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Declining US Natural Interest Rate: Quantifying and Qualifying the Role of Pensions

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Declining US Natural Interest Rate: Quantifying and Qualifying the Role of Pensions[*]

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Abstract

We develop a life-cycle model and calibrate it to the US economy to quantify and qualify the role of the public pension system for the past and future trend of the natural interest rate, the so-called r^*. Between 1970 and 2015, past pension reforms mitigated the secular decline in r^*, raising it by around 1%, mainly through the positive effect of a higher replacement rate. As regards the future, we simulate the demographic trends, expected between 2015 and 2060, combined with alternative pension reforms and productivity growth scenarios. An increase in the effective retirement age delivers the highest r^*, and thus the best welfare results, regardless of future productivity. On the contrary, the effects of a lower replacement rate strongly depend on the future productivity scenario. Under stagnant productivity, such pension system adjustment would exacerbate the fall in $r*$ induced by population ageing, with negative implications for welfare.

Keywords: Natural interest rate, pensions, population ageing
JEL classification: E60, H55

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1 Introduction

Since the US economy experienced more than ten years of zero nominal rates, a growing body of the literature has emphasized the central role of the declining "natural" interest rate, the so-called r^*, used as a benchmark for setting the policy rate, in accounting for such a stylized fact (e.g., Laubach and Williams, 2016; Kiley and Roberts, 2017). The existing literature has explained the declining trend in the US natural interest rate as the result of a widening gap between saving and investment, and it has put forward several candidates as potential drivers of such phenomenon (Summers, 2014), among which, demographics and productivity stand out as the most quantitatively relevant (Eggertsson et al., 2019; Gagnon et al., 2021). However, it has overlooked an economic institution that crucially affects saving behavior: the public pension system.

Yet, the size of the US pension system, officially the Old-Age, Survivors, and Disability Insurance (OASDI) program, is not negligible (7% of GDP, 2019 OECD data).[1] Hence, pensions could have a direct, quantitatively relevant, impact on r^*.

Moreover, the size and rules of the pension system vary in response to demographic trends, which influence the dependency ratio (DR). The US DR between people aged over 65 and those 20-64 has been more stable than that of other advanced countries, due to a more muted increase in life expectancy and decline in the fertility rate (Figure 1), calling for minor and less severe pension adjustments to demographic change.[2] Notwithstanding, the DR is expected to increase in the future, forcing the US government to speed up the reform process.[3] The pension reforms, induced indirectly by demographics, could also have a significant impact on r^*.

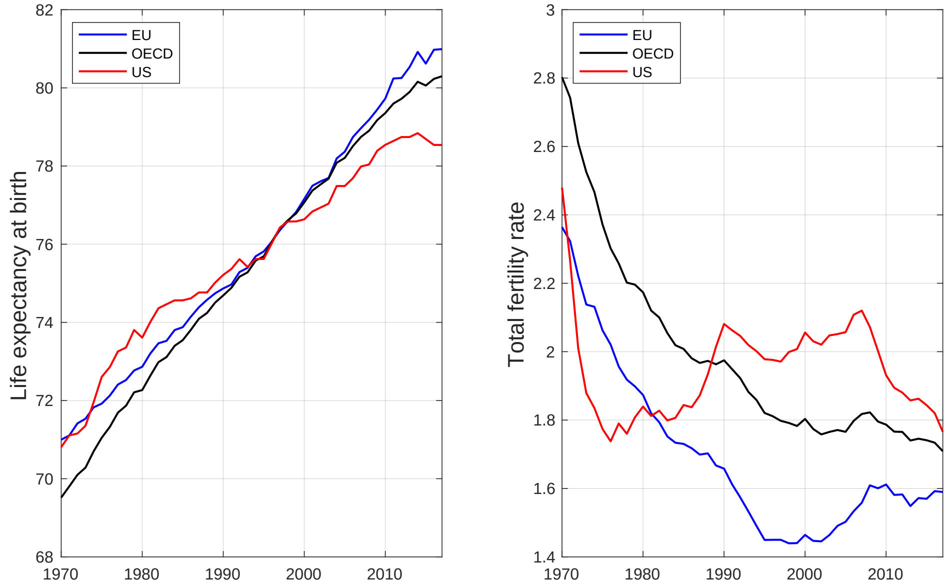
This paper investigates the quantitative importance of the public pension system for the US natural interest rate in the last fifty years, and it carries out a prospective

[1]The OASDI program operates under a pay-as-you-go (PAYG) basis providing benefits to retirees and disabled people. The current workers finance the benefits of the current retirees through payroll taxes, and the pension system keeps the financial resources in two trust funds, the Old-Age and Survivors Insurance Trust Fund (OASI) for retirement and the Disability Insurance Trust Fund (DI) for disability.

[2]A higher DR threatens the financial sustainability of PAYG schemes because fewer workers would finance the benefits of more retirees. Most of the advanced countries already experienced a pronounced rise in the DR and, consequently, implemented major severe reforms to contain expenditure and increase revenues (OECD, 2017). The US DR did not rise markedly (from 19.7% in 1975 to 24.6% 2015), and thus only minor pension reforms were implemented, while major ones (e.g., "Simpson-Bowles" plan) were never adopted (OECD, 2013).

[3]The US DR will reach 40.3% over the period 2015-2050 (OECD, 2017). Consequently, the OASI will exhaust in 2034 (2019 Annual Report of the Board of Trustees of the OASI and DI), and the deterioration in the funding position of the OASI will presumably call for severe pension reforms.

Figure 1: Demographic trends
Source: World Bank



analysis of its future impact, in response to population ageing, under different policy and productivity growth scenarios. Put simply, we aim to identify and measure the effect of the pension system and its reforms on r^* in the past and the future.

To identify the theoretical channels through which pensions affect r^*, we first develop an overlapping generations (OLG) model with three generations. Our stylized model suggests that the existence of a PAYG pension scheme counteracts the fall in r^* by shrinking excess savings. Indeed, the contribution to the scheme decreases the amount of resources available to save, and (future) pension benefits mitigate the incentive to save. Accordingly, a pension reform cutting benefits via a lower replacement rate induces saving more putting further downward pressure on r^*, while a higher contribution mitigates the downward pressure by decreasing savings. Finally, a higher retirement age (RA) influences r^* in the same direction of the contribution reform because, though savings are accumulated across a longer working life, a shorter retirement reduces the optimal stock of saving to smooth consumption.[4]

Once identified the theoretical channels, we build a quantitative life-cycle OLG model to measure the effect of pensions on r^*. Our model replicates a realistic age structure given that the probabilities of retiring and dying are age-dependent, unlike in

[4]Major pension reforms in advanced countries were implemented precisely by cutting the replacement rate and by raising the contribution rate and the retirement age (OECD, 2017).

the “perpetual youth” (PY) models (e.g., Carvalho et al., 2016; Kara and von Thadden, 2016).[5] Furthermore, while it shares its basic features with the one in Eggertsson et al. (2019), it is augmented with a careful representation of the US pension system. This innovation allows us decomposing the decline in r^* between 1970 and 2015, shedding light on the specific quantitative effect of the US pension system disregarded by the previous literature.[6] In particular, our augmented model can disentangle the impact of the single pension reforms on r^*, which is neglected by the general analysis of fiscal policy in Rachel and Summers (2019).

On the one hand, accounting for the pension system, even absent any reform, allows our life-cycle OLG model to quantify properly the contribution of all the other drivers to the fall in r^*, taking into account the ability of pensions to absorb savings. Specifically, the contribution of each driver is substantially smaller once the pension system is modelled. Taking the example of the lower mortality rate, it decreased the natural rate by 64 basis points according to our quantitative model with a pension system, against the 182 basis points obtained by Eggertsson et al. (2019) without it. On the other hand, the past reforms of the US pension system prevented the natural interest rate from dropping further by around 1%, counteracting the downward pressure of demographic and technological trends. This result originates mostly from the replacement rate, which increased substantially in the US over the period considered.

Since the future demographic trends are expected to worsen, with crucial implications for the pension system, we run a second quantitative exercise that simulates them to study the effect of different pension reforms on r^*. In this respect, our analysis is very close to the one of Attanasio et al. (2007) and Krueger and Ludwig (2007),[7] but we study the different pension adjustments under the scenario that the fall in r^* leads to $r < g$, and thus the implicit return of the PAYG asset is larger than the real rate.[8]

More precisely, we simulate the demographic changes underlying the United Nations population projections for the US between 2015 and 2060, and we study the

[5]For more technical details about the overall structure of the PY model and that of the quantitative OLG model employed we refer, respectively, to Gertler (1999) and Auerbach and Kotlikoff (1987).

[6]Most of the literature has studied the interplay between demographics, the pension system and interest rates in Europe, Japan or a joint group of advanced countries. See, e.g., Ikeda and Saito (2014), Carvalho et al. (2016), Cooley and Henriksen (2018), Barany et al. (2018), Bielecki et al. (2020), Papetti (2021), and Auclert et al. (2021).

[7]Other similar works regarding pension reforms are, among others, de Nardi et al. (1999), Börsch-Supan et al. (2006), and Krueger and Kubler (2006).

[8]Blanchard (2019) proves that $r < g$ is a real possibility for advanced economies, including the US, if the declining trend in r^* will persist in the future.

expected evolution of r^* and welfare implied by alternative pension reforms along the transition. Furthermore, to account for uncertain future technological trends, we compare a scenario of "stagnation", in which the labor productivity growth rate is fixed at the 2015 level (0.65%), to a "normal" one in which it grows at 2.02% per year.

Our results indicate that, although the future natural interest rate is subject to high variability depending on the scenario-reform combination, raising the effective RA always delivers the highest value of r^* in both productivity scenarios considered. Moreover, the RA reform determines the largest gains according to our welfare measure, which is based on the consumption equivalent variation relative to a the initial stationary equilibrium. By contrast, the welfare ranking of the two other pension adjustments examined, i.e., lower replacement rate and higher contribution rate, driven by the implied effect on r^*, depends on the evolution of labor productivity. If productivity growth is robust, the two reforms have a similar outcome in terms of on r^* and welfare. Instead, if productivity is stagnant, the replacement rate reform exacerbates the decline in natural interest rate due to population ageing, implying greater welfare losses than a reform of the contribution rate. Finally, in our environment, characterized by $r^* < g$, financing the pension deficit arising from future demographic patterns is feasible and does not lead to a higher debt-to-GDP ratio in the long run. However, such policy may determine a lower r^* in the future and worse welfare implications than other pension adjustments.

The remainder of the paper is organized as follows: Section 2 shows the main theoretical mechanisms at work in a simple three-period OLG model; Section 3 develops a quantitative life-cycle model, which is used in Section 4 and 5 to run our quantitative experiments; Section 6 concludes.

2 Theoretical model

We develop a theoretical model to study the effect of demographic changes, with and without a pension system, and pension reforms on r^*. We describe now our deterministic endowment economy with three overlapping generations and a PAYG pension system.

