricing problem is:

$$P_t^{1-\epsilon_t^p} = (1-\Gamma_p)\left(P_t^*\right)^{1-\epsilon_t^p} + \Gamma_p \left(\pi_{t-1}^{\mu_p} \overline{\pi}_{ss}^{1-\mu_p} P_{t-1}\right)^{1-\epsilon_t^p} \tag{38}$$

where P_t^* is the optimal price level and P_t is the retail price index. Aggregating across individual retailers, we obtain

$$Y_t = \frac{Y_t^{INT}}{\xi_t^p} \ , \tag{39}$$

where ξ_t^p is the standard measure of price dispersion under Calvo pricing.

2.2 Households.

The representative household ι, $\iota \in (0,1)$, maximizes

$$E_t \sum_{i=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma}\left(C_{t+i} - hC_{t+i-1}\right)^{1-\sigma}\right] \exp\left(\psi \frac{\sigma-1}{1+\varphi} \zeta_{t+i}^l l_{t+i}\left(\iota\right)^{1+\varphi}\right) , \tag{40}$$

where ζ_t^l is a labor supply shock

$$\ln(\zeta_t^l) = \rho^l \ln(\zeta_{t-1}^l) + \varepsilon_t^l ; \ \varepsilon_t^l \sim N\left(0, \sigma^l\right) \ , \tag{41}$$

[4] We follow Smets and Wouters (2007) in modeling the price markup shock as an ARMA(1,1) process. This allows to catch high-frequency fluctuations in inflation.

subject to:[5]

$$C_t + I_t + \frac{B_t}{P_t} = w_t(\iota)\, l_t(\iota) + \left(r_t^k - a_t^u\right) U_t K_t + R_{t-1}^n \frac{B_{t-1}}{P_t} + D_t^F \ . \tag{42}$$

C_t is consumption of the retail goods bundle, D_t^F are firm dividends, B_t is a one-period nominally riskless bond with gross remuneration R_t^n, U_t denotes variable capacity utilization, and $a_t^u = \gamma_1 (U_t - 1) + \frac{\gamma_2}{2} (U_t - 1)^2$ defines its adjustment cost.

The capital stock evolves as follows:

$$K_{t+1} = \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t + (1-\delta) K_t \ , \tag{43}$$

where δ is the depreciation rate, $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ defines investment adjustment costs, and μ_t,

$$\ln(\mu_t) = \rho^\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu ;\ \varepsilon_t^\mu \sim N(0, \sigma^\mu) \ , \tag{44}$$

is a shock to the marginal efficiency of investment (MEI shock).

Households face a downward-sloping demand function:

$$l_t(\iota) = \left(\frac{w_t(\iota)}{w_t}\right)^{-\epsilon^w} l_t \ , \tag{45}$$

and Calvo rigidities affect wage setting decisions: each household either optimizes with probability Γ_w or follows the indexation rule

$$w_t(\iota) = \frac{\pi_{t-1}^{\mu_w} \overline{\pi}_{ss}^{1-\mu_w} w_{t-1}(\iota)}{\pi_t}. \tag{46}$$

[5]We implicitly assume that risk-sharing schemes insulate individual consumption from idiosyncratic shocks to the household wage bill.

Wage dynamics are

$$w_t^{1-\epsilon^w} = (1-\Gamma_w)\,(w_t^*)^{1-\epsilon^w} + \Gamma_w \left(\frac{\pi_{t-1}^{\mu_w}\overline{\pi}_{ss}^{1-\mu_w} w_{t-1}}{\pi_t}\right)^{1-\epsilon^w} \;, \tag{47}$$

where w_t^* is the wage set by re-optimizing households.

2.3 Monetary Policy.

We opt for a very simple Taylor rule,[6]

$$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R_{t-1}^n}{R_{ss}^n}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\kappa^\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i} \zeta_t^r \;, \tag{48}$$

where R_{ss}^n is the steady-state nominal interest rate, ζ_t^r is a monetary policy shock

$$\ln(\zeta_t^r) = \rho^r \ln(\zeta_{t-1}^r) + \varepsilon_t^r;\; \varepsilon_t^r \sim N\left(0, \sigma^r\right) \tag{49}$$

and

$$\ln\left(\overline{\pi}_t\right) = (1-\rho^\pi)\left(\overline{\pi}_{ss}\right) + \rho^\pi \ln\left(\overline{\pi}_{t-1}\right) + \varepsilon_t^\pi;\;\; \varepsilon_t^\pi \sim N\left(0, \sigma^\pi\right) \tag{50}$$

is the stochastic inflation target.

2.4 Market clearing.

Market clearing requires:[7]

$$L_t = L_t^{INT} \;, \tag{51}$$

$$K_t = K_t^{INT} \;, \tag{52}$$

$$Y_t = C_t + I_t + G_t \;. \tag{53}$$

[6]We also experimented with the complex rule in Smets and Wouters (2007). Our results were confirmed.
[7]The model is solved up to first order, we therefore neglect price and nominal wage dispersion.

where $G_t = g_t^s Y_{ss}$ denotes public consumption as a fraction of steady-state output and

$$\ln(g_t^s) = (1 - \rho^{g^s}) \ln(g^s) + \rho^{g^s} \ln(g_{t-1}^s) + \varepsilon_t^{g^s};\ \varepsilon_t^{g^s} \sim N\left(0, \sigma^{g^s}\right) \tag{54}$$

denotes a public consumption shock as in Smets and Wouters (2007) (S&W henceforth).

2.5 Shocks and the endogenous persistence of efficiency thresholds.

To support intuition, we discuss here the IRFs to productivity, MEI, and markup shocks (Figures 4).[8] The choice of this specific subset of shocks is motivated by their relative importance for the subsequent analysis of observed entry rate dynamics.

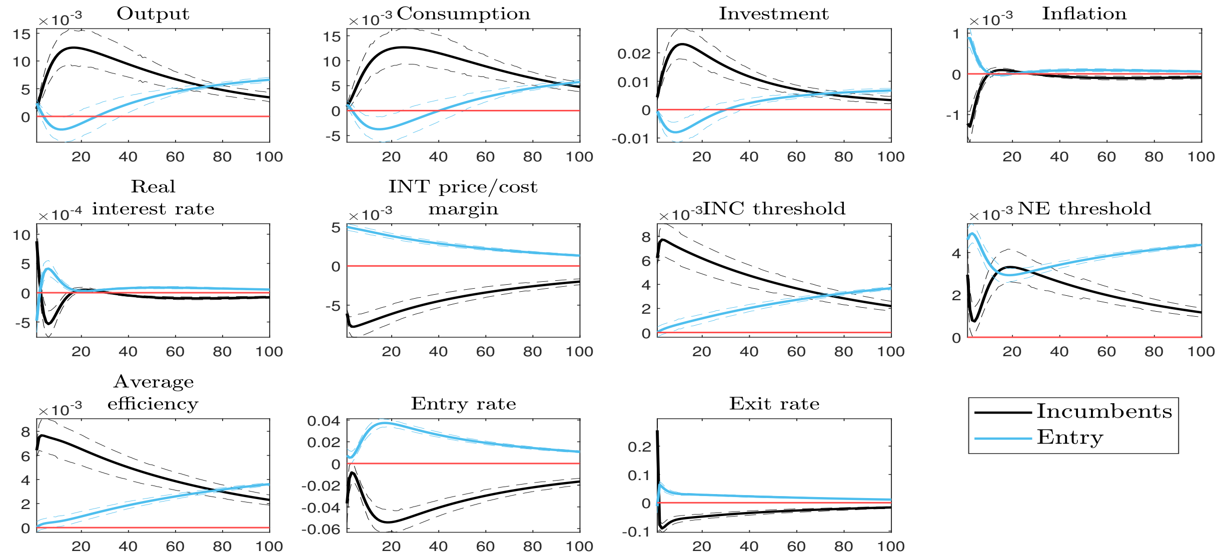
Our purpose is to clarify the endogenous propagation mechanism that drives the response of firms' productivity to exogenous shocks. Right from the outset, note that technology shocks to $NE(INC)$ firms adversely affect the valuation of the other group of firms, and therefore impact on exit(entry) rates. Further, demand and markup shocks affect entry/exit rates through the price/cost margin of INT-firms. This, in turn, matters for average firm productivity that unambiguously falls in the occurrence of INT-sector demand-driven booms and vice-versa.

Consider first a white noise entry shock, ε_t^z. From condition (16) it is easy to see that the shock entails a permanent increase in the new entrants' productivity shifter z_t. There are permanent effects on consumption and investment that materialize at very low frequencies. By looking at the dynamics of the productivity thresholds for both NEs and $INCs$, one can gauge the persistence of the endogenous amplification mechanism, that turns the shock into a permanent increase in average firm productivity which is as large as the initial increase in z_t. Even if ε_t^z has no persistence, increased supply from NEs immediately raises the productivity

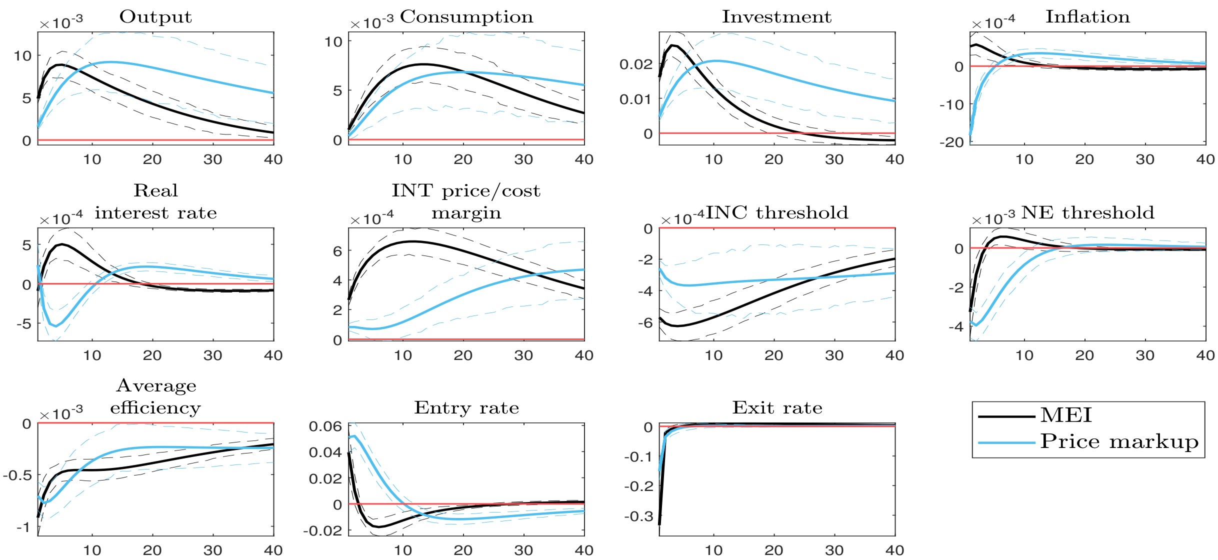
[8]Parameters are calibrated at the posterior-mean values obtained for our baseline model (see section 4 below).

Figure 4: Estimated IRFs (Baseline model)

(a)



(b)



Note: Quarterly estimated mean impulse responses (solid lines) with 90% HPD intervals (dashed lines). Panel (a): one-standard-deviation entry and incumbents' productivity shocks. Panel (b): one-standard-deviation MEI and price markup shocks.

threshold for INC_t firms. This, in turn, triggers a gradual and very persistent upward shift in the support of the $H_t\eta_{t-1}$ mass of surviving incumbents (see condition 21), causing a twofold effect. On the one hand, the incumbents' expected survival probability falls. On the other hand, the increase in $\hat{A}_t^{INC}$ drives the long-term response of output. The short-run transitions require a careful discussion. The increased competition from NEs lowers the present value of potential incumbents and raises the exit rate. In fact, the shock is associated with an episode of "creative destruction". This effect is so strong that the initial surge in the exit rate reduces the extensive margin pushing up the consumption price of INT-goods and the marginal cost for firms in the retail sector. This, in turn, causes a persistent increase in inflation and in real interest rates that initially lowers both consumption and investment. Finally, note that the shock raises the price/cost margin of INT-firms.

The incumbents productivity shock, ε_t^{Ψ}, is by assumption temporary and has an estimated autoregressive coefficient $\rho^{\Psi} = 0.247$ at the posterior mean. The shock affects the bulk of the η_{t-1} firms and therefore has a large effect on the supply of INT-goods. The decrease in $\frac{p_t}{(p_t^z)^{\gamma}}$ causes a persistent fall in the entry rate, and it is initially so strong that the exit rate increases too. Note that ε_t^{Ψ} increases the density of firms characterized by $A_t^{f,INC} > \hat{A}_t^{INC}$. For this reason, after a few quarters the exit rate falls below steady state and the number of incumbents picks up again. The initial reduction in the number of incumbents is associated to shifts to the support of the $H_t\eta_{t-1}$ mass of surviving incumbents (see condition 21), raising the average efficiency of these firms. Due to the persistent fall in $\frac{p_t}{(p_t^z)^{\gamma}}$, both productivity thresholds remain above steady state for a prolonged period. In line with standard productivity shocks, the increased supply of INT-goods has a deflationary effect that triggers an expansionary monetary policy, stimulating both consumption and investment.

The MEI shock drives a standard boom in demand. The increase in $\frac{p_t}{(p_t^z)^{\gamma}}$ raises(lowers) the entry(exit) rate. This has non-negligible implications for the productivity thresholds and for average firm efficiency that persistently fall. Finally, a negative markup shock has an unambiguously expansionary effect. The shock pulls up $\frac{p_t}{(p_t^z)^{\gamma}}$, lowering productivity thresholds and increasing(reducing) the entry(exit) rate. Average efficiency of INT-firms falls. The expansionary monetary policy response to the shock supports the growth of both consumption and investment.

