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Citizens' Protests: causes and consequences*

A Research on Regime Change and Revolutionary Entrepreneurs

by Bueno De Mesquita

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Abstract

Citizens' political participation to protests is a crucial issue for any political system, whether democratic or autocratic. Political systems have different ways of dealing with citizens' protests, determining cost and benefit of public dissent, responding to public requests and allowing different degree of transparency in public information. Also the social characteristics of a country, such as citizens' diversity and radicalization, matter for citizens' political participation. The aim of this paper is to analyze causes and consequences of citizens' protests, focusing on how private and public information affect citizens' opinion and political behavior, and on how they depend on sociopolitical factors as well as on the political regime. In *Regime Change and Revolutionary Entrepreneurs,* Bueno de Mesquita proposed a seminal model to study why revolutionary vanguards might use violence to mobilize citizens against a regime. We claim that the model can be used more generally to investigate citizens' protest. We refer to his model to understand citizens' political behavior, studying the relationship between the model's structural parameters and the causes and consequences of citizens' protests, adopting a partially different approach and extending his results.

Keywords: protests, political regimes, sociopolitical variables
JEL Codes: C72, D74, P48

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1 Introduction

Why is the world protesting so much? The last fifteen years have witnessed a significant increase in mass protest: the number of large protests and demonstrations globally has risen by 36% since the global financial crisis in 2008-2009, from an average of 355 per year in the decade to 2009 to 482 per year in the decade following the Global Financial Crisis.[1] In particular, increases in large protests[2] have been notable across all over the world, from Europe to the Middle East, from Africa to South-America, from Asia to North-America. For example, the number of large protests in Europe increased by 71%, averaging 92 annually in the period 2000-2009 and 157 in 2010-2019. Average annual figures for the Middle East and North Africa region increased by 229 % (22 to 72 per annum), while those for sub-Saharan Africa increased by 48% (59 to 88 per annum).[3] In general, mass protests have characterized countries as different as Bolivia, Brazil, Chile, Ecuador, Guinea, Haiti, Honduras, Hong Kong, India, Iraq, Kazakhstan, Lebanon, Pakistan, United States, often but not always leading to a reverse of government policies.

An important aspect of this stylized facts is that mass protest involves all kind of political regimes, from the most democratic to highest authoritarian. Are we in a historic age of protest? Probably not since the wave of "people power" movements swept Asian and east European countries in the late 1980s and early 1990s has the world experienced such a simultaneous outpouring of popular anger on the streets. Before that, only the global unrest of the late 1960s bears comparison in terms of the number of countries and of the number of people mobilized. However, those two waves of global unrest seemed more joined-up than the present spate of apparently unconnected and spontaneous movements. Protesters in many different countries had similar grievances and aims. This time, some themes inevitably crop up in country after country. A study[4] by a team of researchers of the Friedrich-Ebert-Stiftung (FES) and the Initiative for Policy Dialogue, a nonprofit organization based at Columbia University, looking at more than 900 protest movements or episodes across 101 countries and territories, concludes that we are living through a period of history like the years around 1848, 1917 or 1968 when large numbers of people rebelled against the way things were, demanding change.

But why? Why do protests spread globally at particular points in history? What accounts for the important differences that we find between similar protests movements in different political and social contexts? What explains the shift in strategies, aims, and organizational forms that we regularly observe over the course of protest waves? Is the search for a unifying theory pointless? After all, when you look more closely at the earlier waves, the impression of coherence might seem illusory. They too were more variegated than is often assumed. The global upheavals of the late 1960s ranged from Red Guards in

[1] See the study by (re)insurance group Chaucer https://www.chaucergroup.com/.
[2] Large protest are defined as a protest with at least 100 participants.
[3] Data by Binghampton University, NY.
[4] Ortiz *et al.* 2022.

China, to affluent Western youths who had stumbled on the joys of life without strict old traditional rules. In between were protesters against the Vietnam war, the Soviet domination of eastern Europe and the tedious traditional lectures at universities. Even the people-power revolutions of 20 years later were as marked by their differences as their similarities. Right-wing strongmen such as the Philippines' Ferdinand Marcos or South Korea's Chun Doo-Hwan were a far cry from east European thugs such as Nicolae Ceausescu and Wojciech Jaruzelski.

The difficulty in discerning a pattern has not stopped scholars from trying. In general, on one hand, multiple empirical studies have disproved the common assumption that the greater the grievances, the more likely people will engage in political protests, on the other hand, different theoretical school have emphasized the role of different set of factors such as: the perception and interpretation of grievances and their causes, the expected impact of protest, i.e. the government responsiveness, the commonalities of the protesters, thus the country political diversity and radicalization, the expected benefit from success with respect to the cost from protesting, the context structure including political, economic, cultural opportunities and restrictions on protest, and, more generally, the social, political and cultural characteristics of a country.[5] We believe the answer is to go back to first principles and analyze what makes people take their grievances to the streets. Three reasons seem basic. First, for all its legal and physical dangers, successful protest can be more exciting than the drudgery of daily life. Second, when everybody else is doing it, solidarity becomes the fashion and makes protests effective. The third reason for demonstrating is that, in many situations, using conventional political channels may seem useless. In the protests of the late 1980s, the targets were usually autocratic governments that allowed at best sham elections. In the absence of the ballot box, the street was the only way to demonstrate "people power". Some of these years protests - for example against Abdelaziz Bouteflika in Algeria or Omar al-Bashir in Sudan - have been analogous. But also apparently well-functioning liberal democracies have been affected by large protests. The point is that, for a number of reasons, people may be feeling unusually powerless these days, believing that their votes do not matter and that the government is not accountable.

Whatever the reasons, citizens' political participation to protests is a crucial issue for any political system, whether democratic or autocratic. Of course, the ways, the costs and the effects differ in different political regimes, and actually the government answer to citizens' protests define a crucial characteristic of a polity. But, democratic or autocratic, all polities have some form of public involvement in the political process, if only to accept public policies. And all political systems have different ways of dealing with citizens' participation to political behavior, determining cost and benefit of public dissent, responsiveness to public requests and transparency in public information.

In this work, we are interested in citizens' political protests, i.e. in the

[5] See e.g. Dalton and van Sickle 2005, Finkel *et al.* 1989, Muller 1986 and Van Aelst and Walgrave 2001.

deliberate and public expression of dissent towards a government policy with the intent of influencing a political decision that a group of citizens perceive as having negative consequences for themselves or for their vision of the public good. Protests can refer to any political and social issue that regards the citizens as a collectivity, whether it is a specific policy or a political regime as a whole. The aim of the protest can be narrow or broad, reformist or revolutionary. The forms of protests, too, include a broad range of activities, from writing a petition, to attending a march, from blocking traffic to injuring or even killing people. The aim of this paper is to analyze causes and consequences of citizens' protests. In particular, we want to analyze when and why unsatisfied citizens are able to overcome the collective action problem, protesting against a government and possibly inducing changes in government policy. Specifically, this analysis will focus on how private and public information affect citizens' opinion and political behavior, and how these effects depend on sociopolitical factors as well on the political governance system.

In "*Regime Change and Revolutionary Entrepreneurs*" Bueno de Mesquita proposed an important model to study why revolutionary vanguards might use violence to mobilize a mass of citizens against a regime. His analysis in particular focused on how vanguard's violence, affecting the population sentiment on a regime, may help its overthrown. In the paper Bueno de Mesquita proposes a particular global game with one-sided limit dominance that, differently from the usual global games with two-sided-limit dominance, have multiple equilibria, arguing for selecting one of these equilibria. The selected equilibrium has three possible probabilistic outcomes relative to citizens' protests: one where there is no mobilization, one with insufficient mobilization, and one with successful mobilization and thus protest. We claim that the Bueno de Mesquita model can be used more generally to investigate citizens' behavior within different political regimes and different countries.[6] In particular, the aim of this work is to use and revise the Bueno de Mesquita model to deepen the understanding of causes and consequences of citizens' political behavior, studying the relationship between the model's structural parameters and causes and consequences of citizens' protests, to better understanding the basic principles of how citizens function within the political process across different political systems. Thus, in this paper, we review and revise the model, adopting a new approach to derive some of the paper's main results, discussing the interpretation of the exogenous variables of the model, extending and correcting some of Bueno de Mesquita results.

The paper is organized as follows: in section 2 we present and discuss the model and the interpretation of the exogenous variables, section 3 reviews the properties of the citizens' beliefs, i.e. of a country public opinion, while section 4 derives citizens' behavior. Section 5 derives the possible equilibrium outcomes and discuss the relationships between the sociopolitical variables and the outcome probabilities, while section 6 concludes. All the results and their proofs

[6] As argued in Rubistein 2001, an economic model differs from a purely mathematical model in that it is a combination of mathematical structures and interpretation. Thus, interpretation is a substantial part of an economic model.

are in appendix A, in appendix B are the simulations, while in the main text we report the most relevant results and their interpretations: this should allow a better understanding of the analysis of citizens' political behavior.

2 The Model

We start reviewing Bueno de Mesquita 2010 model, summing up the main building blocks, limiting the model to the protest's stage. We use Bueno de Mesquita notation to simplify the comparison with the original paper, however we will provide a more general interpretation of the variables, so that the old notation might sometimes sound strange.

2.1 The Players

There is a continuum of population members of a given country, the citizens $i \in [0, 1]$.

2.2 The Set of Actions

Each player $i \in [0, 1]$ can decide whether to attack, $a_i = 1$, or not, $a_i = o$. Thus, the set of possible choices for each player is dychotomous: $A_i = \{0, 1\}$

2.3 The Conflict Techonology and the Possible Outcomes

The conflict technology is such that there is a change if and only if enough players attack. Let denote by $R = 0$ the outcome of no change, and $R = 1$ when there is a change. Thus, formally the conflict tecnology is discontinuous in this way:

$$\mathbb{P}\{R|\mathcal{N}\} = \begin{cases} 0 & \text{if } \mathcal{N} < T \\ 1 & \text{if } \mathcal{N} \geq T \end{cases}$$

where $\mathcal{N}$ is the mass of attacking players

$$\mathcal{N} = \int_0^1 a_i di$$

and T is the threshold to obtain a change.

2.4 Players' Payoffs

To conclude the description of the game, we need to define the players' payoffs that, as usual, are defined on the possible outcomes, $R = 0$ and $R = 1$:

	$R = 0 \Leftrightarrow \mathcal{N} < T$	$R = 1 \Leftrightarrow \mathcal{N} \geq T$
$a_i = 0$	0	θ_i
$a_i = 1$	$-k$	$(1 + \gamma)\,\theta_i - k$

where k is the private cost for a citizen to protests, while γ is the advantage of being part of the protests when the protest is successful. In the original Bueno de Mesquita 2010 paper the payoff matrix is

	$\mathcal{N} < T \Leftrightarrow R = 0$	$\mathcal{N} \geq T \Leftrightarrow R = 1$
$a_i = 0$	0	$(1-\gamma)\,\theta_i$
$a_i = 1$	$-k$	$\theta_i - k$

however we prefer our notation, which is strategically equivalent to Bueno de Mesquita,[7] because it emphasizes that the model's results require an intrinsic utility from participating to a successful protest. Actually, γ can be interpreted as a selective incentive to overcome collective action problems.[8]

Hypothesis 1 $\gamma \in (0,1]\,, \;\; k \in [0,\infty)\,.$

Remark 1 *Note that $\gamma \neq 0$ is crucial, otherwise $a_i = 1$ is always a strictly dominated action, and the model would be trivial.*

2.5 The Information Structure

The country is characterized by a common sentiment towards the government policy, $\theta \in \mathbb{R}$, where $\theta < 0$ means a support for the policy, vice-versa $\theta > 0$. The true θ is unknown by the citizens: each citizen i receives a **private signal**

$$\theta_i = \theta + \varepsilon_i,$$

while all the citizens receive a **public signal**

$$v = \theta + t + \eta$$

where t is a parameter not observed by the citizens. All other variables are common knowledge.

Remark 2 *The relationship between v, t, θ and η means that the higher the country's sentiment towards the government θ and the higher the parameter t, the higher the value of the observed v, however these variables cumulate and have a noise η so that θ and t can't be extrapolated from the realization of the random variable v : t and θ are confounded in the observed public signal v. This is a classic example of what Fudenberg and Tirole 1986 call "Signal-Jamming", in the sense that t is a variable that interferes with the citizens' inference on θ, given the private and the public signal (θ_i, v). Examples of signal-jamming by nature is Fudenberg and Tirole 1986, while in Holmstroem 1999, the variable is strategically used by a player.*

[7] It is immediate to see that the best reply correspondences coincide for the two payoff structures.

[8] Apolte 2012, Lichbach 1995; Olson 1965; Popkin 1979; Tullock 1971, 1974.

Consider the following assumptions on these signals, where in general $N(\mu, \sigma^2)$ is the Normal distribution with expected value μ and variance σ^2.

Hypothesis 2

$$\theta \sim N\left(0, \sigma_\theta^2\right), \ \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right), \ \eta \sim N\left(0, \sigma_\eta^2\right), \quad t \in [0, \infty).^9$$

Hypothesis 3 *θ, ε_i and η are independent.*

These assumptions imply

$$\theta_i \sim N\left(0, \sigma_\theta^2 + \sigma_\varepsilon^2\right).$$

Remark 3 *Since θ_i does not provide any information on t, we can denote by t^* the common expectation of t by any citizen $i \in [0, 1]$. Then*

$$v - t^* = \theta + (t - t^*) + \eta$$

*is the **unexpected component of the public signal,** which ex ante is expected to be 0, since $E[t - t^*] = 0$, however ex post its realization can be greater or smaller than expected, depending on the realization of the random variables, so that citizen i can not distinguish whether a big public signal is due to exceptional political activism or to big anti-government sentiment. For this reason, the following analysis will be in terms of $v - t^*$, which is distributed as follows*

$$v - t^* \sim N\left(0, \sigma_\theta^2 + \sigma_\eta^2\right).$$

2.6 Timing and Choices

The timing of players' strategic interaction is as follows.

1. **Nature choices:** nature choose the random variables θ, η, ε_i and the parameter $t \in [0, \infty)$;
2. **Citizens' information:** each citizen i observes a private signal $\theta_i = \theta + \varepsilon_i$ and a public signal $v = t + \theta + \eta$;
3. **Protest stage:** each citizen i decides whether to join the protest, $a_i = 1$, or not, $a_i = 0$;
4. **Final Outcome:** the protest succeed, $R = 1$, if the number of citizens joining the protest, $\mathcal{N}$, is greater or equal to T, otherwise it fails.

Schematically, the timing of the game is:

Because of the citizens' information, a citizen i strategy is a map of the following type

$$s_i(\theta_i, v) : \mathbb{R} \times \mathbb{R} \rightarrow \{0, 1\},$$

while the conflict technology is

$$\mathbb{P}(R = 1; a_i, i \in [0, 1]) = \begin{cases} 1 & if \quad \int_0^1 a_i di = \mathcal{N} \geq T \\ 0 & if \quad \int_0^1 a_i di = \mathcal{N} \leq T. \end{cases}$$

We will call this situation of strategic interaction the **protest game**.

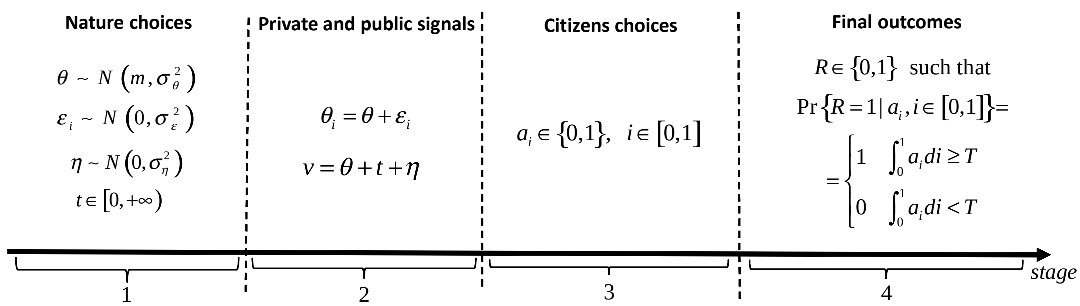

Figure 1: The timing of the game

2.7 Interpretation

Although the model is highly stylized, it admits a variety of interpretations and possible applications. As argued in Rubinstein 2001, the interpretation of a model is an essential ingredient of a model. We interpret the model as one of political change, in which each citizen observes both a private and a public signal, where the public one is related to political vanguard's activism: given the signals (θ_i, v), $i's$ decides whether or not to take actions, $a_i = 1$ or $a_i = 0$, to induce a change in the established policy.

2.7.1 The Random Variables

The stochastic structure of the model contemplate five random variables

$$\theta \sim N\left(0, \sigma_\theta^2\right); \quad \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right); \quad \theta_i \sim N\left(0, \sigma_\theta^2 + \sigma_\varepsilon^2\right); \quad \eta \sim N\left(0, \sigma_\eta^2\right); \quad v{-}t^* \sim N\left(0, \sigma_\theta^2 + \sigma_\eta^2\right).$$

Let consider these variables and their interpretation within this model of citizens' political behavior:

1. $\theta \sim N\left(0, \sigma_\theta^2\right)$ is the country average sentiment towards the government's policy. It is supposed unknown by the agents, and normally distributed. Note that $\theta > 0$ means opposition to the government's policy, while $\theta < 0$ means support for the government's policy, hence to assume $E(\theta) = 0$ means that ex ante the agents expect that the country sentiment towards

the government is neutral. From now on we will write of θ as the **country antigovernment sentiment** since a positive realization of this variable means a positive return from a change in government policies;

2. $\theta_i = \theta + \varepsilon_i \sim N\left(0, \sigma_\theta^2 + \sigma_\varepsilon^2\right)$ is the individual $i's$ sentiment towards the government's policy, i.e., i's private signal on the country common sentiment towards the government's policy. From now on we will write of θ_i as the **individual antigovernment sentiment** since a positive realization of this variable means a positive return to i from a change in government policies;

3. $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$ is what differentiate i's individual antigovernment sentiment from the average country antigovernment sentiment;

4. $v = \theta + t + \eta \sim N\left(t^*, \sigma_\theta^2 + \sigma_\eta^2\right)$ is the public signal on the country average sentiment towards the government's policy, which depends on θ and on the parameter t, which is interpreted below, as an exogenous sociopolitical variable;

5. $\eta \sim N\left(0, \sigma_\eta^2\right)$ is the stochastic noise in the public signal that meddle with the public signal, so that agents are not able, given the expectation of t, to infer the true value of the country antigovernment sentiment.

2.7.2 The Set of Exogenous Sociopolitical Variables

These are the variables related to social and political aspects of a country in a given historical period. The set of exogenous sociopolitical variables are

$$S = \left\{\left(t^*, \sigma_\theta^2, \sigma_\varepsilon^2\right) \in \mathbb{R}_+^3\right\}$$

and a particular vector of these variables is denoted by $\mathbf{s} \in S$. Let consider these variables and their interpretation within this model of citizens' political behavior:

1. $t^* \in [0, \infty)$ is the common expectation of t, a parameter representing the **vanguard's antigovernment activism**, which is unknown by the citizens, but imperfectly observed through the public random signal v. It is a way activists use to send a public signal to the citizens about the unknown common antigovernment sentiment of the country. Since in this paper we are interested in understanding the causes and the consequences of citizens' political behavior, we take the vanguard's activism t as exogenous as well as t^*. In the original Bueno de Mesquita paper, t is the vanguard's violence chosen by the revolutionary entrepreneurs to induce the citizens' revolt, and v is the violence observed by the citizens. We think that this interpretation is not fully convincing since v does not impact directly on players' payoff as it should in violent conflicts: even the extension at the end of the paper where v affects negatively T does not seem to us to catch the effects of the violence on citizens' and government's payoffs;

2. σ_θ^2 is the variance of the country's common antigovernment sentiment, thus a bigger σ_θ^2 implies a greater probability of extreme values for θ, positive and negative, hence we can interpret σ_θ^2 as a **political radicalization parameter**;

3. σ_ε^2 is the variance of the idiosyncratic component of individual antigovernment sentiment θ_i, which has a twofold role in the model, since it is both a private signal and $i's$ sentiment towards the government's policy. Thus a bigger σ_ε^2 implies a reduction in the informativeness of the private signal, but also a greater dispersion in the possible realizations of the individual antigovernment sentiment θ_i : from this second point of view, we can interpret σ_ε^2 as the **country political diversity**.

2.7.3 The Set of Exogenous Policy Variables

These are the variables that characterize a specific political regime. The set of exogenous policy variables is denoted by

$$P = \left\{ \left(T, \gamma, k, \sigma_\eta^2\right) \in (0,1]^2 \times \mathbb{R}_+^2 \right\}$$

and a particular vector of these variables is denoted by $\mathbf{p} \in P$. Let consider these policy variables and their interpretation within this model of citizens' political behavior.

1. T is the threshold such that, once the percentage of protesting citizens exceeds this level, then the government would change its policy; thus the greater T, the more difficult is to induce a change through a public protest. According to the political and economic literature,[10] a government is responsive if it adopts policies that are signaled as preferred by citizens, where signals include various form of direct political actions, such as demonstrations, letter campaigns, and the like. Thus, $(1 - T)$ can be interpreted as the **government responsiveness**;

2. γ is the intrinsic utility the citizens get from participating to successful protests, i.e. it measures the selective incentives to overcome collective action problems, thus we can interpret γ as a measure of the **regime's inclusiveness.** According to the Global State of Democracy Indices[11] that measures democratic performance for 165 countries around the world across 29 aspects of democracy, to ensure inclusive and participatory decision-making at all levels is a crucial aspect to evaluate a political regime: the greater the inclusiveness, the greater the measure of democracy;

3. k is the cost of protesting for the citizens, thus part of a measure of the **repression** by the regime.As we will see, a key parameter in the analysis

[10] See e.g. Przeworski *et al.* 1999.

[11] International Institute for Democracy and Electoral Assistance (IDEA), 2021.

is the cost of protesting with respect to the benefit from successful protets,

$$\frac{k}{\gamma},$$

that we interpret as an index of **government repression**;

4. σ_η^2 is the variance of the noise in the public signal, thus a bigger σ_η^2 implies a looser connection between vanguard's activism, the common antigovernment sentiment and the public signal, hence we interpret σ_η^2 as the country **opacity in public information**: as well known, the effectivity of citizens' checks on the policies of the government depends on the informativeness of public information, and the greater the noise, the smaller the information provided by the public signal.[12]

Finally, we will write a generic function as $f(x; \mathbf{p}, \mathbf{s})$ to emphasize its dependence from the exogenous vector $(\mathbf{p}, \mathbf{s}) \in P \times S$ of sociopolitical and policy variables.

The following table sum up the variables of the model and their interpretation

Variables	**Interpretation**
Endogenous Variables	
$a_i \in \{1, 0\}$	citizen i protests or not
$R \in \{1, 0\}$	success or not of protests
Random Variables	
$\theta \sim N\left(0, \sigma_\theta^2\right)$	country unknown level of antigovernment sentiment
$\theta_i \sim N\left(0, \sigma_\theta^2 + \sigma_\varepsilon^2\right)$	$i's$ level of antigovernment sentiment
$\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$	idiosyncratic component of $i's$ antigovernment sentiment
$v \sim N\left(t^*, \sigma_\eta^2\right)$	public signal on country antigovernment sentiment
$\eta \sim N\left(0, \sigma_\eta^2\right)$	noise in the public signal
Exogenous sociopolitical variables: $S = \left\{\left(t^*, \sigma_\theta^2, \sigma_\varepsilon^2\right) \in \mathbb{R}_+^3\right\}$	
$(t^*)\ t \in [0, \infty)$	(expected) vanguard's antigovernment activism
σ_θ^2	country political radicalization
σ_ε^2	country political diversity
Exogenous policy variables: $P = \left\{\left(T, \gamma, k, \sigma_\eta^2\right) \in (0, 1]^2 \times \mathbb{R}_+^2\right\}$	
$1 - T$	government responsiveness
γ	political inclusiveness
k	cost of protesting
k/γ	government political repression
σ_η^2	opacity in the public information
Table 1: variables and their meanings	

[12] For example, see Besley and Prat 2006 for an analysis of the role of media to insure government accountability towards citizens.

2.8 A Taxonomy of the Political Regimes and of the Country Sociopolitical Characteristics

Using the previous interpretations of the exogenous variables, we propose an intuitive qualitative classification of the possible different political regimes and of the different countries from a sociopolitical point of view. In particular, we consider the possible combination of the policy variables to identify eight different political regimes.

	σ_η^2 *small*	σ_η^2 *big*
$(1-T)$ *big*, $\frac{k}{\gamma}$ *small*	**RRT** responsive, tolerant, transparent	**RTO** responsive, tolerant, opaque
$(1-T)$ *small*, $\frac{k}{\gamma}$ *small*	**UTT** unresponsive, tolerant, transparent	**UTO** unresponsive, tolerant, opaque
$(1-T)$ *big*, $\frac{k}{\gamma}$ *big*	**RIT** responsive, intolerant, transparent	**RIO** responsive, intolerant, opaque
$(1-T)$ *small*, $\frac{k}{\gamma}$ *big*	**UIT** unresponsive, intolerant, transparent	**UIO** unresponsive, intolerant, opaque

Table 2: different political regimes P

These eight possible political regimes are defined on the basis of three dimensions. Leaving aside vanguard's activism that will play a specific role later, a country society can be identified on the basis of the remaining sociopolitical characteristics as follows

	σ_θ^2 *small*	σ_θ^2 *big*
t^* *small*, σ_ε^2 *small*	**QHM** quiet, homogeneous, moderate	**QHR** quiet, homogeneous, radicalized
t^* *big*, σ_ε^2 *small*	**THM** turbulent, homogeneous, moderate	**THR** turbulent, homogeneous, radicalized
t^* *small*, σ_ε^2 *big*	**QDM** quiet, diverse, moderate	**QDR** quiet, diverse, radicalized
t^* *big*, σ_ε^2 *big*	**TDM** turbulent, diverse, moderate	**TDR** turbulent, diverse, radicalized

Table 3: different political societies S

In the following analysis we will consider how the causes and the consequences of citizens' political behavior change in different political regimes and different societies.

3 Solving the Model

3.1 The Public Opinion

Within this model, to study public opinion means to study citizens' beliefs, and the mechanism of its formation according to our model. Does public opinion matter? We will show that public opinion affect citizens' behavior and thus represents an important driver of public policy changes. Of course, political outcomes will depend not only on public opinion, but also on how this reflects on citizens' behavior depending on the political regime and the political society, and in turn citizens' behavior will induce specific political outcomes depending on government responsiveness.

Let we start reviewing citizens' beliefs as implied by previous hypotheses.

- **Citizen's i posterior beliefs on country unknown level of antigovernment sentiment given $i's$ private information**:

$$\theta|\theta_i \sim N\left(\lambda\theta_i, \lambda\sigma_\varepsilon^2\right)$$

where

$$\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}.$$

- **Citizen's i posterior beliefs on country unknown level of antigovernment sentiment given $i's$ private and public information**:

$$\theta|\theta_i, v - t^* \sim N\left(\psi\left(v - t^*\right) + \left(1 - \psi\right)\lambda\theta_i, \psi\sigma_\eta^2\right)$$

where

$$\psi = \frac{\lambda\sigma_\varepsilon^2}{\lambda\sigma_\varepsilon^2 + \sigma_\eta^2} = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\sigma_\varepsilon^2}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\sigma_\varepsilon^2 + \sigma_\eta^2} = \frac{\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\varepsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\varepsilon^2\sigma_\eta^2}.$$

Consider the following intuitive terminology.

