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Forbearance vs foreclosure in a general equilibrium model

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Abstract

We build a business cycle model characterized by endogenous firms dynamics, where banks may prefer debt renegotiation, i.e. non-performing exposures, to outright borrowers default. Debt renegotiations *per se* do not have adverse effects in the event of financial crisis episodes, but a large share of non-performing firms is associated with a sharp deterioration of economic activity if there are congestion effects in banks ability to monitor non-performing loans and the opacity of such loans adversely affects banks' moral hazard problem. Aggressive interest rate reductions and quantitative easing limit defaults and the output contraction caused by a financial crisis, without adverse effects on the entry of new firms. The decline in the natural interest rate, due to slower productivity growth and persistent liquidity shocks, might explain the observed long-run trend in the share of non-performing loans.

Keywords: Non-Performing Loans, DSGE Model, Financial Frictions, Quantitative Easing, Firms Entry.

JEL Codes: E32, E44, E50, E58.

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1 Introduction

We build a DSGE model characterized by endogenous firms dynamics, where the firms' exit rate is mitigated because banks prefer to avoid the default of some borrowers and choose to hold "non-performing" loans in their portfolios. This innovation has implications for the model-predicted effects of shocks and monetary policy actions.

The productivity slowdown that occurred over the past decade attracted growing interest on the consequences of inefficient banks' lending. The phenomenon was first spotted in Japan in the 1990s and the early 2000s (Caballero et al., 2008; Peek and Rosengren, 2005). Adalet McGowan et al. (2018) show that, abstracting from cyclical effects, the rising capital share allocated to non-performing firms creates a congestion effect that limits productivity-enhancing capital reallocation and creates barriers to entry of new firms. According to Banerjee and Hofmann (2018), the phenomenon, exacerbated in economic downturns, is at least partly explained by the downward trend in the interest rates, that gradually reduced the creditors' opportunity cost of "evergreening" loans to insolvent firms.

Within the EMU, the financial crisis caused an unprecedented increase in the share of non-performing and forborne loans[1], raising regulators' concerns for the stability of the financial system (Bofondi and Ropele, 2011) and the potential barriers to productivity growth and innovation created by the potentially insolvent borrowers. Storz et al. (2017) and Andrews et al. (2017) argue that inefficient credit allocation and excessive leveraging of weak firms hampered recovery and productivity growth within the European Monetary Union. Acharya et al. (2019) point at the risk that strongly accommodative monetary policies in the Eurozone might exacerbate the productivity slowdown by distorting loans allocation towards less efficient firms.

So far, empirical research had the lion's share in the field. Our contribution instead is theoretical, and our dynamic stochastic general equilibrium model incorporates the financial frictions that may induce banks to live with non-performing loans. While there is little doubt that in the long run a high share of non-performing loans increases dispersion of firms efficiency and adversely affects the efficiency of the economy, it is less obvious that bank forbearance cannot play a valuable role when bankruptcies would otherwise occur in consequence of a combination of adverse demand shocks and nominal rigidities. This potential trade-off is captured in the empirical work of Gropp et al. (2022) who find that regulatory forbearance on banks is associated both with lower output losses during the crisis and with slower post-crisis output and productivity growth.

To address the potentially controversial effects of banks forbearance we need a specific set of modelling assumptions. First, we need to model firms behaviour, accounting for both endogenous entry and endogenous exit. Second, we must characterize banks' incentives to

[1] From the 2018 *Final Report - Guidelines on management of non-performing and forborne exposures* of the European Banking Authority.

re-negotiate debt contracts. Following Hopenhayn (1992) and Asturias et al. (2017), in our model the technology is characterized by idiosyncratic firm efficiency, decreasing returns to scale, and fixed production costs. In each period, a cohort of new entrants (*NE*s) drives stochastic productivity growth. Before shock realizations are observed, incumbent firms inherit installed productive capacity and the bank loan needed to finance it. The combination of fixed production cost and predetermined loan identifies their profitability threshold but is not sufficient to identify bankruptcy decisions, which depend on banks' incentives to choose between forbearance and foreclosure. Following the standard costly state verification approach (Townsend, 1979), we assume that loans repossession is possible conditional to a liquidation cost. As a result, banks find it profitable to renegotiate debt service payments for a fraction of firms that cannot meet the profitability threshold.

The rest of the model is essentially based on Gertler and Karadi (2011, GK henceforth), where a moral hazard problem imposes an endogenous balance sheet constraint on commercial banks, and nominal rigidities create room for monetary policy actions.

Our model generates a large amplification mechanism. The introduction of endogenous firm dynamics is sufficient to obtain a relatively large fall in investment, amplifying the output loss relative to the standard GK model. The investment drop essentially happens because bankruptcies increase, the flow of new entrants shrinks, decreasing returns to scale strengthen the intermediate goods producers' incentive to scale down their size in response to the shock. Inefficient allocation of factor inputs, due to predetermined capital, plays an important role in deepening the output contraction.