The size of each generation is N_t^i with $i = y, m, o$ and the ratio between the young and middle generation is $(1 + n_t) = \frac{N_t^y}{N_t^m}$, where n_t is the fertility/population growth rate at which grows also the total endowment. On the other hand, the length of the generations is, respectively, $\lambda_t^y = 1$, λ_t^m and $\lambda_t^o \in (0, 1)$ for youth, middle age/working life and old age/retirement, and the total length of life is accordingly[9]

$$\lambda_t = 1 + \lambda^m_{t-1} + \lambda^o_t \leq 3. \tag{1}$$

The representative household's maximization problem is

$$\max_{c^y_t, c^m_{t+1}, c^o_{t+2}} \left\{ \ln c^y_t + \beta \lambda^m_{t+1} \ln c^m_{t+1} + \beta^2 \lambda^o_{t+2} \ln c^o_{t+2} \right\}$$

s.t.

$$c^y_t = a^y_t \tag{2}$$

$$\lambda^m_{t+1} c^m_{t+1} = \lambda^m_{t+1} Y - (1 + r^*_t)\, a^y_t - \lambda^m_{t+1} a^m_{t+1} - \lambda^m_{t+1} T_{t+1} \tag{3}$$

$$\lambda^o_{t+2} c^o_{t+2} = \left(1 + r^*_{t+1}\right) \lambda^m_{t+1} a^m_{t+1} + \lambda^o_{t+2} \nu_{t+2} Y \tag{4}$$

$$a^y_t \leq \frac{\lambda^m_{t+1} D}{(1 + r^*_t)}. \tag{5}$$

The household's utility, discounted at the rate β, is given by the real consumption c^i_t with $i = y, m, o$, and it includes the length of generations. This lifetime utility representation distinguishes the sub-period/yearly utility in each stage of life and the length of each stage of life (Philipson and Becker, 1998).[10] Therefore, all variables are sub-period/yearly variables and need to be multiplied by the relevant λ to obtain their aggregate counterpart.[11]

Young people borrow and consume the real amount a^y_t, equation (2), by issuing a one-period risk-free bond, which pays the real return r^*_t. They face an exogenous debt limit D, which is adjusted by the length of the future middle generation (λ^m_{t+1}) in equation (5). Indeed, a longer middle age implies that the endowment Y is received for more periods, improving the ability to repay debt in the future and thus the borrowing capacity today. As shown by equation (3), middle-aged people are savers investing each sub-period their endowment in real bonds a^m_{t+1} (the private pension) and paying the contribution to the public pension system T_{t+1} (the public pension), as well as repaying the debt accumulated during the young age, $(1 + r^*_t)\, a^y_t$. Finally, the old generation

[9] This assumption is substantially equivalent to the one in Jorgensen and Jensen (2010), with the only difference that the length of life is fully predictable, and thus we do not need the expectation operator.

[10] In Philipson and Becker (1998), the sub-period/yearly utility measures the quality of life, while the length of generations measures the quantity of life. Given the assumed utility function and no discounting within middle age and within old age, the consumption in each sub-period is the same during both middle age and old age.

[11] For example: Y is the endowment in each sub-period of middle age and $\lambda^m_t Y$ is the total endowment.

consumes the private and public pension in equation (4), where the former is given by the (gross) real return, $1 + r^*_{t+1}$, multiplied by the total investment in bonds (= savings), $\lambda^m_{t+1} a^m_{t+1}$, while the latter is given by the replacement rate, ν_{t+2}, multiplied by the endowment, and it is paid out for all the sub-periods of the old age, λ^o_{t+2}. We assume the borrowing constraint is binding,

$$a^y_t = \frac{\lambda^m_{t+1} D}{(1 + r^*_t)}, \tag{6}$$

via a parametric assumption,[12] and the optimality condition for the problem is the standard Euler equation,

$$\frac{1}{c^m_t} = \beta (1 + r^*_t) \frac{1}{c^o_{t+1}}. \tag{7}$$

Finally, there exists a PAYG scheme, whose budget constraint is

$$\lambda^m_t T_t N^m_t = \lambda^o_t \nu_t Y N^o_t. \tag{8}$$

The left-hand side (LHS) is the total contribution, where λ^m_t determines how many sub-periods N^m_t middle-aged agents contribute to the pension system. Instead, the right-hand side (RHS) is the total expenditure, which corresponds to the pension benefit $\nu_t Y$ paid out for the entire length of retirement (λ^o_t) to N^o_t retirees. Although the RA is not explicitly modelled in (8), it is captured by λ^m_t, whose variation influences directly the contribution side (for how many sub-periods T_t is received), and indirectly the expenditure side via the length of the retirement (for how many sub-periods $\nu_t Y$ is paid out), because λ^o_t also varies with λ^m_{t-1} in (1), for a given λ_t. Hence, any increase (decrease) in the length of middle age coincides substantially with an increase (decrease) in the RA, and λ^m_t can be treated as a pension variable set by the government.

2.1 Natural interest rate, demographics and pensions

We focus on the credit market, where the natural interest rate r^* is determined, in particular, on its steady state equilibrium, in which both the demographic variables (λ, λ^o, n) and those related to the pension system (ν, T, λ^m) take a constant value. The credit market is in equilibrium when the demand for credit/borrowing from the young generation equals the supply of credit/saving from the middle one, given the different

[12] $D < \frac{1}{\left[1+\beta\left(\lambda^m_t + \beta \lambda^o_{t+1}\right)\right]} \left[(Y - T_t) + \left(\frac{\lambda^o_{t+1}}{\lambda^m_t}\right) \frac{\nu_{t+1} Y}{(1+r_t)}\right]$.

size and length of the generations. This, at the steady state, amounts to

$$(1+n)\,\lambda^y a^y = \lambda^m a^m. \tag{9}$$

Combining equation (6) and the left-hand side of (9) yields the total demand for borrowing D^c, namely

$$D^c = \left(\frac{1+n}{1+r^*}\right)\lambda^m D. \tag{10}$$

It depends positively on the size of borrowers (young people) relative to savers (middle-aged ones), $1+n$, as well as the exogenous debt limit D adjusted by the length of the middle generation. Instead, we derive the total supply of saving

$$S^c = \lambda^m a^m = \lambda^m \left\{ \frac{\beta\lambda^o}{\lambda^m + \beta\lambda^o} \left[(Y - D - T) - \frac{\nu Y}{\beta\,(1+r^*)} \right] \right\} \tag{11}$$

by using (3), (4), (6), and (7). It depends on the length of middle age λ^m, the extensive margin, and on the sub-period saving a^m, the intensive margin. The gross natural interest rate that equates (10) and (11) is

$$(1+r^*) = \frac{(\lambda^m + \beta\lambda^o)\,(1+n)\,D + \lambda^o \nu Y}{\beta\lambda^o\,(Y - D - T)}. \tag{12}$$

We now carry out a comparative statics analysis to study the effect of changing steady state values for demographic and pension variables on r^*.

Demographics and pensions. The two observed demographic changes are the decrease in fertility and the increase in life expectancy, but we focus exclusively on the latter to save space. We start from the case without a pension system ("No Pension", NP), i.e., $T = \nu = 0$ and $\lambda^m = \overline{\lambda}^m$. Then, we introduce the PAYG scheme ("Pension", P) by assuming a constant positive value for contribution and replacement rate (but no pension reforms), $T = \overline{T} > 0$ and $\nu = \overline{\nu} > 0$.

An increase in life expectancy λ (≤ 3) coincides with a longer retirement period $\lambda^o = \lambda - 1 - \overline{\lambda}^m$, keeping λ^m fixed ($\lambda^m = \overline{\lambda}^m$). A higher λ^o, for a given wealth, reduces consumption in each sub-period of retirement, determining a higher propensity to save in middle age and a lower natural rate:[13]

$$\left(\frac{\partial S^c}{\partial \lambda}\right)^{NP} = \frac{\beta\left(\overline{\lambda}^m\right)^2 (Y - D)}{\left(\overline{\lambda}^m + \beta\lambda^o\right)^2} > 0 \Rightarrow \left(\frac{\partial r^*}{\partial \lambda}\right)^{NP} < 0. \tag{13}$$

The positive impact of higher life expectancy on savings, and thus the resulting negative one on r^*, is mitigated when the PAYG is at work:[14]

$$\left(\frac{\partial S^c}{\partial \lambda}\right)^{P} = \left(\frac{\partial S^c}{\partial \lambda}\right)^{NP} - \frac{\beta\left(\overline{\lambda}^m\right)^2}{\left(\overline{\lambda}^m + \beta\lambda^o\right)^2}\left[\overline{T} + \frac{\overline{\nu}Y}{\beta\left(1 + r^*\right)}\right] > 0. \tag{14}$$

The pension scheme decreases disposable income and savings by levying $\overline{T}$. Furthermore, old households receive the pension benefit $\overline{\nu}Y$ for more time when λ^o increases, with a consequent higher lifetime income that reduces the incentive to save.

Pension reforms. Demographics, as well as influencing S^c and r^*, carries important consequences for the pension system budget through the DR $(= \frac{\lambda^o N_t^o}{\lambda^m N_t^m})$. As a result of longer life expectancy that causes higher λ^o (and lower fertility $1 + n = \frac{N_t^m}{N_t^o}$), the DR rises, generating a potential imbalance between the contribution side (LHS) and the expenditure side (RHS) in equation (8). To preserve the financial sustainability of the PAYG scheme, the government has to vary the policy parameters (ν, T, λ^m), which were treated as fixed above, while now they change one at a time. These pension reforms, in turn, can amplify or mitigate the original downward pressure exerted by demographics on r^*, equations (13) and (14).

A change in the replacement rate ν relates negatively to aggregate savings and positively to the natural interest rate:

$$\left(\frac{\partial S^c}{\partial \nu}\right) = -\left(\frac{\overline{\lambda}^m}{\overline{\lambda}^m + \beta\lambda^o}\right)\frac{\lambda^o Y}{(1 + r^*)} < 0 \Rightarrow \left(\frac{\partial r^*}{\partial \nu}\right) = \frac{Y}{\beta\left(Y - D - \overline{T}\right)} > 0. \tag{15}$$

[13] Any change in D^c affects positively r^*, $\frac{\partial r^*}{\partial D^c} > 0$, while changes in S^c do negatively, $\frac{\partial r^*}{\partial S^c} < 0$ (Appendix A.1, Figure 5). Hence, a reduction of n, and the credit demand (10), has the same negative effect on r^* as a higher λ, and the analysis below applies also to a lower fertility rate. More generally, demographics affects negatively r^* through several channels (Carvalho et al., 2016; Eggertsson et al., 2019; Gagnon et al., 2021). Here we replicate only those interacting with pensions, while our quantitative model, in Section 3, incorporates all of them.

[14] The partial derivative (14) is positive because $\left[\left(Y - D - \overline{T}\right) - \frac{\overline{\nu}Y}{\beta(1+r^*)}\right] > 0$, which is the same condition for having a positive supply of saving (11).

Therefore, a reduction of the replacement rate (negative change), which aims to restore a balanced pension system budget via lower aggregate expenditure, increases saving, putting further downward pressure on r^*, on top of that from demographics. Indeed, a lower ν reduces the lifetime income of the agents, inducing them to build up more savings during middle age.

Instead, if the contribution to the pension scheme T varies, this has a negative impact on savings and a positive one on the natural rate:

$$\left(\frac{\partial S^c}{\partial T}\right) = -\overline{\lambda}^m \left(\frac{\beta\lambda^o}{\overline{\lambda}^m + \beta\lambda^o}\right) < 0 \Rightarrow \left(\frac{\partial r^*}{\partial T}\right) = \frac{\left[\left(\overline{\lambda}^m + \beta\lambda^o\right)(1+n)\, D + \lambda^o \overline{\nu} Y\right]}{\beta\lambda^o \left(Y - D - T\right)^2} > 0. \tag{16}$$

An increase in T, aimed at raising aggregate contribution to the PAYG scheme, reduces the resources available to save at middle age, and the negative effect on aggregate savings counteracts the original downward pressure on r^*.

Finally, the government can raise λ^m through the RA, so that agents pay contributions for more sub-periods of middle age. By taking the new higher λ as given, this reform also shortens the length of retirement λ^o in (1), causing a lower expenditure for pension benefits. A longer middle age has a positive impact on D^c,

$$\left(\frac{\partial D^c}{\partial \lambda^m}\right) = \left(\frac{1+n}{1+r^*}\right) D > 0, \tag{17}$$

because an endowment received for a longer period improves the capacity to repay debt. Instead, the effect of a higher λ^m, and a lower λ^o,[15] on S^c is ambiguous,

$$\left(\frac{\partial S^c}{\partial \lambda^m}\right) = a^m - \lambda^m \left[\frac{\beta\left(\lambda^m + \lambda^o\right)}{\left(\lambda^m + \beta\lambda^o\right)^2}\right] \left[\left(Y - D - \overline{T}\right) - \frac{\overline{\nu} Y}{\beta\left(1 + r^*\right)}\right] \gtreqless 0, \tag{18}$$

given that the extensive and intensive margins of saving work in opposite directions. Since a higher λ^m raises the number of sub-periods agents save, for any given a^m, there is a positive effect on S^c working through the extensive margin (the first term on the RHS). On the other hand, a lower λ^o: *i*) reduces the propensity to save because a lower a^m is necessary to smooth consumption through a shorter retirement; *ii*) induces to increase a^m because pension benefits $\overline{\nu} Y$ are received for fewer sub-periods. Overall, given $\left[\left(Y - D - \overline{T}\right) - \frac{\overline{\nu} Y}{\beta(1+r^*)}\right] > 0$, there is a negative net effect on S^c working through the intensive margin (the second term on the RHS).