Definition 1 *A citizen i is an **extremist** if $\theta_i > E\left(\theta\right)$, i.e. if his/her antigovernment sentiment is greater than the average country's antigovernment sentiment, otherwise he/she is a **moderate.***

Notation 1 *Let denote by $\searrow$ a decreasing behavior, by $\nearrow$ an incresing behavior, by $\nearrow\searrow$ and $\searrow\nearrow$ a non monotonic behavior.*

Then, the following result is obvious but interesting:

Result 1 *The expected country's level of antigovernment sentiment given $i's$ private signal is*

1. *increasing in $i's$ level of antigovernment sentiment;*
2. *when i is a moderate, decreasing in country radicalization and increasing in country diversity;*
3. *when i is an extremist, increasing in country radicalization and decreasing in country diversity;*
4. *almost coinciding with $i's$ antigovernment sentiment, when country radicalization is increasing without limit with a finite amount of country diversity;*

5. *almost degenerated in* 0*, when country diversity is increasing without limit with a finite amount of country radicalization.*

In a table, the expected country's level of antigovernment sentiment given i'*s private signal is*

Socio-pol. var.	**Private signal**	
	Moderate	Extremist
i's antigovernment sentiment	$\nearrow$	$\nearrow$
radicalization	$\searrow$	$\nearrow$
diversity	$\nearrow$	$\searrow$

Table 4: behavior of $E\left(\theta|\theta_i\right)$

This result is simple but interesting because it explains why people think their political position on average is shared by other citizens, and it shows the difference in perceptions between moderates and extremists in a society, in particular on country radicalization and diversity.

The public signal v changes this result and has different consequences for extremists and for moderates; before stating the result, consider the following intuitive terminology.

Definition 2 *The unexpected component of the public signal* $v - t^*$ *and the private signal* θ_i

1. *for a* ***moderate*** *are*

   (a) ***strongly incendiary*** *if* $(v - t^*) > -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$;

   (b) ***incendiary*** *if* $(v - t^*) \in \left[E\left(\theta;\theta_i;\mathbf{p},\mathbf{s}\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i, -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i\right]$;

   (c) ***moderating*** *if* $(v - t^*) \in \left[\frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i, E\left(\theta;\theta_i;\mathbf{p},\mathbf{s}\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i\right]$;

   (d) ***strongly moderating*** *if* $(v - t^*) < \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i$;

2. *for an* ***extremist*** *are*

   (a) ***strongly incendiary*** *if* $(v - t^*) > \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i$;

   (b) ***incendiary*** *if* $(v - t^*) \in \left[E\left(\theta;\theta_i;\mathbf{p},\mathbf{s}\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i, \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i\right]$;

   (c) ***moderating*** *if* $(v - t^*) \in \left[-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i, E\left(\theta;\theta_i;\mathbf{p},\mathbf{s}\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i\right]$;

   (d) ***strongly moderating*** *if* $(v - t^*) < -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$.

This definition is illustrated in the following picture:

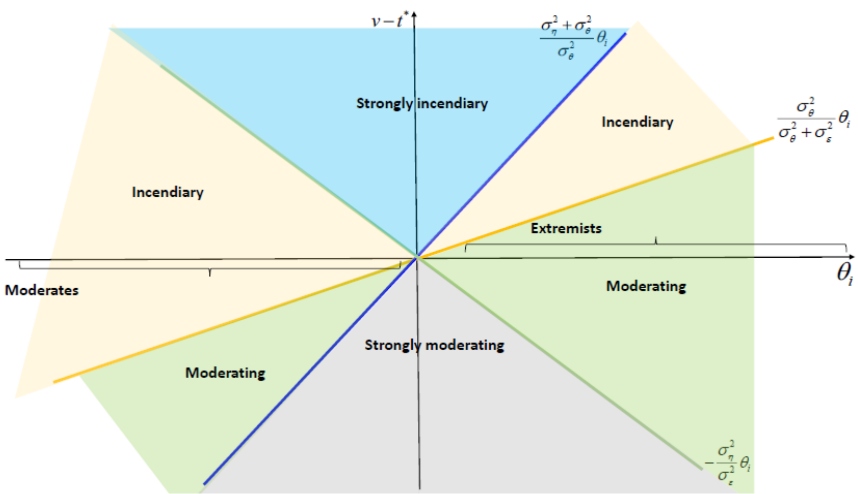

Figure 2: The role of the public signal on citizens' beliefs.

Then, the following result is immediate:

Result 2 *The expected country antigovernment sentiment given i's private and public signals is*

1. *increasing in i's level of antigovernment sentiment and in the unexpected component of the public signal;*
2. *increasing in the opacity of public information if and only if the unexpected component of the public signal is moderating;*
3. *increasing in country radicalization if and only if the unexpected component of the public signal is*
   - *not strongly moderating for an extremist or*
   - *strongly incendiary for a moderate;*
4. *increasing in country political diversity if and only if the unexpected component of the public signal is*
   - *strongly incendiary for an extremist or*
   - *not strongly moderating for a moderate.*

An alternative way of stating this result is the following

Result 3 *The expected country antigovernment sentiment given i's private and public signals is*

1. *increasing in $i's$ level of antigovernment sentiment and in the unexpected component of the public signal;*

2. *increasing in the opacity of public information if and only if the unexpected component of the public signal is smaller than $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i$;*

3. *increasing in country radicalization if and only if the unexpected component of the public signal is greater than $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$;*

4. *increasing in country political diversity if and only if the unexpected component of the public signal is greater than $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}\theta_i$.*

In a table, the expected country's level of antigovernment sentiment given $i's$ private and public signals is

Socio-pol. var.	**Public and private signals**					
	Strong inc	Ext&Inc	Extr&Moder	Strongly moder	Moder&Moder	Moder&Inc
i's antigov sent	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
unexpect activism	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
radicalization	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$
diversity	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$
opacity	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$

Table 5: behavior of $E(\theta|\theta_i, v-t^*)$

The following figure represents the situation:

These results show the different properties of the expected country antigovernment sentiment depending on whether i is moderate or extremist, and the different effects of private and public signal, with their subtle interaction, on the expected country antigovernment sentiment. In particular, it is immediate to derive the following result.

Result 4 *The public signal changes the behavior of citizens' expectations with respect to the case of private signal only when the signals are strongly incendiary or strongly moderating. Moreover, the space of strongly incendiary/moderating signals is shrinking when*

1. *the opacity in public information is increasing;*

2. *country radicalization is decreasing;*

3. *country diversity is decreasing.*

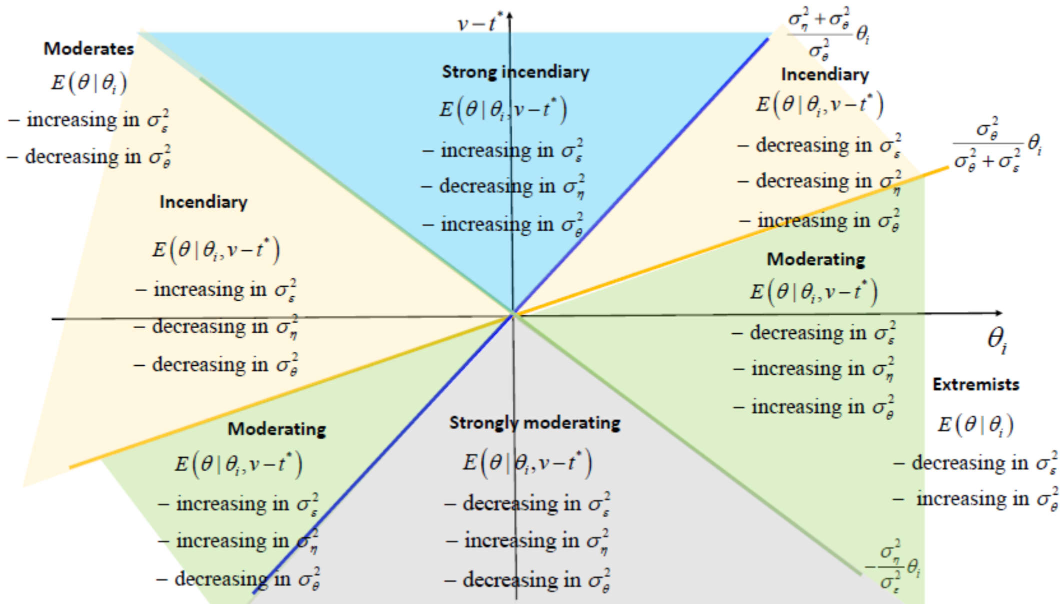

Figure 3: Expected country antigovernment sentiment and the socio-political variables.

The following table illustrates when the public signal changes public opinion behavior with respect to the case of private signal only

Socio-pol.var.	**Public and private signals**					
	Strong inc	Ext&Inc	Extr&Moder	Strong.moder	Moder&Moder	Moder&Inc
radicalization	$\nearrow$ also for Moder	$\nearrow$	$\nearrow$	$\searrow$ also for Extr	$\searrow$	$\searrow$
diversity	$\nearrow$ also for Extr	$\searrow$	$\searrow$	$\searrow$ also for Moder	$\nearrow$	$\nearrow$

Table 6: behavior of $E(\theta|\theta_i, v-t^*)$ and $E(\theta|\theta_i)$

The following figure illustrates the situation

3.2 Citizens' Behavior

We start with a simple but interesting result on citizens' individual rationality, and then we derive their equilibrium behavior following four steps, as in Bueno de Mesquita 2010, however using a slightly different methodology.

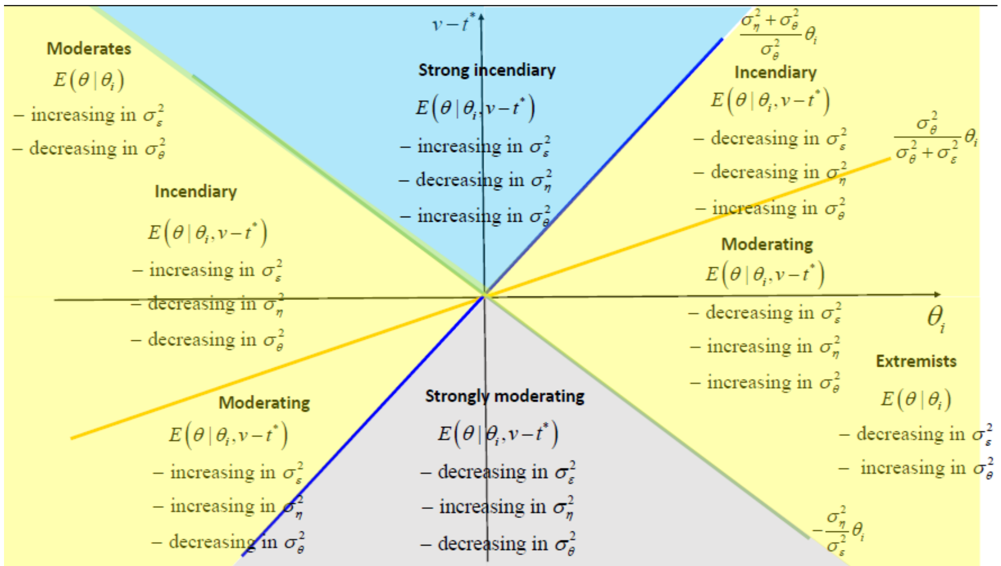

Figure 4: The role of public signals

3.2.1 The Silent Citizens

Denote by $\Phi(\cdot)$ the cumulative distribution function of the standard normal. From the payoff table the following result is immediate:

Result 5 *Any citizen with type* $\theta_i \in \left(-\infty, \frac{k}{\gamma}\right)$ *has a dominant strategy not to participate whatever the private and public signals*

This result justify the following terminology.

Definition 3 *The citizens that will never protest notwithstanding the public and private signals is the **silent group.***

Note that the silent group is different from the moderates, because it also depends on the variables related to the polity, besides the sociopolitical variables as for the moderates. In particular, considerate the following definition.

Definition 4 *A political regime is repressive if* $\frac{k}{\gamma} > \theta$*, tolerant otherwise.*

Then, we can say that the moderates are a subset of the silent citizens in repressive political regimes, and viceversa in tolerant political regimes. The following figure illustrates the situation for the two case of repressive and tolerant political regimes.

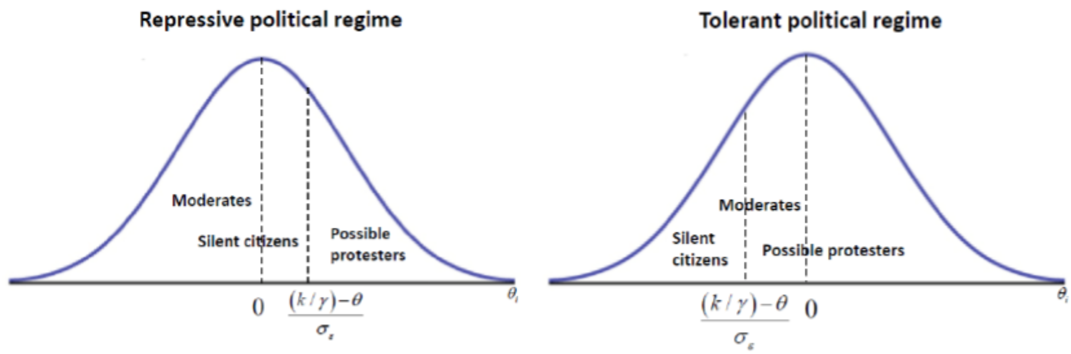

Figure 5: Moderates and silent group as determined by the political regime

Then, the following result is immediate.

Result 6 *The measure of the* ***silent group*** *is* $\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)>0$, *which is*

1. *increasing in the government repression;*
2. *decreasing in the country antigovernment sentiment*;
3. *decreasing in the country diversity* ***for repressive political regimes;***
4. *increasing in the country diversity* ***for tolerant political regimes.***

These results are summed up in the following table

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\searrow$	
	any	
country antigovernment sentiment	$\searrow$	
	Tolerant	Repressive
diversity	$\nearrow$	$\searrow$

Table 7: behavior of the measure of the silent group

Remark 4 *An interesting aspect of this results is that political diversity reduces the measure of the silent group only in repressive polities, which means that in more democratic countries, political participation is increasing for more heterogenous societies.*

A further interesting result is the following:

Result 7

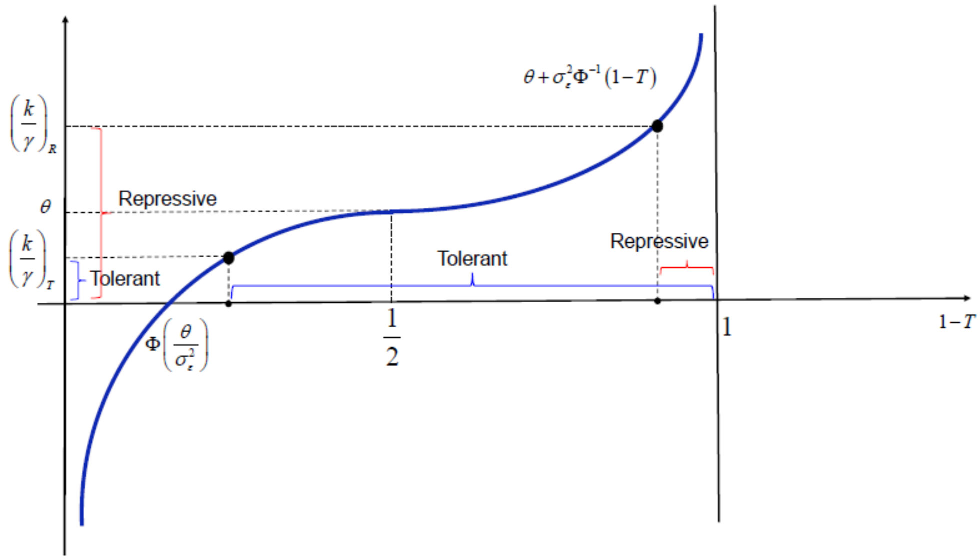

Figure 6: Repression and responsiveness such that protests can succeed

1. *Protests are impossible if and only if*

$$\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\to\infty,$$

hence protests are always possible for tolerant political regimes;

2. *Protests can be successful if and only if*

$$\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)\leq 1-T\Leftrightarrow\frac{k}{\gamma}\leq\theta+\sigma_\varepsilon\Phi^{-1}\left(1-T\right)$$

restricting the set of political regims and societies where protests can succeed. In particular

   (a) *responsiveness should be greater than* $\Phi\left(\frac{\theta}{\sigma_\varepsilon}\right)$, *otherwise protests can't succeed;*

   (b) *when the regime is repressive, responsiveness should be greater than* $\frac{1}{2}$, *otherwise protests can't succeed.*

The following figure illustrates the situation.

3.3 Citizens' Equilibrium Behavior

3.3.1 The Equilibrium Concept

As in Bueno de Mesquita 2010, we consider **cutoff equilibria**, i.e. Perfect Bayesian Equilibria (PBE) in pure strategies, with two restrictions:

1. **cutoff strategies:**

$$s_i\left(\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)=\begin{cases}1 & if \quad \theta_i \geq \widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right) \\ 0 & if \quad \theta_i<\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)\end{cases}$$

2. **common cutoff selection:** when there are multiple equilibria with possible multiple cutoff points, the citizens will choose the same threshold $\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)$.

Remark 5 *The common cutoff selection is requested by the possibility of multiple equilibria in the model,*[13] *while cutoff strategies are commonly used in global games.*[14]

3.3.2 Deriving the Cutoff Equilibria of the Protest Game.

Following Bueno de Mesquita 2010, the analysis is organized in four steps.

Step 1: The Cutoff Rule Let conjecture there exists an increasing map[15]

$$\widehat{\theta}:\left(v-t^*\right) \mapsto\left[\frac{k}{\gamma},+\infty\right]$$

such that

$$s_i\left(\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)=\begin{cases}1 & if \quad \theta_i \geq \widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right) \\ 0 & if \quad \theta_i<\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)\end{cases} \Leftrightarrow$$

$$\Leftrightarrow s_i\left(\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)=\begin{cases}1 & if \quad \varepsilon_i \geq \widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)-\theta \\ 0 & if \quad \varepsilon_i<\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)-\theta .\end{cases}$$

Step 2: i's Beliefs about the Probability of Policy Change From player i's perspective, if all other players use the cutoff rule $\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)$, then j protests if $\varepsilon_j \geq \widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)-\theta$. Thus, the mass of citizens protesting is

$$\mathcal{N}\left(\theta, \widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)\right)=1-\Phi\left(\frac{\widehat{\theta}\left(v-t^*; \mathbf{p}, \mathbf{s}\right)-\theta}{\sigma_{\varepsilon}}\right),$$

[13] See Bueno De Mesquita 2011 for a discussion about the reasons for uniqueness and multiplicity in global games.

[14] See Morris and Shin 2003 for a comprehensive review of global games.

[15] In section 6.3 we will prove it does exist such a map.

so that in equilibrium the protest succeeds if

$$\mathcal{N}\left(\theta,\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)\right)=1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\theta}{\sigma_{\varepsilon}}\right)\geq T.$$

Since $\mathcal{N}\left(\theta,\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)\right)$ is increasing in θ, for any cutoff rule $\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)$ there exists a minimal level of θ, such that the protest is successful, which is the solution of the following equation in θ^{*}

$$\mathcal{N}\left(\theta^{*},\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)\right)=1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\theta^{*}}{\sigma_{\varepsilon}}\right)=T\Rightarrow$$

$$\Rightarrow\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)\in\mathbb{R}.$$

Then, we might conclude as follows.

Conclusion 1 *The* ***minimal level of the country antigovernment sentiment, such that the protest is successful*** *when the common cutoff rule is* $\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)$, *is*

$$\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right),$$

which is

1. *decreasing in the responsiveness of the political regime, such that*
   (a) $\lim_{1-T\to1}\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=-\infty$;
   (b) $\lim_{1-T\to0}\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=\infty$;
   (c) $\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T=\frac{1}{2},\sigma_{\varepsilon}^{2}\right)=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)\geq\frac{k}{\gamma}$;
   (d) $\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T=\Phi^{-1}\left(\frac{\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)}{\sigma_{\varepsilon}}\right),\sigma_{\varepsilon}^{2}\right)=0$;
2. *linearly increasing or decreasing in the diversity of the country depending whether the political regime is unresponsive or responsive;*
3. *linearly increasing in the common cutoff* $\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)$, *such that*
   (a) *the minimum is* $\frac{k}{\gamma}-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)$;
   (b) $\lim_{\widehat{\theta}\left(v-t^{*}\right)\to\infty}\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=\infty$;
   (c) $\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);T,\sigma_{\varepsilon}^{2}\right)=0$ *if and only if* $\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)=\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)$.

Graphically

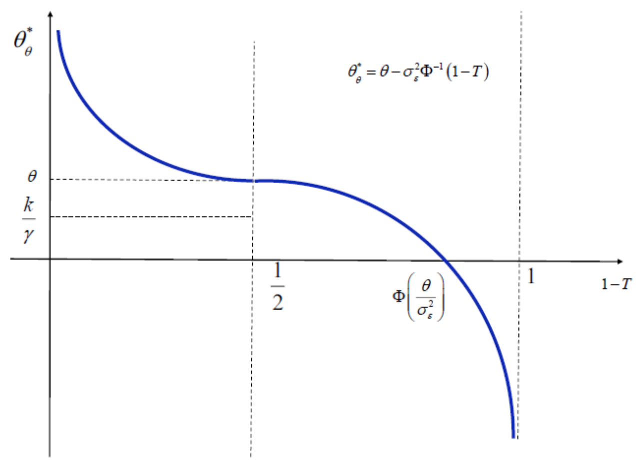

Figure 7: $\theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);T,\sigma^2_\varepsilon\right)$ as a function of T.

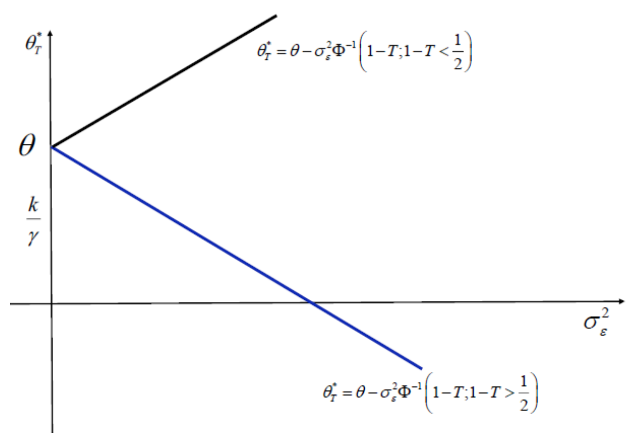

Figure 8: $\theta^*\left(\widehat{\theta}\left(v-t^*|\mathbf{p},\mathbf{s}\right);T,\sigma^2_\varepsilon\right)$ as a function of σ^2_ε.

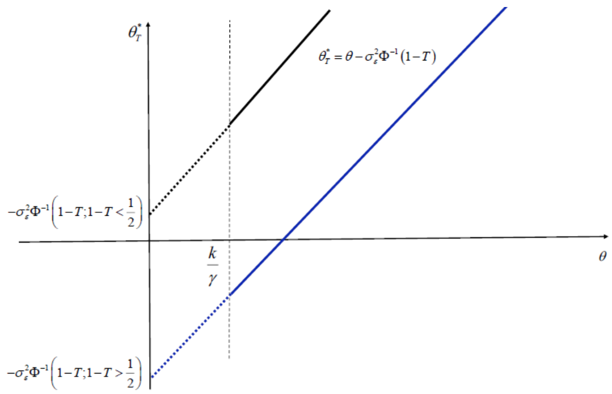

Figure 9: $\theta^* \left(\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right); T, \sigma_\varepsilon^2\right)$ a s a function of $\widehat{\theta}$.

Remark 6 *The previous results show that a more heterogenous country is more (less) likely to have a successful protest if the political regime is responsive (unresponsive). Hence, to evaluate the likelihood of a successful protest, it is important to consider the combination of political and social variables that characterize a country.*

Now, it is possible to evaluate **i's subjective belief about the probability of policy change**, $\mathbb{P}\left(\mathcal{N} \geq T | \theta_i, v - t^*, \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right)$, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)$:

$$\mathbb{P}\left(\mathcal{N} \geq T | \theta_i, v - t^*, \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right) = \mathbb{P}\left(\theta \geq \theta^*\left(\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)\right) | \theta_i, v - t^*; \mathbf{p}, \mathbf{s}\right) =$$

$$= 1 - \Phi\left(\frac{\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right) - \sigma_\varepsilon \Phi^{-1}\left(1 - T\right) - \psi\left(v - t^*\right) - \left(1 - \psi\right)\lambda\theta_i}{\sqrt{\psi\sigma_\eta^2}}; \mathbf{p}, \mathbf{s}\right),$$

since

$$\theta | \theta_i, v - t^* \sim N\left(\psi\left(v - t^*\right) + \left(1 - \psi\right)\lambda\theta_i, \psi\sigma_\eta^2\right).$$

Result 8 ***i's subjective belief about the probability of policy change**, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)$, is*

1. *increasing in the responsiveness of the political regime;*
2. *increasing in the public signal and in its unexpected component;*
3. *increasing in the private signal;*

4. *uncertain in radicalization, diversity and opacity.*

When the formal analysis does not lead to well defined relationships, we investigate the relationships through simulations that can be found in Appendix B. From these simulations we are able to derive the following results.