Our results so far suggest that the increase in the share of non-performing loans looks more like a symptom rather than a cause of the severity of the crisis. We cannot detect an adverse cyclical response due to loan-contract renegotiations unless two additional issues are borne to the forefront. The first one is the existence of some non-negligible congestion effects that raise liquidation costs during the crisis (Fell et al., 2018). The second one is the potential endogeneity of the commercial banks' moral hazard problem. There is a consensus that non-performing loans are opaque and difficult to value. Hence they might hinder a bank's ability in accessing liquidity. Ceteris paribus, a high share of non-performing loans could adversely affect commercial banks' supply of loans (Balgova et al., 2016; Huljak et al., 2022). Our simulations suggest that this latter channel has potentially devastating effects in determining the consequences of a crisis, leading to an increase in the loan-deposit interest rate spread which is very close to what was observed in the Eurozone at the height of the 2011 crisis. We also find that debt renegotiations have adverse effects when permanent technology shocks, modelled as a permanent increase in the productivity distribution of new entrants, hit the economy. Relative to a model where debt renegotiations are artificially forbidden, we observe a much slower pace of convergence to the new, more efficient steady state. This happens because debt renegotiations generate a steady state where non-performing firms are relatively less productive and more exposed to the fall in prices caused by the shock. As a result, we observe a stronger initial increase in bankruptcies as well as a larger entry flow. This is a new, hitherto unexplored implication of the existence of non-performing firms.

Our analysis of monetary policies reaches two key results. The first one is that discretionary monetary expansion stimulates firms turnover and is beneficial for entry flows. Moreover, by raising labor costs it also limits the survival of non-performing firms. The second result is that a strong monetary response to the financial crisis, including quantitative easing actions is beneficial to counteract the adverse effects on firm exit and entry flows. This result carries over to situations where debt renegotiations cause congestion effects of banks' ability to manage non-performing loans and even when non-performing loans are assumed to harm banks access to households' savings. In this regard, the effectiveness of monetary policy is directly linked to its ability to dampen the interest rate spread in the aftermath of the financial crisis.

One important part of the paper is devoted to a discussion of the fundamental drivers of the share of non-performing loans in the steady state of our model economy. Banerjee and Hofmann (2018) document that the number of non-performing firms followed a secular trend and cannot be ascribed to a single financial crisis episode. To rationalize this finding, they emphasize the role of factors that reduced financial pressure, including lower real interest rates. We find that financial pressures, i.e. a smaller return spread between loans and deposit rates, *per se* do not matter in so far as they act symmetrically across all incumbent firms and do not affect the relative demand for loans from non-performing incumbents. We do find, however, that a reduction in the long run growth rate, which permanently lowers interest rates, also has a powerful positive effect on the share of non-performing firms, but this happens because the slower productivity growth weakens firms turnover, limiting exit rates in steady state. We also find that a reduction in fixed production costs, i.e. market deregulation raises the share of non-performing firms. This simply happens because, ceteris paribus, banks entirely appropriate the fixed cost reduction when they choose to avoid the bankruptcy of the insolvent firm.

Studies such as Acharya et al. (2019) document that the fraction of zombie lending in the EMU remained important , in spite of QE policies, Our contribution clarifies that the correlation between low interest rates and a high share of zombie loans arises in consequence of adverse non-monetary shock, but it is in fact weakened by the strength of monetary responses that lower policy rates and the loan-deposit return spread. To further clarify the issue, we discuss the response of non-performing loans to the liquidity shocks introduced in Del Negro et al. (2017). Such shocks are found to bring down loan rates and to raise the share of zombie lending, but the fall in the policy rate and the Quantitative Easing actions have a significant stabilizing effect.

We contribute to the growing field of studies on endogenous entry. The seminal work of Bilbiie et al. (2012) studies the role of endogenous entry in propagating business cycle fluctuations focusing on extensive margins, other studies include different levels of competition

(Jaimovich and Floetotto, 2008; Etro and Colciago, 2010). Siemer (2014) and Bergin et al. (2018) study the interaction between financial shocks and endogenous entry. The distinctive feature of our contribution is that exit decisions are also endogenous and explained by financial frictions. Rossi (2019) also studies the effect of financial frictions on endogenous exit decisions, but her modelling strategy does not allow for non-performing banks exposures. Our characterization of endogenous firm dynamics is akin to Piersanti and Tirelli (2020), but in their model entry/exit flows are restricted to the capital goods producing sector of the economy and financial frictions do not impact on profitability thresholds.