[15]In computing (18) and (19), we plug (1) into (11) to consider also the variation of λ^o.

The forces pushing r^* up (via higher borrowing and lower savings) are relatively stronger than the one pushing it down (via higher savings). As a consequence, the overall impact of a higher RA on the natural rate is positive:

$$\left(\frac{\partial r^*}{\partial \lambda^m}\right) = \frac{(\lambda - 1)}{(\lambda - 1 - \lambda^m)^2}\left[\frac{(1+n)\,D}{\beta\left(Y - D - \overline{T}\right)}\right] > 0. \tag{19}$$

3 Quantitative model

Although our three-period OLG model explains clearly the relationship between a PAYG scheme, and its reforms, and the natural rate, we need a more sophisticated framework to measure the quantitative importance of that relationship. In particular, we need to model a production economy in which the saving decision affects production via capital accumulation, and with a more realistic age structure and pension system. The proposed quantitative framework draws on the one in Eggertsson et al. (2019), which we extend adding a public PAYG pension system that captures the specifics of the US OASDI program.

Here we sketch the behavior of households, firms, and government, referring to Appendix B for the complete model, our calibration strategy and the solution method. The behavior of households and firms is the same assumed in Eggertsson et al. (2019), except for the features interacting with the pension system, our innovation.

3.1 Households

Households enter the economy (and have kids) at age 26, participating to the labor market until their retirement at age RA_t. They die certainly at the maximum possible age of $J = 81$, but they face a positive probability of dying even before. The population growth depends on the fertility rate of every family, f_t. A representative household i aged j gets utility from consumption and bequest to each descendent, $c_t(i,j)$ and $x_t(i,j)$, with the corresponding CRRA utility functions, $u(.)$ and $v(.)$, discounted at the rate β and multiplied by the age-dependent survival probability $s(j)$, and the strength of the bequest motive measured by the parameter μ.[16] A household i entering the economy

[16] By assumption $s(j = 26) = 1$.

at time t maximizes accordingly the lifetime utility

$$U_t(i) = \sum_{j=26}^{J} \left\{ \prod_{m=26}^{j} [s(m)]\, \beta^{j-26} u\big(c_{t-26+j}(i,j)\big) \right\} + \prod_{m=26}^{J} [s(m)]\, \beta^{J-26} \mu v\big(x_{t-26+J}(i,J)\big)$$

subject to the budget constraints

$$c_t(i,j)+\xi_t a_{t+1}(i,j+1) = (1-\tau_t^b-\tau_t^w \mathbf{1}_{i\in\Psi_t})w_t hc(j)+\Pi_t(j)+[r_t^k+\xi_t(1-\delta)]\left[\frac{a_t(i,j)}{s(j)}+q_t(j)\right]$$

$$c_t(i,j)+\xi_t a_{t+1}(i,j+1)+f_{t-j+26}(26)x_t(i,j) = p_t^b(j)\mathbf{1}_{i\in\Psi_t}+[r_t^k+\xi_t(1-\delta)]\left[\frac{a_t(i,j)}{s(j)}+q_t(j)\right].$$

The first constraint holds for $26 \leq j \leq RA_t$, when the household is young and active in the labor market, while the second one holds for $RA_t < j \leq J$, when the household is old and retired. Young households supply inelastically their labor endowment for the labor income $w_t hc(j)$, where w_t is the real wage and $hc(j)$ is the age-dependent labor efficiency level. A proportion τ_t^b of the labor income is paid in form of taxes to the government, while τ_t^w is the contribution rate to the public pension system. The indicator function $\mathbf{1}_{i\in\Psi_t}$ is a dummy that takes value 1 only when $i \in \Psi_t$, i.e., if the household i participates to the pension system, and it is 0 otherwise.[17] Young households also earn firms' profits $\Pi_t(j)$, which are distributed proportionally according to gross labor income.

Agents can save to smooth consumption over their lifetime by purchasing one-period assets, $a_t(i,j)$, which can be physical capital or risk-free bonds. The exogenous price of capital in consumption units is ξ_t, while the return on capital, which depreciates at the rate δ, is r_t^k. Young households can also borrow, but they face the borrowing limit $a_t(i,j)(1+r_t) \geq D_t(j) = d_t w_t hc(j)$, where $-1 \leq d_t \leq 0$.[18] Finally, all households insure against the idiosyncratic risk of death before age J by participating to annuity markets (Ríos-Rull, 1996). Involuntary bequests are accordingly shared among the surviving members of the same cohort, as expressed by the term $\frac{a_t(i,j)}{s(j)}$. Instead, voluntary bequests are left only at age J, while inheritances are received one period after the

[17] Ψ_t is the set of pension scheme participants at time t. The size of Ψ_t is $\psi_t \sum_{j=26}^{J} N_t(j)$, as a constant fraction ψ_t of each cohort j participates to the scheme.

[18] For a no-arbitrage condition, the return from risk-free bonds equals that from capital investment: $1 + r_t = [r_t^k + (1-\delta)\xi_t]/\xi_{t-1}$. The borrowing constraint is expressed on asset accumulation, unlike equation (5) of the theoretical model. Moreover, $D_t(j)$ grows at the rate of productivity growth (due to w_t) and the household's earning potential over the life-cycle (due to $hc(j)$).

death of their parents, $q_t(j = 57)$.[19] After retirement, individual income is given by the proceedings from investing in assets, and for the pension scheme participants, the benefit $p_t^b(j)$.

3.2 Firms

The aggregate production function is of CES-type:

$$Y_t = \left[\alpha(A_k K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{l,t} L_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{20}$$

where $0 < \alpha < 1$. Y_t is aggregate output, σ is the elasticity of substitution between inputs, $A_{l,t}$ is a labor-augmenting technological process growing at the exogenous rate g_t^p, and A_k is the constant capital productivity. Aggregate labor is the sum of the labor productivity of each cohort weighted by its mass, $L_t = \sum_{j=26}^{J} N_t(j)hc(j)$, where $hc(j) = 0$ for $j > RA$. Aggregate capital K_t evolves over time according to the law of motion $K_{t+1} = (1-\delta)K_t + \frac{I_t}{\xi_t}$, where I_t is aggregate investment. The returns to labor and capital are, respectively,

$$w_t = \frac{1}{1+\psi_t \tau_t^f} \frac{\theta_t - 1}{\theta_t} (1-\alpha) A_{l,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} \tag{21}$$

$$r_t^K = \frac{\theta_t - 1}{\theta_t} \alpha A_k^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}, \tag{22}$$

where $\theta_t > 1$ is the elasticity of substitution across final good varieties,[20] and $\psi_t \tau_t^f$ accounts for the employers contribution (at the tax rate τ_t^f) to the pension scheme in favor of the fraction ψ_t of workers participating to the scheme.

[19]Formally, $x_t(i,j) = 0 \quad \forall j \neq J$ and $q_t(j) = 0 \quad \forall j \neq 57$. As shown in the Appendix B.1, we assume that inheritances q_t do not depend on the index i, i.e., they are the same for participants and non-participants to the pension system of the same age.

[20]Although we focus here on the aggregate production sector only, there are three types of firms: monopolistically competitive final goods firms, and competitive intermediate goods and capital goods firms. See Appendix B.2 for more details.

3.3 Government

The government budget constraint is

$$B_{t+1} = (1 + r_{t+1})B_t + G_{t+1} - G^p_{t+1} - T_{t+1}, \tag{23}$$

where G_t is the public expenditure, G^p_t is the pension surplus, B_t denotes public debt, and labor income taxes are $T_t = \tau^b_t w_t L_t$. On the balanced growth path, we assume that the government debt-to-output ratio is constant and the tax rate τ^b varies to keep the government budget balanced.[21]

3.4 Pension system

The pension system plays a central role in our quantitative model, and we tailor it to replicate the salient features of the US public pension system, the OASDI program. While a fraction of households, $\psi_t \leq 1$, participates to the public pension system, contributing the tax rate τ^w_t when active and receiving the pension $p^b_t(j)$ once retired, the remaining fraction of households does not, $\tau^w_t = p^b_t(j) = 0$. The budget constraint of the pension system is

$$G^p_t = \psi_t \tau^p_t w_t \sum_{j=26}^{RA} N_t(j) hc(j) - \sum_{j=RA+1}^{J} \psi_{t+RA-1-j} N_t(j) p^b_t(j), \tag{24}$$

where $\tau^p_t = \tau^w_t + \tau^f_t$ is the total contribution rate from workers and employers. The first term on the RHS is the total contribution from working households, while the second term is the total expenditure for pensions benefits to retirees. Equation (24) is the equivalent of (8). However, now the pension budget is not necessarily balanced. The variable G^p_t denotes the pension system surplus or deficit, depending on whether the total contributions exceed the total benefits or vice versa. This new assumption allows for more precise calibration of the model to the OASDI program, which has run a surplus over the last decades but it is expected to undergo deterioration of its financial conditions due to population ageing and the resulting higher DR.

[21] This assumption implies that whenever $G^p_t < 0$, the tax burden imposed by the pension deficit falls on all working households, including those not covered by the public pension scheme. Similarly, if $G^p_t > 0$, the pension surplus is shared across all working households.

The individual pension benefit of a retiree aged j is

$$p_t^b(j) = \nu_t \phi(RA_t, FRA_t) \frac{w_{t-j+60}}{35} \sum_{z=RA-35+1}^{RA_t} hc(z). \tag{25}$$

Our calculation of the pension benefit, which is a fraction ν_t of the average gross labor income, follows the US Social Security regulation closely. A detailed account of the computation procedure can be found in Appendix B.3. In short, the US Social Security Administration computes the pension benefits according to the Primary Insurance Amount (PIA), which considers only the average labor earnings of the top 35 years of contribution, defined as Average Indexed Monthly Earnings (AIME). Monthly earnings are indexed relative to the average wages of the indexing year, the year in which the contributor turns 60. We calculate the AIME by averaging the gross labor earnings of the last 35 years of work because, given the calibration, they correspond to the top 35 years of earnings. On the other hand, we index individual wages relative to the economy-average wage in the year in which the agent turned 60, i.e., w_{t-j+60}. As the US regulation applies a penalty to benefits in case of early retirement, the function $0 < \phi(RA_t, FRA_t) \leq 1$ gives the penalty, in terms of replaced contributions, if the common retirement age of agents RA_t is lower than the full retirement age FRA_t.

Table 1: Calibration

Parameters	Symbol	1970	2015	
Estimated from the data				Source
Income profile	hc_j			Gourinchas and Parker (2002)
Mortality profile	s_j			US mortality tables, CDC
Total fertility rate	n	2.8	1.88	UN fertility data
Productivity growth	g^p	2.02%	0.65%	Fernald (2012)
Government spending (percent of GDP)	G	21.3%	21.3%	CEA
Public debt (percent of GDP)	$\frac{B}{Y}$	42%	118%	Flow of Funds
Replacement rate	ν	32.3%	40.8%	US Social Security
Retirement age	RA	63	65	US Census Bureau
Pension coverage	ψ	90%	96%	US Social Security
Pension contribution rate	τ^p	8.4%	12.4%	US Social Security
Taken from the literature				Source
Elasticity of intertemporal substitution	ρ	0.75	0.75	Gourinchas and Parker (2002)
Capital/labor elasticity of substitution	σ	0.6	0.6	Antras (2004)
Depreciation rate	δ	12%	12%	Jorgenson (1996)
Price of investment goods	ξ	1.3	1	Fernald (2012)
Calibrated to match some data moments				
Rate of time preference	β	0.96	1.01	
Borrowing limit (percent of annual gross labor income)	d	9.52%	50.22%	
Bequests parameter	μ	55.06	9.12	
Retailer elasticity of substitution	θ	8.6	4.89	
Capital share parameter	α	0.19	0.24	
Data moments				Source
Natural interest rate		2.62%	-1.47%	FED
Investment-to-output ratio		16.8%	15.9%	NIPA
Consumer-debt-to-output ratio		4.2%	6.3%	Flow of Funds
Labor share		72.4%	66%	Elsby et al. (2013)
Bequests-to-output ratio		3%	3%	Hendricks (2001)

4 The natural interest rate in the past

We employ our quantitative model to disentangle the contribution of several determinants of the declining trend in the US natural interest rate between 1970 and 2015. Since our model is an extended version, *with* pensions, of the one in Eggertsson et al. (2019), and we follow their procedure to decompose the fall in r^*, we use the latter model as the benchmark *without* pensions to compare our results.