Result 9 ***i's subjective belief about the probability of policy change,*** *given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)$, is*

1. *increasing in opacity unless the political regime is responsive and the society is radicalized and heterogenous;*

2. *increasing in diversity unless the political regime is unresponsive;*

3. *increasing in radicalization unless the political regime is responsive.*

The result is summed up in the following table:

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\searrow$	
	any	
responsiveness	$\nearrow$	
	DR&R	other
opacity	$\searrow$	$\nearrow$
	R	other
radicalization	$\searrow$	$\nearrow$
	R	other
diversity	$\nearrow$	$\searrow$
	any	
unexp activism	$\nearrow$	

Table 8: i's belief about the probability of policy change

Step 3: Citizens that Will Participate to Protests A player i who believes that everyone else is using the cutoff rule $\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)$ will participate to protests if and only if

$$E\left[U_i\left(1, R\right); \mathbf{p}, \mathbf{s}\right] \geq E\left[U_i\left(0, R\right); \mathbf{p}, \mathbf{s}\right] \Leftrightarrow \mathbb{P}\left(\mathcal{N} \geq T | \theta_i, v - t^*, \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)\right) \gamma\theta_i \geq k.$$

$$\Leftrightarrow \mathbb{P}\left(\mathcal{N} \geq T | \theta_i, v - t^*, \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)\right) \gamma\theta_i \geq k \Leftrightarrow s_i\left(\theta_i, v; \mathbf{p}, \mathbf{s}\right) = 1 \Leftrightarrow$$

$$\Leftrightarrow \left[1 - \Phi\left(\frac{\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right) - \sigma_\varepsilon \Phi^{-1}\left(1 - T\right) - \psi\left(v - t^*\right) - \left(1 - \psi\right)\lambda\theta_i}{\sqrt{\psi\sigma_\eta^2}}; \mathbf{p}, \mathbf{s}\right)\right] \gamma\theta_i \geq k.$$

Step 4: Citizens' Equilibrium Behavior Let define $i's$ expected incremental benefit (IB) from protesting when i is expecting the same cutoff behavior from the other citizens as

$$E\left[U_i\left(1,R;\mathbf{p},\mathbf{s}\right)\right] =: IB\left(\theta_i,\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right),v-t^*;\mathbf{p},\mathbf{s}\right) =$$

$$= \left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^*\right)+\left(1-\psi\right)\lambda\theta_i\right]}{\sqrt{\psi\sigma_\eta^2}}\right)\right]\gamma\theta_i.$$

Equilibrium requires $\widehat{\theta}$ to be the solution of the following equation

$$IB\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right),\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right),v-t^*;\mathbf{p},\mathbf{s}\right) = k.$$

Let define

$$\widehat{IB}\left(\widehat{\theta},v-t^*;\mathbf{p},\mathbf{s}\right) := IB\left(\widehat{\theta}\left(v-t^*\right),\widehat{\theta}\left(v-t^*\right),v-t^*;\mathbf{p},\mathbf{s}\right).$$

Then a generic $\widehat{\theta}$ is the equilibrium cutoff if and only if

$$\widehat{IB}\left(\widehat{\theta},v-t^*;\mathbf{p},\mathbf{s}\right) = k \Leftrightarrow$$

$$\Leftrightarrow \left[1-\Phi\left(\frac{\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^*\right)+\left(1-\psi\right)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_\eta^2}}\right)\right]\gamma\widehat{\theta} = k \Leftrightarrow$$

$$\Leftrightarrow \left[1-\Phi\left(\frac{\left[1-\left(1-\psi\right)\lambda\right]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)\right]\gamma\widehat{\theta} = k \Leftrightarrow$$

$$\Leftrightarrow \widehat{\theta} = \frac{k}{\gamma}\frac{1}{1-\Phi\left(\frac{\left[1-\left(1-\psi\right)\lambda\right]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)} \Leftrightarrow$$

$$\Leftrightarrow \widehat{\theta} = \frac{k}{\gamma}F\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right), \quad \text{where} \quad F\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right) := \frac{1}{1-\Phi\left(\frac{\left[1-\left(1-\psi\right)\lambda\right]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)}.$$

Note that

$$F\left(0;\mathbf{p},\mathbf{s}\right) = \frac{1}{1-\Phi\left(-\frac{\sigma_\varepsilon\Phi^{-1}\left(1-T\right)+\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)} > 1$$

$$\lim_{\widehat{\theta}\to\infty}F\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right) = \infty, \quad \lim_{\widehat{\theta}\to-\infty}F\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right) = 1$$

$$\frac{\partial F\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)}{\partial\widehat{\theta}} = \frac{\phi\left(\frac{\left[1-\left(1-\psi\right)\lambda\right]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)\frac{\left[1-\left(1-\psi\right)\lambda\right]}{\sqrt{\psi\sigma_\eta^2}}}{\left[1-\Phi\left(\frac{\left[1-\left(1-\psi\right)\lambda\right]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}}\right)\right]^2} > 0.$$

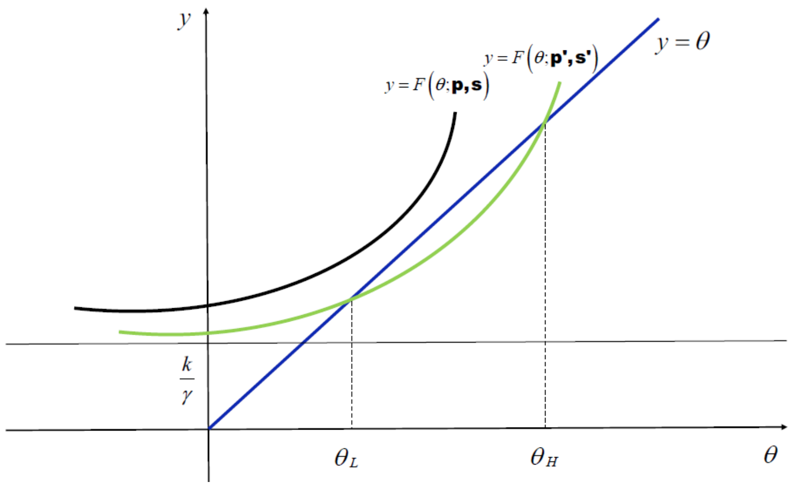

Figure 10: The equilibrium cutoff

Thus, the situation is represented in the following figure that shows the existence of 0 or 2 cutoff equilibria, depending on the values of $\mathbf{p}, \mathbf{s}$.

Thus, the following result is quite immediate.

Result 10 *Any possible equilibrium cutoff satisfies the following restriction*

$$\widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right) \in \left[\frac{k}{\gamma}, \infty\right].$$

According to the previous result, any equilibrium cutoff satisfies the condition of individual rationality, i.e. $\widehat{\theta} \geq \frac{k}{\gamma}$. Moreover, any $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ can be an equilibrium cutoff for certain combinations of the exogenous variables.

3.3.3 Vanguard Activism, Public Signal and Equilibrium Cutoff

In this subsection, which is original with respect to Bueno de Mesquita 2010, we study the relationship between the level of $v - t^*$ and the citizens' equilibrium behavior such that a generic $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ is an equilibrium cutoff, in order to characterize such equilibrium cutoff $\widehat{\theta}$.

A generic $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ is an equilibrium cutoff if and only if

$$\widehat{IB}\left(\widehat{\theta}, v - t^*; \mathbf{p}, \mathbf{s}\right) = k \Leftrightarrow$$

$$\Leftrightarrow \Phi\left(\frac{\widehat{\theta}-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^{*}\right)+\left(1-\psi\right)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)=1-\frac{k}{\gamma\widehat{\theta}}\Leftrightarrow$$

$$\Leftrightarrow \frac{\widehat{\theta}-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^{*}\right)+\left(1-\psi\right)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_{\eta}^{2}}}=-\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\Leftrightarrow$$

$$\Leftrightarrow v-t^{*}=\left(\sqrt{\frac{\sigma_{\eta}^{2}}{\psi}}\right)\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{\left[1-\left(1-\psi\right)\lambda\right]}{\psi}\widehat{\theta}-\frac{\sigma_{\varepsilon}}{\psi}\Phi^{-1}\left(1-T\right)$$

Definition 5 *Let define*

$$f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right):=\left(\sqrt{\frac{\sigma_{\eta}^{2}}{\psi}}\right)\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{\left[1-\left(1-\psi\right)\lambda\right]}{\psi}\widehat{\theta}-\frac{\sigma_{\varepsilon}}{\psi}\Phi^{-1}\left(1-T\right)$$

so that

$$v-t^{*}=f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$$

is the level of public signal (and of its unexpected component) such that a generic $\widehat{\theta}\in\left[\frac{k}{\gamma},\infty\right]$ *is an equilibrium cutoff.*

Let we state some properties of $f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$.

Lemma 1 $f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ *is*

1. *U shaped in* $\widehat{\theta}\in\left(\frac{k}{\gamma},\infty\right)$, *reaching a global minimum denoted by*

$$\widehat{\theta}^{*}\left(\mathbf{p},\mathbf{s}\right)=\arg\min_{\widehat{\theta}}f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right);$$

2. *decreasing in the responsiveness of the political regime;*
3. *increasing in the repression of the political regime*;
4. *uncertain in country diversity, radicalization and in information opacity.*

Using simulations, we are also able to derive the following result.

Result 11 $f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ *is*

1. *increasing in opacity unless the political regime is responsive and tolerant;*
2. *increasing in diversity unless the political regime is responsive;*

3. *decreasing in radicalization unless the political regime is responsive but opaque and the society heterogenous.*

The results are summed up in the following table

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\nearrow$	
	any	
responsiveness	$\searrow$	
	RT	other
opacity	$\searrow$	$\nearrow$
	D&RO	other
radicalization	$\nearrow$	$\searrow$
	R	other
diversity	$\searrow$	$\nearrow$
	any	
unexp activism	constant	

Table 9: $f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ **and the sociopolitical variables**

In conclusion, we can say that there exist and it is unique a $\widehat{\theta}(\mathbf{p},\mathbf{s}) = \widehat{\theta}^*(\mathbf{p},\mathbf{s})$ for which the level of unexpected activism $v - t^*$ required for $\widehat{\theta}(\mathbf{p},\mathbf{s}) = \widehat{\theta}^*(\mathbf{p},\mathbf{s})$ to be an equilibrium cutoff is minimal. Let we stress that while $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ does not depend on T, however $f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$ does depend on T.

The following figure shows the relationship between $v - t^*$ and $\widehat{\theta}$ in $v - t^* = f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$:

Consider the properties of

$$\widehat{\theta}^*(\mathbf{p},\mathbf{s}) = \arg\min_{\widehat{\theta}} f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right):$$

Lemma 2 $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ *is*

1. *independent from responsiveness;*

2. *increasing in the level of government repression;*

3. *decreasing in the country diversity;*

4. *increasing in the country radicalization;*

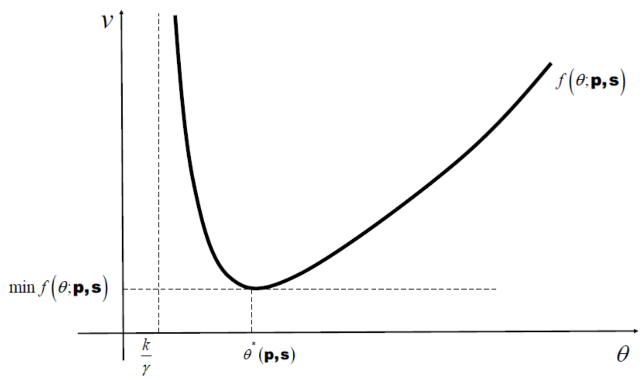

Figure 11: The function $v - t^* = f\left(\widehat{\theta}|\mathbf{p}, \mathbf{s}\right)$.

5. *increasing in opacity.*

The results are summed up in the following table

Socio-pol. Var	**Socio political situation**
	any
repression	$\nearrow$
	any
responsiveness	*constant*
	any
opacity	*constant*
	any
radicalization	$\nearrow$
	any
diversity	$\searrow$
	any
unexp activism	*constant*

Table 10: $\widehat{\theta}^*(\mathbf{p}, \mathbf{s})$ **and the sociopolitical variables**

The above results have the following implications for the possible equilibrium cutoff $\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s})$.

Result 12 *Consider the possible equilibrium payoff* $\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s})$:

1. *If* $(v - t^*)$ *is small enough, i.e. if* $(v - t^*) < f\left(\widehat{\theta}^*(\mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right)$, *then there exists no finite equilibrium cutoff, thus* $\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s}) = \infty$;
2. *If* $(v - t^*) = f\left(\widehat{\theta}^*(\mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right)$, *then there exists one finite equilibrium cutoff and* $\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s}) = \widehat{\theta}^*(\mathbf{p}, \mathbf{s})$;

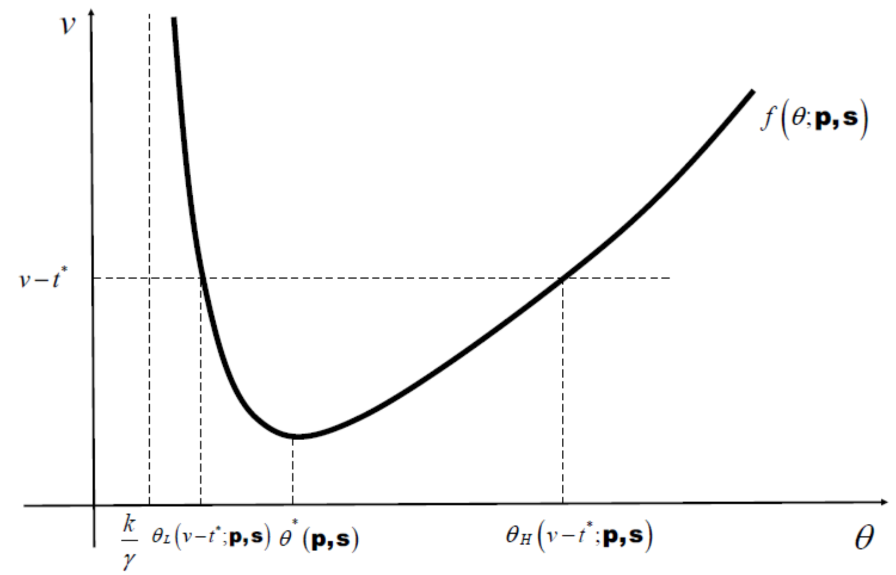

Figure 12: The equilibrium cutoffs.

3. *If $(v-t^*)$ is big enough, i.e. if $(v-t^*) > f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$, then there exists two finite equilibrium cutoff, thus $\widehat{\theta}(v-t^*;\mathbf{p},\mathbf{s}) \in \left\{\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s}), \widehat{\theta}_H(v-t^*;\mathbf{p},\mathbf{s})\right\}$ with*
$$\frac{k}{\gamma} < \widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s}) < \widehat{\theta}^*(\mathbf{p},\mathbf{s}) < \widehat{\theta}_H(v-t^*;\mathbf{p},\mathbf{s}).$$

The following figure illustrate the result:

Note that the necessary and sufficient condition to get two finite equilibrium cutoff $\widehat{\theta}_L \leq \widehat{\theta}_H$ is
$$(v-t^*) > f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right),$$
so that for given $(v-t^*)$,this inequality restricts the set of possible sociopolitical and policy variables $(\mathbf{p},\mathbf{s}) \in P \times S$.

Definition 6 *Let define*
$$\mathcal{PS}$$
the set of $(\mathbf{p},\mathbf{s})$ such that for a given $(v-t^*)$, $(v-t^*) > f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$. Formally
$$\mathcal{PS} := \left\{(\mathbf{p},\mathbf{s}) : \quad (v-t^*) > min_{\widehat{\theta}} f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)\right\}.$$

Therefore, we get the following result.

Proposition 1 *A strategy profile, where all citizens use the same strategy*

$$s : \mathbb{R} \times \mathbb{R} \to \{0, 1\}$$

which is not identically 0, *i.e. never participate to the protest, is consistent with a cutoff equilibrium if and only if:*

$$s(\theta_i, v - t^*; \mathbf{p}, \mathbf{s}) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s}) \\ 0 & \qquad otherwise \end{cases}$$

with $\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s})$ *satisfying the following conditions*

1. $v - t^* = f\left(\widehat{\theta}(v - t^*; \mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right);$
2. $(\mathbf{p}, \mathbf{s}) \in \mathcal{PS}$;
3. *continuity in* $(\mathbf{p}, \mathbf{s})$.

To guarantee positive participation for a given realization of $v - t^*$, we make the following assumptions.

Assumption 1 $(\mathbf{p}, \mathbf{s}) \in \mathcal{PS}$.

It is now possible to characterize the citizens' equilibrium behavior in the protest stage.

Proposition 2 *There are three strategies for the citizens that are consistent with a cutoff equilibrium of the full game:*

1. $$s^{\infty}(\theta_i, v - t^*; \mathbf{p}, \mathbf{s}) = 0 \quad for\ all \ \ \theta_i \ \ and \ \ v - t^*;$$

2. $$s^M(\theta_i, v - t^*; \mathbf{p}, \mathbf{s}) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}_H(v - t^*; \mathbf{p}, \mathbf{s}) \\ 0 & if \qquad otherwise \end{cases}$$

3. $$s^L(\theta_i, v - t^*; \mathbf{p}, \mathbf{s}) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) \\ 0 & if \qquad otherwise. \end{cases}$$

This means that, for any level of activism above the minimum $\widehat{\theta}^*(\mathbf{p}, \mathbf{s})$, there are two compatible equilibrium cutoffs $\widehat{\theta}_L$ and $\widehat{\theta}_H$.

Remark 7 *Note that* $\widehat{\theta}_L$ *decreases towards* $\frac{k}{\gamma}$ *as the level of unexpected activism increases, while* $\widehat{\theta}_H$ *grows towards infinity as activism increases. Intuitively, the growth in unexpected activism, net of the other variables in the game, should prompt citizens characterized by more restrained anti-government sentiment to join the protest. In this perspective it is implausible that as unexpected activism increases the level of anti-government sentiment for which one is indifferent to participate increases, as it happens with* $\widehat{\theta}_H$. *For this reason Bueno de Mesquita 2010 considers only the lower cutoff* $\widehat{\theta}_L$.

Assumption 2 *The space of possible equilibrium cutoff $\widehat{\theta}$ is restricted to*

$$\widehat{\theta} \in \left(\frac{k}{\gamma}, \widehat{\theta}^{*} \left(\mathbf{p}, \mathbf{s} \right) \right)$$

so that citizens do no play the equilibrium strategy $s^{M} \left(\theta_i, v - t^{*}; \mathbf{p}, \mathbf{s} \right)$.

As a result of these assumptions, for each level of activism we will have a single finite equilibrium cutoff $\widehat{\theta}_L \left(v - t^{*}; \mathbf{p}, \mathbf{s} \right)$ with the following properties, that derives immediately from the previous characterization.

Result 13 *The finite equilibrium cutoff $\widehat{\theta}_L \left(v - t^{*}; \mathbf{p}, \mathbf{s} \right)$ is*

1. *decreasing in the unexpected component of the public signal $v - t^{*}$;*
2. *decreasing in the responsiveness of the political regime*;
3. *increasing in the repression of the political regime;;*
4. *uncertain in country diversity, radicalization and in information opacity.*

Using simulations, we are able to derive the following result.

Result 14 *The finite equilibrium cutoff $\widehat{\theta}_L \left(v - t^{*}; \mathbf{p}, \mathbf{s} \right)$ is*

1. *increasing in opacity unless the political regime is responsive and tolerant;*
2. *increasing in diversity unless the political regime is responsive;*
3. *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up this results

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\nearrow$	
	any	
responsiveness	$\searrow$	
	RT	other
opacity	$\searrow$	$\nearrow$
	D&RO	other
radicalization	$\nearrow$	$\searrow$
	R	other
diversity	$\searrow$	$\nearrow$
	any	
unexp activism	constant	

Table 9: cutoff and the sociopolitical variables

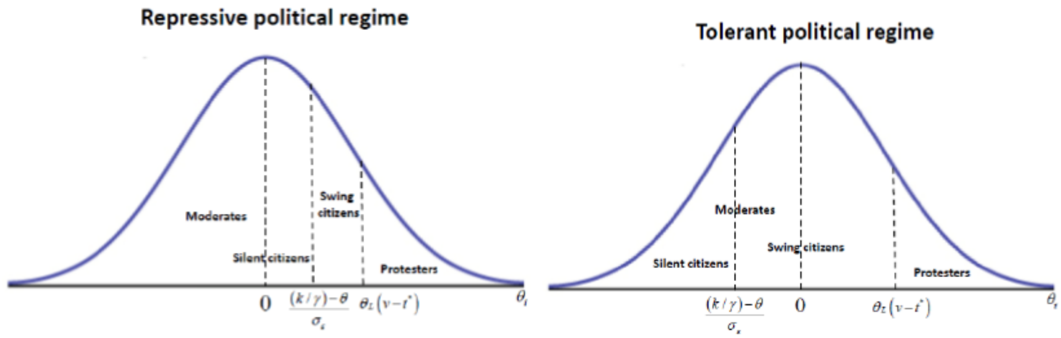

Figure 13: Moderates, Silent, Swing and Protesting Citizens

Definition 7 *The citizens that are not part of the silent group, but that do not demonstrate given the threshold* $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ *are called* ***swing citizens,***[16] *because they can swing to protest if there are changes in the exogenous variables*

Remark 8 *The following figure shows how the citizens distribute among different categories depending on the political regime*

The previous result on $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ and the previous definitions imply the following result.

Result 15 *The percentage of* ***protesting citizens*** *is*

$$1 - \Phi\left(\frac{\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) - \theta}{\sigma_\varepsilon}\right),$$

which is

1. *increasing in the antigovernment sentiment;*
2. *increasing in the unexpected component of the public signal* $v - t^*$*;*
3. *increasing in the responsiveness of the political regime*;
4. *decreasing in the repression of the political regime;*
5. *decreasing in the opacity of the political regime unless the political regime is responsive and tolerant;*
6. *decreasing in diversity unless the political regime is responsive when* $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) < \theta$ *and after this threshold is increasing with upper limit* $\frac{1}{2}$*;*

[16] Of course, the name is taken from the literature on swing voters.

7. *increasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up these results

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\searrow$	
	any	
responsiveness	$\nearrow$	
	RT	other
opacity	$\nearrow$	$\searrow$
	D&RO	other
radicalization	$\searrow$	$\nearrow$
	R	other
diversity	$\nearrow$	$\searrow$
	any	
unexp activism	$\nearrow$	
	any	
antigovernment sentiment	$\nearrow$	

Table 12: protesting citizens and sociopolitical variables

This result is interesting because it states that the more democratic a political regime is, the greater is the percentage of protesting citizens, with an interesting exception: an increase in transparence would reduce participation when the political regime is highly democratic (D1 and D2). Hence in authoritarian regimes, the control of public information complements repression and unresponsiveness. As usual, the interaction between political and social variables is more complex: homogenous society are more likely to witness highly participated protests unless the political regime is partially democratic, i.e. responsive (regimes D1, D2, D4, D6 in table 2).

Result 16 *The percentage of* ***swing citizens*** *is*

$$\Phi\left(\frac{\widehat{\theta}_L\left(v-t^*;\mathbf{p},\mathbf{s}\right)-\theta}{\sigma_\varepsilon}\right)-\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right),$$

which is

1. *first increasing, then decreasing in the antigovernment sentiment: the maximum is reached when*

$$\theta=\frac{1}{2}\left[\widehat{\theta}_L\left(v-t^*;\mathbf{p},\mathbf{s}\right)+\frac{k}{\gamma}\right];$$

2. *decreasing in the unexpected component of the public signal $v - t^*$;*
3. *decreasing in the responsiveness of the political regime*;
4. *first increasing and then decreasing in the level of government repression;*
5. *increasing in opacity unless the political regime is responsive and the society radicalized;*
6. *decreasing in diversity unless the political regime is tolerant and the society radicalized or the political regime is repressive and the society moderate, when it is first increasing and then decreasing;*
7. *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up these results

Socio-pol. Var	**Socio political situation**		
	any		
repression	$\nearrow\searrow$		
	any		
responsiveness	$\searrow$		
	R&R	other	
opacity	$\searrow$	$\nearrow$	
	D&RO	other	
radicalization	$\nearrow$	$\searrow$	
	R&T	M&R	other
diversity	$\nearrow\searrow$		$\searrow$
	any		
unexp activism	$\searrow$		
	any		
antigovernment sentiment	$\nearrow\searrow$		

Table 13: swing citizens and sociopolitical variables

3.3.4 Vanguard Activism, Public Signal and Citizens' Protests

The aim of this subsection is to specify the situations characterized by point (η^*, θ) such that given vanguard's activism $(v - t^*)$, and the exogenous variables $(\mathbf{p}, \mathbf{s})$, some citizens will protest. These situations will be delimited by a curve representing the locus of points (η^*, θ) for which the participation to the protest showed by the citizens is strictly positive.

If the citizens play the strategy $s^{\infty}(\theta_i, v - t^*; \mathbf{p}, \mathbf{s})$ so that no one ever participates, then vanguard activism has no effect on population members' behavior. But if population members play $s^L(\theta_i, v - t^*; \mathbf{p}, \mathbf{s})$, activism can affect their behavior. The higher $v-t^*$, the more antigovernment sentiment each population member believes there is in society, since the expected average anti-government sentiment

$$E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)$$

is increasing in $v-t^*$. Moreover, for a given cutoff rule, the higher θ, the more people will participate. Hence, higher levels of $v-t^*$ make population members believe that protest is more likely to succeed. This increases the incremental benefit of participation.

Consider the minimal level $v-t^*$ for which it is possible the existence of an equilibrium cutoff, i.e.

$$v-t^* \geq f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right):$$

since

$$v-t^* = \theta + \underbrace{t-t^*+\eta}_{\eta^*} = \theta + \eta^*$$

then

$$\begin{aligned} v-t^* &\geq f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right) \Leftrightarrow \theta + \eta^* \geq f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right) \Leftrightarrow \\ &\Leftrightarrow \theta \geq f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right) - \eta^*. \end{aligned}$$

Thus

Definition 8 *The function*

$$\mathring{\theta} := f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right) - \eta^* =: \mathring{\theta}\left(\eta^*; \left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right)\right); \mathbf{p}, \mathbf{s}\right)$$

represents the locus of points (η^*, θ) *such that activism is equal to the minimum level requested to have a finite equilibrium cutoff* $\widehat{\theta}_L\left(v-t^*; \mathbf{p}, \mathbf{s}\right)$.

From this definition and previous results, it is immediate to derive the following properties of this function

Lemma 3 *The function*

$$\mathring{\theta}\left(\eta^*|\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right)\right), \mathbf{p}, \mathbf{s}\right)$$

has the following properties:

1. *it is a straight line with domain* $\mathbb{R}$*, codomain* $\mathbb{R}$*, slope -1 and vertical intercept*

$$f\left(\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right); \mathbf{p}, \mathbf{s}\right)$$

   defined for $\widehat{\theta}^*\left(\mathbf{p}, \mathbf{s}\right) \geq \frac{k}{\gamma}$;

2. *all the points* (η^*, θ) *of the curve* $\mathring{\theta}\left(\eta^*|\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right),\mathbf{p},\mathbf{s}\right)$ *are characterized by the same equilibrium cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ *which is the cutoff that requires the lowest level of activism in order to be an equilibrium;*

3. *as repression increases, the points on the line will be characterized by a higher level of unexpected activism* $v - t^*$ *and a higher cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$;

4. *as responsiveness decreases, the points on the line will be characterized by a higher level of activism, but with the same cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$;

5. *the vertical intercept*
$$f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$$
is

   (a) *decreasing in the responsiveness of the political regime;*

   (b) *increasing in the repression of the political regime*;

   (c) *increasing in opacity unless the political regime is responsive and tolerant;*

   (d) *increasing in diversity unless the political regime is responsive;*

   (e) *decreasing in radicalization unless the political regime is responsive but opaque and the society heterogenous.*

The function $\mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right);\mathbf{p},\mathbf{s}\right)$is linear because, given values of $(\mathbf{p},\mathbf{s})$, there is a minimum level of activism so that the cutoff associated with it is $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$. Given this equilibrium cutoff any real value of θ is compatible with the equilibrium, provided that η^* varies along with it while keeping the minimal level of activism constant. This leads to conclude that in this context θ is unrelated to the equilibrium cutoff $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ for which the level of activism is minimal; therefore despite the presence of the term $\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)$ in the formulation of $f(\widehat{\theta};\mathbf{p},\mathbf{s})$ the function $\mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right),\mathbf{p},\mathbf{s}\right)$ is linear..

3.3.5 Vanguard Activism, Public Signal and Successful Protests

The aim of this subsection is to specify the situations characterized by point (η^*, θ) such that given vanguard's activism $(v - t^*)$, and the exogenous variables $(\mathbf{p},\mathbf{s})$, enough citizens will protest so that the protest is successful. These situations will be delimited by a curve representing the locus of points (η^*, θ) such that the participation to the protest showed by the citizens is equal to T. In the previous subsections, it has been shown that a generic $\widehat{\theta} \geq \frac{k}{\gamma}$ is the equilibrium cutoff if and only if

$$v - t^* = \theta + \eta = \frac{[1-(1-\psi)\lambda]\,\widehat{\theta} + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) - \sigma_\varepsilon\Phi^{-1}(1-T)}{\psi}.$$

Moreover, the participation is exactly equal to T if and only if

$$\mathbb{P}(\theta_i \geq \widehat{\theta}) = T \Rightarrow 1 - \Phi\left(\frac{\widehat{\theta} - \theta}{\sigma_\varepsilon}\right) = T \Rightarrow \widehat{\theta} = \theta + \sigma_\varepsilon \Phi^{-1}(1-T)_\varepsilon.$$

From these equations, it follows that

$$\theta + \eta = \frac{[1-(1-\psi)\lambda]\left[\theta + \sigma_\varepsilon \Phi^{-1}(1-T)\right] + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right) - \sigma_\varepsilon\Phi^{-1}(1-T)}{\psi} \Rightarrow$$

$$\Rightarrow \eta^* = -\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi} + \frac{[(1-\psi)(1-\lambda)]\,\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi}.$$

Then we get the following definition.

Definition 9 *The function*

$$\eta^*(\theta; \mathbf{p}, \mathbf{s}) = \frac{[(1-\psi)(1-\lambda)]\,\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi} - \frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}$$

represents the locus of points (η^*, θ) such that the citizens' participation to the protest is equal to T.

The following result describes the properties of $\eta^*(\theta; \mathbf{p}, \mathbf{s})$.