Theoretical work on the role of non-performing loans in business cycle models is still in its infancy. Much research has linked zombie lending to weakly capitalised banks. According to Bruche and Llobet (2014), the limited liability constraint incentivizes inefficient banks banks to engage in evergreening and "gamble for resurrection". Acharya et al. (2021) focus on the adverse effects of policies that, by keeping inefficent banks alive, cause the misallocation of credit towards low-quality firms. In Faria-e Castro et al. (2021) evergreening occurs in consequence of relationship lending. Our approach is different because the pre-determined capital friction generates a systemic effect that acts symmetrically across banks. Our approach is somewhat closer to Begenau et al. (2020), who emphasize banks' delayed loss recognition, but we cast the predetermined capital friction in a general equilibrium model which accounts for endogenous firm dynamics and for the endgenous response of the economy to standard business cycle shocks.

The rest of the paper is organized as follows: Section 2 describes the model economy, section 3 presents the results and section 4 concludes.

2 The Model Economy[2]

The backbone of the model essentially follows GK. In this economy, households consume, supply labor services and hold their wealth in the form of bank deposits. Bank loans to intermediate good producers are used to purchase capital goods. Commercial banks are subject to a moral hazard problem, and this introduces a wedge between the real return on bank deposits and the expected return from loans. Monopolistically competitive retailers allow introducing nominal rigidities. Capital goods producers and consumers demand the same bundle of retail goods.

The sequence of events is as follows (see Figure 1). At the end of time $t-1$, the η_{t-1} intermediate goods producers borrow from financial intermediaries in order to buy capital. Their decisions are based on expectations of idiosyncratic efficiency and systemic shocks. Immediately thereafter, systemic shocks are observed and intermediate producers, including potential new entrants, learn their idiosyncratic productivity. Some new firms decide to enter

[2]See the Online Appendix for a full derivation of the model.

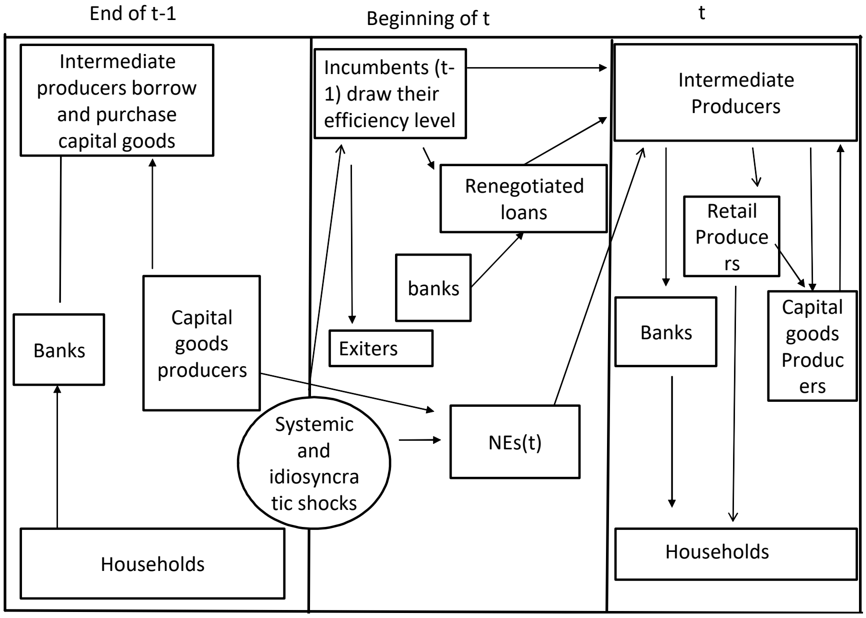


Figure 1: Sequence of events.

the market and the mass of operating firms is

$$\eta_t = NE_t + INC_t$$

where INC_t defines incumbents surviving out of the η_{t-1} producers. The remaining firms go bankrupt. This is the crucial time when debt renegotiations occur, and some firms manage to survive even if they cannot fully honour the loan contract.

Upon entry, *NEs* obtain the bank loans needed to purchase capital goods at the price Q_{t-1}. Then all the η_t firms hire labor at the consumption wage rate w_t, produce and sell their goods to retailers at the relative price p_t^m, households and capital goods producers purchase the retail goods, the intermediate firms sell their depreciated capital to capital goods producers at the relative price Q_t and repay their loans.

The modelization of firm dynamics is based on Asturias et al. (2017) but we allow firms to use both labor and capital inputs. Following Piersanti and Tirelli (2020), in every period a stochastic trend drives average efficiency of potential new entrants, whereas the average efficiency of incumbent firms is subject to a gradual depreciation.[3] In addition, we introduce substantial innovations concerning firms interactions with the financial sector, and we give insights on the specific role played by commercial banks in driving exit decisions. The set of *INCs* includes some low-productivity firms that cannot repay their capitalized loans but still yield a return that is higher than the one the bank would obtain from foreclosure. In this

[3]This assumption facilitates the calibration of steady state entry/exit flows.

case, forbearance applies and the loan is *de facto* renegotiated.