We decompose the decline of -4.09 percentage points in r^* (from 2.62% in 1970 to -1.47% in 2015) through comparative statics between balanced growth path-stationary equilibria.[22] We treat 1970 and 2015 as two balanced growth path-stationary equilibria,[23] and we calibrate our quantitative model accordingly (Table 1).[24] We first take the 2015 stationary equilibrium. Then, we shock, one at a time, the parameters associated with the drivers of r^*, assigning them the 1970 value, and keeping all the other parameters constant. We interpret the implied variation in the natural interest rate as the effect of each single driver on the evolution of r^* over 1970-2015. If the implied variation is positive, the effect of the driver is negative and vice versa. The top part of Table 2 shows the results of our decomposition for the single drivers, that is the "single effect". In general, this decomposition procedure follows the logic of the comparative statics analysis in Section 2, allowing us to provide our quantitative results with a theoretical intuition. We get two key results.

First, the pension scheme is crucial to quantify the contribution of different drivers to the decline in r^*. In our model, the main drivers are the same, lower mortality/fertility rates and productivity growth, but their impact on the past pattern of r^* falls greatly in absolute value compared to Eggertsson et al. (2019) (the two columns in the top of Table 2).[25]

[22]Our model replicates precisely the US natural interest rate in 1970 (2.62%), unlike the benchmark of Eggertsson et al. (2019) ($r^* = 2.55\%$). Therefore, the total variation of r^* in the two models is slightly different in the top of Table 2.

[23]A definition of competitive stationary equilibrium is given in Appendix B.4. To treat 2015 as a stationary equilibrium, we implicitly assume, as in Eggertsson et al. (2019), the US output gap was zero at that time, and thus the natural and real interest rates coincided at -1.47%.

[24]To be precise, the first set of parameters are taken from 1970 and 2015 data; the second set consists of constant parameters from the literature; and only the parameters in the third set are calibrated to match some key moments in 1970 and 2015 data. We describe our calibration in Appendix C.1.

[25]Given our decomposition procedure, the sum of all the amounts in the columns could not equal the total variation in r^* due to interaction effects. The effects of changes in the labor share and in the borrowing limit are not reported in the table as they do not represent of the focus of our analysis. Their impact on r^* is -0.43% and +0.21% respectively (-0.52% and +0.13% in Eggertsson et al., 2019).

Table 2: Decomposition of the decline in the natural interest rate

	Our model	EMR (2019)
Total natural interest rate variation	-4.09%	-4.02%
Single effect		
Mortality rate	-0.64%	-1.82%
Fertility rate	-0.38%	-1.84%
Productivity growth	-1.44%	-1.90%
Government debt (percent of GDP)	+0.99%	+2.11%
Relative price of investment goods	-0.27%	-0.44%
Replacement rate	+0.50%	-
Retirement age	+0.44%	-
Coverage	+0.16%	-
Contribution rate	-0.05%	-
Combined effect		
All pension reforms	+1.10%	-
Demographics	-0.99%	-
Demographics + All pension reforms	-0.21%	-
Demographics + Replacement rate	-0.71%	-
Demographics + Retirement age	-0.59%	-
Demographics + Coverage	-0.90%	-
Demographics + Contribution rate	-1.02%	-
Demographics + Productivity growth	-2.69%	-
Demographics + Productivity growth + All pension reforms	-1.90%	-

This quantitative finding derives from the pension system ability to absorb savings and raise the natural interest rate, described by (13) and (14) in the three-period OLG model. Contributing to the pension scheme reduces disposable income and crowds out private savings and bequests. Moreover, the PAYG entitlement offers a different type of insurance relative to the other assets in the quantitative model, disincentivizing saving.

Second, the effect of the pension reforms on the natural interest rate is quantitatively significant and positive overall, counteracting so the downward pressure exerted by demographic and technological trends. When we consider every single pension reform in the top part of Table 2 (last four rows), not surprisingly, the sign of the effect on r^* is the one theoretically identified. The only exception is the contribution rate, whose impact is close to zero since it changed marginally from 1970 (8.4%) to 2015 (12.4%). More interestingly, the replacement rate rose in the US from 32.3% to 40.8% in the period considered, discouraging private savings and increasing r^* (see equation (15)) by 0.5 percentage points. The impact of a higher RA, from 63 to 65 years, on r^* has the same sign (see (19)), because each household needs fewer savings to support the same consumption at old age, but a slightly smaller size, +0.44 percentage points.[26] Finally, the effect of a higher pension scheme coverage is positive and even smaller, +0.16%, consistently with the marginal increase in the OASDI coverage (from 90% to 96%).

As well as the single drivers of the r^* trend, we also consider several drivers simultaneously (the "combined effect"), such as all the pension reforms implemented, and their interaction with lower mortality and fertility rates ("demographics") and lower productivity growth. The bottom part of Table 2, in the first line, shows that the changes to replacement and contribution rates, RA, and coverage raised jointly the natural interest rate by approximately 1%. This is roughly the same extent[27] to which all the pension reforms shrank the negative impact on r^* of demographics alone (-0.21% vs -0.99%) and in conjunction with lower productivity (-1.9% vs -2.69%). Again, the single pension reforms that drive these results are the reform to the replacement rate and to the RA.

[26]Given the uncertainty surrounding the effective RA in 1970, in Appendix C.2, we calibrate a higher initial RA, 64 years, and thus a lower variation of this pension parameter over the period considered. This alternative calibration confirms our result qualitatively (the sign), but it points to a smaller quantitative impact of a higher RA on r^*, +12 basis points, though the overall effect of all pension reforms is still significantly high, +0.77 percentage points.

[27]It cannot be precisely the same due to interaction, as already discussed. For the same reason, the single effects of the pension reforms do not add up.

5 The natural interest rate in the future

The drivers of the historical decline in r^*, investigated in the previous section, will likely have an impact in the future as well. In particular, population is expected to age, with crucial implications for the natural interest rate and the pension system. In this section, we use our quantitative model, augmented with endogenous labor supply,[28] to simulate the US economy's transition in response to the future demographic projections of the UN for the period 2015-2060, taking as a starting point the 2015 stationary equilibrium.[29] In 2016, the economy is shocked unexpectedly by changes in mortality, fertility, and productivity growth, so that the living cohorts revise their optimal decisions of consumption and saving over the life-cycle. Just after the unexpected shock, all agents have perfect foresight on the future evolution of all exogenous variables, which return constant after 2060, when the economy has converged to a new stationary equilibrium. Overall, changes in longevity and fertility cause a variation in the DR from 42.4% in 2015 to 46.9% in 2060. This setting allows us to study the natural interest rate and the welfare of all the generations involved in the transition under two productivity scenarios and subject to different pension reforms.

We compare a "stagnation" scenario in which the labor productivity permanently grows at the 2015 level, $g^p = 0.65\%$, with a "normal" one in which it does at the average growth rate of the real GDP in the postwar period, $g^p = 2.02\%$. As the real rate equals the natural one, which is negative in the initial equilibrium (-1.47%), we account explicitly for the economic context where the real interest rate is lower than the economy's growth rate, $r^* < g = n + g^p$, where n is the population growth rate deriving from the abovementioned fertility rate f.[30] We view this environment as the most likely for the US economy in the future, given the past pattern of r and g (Blanchard, 2019).

We study three pension reforms: an increase in the tax rate τ^p, a decrease in the replacement rate ν, an increase by one year of the effective RA; in addition to these, we

[28]This innovation is outlined in Appendix D.1. In the previous exercise, for comparability reasons, we preferred not to add this feature so that the different results with respect to Eggertsson et al. (2019) are exclusively driven by the pension system. Instead, here the intertemporal allocation of labor, and its interaction with retirement age, is crucial for the welfare analysis.

[29]More precisely, we inform the model with the available estimates of demographic trends between 2015 and 2060, including year-by-year age-dependent survival probabilities and fertility rates. For further details about the calibration see Appendix D.2.

[30]Since we are interested in long run (real) phenomena, we abstract from (nominal) frictions that play a significant role in the short run, and thus from temporary deviations of the real rate from its natural counterpart.

Table 3: Main variables and pension parameters

	2015	New stationary equilibrium							
Reform	/	contribution rate		replacement rate		retirement age		debt-financing	
Scenario	/	stagnation	normal	stagnation	normal	stagnation	normal	stagnation	normal
r^*	-1.47%	-2.06%	-0.55%	-2.41%	-0.57%	-1.85%	-0.33%	-2.18%	-0.97%
τ^p	12.4%	15.5%	12.6%	12.4%	12.4%	12.4%	12.4%	12.4%	12.4%
ν	40.8%	40.8%	40.8%	32.4%	40.2%	40.8%	40.8%	40.8%	40.8%
RA	65	65	65	65	65	66	66	65	65
$\frac{B}{Y}$	118%	118%	118%	118%	118%	118%	118%	85%	32.1%
τ^b	35.3%	31.33%	31.63%	30.08%	31.56%	34.48%	32.36%	35.3%	35.3%
ϕ	93.3%	86.6%	86.6%	86.6%	86.6%	93.3%	93.3%	86.6%	86.6%
DR	42.4%	46.9%	46.9%	46.9%	46.9%	43.5%	43.5%	46.9%	46.9%

analyze the case of "debt-financing" in which there are no reforms and the pension deficit is debt-financed. For each of these adjustments, we shock the corresponding parameter keeping constant all the others, as shown in Table 3.[31] In absence of any pension reform, population ageing would bring about an increase in the pension balance deficit due to a larger DR. Under the first two adjustments considered, such deficit is neutralized by a change in the corresponding policy parameter. In the case of an increase in effective retirement age, the larger pension deficit is only partly offset by an increase in the number of working relative to retirement years.[32]

Table 3 displays the main variables and pension parameters in the initial and in the new long run stationary equilibrium, distinguishing the two productivity scenarios

[31] Apart from the change in the age of retirement, we assume that the effective RA remains at 65 years. However, given the automatic adjustment of the FRA to life expectancy in the US legislation, we impose that the penalty associated with early retirement increases as FRA reaches 67 years, and consequently $\phi(RA, FRA)$ in (25) declines from 93.3% to 86.6% for all the different pension system adjustments, except for the case of an increase in RA.

[32] In order to neutralize the effect of other fiscal variables on the government budget constraint, we assume that an adjustment of τ^b in (23) maintains the debt-to-GDP ratio constant at the 2015 level, $\frac{B}{Y} = 118\%$, for each of the three pension reforms. In the fourth case instead, we keep τ^b constant so that the increase in the pension deficit due to ageing has an impact on public debt. In Appendix D.3, we assume a constant payroll tax rate and an endogenous debt-to-GDP ratio for the three reforms. This exercise confirms our results regarding the impact of future pension reforms on r^*.

with grey and white columns. Not surprisingly, r^* is lower than the initial equilibrium value under "stagnation" (first row, grey columns), while it is higher than -1.47%, but still negative, in the "normal" productivity scenario (first row, white columns).[33] In general, the natural interest rate in the new equilibrium varies greatly according to the scenario-reform combination ranging from +114 (normal-retirement age) to -94 basis points (stagnation-replacement rate) relative to the initial 2015 level.