Lemma 4 *The curve*

$$\eta^*(\theta; \mathbf{p}, \mathbf{s})$$

has the following properties:

1. *$\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon\Phi^{-1}(1-T), \widehat{\theta}^*(\mathbf{p}, \mathbf{s}) - \sigma_\varepsilon\Phi^{-1}(1-T)\right]$, while $\eta^* \in (-\infty, \infty)$. Note that $\frac{k}{\gamma} - \sigma_\varepsilon\Phi^{-1}(1-T)$ is positive if and only if $1 - T \leq \Phi(\frac{k}{\gamma\sigma_\varepsilon})$;*
2. *it has a minimum in $\widetilde{\theta}$;*
3. *it has an asymptote for $\theta = \frac{k}{\gamma} - \sigma_\varepsilon\Phi^{-1}(1-T)$;*
4. *it is convex;*[17]
5. *all points (η^*, θ) of the curve manifest a level of activism composed by a fixed part that varies in $(\mathbf{p}, \mathbf{s})$, and a random part that varies in $(\theta, \mathbf{p}, \mathbf{s})$;*
6. *if the government repression grows, all the points (η^*, θ) of the curve will be characterized by a higher level of activism and a higher cutoff;*

[17] This result implies that Figure 4 in De Mesquita 2020 is wrong because it depicts a function with an inflection point. Actually, $\eta^*(\theta; \mathbf{p}, \mathbf{s})$ is a function as a function of θ, but it is not invertible, because for some values of η, ther are two values of θ.

7. *if responsiveness decreases, all the points (η^*, θ) of the curve will be characterized by a higher level of activism and the same cutoff;*

8. *the relationship between all the points (η^*, θ) of the curve and the country radicalization, diversity and opacity in public information is uncertain: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters. Using simulations, we are able to derive the following results: (η^*, θ) is*

   (a) *increasing in opacity unless the political regime is responsive and the society is radicalized and homogeneous;*

   (b) *increasing in diversity;*

   (c) *decreasing in radicalization.*

4 The Equilibrium Outcomes

Which outcome will prevail depends on the citizen's behavior, which in turn will depend on the combinations of the functions

$$\mathring{\theta}\left(\eta^0; \widehat{\theta}^*(\mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right) := f\left(\widehat{\theta}^*(\mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right) - \eta^0 =$$

$$= \frac{[1-(1-\psi)\lambda]\widehat{\theta}^* + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}^*}\right) - \sigma_\varepsilon\Phi^{-1}(1-T)}{\psi} - \eta^0 \Leftrightarrow$$

$$\Leftrightarrow \eta^0 = -\mathring{\theta} + \left[\frac{1-(1-\psi)\lambda}{\psi}\right]\widehat{\theta}^* + \sqrt{\frac{\sigma_\eta^2}{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}^*}\right) + \frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(1-T)$$

and

$$\eta^*(\theta; \mathbf{p}, \mathbf{s}) = \frac{[(1-\psi)(1-\lambda)]\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma(\theta - \Phi^{-1}(T)\sigma_\varepsilon)}\right)}{\psi} + \frac{(1-\psi)\lambda\Phi^{-1}(T)\sigma_\varepsilon}{\psi}$$

in the space

$$(\theta, \eta).$$

The following result describes the relationship between the two curves:

Result 17

1. $\eta^*(\theta; \mathbf{p}, \mathbf{s}) \geq \eta^0(\theta; \mathbf{p}, \mathbf{s})$ *for any* θ;

2. $\eta^*(\theta; \mathbf{p}, \mathbf{s}) = \eta^0(\theta; \mathbf{p}, \mathbf{s})$ *when* $\theta = \widehat{\theta}^* - \sigma_\varepsilon\Phi^{-1}(1-T)$.

For any level of vanguard's activism $(v - t^*)$, there are three possible equilibrium outcomes:

1. **No protest**, when no citizens joins the protests against the government;

2. **Failed protest,** when some citizens protest, but they are not enough to change the policy;

3. **Successful protest**, when enough citizens join the protest and thus are able to change the policy.

Which outcome will prevail depends on the citizen's behavior, which in tun will depend on the realization of vanguard's activism, i.e. on $(v - t^*)$: since $v - t^* = \theta + t + \eta - t^*$, for given vanguard's effort and expected effort (t, t^*), then the citizens' behavior depend on the combinations of the behavioral functions described before

$$\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) ; \mathbf{p}, \mathbf{s}\right) \quad \text{and} \quad \eta^* (\theta; \mathbf{p}, \mathbf{s})$$

in the space

$$(\eta, \theta) .$$

To characterize these three possible outcomes, consider the properties of these two functions.

Result 18 *There exist a unique*

$$\eta_0^* = f\left(\widehat{\theta}^* ; \mathbf{p}, \mathbf{s}\right) - \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) + \sigma_\varepsilon \Phi^{-1}(1 - T)$$

such that

$$\mathring{\theta}\left(\eta_0^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) , \mathbf{p}, \mathbf{s}\right) = \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) - \sigma_\varepsilon \Phi^{-1}(1 - T) = \theta_0.$$

Moreover the point (η_0^*, θ_0)

1. *belongs to the curve* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) , \mathbf{p}, \mathbf{s}\right)$*;*

2. *it is the unique point of the curve characterized by participation exactly equal to* T*;*

3. *all the points of the curve characterized by* $\eta^* > \eta_0^*$ *will exhibit participation strictly less than* T*;*

4. *all the points of the curve characterized by* $\eta^* < \eta_0^*$ *will exhibit participation strictly greater than* T;

5. *it is the unique intersection with the curve* $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ *.*

Result 19 *The function* $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ *has the following properties*

1. *for a given* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1 - T), \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) - \sigma_\varepsilon \Phi^{-1}(1 - T)\right]$ *, the points of* $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ *are always on the east of* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) , \mathbf{p}, \mathbf{s}\right)$*,therefore* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) , \mathbf{p}, \mathbf{s}\right)$ *is dominated by* $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ *for all* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1 - T), \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) - \sigma_\varepsilon \Phi^{-1}(1 - T)\right]$*;*

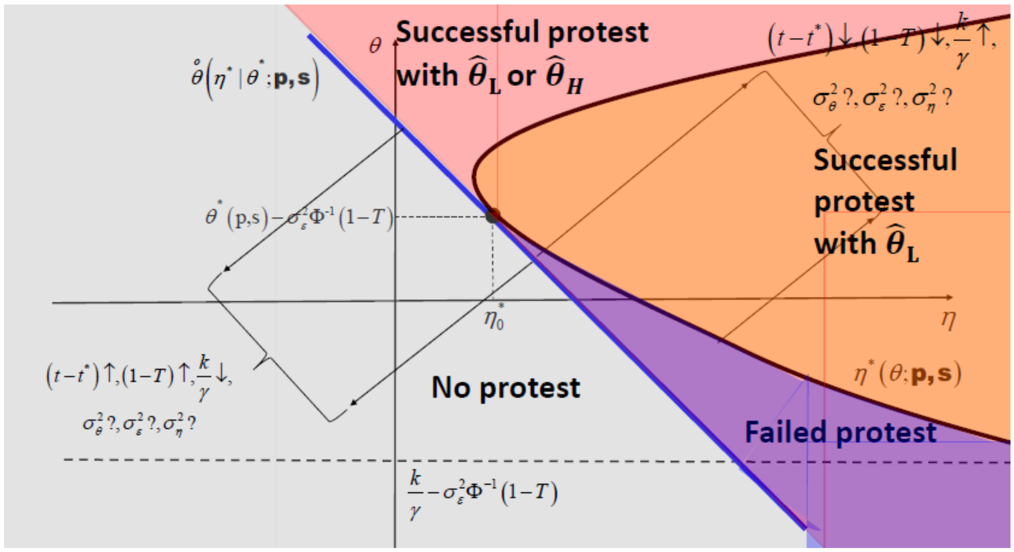

Figure 14: The possible equilibrium outcomes

2. *For descending values of* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T), \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) - \sigma_\varepsilon \Phi^{-1}(1-T)\right]$ *the points of* $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ *exhibit ascending values of activism.*

These results justify the following picture, where we combine them.

4.1 The Probabilities of No Protest, of Failed Protest, and of Successful Protest

Our aim in this subsection is to evaluate the probabilities of the three possible equilibrium outcomes:

1. no protest, the grey area;
2. unsuccessful protest, the viola area;
3. successful protest, the orange and the pink areas: orange, when the protest is successful only with the equilibrium cutoff $\widehat{\theta}_L (v - t^*; \mathbf{p}, \mathbf{s})$, pink when the protest is successful also with the equilibrium cutoff $\widehat{\theta}_H (v - t^*; \mathbf{p}, \mathbf{s})$.

Because of the results of previous subsection, there exists a unique $\eta^* = \eta_0^*$ such that

1. $\eta_0^* = f\left(\widehat{\theta}^* (\mathbf{p}, \mathbf{s}); \mathbf{p}, \mathbf{s}\right) - \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) - \Phi^{-1}(T)\sigma_\varepsilon$

2. $\mathring{\theta}\left(\eta_0^*; \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) = \theta_0$

3. $\eta^* (\theta_0; \mathbf{p}, \mathbf{s}) = \eta_0^*$

4. $\eta^* (\theta; \mathbf{p}, \mathbf{s})$ is defined for $\eta^* \geq \eta_0^*$

5. $\eta^* \geq \eta_0^* \Rightarrow \eta^* (\theta; \mathbf{p}, \mathbf{s}) \geq \mathring{\theta}\left(\eta^*; \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$.

A scenario where $\eta^* (\theta; \mathbf{p}, \mathbf{s}) \leq \mathring{\theta}\left(\eta^*; \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$ for $\eta^* \geq \eta_0^*$ is impossible, because if this were the case along the line $\eta^* (\theta; \mathbf{p}, \mathbf{s}) \geq \mathring{\theta}\left(\eta^*; \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$ for descending θ we would observe increasing participation at the same cutoff and activism level.

4.1.1 The Probability of Citizens' Protest

Result 20 *The probability of citizens' protest in equilibrium is*

1. *increasing in the responsiveness of the political regime*;
2. *decreasing in the repression of the political regime;*
3. *uncertain in country radicalization, diversity and opacity.*

Using simulations, we are able to derive the following result:

Result 21 *The probability of citizens' protest in equilibrium is*

1. *has no clear trend in opacity, even if responsiveness seems to induce an increasing trend;*
2. *increasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*
3. *increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

The following table sum up these results:

Socio-pol. Var	Socio political situation		
	any		
repression	$\searrow$		
	any		
responsiveness	$\nearrow$		
	D&RT	R&R	other
opacity	$\searrow$		$\nearrow$
	D&RT		other
radicalization	$\searrow$		$\nearrow$
	R&RRO	UTO	other
diversity	$\searrow$	$\searrow$	$\nearrow$
	any		
unexp activism	$\nearrow$		
	any		
antigovernment sentiment	$\nearrow$		

Table 14: probability of protest and sociopolitical variables

4.1.2 The Probability of No Protest

This result is the mirror image of previous one, however it has been proved independently and it interesting in itself.

Result 22 *The probability of no protest in equilibrium is*

1. *decreasing in the responsiveness of the political regime*;
2. *increasing in the repression of the political regime;*
3. *uncertain in country radicalization, diversity and public information opacity.*

Using simulations, we are able to derive the following result:

Result 23 *The probability of no protest in equilibrium*

1. *has no clear trend in opacity, even if responsiveness seems to induce a decreasing trend;*
2. *is decreasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*
3. *is decreasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

4.1.3 The Probability of Successful Protest

Result 24 *The probability of successful protest is*

1. *tending to increase in the responsiveness of the political regime*;
2. *decreasing in the repression of the political regime;*
3. *uncertain in* σ_θ^2, σ_ε^2 *and* σ_η^2*: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters.*

Using simulations, we are able to derive the following result.

Result 25 *The probability of successful protest is*

1. *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
2. *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
3. *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

The following table sum up our results:

Socio-pol. Var	**Socio political situation**		
	any		
repression	$\searrow$		
	any		
responsiveness	$\nearrow$		
	R&R	D&R	other
opacity	$\nearrow$	$\nearrow\searrow$	$\searrow$
	D&RT		other
radicalization	$\searrow$		$\nearrow$
	R&UT		other
diversity	$\searrow$		$\nearrow$
	any		
unexp activism	$\nearrow$		
	any		
antigovernment sentiment	$\nearrow$		

Table 15: probability of successful protest and sociopolitical variables

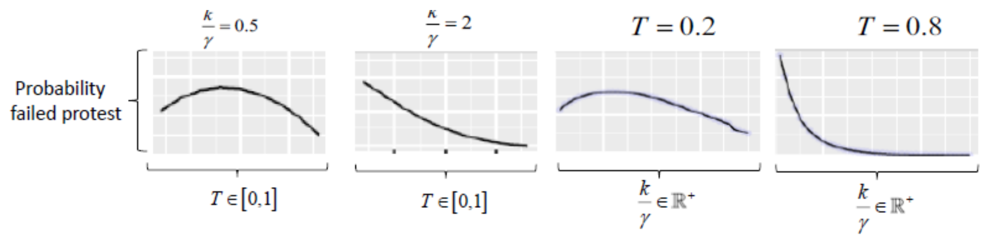

Figure 15: Probability of failed protest as function of T and of $\frac{k}{\gamma}$.

4.1.4 The Probability of Failed Protest

Result 26 *The probability of positive mobilization but failed protest has no clear monotone relationship with the responsiveness and the repression of the political regime, however our simulations show that the relations is*

1. *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogenous, otherwise is not monotonic, first decreasing and increasing*

2. *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*

3. *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*

4. *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*

5. *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

The following figure represents some interesting results of our simulations:

The following table sum up these results:

Socio-pol. Var	**Socio political situation**			
	RO			any
repression	$\nearrow\searrow$			$\searrow$
	D&T	R&TO		other
responsiveness	$\searrow\nearrow$			$\nearrow$
	D&T	R&T	R&R	other
opacity	$\nearrow$			$\searrow$
	H&R			other
radicalization	$\nearrow\searrow$			$\nearrow$
	UO	M&RT		other
diversity	$\nearrow$	$\nearrow\searrow$		$\searrow$
	any			
unexp activism	$\nearrow\searrow$			
	any			
antigovernment sentiment	$\nearrow\searrow$			

Table 16: probability of failed protest and sociopolitical variables

To understand the reason for the non monotonicity in responsiveness, consider an increase in T. As shown, the mass of citizens protesting $\mathcal{N}\left(\theta, \widehat{\theta}\left(v-t^{*}\right)\right)$ is

$$\mathcal{N}\left(\theta, \widehat{\theta}_{L}\left(v-t^{*} ; \mathbf{p}, \mathbf{s}\right)\right)=1-\Phi\left(\frac{\widehat{\theta}_{L}\left(v-t^{*} ; \mathbf{p}, \mathbf{s}\right)-\theta}{\sigma_{\varepsilon}}\right)$$

which is decreasing in $\widehat{\theta}_{L}\left(v-t^{*} ; \mathbf{p}, \mathbf{s}\right)$, which in turn is increasing in T. Thus the number of people revolting is decreasing as T increases, however also the threshold such that the protest is successful is increasing, thus it is possible that the probability of unsuccessful protest increases depending of which of the two factors is prevailing. The reason for the non monotonicity in $\frac{k}{\gamma}$ is similar.

5 Conclusion

The aim of this paper was to use the Bueno de Mesquita 2010 model to investigate the causes and the consequences of citizens' protests. In particular, our aim was to analyze the complex interaction between political regimes and countries' social characteristics. Let we sum up our main findings.

5.1 Causes

The analysis of the causes of citizens' protests can be analyzed considering the effects of our sociopolitical variables on the number of protesting citizens and on the probability of protests.

5.1.1 The Number of Protesting Citizens

In general, an increase in the common antigovernment sentiment and of the activism of vanguards increases the number of protesting citizens, whatever the political regime.

Similarly, an increase in the democratic dimensions of a polity - responsiveness, tolerance and transparence - increases the number of protesting citizens, apart from a paradoxical effect of transparency that reduces the number of protesting citizens in the most democratic polity.

Finally, both an increase in country diversity increases the number of protesting citizens in democratic regimes, otherwise reduces such number, while radicalization increases this number, unless the polity is responsive, but opaque and the country heterogenous.

5.1.2 The Probability of Protests

In general, an increase in the responsiveness and tolerance of the political regime increases the probability of protests, whatever the political regime, while an increase in transparency increases the probability of protests only if the polity is responsive, tolerant and the country heterogeneous or the polity is responsive and the country radicalized, otherwise the probability of protests is decreasing in transparence.

An increase in country diversity increases the probability of protests in responsive regimes in radicalized countries or in unresponsive, opaque but tolerant regimes, otherwise reduces such probability, while radicalization decreases this probability in responsive and tolerant regimes with heterogenous countries, otherwise the probability of protests is increasing.

5.2 Consequences

The analysis of the consequences of citizens' protests can be analyzed considering the effects of our sociopolitical variables on the probability of successful and of failed protests.

5.2.1 Successful Protests

A protest is successful if it is able to induce the government to change policy.

The probability of successful protests is usually increasing in our political and social variables, i.e. in responsiveness, tolerance, transparency, diversity and radicalization, apart from some interesting cases. Transparency, in the case of responsive regimes and heterogenous countries has a non monotonic effect, first decreasing and then increasing the probability of successful protests, and a decreasing relationship in the case of responsive regimes and radicalized countries. An increase in diversity decreases the probability of successful protests in an unresponsive but tolerant regime in a radicalized country, while an increase in radicalization reduces the probability of successful protests when the political regime is responsive and tolerant in a heterogenous country.

5.2.2 Failed Protests

A protest fails when the political regimes decides not to change policy notwithstanding the protests.

The probability of failed protests is usually increasing in responsiveness unless the political regimes is tolerant and the society heterogenous or tolerant but opaque and the society radicalized where this probability is first decreasing and then increasing. Similarly for tolerance, where its effects are non monotonic in a responsive but opaque regime. Transparency has a complex effect on this probability, which is usually increasing apart from the case of a tolerant political regime in a heterogenous or radicalized society or of a responsive regime in a radicalized society when its increase induces a reduction in this probability. An increase in diversity induces a decrement in this probability unless the political regime is unresponsive and opaque, or responsive and tolerant in a moderated society, when increments in diversity induce first an increment and then a decrement in this probability. Finally, an increment in radicalization generates a reduction in such probability, unless the regime is responsive and the society homogenous when first there is an increment and then a reduction.

5.3 Future Works

We believe that this paper has shown the efficacy of this model to analyze the causes and consequences of citizens' protests, extending Bueno de Mesquita analysis. On the other hand, according to our view, the main limitation of this model to analyze citizens' political behavior is twofold. First, it limits citizens' political behavior to a dual choice, whether to protest or not, while also the intensity of protests matters. Second, it limits citizens' political behavior to one dimension, while, beside protesting, citizens have other ways of dealing with public policies, for example voting or using violent tools. This notwithstanding, we believe this paper is a step towards a theory of how citizens come to political choices depending on different political and social settings, and how these choices affect the possible political outcomes.

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Part I
Appendices

7 Appendix A

This Appendix contains the analytical proofs of the results, that have been omitted by the main text to make it more readable.

8 Solving the Model

8.1 The Public Opinion

Result 1 *The expected country's level of antigovernment sentiment given i's private signal is*

1. *increasing in i's level of antigovernment sentiment;*
2. *when i is a moderate, decreasing in country radicalization and increasing in country diversity;*
3. *when i is an extremist, increasing in country radicalization and decreasing in country diversity;*
4. *almost coinciding with i's antigovernment sentiment, when country radicalization is increasing without limit with a finite amount of country diversity;*
5. *almost degenerated in 0, when country diversity is increasing without limit with a finite amount of country radicalization.*

Proof. Since[18]

$$\theta|\theta_i \sim N(\lambda\theta_i, \lambda\sigma_\varepsilon^2) \quad \text{where} \quad \lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}$$

1.
$$\frac{\partial E\left(\theta|\theta_i; \mathbf{p}, \mathbf{s}\right)}{\partial \theta_i} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} > 0$$

[18] See for example DeGroot 1970.

2. If $\theta_i < 0$
$$\frac{\partial E\left(\theta|\theta_i;\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\varepsilon^2} = -\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\ \theta_i > 0$$
$$\frac{\partial E\left(\theta|\theta_i;\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\theta^2} = \frac{\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\ \theta_i < 0$$
3. If $\theta_i > 0$
$$\frac{\partial E\left(\theta|\theta_i;\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\varepsilon^2} = -\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\ \theta_i < 0$$
$$\frac{\partial E\left(\theta|\theta_i;\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\theta^2} = \frac{\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\ \theta_i > 0$$
4.
$$\lim_{\sigma_\theta^2\to+\infty}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)\ \theta_i = \lim_{\sigma_\theta^2\to+\infty}\left(\frac{1}{1+\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}}\right)\ \theta_i = \left(\frac{1}{1+0}\right)\theta_i = \theta_i$$
5.
$$\lim_{\sigma_\varepsilon^2\to+\infty}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)\ \theta_i = \left(\frac{1}{1+\infty}\right)\theta_i = 0.$$

■

Result 2 *The expected country antigovernment sentiment given i's private and public signals is*

1. *increasing in i's level of antigovernment sentiment and in the unexpected component of the public signal;*
2. *increasing in the opacity of public information if and only if the unexpected component of the public signal is moderating*
3. *increasing in country radicalization if and only if the unexpected component of the public signal is*
   - *not strongly moderating for an extremist or*
   - *strongly incendiary for a moderate;*
4. *increasing in country political diversity if and only if the unexpected component of the public signal is*
   - *strongly incendiary for an extremist or*
   - *not strongly moderating for a moderate.*

Proof. Since[19]

$$\theta|\theta_i, v-t^* \sim N(\psi(v-t^*)+(1-\psi)\lambda\theta_i, \psi\sigma_\eta^2) \quad \text{where} \quad \psi = \frac{\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\varepsilon^2}$$

1.
$$\frac{\partial E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)}{\partial(v-t^*)} = \psi > 0$$
$$\frac{\partial E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)}{\partial\theta_i} = (1-\psi)\lambda > 0$$

2.
$$\frac{\partial E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)}{\partial\sigma_\eta^2} = \frac{(\sigma_\varepsilon^2+\sigma_\theta^2)(\sigma_\varepsilon^2\sigma_\theta^2)(\lambda\theta_i-(v-t^*))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* < \frac{\sigma_\theta^2\theta_i}{\sigma_\theta^2+\sigma_\varepsilon^2}$$

3.
$$\frac{\partial E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)}{\partial\sigma_\theta^2} = \frac{\sigma_\varepsilon^2\sigma_\eta^2(\sigma_\varepsilon^2(v-t^*)+\sigma_\eta^2(\theta_i))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* > -\frac{\theta_i\sigma_\eta^2}{\sigma_\varepsilon^2}$$

4.
$$\frac{\partial E\left(\theta|\theta_i, v-t^*; \mathbf{p}, \mathbf{s}\right)}{\partial\sigma_\varepsilon^2} = \frac{\sigma_\theta^2\sigma_\eta^2(\sigma_\theta^2(v-t^*-\theta_i)+\sigma_\eta^2(-\theta_i))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* > \frac{\theta_i(\sigma_\theta^2+\sigma_\eta^2)}{\sigma_\theta^2}.$$

■

Result 3 *The expected country antigovernment sentiment given i's private and public signals is*

1. *increasing in i's level of antigovernment sentiment and in the unexpected component of the public signal;*
2. *increasing in the opacity of public information if and only if the unexpected component of the public signal is smaller than $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i$, i.e., if the unexpected component of the public signal is moderating or strongly moderating;*
3. *increasing in country radicalization if and only if the unexpected component of the public signal is greater than $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$, i.e., if the unexpected component of the public signal is strong incendiary or for extremists is not strongly moderating*;

[19] See for example DeGroot 1970.

4. *increasing in country political diversity if and only if the unexpected component of the public signal is greater than* $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}\theta_i$*, i.e., if the unexpected component of the public signal is incendiary or strongly incendiary or for moderates is not strongly moderating.*

Proof. Analogous to the proof of result 2. ■

Result 4 *The public signal changes the behavior of citizens' expectations with respect to the case of private signal only when the signals are strongly incendiary or strongly moderating. Moreover, the space of strongly incendiary/moderating signals is shrinking when*

1. *the opacity in public information is increasing;*
2. *country radicalization is decreasing;*
3. *country diversity is decreasing.*

Proof.

1. If σ_η^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ decreases, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ increases shrinking the space of strongly incendiary/moderating signals;
2. If σ_θ^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ does not vary, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ decreases, therefore the space of strongly incendiary/moderating signals is shrinking when country radicalization is decreasing;
3. If σ_ε^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ increases, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ does not vary, therefore the space of strongly incendiary/moderating signals is shrinking when country diversity is decreasing.

■

8.2 Citizens' Behavior

Result 5 *Any citizen with type* $\theta_i \in \left(-\infty, \frac{k}{\gamma}\right)$ *has a dominant strategy not to participate whatever the private and public signals*

Proof. If the protest is not successful, the optimal choice is always $a_i = 0$ because:

$$k > 0 \Rightarrow 0 > -k$$

If the protest is successful the citizens choose $a_i = 0$ if and only if:

$$(1-\gamma)\theta_i > \theta_i - k \Leftrightarrow \theta_i < \frac{k}{\gamma}$$

Therefore $a_i = 0$ is a dominant strategy fo those citizens characterized by $\theta_i < \frac{k}{\gamma}$. The strategy $a_i = 1$ can not be a dominant strategy because if the protest is not successful the optimal choice does not depend on θ_i and it is always $a_i = 0$. ■

Result 6 *The measure of the silent group is* $\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right) > 0$, *which is*

1. *increasing in the government repression;*
2. *decreasing in the country antigovernment sentiment*;
3. *decreasing in the country diversity for repressive political regimes;*
4. *increasing in the country diversity for tolerant political regimes*

Proof.

1. $$\frac{\partial\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)}{\partial\frac{k}{\gamma}} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)\frac{1}{\sigma_\varepsilon} > 0$$

2. $$\frac{\partial\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)}{\partial\theta} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)\left(-\frac{1}{\sigma_\varepsilon}\right) < 0$$

3. $$\frac{\partial\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\varepsilon} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)\left(-\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon^2}\right) < 0 \Leftrightarrow \frac{k}{\gamma}-\theta > 0$$

   In this case, as σ_ε increases, the mass of silent citizens decreases with lower limit $\frac{1}{2}$. If $\sigma_\varepsilon \to 0$ the mass of silent citizens becomes equal to 1.

4. $$\frac{\partial\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)}{\partial\sigma_\varepsilon} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)\left(-\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon^2}\right) > 0 \Leftrightarrow \frac{k}{\gamma}-\theta < 0$$

   In this case, as σ_ε increases, the mass of silent citizens increases with upper limit $\frac{1}{2}$. If $\sigma_\varepsilon \to 0$ the mass of silent citizens becomes equal to 0.

■

Result 7 *Protests*

1. *are impossible if and only if*

$$\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon} \to \infty,$$

hence protests are always possible for tolerant political regimes;

2. *can be successful if and only if*

$$\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right) \leq 1-T \Leftrightarrow \frac{k}{\gamma} \leq \theta+\sigma_\varepsilon\Phi^{-1}\left(1-T\right)$$

restricting the set of political regims and societies where protests can succeed. In particular

   (a) *responsiveness should be greater than* $\Phi\left(\frac{\theta}{\sigma_\varepsilon}\right)$, *otherwise protests can't succeed;*

   (b) *when the regime is repressive, responsiveness should be greater than* $\frac{1}{2}$, *otherwise protests can't succeed.*

Proof.