Right from the outset, we wish to clarify the rule adopted for the identification of non-performing firms. Models that abstract from financial frictions typically allow firms to operate with negative current profits insofar as the present value of discounted profits is non-negative. Assuming that debt renegotiations may occur when the present value of the firm is negative would require keeping track of firm-specific debt dynamics. To preserve tractability, we posit that all bank loans are short term, i.e. last one period, and that firms cannot carry over unserviced debt. By way of contrast, banks may offer within-period interest rate renegotiations to firms whose current profits would be negative at the contractual loan rate.

To preserve simplicity, we assume that renegotiations are based on a standard scheme, where the firm optimally chooses the scale of production and the bank receives all revenues available after payment of the fixed cost and of the wage bill. The bank incentive to renegotiate lies in the higher expected payoff relative to the alternative of repossessing the loan and paying the liquidation cost. The firm incentive to produce lies in the expectation of non-negative profits in future periods.

Finally, we assume that *NEs* optimally choose their capital stock at the beginning of each period. Relative to *INCs*, who inherit a bank loan whose amount was chosen before observing shock realizations, *NEs* have a second-mover advantage in the choice of their capital stock. We made this choice to neglect the accumulation of bad loans to potential new entrants and to sharpen the focus on renegotiations of loans to incumbent firms, in line with the empirical literature on NPLs.[4]

2.1 Households

Household members can be workers or bankers. Workers supply labor, l_t, at the competitive real wage rate, w_t. Bankers and workers randomly switch roles. Centralized decisions implement full consumption risk sharing within the household. Individual preferences are based on *t*he standard consumption bundle c_t and on labor effort

$$E_t \sum_{j=0}^{\infty} \beta^t \left(ln(c_{t+j} - hc_{t+j-1}) - \psi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

The flow budget constraint is:

$$c_t + D_t + B_t = w_t l_t + r_{t-1}^d D_{t-1} + \frac{r_t^n B_{t-1}}{\pi_t} + \Pi_t^{B,F} - LST_t \tag{2}$$

Where r_t^d is the risk-free real remuneration on bank deposits D_t, B_t is a nominally riskless government bond, r_{t-1}^n and π_t respectively define the policy and the inflation rates, $\Pi_t^{B,F}$ is the

[4]Our results carry over to a version of the model where *NEs* capital is predetermined to shocks realizations.

flow of profits from bank ownership and LST_t are lump-sum taxes that ensure government solvency. Standard first order conditions apply:

$$\lambda_t = \frac{1}{c_t - hc_{t-1}} - \frac{\beta h}{c_{t+1} - hc_t} \tag{3}$$

$$l_t = \left(\frac{\lambda_t w_t}{\psi}\right)^{\frac{1}{\varphi}} \tag{4}$$

$$\lambda_t = \beta \frac{r_t^n}{\pi_{t+1}} E_t\{\lambda_{t+1}\} \tag{5}$$

$$\lambda_t = \beta r_t^d E_t\{\lambda_{t+1}\} exp\left(\varsigma_t\right) \tag{6}$$

$$\varsigma_t = \rho^{\varsigma} \varsigma_{t-1} + \varepsilon_t^{\varsigma} \tag{7}$$

Following Del Negro et al. (2017), the "liquidity" shock, ς_t, drives a wedge between the desired return on deposits and the return on treasuries, which is driven by the policy rate.[5]

2.2 The Intermediate Sector

In a perfectly competitive market, the intermediate good producer j, f, $(f = NE, INC)$, is characterized by the following production function:

$$\begin{aligned} y_t^{j,f} &= A_t^{j,f}\left(z_t^{j,f}\right)^{\gamma} : \\ z_t^{j,f} &= [(\Xi_t k_t^{j,f})^{\alpha}(l_t^{j,f})^{1-\alpha}] \end{aligned} \tag{8}$$

where $\gamma < 1$ defines decreasing return to scale, $A_t^{j,f}$ is the idiosyncratic efficiency level, $z_t^{j,f}$ is a standard bundle of capital $\left(k_t^{j,f}\right)$ and labor $\left(l_t^{j,f}\right)$ inputs, Ξ_t is a stochastic measure of capital quality.

$$\ln\left(\Xi_t\right) = \rho^{\Xi}\ln\left(\Xi_{t-1}\right) + \sigma^{\Xi}\varepsilon_t^{\Xi}, \quad \varepsilon_t^{\Xi} \sim \epsilon(0,1). \tag{9}$$

Firm profits in terms of retail goods are:

$$\Pi_t^{j,f} = p_t^m y_t^{j,f} - r_t^k b_t^{j,f} + Q_t \Xi_t k_t^{j,f}(1-\delta) - w_t l_t^{j,f} - \phi_t^f \tag{10}$$

Where p_t^m is the intermediate-good relative price in terms of retail goods, r_t^k is the real interest rate on bank loans b_t^j used to purchase the capital stock. All firms choose to supply goods up to the point where the marginal cost equals p_t^m. Finally, ϕ^f is a fixed cost that grows at the deterministic rate g^{ϕ^f}. No firm operates at $\Pi_t^{j,f} < 0$.