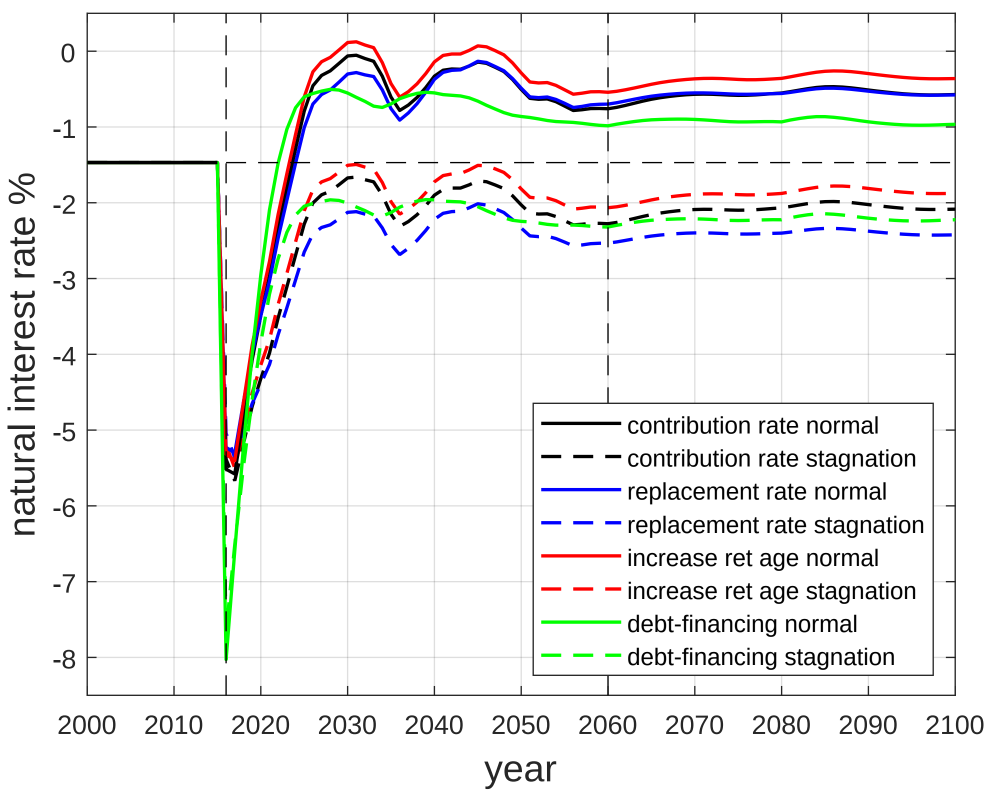
Looking at the specific pension reforms, a higher effective RA delivers the highest value of r^* regardless of the productivity scenario, as it counteracts the fall in the labor-capital ratio imposed by ageing. Furthermore, under the more optimistic scenario, a lower replacement rate and a higher contribution rate determine a similar increase in r^* (to -0.57% and -0.55% respectively). Instead, under the more pessimistic productivity scenario, decreasing the replacement rate determines a much larger fall in r^* relative to raising the contribution rate (-2.41% vs -2.06%), which results in the lowest value of the natural rate across simulations. Intuitively, a decrease of the replacement rate always provides an incentive to increase savings, with a consequent downward pressure on r^*. However, if the productivity growth is robust, the stronger wage growth reduces the need of a large adjustment to neutralize the impact of demographics on the balance between pension revenues and expenses, by increasing the total amount of contributions to the scheme. By contrast, under stagnant labor productivity growth, low wages combined with a shrinking working population call a much larger pension adjustment and, in the case of a replacement rate reform, a larger impact on overall savings.

These results are perfectly in line with those of Section 4, which highlights a significant effect of RA and ν on r^* in the past: while the replacement rate increase observed in the past mitigated the downward pressure on r^* exerted by demographics and technology, implementing a reduction of the replacement rate in the future under more adverse demographic and technological trends exacerbates their negative impact on r^*.

Finally, we analyze the case of "debt-financing". When productivity growth is low (grey columns), we observe a less marked decline in r^* than under the replacement rate reform, but "debt-financing" brings about also the smallest increase in r^* when labor productivity grows at 2.02% per year (white columns). The last result could appear

[33] As a consequence of population ageing, agents have an incentive to increase savings and aggregate labor as a factor of production becomes scarcer, with a negative impact on the labor-capital ratio and on r^* via the neoclassical production function. This phenomenon, defined as "capital-deepening", is more than offset by the increase in labor-augmenting technological progress under the normal productivity growth scenario.

Figure 2: Natural interest rate along the transition



surprising because a higher public debt would absorb, even substantially, the excess of saving pushing the natural rate up. However, the positive effect of a larger primary deficit on the debt-to-GDP ratio is more than compensated by the negative effect due to a positive differential between the debt refinancing cost and the GDP growth rate, (i.e. $r^* < g$). As a consequence, the debt-to-GDP ratio, $\frac{B}{Y}$, falls dramatically in both productivity scenarios, and a lower $\frac{B}{Y}$ reduces r^* given the relative abundance of savings over assets.

In order to complement our analysis, we now investigate the entire time path of r^* along the transition in Figure 2, where solid lines correspond to the "normal" case and dashed lines to "stagnation". For each reform, we can interpret the distance between the full and dashed lines of the same color in a given year of the transition as an approximated measure of the natural interest rate "variability" associated with the two productivity scenarios.

The variability in the dynamics of r^* implied by the replacement rate reform, measured by the vertical distance between solid and dashed blue lines, is much larger than the one due to the RA and contribution rate adjustments (red and black lines respectively). Overall, the range of values of r^* identified by the two productivity scenarios with "debt-financing", which causes a dramatic fall of the natural interest rate in the early years of the transition, appears as the narrowest (i.e. the vertical distance between the green lines is the smallest compared to the other pension reforms). Consistently with the results of Table 3, the effect of a lower replacement rate on the future dynamics of r^* varies substantially with the productivity scenario. Again, the reform of the replacement rate brings about an evolution of the natural interest rate almost identical to the one implied by the contribution rate adjustment with "normal" productivity growth, but the former has a much stronger negative impact on r^* than the latter with "stagnation".

5.1 Welfare analysis

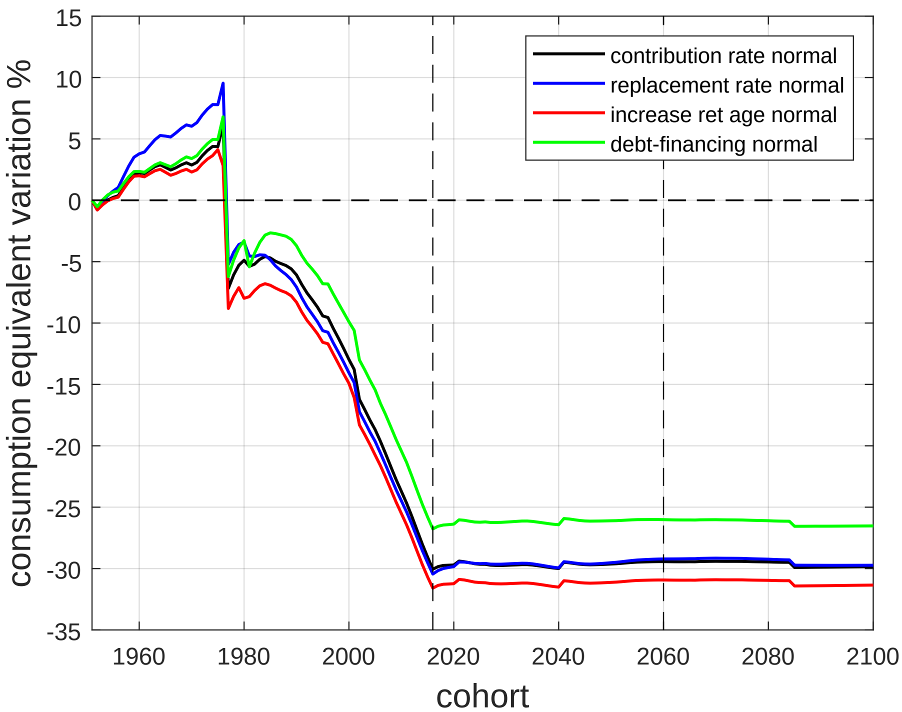
The analysis of the transition dynamics allows us to determine the impact of the pension reforms and "debt-financing" on the welfare of all the generations involved. Our welfare criterion is the lifetime utility of the generation aged 26 at time t under the policy adjustment $z = rr, cr, ra, df$ and the productivity growth scenario $i = n, s$, namely

$$W = \sum_{j=26}^{J} \left\{ \prod_{m=26}^{j} \left[s_{t+m-1}(m)\tilde{\beta}_{t+m-1} \right] u\big((1 + CEV^{z,i})\tilde{c}^{z,i}_{t+j-1}(j), l^{z,i}_{t+j-1}(j)\big) \right\}$$
$$+ \prod_{m=26}^{J} \left[s_{t+m-1}(m)\tilde{\beta}_{t+m-1} \right] \mu v\big(\tilde{x}^{z,i}_{t+J-1}(j)\big),$$

where $s(j)$ is the age-dependent survival probability, $\tilde{\beta}_t = \beta(1 + g_t)^{1-\frac{1}{\rho}}$ is the growth-adjusted discount factor, $l_t(j)$ is the fraction of hours worked at age j and $\tilde{c}_t(j) = \frac{c_t(j)}{A_{l,t}}$ and $\tilde{x}_t(j) = \frac{x_t(j)}{A_{l,t}}$ are the labor productivity-adjusted levels of consumption and intended bequests.[34] For the sake of comparability, we report the results in terms of growth-adjusted consumption equivalent variation (CEV) relative to a benchmark welfare level. A *positive* CEV corresponds to a *welfare-reducing* productivity-reform combination because a positive consumption compensation is necessary to deliver the benchmark level of welfare. Vice versa, a *negative* CEV corresponds to a *welfare-enhancing* productivity-reform combination. As a benchmark for welfare, we take the lifetime utility of the generation dying in 2015, the year before the transition. This is

[34]We assume that the first term of the product, i.e. the effective discount factor of utility at age 26 $\left[s_{t+25}(26)\tilde{\beta}_{t+25} \right] = 1$, for every t.

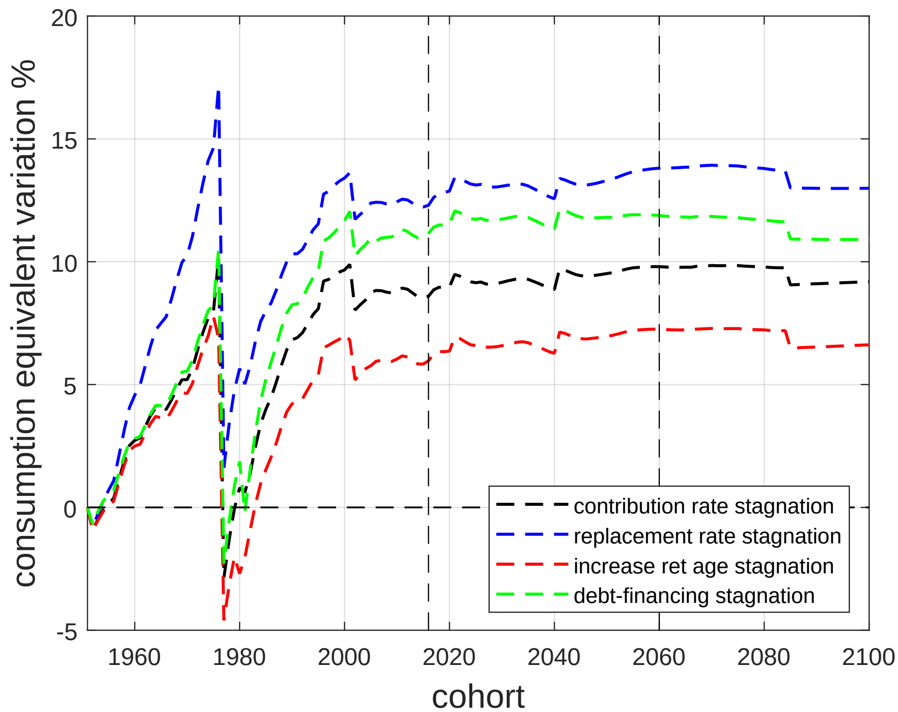
Figure 3: Growth-adjusted CEV under the "normal" scenario



the generation aged 26 in 1952, and, by using this reference level, we can identify the role of productivity growth for the ranking of the alternative pension adjustments.