1. Protests are impossible if the population is composed only by silent citizens therefore when:

$$P\left(\theta_i<\frac{k}{\gamma}|\theta\right)=1 \Leftrightarrow \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right)=1 \Leftrightarrow \frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}=+\infty \Leftrightarrow \frac{k}{\gamma}-\theta>0$$

2. The necessary but not sufficient condition for the protest to be successful is that the mass of silent citizens is less than $1-T$ as there would be at least a potential T portion of participants

$$P\left(\theta_i<\frac{k}{\gamma}|\theta\right) \leq 1-T \Leftrightarrow \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};\mathbf{p},\mathbf{s}\right) \leq 1-T \Leftrightarrow \frac{k}{\gamma} \leq \theta+\sigma_\varepsilon\Phi^{-1}\left(1-T\right)$$

   (a) Given $\frac{k}{\gamma}>0$ if $T>\Phi\left(\frac{\theta}{\sigma_\varepsilon}\right)$ protests can not succeed because if:

$$T>\Phi\left(\frac{\theta}{\sigma_\varepsilon}\right) \Rightarrow \theta+\sigma_\varepsilon\Phi^{-1}\left(1-T\right)<0$$

   and therefore to observe a winning protest, $\frac{k}{\gamma}$ would need to be less than a negative value

(b) The protests is not impossible if:

$$\frac{k}{\gamma} - \theta \leq \sigma_\varepsilon \Phi^{-1}(1-T)$$

If $\frac{k}{\gamma} > \theta$ in order to observe successful protests $\sigma_\varepsilon \Phi^{-1}(1-T)$ must be positive hence:

$$\sigma_\varepsilon \Phi^{-1}(1-T) > 0 \Leftrightarrow 1-T > \frac{1}{2}.$$

■

8.3 Citizens' Equilibrium Behavior

Conclusion 2 *The minimal level of the country antigovernment sentiment, such that the protest is successful when the common cutoff rule is* $\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right)$, *is*

$$\theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\ \mathbf{s}\right);T,\sigma_\varepsilon^2\right) = \widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right) - \sigma_\varepsilon \Phi^{-1}\left(1-T\right),$$

which is

1. *decreasing in the responsiveness of the political regime, such that*
   - *(a)* $\lim_{1-T\to 1} \theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);T,\sigma_\varepsilon^2\right) = -\infty$;
   - *(b)* $\lim_{1-T\to 0} \theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);T,\sigma_\varepsilon^2\right) = \infty$;
   - *(c)* $\theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);1-T=\frac{1}{2},\sigma_\varepsilon^2\right) = \widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right) \geq \frac{k}{\gamma}$;
   - *(d)* $\theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);1-T=\Phi^{-1}\left(\frac{\widehat{\theta}(v-t^*;\mathbf{p},\mathbf{s})}{\sigma_\varepsilon}\right),\sigma_\varepsilon^2\right) = 0$;
2. *linearly increasing or decreasing in the diversity of the country depending whether the political regime is unresponsive or responsive;*
3. *linearly increasing in the common cutoff* $\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right)$, *such that*
   - *(a) the minimum is* $\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}\left(1-T\right)$;
   - *(b)* $\lim_{\widehat{\theta}(v-t^*)\to\infty} \theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);T,\sigma_\varepsilon^2\right) = \infty$;
   - *(c)* $\theta^*\left(\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right);T,\sigma_\varepsilon^2\right) = 0$ *if and only if* $\widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right) = \sigma_\varepsilon \Phi^{-1}\left(1-T\right)$.

Proof.

1. (a)

$$\lim_{1-T\to 1} \widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right) - \sigma_\varepsilon \Phi^{-1}\left(1-T\right) = \widehat{\theta}\left(v-t^*;\mathbf{p},\mathbf{s}\right) - \infty = -\infty$$

(b)

$$\lim_{1-T\to 0}\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)+\infty=+\infty$$

(c)

$$\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T=\frac{1}{2},\sigma_{\varepsilon}^{2}\right)=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-0=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)$$

(d)

$$\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T=\Phi^{-1}\left(\frac{\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)}{\sigma_{\varepsilon}}\right),\sigma_{\varepsilon}^{2}\right)=$$
$$=\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)-\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)=0$$

2.

$$\frac{\partial\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T,\sigma_{\varepsilon}^{2}\right)}{\partial\sigma_{\varepsilon}}=-\Phi^{-1}(1-T)>0\Leftrightarrow 1-T<\frac{1}{2}$$

3.

(a)

$$\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)\geq\frac{k}{\gamma}\Rightarrow\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T,\sigma_{\varepsilon}^{2}\right)\geq\frac{k}{\gamma}-\sigma_{\varepsilon}\Phi^{-1}(1-T)$$

(b)

$$\lim_{\widehat{\theta}(v-t^{*})\to\infty}\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T,\sigma_{\varepsilon}^{2}\right)=\infty-\sigma_{\varepsilon}\Phi^{-1}(1-T)=+\infty$$

(c)

$$\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)=\sigma_{\varepsilon}\Phi^{-1}(1-T)$$
$$\Rightarrow\theta^{*}\left(\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right);1-T,\sigma_{\varepsilon}^{2}\right)=\sigma_{\varepsilon}\Phi^{-1}(1-T)-\sigma_{\varepsilon}\Phi^{-1}(1-T)=0.$$

■

Result 8 *i's subjective belief about the probability of policy change, given the private and the public signals and the belief that all other players j participate if and only if* $\theta_j\geq\widehat{\theta}\left(v-t^{*};\mathbf{p},\mathbf{s}\right)$, *is*

1. *increasing in the responsiveness of the political regime;*
2. *increasing in the public signal and in its unexpected component;*

3. *increasing in the private signal;*

4. *uncertain in radicalization, diversity and opacity.*

Proof. Consider i's subjective belief about the probability of policy change:

$$1 - \Phi\left(\frac{\widehat{\theta}\left(v - t^{*}; \mathbf{p}, \mathbf{s}\right) - \sigma_{\varepsilon}\Phi^{-1}\left(1 - T\right) - \psi\left(v - t^{*}\right) - \left(1 - \psi\right)\lambda\theta_{i}}{\sqrt{\psi\sigma_{\eta}^{2}}}; \mathbf{p}, \mathbf{s}\right)$$

1. If $1 - T$ grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.

2. If $v - t^{*}$ grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.

3. If θ_i grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.

4. The uncertainty is related to the fact that that trends vary depending on the values assumed by $T, v - t^{*}, \theta_i, \sigma_{\varepsilon}, \sigma_{\theta}, \sigma_{\eta}$ jointly.

■

Result 9 *i's subjective belief about the probability of policy change, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v - t^{*}; \mathbf{p}, \mathbf{s}\right)$, is*

1. *increasing in opacity unless the political regime is responsive and the society is radicalized and heterogenous;*

2. *increasing in diversity unless the political regime is unresponsive;*

3. *increasing in radicalization unless the political regime is responsive.*

Proof. see Appendix B. ■

Result 10 *Any possible equilibrium cutoff satisfies the following restriction*

$$\widehat{\theta}\left(v - t^{*}; \mathbf{p}, \mathbf{s}\right) \in \left[\frac{k}{\gamma}, \infty\right].$$

Proof. The equilibrium condition is:

$$1 - \Phi\left(\frac{\widehat{\theta}\left(v - t^{*}; \mathbf{p}, \mathbf{s}\right)\left(1 - (1 - \psi)\lambda\right) - \sigma_{\varepsilon}\Phi^{-1}\left(1 - T\right) - \psi\left(v - t^{*}\right)}{\sqrt{\psi\sigma_{\eta}^{2}}}; \mathbf{p}, \mathbf{s}\right) = \frac{k}{\gamma\widehat{\theta}\left(v - t^{*}; \mathbf{p}, \mathbf{s}\right)}$$

If $\widehat{\theta}(v-t^*;\mathbf{p},\mathbf{s}) < \frac{k}{\gamma}$ it would be impossible to solve the equation because the equilibrium condition would require a probability strictly greater than the unit. For this reason, the equilibrium condition by construction has as its solution in terms of equilibrium cutoff values greater than $\frac{k}{\gamma}$. ■

Lemma 5 *$f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ is*

1. *U shaped in $\widehat{\theta} \in \left(\frac{k}{\gamma},\infty\right)$, reaching a global minimum denoted by*

$$\widehat{\theta}^*(\mathbf{p},\mathbf{s}) = \arg\min_{\widehat{\theta}} f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right);$$

2. *decreasing in the responsiveness of the political regime;*
3. *increasing in the repression of the political regime*;
4. *uncertain in country diversity, radicalization and in information opacity.*

Proof.

1.
$$\frac{\partial f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)}{\partial\widehat{\theta}} = \frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi} =$$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\frac{1}{\sqrt{2\pi}}e^{-\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi} =$$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}$$

Since

$$\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\left(\frac{1}{\widehat{\theta}^2}\right) \leq \frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\psi}}{\sigma_\eta}$$

then, consider the function

$$g(\widehat{\theta}) = e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\left(\frac{1}{\widehat{\theta}^2}\right):$$

- $g(\widehat{\theta})$ is a continuous function

- $g(\widehat{\theta})$ has domain $\left(\frac{k}{\gamma},\infty\right)$ and codomain $(0,+\infty)$
- $g(\widehat{\theta})$ is an injective function
- $g(\widehat{\theta})$ is monotonically decreasing in $\widehat{\theta}$ along the entire domain with $\lim_{\widehat{\theta}\to\frac{k}{\gamma}} g(\widehat{\theta})=\infty$ and $\lim_{\widehat{\theta}\to\infty} g(\widehat{\theta})=0$.

Moreover
$$\frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\psi}}{\sigma_\eta}\in(0,+\infty)$$
Therefore there exist an unique $\widehat{\theta}(\gamma,k)=\widehat{\theta}^*(\gamma,k)$ such that
$$g(\widehat{\theta})\leq\frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\psi}}{\sigma_\eta}\Leftrightarrow\widehat{\theta}\geq\widehat{\theta}^*(\gamma,k).$$
Therefore
$$min_{\widehat{\theta}}f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)=f\left(\widehat{\theta}^*\left(\mathbf{p},\mathbf{s}\right),\mathbf{p},\mathbf{s}\right).$$
Since the derivative is positive, once it passes the value where it is null $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ it will be a point of minimum and the function will have a U-shape.

2. $$\frac{\partial f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)}{\partial T}=\frac{\partial}{\partial T}\left[\frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(T)\right]=\frac{\sigma_\varepsilon}{\psi}\frac{1}{\phi[\Phi^{-1}(T)]}>0$$
   Since it is increasing in T it will be decreasing in $1-T$

3. $$\frac{\partial f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)}{\partial\frac{k}{\gamma}}=\frac{\partial}{\partial\frac{k}{\gamma}}\left[\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]=\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)]}\frac{1}{\widehat{\theta}}>0$$

4. The uncertainty is related to the fact that that trends vary depending on the values assumed by $T,\theta_i,\sigma_\varepsilon,\sigma_\theta,\sigma_\eta$ jointly

■

Result 11 $f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ *is*

1. *increasing in opacity unless the political regime is responsive and tolerant;*
2. *increasing in diversity unless the political regime is responsive;*
3. *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Proof. see Appendix B. ■

Lemma 6 $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$ *is*

1. *independent from responsiveness;*
2. *increasing in the level of government repression;*
3. *decreasing in the country diversity;*
4. *increasing in the country radicalization;*
5. *increasing in opacity.*

Proof. $\hat{\theta}^{*}$ is derived by:

$$\frac{df(\hat{\theta}|...)}{d\hat{\theta}} = 0$$

$$\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\left(-\frac{k}{\gamma\hat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=0$$

1. T is not involved within the equation $\Rightarrow \hat{\theta}^{*}$ does not depend on T
2. Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

$$f\left(\widehat{\theta}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

   If $\frac{k}{\gamma}$ grows the asymptote of the curve in $\widehat{\theta}(\mathbf{p},\mathbf{s})$ grows. In addition, each $\widehat{\theta}(\mathbf{p},\mathbf{s})$ will be characterized by a greater value, so the curve not only shifts to the right due to the asymptote, but also grows upwards. Consequently given the U-shape of the curve and this transformation $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$ increases

3. Consider the condition useful to derive $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$:

$$\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\left(-\frac{k}{\gamma\hat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=0$$

   Can be rewritten as

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2}=\frac{[1-(1-\psi)\lambda]}{\sqrt{\psi}}\frac{\gamma}{k}\frac{1}{\sigma_\eta}$$

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2}=\frac{\sigma_\theta^2\sigma_\epsilon+\sigma_\eta^2\sigma_\epsilon}{\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}}\frac{1}{\sigma_\eta\sigma_\theta}\frac{\gamma}{k}$$

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2}=A$$

Only the right-hand member (called A) depends on $\sigma_\epsilon, \sigma_\eta, \sigma_\theta$

$$\frac{dA}{d\sigma_\epsilon}=\frac{\frac{2(\sigma_\theta^2+\sigma_\eta^2)(\sigma_\theta^2\sigma_\eta^2)}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}\right)}}{\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}\right)^2}=\frac{2(\sigma_\theta^2+\sigma_\eta^2)(\sigma_\theta^2\sigma_\eta^2)}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}\right)^3}\geq 0$$

If σ_ϵ increases $\Rightarrow A$ increases and therefore since the left member is not affected by the variation of $\sigma_\epsilon \Rightarrow \hat{\theta}^*$ decreases

4.
$$\frac{dA}{d\sigma_\theta}=\frac{1}{\sigma_\theta\sigma_\eta}\frac{-2\sigma_\eta^4\sigma_\epsilon^3-4\sigma_\eta^4\sigma_\theta^2\sigma_\epsilon-2\sigma_\theta^2\sigma_\epsilon^3\sigma_\eta^2}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}\right)^3}<0$$

If σ_θ increases $\Rightarrow A$ decreases and therefore since the left member is not affected by the variation of $\sigma_\theta \Rightarrow \hat{\theta}^*$ increases

5. Similarly to the previous case if σ_η increases $\Rightarrow A$ decreases and therefore $\hat{\theta}^*$ increases.

■

Result 12 *Consider the possible equilibrium payoff* $\widehat{\theta}\,(v-t^*;\mathbf{p},\mathbf{s})$:

1. *If* $(v-t^*)$ *is small enough, i.e. if* $(v-t^*)<f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$*, then there exists no finite equilibrium cutoff, thus* $\widehat{\theta}\,(v-t^*;\ \mathbf{p},\mathbf{s})=\infty$;

2. *If* $(v-t^*)=f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$*, then there exists one finite equilibrium cutoff and* $\widehat{\theta}\,(v-t^*;\mathbf{p},\mathbf{s})=\widehat{\theta}^*(\mathbf{p},\mathbf{s})$;

3. *If* $(v-t^*)$ *is big enough, i.e. if* $(v-t^*)>f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$*, then there exists two finite equilibrium cutoff, thus* $\widehat{\theta}\,(v-t^*;\ \mathbf{p},\mathbf{s})\in\left\{\widehat{\theta}_L\,(v-t^*;\mathbf{p},\mathbf{s}),\widehat{\theta}_H\,(v-t^*;\mathbf{p},\mathbf{s})\right\}$ *with*
$$\frac{k}{\gamma}<\widehat{\theta}_L\,(v-t^*;\mathbf{p},\ \mathbf{s})<\widehat{\theta}^*(\mathbf{p},\mathbf{s})<\widehat{\theta}_H\,(v-t^*;\mathbf{p},\mathbf{s})\,.$$

Proof. The function $f\left(\widehat{\theta}\,(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$ is U-shaped in $\widehat{\theta}\,(\mathbf{p},\mathbf{s})$ therefore there is an equilibrium cutoff $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ that requires the minimum level of violence $f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$.

1. If $v-t^*$ is below this level no finite cutoff can be the equilibrium, because all the finite cutoffs in order to be the equilibrium require a level of violence greater than $f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$.
2. If $v-t^*$ is exactly equal to this level the equilibrium cutoff will be exactly that finite value which requires the minimum level of violence therefore $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$.
3. If $v-t^*$ exceeds this value then given its U-shape there will exist two finite equilibrium values.

■

Result 13 *The finite equilibrium cutoff* $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ *is*

1. *decreasing in the unexpected component of the public signal;*
2. *decreasing in the responsiveness of the political regime*;
3. *increasing in the repression of the political regime*;*;*
4. *uncertain in country diversity, radicalization and in information opacity.*

Proof.

1. By setting the level of $v-t^*$ above the minimum level required to observe a finite equilibrium cutoff, we will observe two equilibrium cutoffs. Since the curve is U-shaped as the level of $v-t^*$ decreases the two solutions will come closer together until they coincide into one when the violence reaches the compatibility minimum. Therefore the lower cutoff will increase towards $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ while the upper cutoff will decrease towards $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ as $v-t^*$ decreases. Consequently, the lower cutoff grows as $v-t^*$ decreases.
2. Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

$$f\left(\widehat{\theta}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

   If T increases the curve is shifted upwards then fixed a level of violence $v-t^*$ given the U-shape of the curve we will observe two new solutions closer together, in other words the lower equilibrium cutoff will be higher, the upper one lower. Therefore the lower equilibrium cutoff is increasing in T and decreasing in $1-T$.
3. Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

$$f\left(\widehat{\theta}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

If $\frac{k}{\gamma}$ grows the asymptote of the curve in $\widehat{\theta}(\mathbf{p},\mathbf{s})$ grows. In addition, each $\widehat{\theta}(\mathbf{p},\mathbf{s})$ will be characterized by a greater value, so the curve not only shifts to the right due to the asymptote, but also grows upwards. Consequently given the U-shape of the curve fixed a level of violence $v-t^*$ we will have both solutions increased. While when $\frac{k}{\gamma}$ decreases the situation is the opposite therefore the lower cutoff is increasing in $\frac{k}{\gamma}$.

4. The uncertainty is related to the fact that that trends vary depending on the values assumed by $T, \sigma_\varepsilon, \sigma_\theta, \sigma_\eta$ jointly.

■

Result 14 *The finite equilibrium cutoff* $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ *is*

1. *increasing in opacity unless the political regime is responsive and tolerant;*
2. *increasing in diversity unless the political regime is responsive;*
3. *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Proof. see Appendix B. ■

Result 15 *The percentage of* ***protesting citizens*** *is*

$$1-\Phi\left(\frac{\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})-\theta}{\sigma_\varepsilon}\right),$$

which is

1. *increasing in the antigovernment sentiment;*
2. *increasing in the unexpected component of the public signal* $v-t^*$*;*
3. *increasing in the responsiveness of the political regime*;
4. *decreasing in the repression of the political regime;*
5. *decreasing in the opacity of the political regime unless the political regime is responsive and tolerant;*
6. *decreasing in diversity unless the political regime is responsive when* $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})<\theta$ *and after this threshold is increasing with upper limit* $\frac{1}{2}$*;*
7. *increasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Proof.

1. If θ increases the argument of $\Phi()$ decreases therefore the percentage of protesting citizens increases

2. If $v - t^*$ increases $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ decreases (Result 13), the argument of $\Phi()$ decreases therefore the percentage of protesting citizens increases

3. If T increases $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ increases (Result 13), the argument of $\Phi()$ increases therefore the percentage of protesting citizens decreases

4. If $\frac{k}{\gamma}$ increases $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ increases (Result 13), the argument of $\Phi()$ increases therefore the percentage of protesting citizens decreases

5. $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ is increasing in opacity unless the political regime is responsive, tolerant and the society is radicalized or the political regime is responsive and the society diverse (Result 14) therefore in these scenarios the percentage of protesting citizens decreases because the argument of $\Phi()$ increases

6. $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ is increasing in diversity unless the political regime is responsive (Result 14) therefore in these scenarios the percentage of protesting citizens decreases rapidly since $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) < \theta$ because the argument of $\Phi()$ is negative and increasing and after this threshold the the argument of $\Phi()$ is positive and decreasing with upper limit $\frac{1}{2}$

7. $\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s})$ is decreasing in radicalization unless the political regime is responsive but opaque and the society diverse (Result 14) therefore in these scenarios the percentage of protesting citizens increases because the argument of $\Phi()$ decreases.

■

Result 16 *The percentage of* ***swing citizens*** *is*

$$\Phi\left(\frac{\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) - \theta}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\frac{k}{\gamma} - \theta}{\sigma_\varepsilon}\right),$$

which is

1. *first increasing, then decreasing in the antigovernment sentiment: the maximum is reached when*

$$\theta = \frac{1}{2}\left[\widehat{\theta}_L(v - t^*; \mathbf{p}, \mathbf{s}) + \frac{k}{\gamma}\right];$$

2. *decreasing in the unexpected component of the public signal $v - t^*$;*

3. *decreasing in the responsiveness of the political regime*;
4. *uncertain in the level of government repression;*
5. *increasing in opacity unless the political regime is responsive and the society radicalized;*
6. *decreasing diversity unless the political regime is tolerant and the society radicalized or the political regime is repressive and the society moderate, when it is first increasing and then decreasing;*
7. *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Proof.

1. $$\frac{\partial\left(\Phi\left(\frac{\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})-\theta}{\sigma_\varepsilon}\right)-\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)\right)}{\partial\theta}=\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\left[-e^{-\left(\frac{\hat{\theta}-\theta}{\sigma_\varepsilon}\right)^2}+e^{-\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)^2}\right]\geq 0\Leftrightarrow$$
$$\Leftrightarrow-\left(\frac{\hat{\theta}-\theta}{\sigma_\varepsilon}\right)^2\leq-\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)^2\Leftrightarrow\theta\leq\frac{1}{2}\left[\hat{\theta}+\frac{k}{\gamma}\right]$$
2. If $v-t^*$ increases $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ decreases (Result 13) and therefore the first member decreases while the second does not vary hence the percentage decreases.
3. If T increases $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ increases (Result 13) and therefore the first member increases while the second does not vary hence the percentage increases. Clearly it is the opposite for $1-T$.
4. If $\frac{k}{\gamma}$ increases both member increases therefore initially the percentage increases, but after a certain threshold it decreases
5. $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ is increasing in opacity unless the political regime is responsive and the society radicalized (Result 14) therefore in these scenarios the first member increases while the second does not vary hence the percentage increases
6. $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ is increasing in diversity unless the political regime is responsive (Result 14) therefore in these scenarios the percentage initially increases and then it decreases because as the diversity increases both members tends to $\frac{1}{2}$
7. $\widehat{\theta}_L(v-t^*;\mathbf{p},\mathbf{s})$ is decreasing in radicalization unless the political regime is responsive but opaque and the society diverse (Result 14) therefore in these scenarios the first member decreases while the second does not vary hence the percentage decreases.

■

Lemma 7 *The function*

$$\mathring{\theta}\left(\eta^*|\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right),\mathbf{p},\mathbf{s}\right)$$

has the following properties:

1. *it is a straight line with domain* $\mathbb{R}$*, codomain* $\mathbb{R}$*, slope -1 and vertical intercept*
$$f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$$
defined for $\widehat{\theta}^*(\mathbf{p},\mathbf{s}) \geq \frac{k}{\gamma}$;

2. *all the points* (η^*,θ) *of the curve* $\mathring{\theta}\left(\eta^*|\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right),\mathbf{p},\mathbf{s}\right)$ *are characterized by the same equilibrium cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ *which is the cutoff that requires the lowest level of activism in order to be an equilibrium;*

3. *as repression increases, the points on the line will be characterized by a higher level of unexpected activism* $v-t^*$ *and a higher cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$;

4. *as responsiveness decreases, the points on the line will be characterized by a higher level of activism, but with the same cutoff* $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$;

5. *the vertical intercept*
$$f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$$
is

   (a) *decreasing in the responsiveness of the political regime;*

   (b) *increasing in the repression of the political regime*;

   (c) *increasing in opacity unless the political regime is responsive and tolerant;*

   (d) *increasing in diversity unless the political regime is responsive;*

   (e) *decreasing in radicalization unless the political regime is responsive but opaque and the society heterogenous.*

Proof.

$$\mathring{\theta} := f\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)-\eta^* =: \mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*(\mathbf{p},\mathbf{s})\right);\mathbf{p},\mathbf{s}\right)$$

1. $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s}) \in \left(\frac{k}{\gamma},\infty\right)$ is a fixed value derived by the realization of the exogenous parameters $T,k,\gamma,\sigma_{\varepsilon},\sigma_{\theta},\sigma_{\eta}$ hence also $f\left(\widehat{\theta}^{*}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right) \in \mathbb{R}$ is a fixed value. Consequently the function is a straight line because it generates θ translating $-\eta^{*}$ by a fixed amount $f\left(\widehat{\theta}^{*}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$. Since η^{*},θ are drawn by a normal distribution with support $\mathbb{R}$ the domain and the codomain of the function must be the set $\mathbb{R}$. The slope is the coefficient assigned to η^{*} hence -1, while the intercept is the value obtained when $\eta^{*}=0$ hence $f\left(\widehat{\theta}^{*}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$

2. All the points of the curve are characterized by the same level of violence

$$\theta+\eta^{*}=f\left(\widehat{\theta}^{*}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$$

   That is the level of violence associated to the cutoff $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$, therefore they are all characterized by the same cutoff $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$

3. The function $f\left(\widehat{\theta}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$ is increasing in $\frac{k}{\gamma}$ (Lemma 1) therefore its minimum value would be higher as $\frac{k}{\gamma}$ increases, consequently every point will be characterized by a higher level of violence. Moreover also $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$ is increasing in $\frac{k}{\gamma}$ (Lemma 2) consequently every point will be characterized by a higher cutoff.

4. The function $f\left(\widehat{\theta}(\mathbf{p},\mathbf{s});\mathbf{p},\mathbf{s}\right)$ is decreasing in $1-T$ (Lemma 1) therefore its minimum value would be lower as $1-T$ increases, consequently every point will be characterized by a lower level of violence. Moreover $\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})$ is independent from $1-T$ (Lemma 2) consequently every point will be characterized by the same cutoff.

5. Follows from Result 11 and Lemma 1.

■

Lemma 8 *The curve*

$$\eta^{*}(\theta;\mathbf{p},\mathbf{s})$$

has the following properties:

1. $\theta \in \left(\frac{k}{\gamma}-\sigma_{\varepsilon}\Phi^{-1}(1-T),\widehat{\theta}^{*}(\mathbf{p},\mathbf{s})-\sigma_{\varepsilon}\Phi^{-1}(1-T)\right]$, *while* $\eta^{*}\in(-\infty,\infty)$. *Note that* $\frac{k}{\gamma}-\sigma_{\varepsilon}\Phi^{-1}(1-T)$ *is positive if and only if* $1-T\leq\Phi(\frac{k}{\gamma\sigma_{\varepsilon}})$;

2. *it has a minimum in* $\widetilde{\theta}$;

3. *it has an asymptote for* $\theta=\frac{k}{\gamma}-\sigma_{\varepsilon}\Phi^{-1}(1-T)$;

4. *it is convex;*[20]

5. *all points* (η^*, θ) *of the curve manifest a level of activism composed by a fixed part that varies in* $(\mathbf{p}, \mathbf{s})$, *and a random part that varies in* $(\theta, \mathbf{p}, \mathbf{s})$;

6. *if the government repression grows, all the points* (η^*, θ) *of the curve will be characterized by a higher level of activism and a higher cutoff;*

7. *if responsiveness decreases, all the points* (η^*, θ) *of the curve will be characterized by a higher level of activism and the same cutoff;*

8. *the relationship between all the points* (η^*, θ) *of the curve and the country radicalization, diversity and opacity in public information is uncertain: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters. Using simulations, we are able to derive the following results:* (η^*, θ) *is*

   (a) *increasing in opacity unless the political regime is responsive and the society is radicalized and homogeneous;*

   (b) *increasing in diversity;*

   (c) *decreasing in radicalization.*

Proof.

$$\eta^*(\theta; \mathbf{p}, \mathbf{s}) = \frac{[(1-\psi)(1-\lambda)]\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi} - \frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}$$

1. The argument of $\Phi^{-1}()$ must be contained in the set $(0,1)$ therefore $\theta \geq \frac{k}{\gamma} - \sigma_\varepsilon\Phi^{-1}(1-T)$, while there are no constraints on the codomain of the function

2.

$$\frac{\partial\eta^*()}{\partial\theta} = \frac{(1-\psi)(1-\lambda)}{\psi} + \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}}\left(-\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) \geq 0 \Leftrightarrow$$

$$\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) \leq \frac{(1-\psi)(1-\lambda)}{\psi}$$

   It is possible to observe that

$$\frac{(1-\psi)(1-\lambda)}{\psi}$$

[20] This result implies that Figure 4 in De Mesquita 2020 is wrong because it depicts a function with an inflection point. Actually, $\eta^*(\theta; \mathbf{p}, \mathbf{s})$ is a function as a function of θ, but it is not invertible, because for some values of η, ther are two values of θ.

is positive and fixed

$$\lim_{\theta-\Phi^{-1}(T)\sigma_\epsilon\to\frac{k}{\gamma}} \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma\left(\theta-\Phi^{-1}(T)\sigma_\varepsilon\right)}\right)\right)^2}{2}}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right)=+\infty$$

$$\lim_{\theta-\Phi^{-1}(T)\sigma_\epsilon\to+\infty} \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma\left(\theta-\Phi^{-1}(T)\sigma_\varepsilon\right)}\right)\right)^2}{2}}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right)=0^+$$

Therefore there must be a $\overline{\theta}$ such that when $\theta\geq\overline{\theta}$ the inequality is satisfied hence $\overline{\theta}$ is the minimum

3. Follows from point 1

$$\frac{\partial^2\eta^*()}{\partial\theta^2}=\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma\left(\theta-\Phi^{-1}(T)\sigma_\varepsilon\right)}\right)\right)^2}{2}}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^3}\right)\times$$
$$\times\left(2+e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma\left(\theta-\Phi^{-1}(T)\sigma_\varepsilon\right)}\right)\right)^2}{2}}\left(\frac{k\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^3}\right)\right)$$

is always positive hence the function is strictly convex if we consider the lower cutoff

4.

$$\eta^*()=-\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}+\frac{\left[(1-\psi)\left(1-\lambda\right)\right]\theta+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi}.$$

5.

$$\frac{\partial\eta^*\left(\theta;\mathbf{p},\mathbf{s}\right)}{\partial\frac{k}{\gamma}}=\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)\right)}\frac{1}{\theta+\sigma_\varepsilon\Phi^{-1}(1-T)}>0$$

When $\frac{k}{\gamma}$ increase the curve increases therefore all the points will be characterized by a higher level of violence and higher cutoff

6.

$$\frac{\partial\eta^*\left(\theta;\mathbf{p},\mathbf{s}\right)}{\partial T}=\frac{\lambda(1-\psi)\sigma_\varepsilon}{\psi}\frac{1}{\phi(\Phi^{-1}(T))}>0$$

When T increases the curve is positively shifted on the η^* axes hence every point will be characterized by the same cutoff and a higher level of violence

7. see appendix B.