3 Bayesian estimation.

We estimate the model on U.S. data spanning from 1966:I to 2019:IV (with a presample of four quarters starting in 1965:I). The dataset consists of the *yearly* firm entry rate measured by the Business Dynamics Statistics (BDS), and of seven standard macroeconomic variables observed at a quarterly frequency: worked hours, the Fed funds rate, the inflation rate (GDP deflator), and the growth rates of GDP, investment, consumption and wages in real terms. The macroeconomic observables and the initial date are the same considered by S&W, whose results we take as benchmark reference for business cycle analysis.[9]

As regards our measure of firm entry, we choose the BDS database, that gathers information on the entire universe of U.S. firms.[10] This source has been widely employed to study various features of business dynamism. Examples include Hathaway and Litan (2014), who analyze the geographical aspects of the decline in U.S. business dynamism, Decker et al. (2014), who study the role played by entrepreneurship (in the form of startup rates) in U.S. job creation, Gourio et al. (2016), who use a VAR to estimate the effects of a shock to the number of startups, and Karahan et al. (2019), who link the fall in firm entry to the slowing pace of labor supply growth. An alternative source for data on startups is provided by the Bureau of Labor Statistics (BLS), whose records are available at a quarterly frequency. However, the BLS data are characterized by two crucial features that make the BDS more suitable for our purposes. First, its sample starts in 1992 and does not allow us to study the persistent decline in the entry rate. Second, the BLS provides data on *establishment*

[9] Appendix B.1 contains a detailed discussion of data sources, definitions, and transformations.

[10] The BDS data are aggregated starting from the Longitudinal Business Database (LBD), that tracks single establishments and firms since 1976. These micro-data are used, among the others, by Decker et al. (2020) to discriminate between possible reasons behind the firm entry decline.

entry rates: while these might be more useful to investigate job creation and destruction, they are arguably less relevant for new business formation, as new business establishments do not necessarily represent new firms. Conversely, the BDS database does distinguish between firms and establishments.[11]

In order to deal with the mixed-frequency nature of our dataset, we construct an annualized model-implied measure of firm entry.[12] In particular, we take the sum of new entrants over the periods $t : t-3$ divided by the average of total firms over the periods $t : t-7$, where each period t denotes a quarter. This is consistent with the BDS measure of firm entry, which is defined as the number of firm births in each year divided by the average number of firms in that and in the previous year. Then, our model-implied variable is matched with the observed BDS data only in the final quarter of each year. The values for the remaining quarters are treated as missing observations and inferred by the Kalman filter.

Hirose and Inoue (2016) suggest that ZLB periods may bias the estimates of some parameters and shocks. To check for this, we estimated the model over the subsample 1966:I-2007:III. Further, we estimated the model over the full sample after replacing the Fed funds rate with the shadow rate, obtained by Wu and Xia (2016), from 1990:I up to the end of our sample.

Another important issue concerns the analysis of unconventional monetary policies, which are not considered in our model. To some extent these policy actions might be captured by MEI shocks, which may be interpreted as disturbances that affect the financial system ability to turn savings into capital (Justiniano et al., 2011).[13]

[11] BLS data are used by Casares et al. (2020) to estimate a model with endogenous entry and exit. Differently from our long-term perspective, their focus is on the period following the financial crisis and on the relationship between U.S. business cycle fluctuations and the extensive margin.

[12] See Pfeifer (2013) for references on methodology.

[13] We also experimented with an additional risk-premium shock that did not play any significant role in our estimates.

Table 1: Calibrated parameters (Baseline model)

Parameter	L_{ss}	g^s	ϵ^p	ϵ^w	α	γ	δ	ξ	$entry$	$w_{ss}\phi^{INC}$	ϕ^{ratio}	ρ^π
Value	0.33	0.18	6	21	0.33	0.9	0.025	15	0.025	0.05	0.7	0.99

3.1 Calibration and priors.

Following common practice, we calibrate some parameters that are hard to identify (Table 1). These include the capital depreciation rate, $\delta = 0.025$, corresponding to a 10% depreciation rate per year; the capital share $\alpha = 0.33$, corresponding to a steady-state share of capital income roughly equal to 30%; the labor disutility parameter ψ is calibrated to pin down the steady-state level of worked hours at 0.33; the steady-state product and labor market elasticities, ϵ^p and ϵ^w, are set at 6 and 21, implying steady-state markups of 20% and 5% respectively, as in Christiano et al. (2014). The share of government spending in aggregate output, $g^s = 0.18$, and the AR(1) parameter in (50), $\rho^\pi = 0.99$, are borrowed from Del Negro et al. (2015).

We set firms' return, $\gamma = 0.9$, in the range of Basu and Fernald (1997) estimates, and the tail index of the Pareto distribution, $\xi = 15$, following Asturias et al. (2017).

We set the detrended support of the NEs distribution, z, the depreciation rate of firms efficiency, δ^{INC}, the detrended and wage-adjusted fixed production costs, $w_{ss}\phi^j$, to calibrate some steady-state variables that characterize firm dynamics and the structure of the INT-goods sector. We set the firm entry rate, $\frac{NE}{\eta} = 2.5\%$, to match the 10% average yearly entry rate observed over the period 1978-2019. The steady-state number of firms, η, is normalized at 1. The fixed costs of production in labor units, $w\left(\phi^{NE} + \phi^{INC}\right)$, amount to $13,8\%$ of total GDP (Bilbiie et al., 2012; Colciago and Etro, 2010). The relative production size of NEs, that ultimately depends on the fixed costs ratio, is 0.7, close to the value reported in Clementi and Palazzo (2016). The remaining parameters are estimated with Bayesian techniques. Priors are in line with those adopted in previous empirical DSGE models. In particular, most prior distributions are borrowed from S&W with few minor differences. We slightly reduce the prior standard deviation of φ, the inverse Frisch elasticity parameter. The prior for $\bar{\pi}_{ss}$ is looser and centered on a higher mean, following Del Negro et al. (2015). Finally, the Taylor rule response to GDP growth and the two Calvo parameters are assigned a higher prior mean, closer to Christiano et al. (2014) and Justiniano et al. (2011).

Table 2: Estimated parameters and structural shocks (Baseline model)

		Prior			Posterior			
	Description	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	Inverse EIS	norm	1.500	0.3750	1.173	0.0415	1.1048	1.2389
φ	Inverse Frisch elasticity	norm	2.000	0.5000	2.354	0.4387	1.6174	3.0601
h	Consumption habits	beta	0.700	0.1000	0.840	0.0331	0.7860	0.8940
$100(\beta^{-1}-1)$	Discount factor	gamm	0.250	0.2000	0.169	0.0798	0.0332	0.2887
$\bar{\pi}_{ss}$	SS inflation rate	gamm	0.750	0.4000	0.716	0.3398	0.1743	1.2297
$100(g^z-1)$	Deterministic trend	norm	0.400	0.1000	0.350	0.0437	0.2791	0.4230
κ_π	Taylor rule coeff. on π	norm	1.500	0.2500	1.736	0.1767	1.4447	2.0237
κ_y	Taylor rule coeff. on y	norm	0.200	0.0500	0.239	0.0453	0.1647	0.3136
ρ_i	Policy rate per.	beta	0.750	0.1000	0.764	0.0274	0.7196	0.8094
Γ_p	Price rigidity	beta	0.650	0.1000	0.834	0.0215	0.7982	0.8688
μ_p	Price indexation	beta	0.500	0.1500	0.247	0.0971	0.0923	0.4000
Γ_w	Wage rigidity	beta	0.650	0.1000	0.815	0.0493	0.7345	0.8988
μ_w	Wage indexation	beta	0.500	0.1500	0.315	0.1457	0.0924	0.5217
γ_I	Investment adjustment costs	norm	4.000	1.5000	9.415	1.1286	7.5649	11.2810
σ_a	Capital utilization elasticity	beta	0.500	0.1500	0.884	0.0483	0.8092	0.9601
ρ^μ	MEI shock per.	beta	0.500	0.2000	0.556	0.0683	0.4554	0.6648
ρ^r	Monetary shock per.	beta	0.500	0.2000	0.305	0.0617	0.2064	0.4099
ρ^p	Price markup shock per.	beta	0.500	0.2000	0.984	0.0064	0.9742	0.9943
η^p	Price markup shock MA par.	beta	0.500	0.2000	0.363	0.0682	0.2513	0.4757
ρ^l	Labor supply shock per.	beta	0.500	0.2000	0.172	0.0636	0.0666	0.2739
ρ^Ψ	Incumbents' prod. shock per.	beta	0.500	0.2000	0.247	0.0667	0.1378	0.3566
ρ^{g^s}	Gov. spending shock per.	beta	0.500	0.2000	0.961	0.0106	0.9442	0.9779
σ^z	Entry shock s.d.	gamm	0.100	0.0500	0.005	0.0003	0.0047	0.0055
σ^μ	MEI shock s.d.	gamm	0.100	0.0500	0.090	0.0154	0.0655	0.1145
σ^r	Monetary policy shock s.d.	gamm	0.100	0.0500	0.002	0.0001	0.0022	0.0026
σ^p	Price markup shock s.d.	gamm	0.100	0.0500	0.079	0.0104	0.0625	0.0959
σ^l	Labor supply shock s.d.	gamm	0.100	0.0500	0.158	0.0363	0.0995	0.2146
σ^Ψ	Incumbents' prod. shock s.d.	gamm	0.100	0.0500	0.006	0.0003	0.0053	0.0063
σ^{g^s}	Gov. spending shock s.d.	gamm	0.100	0.0500	0.033	0.0017	0.0306	0.0360
σ^π	Inflation target shock s.d.	gamm	0.100	0.0500	0.001	0.0002	0.0008	0.0013

Note: The last two columns report the lower (HPD inf) and the upper bound (HPD sup) of the parameter's 90% highest posterior density interval.

4 Results.

Table 2 describes parameters and shock processes and reports our posterior estimates for the baseline, full sample model.[14] Consumption habits are in line with Justiniano et al. (2011), whereas both Calvo parameters are close to the values reported by Del Negro et al. (2015), and are substantially smaller than in Del Negro et al. (2017) and Casares et al. (2020). The elasticity of capital utilization costs is slightly higher than in Casares et al. (2020) and Justiniano et al. (2011) who find a value of 0.84. Lastly, investment adjustment costs are close to Lewis and Stevens (2015) and below the estimate obtained by Christiano et al. (2014).

The first step in our analysis is a comparison with alternative estimates: an "uninformed" model that excludes the entry rate from the set of observables (unobserved-entry model, UEM) and a standard NK model following S&W (SNK). We also implement robustness checks restricting the sample to the pre-GFC period, *i.e.* the estimation sample is truncated at 2007:III (short-sample model, SSM), and substituting the shadow rate for the observed interest rate (SRM).[15]

Relative to BM, posterior estimates are virtually unchanged in SRM. Under SSM we see a decrease in investment adjustment (γ_I) and capital utilization (σ_a) costs. All remaining parameters are substantially stable.

Relatively to SNK, in BM we obtain higher internal persistence: consumption habits (h), price and wage stickiness (Γ_p and Γ_w), investment adjustment and capital utilization costs are somewhat larger, but fall in the ballpark of existing estimates in the DSGE literature. On the contrary, the exogenous persistence of the standard NK model, identified in the shocks' autocorrelation coefficients, is generally more pronounced than in our model.

The UEM estimation produces a higher degree of wage indexation with respect to BM. Also, the elasticity of investment adjustment costs and the MEI-shock autoregressive parameter are closer to SNK than to BM. It follows that our model requires a larger γ_I but a smaller

[14] See Appendix B.2 for a detailed discussion of parameters' identification and convergence.

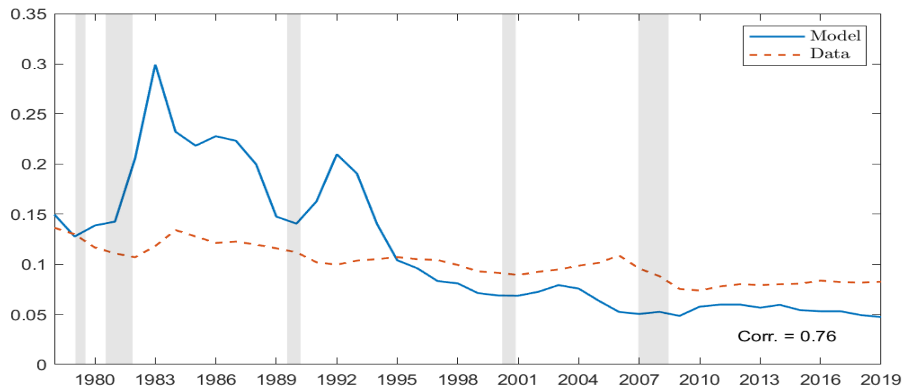
[15] See Table A1 in the Appendix.

ρ^{μ} in order to explain entry rate dynamics.

4.1 Drivers of the entry rate decline.

A preliminary step in our analysis is a discussion of the model-implied entry-rate series (MI-ER) obtained in UEM. In this "uninformed" model, MI-ER grossly overpredicts the response of observed entry rates (O-ER) to post-recession recoveries up to the mid-90s (see Figure 5). Nevertheless, information coming from the standard set of observed macro variables and the need to match the U.S. business cycle data is sufficient for the model to predict a long term decline in MI-ER, and its correlation with the O-ER series is rather large (0.76).

Figure 5: MI-ER, 1978:I-2019:IV (UEM)

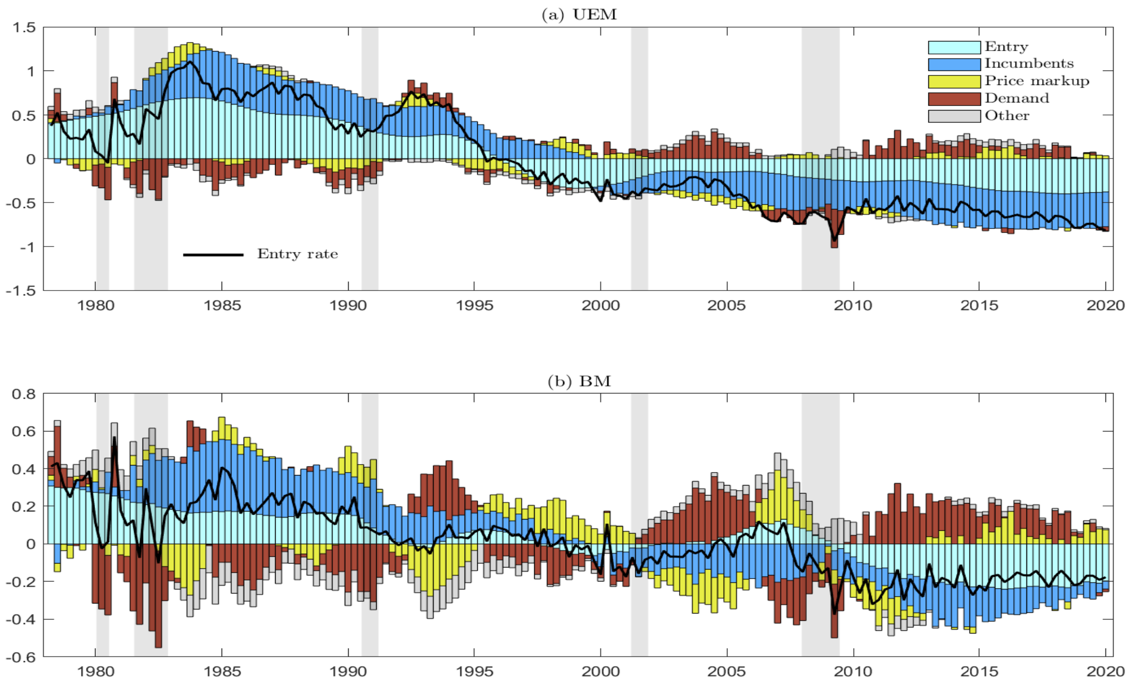


Note: The solid line shows the annualized smoothed estimate of the model-implied entry rate at the posterior mean (UEM estimation). The dashed line shows the annual firm entry rate in the data (BDS).