Figure 3 plots the CEV under the "normal" scenario. As expected, the higher level of productivity growth relative to the initial stationary equilibrium brings about welfare gains for future generations despite the impact of ageing and regardless of the specific pension adjustment. However, the effect is heterogeneous on the generations alive in 2015, at the beginning of the transition, due to the interaction between ageing, productivity and each of the different pension adjustments. All the cohorts born between 1953 and 1977 suffer from a welfare loss, while those born afterwards experience a welfare gain. Indeed, the former correspond to the cohorts that are already retired at the beginning of the transition and therefore do not benefit from the higher wages resulting from higher labor productivity. Within such group of households, the pension reform that delivers the worst welfare implications is the replacement rate adjustment, as it imposes a lower pension given the total amount of pension contributions. The welfare ranking across pension reforms changes for those born after 1977, with "debt-financing" becoming the adjustment that delivers the smallest benefit and a replacement rate reform being relatively more desirable than an increase in the contribution rate.

Figure 4: Growth-adjusted CEV under the "stagnation" scenario



Anyway, a higher retirement age is the reform that implies the largest welfare gain for all generations (the red line is always below the others in Figure 3). The welfare ranking among the different reforms does not necessarily coincide in the two productivity cases. In fact, in the presence of stagnant growth, the ranking is stable across the different generations involved in the transition, with an increase in the retirement age resulting the most desirable adjustment and the reform of the replacement rate the most detrimental one, while a higher contribution rate is better in terms of welfare than "debt-financing" (Figure 4).

To summarize the overall results of this section, we observe that the pension adjustments associated with the largest drops in the natural interest rate correspond to the ones with the largest welfare losses. This result suggests that *the natural interest rate is a sufficient metric for welfare* in our quantitative model, because it crucially affects the gap between the return from investment r^* and the economy growth rate g: the lower r^*, the larger the gap between r^* and g, and the lower will be welfare. In other terms r^* is a key determinant of the generations' welfare, with crucial implications for the most desirable pension reform in response to demographic changes.

6 Conclusions

The US public pension system is smaller than that of the major European countries but it has not gone through radical reforms to contain expenditures and increase revenues, thanks to a slower transition towards an ageing society.

The past evolution of the US pension system has played a crucial role for the natural interest rate (r^*) trend between 1970 and 2015. Our quantitative life-cycle OLG model allows us to *quantify* the combined effect of all past pension adjustments, which raised r^* approximately by 1%; and to *qualify* the past increase in the replacement rate as the main driver of this positive effect.

This suggests that the higher natural interest rate in the US compared to the other advanced countries (see, Holston et al., 2017) may not be only the result of a delayed demographic transition, but also the product of its interaction with costly pension adjustments, like a higher replacement rate, which are affordable under temporarily less adverse demographic trends. However, in the future, whether the pension system will keep pushing r^* up or not will depend on the specific pension reforms implemented in response to evolving demographic and technological dynamics.

What we have found is that an increase in the retirement age, independently of future technological patterns, would exert a substantial upward pressure on r^*, counteracting the downward pressure due to the adverse demographic trends expected between 2015 and 2060. To a lesser extent, the same would happen with a future pension reform raising the contribution rate, but not with a reform lowering the replacement rate.

These results are particularly interesting because relatively higher (lower) values for r^* are associated with welfare gains (losses) in the long run, if the real rate stays below the economy growth rate, as occurred in the past (see, e.g., Blanchard, 2019). As a consequence, the government needs to choose carefully the future pension adjustments in response to demographic changes if $r < g$. Within our framework, in the case that raising the already high age of retirement was not viable, an increase in the contribution rate, which the US government has not updated since 1990,[35] would look like a safe alternative.

In any case, the future pension reform that is mostly discouraged by the results of our analysis is the one of the replacement rate, whose impact on r^* varies starkly with the productivity scenario. Indeed, with an ageing population, the resulting pension

[35] Data available at https://www.ssa.gov/OACT/ProgData/taxRates.html.

deficit calls for a lower, not higher, replacement rate. Hence, this pension channel would work in the opposite direction with respect to the past, exacerbating the decline in r^* and imposing welfare losses under stagnant productivity growth. Furthermore, the welfare losses would be concentrated within the initial old generations, which could oppose forcefully to the replacement rate reform.

Finally, our analysis highlights the limited effectiveness of a future debt-financed pension deficit in mitigating the decline in r^*. When $r^* < g$, as pointed out in the existing literature, the fiscal costs of a public debt expansion, in terms of debt-to-GDP ratio, may be small or even zero. The quantitative exercise where the increasing pension deficit due to an older population is financed by issuing public debt confirms this results, as it features a lower debt-to-GDP ratio in the long run. However, such policy leads to worse welfare implications and a lower natural rate than other pension adjustments, because the increase in public debt is not sufficient to absorb the increased aggregate savings.

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Appendices

A Theoretical model

A.1 Credit market equilibrium: a graphical illustration

For the sake of convenience, we report again here the total demand for credit D^c,

$$D^c = \left(\frac{1+n}{1+r^*}\right)\lambda^m D,$$

the total credit supply,

$$S^c = \lambda^m a^m = \lambda^m \left\{ \frac{\beta\lambda^o}{\lambda^m + \beta\lambda^o} \left[(Y - D - T) - \frac{\nu Y}{\beta(1+r)} \right] \right\},$$

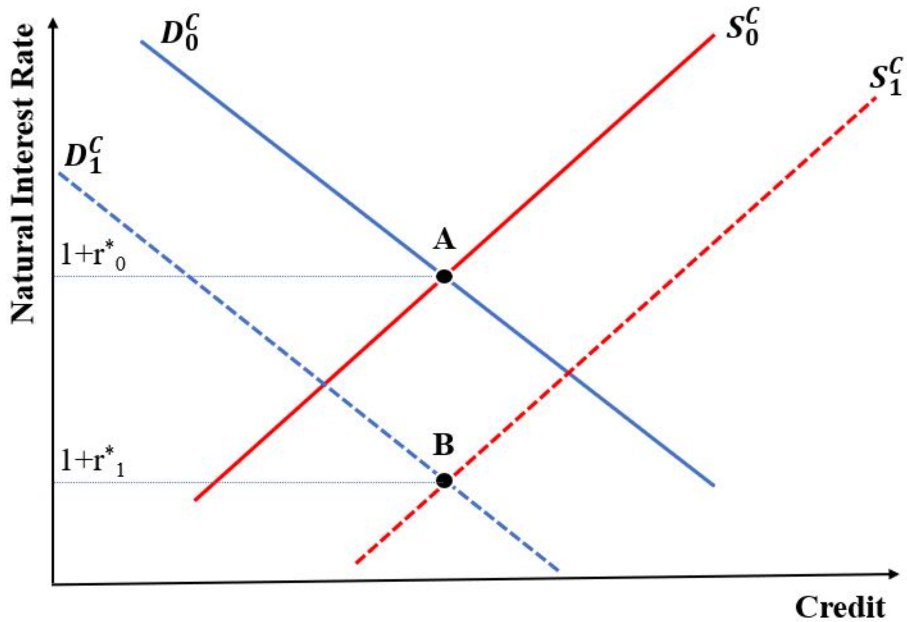
and the equilibrium/natural interest rate,

$$(1+r^*) = \frac{(\lambda^m + \beta\lambda^o)(1+n)D + \lambda^o \nu Y}{\beta\lambda^o (Y - D - T)} = \frac{(\lambda^m + \beta\lambda^o)(1+n)D}{\beta\lambda^o (Y - D - T)} + \frac{\nu Y}{\beta(Y - D - T)}.$$

As depicted in Figure 5, the credit demand D^c relates negatively to $1+r^*$ and the corresponding curve is accordingly downward-sloping. By contrast, the credit supply S^c relates positively to the natural interest rate and the associated curve is upward-sloping.[36] Furthermore, a reduction in the credit demand that shifts the corresponding curve downward, from D_0^C to D_1^C in Figure 5, results in a lower equilibrium interest rate. The same happens for an increase in the supply of saving, which shifts the credit supply curve downward, for example, from S_0^C to S_1^C in the figure. On the contrary, an increase in the credit demand and a decrease in saving would shift the corresponding curve upward, increasing the natural interest rate. In mathematical terms, this means that $\frac{\partial r^*}{\partial D^c} > 0$ and $\frac{\partial r^*}{\partial S^c} < 0$.

[36] The relationship between saving and the natural interest rate relies on the effect of $1+r^*$ on the discounted value of future pension benefits. This relationship would not be necessarily positive with different preferences, but we view the case considered as the most relevant empirically, as Eggertsson et al. (2019).

Figure 5: Credit Market Equilibrium



B Quantitative model

B.1 Households

Demographics

Households die certainly at age $J = 81$, but each period they face a positive probability to die $1 - s_t(j)$, where $s_t(j)$ is the survival probability between age j and $j + 1$. The age-specific probabilities $[s_t(j)]_{j=26}^{J-1}$, along with the fertility rate f_t, determine the DR between workers and retirees. Indeed, given $N_t(j)$ households aged j at time t, there will be $N_{t+1}(j + 1) = s_t(j)N_t(j)$ households aged $j + 1$ at time $t + 1$, while the total population at time t is $N_t = \sum_{j=26}^{J} N_t(j)$. The population entering the economy at time t is given by the population of their parents times the fertility rate of the parents, $N_t(26) = N_{t-25}(26) * f_{t-25}(26)$. In what follows, we study a stationary equilibrium in which the total fertility rate and the survival probabilities do not depend on time. In this equilibrium, f determines the population growth rate n through the equation $n = f^{\frac{1}{25}} - 1$, and $(1+n)$ is the ratio between the size of the newborn generation and that of the previous period. Moreover, the relationship $N(j + 1) = s(j)N(j)/(1 + n)$ holds for $j \in [26, J - 1]$ and a given $N(26)$. We normalize $N(26)$ such that total population equals 1 and so the mass of each cohort is also the corresponding share of the overall population, $N(j)/N$.

Utility

Utility from consumption and bequests has the same CRRA functional form and the same intertemporal elasticity of substitution ρ:

$$u\left(c_t(i,j)\right) = \frac{c_t(i,j)^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} \tag{26}$$

$$v\left(x_t(i,J)\right) = \frac{x_t(i,J)^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}}. \tag{27}$$

Bequests

A fraction $0 \leq \psi_t \leq 1$ of households at each age is covered by the public pension system. The remaining fraction $(1-\psi_t)$ is not and builds value for old-age consumption via private savings uniquely. Therefore, these two types of agents make different consumption/saving decisions over their lifetime and leave different amount of bequests to their descendants. However, we assume that participants and non-participants of each cohort pool together their bequests so that their offspring receives the same inheritance:

$$q_{t+1}(j=57) = \frac{N_t(J)}{N_{t+1}(57)} f_{t-J+26}\left[\psi_t x_t(i \in \Psi_t, J) + (1-\psi_t)\, x_t(i \notin \Psi_t, J)\right], \tag{28}$$

where Ψ_t is the set of participants to the public pension system at time t.

B.2 Firms

In the economy, there are three types of firms producing respectively final goods, intermediate goods, and capital goods.

Final goods firms

Final goods firms operate in a regime of monopolistic competition. They purchase intermediate goods y_t^m at the price p_t^{int}, and transform them in differentiated final goods via a linear production function, $y_t^f(i) = y_t^m$. The different varieties of final goods are combined into aggregate output through a CES aggregator:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}},$$

where $\theta_t > 1$ is the elasticity of substitution across different varieties. The final good producer sets the price $p_t(i)$ in each period by solving the problem:

$$\max_{p_t(i)} \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^m$$

s.t.

$$y_t^f(i) = y_t^m = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}, \tag{29}$$

where $P_t = \left[\int_0^1 p_t(i)^{1-\theta_t} di \right]^{\frac{1}{1-\theta_t}}$ is the aggregate price level and the constraint (29) represents the demand curve of the differentiated final good. The optimal price,

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t}, \tag{30}$$

is given by a mark-up $\frac{\theta_t}{\theta_t - 1}$ over the marginal cost. As the intermediate good is homogeneous, all final goods producers set the same price, $p_t(i) = P_t$. Hence,

$$\frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t} \tag{31}$$

and aggregate profits are $\Pi_t = \frac{Y_t}{\theta_t}$. These profits are distributed among households in proportion to their labor income:

$$\frac{Y_t}{\theta_t} = \sum_{j=26}^{J} N_{j,t} \Pi_{j,t}. \tag{32}$$

Finally, using $p_t(i) = P_t$ into (29) yields

$$y_t^f(i) = y_t^m = Y_t. \tag{33}$$

Intermediate goods firms

Intermediate goods firms operate in perfect competition, combining labor and capital through the following CES production technology:

$$Y_t = y_t^m = \left[\alpha (A_k K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{l,t} L_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where the output of each intermediate (and final) goods firm coincides with aggregate output, given (33). These firms also contribute to the pension scheme paying a tax τ_t^f on each unit of labor involved in the scheme, and their maximization problem is

$$\max_{L_t, K_t} \quad \frac{p_t^{int}}{P_t} Y_t - (1 + \psi_t \tau_t^f) w_t L_t - r_t^K K_t,$$

with the following optimality conditions

$$w_t = \frac{1}{1 + \psi_t \tau_t^f} \frac{p_t^{int}}{P_t} (1-\alpha) A_{l,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}$$

$$r_t^K = \frac{p_t^{int}}{P_t} \alpha A_k^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}},$$

which become (21) and (22), respectively, after using (31).