■

9 The Equilibrium Outcomes

Result 17

1. *$\eta^*(\theta;\mathbf{p},\mathbf{s}) \geq \eta^0(\theta;\mathbf{p},\mathbf{s})$ for any θ;*

2. *$\eta^*(\theta;\mathbf{p},\mathbf{s}) = \eta^0(\theta;\mathbf{p},\mathbf{s})$ when $\theta = \widehat{\theta}^* - \sigma_\varepsilon \Phi^{-1}(1-T)$.*

Proof.

1. Points characterised by a level of violence below the minimum compatible with equilibrium by construction do not allow mobilization by citizens to be observed because there would be no participation rule. Consequently, the points on curve $\eta^*(\theta;\mathbf{p},\mathbf{s})$ that guarantee exactly T participation will always be above the points on curve $\eta^0(\theta;\mathbf{p},\mathbf{s})$ for any θ

2. The two curves are equivalent at the point where the minimum violence level guarantees participation equal to T. The curve $\eta^0(\theta;\mathbf{p},\mathbf{s})$ is characterized by equilibrium cutoff $\widehat{\theta}^*$, the participation is exactly equal to T when is satisfied the condition $\theta = \hat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)$, therefore in the point $\theta = \widehat{\theta}^* - \sigma_\varepsilon \Phi^{-1}(1-T)$. the curve $\eta^0(\theta;\mathbf{p},\mathbf{s})$ shows participation equal to T.

■

Result 18 *There exist a unique*

$$\eta_0^* = f\left(\widehat{\theta}^*;\mathbf{p},\mathbf{s}\right) - \widehat{\theta}^*(\mathbf{p},\mathbf{s}) + \sigma_\varepsilon \Phi^{-1}(1-T)$$

such that

$$\mathring{\theta}\left(\eta_0^* | \widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) = \widehat{\theta}^*(\mathbf{p},\mathbf{s}) - \sigma_\varepsilon \Phi^{-1}(1-T) = \theta_0.$$

Moreover the point (η_0^*, θ_0)

1. *belongs to the curve $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$;*

2. *it is the unique point of the curve characterized by participation exactly equal to T;*

3. *all the points of the curve characterized by $\eta^* > \eta_0^*$ will exhibit participation strictly less than T;*

4. *all the points of the curve characterized by $\eta^* < \eta_0^*$ will exhibit participation strictly greater than T;*

5. *it is the unique intersection with the curve $\eta^*(\theta;\mathbf{p},\mathbf{s})$.*

Proof.

1. Since the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ has codomain $\mathbb{R}$ and $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$ is a unique fixed value in the interval $\left(\frac{k}{\gamma},\infty\right)$ there must be a point in which the curve is equivalent to $\widehat{\theta}^*(\mathbf{p},\mathbf{s})-\sigma_\varepsilon\Phi^{-1}(1-T)$

2. All the points of the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ are characterized by the lowest level of violence compatible with equilibrium and equilibrium cut-off $\widehat{\theta}^*(\mathbf{p},\mathbf{s})$. The participation is exactly equal to T when is satisfied the condition $\theta=\hat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)$, therefore in the point (η_0^*,θ_0) the participation is exactly equal to T.

3. If $\eta^*>\eta_0^*$ the curve returns a level of θ strictly less than θ_0 and therefore a level of participation strictly less than T.

$$\eta^*>\eta_0^*\Rightarrow\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)<\theta_0=\widehat{\theta}^*(\mathbf{p},\mathbf{s})+\Phi^{-1}(T)\sigma_\varepsilon$$

4. If $\eta^*<\eta_0^*$ the curve returns a level of θ strictly greater than θ_0 and therefore a level of participation strictly greater than T.

$$\eta^*<\eta_0^*\Rightarrow\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)>\theta_0=\widehat{\theta}^*(\mathbf{p},\mathbf{s})+\Phi^{-1}(T)\sigma_\varepsilon.$$

5. Finally, these inequalities imply that (η_0^*,θ_0) is the only point on the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ such that the citizens' participation to protest is exactly equal to T, thus point 5 follows.

■

Result 19 *The function $\eta^*(\theta;\mathbf{p},\mathbf{s})$ has the following properties*

1. *for a given $\theta\in\left(\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T),\widehat{\theta}^*(\mathbf{p},\mathbf{s})-\sigma_\varepsilon\Phi^{-1}(1-T)\right]$, the points of $\eta^*(\theta;\mathbf{p},\mathbf{s})$ are always on the east of $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$, therefore $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ is dominated by $\eta^*(\theta;\mathbf{p},\mathbf{s})$ for all $\theta\in\left(\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T),\widehat{\theta}^*(\mathbf{p},\mathbf{s})-\sigma_\varepsilon\Phi^{-1}(1-T)\right]$;*

2. *For descending values of $\theta\in\left(\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T),\widehat{\theta}^*(\mathbf{p},\mathbf{s})-\sigma_\varepsilon\Phi^{-1}(1-T)\right]$ the points of $\eta^*(\theta;\mathbf{p},\mathbf{s})$ exhibit ascending values of activism.*

Proof. Consider the points

1. $O=\left(\eta_0^*,\mathring{\theta}\left(\eta_0^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)\right)=(\eta_0^*,\theta_0)$

2. $A = \left(\eta_1^*, \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)\right)$

3. $B = \left(\eta_2^*, \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)\right)$

where

1. $\eta_0^* < \eta_1^* < \eta_2^*$

2. $\mathring{\theta}\left(\eta_2^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) < \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) < \theta_0$.

All the points belong to the curve $\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$, and for this reason they are characterized by the same level of activism ($f\left(\widehat{\theta}^*; \mathbf{p}, \mathbf{s}\right)$) and cutoff ($\widehat{\theta}^* (\mathbf{p}, \mathbf{s})$). Consider the point O, where by construction the citizens' participation to protest is T. On the other hand, in A the cutoff is the same of O ($\widehat{\theta}^* (\mathbf{p}, \mathbf{s})$), but

$$\mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) < \theta_0 = \widehat{\theta}^* (\mathbf{p}, \mathbf{s}) + \Phi^{-1}(T)\sigma_\varepsilon,$$

therefore the citizens' participation to the protest in A is clearly less than T. In order to observe participation equal to T for $\theta = \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$ it is necessary to have an higher level of activism that induces a lower equilibrium cutoff $\widehat{\theta} < \widehat{\theta}^* (\mathbf{p}, \mathbf{s})$ such that

$$\mathring{\theta}\left(\eta_1^* \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) = \widehat{\theta} + \Phi^{-1}(T)\sigma_\varepsilon.$$

Then, there exist a unique

$$\eta_A^* > \eta_1^*$$

that, for a fixed $\theta = \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$, there is a higher level of activism such that

$$\eta_A^* + \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) >$$

$$> \eta_1^* + \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) = f\left(\widehat{\theta}^*; \mathbf{p}, \mathbf{s}\right)$$

so that in the point $A_1 = (\eta_A^*, \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right))$ the participation to protest is T. Obviously the point A_1 lies on the east of A, because they show the same level of θ and $\eta_A^* > \eta_1^*$. The line $\mathring{\theta}\left(\eta^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right)$ translated by $\eta_A^* - \eta_1^*$ represents all the combinations of θ and η^* for which we observe this new higher level of activism $\eta_A^* + \mathring{\theta}\widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}$ and the equilibrium cutoff along this line will be

$$\mathring{\theta}\left(\eta_1^* | \widehat{\theta}^* (\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}\right) - \Phi^{-1}(T)\sigma_\varepsilon.$$

Similarly, in B the cutoff is the same of O ($\widehat{\theta}^*(\mathbf{p},\mathbf{s})$), but

$$\mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) < \theta_0 = \widehat{\theta}^*(\mathbf{p},\mathbf{s}) + \Phi^{-1}(T)\sigma_\varepsilon,$$

therefore the participation in B is less than T. Similarly to the case of point A, there exists a unique

$$\eta_B^* > \eta_2^*$$

such that in the point $B_2 = \left(\eta_B^*, \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)\right)$ the participation is equal to T. Moreover, note that

$$\eta_B^* - \eta_2^* > \eta_A^* - \eta_1^*.$$

If we translate the point B by $\eta_A^* - \eta_1^*$ (r_1) we obtain the point B_1 on the line r_1, because A and B are characterized by the same level of activism and A_1 and B_1 are their translation for a common value. The point B_1 is characterized by the same level of activism of A_1 and $\theta = \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$, but on the line r_1 the participation T is gained for $\mathring{\theta}\left(\eta_1^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) > \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$, therefore the participation T for $\theta = \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ is observed in the point B_2 in the east of B and B_1 where the level of activism is higher than the one observed on the line r_1 and the cutoff is lower:

$$\begin{aligned}
&\eta_B^* + \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) > \\
&> \eta_A^* + \mathring{\theta}\left(\eta_1^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) > \\
&> \eta_1^* + \mathring{\theta}\left(\eta_1^*\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right).
\end{aligned}$$

Then, the line $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ translated by $\eta_B^* - \eta_2^*$ (r_2) represents all the combinations of θ and η^* for which we observe this new higher level of activism ($\eta_B^* + \mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$), and the equilibrium cutoff on this line will be

$$\mathring{\theta}\left(\eta_2^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right) - \Phi^{-1}(T)\sigma_\varepsilon.$$

In conclusion if we consider all the θ for which $\eta^*(\theta;\mathbf{p},\mathbf{s})$ is defined and we iterate the above reasoning considering new starting points of $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*(\mathbf{p},\mathbf{s}),\mathbf{p},\mathbf{s}\right)$ characterized by descending values of θ we get the lemma. ■

9.1 The Probabilities of No Protest, of Failed Protest, and of Successful Protest

9.1.1 The Probability of Citizens' Protest

Result 20 *The probability of citizens' protest in equilibrium is*

1. *increasing in the responsiveness of the political regime*;
2. *decreasing in the repression of the political regime;*
3. *uncertain in country radicalization, diversity and opacity.*

Proof. The probability of the outcome "mobilization" is given by:

$$1-\frac{1}{2}(\alpha^*+\beta^*)=$$

$$=1-\frac{1}{2}\Phi\left(\frac{\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)+\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right)+$$

$$-\frac{1}{2}\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)|T,k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)-\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)-\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right).$$

1. The term α^* is clearly increasing in T. The term β^* is increasing in T since the numerator of the argument of $\Phi()$ can be rewritten as

   $$\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\hat{\theta}^*+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)-\hat{\theta}^*-\Phi^{-1}(T)\sigma_\epsilon=$$

   $$=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi}-1\right)\hat{\theta}^*+\Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi}-1\right)\right)$$

   Consequently the probabilty of "mobilization" is decreasing in T and therefore increasing in $1-T$

2. Since $\widehat{\theta}^*$ is increasing in $\frac{k}{\gamma}$ the term α^* is increasing in $\frac{k}{\gamma}$. Regard the term β^* if we look at the numerator of the argument of $\Phi()$

   $$=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi}-1\right)\hat{\theta}^*+\Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi}-1\right)\right)$$

   when $\frac{k}{\gamma}$ increases $\widehat{\theta}^*$ increases therefore the second addend increases, while when $\frac{k}{\gamma}$ also the first addend increases similarly as we shown in previous results therefore β^* is increasing in $\frac{k}{\gamma}$ and the overall probability must be decreasing in $\frac{k}{\gamma}$

3. with regard to the trends for these quantities, it is necessary to investigate them as they vary with the values assumed by the other exogenous variables.

■

Result 21 *The probability of citizens' protest in equilibrium is*

1. *has no clear trend in opacity, even if responsiveness seems to induce an increasing trend;*
2. *increasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*
3. *increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

Proof. see Appendix B. ■

9.1.2 The Probability of No Protest

Result 22 *The probability of no protest in equilibrium is*

1. *decreasing in the responsiveness of the political regime*;
2. *increasing in the repression of the political regime;*
3. *uncertain in country radicalization, diversity and public information opacity.*

Proof. The probability of the outcome "no mobilization" is given by:

$$\frac{1}{2}(\alpha^* + \beta^*) =$$

$$= \frac{1}{2}\Phi\left(\frac{\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right) + \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right) +$$

$$+\frac{1}{2}\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)|T,k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right) - \widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right) - \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right).$$

1. The term α^* is clearly increasing in T. The term β^* is increasing in T since the numerator of the argument of $\Phi()$ can be rewritten as

$$\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\hat{\theta}^*+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)-\hat{\theta}^*-\Phi^{-1}(T)\sigma_\epsilon=$$

$$=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi}-1\right)\hat{\theta}^*+\Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi}-1\right)\right)$$

Consequently the probabilty of "no mobilization" is increasing in T and therefore decreasing in $1-T$.

2. Since $\widehat{\theta}^*$ is increasing in $\frac{k}{\gamma}$ the term α^* is increasing in $\frac{k}{\gamma}$. Regard the term β^* if we look at the numerator of the argument of $\Phi()$

$$=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right)+\hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi}-1\right)\hat{\theta}^*+\Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi}-1\right)\right)$$

when $\frac{k}{\gamma}$ increases $\widehat{\theta}^*$ increases therefore the second addend increases, while when $\frac{k}{\gamma}$ also the first addend increases similarly as we shown in previous results therefore β^* is increasing in $\frac{k}{\gamma}$ and the overall probability is increasing in $\frac{k}{\gamma}$.

3. with regard to the trends for these quantities, it is necessary to investigate them as they vary with the values assumed by the other exogenous variables.

■

Result 23 *The probability of no protest in equilibrium*

1. *has no clear trend in opacity, even if responsiveness seems to induce a decreasing trend;*

2. *is decreasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*

3. *is decreasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

Proof. see Appendix B. ■

9.1.3 The Probability of Successful Protest

Result 24 *The probability of successful protest is*

1. *tending to increase in the responsiveness of the political regime*;
2. *decreasing in the repression of the political regime;*
3. *uncertain in* σ^2_θ, σ^2_ε *and* σ^2_η*: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters.*

Proof. The probability of the outcome "successful protest" is given by:

$$(1-\alpha^*)(1-\beta^*)+\frac{1}{2}(1-\alpha^*)\beta^*+\delta(1-\rho)\frac{1}{2}\alpha^*(1-\beta^*)=$$

$$=1-\beta^*-\alpha^*+\alpha^*\beta^*+\frac{1}{2}\beta^*-\frac{1}{2}\alpha^*\beta^*+\frac{1}{2}\alpha^*\delta-\frac{1}{2}\alpha^*\delta\beta^*-\frac{1}{2}\alpha^*\delta\rho+\frac{1}{2}\alpha^*\delta\rho\beta^*=$$

$$=1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho)$$

where[21]

$$\alpha^*=\Phi\left(\frac{\widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right)+\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right)$$

$$\beta^*=\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right)|T,k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right)-\widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right)-\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right)$$

Moreover

$$\frac{\partial(1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho))}{\partial\beta^*}=-\frac{1}{2}+\frac{1}{2}\alpha^*(1-\delta+\delta\rho)>0\Leftrightarrow$$

$$\alpha^*(1-\delta+\delta\rho)>1$$

Since the condition is not verifiable as both factors are strictly less than 1, the probability is decreasing in β^*.

$$\frac{\partial(1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho))}{\partial\alpha^*}=\left(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2}\right)+\frac{1}{2}\beta^*(1-\delta+\delta\rho)$$

$$(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\beta^*(1-\delta+\delta\rho)>0\Leftrightarrow\beta^*>\frac{2-\delta+\delta\rho}{1-\delta+\delta\rho}$$

Since the condition is not verifiable because the right member i greater than 1, the probability is decreasing in α^*.

1. The probability is decreasing in both α^*,β^*, we have shown that both terms are increasing in T, therefore the overall probability is increasing in $1-T$. However when $\sigma^2_\theta,\sigma^2_\eta$ are high and σ^2_ε is low there is initially a little non-monotonic effect due to the trend of ρ with respect to T.

[21] ρ is derived by a precise probability ratio

2. The probability is decreasing in both α^*, β^*, we have shown that both terms are increasing in $\frac{k}{\gamma}$ therefore the overall probability is decreasing in $\frac{k}{\gamma}$.
3. with regard to the trends for these quantities, it necessary to investigate them as they vary with the values assumed by the other exogenous variables.

■

Result 25 *The probability of successful protest is*

1. *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
2. *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
3. *increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

Proof. see Appendix B. ■

9.1.4 The Probability of Successful Protest

Result 26 *The probability of positive mobilization but failed protest has no clear monotone relationship with the responsiveness and the repression of the political regime, however our simulations show that the relations is*

1. *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogenous, otherwise is not monotonic, first decreasing and increasing*
2. *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*
3. *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*
4. *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*
5. *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

Proof. see appendix B. ■

10 Appendix B

This Appendix contains results, simulations' codes and graphs that have been omitted by the main text to make it more readable.

11 Result 9

i's subjective belief about the probability of policy change, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v - t^*; \mathbf{p}, \mathbf{s}\right)$, is

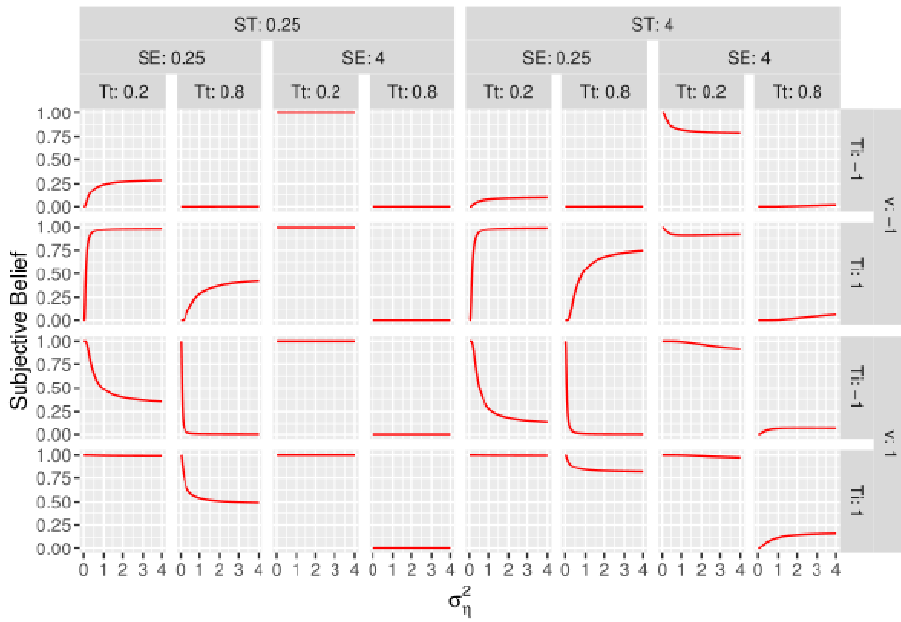
1. increasing in opacity unless the political regime is responsive and the society is radicalized and heterogenous;
2. increasing in diversity unless the political regime is unresponsive;
3. increasing in radicalization unless the political regime is responsive.

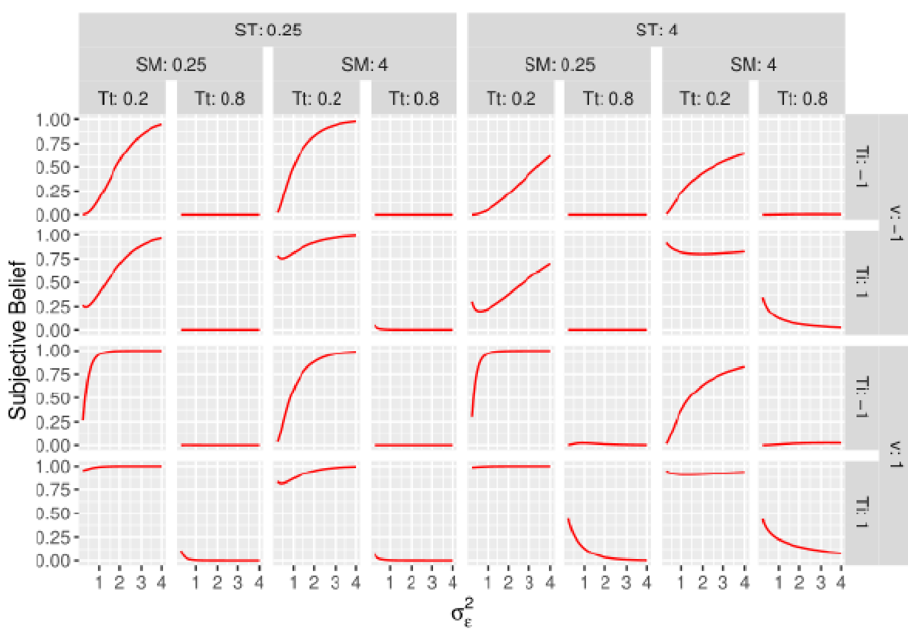
11.1 Simulation's Code

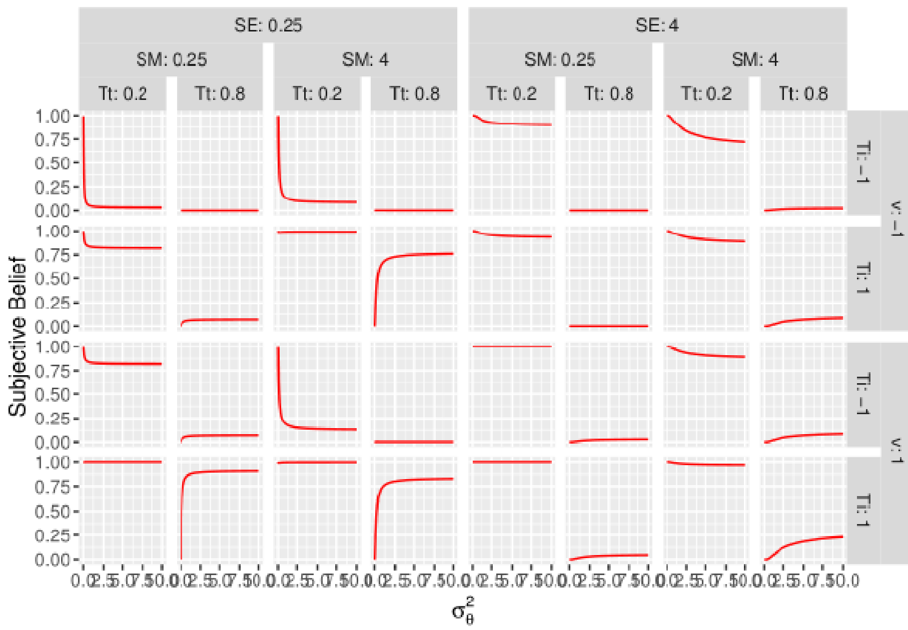
```
SM=seq(0.01,4,0.01)
SE=c(0.25,4)
ST=c(0.25,4)
Tt=c(0.2,0.8)
h_t=0.1
gamma=0.5
v=c(-1,1)
Ti=c(-1,1)
a=crossing(SE,ST,Tt,h_t,gamma,SM,v,Ti)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=a$h_t+sqrt(a$SE)*qnorm(a$Tt)-a$psi*a$v-(1-a$psi)*(a$lambda*a$Ti)
a$Y2=sqrt(a$psi*a$SM)
a$Y3=1-pnorm(a$Y1/a$Y2)
a %>%
ggplot(aes(x=SM, y=Y3)) +
geom_line(color="red") +
facet_nested(v+Ti  ~ ST+SE+Tt , labeller = label_both) +
labs(y="Subjective Belief",x=expression(sigma[eta]^2))

SE=seq(0.25,4,0.01)
SM=c(0.25,4)
ST=c(0.25,4)
```

```
Tt=c(0.2,0.8)
h_t=0.6
gamma=0.5
v=c(-1,1)
Ti=c(-1,1)
a=crossing(SE,ST,Tt,h_t,gamma,SM,v,Ti)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=a$h_t+sqrt(a$SE)*qnorm(a$Tt)-a$psi*a$v-(1-a$psi)*(a$lambda*a$Ti)
a$Y2=sqrt(a$psi*a$SM)
a$Y3=1-pnorm(a$Y1/a$Y2)
a %>%
ggplot(aes(x=SE, y=Y3)) +
geom_line(color="red") +
facet_nested(v+Ti  ~ ST+SM+Tt , labeller = label_both) +
labs(y="Subjective Belief",x=expression(sigma[epsilon]^2))

ST=seq(0.01,10,0.01)
SM=c(0.25,4)
SE=c(0.25,4)
Tt=c(0.2,0.8)
h_t=0.1
gamma=0.5
v=c(-1,1)
Ti=c(-1,1)
a=crossing(SE,ST,Tt,h_t,gamma,SM,v,Ti)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=a$h_t+sqrt(a$SE)*qnorm(a$Tt)-a$psi*a$v-(1-a$psi)*(a$lambda*a$Ti)
a$Y2=sqrt(a$psi*a$SM)
a$Y3=1-pnorm(a$Y1/a$Y2)
a %>%
ggplot(aes(x=ST, y=Y3)) +
geom_line(color="red") +
facet_nested(v+Ti  ~ SE+SM+Tt , labeller = label_both) +
labs(y="Subjective Belief",x=expression(sigma[theta]^2))
```

11.2 Simulation's results


ST: 0.25
ST: 4
SE: 0.25
SE: 4
Tt: 0.2
Tt: 0.8
Ti: −1
Ti: 1
v: −1
v: 1
Subjective Belief
σ_η^2


ST: 0.25
ST: 4
SM: 0.25
SM: 4
Tt: 0.2
Tt: 0.8
Ti: −1
Ti: 1
v: −1
v: 1
Subjective Belief
σ_ε^2



12 Result 11

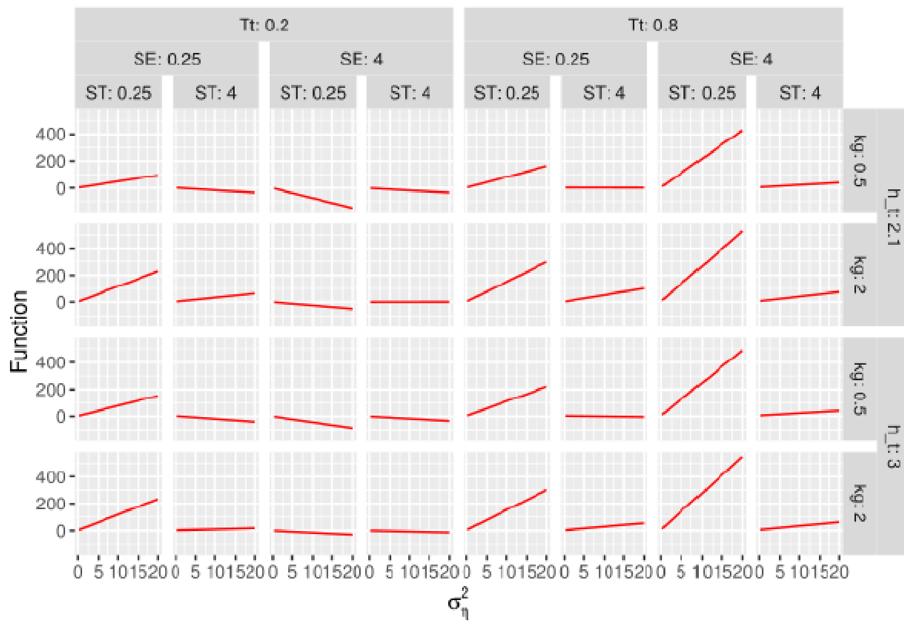
$f\left(\widehat{\theta};\mathbf{p},\mathbf{s}\right)$ is

1. increasing in opacity unless the political regime is responsive and tolerant;
2. increasing in diversity unless the political regime is responsive;
3. decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.