[5] Fisher (2015) offers a microfoundation, showing how the Smets and Wouters (2007) risk premium shock is formally equivalent to a shock to the preference for holding risk-free assets.

2.2.1 New Entrants

Potential *NEs* draw their idiosyncratic efficiency levels from a Pareto distribution:

$$f_t(A_t^{j,NE}) = \int_{e_t}^{+\infty} \frac{\xi e_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{j,NE}) = 1; \tag{11}$$

Note that

$$e_t = e_{t-1} g_t^e$$

and

$$ln(g_t^e) = (1-\rho^z) ln(g^e) + \rho^z ln(g_{t-1}^e) + \sigma^{g^e} \varepsilon_t^e, \quad \varepsilon_t^e \sim \mu(0,1) \tag{12}$$

define the support of the technology frontier and its stochastic trend. The capital and labor inputs used by *NE* firms are

$$k_t^{j,NE} = \frac{\alpha \gamma p_t^m \left(y_t^{j,NE}\right)}{\left[Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t\right]} \tag{13}$$

$$l_t^{j,NE} = \frac{(1-\alpha) \gamma p_t^m \left(y_t^{j,NE}\right)}{w_t} \tag{14}$$

Using conditions (8), (10), (13), (14), we obtain the cost per unit of the bundle $z_t^{j,f}$, p_t^z:

$$p_t^z = \left[\frac{(Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}, \tag{15}$$

defined in retail goods, and the firm supply function

$$y_t^{j,NE} = \left(A_t^{j,NE}\right)^{\frac{1}{1-\gamma}} \left\{\frac{\gamma \left(p_t^m\right)}{p_t^z}\right\}^{\frac{\gamma}{1-\gamma}} \tag{16}$$

Using conditions (8), (13), (14), and the firm supply when profits are nil:

$$y_t^{j,NE} \left\{\Pi_t = 0\right\} = \frac{\phi_t^{NE}}{p_t^m (1-\gamma)}, \tag{17}$$

we obtain the idiosyncratic efficiency cutoff associated to the zero-profit condition:

$$\hat{A}_t^{NE} = \left[\frac{\phi_t^{NE}}{p_t^m (1-\gamma)}\right]^{1-\gamma} \left(\frac{p_t^z}{p_t^m \gamma}\right)^{\gamma}. \tag{18}$$

The interpretation of (18) is now straightforward; the idiosyncratic efficiency of the marginal firm increases in the production values of fixed and variable costs, respectively $\frac{\phi_t^{NE}}{p_t^m}$ and $\frac{p_t^z}{p_t^m}$.

The subset of the $f_t(A_t^{j,NE})$ firms characterized by $A_t^{j,NE} \geq \hat{A}_t^{NE}$ defines the *NEs* mass:

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi e_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{j,NE}) = \left(\frac{e_t}{\hat{A}_t^{NE}}\right)^{\xi}; \hat{A}_t^{NE} \geq e_t \tag{19}$$

Condition (19) shows that the mass of New entrants increases in the support of the technology frontier e_t and decreases in the productivity cutoff $\hat{A}_t^{NE}$, where the latter is essentially driven by variations in p_t^m and p_t^z. We obtain total *NEs* production using conditions (16) and (19)[6]:

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{jNE}\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma\left(p_t^m\right)}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} \frac{\xi e_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{jNE}) = \frac{NE_t \xi \phi_t^{NE}}{\xi(1-\gamma)-1} \tag{20}$$

Thus, variations in NE_t directly map into Y_t^{NE} and the fixed cost, ϕ_t^{NE}, has a twofold effect. On the one hand, it raises the productivity threshold $\hat{A}_t^{NE}$ (see condition 18) depressing the mass of NE firms and Y_t^{NE}. On the other hand, the efficiency of NE firms increases in ϕ_t^{NE} and this raises their output. From (19) it is easy to see that the former effect prevails: an increrase in the fixed cost lowers NEs output.

2.2.2 Incumbents

At the end of period $t-1$, the

$$\eta_{t-1} = NE_{t-1} + INC_{t-1} \tag{21}$$

firms borrow from commercial banks the amount

$$b_{t-1}^{j,\eta} = Q_{t-1} k_t^{j,\eta},$$

that is used to purchase capital goods from the capital goods producers. The choice of $b_{t-1}^{j,\eta}$ depends on expected shocks realizations. Then, at the beginning of period t systemic shocks are observed and the η_{t-1} firms draw their idiosyncratic $A_t^{j,\eta}$ from the following Pareto distribution

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^I(1-\delta^{inc})}^{+\infty} \frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^{\xi}}{(A_t^{j,I})^{\xi+1}} d(A_t^{j,I}), \tag{22}$$

where $\hat{A}_{t-1}^I$ denotes the lower bound of the distribution that characterized the INC_{t-1} firms, and the term $(1-\delta^{inc}) < 1$ implies that, on average, the knowledge capital of incumbent firms' is subject to obsolescence, as in Piersanti and Tirelli (2020).