Capital goods firms

In a perfectly competitive investment-specific production sector, the composite final good is converted into capital goods, using a linear production function. The maximization problem of capital goods firms is

$$\max_{Y_t^K} \quad \xi_t K_t - Y_t^K$$

s.t.

$$K_t = z_t Y_t^K,$$

where Y_t^K is the final good employed for capital production, z_t is the productivity of the investment-specific production sector, and ξ_t is the price of capital goods.

B.3 Pension benefit formula

To replicate the main features of the OASDI program, in particular the pension benefits calculation, we follow the US Social Security regulation, which establishes that:

- pension benefits are computed according to the Primary Insurance Amount (PIA), which considers only the average labor earnings of the top 35 years of contribution to the scheme, defined as Average Indexed Monthly Earnings (AIME);
- monthly earnings are indexed relative to the average wages of the indexing year, which is the year in which the agent turns 60, to factor in wage growth;
- once the AIME is determined, the PIA is computed using some *bend points*, which are adjusted every year;
- a penalty to benefits is applied whenever agents retire before the full retirement age.

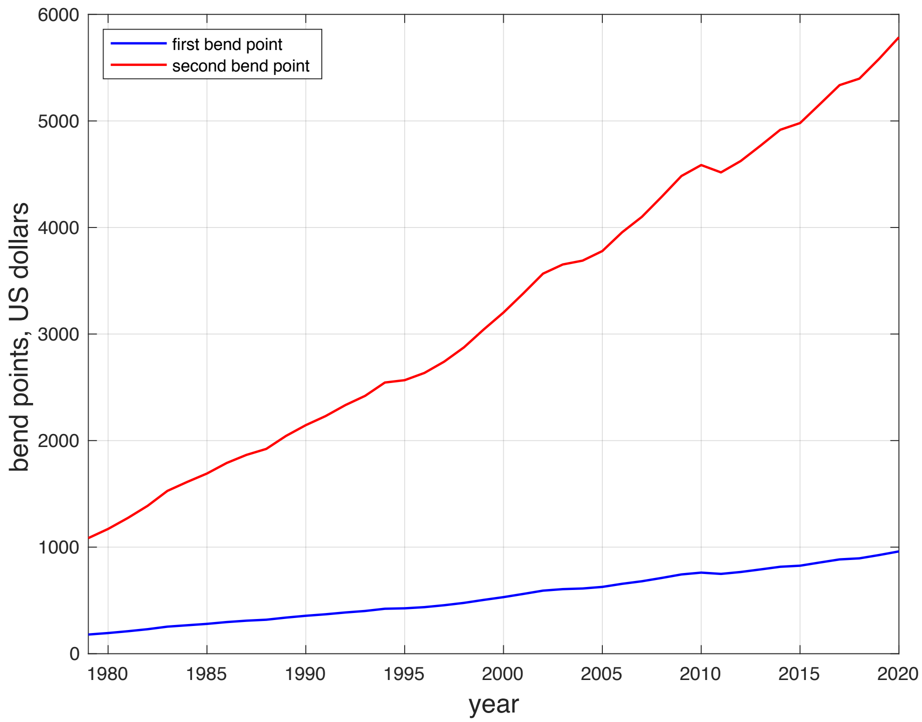
The bend points are some dollar amounts that, combined with some fixed percentages (in practice: 90%, 32%, and 15%), establish the implicit replacement rate, i.e., the fraction of average earnings are replaced by the pension benefits. The bend points for 2020 are $960 and $5785. Then, the PIA is the sum of the 90% from the first $960 of the AIME, the 32% from earnings between $960 and $5,785, and the 15% of monthly earnings over $5,785. Such calculation implies that poorer agents enjoy a larger fraction of their AIME as a pension transfer and that the program produces a redistribution from high to low earners. Figure 6 shows the evolution of the bend points over time.[37]

Since our quantitative model does not account for earnings heterogeneity, it cannot capture such redistribution. However, we can replicate the implicit replacement rates for medium earners of different cohorts, which are estimated by the Social Security Administration, during the periods under analysis.

In our notation, the contribution rate is τ_t^p, and it is the sum of the employee and employer contribution rates, τ_t^w and τ_t^f, respectively. The effective retirement age is RA_t and the replacement rate is ν_t. The latter is the fraction of the average gross labor income (subject to the indexation mentioned above) earned during working age that each entitled retiree receives as a benefit. The individual pension benefit of a retiree i

[37]Source: https://www.ssa.gov/oact/cola/bendpoints.html.

Figure 6: OASDI bend points over time



aged j at time t is

$$p_t^b(i,j) = \nu_t \phi(RA_t(i), FRA_t) \frac{1}{35} \sum_{z=RA(i)-35+1}^{RA_t(i)} \left(\frac{w_{t-j+60}}{w_{t-j+z}}\right) \big(w_{t-j+z}(i) hc(z)\big).$$

The AIME is calculated averaging the gross labor earnings of the last 35 years of work before retirement age $RA_t(i)$ because, given the calibration, they correspond to the top 35 years of earnings during the working life. Furthermore, individual wages are indexed with respect to the economy-average wage in the year in which agent i, aged j at time t, turned 60, i.e., w_{t-j+60}. Finally, the function $0 < \phi(RA_t(i), FRA_t) \leq 1$ gives the penalty, in terms of replaced contributions, applied whenever the individually chosen retirement age $RA_t(i)$ is lower than the full retirement age FRA_t. The penalty associated to early retirement is increasing in the months of work foregone with respect to the full retirement age. In particular, a pension *"benefit is reduced $\frac{5}{9}$ of one percent for each month before full retirement age, up to 36 months. If the number of months exceeds 36, then the benefit is further reduced $\frac{5}{12}$ of one percent per month"*[38].

[38] US Social Security Online, www.socialsecurity.gov

As economy-average wage equals the individual wage in the same year, due to the assumption that each cohort is homogeneous in terms of gross labor income and choice of age of retirement, we have that $\forall i$:

$$w_{t-j+z}(i) = w_{t-j+z}$$

$$RA_t(i) = RA_t.$$

This implies that we can drop the identifier i from the pension benefit equation, which collapses to:

$$p_t^b(j) = \nu_t \phi(RA_t, FRA_t) \frac{w_{t-j+60}}{35} \sum_{z=RA-35+1}^{RA} hc(z).$$

B.4 Competitive stationary equilibrium

The model features CRRA preferences and exogenous growth, given by population growth and labor-augmenting technological progress. Therefore, we adjust all the variables to solve for the stationary equilibrium of the model. This means that we need to divide all the aggregate variables by $(1+g^p)^t(1+n)^t$, while we need to divide cohort variables, pension benefits and wages by $(1+g^p)^t$. Then, a stationary competitive equilibrium of this economy can be defined as the marginal return to capital r^K, the wage rate w, aggregate output Y, aggregate capital K, aggregate labor L, bequests given $x(i)$ and received q, the tax rates τ^w, τ^f and τ^b, the pension benefit p^b and the age profiles of consumption $[c(i,j)]_{j=26}^{J}$ and assets $[a(i,j)]_{j=26}^{J}$ $\forall i$ such that:

- lifetime utility is maximized subject to all period budget and borrowing constraints, given initial asset holdings $a(i, 26)$ $\forall i$;
- total bequests given equal total bequests received;
- demographic phenomena follow the dynamics described in Appendix B.1;
- capital evolves over time according to its law of motion;
- final, intermediate and capital goods producers maximize their profits subject to their technological constraints and the relevant market structure, as described in Appendix B.2;
- the government satisfies its budget constraint;

- the pension benefit is computed according to the formula defined in Appendix B.3;
- asset markets clear, so that $\sum_i \sum_{j=26}^{J} N_j \xi a(i,j) = \xi K + b$.

B.5 Solution method

The solution of the stationary equilibrium and transition dynamics of the model requires standard numerical methods, given the large systems of non-linear equations. First of all, we need to re-scale the model for growth, so that it is stationary, as described in the previous subsection. Then, a standard algorithm is implemented. The iterative procedure we follow can be summarized in a series of steps:

1. guess a set of endogenous variables, including aggregate capital K, bequests received q and the adjusting fiscal policy parameter, in the simulation;
2. given these endogenous variables, calculate prices w and r^K, pension benefits p^b, profits Π and aggregate output Y;
3. solve households' lifetime utility maximization problem to retrieve the optimal age profile of consumption and assets, as well as bequests given;
4. compute the ex-post value of bequests received q, given the equality between total bequests given and received;
5. combine individual choices to determine the ex-post level of aggregate capital, taking into account the economy age structure;
6. calculate the ex-post fiscal policy parameter that makes the government budget balanced;
7. verify convergence of ex-post variables to the iteration initial guesses;
8. if convergence is not achieved yet, update the guess for the next iteration and repeat all the steps until convergence.

Solving for the entire perfect foresight transition path requires choosing a sufficiently large number of transition periods to ensure the convergence to a new stationary equilibrium (or balanced growth path). We solve the transition forward, which means that we solve the lifetime utility maximization problem for each generation involved in the transition taking into account the evolution of prices along her lifetime, starting from the generation that reaches age J in the first period of the transition. Her optimal bequests given x_t are used to determine the guess for the bequests received by their descendants in the next period, q_{t+1}.

One of the most delicate steps of the procedure is the one where we compute the optimal path of consumption and savings over the life-cycle, because of the presence of borrowing constraints. For this task, we employ the endogenous grid method proposed by Carroll (2006).

C The natural interest rate in the past

C.1 Calibration

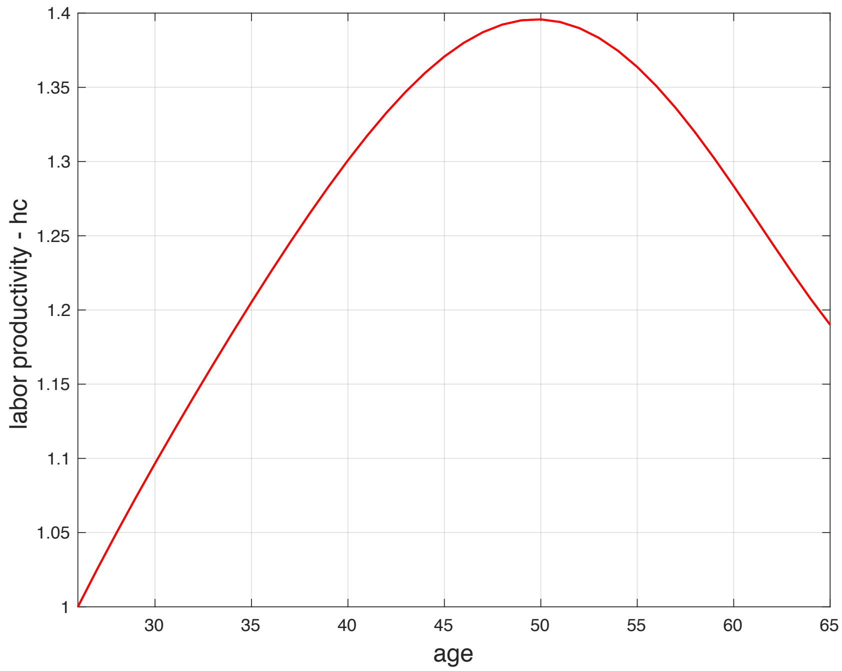
Most of parameters in Table 1 are calibrated in the same way as in Eggertsson et al. (2019). In particular, our calibration for the age profile of labor productivity $[hc(j)]_{j=26}^{RA_t}$ takes the same values as in that paper, where the vector $[hc(j)]_{j=26}^{RA_t}$ is obtained matching the earnings profile estimated from the data by Gourinchas and Parker (2002), and characterized by the typical inverted U-shape (Figure 8). For further details about the other common parameters and the calibration procedure, we refer the reader to Eggertsson et al. (2019), while we describe here the calibration of the pension parameters, which distinguish our model from the benchmark one.