It is also interesting to look at the historical decomposition of MI-ER (Figure 6, Panel (a)), where the estimated technology shocks that persistently lower(raise) the productivity of NEs($INCs$) also imply the prediction of a long term decline in the entry rate.

Figure 6, Panel (b), presents the historical decomposition of the O-ER obtained in our baseline model. Relative to MI-ER obtained in UEM, the contribution of technology shocks is virtually unchanged, confirming that productivity growth of potential NEs gradually declined over the sample, and that NEs were crowded out by the technology shocks that raised $INCs$ productivity. Price-markup and other, mainly demand, shocks contribute to predict the observed entry rate, essentially compensating the gap between MI-ER and O-ER. These shocks bring down the entry rate when it is relatively high (between 1978 and the mid-90s), and tend to raise it thereafter. These results are in contrast with Gutiérrez and Philippon

Figure 6: Historical shock decomposition: MI-ER and O-ER, 1978:I-2019:IV



Note: The solid line is firm entry rate in log-deviations from its steady state (quarterly estimate at the posterior mean). The colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values). Panel (a): UEM estimation. Panel (b): BM estimation (firm entry rate coincides with the observed one).

(2017), who argue that the increase in price markups might have been at the root of the observed decline in entry rates. In fact, our model predicts a long term increase in price markups and in "pure" profits in line with evidence reported in Traina (2018). Further, the historical decomposition of price markups highlights the persistent decline in the demand elasticity of retail goods, ϵ_t^p, as the main driver of long-run markup dynamics, but this latter effect has only transitory implications for the entry rate.[16]

[16]See Figure A1 in the Appendix.

We estimate a strong correlation (78%) between the entry rate and the price/cost margin index $\frac{p_t}{(p_t^z)}$. The historical decomposition of $\frac{p_t}{(p_t^z)}$ is essentially driven by technology shocks, whereas those demand shocks that matter for the entry rate bear nearly symmetrical effects on the prices of the intermediate goods and of the Z bundle. This is a novel result, whereby $\frac{p_t}{(p_t^z)}$ can be interpreted as a summary statistics capturing the occurrence of technology shocks.[17]

To conclude this discussion, we highlight the reasons why technology shocks may have similar implications for the entry rate and for the price/cost margin in the INT-sector. Consider first the case of an adverse entry shock (Figure 4).[18] In this case, the short-run effect of the shock temporarily raises output and demand for labor and capital but depresses the consumption price of intermediate goods and lowers $\frac{p_t}{(p_t^z)}$. Then, consider a favorable shock to the productivity of incumbents. For reasons already discussed in section 2.5 above, the entry rate inevitably falls, but the sustained output expansion raises both the wage rate and the rate of return on capital. As a result, p_t^z increases and the price/cost margin of INT-firms inevitably falls.

Table 3: Historical shock decomposition "summary": GDP growth, 1978:I-2019:IV

	Shocks				
	Incumbents	Entry	Price markup	Demand	Other
UEM	16.3%	11.9%	11.3%	48.0%	12.4%
BM	10.2%	7.1%	9.8%	60.7%	12.2%

Note: For each shock group, the percentage terms refer to its average contribution to GDP growth, as obtained from the historical shock decomposition (at the posterior mean), over the period 1978:I-2019:IV. Specifically, for each quarter, we derive the absolute value of a shock's contribution to deviations of GDP growth from its mean. We then calculate the ratio of this value to the sum of all shocks' contributions (again in absolute value), and we average the ratios over quarters.

As a final remark, Table 3 summarizes the contributions of shocks to observed GDP growth in the estimation of BM and UEM since 1978, *i.e.* the first year when BM is "constrained" to match the entry rate. Technology shocks play a lesser role in BM, and are replaced by

[17]See Figure A2 in the Appendix.
[18]Note that the IRFs in Figure 4 display the effects of a *positive* entry shock.

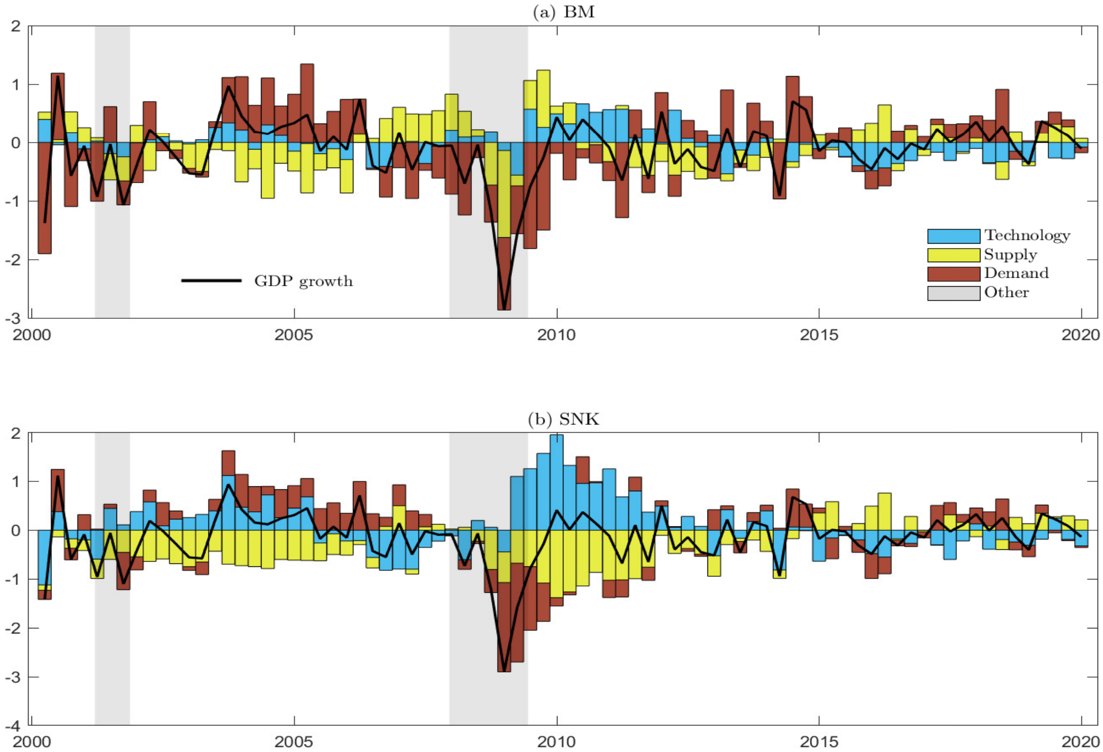
demand shocks. Furthermore, the two models agree about the relative contributions of the technology shocks that affect NEs and $INCs$.

4.2 Baseline vs SNK model.

The purpose here is to benchmark our interpretation of the U.S. business cycle against the established narrative based on the SNK model. With respect to the common shocks, the IRFs of the two models are very similar (reported in Figure A3 in the Appendix). An interesting comparison concerns the historical decomposition of GDP growth obtained in the baseline and in the SNK model (Figure 7).

To sharpen the analysis we focus on the post-2000 period. After 2013, the two models convey similar messages, but in the previous years important differences are easy to spot. According to the SNK estimates, markup shocks are persistently contractionary, whereas technology shocks pull in the opposite direction with an almost symmetrical pattern. Thus the SNK model conveys a story where pre-2013 growth is determined by a combination of technology improvements and persistently adverse markup shocks. These contemporaneous and opposite effects are particularly large in the occurrence and in the immediate aftermath of the GFC. The contribution of technology shocks to the post 2008:IV recovery appears implausibly large and in sharp contrast with results obtained in contributions such as Fernald (2014) and Vinci and Licandro (2021). By contrast, our baseline model does not generate equally persistent patterns and technology shocks play a lesser role. Demand shocks are relatively more important. Their positive contribution to growth in the 2003-2006 period is consistent with the popular narrative about the importance of the credit boom in the run-up to the GFC.

Figure 7: Historical shock decomposition comparison: GDP growth, 2000:I-2019:IV



Note: The solid line is observed GDP growth in log-deviations from its estimated steady state. The colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks for panel (a) and (b), and risk premium shocks for panel (b); "Supply" includes price markup shocks for panel (a) and (b), labor supply shocks for panel (a), and wage markup shocks for panel (b); "Other" includes contribution from initial values). Panel (a): BM estimation. Panel (b): SNK estimation.

4.3 Firm efficiency and total factor productivity in the long-run.

We define TFP, average firm efficiency and efficiency dispersion respectively as

$$TFP_t = \int_{\hat{A}_t^{NE}}^{\infty} A_t^{f,NE} + \int_{\hat{A}_t^{INC}}^{\infty} A_t^{f,INC} = \frac{\xi}{\xi - 1}\left(NE_t \hat{A}_t^{NE} + INC_t \hat{A}_t^{INC}\right) , \tag{55}$$

$$\hat{A}_t^{av} = \frac{TFP_t}{\eta_t} , \tag{56}$$

$$\Sigma_t^A = \frac{\xi}{(\xi - 2)\,\eta_t}\left[NE_t\left(\hat{A}_t^{NE}\right)^2 + INC_t\left(\hat{A}_t^{INC}\right)^2\right] . \tag{57}$$

Both technology and non-technology shocks explain the volatility of the growth rates of TFP_t, $\hat{A}_t^{av}$ and Σ_t^A.[19] Shocks to incumbents' productivity determine about 92% of average efficiency growth volatility, while the contribution of non-technology shocks is less than 8%. We observe a similar variance decomposition for the growth rate of efficiency dispersion. By contrast, demand shocks have predominant effects on TFP growth through their impact on the extensive margin.

In Figure 8 we show the model estimates for (55), (56) and (57). Panel (a) reports the sample evolution of $\hat{A}_t^{av}$ and of the z_t shifter that drives the efficiency growth rate for potential new entrants. Taking z_t as a reference, a marked slowdown in $\hat{A}_t^{av}$ characterizes the years between the recessions that hit the U.S. economy in the mid-70s and at the beginning of the 80s. Then the trend is gradually reversed and, since 2008, $\hat{A}_t^{av}$ lies above the z_t shifter. Indeed, average growth rates over subperiods convey the unambiguous message of a gradual slowdown in g_t^z (see Table 4).[20]

Table 4: Annualized average growth rates (BM)

	1966:I-1975:I	1975:II-1982:IV	1983:I-2008:IV	2009:I-2019:IV
z_t shifter	1.52%	0.93%	0.91%	0.72%
Average firm efficiency	0.76%	0.56%	1.19%	1.06%

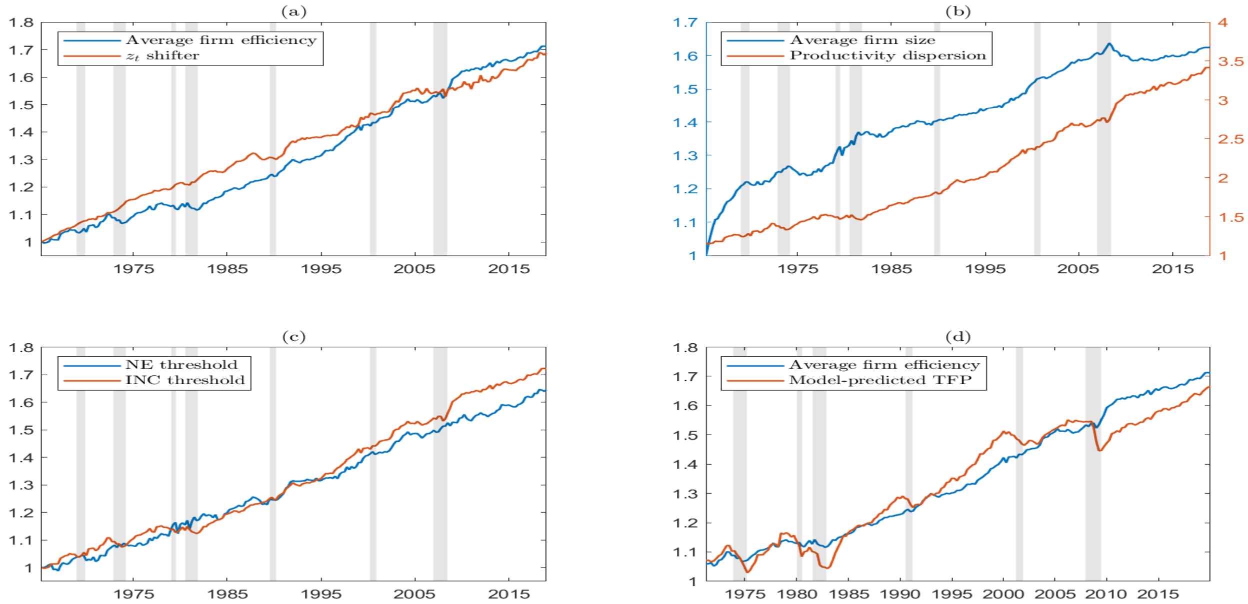
Note: Annualized average growth rates of the smoothed estimates of z_t and $\hat{A}_t^{av}$ at the posterior mean.

Panel (b) of Figure 8 plots the post-1966 series of firm size, proxied by the average size of the Z_t bundle, and firm efficiency dispersion. Hopenhayn et al. (2018) find that average firm size, measured by the number of employees, rose by 20% between 1977 and 2014. Our measure, that also accounts for capital accumulation, predicts a 28% increase over the same period. As for productivity dispersion, Kehrig (2015) reports that it "doubled" over the

[19]See Table A2 in the Appendix for the unconditional variance decomposition.