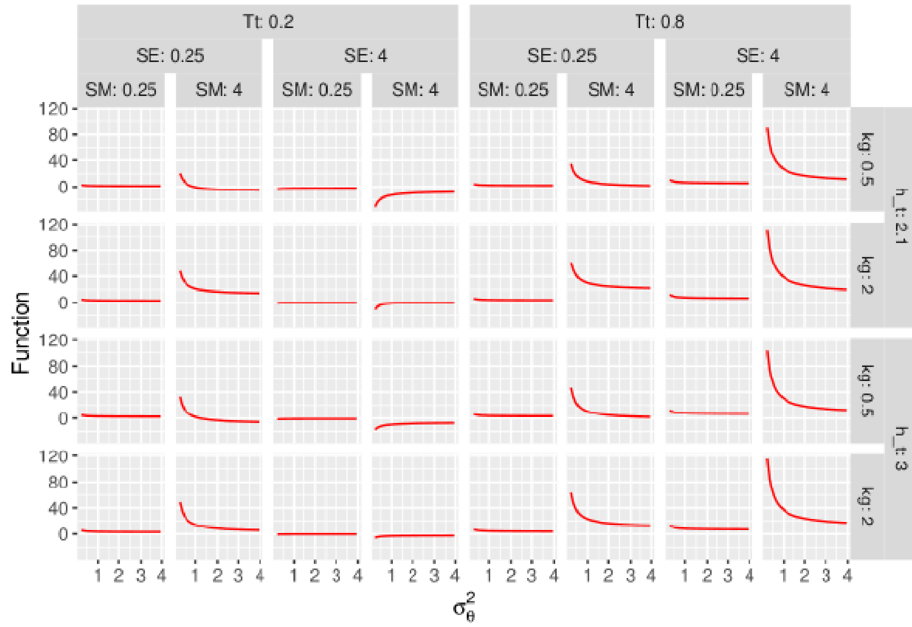
12.1 Simulation's code

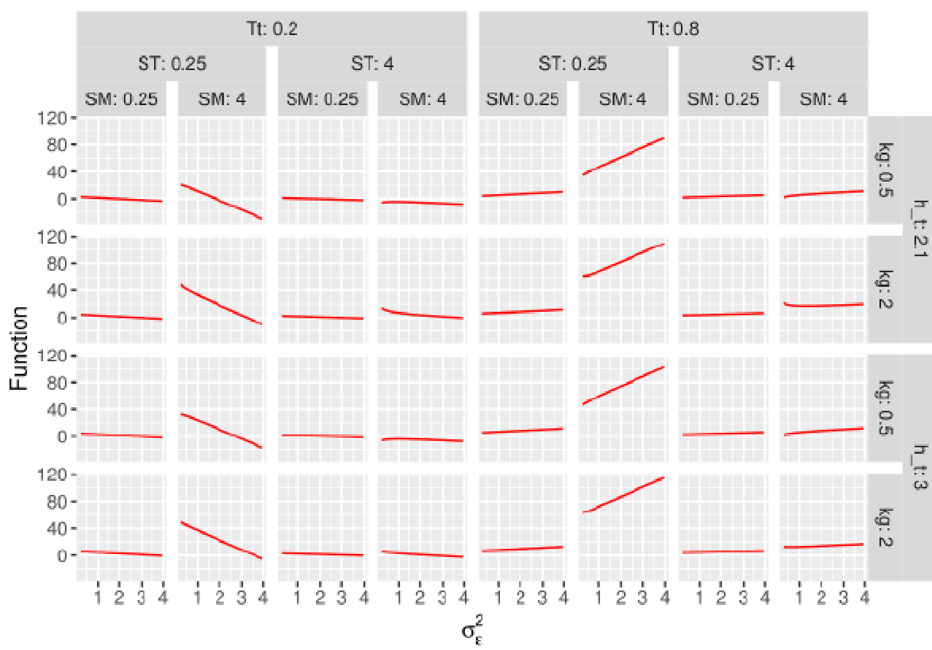
```
SM=seq(0.25,20,0.1)
ST=c(0.25,4)
SE=c(0.25,4)
kg=c(0.5,2)
Tt=c(0.2,0.8)
h_t=c(2.1,3)
a=crossing(SE,ST,SM,h_t,Tt,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$V1=sqrt(a$SM/a$psi)*qnorm(a$kg/a$h_t)
```

```
a$V2=(1-(1-a$psi)*a$lambda)/a$psi*a$h_t
a$V3=a$SE/a$psi*qnorm(a$Tt)
a$Y=a$V1+a$V2+a$V3
a %>%
ggplot(aes(x=SM, y=Y)) +
geom_line(col="red") +
facet_nested(  h_t+kg ~ Tt+SE+ST , labeller = label_both) +
labs(y="Function",x=expression(sigma[eta]^2))

ST=seq(0.25,4,0.1)
SM=c(0.25,4)
SE=c(0.25,4)
kg=c(0.5,2)
Tt=c(0.2,0.8)
h_t=c(2.1,3)
a=crossing(SE,ST,SM,h_t,Tt,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$V1=sqrt(a$SM/a$psi)*qnorm(a$kg/a$h_t)
a$V2=(1-(1-a$psi)*a$lambda)/a$psi*a$h_t
a$V3=a$SE/a$psi*qnorm(a$Tt)
a$Y=a$V1+a$V2+a$V3
a %>%
ggplot(aes(x=ST, y=Y)) +
geom_line(col="red") +
facet_nested(  h_t+kg ~ Tt+SE+SM , labeller = label_both) +
labs(y="Function",x=expression(sigma[theta]^2))

SE=seq(0.25,4,0.1)
SM=c(0.25,4)
ST=c(0.25,4)
kg=c(0.5,2)
Tt=c(0.2,0.8)
h_t=c(2.1,3)
a=crossing(SE,ST,SM,h_t,Tt,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$V1=sqrt(a$SM/a$psi)*qnorm(a$kg/a$h_t)
a$V2=(1-(1-a$psi)*a$lambda)/a$psi*a$h_t
a$V3=a$SE/a$psi*qnorm(a$Tt)
a$Y=a$V1+a$V2+a$V3
a %>%
ggplot(aes(x=SE, y=Y)) +
geom_line(col="red") +
facet_nested(  h_t+kg ~ Tt+ST+SM , labeller = label_both) +
labs(y="Function",x=expression(sigma[epsilon]^2))
```



12.2 Simulation's results


Tt: 0.2
Tt: 0.8
SE: 0.25
SE: 4
SM: 0.25
SM: 4
kg: 0.5
kg: 2
h_t: 2.1
h_t: 3
Function
120
80
40
0
1 2 3 4
σ^2_θ


Tt: 0.2
Tt: 0.8
ST: 0.25
ST: 4
SM: 0.25
SM: 4
kg: 0.5
kg: 2
h_t: 2.1
h_t: 3
Function
120
80
40
0
1 2 3 4
σ^2_ε

13 Result 14

The finite equilibrium cutoff $\widehat{\theta}_L (v - t^*; \mathbf{p}, \mathbf{s})$ is

1. increasing in opacity unless the political regime is responsive and tolerant;
2. increasing in diversity unless the political regime is responsive;
3. decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.

If $f\left(\widehat{\theta}; \mathbf{p}, \mathbf{s}\right)$ grows given a fixed level of $v - t^*$ the value of $\widehat{\theta}_L (v - t^*; \mathbf{p}, \mathbf{s})$ increases therefore this result is a consequence of the previous one.

14 Result 21

The probability of citizens' protest in equilibrium is

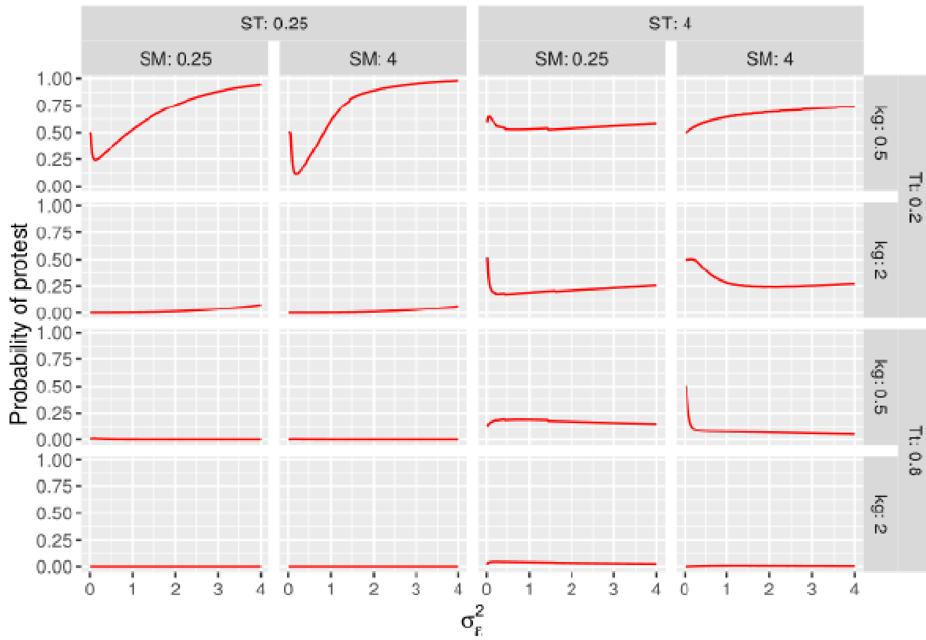
1. has no clear trend in opacity, even if responsiveness seems to induce an increasing trend;
2. increasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;
3. increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.

14.1 Simulation's code

```
m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SM=c(0.25,4)
SE=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
```

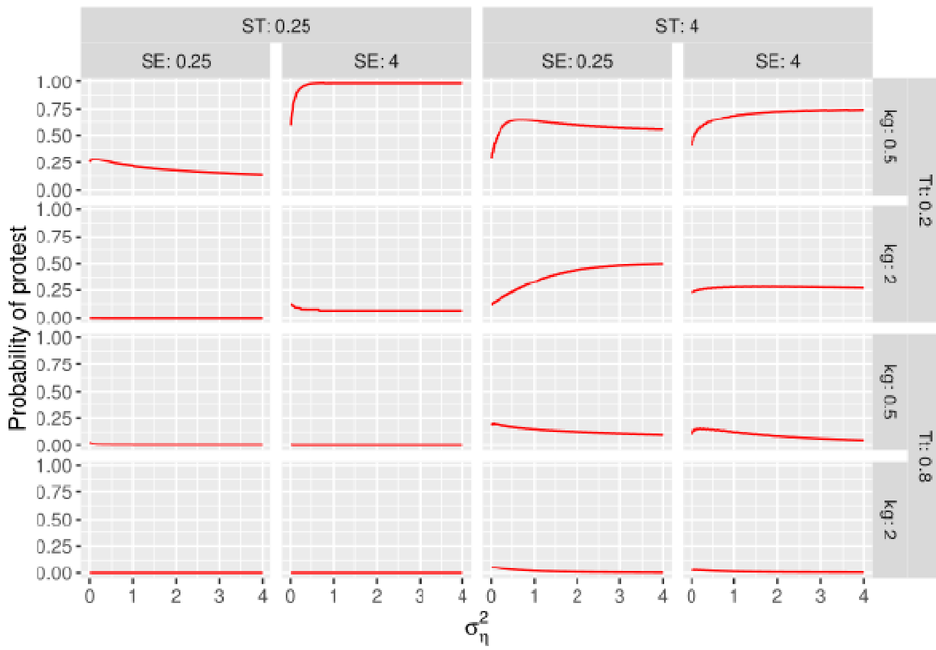
```
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=1-(0.5*P1+0.5*P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SE, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg~ ST+SM ,labeller = label_both)+
labs(y="Probability of protest",x=expression(sigma[epsilon]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SE=c(0.25,4)
SM=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
```

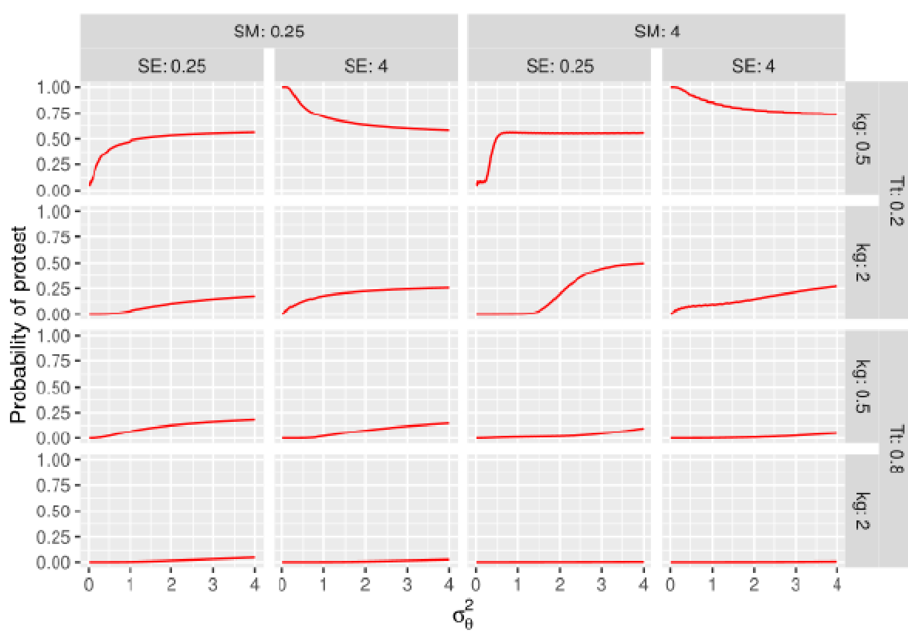
```
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=1-(0.5*P1+0.5*P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SM, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg~ ST+SE ,labeller = label_both)+
labs(y="Probability of protest",x=expression(sigma[eta]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
SM=c(0.25,4)
SE=c(0.25,4)
ST=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=1-(0.5*P1+0.5*P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=ST, y=P)) +
geom_line(col="red",size=0.5) +
```




```
facet_nested(Tt+kg~ SM+SE ,labeller = label_both)+
labs(y="Probability of protest",x=expression(sigma[theta]^2))
```

14.2 Simulation's results


ST: 0.25
ST: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
Probability of protest
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
σ^2_η


SM: 0.25
SM: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
Probability of protest
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
σ^2_θ

15 Result 23

The probability of no protest in equilibrium

1. has no clear trend in opacity, even if responsiveness seems to induce a decreasing trend;
2. is decreasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;
3. is decreasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.

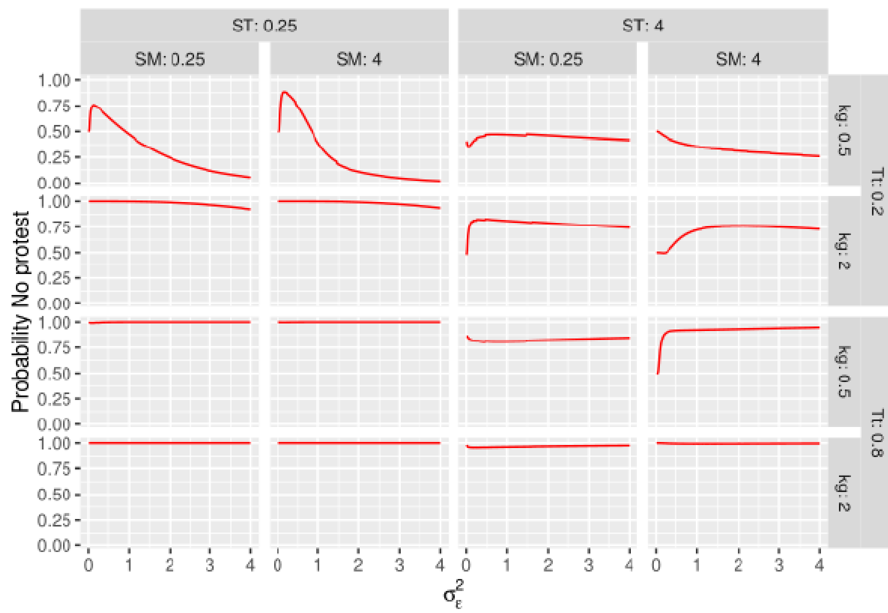
15.1 Simulation's code

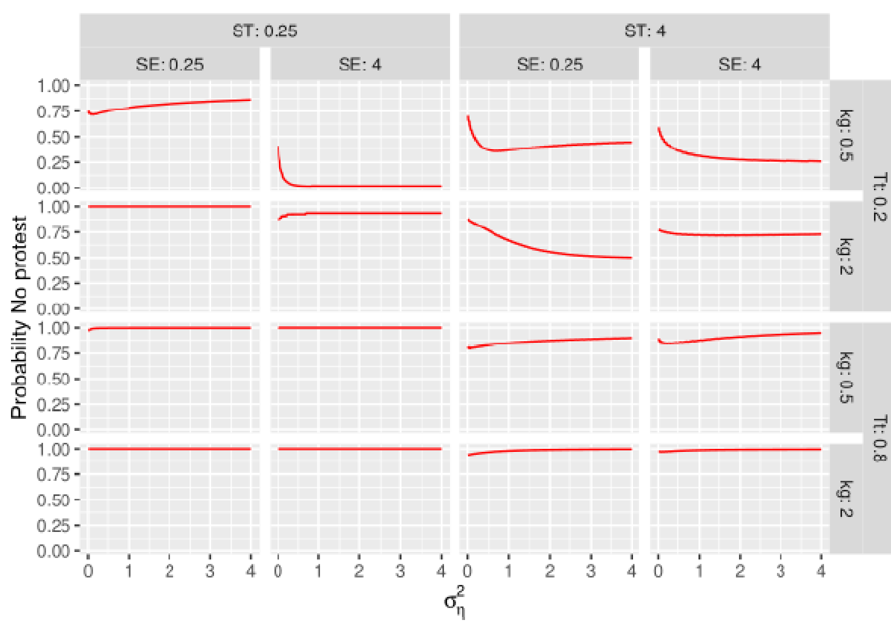
```
m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SM=c(0.25,4)
SE=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=0.5*P1+0.5*P2
}
K=cbind(a,P)
```

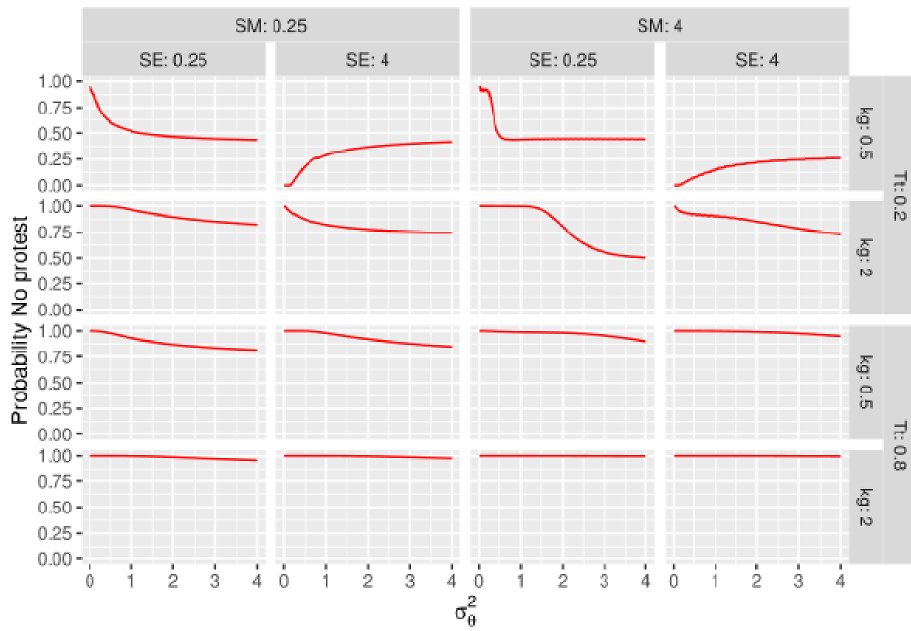
```
K %>%
ggplot(aes(x=SE, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg~ ST+SM ,labeller = label_both)+
labs(y="Probability No protest",x=expression(sigma[epsilon]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SE=c(0.25,4)
SM=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=0.5*P1+0.5*P2
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SM, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg~ ST+SE ,labeller = label_both)+
labs(y="Probability No protest",x=expression(sigma[eta]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
```

```
SM=c(0.25,4)
SE=c(0.25,4)
ST=seq(0.01,4,0.01)
kg=c(0.5,2)
P=c()
a=crossing(SM,kg,Tt,ST,SE)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
P[i]=0.5*P1+0.5*P2
}
K=cbind(a,P)
K %>%
ggplot(aes(x=ST, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg~ SM+SE ,labeller = label_both)+
labs(y="Probability No protest",x=expression(sigma[theta]^2))
```

15.2 Simulation's results


ST: 0.25
ST: 4
SM: 0.25
SM: 4
SM: 0.25
SM: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability No protest
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
σ^2_ε


ST: 0.25
ST: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability No protest
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
σ^2_η



16 Result 25

The probability of successful protest is

1. decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;

2. increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;

3. increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.

16.1 Simulation's code

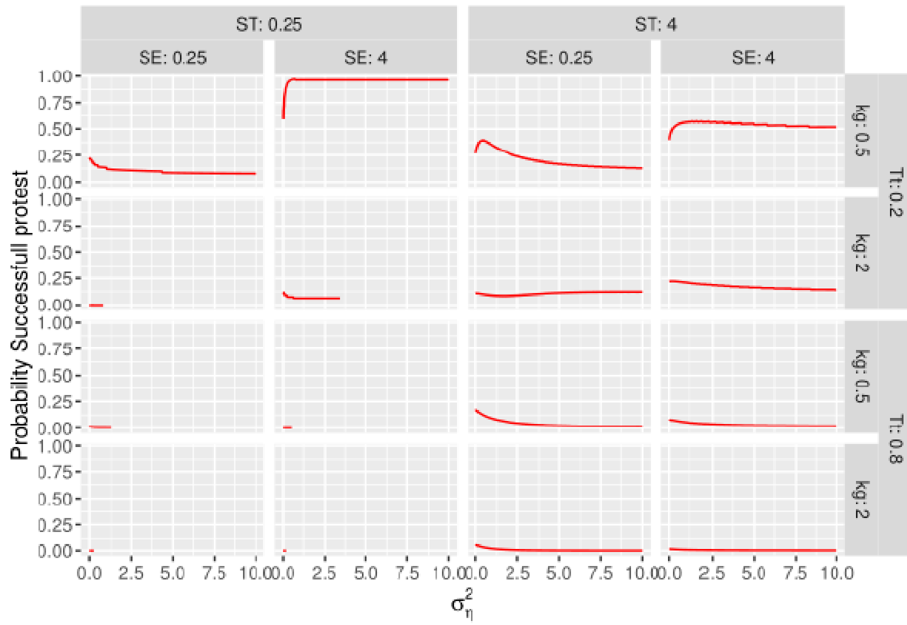
```
m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SM=seq(0.01,10,0.01)
SE=c(0.25,4)
kg=c(0.5,2)
```

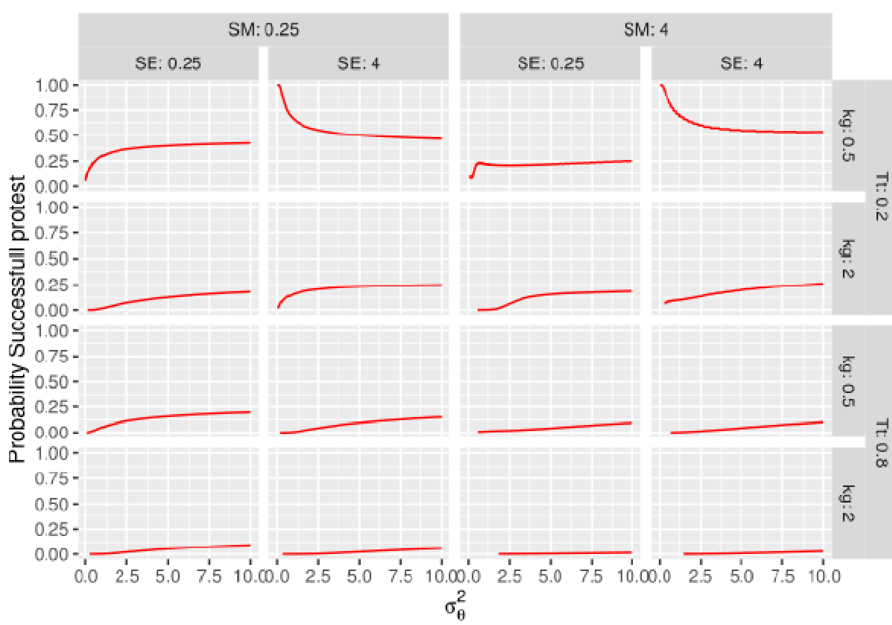
```
delta=0.3
P=c()
a=crossing(SM,kg,Tt,ST,SE,delta)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
delta=as.numeric(a[i,6])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=(1-P1)*(1-P2)+0.5*(1-P1)*P2+delta*rho*0.5*P1*(1-P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SM, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~ST+SE ,labeller = label_both)+
labs(y="Probability Successfull protest",x=expression(sigma[eta]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
SM=c(0.25,4)
ST=seq(0.01,10,0.01)
SE=c(0.25,4)
kg=c(0.5,2)
delta=0.4
P=c()
```

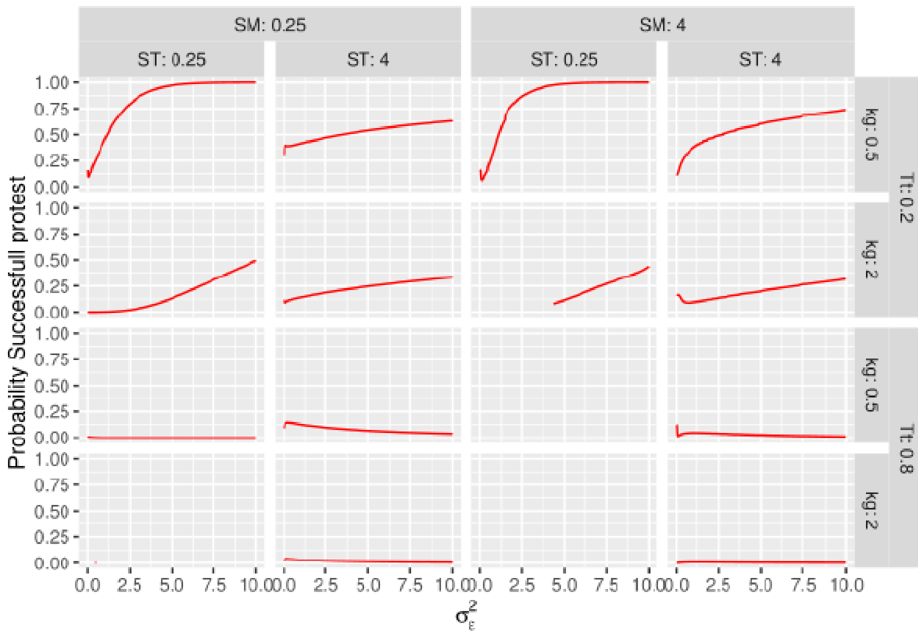
```
a=crossing(SM,kg,Tt,ST,SE,delta)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
delta=as.numeric(a[i,6])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=(1-P1)*(1-P2)+0.5*(1-P1)*P2+delta*rho*0.5*P1*(1-P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=ST, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~SM+SE ,labeller = label_both)+
labs(y="Probability Successfull protest",x=expression(sigma[theta]^2))

m=0
h_t=seq(0,100,0.05)
Tt=c(0.2,0.8)
SM=c(0.25,4)
SE=seq(0.01,10,0.01)
ST=c(0.25,4)
kg=c(0.5,2)
delta=0.4
P=c()
a=crossing(SM,kg,Tt,ST,SE,delta)
for (i in 1:nrow(a)){
```

```
SM=as.numeric(a[i,1])
kg=as.numeric(a[i,2])
Tt=as.numeric(a[i,3])
ST=as.numeric(a[i,4])
SE=as.numeric(a[i,5])
delta=as.numeric(a[i,6])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=(1-P1)*(1-P2)+0.5*(1-P1)*P2+delta*rho*0.5*P1*(1-P2)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SE, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~SM+ST ,labeller = label_both)+
labs(y="Probability Successfull protest",x=expression(sigma[epsilon]^2))
```

16.2 Simulation's results


ST: 0.25
ST: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability Successfull protest
σ^2_η


SM: 0.25
SM: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability Successfull protest
σ^2_θ



17 Result 26

The probability of positive mobilization but failed protest has no clear monotone relationship with the responsiveness and the repression of the political regime, however our simulations show that the relations is

1. increasing in responsiveness when the political regime is intolerant or tolerant and the society homogenous, otherwise is not monotonic, first decreasing and increasing

2. decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;

3. decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;

4. decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;

5. increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not

monotonic, but first increasing and then decreasing.