This sequence of events allows to characterize the optimal $b_{t-1}^{\eta} = Q_{t-1} k_t^{j,\eta}$ as the solution

[6]Condition $\xi(1-\gamma)-1>0$ is necessary to ensure that $\int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{j,NE}\right)^{\frac{1}{1-\gamma}} \left[\frac{\gamma(p_t^m)}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} \frac{\xi e_t^{\xi}}{(A_t^{j,NE})^{\xi+1}} d(A_t^{j,NE})$ converges to a finite value.

to the representative η_{t-1} firm problem that maximizes:

$$E_{t-1}\{\Pi_t^\eta\} = E_{t-1}\{p_t^m y_t^\eta - r_t^k b_{t-1}^\eta + Q_t \Xi_t k_t^\eta (1-\delta) - w_t l_t^\eta\} - \phi_t^I \tag{23}$$

subject to the expected value of (8).

Since firms do not bear insolvency costs, their choice of capital is based on the expected efficiency of the average incumbent firm that will earn non-negative profits in period t. The solution to the problem therefore is

$$\frac{b_{t-1}^\eta}{Q_{t-1}} = \alpha\gamma E_{t-1}\left\{\frac{p_t^m y_t^{j,I^P}}{r_t^k - \frac{Q_t}{Q_{t-1}}\Xi_t(1-\delta)}\right\} \tag{24}$$

Profitable Incumbents. Knowing that each incumbent will produce according to

$$y_t^{j,I^P} = \left[A_t^{j,I^P}\left(\frac{b_{t-1}^\eta}{Q_{t-1}}\right)^{\alpha\gamma}\left(l_t^{j,I^P}\right)^{(1-\alpha)\gamma}\right], \tag{25}$$

and that, after shocks realizations, profitable firms will optimally hire labor conditionally to their idiosyncratic efficiency, to the capital stock choice, and to the real wage in production units

$$l_t^{j,I^P} = \frac{(1-\alpha)\gamma p_t^m y_t^{j,I^P}}{w_t} \tag{26}$$

where $\hat{A}_t^{I^P}$ is the efficiency cut-off, such that the incumbent's expected profits are nil. The profitability threshold, $\hat{A}_t^{I^P}$, is obtained by plugging (25) and (26) into the zero profit condition

$$\Pi_t^\eta = p_t^m y_t^\eta - r_t^k b_{t-1}^\eta + Q_t \Xi_t k_t^\eta(1-\delta) - w_t l_t^\eta - \phi_t^I = 0 \tag{27}$$

such that,

$$\hat{A}_t^{I^P} = \frac{\left\{\phi_t^I + \left[r_t^k - \frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right] b_{t-1}^\eta\right\}^{1-(1-\alpha)\gamma}\left[\frac{w_t}{(1-\alpha)\gamma}\right]^{(1-\alpha)\gamma}}{p_t^m\left[1-(1-\alpha)\gamma\right]^{1-(1-\alpha)\gamma}\left(\frac{b_{t-1}^\eta}{Q_{t-1}}\right)^{\alpha\gamma}}. \tag{28}$$

Note that when the capital choice is not predetermined the productivity cutoff amounts to

$$\hat{A}_t^{I,EA} = \frac{1}{p_t^m}\left[\frac{\phi_t^I}{(1-\gamma)}\right]^{1-\gamma}\left(\frac{p_t^z}{\gamma}\right)^\gamma. \tag{29}$$

In comparison with (29), condition (28) pinpoints the twofold role of predetermined debt. First, the loan repayment net of the residual value of capital is akin to the fixed cost, unambiguously raising the productivity cutoff. Thus, the larger the size of the loan, the higher the productivity cutoff. Second, the larger the stock of borrowed capital the greater the firm size and its ability to generate revenues. This latter effect brings down $\hat{A}_t^{I^P}$. An increase in the wage rate works in the opposite direction, because it adversely affects the labor intensity of the firm production function, reducing its profitability for any given capital stock. Finally, an increase in the price of intermediate goods raises allows less efficient firms to remain profitable.

Finally, $\hat{A}_t^{I,EA} < \hat{A}_t^{I^P}$ because the predetermined capital stock is based on the expected efficiency of profitable firms, and its size is inevitably too large for the relatively less efficient incumbent firms.