[20]These results are broadly consistent with the historical decomposition of observed entry rates, essentially driven in the long run by the adverse(favorable) shocks to $z_t(\Psi_t)$.

Figure 8: Baseline model predictions for TFP, firm efficiency and firm size



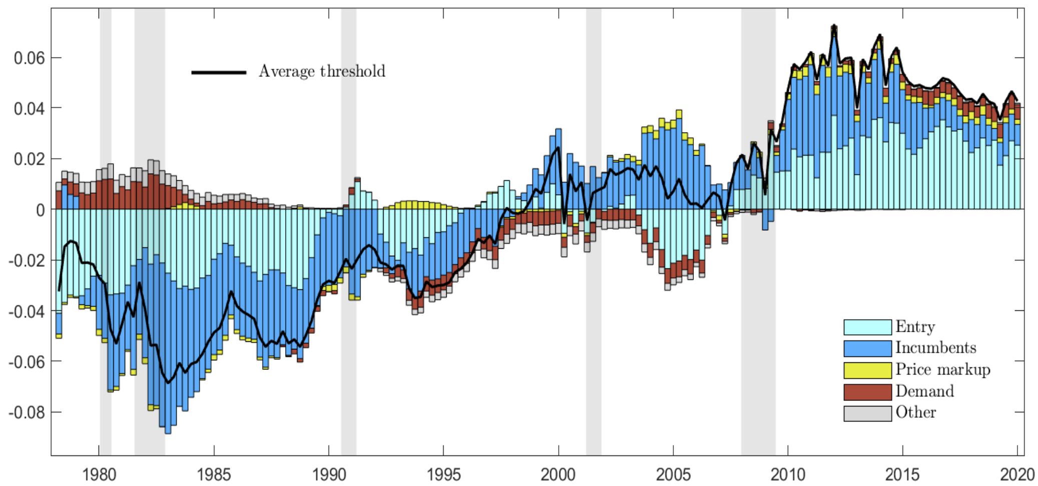
Note: All lines depict quarterly smoothed estimates at the posterior mean (BM estimation). Panel (a): average firm efficiency $\hat{A}_t^{av}$ (blue) and productivity shifter z_t (orange); 1966:I-2019:IV (both series are normalized at 1 in 1966:I). Panel (b): Average firm size (blue), proxied by the ratio of factor-inputs bundle (Z_t) over the number of firms (η_t), and productivity dispersion (orange), *i.e.* weighted average of the productivity dispersion of NEs and $INCs$ (dispersion of INC (NE) firm productivities is computed as the variance of the left-truncated Pareto distribution INC (NE) firms draw efficiency from; the truncation is given by the respective productivity threshold that varies over time, while the shape of the distribution is constant); 1966:I-2019:IV (the smoothed series of Z_t and η_t are normalized at 1 in 1966:I). Panel (c): NE productivity threshold $\hat{A}_t^{NE}$ (blue) and INC productivity threshold $\hat{A}_t^{INC}$ (orange); 1966:I-2019:IV (both series are normalized at 1 in 1966:I). Panel (d): average firm efficiency $\hat{A}_t^{av}$ (blue) and model-predicted TFP $\eta_t \hat{A}_t^{av}$ (orange); 1971:I-2019:IV ($\hat{A}_t^{av}$ is normalized at 1 in 1966:I).

period 1972-2009. Our estimation implies that in 2009 the productivity-dispersion measure was about 2.3-times larger than in 1972.

Panel (c) of Figure 8 reports the productivity thresholds $\hat{A}_t^j$, showing that the efficiency gap between incumbents and new entrants has gradually increased since the mid-1980s. This is consistent with the evidence that incumbents accounted for the lion's share of innovation in the latest decades (Hsieh and Klenow, 2018; Garcia-Macia et al., 2019).

Kehrig (2015) shows that the dispersion of firms' total factor productivity in U.S. manufacturing is greater in recessions than in booms. He builds on this result to discriminate between "Schumpeterian" models that unambiguously praise the cleansing effect of recessions, and the "sullying view" supported by models in the tradition of Melitz (2003), where the procyclical pattern of input costs generates opposite effects of efficiency dispersion. Panel (b) of Figure 8 shows that, even if our INT-firm sector is inherently "Schumpeterian", the estimated pattern of efficiency dispersion in recessions is ambiguous. According to our estimates (Figure 9), during recession episodes in 2001 and 2007, the productivity thresholds are almost entirely determined by technology shocks, but these shocks did not play a key role in determining the recession (see Panel (a) of Figure 7).[21]

Figure 9: Historical shock decomposition: detrended average threshold, 1978:I-2019:IV (BM)



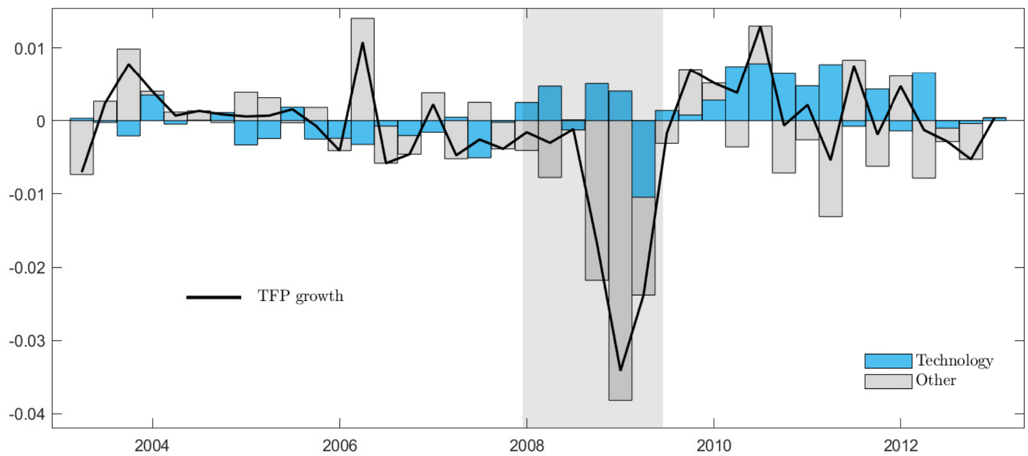
Note: The solid line is the detrended average productivity threshold in log-deviations from its steady state (quarterly estimate at the posterior mean); the colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values).

Extensive margin dynamics drive our estimated TFP measure, as shown in Panel (d) of Figure 8 where the wedge between TFP and average efficiency is entirely accounted for by

[21] To rationalize this result, consider that, because of the predominant presence of incumbents in the market, $\hat{A}_t^{av}$ is mainly affected by the $INCs$' threshold and the latter is strongly sensitive to the price/cost margin which does not necessarily decrease during recessions. See Figure A2 in the Appendix for the smoothed path and the historical decomposition of price/cost margins. We further discuss the determinants of productivity thresholds in Appendix C.3.4.

variations in the mass of firms. Consistently with previous evidence (Field, 2010, 2011), TFP growth is strongly procyclical, it has declined since 2005, but the GFC apparently marked a watershed, as pointed out in studies such as Anzoategui et al. (2019) and Bianchi et al. (2019).[22] Differently from these studies, our estimates interpret the TFP slowdown during the GFC as the consequence of adverse non-technology shocks that mainly operated through the extensive margin (Figure 10).

Figure 10: Historical shock decomposition: model-predicted TFP growth rate, 2003:I-2012:IV (BM)



Note: The solid line is TFP growth in log-deviations from its steady state (quarterly estimate at the posterior mean). The colored bars are the contributions of the grouped shocks ("Other" includes contribution from initial values).

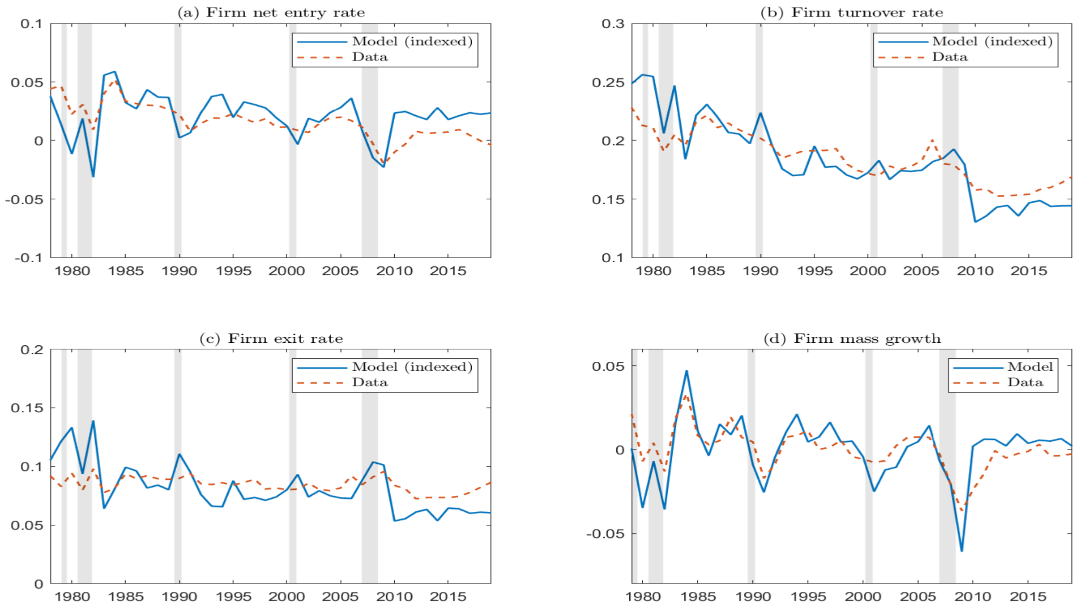
4.4 Other measures of business dynamism.

In addition to the entry rate, our model bears predictions for other measures of business dynamism, such as net entry and turnover, respectively $\frac{NE_t - EX_t}{\eta_t}$ and $\frac{NE_t + EX_t}{\eta_t}$. The model does a reasonably good job in predicting either variable (see Panel (a) and (b) of Figure 11), but there is a tendency to predict a pro(counter)cyclical pattern that is difficult to detect in the observed series for net entry(turnover). In fact, these results are driven by the gap

[22] By contrast, Fernald (2014) points out that productivity behaved similarly to previous episodes of severe recession, but recovered strongly once the recession ended.

between the model-predicted and the observed exit rate series, as shown in Panel (c) of Figure 11.

Figure 11: Firm-mass growth (% deviations from sample mean), 1979-2019 (BM)



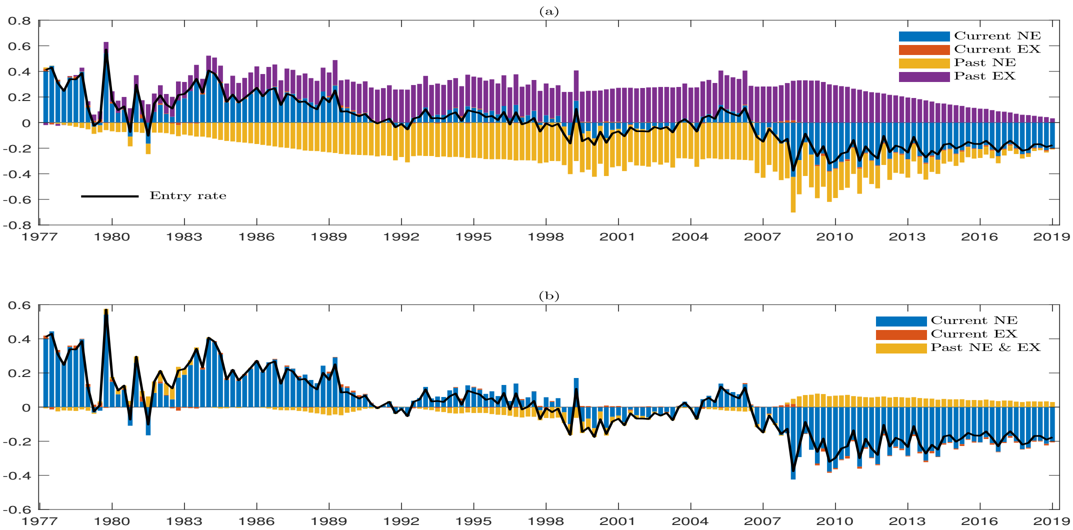
Note: Solid lines show annualized smoothed estimates at the posterior mean (BM estimation); dashed lines display the respective counterparts in the data (BDS). The model-implied series in panels (a), (b) and (c) are indexed at the 2000:I observed values. Both series in Panel (d) are expressed in percentage deviations from their sample mean.

Since exit flows contribute to determine the number of firms, one might wonder whether this bias could have implications for the interpretation of the entry-rate drivers. Up to first-order approximation, condition (3) can be decomposed as follows

$$\widehat{entry} = H\widehat{NE}_t - (1-H)\,\widehat{EX}_t - (1-H)\sum_{j=1}^{T_0}\left(\widehat{NE}_{t-j} - \widehat{EX}_{t-j}\right), \tag{58}$$

where $\hat{x}_t$ denotes log-deviations from the steady state, H is the survival probability of firms in the deterministic steady state, T_0 denotes the initial sample period. Figure 12 suggests that the current estimate of exit has virtually no effect on our interpretation of the observed entry rate. The accumulated past balance between exit and entry has a limited effect and matters only after the GFC. In this period it is driven by the accumulation of exit flows, but its importance in determining $\widehat{entry}$ remains limited.

Figure 12: Entry rate decomposition, 1978:I-2019:IV (BM)



Note: In both panels, the solid line depicts log-deviations from the steady state of the quarterly entry-rate smoothed estimate at the posterior mean (BM estimation). The colored bars display the model-implied contribution to those deviations coming from (i) current new entrants, (ii) current firm exits, (iii) past accumulated new entrants, and (iv) past accumulated firm exits. Panel (b) groups contributions from (iii) and (iv).

Due to the overestimated countercyclical pattern of the exit rate, our model also exaggerates the procyclicality of $\Delta\eta_t$ (see Panel (d) of Figure 11). This effect is particularly strong in occasion of the recessions that marked the beginning and the end of the Great Moderation period. This suggest that our results concerning the deep TFP fall in occasion of the GFC should be taken with some caution.

5 Conclusions.

The paper establishes a strong connection between the long-term decline in the entry rate and the asymmetric technology shocks that persistently hit new entrants and incumbent firms. By contrast, the model-implied cumulative increase in price markups did not contribute to the concurrent fall in the entry rate. Importantly, these results are confirmed even if we exclude the entry rate from the observed variables.