The US Social Security regulation established that the FRA was around 66 years in 2015, but early retirement was possible at 62 years. To take into account the effect of early retirement, we calibrate the effective RA at 65, which is slightly lower than 66 but consistent with Eggertsson et al. (2019). Regarding 1970, we opt for a more conservative calibration of the RA, 63 years. Indeed, though the average RA of men was roughly 65 years (US Census Bureau), there was a large fraction of women excluded from the official labor force in 1970, and average statistics can be misleading because of a left-skewed distribution.[39] Given our baseline calibration of the RA in 1970 and 2015, the pension benefit formula, equation (25), implies a reduction of 13.3% of the replacement rate for early retirement in 1970 and 6.66% in 2015.

The contribution rate for the OASDI in 1970 was 4.2% of the gross labor income

[39]Moreover, the so-called survivorship bias and the decision to retire early of sick people significantly affect the data. In any case, we perform a robustness check in Appendix C.2, where we assume a higher RA in 1970, 64 years, and so a lower variation in the RA over the period 1970-2015.

Figure 7: Age profile of labor productivity



for both employer and employee, while it was 6.2% in 2015. Hence, $\tau^p = \tau^w + \tau^f$ was 8.4% in 1970 and 12.4% in 2015. We calibrate the replacement rate using the US Social Security Administration data on the medium earner's pension benefit, whose net replacement rate was around 32.3% in 1970 and approximately 40.8% in 2015.[40] Finally, the OASDI program underwent a substantial expansion in terms of coverage, as documented in the historical accounts of the US Social Security Administration. While in 1970 Social Security involved around 90% of civilian workers, this proportion increased to 96% in 2015.

C.2 Alternative calibration of the retirement age

We perform a robustness check with an alternative assumption about the increase of the RA over 1970-2015. The same decomposition exercise of Section 4 is repeated under the assumption that the increase in the effective RA is one year, from 64 to 65, between 1970 and 2015. The reduction in benefits due to early retirement becomes accordingly 6.66% in 1970, while all the other parameters, including those related to the pension system, are at their baseline calibration (see Table 1 and Appendix C.1). For the sake

[40]Data available at https://www.ssa.gov/oact/NOTES/ran9/index.html.

Table 4: Decomposition of the decline in the natural interest rate

	Robustness	Baseline
Total natural interest rate variation	-4.09%	-4.09%
Retirement age	+0.12%	+0.44%
All pension reforms	+0.77%	+1.10%
Demographics + All pension reforms	-0.48%	-0.21%
Demographics + Retirement age	-0.85%	-0.59%
Demographics + Productivity growth + All pension reforms	-2.20%	-1.90%

of exposition, in Table 4, we report only the results regarding the pension parameters ("Robustness"), which are compared with the ones of the baseline exercise ("Baseline"), but we do not report those regarding all the other drivers of r^*, which are unchanged compared to the baseline.

A more modest increase in the effective RA generates a smaller mitigation effect of this pension reform on the fall in the natural interest rate, +0.12% against +0.44% (first row of Table 4). Notwithstanding, the overall result regarding the positive and significant impact of the past pension reforms on r^* is substantially unaffected by our alternative calibration of the RA in 1970, as shown by the last four rows in Table 4.

D The natural interest rate in the future

D.1 Households

Utility

In our prospective exercise, we slightly modify the model of Section 3 to account for endogenous labor supply. Therefore, utility does not only depend on consumption and bequests, but also on leisure $(1 - l_t(j))$, where $l_t(j)$ is the fraction of hours worked at age j by individual i, and it is nil after retirement, i.e. $l_t(j) = 0$ for $j \geq 65$. Moreover, we assume that all agents participate to the pension system, i.e. $\psi_t = 1$ for all t, so we can drop the index i in our notation. In order to ensure that preferences remain balanced growth path consistent, we assume the following utility for consumption and leisure:

$$u\left(c_t(j), l_t(j)\right) = \frac{\left[c_t(j)\left(1 - l_t(j)\right)^{\eta}\right]^{1-\frac{1}{\rho}}}{1 - \frac{1}{\rho}}, \tag{34}$$

which replaces (26) and where η is the parameter governing the Frisch elasticity. The inclusion of endogenous labor supply alters also the temporal budget constraint of the working cohorts, those with $26 \leq j \leq RA_t$, such that:

$$c_t(i) + \xi_t a_{t+1}(j+1) = (1 - \tau_t^b - \tau_t^w) w_t h(j) l_t(j) + \Pi_t(j) + [r_t^k + \xi_t(1-\delta)]\left[\frac{a_t(j)}{s(j)} + q_t(j)\right],$$

where $h(j)$ is the age-specific labor productivity. The resulting pension benefit for the household retiring at time t is:

$$p_t^b(j) = \nu_t \phi(RA_t, FRA_t) \frac{w_{t-j+60}}{35} \sum_{z=RA-35+1}^{RA} h(z) l_{t-z+RA-1}(z).$$

D.2 Calibration

According to the UN *medium variant* projections, life expectancy at birth will rise from 78.7 years in 2015 to 85.3 years in 2060. We replicate the positive longevity shock as a gradual increase in the age-dependent survival probabilities, $[s_t(j)]_{j=26}^{J-1}$ with $2015 \leq t \leq 2060$. The UN also predicts a higher US total fertility rate, from 1.875 to 2.017 children per woman, in the same period. In our model, the combination of changes in longevity and fertility causes a variation in the $DR = \frac{Pop(RA+)}{Pop(26-RA)}$ from around 42.4%

in 2015 to 46.9% in 2060 (in absence of a retirement age reform). Moreover, given the automatic adjustment of the FRA to life expectancy in the OASDI program, we assume that the simulated population ageing determines a 1-year increase in FRA, from 66 to 67. As a consequence, in the simulations with constant effective RA, i.e. $RA = 65$, the penalty associated with early retirement increases, such that the variable $\phi(RA, FRA)$ in (25) declines from 93.3% to 86.6%, while the same variable remains constant after a 1-year increase in RA.[41]

Compared to Section 4, we slightly modify the calibration to fully capture the quantitative impact of ageing on pension expenditure. Agents can now reach a higher maximum age that is $J = 90$.[42] Moreover, we assume that the pension system covers all the population, $\psi = 1$, starting from 2015 and during the transition. This assumption, which simplifies the analysis, is realistic because the coverage of the US pension system was very close to 100% in 2015. We calibrate all the other parameters in the same way outlined of Section 4 (Table 1), except for the parameter d, as we assume, for tractability, that agents face no borrowing constraints, without any major implications for our results. As the model features endogenous labor supply, we need to calibrate some additional parameters. In particular, we calibrate the parameter η governing the Frisch elasticity such that the average amount of hours worked is equal to 35% of the time endowment, and we calibrate the vector of age-specific labor productivity $[h(j)]_{j=26}^{RA_t}$ such that the gross labor income for each cohort is the same in the 2015 stationary equilibrium as in Eggertsson et al. (2019). The fit of our calibration is shown in Figure 8. Finally, Table 5 reports the values of the parameters that are set to match the usual data moments in our calibration procedure.

[41]We assume that the early retirement penalty raises only for the newly paid pensions, so that the benefit of the cohorts who were already retired before 2015 is unaffected in this regard.

[42]As agents leave bequests at age J and have children at age 26, raising the maximum age from 81 to 90 implies that households receive their inheritance at age 66 and not at age 57.

Table 5: Calibration

Parameters calibrated to match data moments	Symbol	2015 value
Rate of time preference	β	1.01
Bequests parameter	μ	36.97
Frisch elasticity parameter	η	2.19
Retailer elasticity of substitution	θ	4.08
Capital share parameter	α	0.25

Figure 8: Age profile of labor productivity

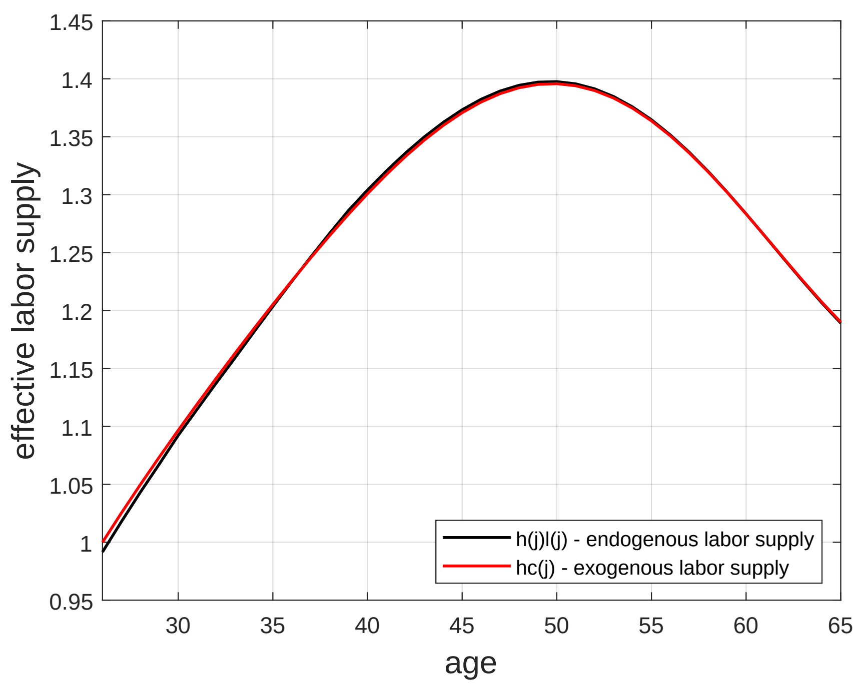


Table 6: Main variables and pension parameters

	2015	New stationary equilibrium					
Reform	/	contribution rate		replacement rate		retirement age	
Scenario	/	stagnation	normal	stagnation	normal	stagnation	normal
r^*	-1.47%	-2.47%	-1%	-2.92%	-1.05%	-1.95%	-0.72%
τ^p	12.4%	15.7%	12.7%	12.4%	12.4%	12.4%	12.4%
ν	40.8%	40.8%	40.8%	31.9%	39.7%	40.8%	40.8%
RA	65	65	65	65	65	66	66
$\frac{B}{Y}$	118%	27%	26.5%	12.6%	24.8%	94.4%	39%
τ^b	35.3%	35.3%	35.3%	35.3%	35.3%	35.3%	35.3%
ϕ	93.3%	86.6%	86.6%	86.6%	86.6%	93.3%	93.3%
DR	42.4%	46.9%	46.9%	46.9%	46.9%	43.5%	43.5%

D.3 Endogenous debt-to-GDP ratio

We perform an exercise in which, during the transition, the government does not adjust the tax rate τ^p, to neutralize the impact of ageing, productivity, and pension reforms on the public debt-to-GDP ratio, $\frac{B}{Y}$. We keep τ^b at its initial level in 2015, 35.3%, and let $\frac{B}{Y}$ to be determined endogenously. It follows that, in the long run, not only each scenario-reform combination leads to different levels of r^* but also to different values of $\frac{B}{Y}$. Moreover, the natural interest rate falls a bit deeper into negative territory in both productivity scenarios because the debt dynamics imply a significant reduction in $\frac{B}{Y}$, as reported in Table 6. Unsurprisingly, a 1-year increase in the effective retirement age is the pension reform that mitigates the most the impact of ageing on r^*, while a change in the contribution rate and a change in the replacement rate bring about similar values for r^* in the "normal" scenario, but very different ones in the "stagnation" scenario.