Debt forbearance, "non-performing" and defaulting firms. When a firm does not meet the profitability condition $\hat{A}_t^{I^P}$, the financial intermediary retains the option of repossessing the capitalized loan, $r_t^k b_{t-1}^\eta$, conditional to payment of a stochastic liquidation cost μ_t.

$$\ln(\mu_t) = (1-\rho^\mu)\ln(\mu_t^*) + \rho^\mu \ln(\mu_{t-1}) + \sigma^\mu \varepsilon_t^\mu, \quad \varepsilon_t^\mu \sim \epsilon(0,1), \quad corr\left(\varepsilon_t^\mu, \varepsilon_t^\Xi\right) \geq 0$$

where $\mu_t^* = g^{\mu^*}\mu_{t-1}^*$ denotes the deterministic component of the liquidation cost. The correlation between ε_t^μ and ε_t^Ξ captures possible congestion effects on liquidation costs due to the severity of financial crisis episodes. The alternative option to outright default is a renegotiation of the "debt contract". One might characterize debt re-negotiation as the outcome of a potentially complex bargaining process. Here we focus on a simpler alternative where we posit that the bank appropriates all t-period revenues net of operational costs. The bank accepts debt renegotiation when proceedings from renegotiation are no less than expected returns from the alternative option of paying μ_t and repossessing the invested capital

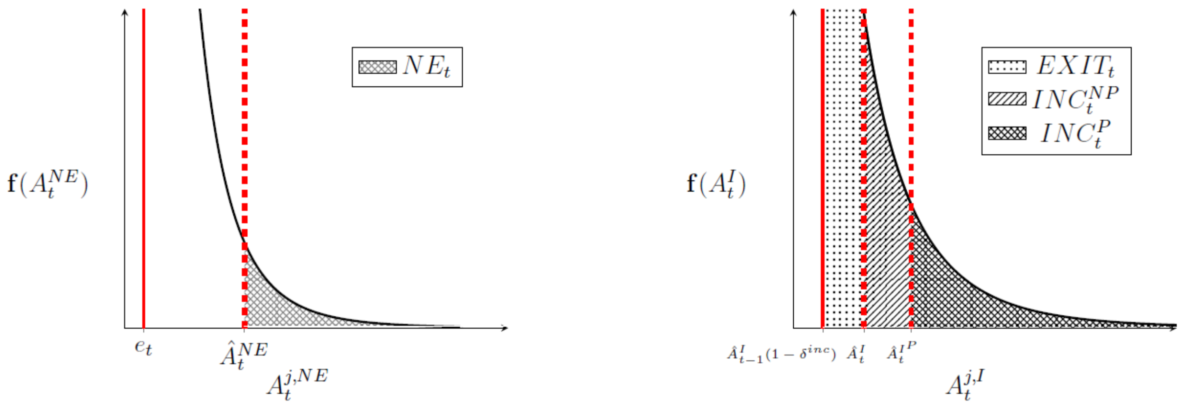
$$p_t^m y_t^{j,I^{NP}} - w_t l_t^{j,I^{NP}} + \frac{\Xi_t Q_t(1-\delta)b_{t-1}^\eta}{Q_{t-1}} - \phi_t^I \geq r_t^k b_{t-1}^\eta - \mu_t.$$

Loan renegotiations occur if $\hat{A}_t^{I^P} > A_t^j > \hat{A}_t^I$, where $\hat{A}_t^I$ denotes the productivity cutoff such that the bank is indifferent between renegotiation and repossession:

$$\hat{A}_t^I = \hat{A}_t^{I^P}\left\{1 - \frac{\mu_t}{\phi_t^I + \left[r_t^k - \frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right]b_{t-1}^\eta}\right\}^{1-(1-\alpha)\gamma}. \tag{30}$$

The efficiency cutoff for NP firms falls short of $\hat{A}_t^{I^P}$ to the extent that the liquidation cost is large relative to the pre-determined cost that profitable incumbents are confronted with.

Figure 2 provides a graphical representation of how NEs and $INCs$ are distributed. Panel (a) identifies the entrants that choose to operate in t. In Panel (b) we represent the distribution of the depreciated knowledge capital inherited by the η_{t-1} firms: $\hat{A}_t^{I^P}$, $\hat{A}_t^I$ and $\hat{A}_{t-1}^I(1-\delta^{inc})$ split the support between profitable, non-performing and exiting η_{t-1} firms.



(a) NEs in t

(b) INCs in t

Figure 2: Firm Dynamics

The distribution of incumbent firms. The functional forms of the efficiency cutoffs $\hat{A}_t^{I^P}$ and $\hat{A}_t^I$ allow to identify the mass of incumbent, performing and non-performing firms respectively:

$$INC_t = \int_{\hat{A}_t^I}^{+\infty} \frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^\xi}{(A_t^{j,I})^{\xi+1}} d(A_t^{j,I}) = \eta_{t-1}\left(\frac{\hat{A}_{t-1}^I}{\hat{A}_t^I}(1-\delta^{inc})\right)^\xi \tag{31}$$