Our results emphasize the importance of the extensive margin in determining the long-term slowdown in TFP growth. The extensive margin also introduces a hitherto unexplored channel for the transmission of non-technology shocks to the cyclical component of aggregate TFP.

The model challenges popular wisdom on the "cleansing" effect of recessions: demand-driven recessions do not necessarily generate survival of the fittest.

Finally, we highlight the reduction in price/cost margins of INT-firms as a single statistic that captures the effects of technology shocks on entry decisions. Micro-econometric analysis should investigate the responsiveness of the entry rate to price/cost margins. We leave this for future work.

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Appendices

A Additional Tables and Figures.

A.1 Tables.

Table A1: Posterior estimates comparison

	Prior			Posterior mean				
	Dist.	Mean	Stdev.	BM	SRM	SSM	UEM	SNK
σ	norm	1.500	0.3750	1.173	1.168	1.233	1.116	1.118
φ	norm	2.000	0.5000	2.354	2.297	2.323	2.039	2.131
h	beta	0.700	0.1000	0.840	0.847	0.842	0.799	0.813
$100(\beta^{-1}-1)$	gamm	0.250	0.2000	0.169	0.149	0.198	0.181	0.389
$\bar{\pi}_{ss}$	gamm	0.750	0.4000	0.716	0.707	0.720	0.715	0.705
$100(g^z-1)$	norm	0.400	0.1000	0.350	0.351	0.430	0.311	0.379
κ_π	norm	1.500	0.2500	1.736	1.762	1.638	1.651	1.787
κ_y	norm	0.200 (0.125)	0.0500	0.239	0.240	0.222	0.246	0.006
κ_{Δ_y}	norm	0.125	0.0500	-	-	-	-	0.025
ρ_i	beta	0.750	0.1000	0.764	0.770	0.750	0.751	0.784
Γ_p	beta	0.650	0.1000	0.834	0.833	0.804	0.813	0.711
μ_p	beta	0.500	0.1500	0.247	0.271	0.371	0.296	0.206
Γ_w	beta	0.650	0.1000	0.815	0.821	0.876	0.805	0.788
μ_w	beta	0.500	0.1500	0.315	0.330	0.336	0.774	0.564
γ_I	norm	4.000	1.5000	9.415	9.694	8.574	7.515	7.542
σ_a	beta	0.500	0.1500	0.884	0.887	0.777	0.863	0.766
ϕ_p	norm	1.250	0.1250	-	-	-	-	1.663
Δ_L	norm	1.000 (0.000)	2.0000	-	-	1.106	-	-2.735
ρ^μ	beta	0.500	0.2000	0.556	0.548	0.460	0.795	0.815
ρ^r	beta	0.500	0.2000	0.305	0.325	0.288	0.239	0.273
ρ^p	beta	0.500	0.2000	0.984	0.982	0.976	0.964	0.973
η^p	beta	0.500	0.2000	0.363	0.359	0.401	0.482	0.845
ρ^l	beta	0.500	0.2000	0.172	0.173	0.134	0.225	-
ρ^w	beta	0.500	0.2000	-	-	-	-	0.933
η^w	beta	0.500	0.2000	-	-	-	-	0.874
ρ^Ψ	beta	0.500	0.2000	0.247	0.240	0.255	0.173	-
ρ^a	beta	0.500	0.2000	-	-	-	-	0.930
ρ^{g^s}	beta	0.500	0.2000	0.961	0.961	0.975	0.966	0.957
ρ^{rp}	beta	0.500	0.2000	-	-	-	-	0.147

Note: Terms in round brackets refer to the prior specifications used in the estimation of the benchmark NK model, when different from ours.
The interpretation of κ_y differs between the NK benchmark and our model. In the former, κ_y determines the Taylor rule response to output gap deviations from the steady state, where the output gap is defined as the difference between the actual and the flexible-price output level. κ_{Δ_y} stands for the monetary policy weight on output gap growth. Conversely, monetary policy in our model targets output growth through κ_y.
ϕ_p is the estimated share of fixed costs and ρ^{rp} is the persistence of risk premium shocks, both absent in our

model, while ρ^w and ρ^a are the autocorrelation coefficients of the wage markup and stationary technology processes, respectively. These two shocks can be thought of as counterparts of our labor supply and incumbent shocks.

Δ_L enters the observation equation of hours worked as a "correction" term, when the steady state of L does not equal the observed sample mean.

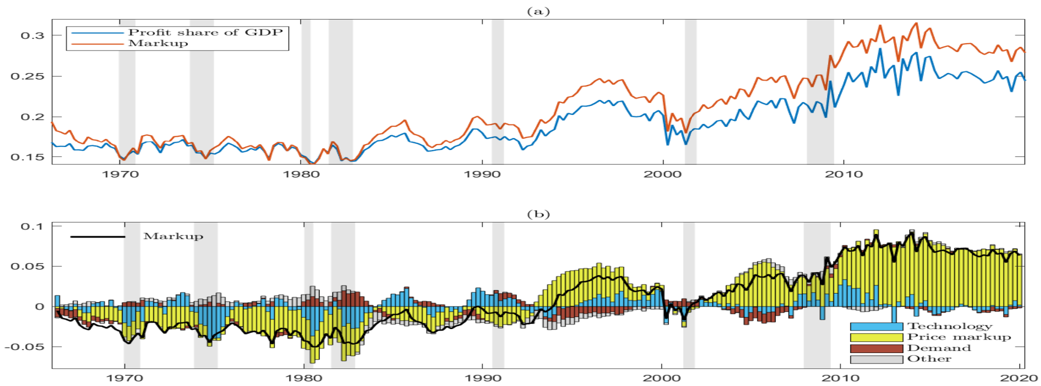
Table A2: Productivity measures variance decomposition (BM)

	Shocks			
	Incumbents	Entry	Supply	Demand
TFP growth	8.2%	2.4%	22.0%	67.4%
Average efficiency growth	91.9%	0.4%	1.5%	6.3%
Efficiency dispersion growth	93.1%	0.4%	1.3%	5.3%

Note: Unconditional variance decomposition at the posterior mean. Non-technology shocks are grouped into two categories: supply and demand. "Supply" includes price markup and labor supply shocks; "Demand" includes monetary policy, inflation target, MEI, and government spending shocks.

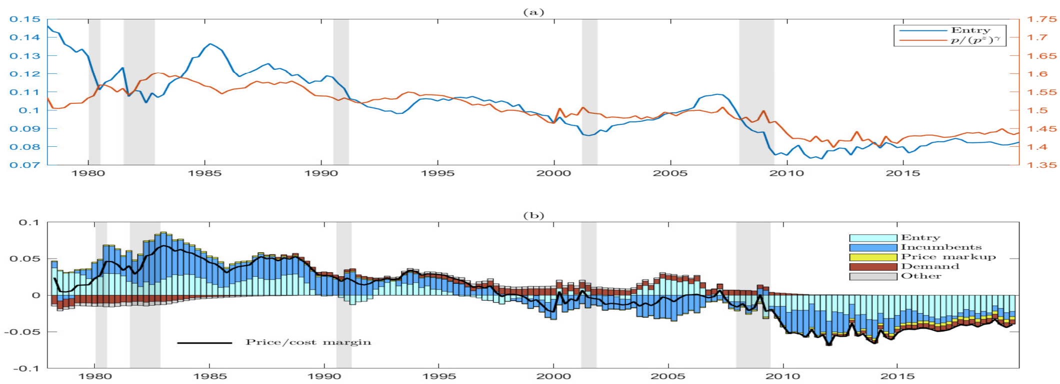
A.2 Figures.

Figure A1: BM predictions: profit share of GDP and markups, 1966:I-2019:IV



Note: Panel (a): quarterly smoothed estimates, at the posterior mean, of profits of INT-firms and retailers as a share of GDP (blue), and of retailers' price markup over marginal costs (orange). Panel (b): the solid line is markup in log-deviations from its steady state (quarterly estimate at the posterior mean); the colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values).

Figure A2: BM predictions: price/cost margin of INT-firms and observed entry rate 1978:I-2019:IV



Note: Panel (a): quarterly smoothed estimates, at the posterior mean, of the price/cost margin of INT-firms (orange), and of the entry rate (blue). Panel (b): the solid line is the price/cost margin in log-deviations from its steady state (quarterly estimate at the posterior mean); the colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values).

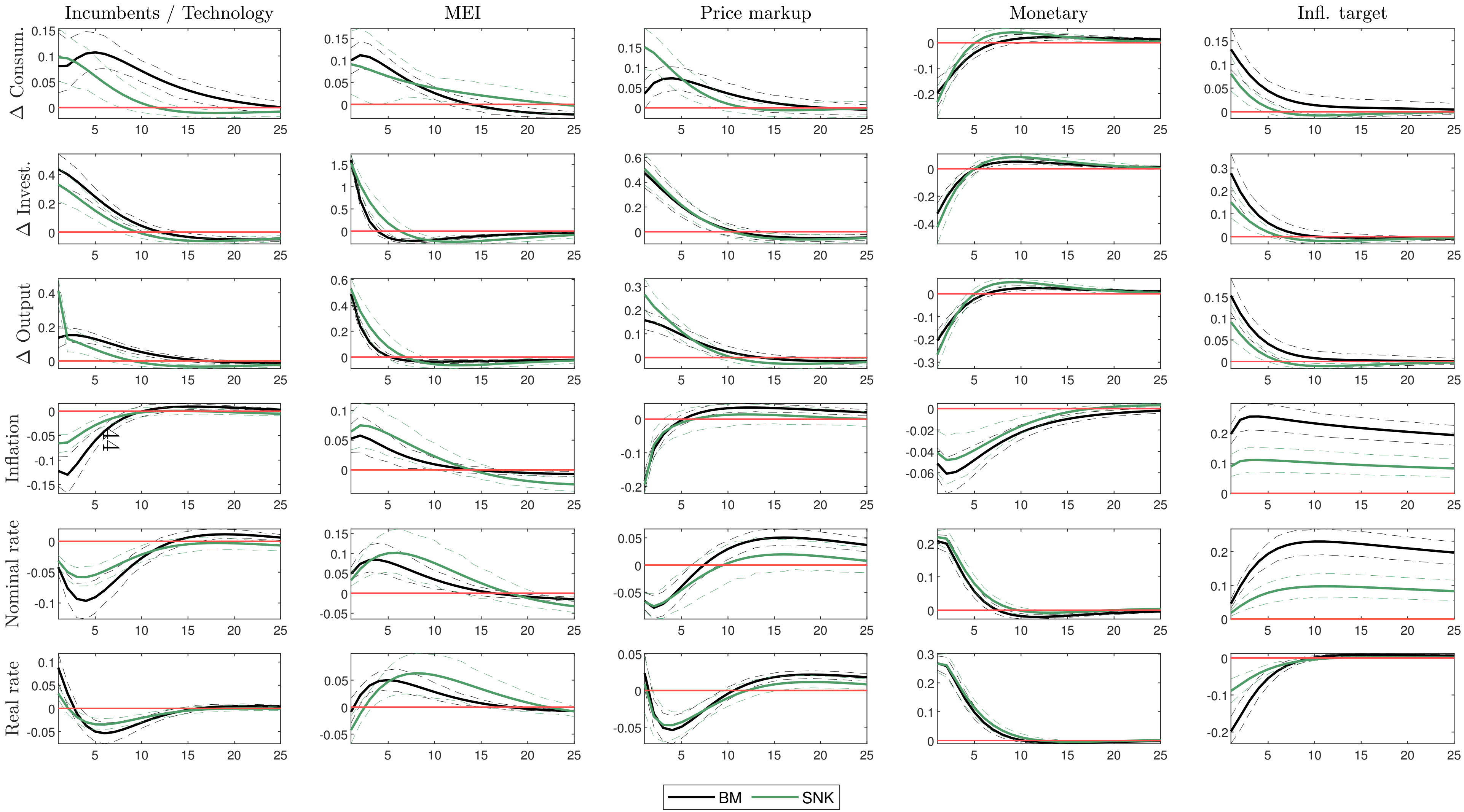


Figure A3: IRFs comparison (BM vs SNK)

Note: Quarterly estimated mean impulse responses (solid lines) with 90% HPD intervals (dashed lines) to one-standard-deviation shocks.

B Estimation technical appendix.

B.1 Data.

Data on real GDP (GDPC1), the GDP deflator (GDPDEF), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are produced at a quarterly frequency by the Bureau of Economic Analysis, and are included in the National Income and Product Accounts (NIPA). Average weekly hours in the nonfarm business sector (PRS85006023) and hourly compensation in the nonfarm business sector (PRS85006103) are produced by the Bureau of Labor Statistics (BLS) at a quarterly frequency. The civilian employment level (CE16OV) and the civilian non-institutional population (CNP16OV) are also produced by the BLS at a monthly frequency. We take quarterly averages of the monthly data. The federal funds rate (FEDFUNDS) is obtained from the Federal Reserve Board's H.15 release at a business day frequency. We take quarterly averages of the annualized daily data. All these data are collected from FRED (except for hourly wages, retrieved from the BLS database), and are transformed following S&W. Data on total firms and firm births (defined as firms born during the last 12 months) are produced by the Census Bureau, within the Business Dynamics Statistics (BDS) survey, at an annual frequency. In a robustness estimation, we use shadow rate data from Wu and Xia (2016).[23]

Table B1: Data Transformation.

Data	Transformation
Output growth	$100\Delta \ln \left(\frac{GDPC1}{CNP16OV_{index}} \right)$
Investment growth	$100\Delta \ln \left[\left(\frac{FPI}{GDPDEF} \right) \frac{1}{CNP16OV_{index}} \right]$
Consumption growth	$100\Delta \ln \left[\left(\frac{PCEC}{GDPDEF} \right) \frac{1}{CNP16OV_{index}} \right]$
Wages growth	$100\Delta \ln \left(\frac{PRS85006103}{GDPDEF} \right)$
Hours worked	$100 \ln \left[\left(\frac{PRS85006023}{CNP16OV_{index}} \right) \left(\frac{CE16OV_{index}}{100} \right) \right]$
Inflation	$100\Delta \ln (GDPDEF)$
Nominal interest rate	$FEDFUNDS/4 \quad [SHADOWRATE/4]$
Entry rate	$100 \ln \left(1 + \frac{BIRTHS_y}{(FIRMS_y + FIRMS_{y-1})/2} \right)$

B.2 Estimation.