17.1 Simulation's code

```
m=0
h_t=seq(1,50,0.01)
Tt=seq(0.1,0.9,0.01)
ST=c(0.25,4)
SM=c(0.25,4)
SE=c(0.25,4)
kg=c(0.5,2)
delta=c(0.3,0.8)
P=c()
a=crossing(SM,Tt,ST,SE,kg,delta)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
Tt=as.numeric(a[i,2])
ST=as.numeric(a[i,3])
SE=as.numeric(a[i,4])
kg=as.numeric(a[i,5])
delta=as.numeric(a[i,6])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=P1*(0.5-0.5*delta+0.5*delta*rho)-P1*P2*(0.5-0.5*delta+0.5*delta*rho)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=Tt, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(kg+delta ~ST+SM+SE,labeller = label_both)+
```

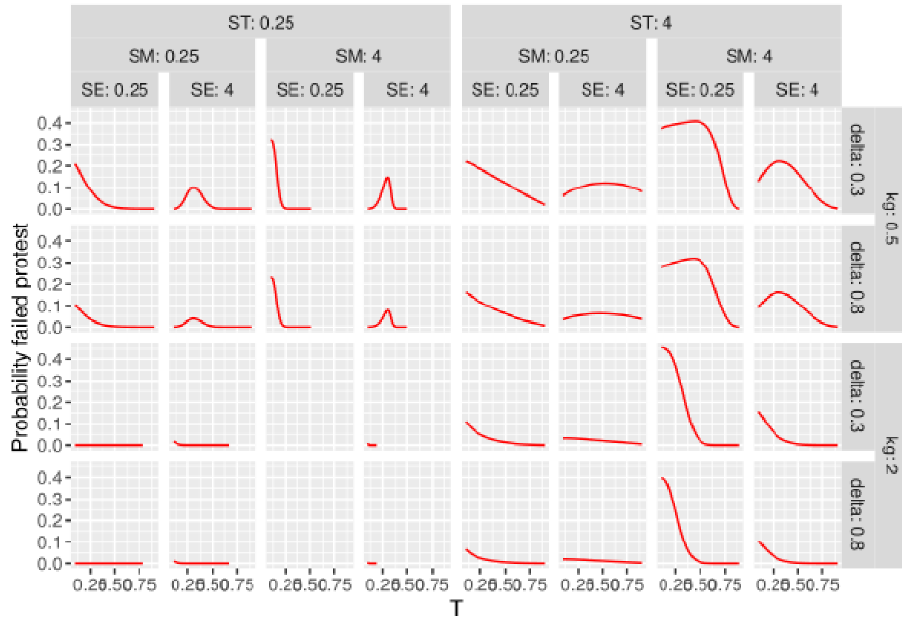
```
labs(y="Probability failed protest",x="T")

m=0
h_t=seq(1,50,0.01)
Tt=c(0.2,0.8)
ST=c(0.25,4)
SM=c(0.25,4)
SE=c(0.25,4)
kg=seq(0.01,5,0.01)
delta=0.3
P=c()
a=crossing(SM,Tt,ST,SE,kg,delta)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
Tt=as.numeric(a[i,2])
ST=as.numeric(a[i,3])
SE=as.numeric(a[i,4])
kg=as.numeric(a[i,5])
delta=as.numeric(a[i,6])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=P1*(0.5-0.5*delta+0.5*delta*rho)-P1*P2*(0.5-0.5*delta+0.5*delta*rho)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=kg, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+delta ~ST+SM+SE,labeller = label_both)+
labs(y="Probability failed protest",x=expression(k/gamma))
```

```
m=0
h_t=seq(1,50,0.01)
Tt=c(0.2,0.8)
ST=seq(0.25,10,0.01)
SM=c(0.25,4)
SE=c(0.25,4)
kg=c(0.5,2)
delta=0.3
P=c()
a=crossing(SM,Tt,ST,SE,kg)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
Tt=as.numeric(a[i,2])
ST=as.numeric(a[i,3])
SE=as.numeric(a[i,4])
kg=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=P1*(0.5-0.5*delta+0.5*delta*rho)-P1*P2*(0.5-0.5*delta+0.5*delta*rho)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=ST, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~SM+SE,labeller = label_both)+
labs(y="Probability failed protest",x=expression(sigma[theta]^2))

m=0
h_t=seq(1,50,0.01)
Tt=c(0.2,0.8)
```

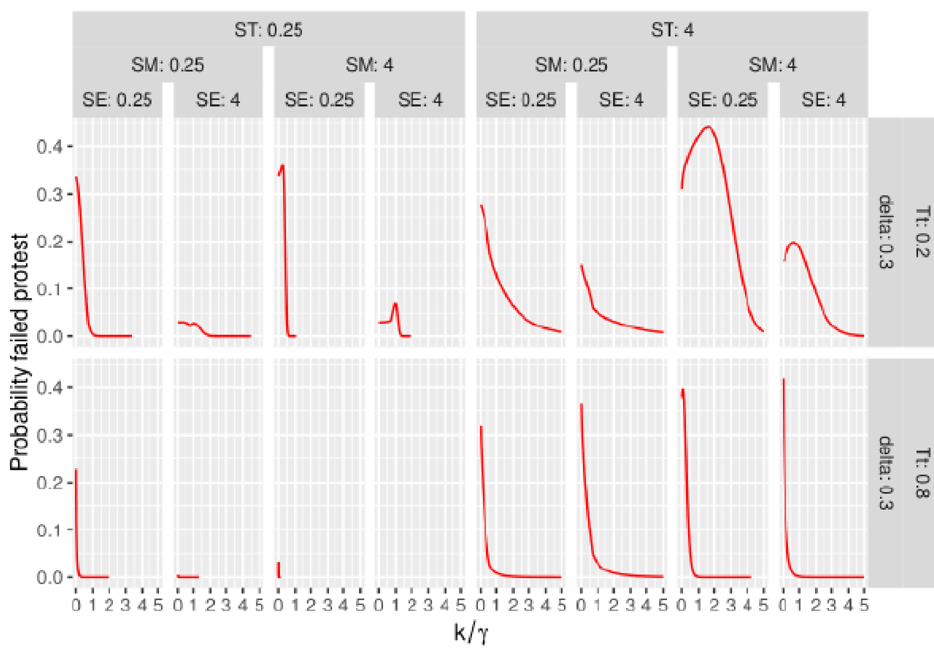
```
SM=seq(0.25,10,0.01)
ST=c(0.25,4)
SE=c(0.25,4)
kg=c(0.5,2)
delta=0.3
P=c()
a=crossing(SM,Tt,ST,SE,kg)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
Tt=as.numeric(a[i,2])
ST=as.numeric(a[i,3])
SE=as.numeric(a[i,4])
kg=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=P1*(0.5-0.5*delta+0.5*delta*rho)-P1*P2*(0.5-0.5*delta+0.5*delta*rho)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SM, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~ST+SE,labeller = label_both)+
labs(y="Probability failed protest",x=expression(sigma[eta]^2))

m=0
h_t=seq(1,50,0.01)
Tt=c(0.2,0.8)
SE=seq(0.25,10,0.01)
ST=c(0.25,4)
SM=c(0.25,4)
```

```
kg=c(0.5,2)
delta=0.3
P=c()
a=crossing(SM,Tt,ST,SE,kg)
for (i in 1:nrow(a)){
SM=as.numeric(a[i,1])
Tt=as.numeric(a[i,2])
ST=as.numeric(a[i,3])
SE=as.numeric(a[i,4])
kg=as.numeric(a[i,5])
lambda=ST/(ST+SE)
psi=lambda*SE/(lambda*SE+SM)
F1=sqrt(SM/psi)*qnorm(kg/h_t)
F2=((1-(1-psi)*lambda)/psi)*h_t
F3=(sqrt(SE)/psi)*qnorm(Tt)
F4=((1-psi)*(1-lambda)/psi)*m
M=round(F1+F2+F3-F4,10)
MM=data.frame(v=M,h_t)
d=which.min(MM$v)
V=MM$v[d]
H=MM$h_t[d]
P1=pnorm((H+qnorm(Tt)*sqrt(SE)-m)/sqrt(ST))
P2=pnorm((V-H-qnorm(Tt)*sqrt(SE))/sqrt(SM))
R1=pnorm((kg-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R2=1-pnorm((V-kg+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
R3=pnorm((H-qnorm(1-Tt)*sqrt(SE)-m)/(sqrt(ST)))
R4=1-pnorm((V-H+qnorm(1-Tt)*sqrt(SE))/(sqrt(SM)))
rho=(R1*R2)/(R3*R4)
P[i]=P1*(0.5-0.5*delta+0.5*delta*rho)-P1*P2*(0.5-0.5*delta+0.5*delta*rho)
}
K=cbind(a,P)
K %>%
ggplot(aes(x=SE, y=P)) +
geom_line(col="red",size=0.5) +
facet_nested(Tt+kg ~ST+SM,labeller = label_both)+
labs(y="Probability failed protest",x=expression(sigma[epsilon]^2))
```

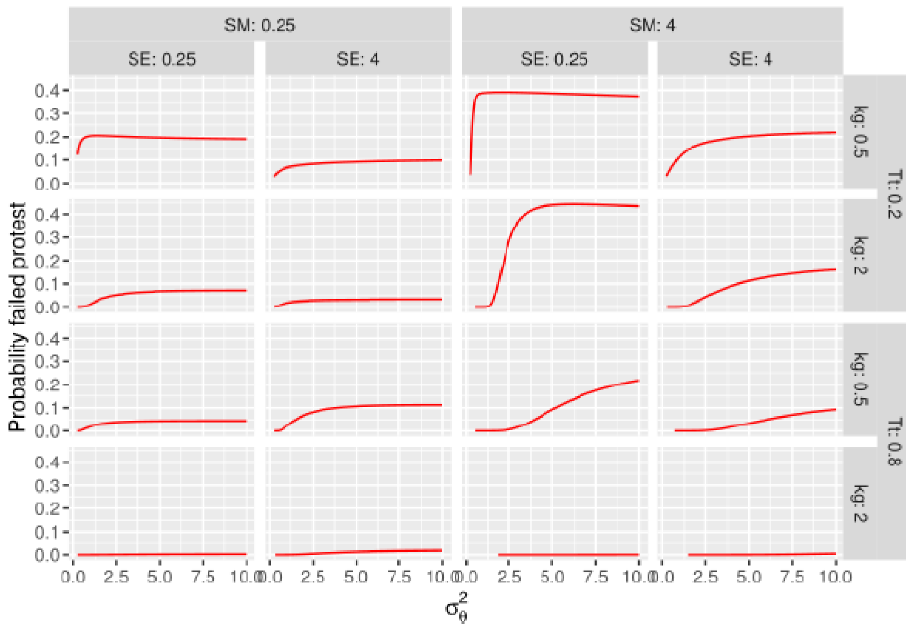
17.2 Simulation's results

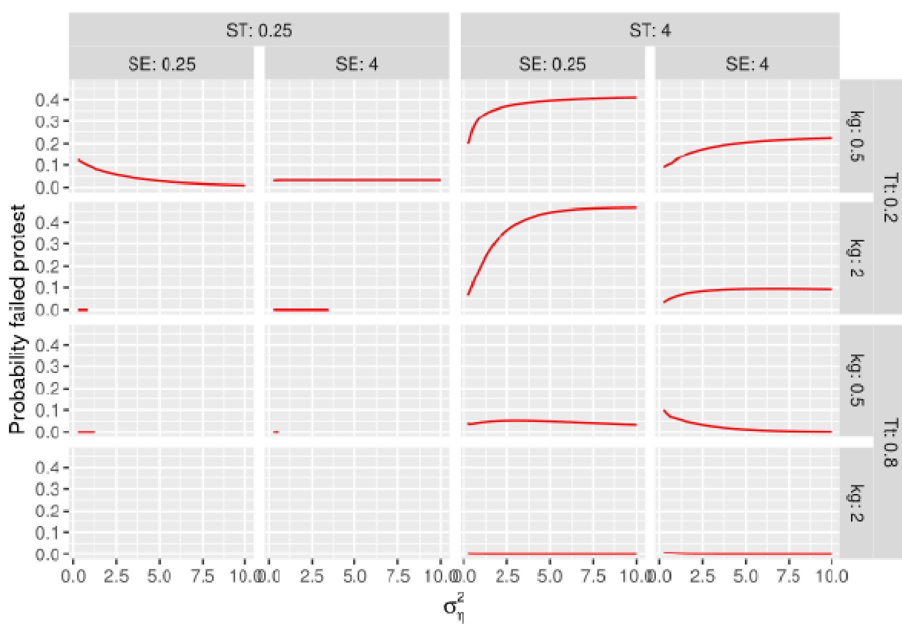


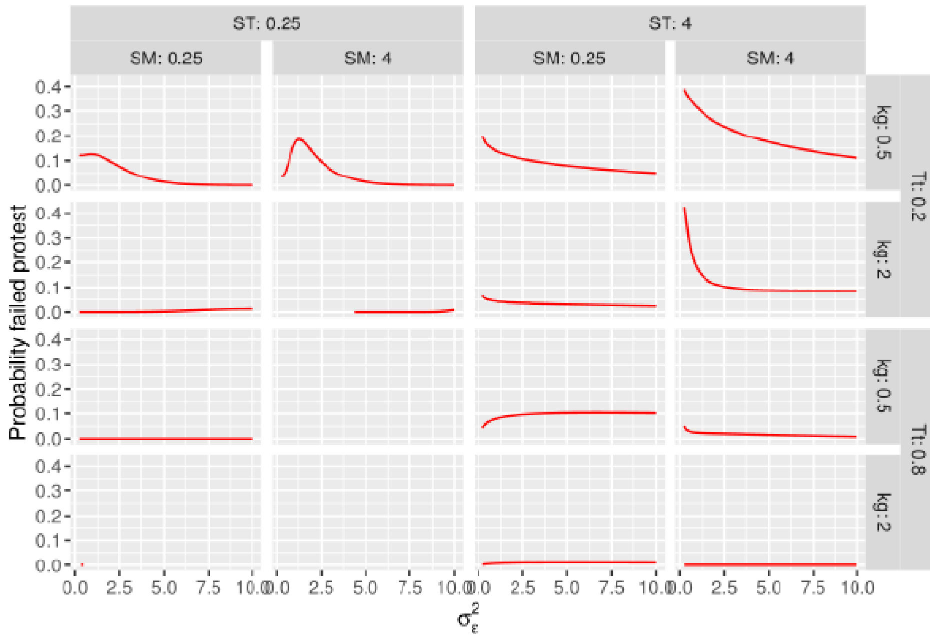
ST: 0.25
ST: 4
SM: 0.25
SM: 4
SE: 0.25
SE: 4
delta: 0.3
delta: 0.8
kg: 0.5
kg: 2
Probability failed protest
T



ST: 0.25
ST: 4
SM: 0.25
SM: 4
SE: 0.25
SE: 4
delta: 0.3
Tt: 0.2
Tt: 0.8
Probability failed protest
k/γ


SM: 0.25
SM: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability failed protest
0.0
0.1
0.2
0.3
0.4
0.0
2.5
5.0
7.5
10.0
σ^2_θ


ST: 0.25
ST: 4
SE: 0.25
SE: 4
SE: 0.25
SE: 4
kg: 0.5
kg: 2
kg: 0.5
kg: 2
Tt: 0.2
Tt: 0.8
Probability failed protest
0.0
0.1
0.2
0.3
0.4
0.0
2.5
5.0
7.5
10.0
σ^2_η



18 Lemma 4.8

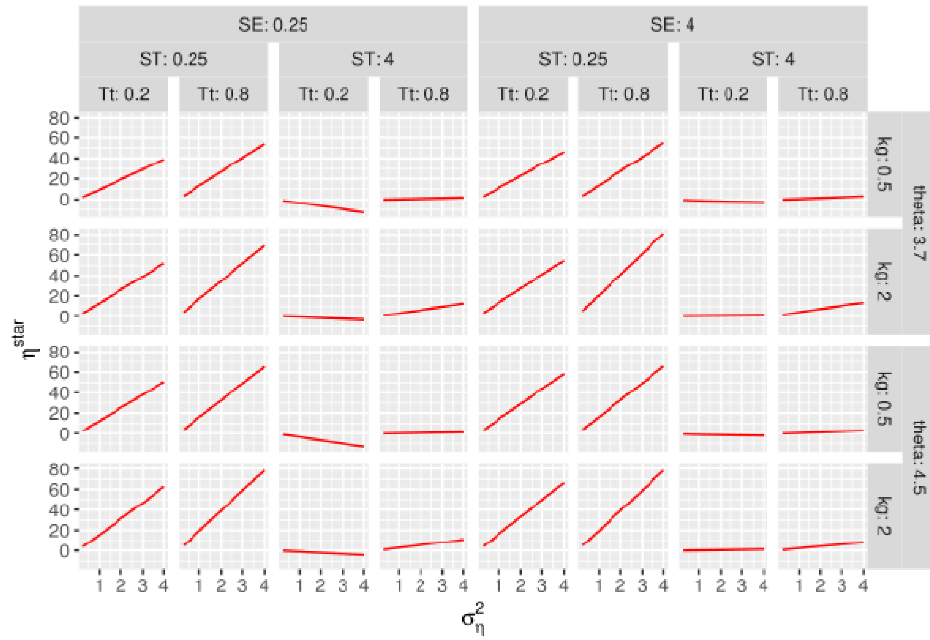
The relationship between all the points (η^*, θ) of the curve and the country radicalization, diversity and opacity in public information is uncertain: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters. Using simulations, we are able to derive the following results: (η^*, θ) is

1. increasing in opacity unless the political regime is responsive and the society is radicalized and homogeneous;
2. increasing in diversity;
3. decreasing in radicalization.

18.1 Simulation's code

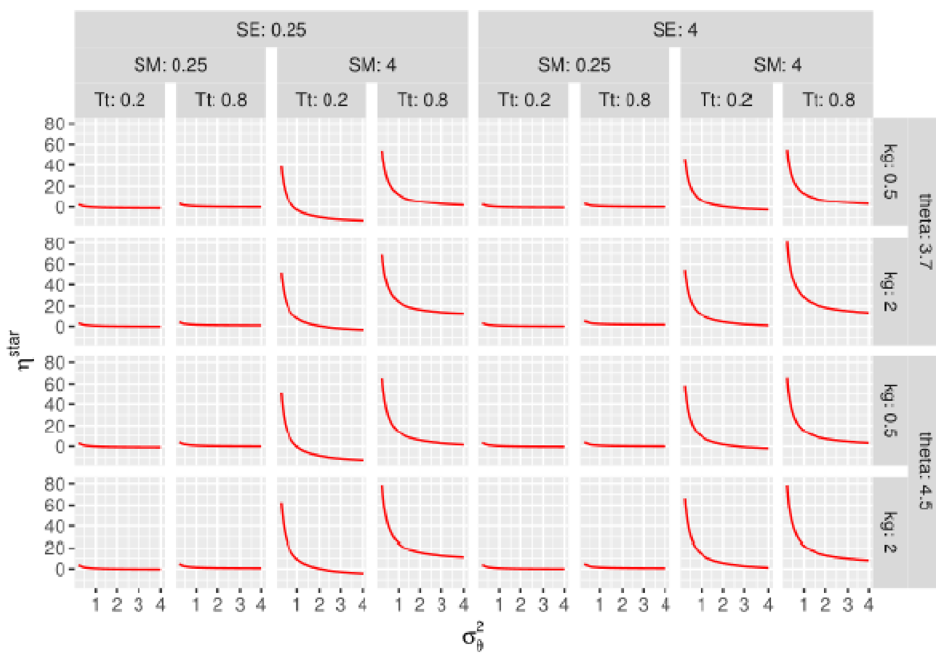
```
ST=c(0.25,4)
SE=c(0.25,4)
SM=seq(0.25,4,0.01)
Tt=c(0.2,0.8)
theta=c(3.7,4.5)
kg=c(0.5,2)
```

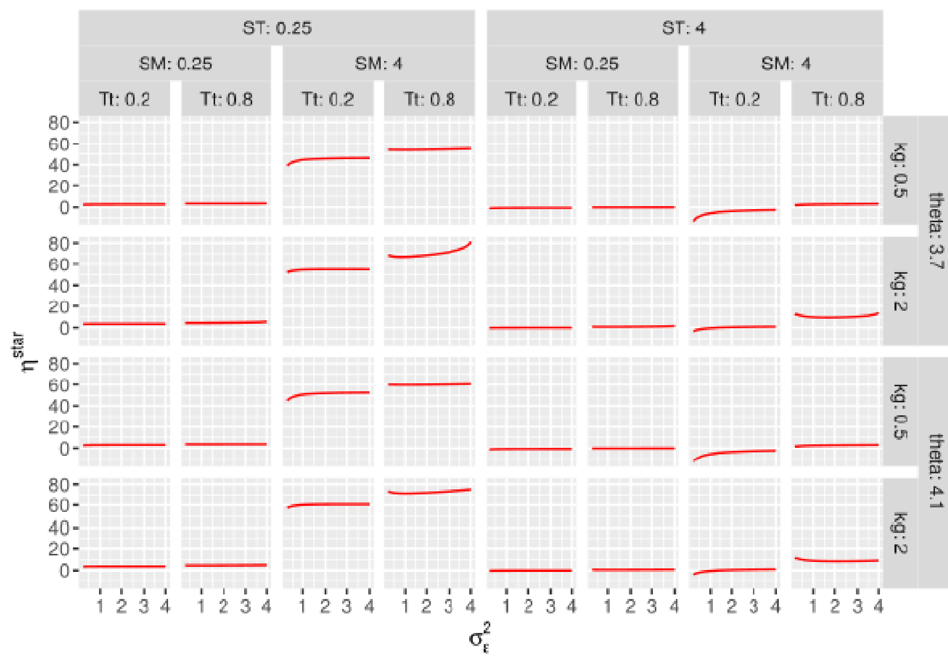
```
a=crossing(ST,SE,SM,Tt,theta,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=(1-a$psi)*(1-a$lambda)/a$psi * a$theta
a$Y2=sqrt(a$SM/a$psi)*qnorm(a$kg/(a$theta-sqrt(a$SE)*qnorm(a$Tt)))
a$Y3=(1-a$psi)*a$lambda*sqrt(a$SE)*qnorm(a$Tt)/a$psi
a$Y4=a$Y1+a$Y2+a$Y3
a %>%
ggplot(aes(x=SM, y=Y4)) +
geom_line(col="red") +
facet_nested( theta+kg ~ SE+ST+Tt , labeller = label_both) +
labs(y=expression(eta^star),x=expression(sigma[eta]^2))

SM=c(0.25,4)
SE=c(0.25,4)
ST=seq(0.25,4,0.01)
Tt=c(0.2,0.8)
theta=c(3.7,4.5)
kg=c(0.5,2)
a=crossing(ST,SE,SM,Tt,theta,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=(1-a$psi)*(1-a$lambda)/a$psi * a$theta
a$Y2=sqrt(a$SM/a$psi)*qnorm(a$kg/(a$theta-sqrt(a$SE)*qnorm(a$Tt)))
a$Y3=(1-a$psi)*a$lambda*sqrt(a$SE)*qnorm(a$Tt)/a$psi
a$Y4=a$Y1+a$Y2+a$Y3
a %>%
ggplot(aes(x=ST, y=Y4)) +
geom_line(col="red") +
facet_nested( theta+kg ~ SE+SM+Tt , labeller = label_both) +
labs(y=expression(eta^star),x=expression(sigma[theta]^2))

SM=c(0.25,4)
ST=c(0.25,4)
SE=seq(0.25,4,0.01)
Tt=c(0.2,0.8)
theta=c(3.7,4.1)
kg=c(0.5,2)
a=crossing(ST,SE,SM,Tt,theta,kg)
a$lambda=a$ST/(a$ST+a$SE)
a$psi=a$lambda*a$SE/(a$lambda*a$SE+a$SM)
a$Y1=(1-a$psi)*(1-a$lambda)/a$psi * a$theta
a$Y2=sqrt(a$SM/a$psi)*qnorm(a$kg/(a$theta-sqrt(a$SE)*qnorm(a$Tt)))
a$Y3=(1-a$psi)*a$lambda*sqrt(a$SE)*qnorm(a$Tt)/a$psi
a$Y4=a$Y1+a$Y2+a$Y3
a %>%
```




```
ggplot(aes(x=SE, y=Y4)) +
geom_line(col="red") +
facet_nested( theta+kg ~ ST+SM+Tt , labeller = label_both) +
labs(y=expression(eta^star),x=expression(sigma[epsilon]^2))
```

18.2 Simulation's results


SE: 0.25
SE: 4
SM: 0.25
SM: 4
Tt: 0.2
Tt: 0.8
kg: 0.5
kg: 2
theta: 3.7
theta: 4.5
0
20
40
60
80
1
2
3
4
η^{star}
σ_θ^2


ST: 0.25
ST: 4
SM: 0.25
SM: 4
Tt: 0.2
Tt: 0.8
kg: 0.5
kg: 2
theta: 3.7
theta: 4.1
η^{star}
σ_ε^2