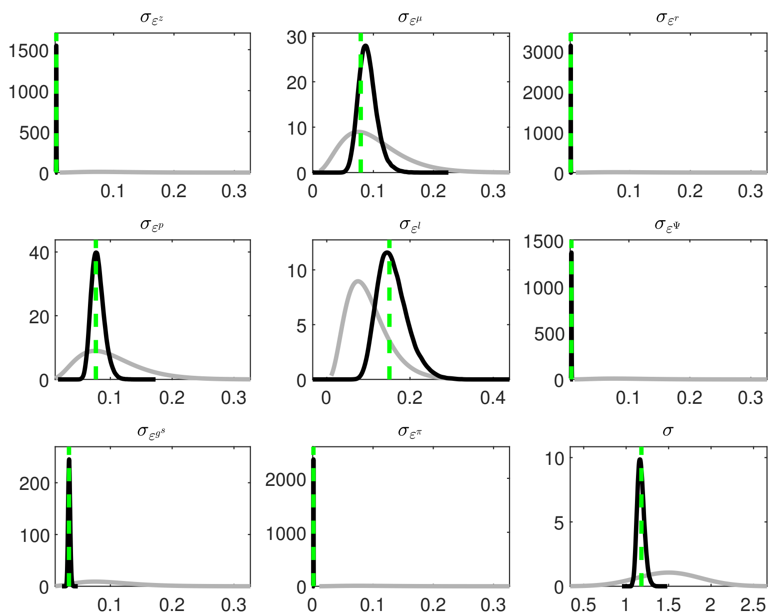
The model is solved using a first-order approximation around the deterministic steady state and is estimated using Dynare 4.6.2 (Adjemian et al., 2022). The baseline estimation is run with a single Markov Chain of 2 million draws, of which we discard the first 400 thousands.

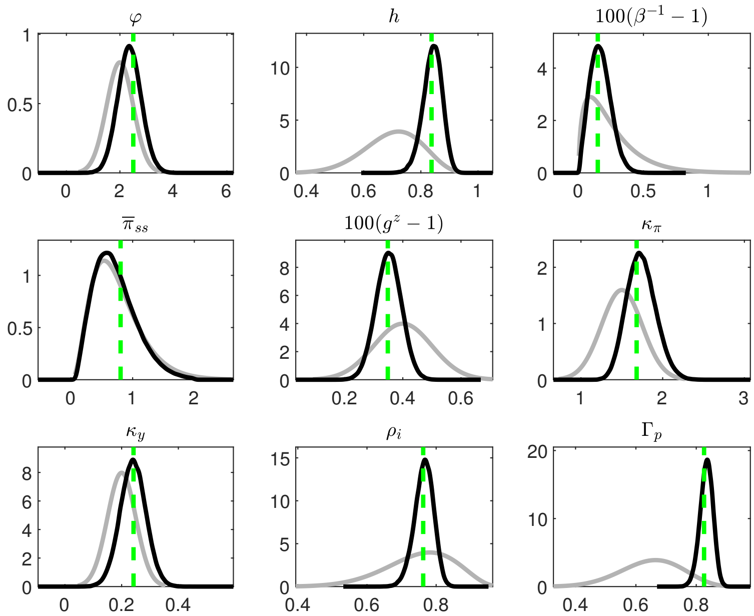
[23]`https://sites.google.com/view/jingcynthiawu/shadow-rates`

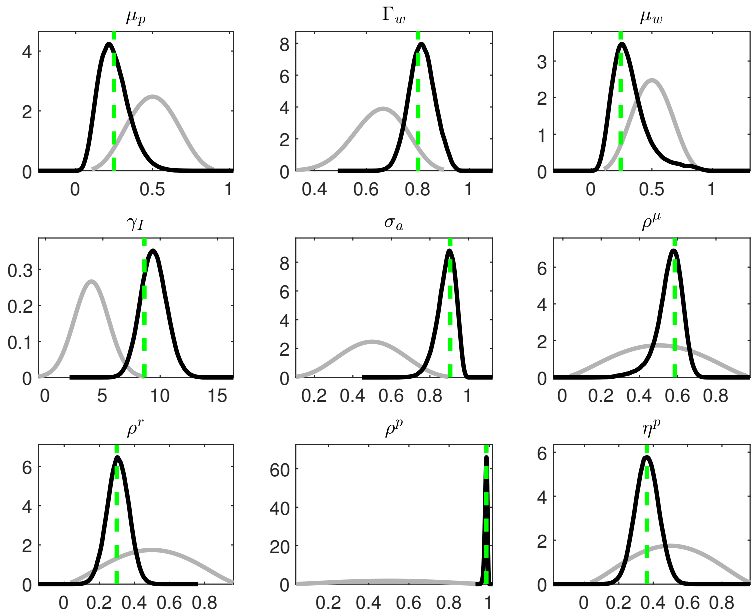
The overall acceptance ratio of the Metropolis-Hastings algorithm is close to 26%. Estimation results are virtually identical if we run four chains of 500 thousand draws each.

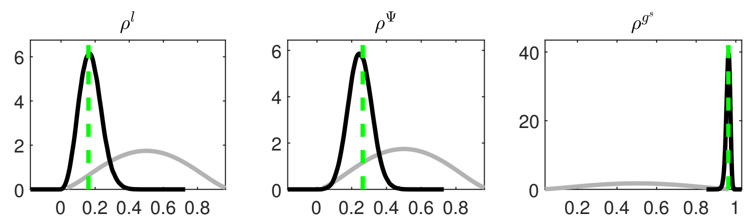
B.2.1 Prior-posterior plots.

All posterior distributions are well-shaped and tighter than the respective priors, with the exception of the steady-state inflation rate whose prior and posterior distributions almost overlap, indicating a weak identification for this parameter. We do not consider this a major weakness of our estimation, since the posterior mean is close to the average inflation rate in the data and to the estimates in the DSGE literature.




φ
h
$100(\beta^{-1}-1)$
$\overline{\pi}_{ss}$
$100(g^z-1)$
κ_π
κ_y
ρ_i
Γ_p


μ_p
Γ_w
μ_w
γ_I
σ_a
ρ^μ
ρ^r
ρ^p
η^p


ρ^l
ρ^Ψ
ρ^{g^s}

B.2.2 Convergence.

We consider two convergence diagnostics tests. The Geweke test uses a χ-square test to compare the means of draws from 400,000 to 720,000 and from 1,200,000 to 2,000,000. The null hypothesis is that the two sample means are equal, suggesting that draws from the two samples come from the same distribution, and thus that the chain has converged. In order to tackle the impact of draws' correlation on the estimates, a Newey and West (1987)-type estimator is used that tapers spectral density. The Raftery and Lewis test identifies the number of burn-in and the number of draws after burn-in required to estimate the q=0.025 percentile (corresponding to a 95% HPDI) with a precision of 0.5% with 95% certainty. If the number of burn-in and required draws is below the number of draws considered in the estimation, we can conclude that the chain has converged.
Looking at the p-values accounting for serial correlation (with taper), the null hypothesis for equality of means of the Geweke test (Table B2) is not rejected for all parameters but κ_π and ρ^l, at a 5% significance level. On the contrary, the Raftery and Lewis test (Table B3) delivers a maximum number of required draws well below the 2 million used in our estimation. In order to further examine the convergence of κ_π and ρ^l, we look at the trace plots of the two parameters (see Figures B1 and B2): in neither case we spot evident drifts or jumps to other modes. Therefore, we are led to conclude that the Markov Chain has converged to the ergodic distribution.

Table B2: Geweke (1992) Convergence Tests, based on means of draws 400000 to 720000 vs 1200000 to 2000000. p-values are for χ^2-test for equality of means.

	Posterior		p-values			
Parameter	*Mean*	*Std*	*No Taper*	4% *Taper*	8% *Taper*	15% *Taper*
σ_{ε^z}	0.0051	0.0003	0.3410	0.9308	0.9232	0.9069
σ_{ε^μ}	0.0906	0.0154	0.0000	0.5877	0.5327	0.5003
σ_{ε^r}	0.0024	0.0001	0.0000	0.1742	0.1723	0.1592
σ_{ε^p}	0.0795	0.0105	0.0006	0.8070	0.7835	0.7506
σ_{ε^l}	0.1584	0.0370	0.0000	0.3895	0.3909	0.3583
$\sigma_{\varepsilon^\Psi}$	0.0058	0.0003	0.0000	0.1566	0.1995	0.2521
$\sigma_{\varepsilon^{g^s}}$	0.0333	0.0017	0.0000	0.1229	0.0868	0.0659
σ_{ε^π}	0.0011	0.0002	0.0000	0.3065	0.2557	0.2323
σ	1.1723	0.0418	0.0000	0.4398	0.4220	0.3782
φ	2.3524	0.4382	0.0000	0.4047	0.3570	0.3561
h	0.8402	0.0331	0.0000	0.7524	0.7516	0.7310
$100(\beta^{-1}-1)$	0.1687	0.0797	0.0000	0.1027	0.1037	0.0663
$\bar{\pi}_{ss}$	0.7145	0.3385	0.0000	0.1459	0.1240	0.1242
$100(g^z-1)$	0.3508	0.0437	0.8886	0.9892	0.9877	0.9866
κ_π	1.7367	0.1770	0.0000	0.0301	0.0155	0.0081

(Continued on next page)

Table B2: (continued)

	Posterior		p-values			
Parameter	*Mean*	*Std*	*No Taper*	4% *Taper*	8% *Taper*	15% *Taper*
κ_y	0.2391	0.0453	0.0000	0.3856	0.3896	0.4003
ρ_i	0.7642	0.0274	0.0000	0.2307	0.3054	0.3418
Γ_p	0.8337	0.0215	0.0002	0.8269	0.8024	0.7629
μ_p	0.2471	0.0975	0.0000	0.0961	0.0838	0.0568
Γ_w	0.8153	0.0493	0.0000	0.4520	0.4308	0.4062
μ_w	0.3161	0.1458	0.0000	0.1583	0.1291	0.1166
γ_I	9.4144	1.1290	0.9057	0.9924	0.9915	0.9904
σ_a	0.8844	0.0481	0.0000	0.5240	0.5560	0.5497
ρ^μ	0.5555	0.0686	0.0000	0.3768	0.3432	0.3348
ρ^r	0.3059	0.0617	0.0000	0.7046	0.7270	0.7181
ρ^p	0.9841	0.0064	0.0000	0.4358	0.4603	0.4485
η^p	0.3625	0.0686	0.0000	0.5989	0.5790	0.5659
ρ^l	0.1722	0.0636	0.0000	0.0203	0.0084	0.0003
ρ^Ψ	0.2472	0.0666	0.0000	0.5467	0.5078	0.4864
ρ^{g^s}	0.9609	0.0105	0.1021	0.9080	0.8946	0.8667

Table B3: Raftery/Lewis (1992) Convergence Diagnostics, based on quantile q=0.025 with precision r=0.005 with probability s=0.950.

$Variables$	$M(burn-in)$	$N(req.draws)$	$N+M(totaldraws)$	$k(thinning)$
σ_{ε^z}	56	60622	60678	1
σ_{ε^μ}	68	73755	73823	1
σ_{ε^r}	90	95568	95658	11
σ_{ε^p}	72	77938	78010	1
σ_{ε^l}	253	293418	293671	18
$\sigma_{\varepsilon^\Psi}$	65	70697	70762	1
$\sigma_{\varepsilon^{g^s}}$	83	85280	85363	8
σ_{ε^π}	91	98169	98260	1
σ	1040	1123633	1124673	43
φ	77	83752	83829	1
h	236	251685	251921	15
$100(\beta^{-1}-1)$	46	50096	50142	1
$\bar{\pi}_{ss}$	37	39767	39804	1

(Continued on next page)

Table B3: (continued)

$Variables$	$M(burn-in)$	$N(req.draws)$	$N+M(totaldraws)$	$k(thinning)$
$100(g^z-1)$	68	73880	73948	1
κ_π	54	58891	58945	1
κ_y	87	94815	94902	1
ρ_i	111	121169	121280	1
Γ_p	106	115155	115261	1
μ_p	53	57460	57513	1
Γ_w	77	83610	83687	1
μ_w	59	63990	64049	1
γ_I	105	109813	109918	11
σ_a	180	195344	195524	1
ρ^μ	375	411367	411742	1
ρ^r	81	87327	87408	1
ρ^p	109	118552	118661	1
η^p	64	69629	69693	1
ρ^l	53	56834	56887	1
ρ^Ψ	78	84465	84543	1
ρ^{g^s}	173	189324	189497	1
$Maximum$	1040	1123633	1124673	43

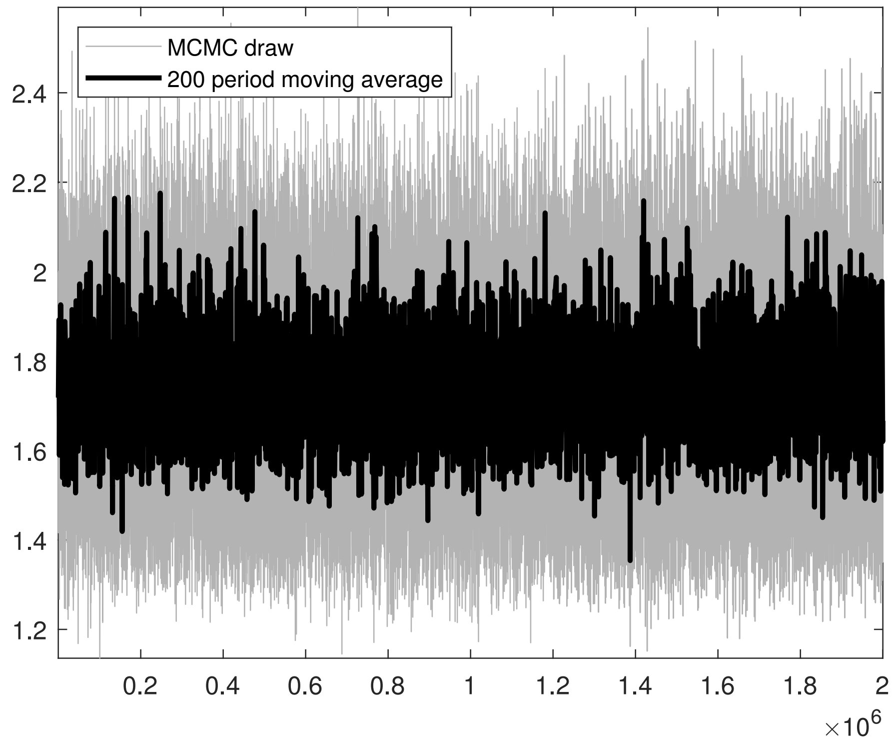


Figure B1: Trace plot for parameter κ_π

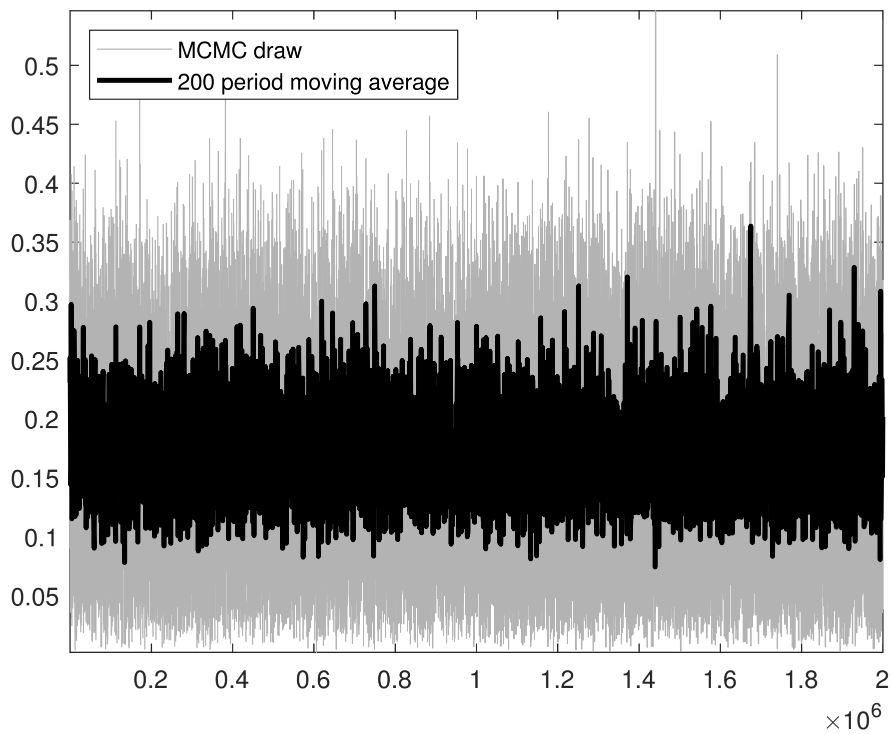


Figure B2: Trace plot for parameter ρ^l

C Theoretical DSGE model

C.1 Set of dynamic equations.

Tables from (C1) to (C3) summarize the system of dynamic equations:

	Descriptions	Equations
1)	Marginal utility of consumption	$\lambda_t = (C_t - hC_{t-1})^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi}\zeta_t^l L_t^{1+\varphi}\right),$
2)	Marginal rate of substitution	$MRS_t = (C_t - hC_{t-1})\left(\psi\zeta_t^l L_t^{\varphi}\right),$
3)	Euler equation from capital	$\frac{\lambda_t}{E_t\{\lambda_{t+1}\}} = \beta\left[E_t\frac{\{r_{t+1}^k\}}{Q_t} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right],$
4)	Euler equation	$i_t = (\pi_{t+1})\ \frac{\lambda_t}{\lambda_{t+1}\beta},$
5)	FOC variable capital utilization	$r_t^k = \gamma_1 + \gamma_2\left(U_t - 1\right),$
6)	Euler equation for investments	$1 = Q_t\mu_t^i\left[1 - \left(S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}}\right)\right)\right] + \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}\mu_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right],$
7)	Capital law of motion	$K_{t+1} = \mu_t^i\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t + (1-\delta)K_t,$
8)	Production bundle cost	$p_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)},$
9)	Incumbents' productivity threshold	$\hat{A}_t^{INC} = \left[\frac{w_t\phi^{INC} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^{\gamma}}{p_t},$
10)	New entrants' productivity threshold	$\hat{A}_t^{NE} = \left[\frac{w_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^{\gamma}}{p_t},$
11)	Discounted value of future dividends	$V_{t+1}^{av} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\hat{A}_{t+1}^{INC}\gamma}{\left(p_{t+1}^z\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi^{INC} + E_{t+1}\left[\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right],$
12)	Survival probability	$H_t = \left(\frac{\hat{A}_{t-1}^{INC}g^z\left(1-\delta^{INC}\right)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi},$

Table C1: List of dynamic equations

	Descriptions	Equations
13)	New entrants	$NE_t = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi},$
14)	Incumbents	$INC_t = \eta_{t-1} H_t,$
15)	Exit	$EX_t = \eta_{t-1}\left(1 - H_t\right),$
16)	Active firms	$\eta_t = NE_t + INC_t,$
17)	Share of entry	$entry_t = \frac{NE_t}{\eta_t},$
18)	Share of exit	$exit_t = \frac{EX_t}{\eta_t},$
19)	INT-output	$Y_t^{INT} = \frac{\xi\left\{\left[w_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]\right]NE_t + \left[w_t\phi^{INC} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]\right]INC_t\right\}}{p_t[\xi(1-\gamma)-1]},$
20)	Capital demand	$K_t = K_t^{INT} = \frac{\alpha\gamma}{r_t^k} p_t Y_t^{INT},$
21)	Labor demand	$L_t = L_t^{INT} = \frac{(1-\alpha)\gamma}{w_t} p_t Y_t^{INT} + NE_t\phi^{NE} + INC_t\phi^{INC},$
22)	TFP	$TFP_t = \frac{\xi}{\xi-1}\left(NE_t\hat{A}_t^{NE} + INC_t\hat{A}_t^{I}\right),$
23)	Average firms' efficiency	$\hat{A}_t^{av} = \frac{TFP_t}{\eta_t}$
24)	Efficiency dispersion	$\Sigma_t^A = \frac{\xi}{(\xi-2)\eta_t}\left[NE_t\left(\hat{A}_t^{NE}\right)^2 + INC_t\left(\hat{A}_t^{INC}\right)^2\right]$
25)	Solow Residual	$SR_t = \frac{Y_t}{\left[(K_t)^{\alpha}(L_t)^{1-\alpha}\right]^{\gamma}},$
26)	Set of Calvo price eq. (1)	$a_{1,t} = Y_t\,\left(\Pi_t^{C*}\right) + \beta\,\Gamma_p\,\frac{\Pi_t^{C*}}{\Pi_{t+1}^{C*}}\,\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{1-\epsilon_t^p}\,\frac{\lambda_{t+1}}{\lambda_t}\,a_{1,t+1},$
27)	Set of Calvo price eq. (2)	$a_{2,t} = \tilde{P}_t\,Y_t + \frac{\lambda_{t+1}}{\lambda_t}\,\beta\,\Gamma_p\,\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{\left(-\epsilon_t^p\right)}\,a_{2,t+1},$
28)	Set of Calvo price eq. (3)	$a_{1,t} = \frac{\epsilon_t^p\,a_{2,t}}{\epsilon_t^p - 1},$
29)	Set of Calvo price eq. (4)	$1 = (1-\Gamma_p)\,\left(\Pi_t^{C*}\right)^{1-\epsilon_t^p} + \Gamma_p\,\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{1-\epsilon_t^p},$
30)	Set of Calvo price eq. (5)	$\xi_t^p = (1-\Gamma_p)\,\left(\Pi_t^{C*}\right)^{\left(-\epsilon_t^p\right)} + \Gamma_p\,\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{\left(-\,\epsilon_t^p\right)}\,\xi_{t-1}^p,$

Table C2: List of dynamic equations

	Descriptions	Equations
31)	Set of Calvo wages eq. (1)	$a^w_{1,t} = \lambda_t w_t^{\epsilon^w} L_t + \beta\Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{\epsilon^w-1} a^w_{1,t+1},$
32)	Set of Calvo wages eq. (2)	$a^w_{2,t} = \varphi w_t^{(1+\theta)\epsilon^w} L_t^{1+\theta} + \beta\ \Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{(1+\theta)\epsilon^w} a^w_{2,t+1},$
33)	Set of Calvo wages eq. (3)	$(w_t^{\#})^{1+\epsilon^w\theta} = \frac{\epsilon^w}{\epsilon^w-1}\frac{a^w_{2,t}}{a^w_{1,t}},$
34)	Set of Calvo wages eq. (4)	$w_t^{1-\epsilon^w} = (1-\Gamma_w)\left(w_t^{\#}\right)^{1-\ \epsilon^w} + \Gamma_w\left(w_{t-1}\frac{\pi_{t-1}^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_t}\right)^{1-\epsilon^w},$
35)	Monetary policy rule	$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R^n_{t-1}}{R^n_{ss}}\right)^{\rho_i}\left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\kappa^\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i}\zeta_t^r,$
36)	Aggregate resources constraint	$Y_t = \frac{Y_t^{INT}}{\xi_t^p} = C_t + I_t + g_t^S Y,$
37)	Technology frontier evolution (NEs)	$z_t = g_t^z\, z_{t-1},$
38)	Shock to NEs' technology	$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z\ln(g_{t-1}^z) + \varepsilon_t^z,$
39)	Shock to INCs' technology	$\ln(\Psi_t) = \rho^\Psi\ln(\Psi_{t-1}) + \varepsilon_t^\Psi,$
40)	Shock to inflation target	$\ln(\overline{\pi}_t) = \ln(1-\rho^\pi)\,\overline{\pi}_{ss} + \rho^\pi\ln(\overline{\pi}_{t-1}) + \varepsilon_t^\pi,$
41)	Shock to monetary policy	$\ln(\zeta_t^r) = \rho^r\ln(\zeta_{t-1}^r) + \varepsilon_t^r,$
42)	Shock to labot supply	$\ln(\zeta_t^l) = \rho^l\ln(\zeta_{t-1}^l) + \varepsilon_t^l,$
43)	Shock to MEI	$\ln(\mu_t) = \rho^\mu\ln(\mu_{t-1}) + \varepsilon_t^\mu,$
44)	Shock to public expenditure	$\ln(g_t^S) = \rho^{g^s}\ln(g_{t-1}^S) + \varepsilon_t^{g^s}.$

Table C3: List of dynamic equations

C.2 The de-trended model.

The model economy follows a Balanced Growth Path (BGP). Output Y_t, consumption C_t, capital K_t , investment I_t and wage w_t grow at the endogenous rate g^t; Further, the technology frontier z_t and the technology thresholds $\hat{A}_t^j$ grow at the exogenous rate g_z^t. The remaining variables are stationary. In order to compute the deterministic steady state and the de-trended model, we have to identify the relation that binds the different growth rates.

C.2.1 Households.

We can start our computation from the Households first order conditions. Since we know that C grows at the same rate of Y, we can show that the Lagrangian multiplier s.s. follows this path:

$$\widetilde{\lambda}_t = \frac{\lambda_t}{g_t} = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi}L_t^{1+\varphi}\right)$$

$$\widetilde{MRS}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)(\psi L_t^{\varphi})$$

From the Households Euler conditions, we can find the de-trended Euler equations on capital:

$$\frac{\lambda_t}{E_t\{\lambda_{t+1}\}} = \beta\left[E_t\frac{\{r_{t+1}^k\}}{Q_t} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]$$

$$\frac{\widetilde{\lambda}_t g_{t+1}}{\beta\, E_t\left\{\widetilde{\lambda}_{t+1}\right\}} = \left(\frac{Q_{t+1}r_{t+1}^k}{Q_t} + \frac{Q_{t+1}}{Q_t}(1-\ \delta)\right)$$

C.2.2 INT-firms.

Once that we have defined the costs of production we can compute the productivity thresholds:

$$\hat{A}_t^j = \left[\frac{w_t\phi^j - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^{\gamma}}{p_t}$$

This implies:

$$\hat{A}_t^j = \left[\frac{g_t w_t\phi^j - E_t\left\{g_t\frac{\widetilde{\Lambda}_{t+1}}{g_{t+1}}H_{t+1}\widetilde{V}_{t+1}^{av,s}g_{t+1}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma_s}\cdot\left[\frac{\left[\frac{\widetilde{r}_t^k}{\alpha}\right]^{\alpha}\left[\frac{g_t\widetilde{w}_t}{(1-\alpha)}\right]^{(1-\alpha)}}{p_t^{\frac{1}{\gamma}}}\right]^{\gamma}$$

$$\hat{A}_t^j = g_t^{1-\alpha\gamma} \left[\frac{w_t \phi^j - E_t \left[\widetilde{\Lambda}_{t+1} H_{t+1} \widetilde{V}_{t+1}^{av} \right\}}{\gamma^{\frac{\gamma}{1-\gamma}} (1-\gamma)} \right]^{1-\gamma} \frac{(\tilde{p}_t^z)^\gamma}{p_t}$$

$$\widetilde{\hat{A}_t^{NE,INC}} = \left[\frac{\tilde{w}_t \phi^j - E_t \left\{ \widetilde{\Lambda}_{t+1} H_{t+1} \widetilde{V}_{t+1}^{av} \right\}}{\gamma^{\frac{\gamma}{1-\gamma}} (1-\gamma)} \right]^{1-\gamma} \frac{(\tilde{p}_t^z)^\gamma}{p_t}$$

Since the number of firms is assumed to be stationary it will follow that $g_t^{1-\alpha\gamma} = g_t^z$

$$\eta_t = \left(\frac{z}{\widetilde{\hat{A}_t^{NE}}} \right)^{\xi_S} + \eta_{t-1} \left(\frac{\widetilde{\hat{A}_{t-1}^I} g^z \left(1 - \delta^{INC}\right) \Psi_t}{g_t^z \widetilde{\hat{A}_t^I}} \right)^{\xi_S}$$

The remaining de-trendization is straightforward.

C.2.3 Set of de-trended equations.

Tables from (C4) to (C6) summarize the system of de-trended equations:

	Descriptions	Equations
1)	Marginal utility of consumption	$\widetilde{\lambda}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi} L_t^{1+\varphi}\right),$
2)	Marginal rate of substitution	$\widetilde{MRS}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)\left(\psi L_t^{\varphi}\right),$
3)	Euler equation from capital	$\frac{\widetilde{\lambda}_t g_{t+1}}{\beta\, E_t\left\{\widetilde{\lambda}_{t+1}\right\}} = \left(\frac{Q_{t+1} r_{t+1}^k}{Q_t} + \frac{Q_{t+1}}{Q_t}\left(1-\ \delta\right)\right),$
4)	Euler equation	$i_t = \frac{\pi_{t+1}}{\beta}\frac{g_{t+1}\widetilde{\lambda}_t}{\widetilde{\lambda}_{t+1}},$
5)	FOC variable capital utilization	$r_t^k = \gamma_1 + \gamma_2\left(U_t - 1\right),$
6)	Euler equation for investments	$1 = Q_t\mu_t^i\left[1-\left(S'\left(\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}}\right)\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}} + S\left(\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}}\right)\right)\right] + \beta E_t\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t g_{t+1}} Q_{t+1}\mu_{t+1}^i S'\left(\frac{\widetilde{I}_{t+1} g_{t+1}}{\widetilde{I}_t}\right)\left(\frac{\widetilde{I}_{t+1} g_{t+1}}{\widetilde{I}_t}\right)^2\right],$
7)	Capital law of motion	$\widetilde{K}_{t+1} = \mu_t^i\left(1 - S\left(\frac{\widetilde{I}_t g_t^K}{\widetilde{I}_{t-1}}\right)\right)\widetilde{I}_t + \frac{(1-\delta)\widetilde{K}_t}{g_t^K},$
8)	Production bundle cost	$\widetilde{p}_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{w}_t}{(1-\alpha)}\right]^{(1-\alpha)},$
9)	Incumbents' productivity threshold	$\widetilde{\hat{A}}_t^{INC} = \left[\frac{\tilde{w}_t\phi^{INC} - E_t\left\{\widetilde{\Lambda}_{t+1} H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\tilde{p}_t^z)^{\gamma}}{p_t},$
10)	New entrants' productivity threshold	$\widetilde{\hat{A}}_t^{NE} = \left[\frac{\tilde{w}_t\phi^{NE} - E_t\left\{\widetilde{\Lambda}_{t+1} H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\tilde{p}_t^z)^{\gamma}}{p_t},$
11)	Discounted value of future dividends	$\widetilde{V}_{t+1}^{av} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\widetilde{\hat{A}}_{t+1}^{INC}\ \gamma}{\left(\widetilde{p}_{t+1}^z\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - \widetilde{w}_{t+1}\phi^{INC} + E_{t+1}\left[\widetilde{\Lambda}_{t+2} H_{t+2}\widetilde{V}_{t+2}^{av}\right],$
12)	Survival probability	$H_t = \left(\frac{\widetilde{\hat{A}}_{t-1}^{INC}\ g^z\left(1-\delta^{INC}\right)\Psi_t}{\widetilde{\hat{A}}_t^{INC}\ \ g_t^z}\right)^{\xi},$

Table C4: List of de-trended equations

	Descriptions	Equations
13)	New entrants	$NE_t = \left(\frac{z}{\widetilde{\hat{A}}_t^{NE}}\right)^{\xi}$,
14)	Incumbents	$INC_t = \eta_{t-1}H_t$,
15)	Exit	$EX_t = \eta_{t-1}\left(1 - H_t\right)$,
16)	Active firms	$\eta_t = NE_t + INC_t$,
17)	Share of entry	$entry_t = \frac{NE_t}{\eta_t}$,
18)	Share of exit	$exit_t = \frac{EX_t}{\eta_t}$,
19)	INT-output	$\widetilde{Y}_t^{INT} = \frac{\xi\left\{\left[\widetilde{w}_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}\widetilde{V}^{av}_{t+1}\right\}\right]NE_t + \left[\widetilde{w}_t\phi^{INC} - E_t\left\{\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}^{av}_{t+1}\right\}\right]INC_t\right\}}{p_t[\xi(1-\gamma)-1]}$,
20)	Capital demand	$\widetilde{K}_t = \widetilde{K}_t^{INT} = \frac{\alpha\gamma}{r_t^k}p_t\widetilde{Y}_t^{INT}$,
21)	Labor demand	$L_t = L_t^{INT} = \frac{(1-\alpha)\gamma}{\widetilde{w}_t}p_t\widetilde{Y}_t^{INT} + NE_t\phi^{NE} + INC_t\phi^{INC}$,
22)	Average productivity	$\widetilde{TFP}_t = \frac{\xi}{\xi-1}\left(NE_t\widetilde{\hat{A}}_t^{NE} + INC_t\widetilde{\hat{A}}_t^{I}\right)$,
23)	Average firms' efficiency	$\widetilde{\hat{A}}_t^{av} = \frac{\widetilde{TFP}_t}{\eta_t}$
24)	Efficiency dispersion	$\widetilde{\Sigma^A}_t = \frac{\xi}{(\xi-2)\eta_t}\left[NE_t\left(\widetilde{\hat{A}}_t^{NE}\right)^2 + INC_t\left(\widetilde{\hat{A}}_t^{INC}\right)^2\right]$
25)	Solow Residual	$\widetilde{SR}_t = \frac{\widetilde{Y}_t}{\left[\left(\widetilde{K}_t\right)^{\alpha}(L_t)^{1-\alpha}\right]^{\gamma}}$,
26)	Set of Calvo price eq. (1)	$\widetilde{a}_{1,t} = \widetilde{Y}_t\left(\Pi_t^{C*}\right) + \beta\,\Gamma_p\frac{\Pi_t^{C*}}{\Pi_{t+1}^{C*}}\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{1-\epsilon_t^p}\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\widetilde{a}_{1,t+1}$,
27)	Set of Calvo price eq. (2)	$\widetilde{a}_{2,t} = \tilde{P}_t\widetilde{Y}_t + \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\beta\,\Gamma_p\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{(-\epsilon_t^p)}\widetilde{a}_{2,t+1}$,
28)	Set of Calvo price eq. (3)	$\widetilde{a}_{1,t} = \frac{\epsilon_t^p\widetilde{a}_{2,t}}{\epsilon_t^p-1}$,
29)	Set of Calvo price eq. (4)	$1 = (1-\Gamma_p)\left(\Pi_t^{C*}\right)^{1-\epsilon_t^p} + \Gamma_p\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{1-\epsilon_t^p}$,
30)	Set of Calvo price eq. (5)	$\xi_t^p = (1-\gamma)\left(\Pi_t^{C*}\right)^{(-\epsilon_t^p)} + \gamma\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{(-\epsilon_t^p)}\xi_{t-1}^p$,

	Descriptions	Equations
31)	Set of Calvo wages eq. (1)	$\widetilde{a}^w_{1,t} = \widetilde{\lambda}_t \widetilde{w}_t^{\epsilon^w} L_t + \beta\Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{\epsilon^w-1} \widetilde{a}^w_{1,t+1},$
32)	Set of Calvo wages eq. (2)	$\widetilde{a}^w_{2,t} = \varphi \widetilde{w}_t^{(1+\theta)v_w} L_t^{1+\theta} + \beta\ \Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{(1+\theta)\epsilon^w} \widetilde{a}^w_{2,t+1},$
33)	Set of Calvo wages eq. (3)	$(\widetilde{w}_t^{\#})^{1+\epsilon^w\theta} = \frac{\epsilon^w}{\epsilon^w-1}\frac{\widetilde{a}^w_{2,t}}{\widetilde{a}^w_{1,t}},$
34)	Set of Calvo wages eq. (4)	$\widetilde{w}_t^{1-\epsilon^w} = (1-\Gamma_w)\left(\widehat{w}_t^{\#}\right)^{1-\ \epsilon^w} + \Gamma_w\left(\frac{\widetilde{w}_{t-1}\pi_{t-1}^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{g_t\pi_t}\right)^{1-\epsilon^w},$
35)	Monetary policy rule	$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R_{t-1}^n}{R_{ss}^n}\right)^{\rho_i}\left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\kappa^\pi}\left(\frac{\widetilde{Y}_t}{g_t\widetilde{Y}_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i}\zeta_t^r,$
36)	Aggregate resources constraint	$\widetilde{Y}_t = \frac{\widetilde{Y}_t^{INT}}{\xi_t^p} = \widetilde{C}_t + \widetilde{I}_t + g_t^S Y,$
37)	Growth rates	$g_t = (g_t^z)^{\frac{1}{1-\alpha\gamma}},$
38)	Shock to NEs' technology	$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z\ln(g_{t-1}^z) + \varepsilon_t^z,$
39)	Shock to INCs' technology	$\ln(\Psi_t) = \rho^\Psi\ln(\Psi_{t-1}) + \varepsilon_t^\Psi,$
40)	Shock to inflation target	$\ln(\overline{\pi}_t) = \ln(1-\rho^\pi)\overline{\pi}_{ss} + \rho^\pi\ln(\overline{\pi}_{t-1}) + \varepsilon_t^\pi,$
41)	Shock to monetary policy	$\ln(\zeta_t^r) = \rho^r\ln(\zeta_{t-1}^r) + \varepsilon_t^r,$
42)	Shock to labor supply	$\ln(\zeta_t^l) = \rho^l\ln(\zeta_{t-1}^l) + \varepsilon_t^l,$
43)	Shock to MEI	$\ln(\mu_t) = \rho^\mu\ln(\mu_{t-1}) + \varepsilon_t^\mu,$
44)	Shock to public expenditure	$\ln(g_t^S) = \rho^{g^s}\ln(g_{t-1}^S) + \varepsilon_t^{g^s}.$

Table C6: List of de-trended equations

C.3 Key derivations.

C.3.1 Equation (25) - Firms' continuation value.

We start from the (12) and (13) to get

$$
\begin{aligned}
V_{t+1}^{av} &= E_t\left\{V\left(A_{t+1}^{j}\right)\right\} = \\
&= \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} V_{t+1}\left(A_{t+1}^{INC,j}\right) f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) = H_{t+1}V_{t+1}^{av} \\
&= \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} (1-\gamma)\left[A_{t+1}^{f,j}\frac{p_{t+1}\gamma^{\gamma}}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}} f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}-\xi-1} d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}-\xi-1} d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\hat{A}_{t+1}^{INC}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi_{t+1}^{INC} + E_{t+1}\left\{\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right\}.
\end{aligned}
$$

C.3.2 Equation (26) - Productivity thresholds.

Also in this case, we get the condition from (12) and (13)

$$
\begin{aligned}
V_t\left(\hat{A}_t^{j}\right) &= (1-\gamma)\left[\hat{A}_t^{j}\frac{p_t\gamma^{\gamma}}{\left(p_t^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_t\phi^{j} + E_t\left\{\Lambda_{t+1}V_{t+1}^{av}\right] = 0 \\
& (1-\gamma)\left[\hat{A}_t^{j}\frac{p_t\gamma^{\gamma}}{\left(p_t^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} = w_t\phi^{j} - E_t\left\{\Lambda_{t+1}V_{t+1}^{av}\right] \\
\hat{A}_t^{j} &= \left[\frac{w_t\phi^{j} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{(1-\gamma)\,p_t}\right]^{1-\gamma}\left[\frac{p_t^{z}}{p_t\gamma}\right]^{\gamma}
\end{aligned}
$$

C.3.3 Equation (32) - Aggregate INT-output.

We start from the idiosyncratic production function.

$$
y_t^{f,j} = A_t^{f,j}\left[(k_t^{f,j})^{\alpha}(l_t^{f,j})^{(1-\alpha)}\right]^{\gamma}
$$

Aggregating for NEs idiosyncratic productivity we get

$$\begin{aligned}
Y_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE}\left[\left(k_t^{f,NE}\right)^{\alpha}\left(l_t^{f,NE}\right)^{1-\alpha}\right]^{\gamma} dF\left(A_t^{f,NE}\right)\\
&= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE}\left[\frac{p_t}{p_t^z}A_t^{f,NE}\gamma\right]^{\frac{\gamma}{1-\gamma}} dF\left(A_t^{f,NE}\right)\\
&= \left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{NE}}^{+\infty}\left(A_t^{f,NE}\right)^{\frac{1}{1-\gamma}} dF\left(A_t^{f,NE}\right)\\
&= \xi z_t^{\xi}\left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{NE}}^{+\infty}\left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}-\xi-1} d\left(A_t^{f,NE}\right)\\
&= \frac{\xi\left(1-\gamma\right)}{\xi(1-\gamma)-1}NE_t\left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}},
\end{aligned}$$

and, aggregating for INCs idiosyncratic productivity we get

$$\begin{aligned}
Y_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC}\left[\left(k_t^{f,INC}\right)^{\alpha}\left(l_t^{f,INC}\right)^{1-\alpha}\right]^{\gamma} dF\left(A_t^{f,INC}\right)\\
&= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC}\left[\frac{p_t}{p_t^z}A_t^{f,INC}\gamma\right]^{\frac{\gamma}{1-\gamma}} dF\left(A_t^{f,INC}\right)\\
&= \left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{INC}}^{+\infty}\left(A_t^{f,INC}\right)^{\frac{1}{1-\gamma}} dF\left(A_t^{f,INC}\right)\\
&= \xi\left[\hat{A}_{t-1}^{INC}g_z\left(1-\delta\right)\Psi_t\right]^{\xi}\left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{INC}}^{+\infty}\left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\gamma}-\xi-1} d\left(A_t^{f,INC}\right)\\
&= \frac{\xi\left(1-\gamma\right)}{\xi(1-\gamma)-1}INC_t\left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}}.
\end{aligned}$$

C.3.4 Elasticity of productivity thresholds.

In order to measure the sensitivity of the two productivity thresholds to their different components, we compute the first-order approximation of equation (26):

$$\widetilde{\hat{A}}_t^{j} = \left[\frac{\widetilde{w}_t\phi^j - E_t\left\{\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}\left(1-\gamma\right)}\right]^{1-\gamma}\frac{(\widetilde{p}_t^z)^{\gamma}}{p_t}.$$

In the detrended steady state,

$$\widetilde{\hat{A}}^{j} = \left[\frac{\widetilde{w}\phi^j - H\widetilde{V}^{av}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\widetilde{p}^z)^{\gamma}}{p}.$$

Log-linearizing,[24]

$$\hat{a}_t^j = \frac{1-\gamma}{w\phi^j - HV^{av}}\left[\phi^j \hat{w}_t - HV^{av}(\hat{\Lambda}_{t+1} + \hat{H}_{t+1} + \hat{V}_{t+1}^{av})\right] - (\hat{p}_t - \gamma \hat{p}_t^z).$$

From our calibration, $\phi^{NE} < \phi^{INC}$ implies that the sensitivity of the thresholds to the approximated wedge between participation costs and expected future profits is larger for NEs. On the other hand, sensitivity to the price/cost margin is the same for each type of firm.

[24]Where $\hat{x}_t$ is the detrended log-deviation of the generic variable x_t. For the sake of a clear notation the log-deviation of $\hat{A}_t^j$ is labeled by $\hat{a}_t^j$.