The mass of profitable incumbents is:

$$INC_t^P = \int_{\hat{A}_t^{I^P}}^{+\infty} \frac{\xi(\hat{A}_{t-1}^I(1-\delta^{inc}))^\xi}{(A_t^{j,I^P})^{\xi+1}} d(A_t^{j,I^P}) = \eta_{t-1}\left(\frac{\hat{A}_{t-1}^I}{\hat{A}_t^{I^P}}(1-\delta^{inc})\right)^\xi \tag{32}$$

$$INC_t^{NP} = \eta_{t-1}\left(\frac{\hat{A}_{t-1}^I}{\hat{A}_t^I}(1-\delta^{inc})\right)^\xi \left(1-\frac{\hat{A}_t^I}{\hat{A}_t^{I^P}}\right)^\xi \tag{33}$$

Note that the fraction of non-performing incumbents amounts to

$$\frac{INC_t^{NP}}{INC_t} = \left(1-\frac{\hat{A}_t^I}{\hat{A}_t^{I^P}}\right)^\xi \tag{34}$$

where $\frac{\hat{A}_t^I}{\hat{A}_t^{I^P}}$ is identified in (30). Hence, the the fraction of non-performing incumbents essentially depends on how large the liquidation cost μ_t is relative to the pre-determined cost that affects the efficiancy cutoff of profitable incumbents. Exiting firms, EX_t, are :

$$EX_t = \eta_{t-1}\left[1 - \left(\frac{\hat{A}^I_{t-1}(1-\delta^{inc})}{\hat{A}^I_t}\right)^{\xi}\right]$$

Production Total production originating from the incumbent firms is

$$Y_t^{I^P} = \int_{\hat{A}_t^{I^P}}^{+\infty}\left[A_t^{j,I^P}\left(\frac{b^\eta_{t-1}}{Q_{t-1}}\right)^{\alpha\gamma}\left(l_t^{j,I^P}\right)^{(1-\alpha)\gamma}\right]d(A_t^{j,I^P}) = \frac{\xi INC_t^P}{\xi\left[1-(1-\alpha)\gamma\right]-1}\left(\frac{\phi_t^I + \left[r_t^k - \frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right]b^\eta_{t-1}}{p_t^m}\right) \tag{35}$$

$$Y_t^{I} = \int_{\hat{A}_t^{I}}^{+\infty}\left[A_t^{j,I^P}\left(\frac{b^\eta_{t-1}}{Q_{t-1}}\right)^{\alpha\gamma}\left(l_t^{j,I^P}\right)^{(1-\alpha)\gamma}\right]d(A_t^{j,I}) = \frac{\xi INC_t}{\xi\left[1-(1-\alpha)\gamma\right]-1}\left(\frac{\phi_t^I - \mu_t + \left[r_t^k - \frac{\Xi_t Q_t(1-\delta)}{Q_{t-1}}\right]b^\eta_{t-1}}{p_t^m}\right) \tag{36}$$

$$Y_t^{I^{NP}} = Y_t^I - Y_t^{I^P} \tag{37}$$

2.3 Capital misallocation and firm distribution

Let us now compare the allocation of capital under the pre-determined capital friction with the outcome that, ceteris paribus, would obtain under the efficient allocation. In this latter case firm's j demand for capital amounts to

$$k_t^{j,EA} = \frac{\alpha\,\gamma p_t^m\left(y_t^{j,EA}\right)}{\left[Q_{t-1}r_t^k - \Xi_t(1-\delta)Q_t\right]} \tag{38}$$

$$y_t^{j,EA} = \left(A_t^j\right)^{\frac{1}{1-\gamma}}\left[\frac{\gamma p_t^m}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}} \tag{39}$$

Under the EA, the capital allocation grows in the idiosyncratic efficiency of the firm (see Figure 3 panel (b)). For any given capital stock $\frac{b^\eta_{t-1}}{Q_{t-1}}$, the pre-determined capital friction implies that too little(much) capital is allocated to more(less) efficient firms. By contrast, predetermined capital is inefficiently concentrated in the hands of the less efficient firms, and capital misallocation limits the scale of production that the more efficient firms can attain.

The inefficient allocation of capital impacts on the distribution of output at different levels of firm efficiency (Figure 3 panel (a)): the predetermined capital frictions limits the contribution of more efficient firms to total production.[7]

Define $A_t^{j^*}$ as the efficiency level that makes firm j^* indifferent between inheriting $\frac{b_{t-1}^{\eta}}{Q_{t-1}}$ and choosing its optimal capital stock according to conditions (38) and (39):

$$A_t^{j^*} = \left\{ \frac{b_{t-1}^{\eta}}{Q_{t-1}} \frac{\left[Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t\right]}{\alpha} \right\}^{1-\gamma} \frac{(p_t^z)^{\gamma}}{\gamma p_t^m} \tag{40}$$

For any given set of the variables p_t^m, p_t^z, $\hat{A}_{t-1}^I$, $\left[Q_{t-1} r_t^k - \Xi_t(1-\delta)Q_t\right]$, the total allocations to the most efficient firms, such that $A_t^j > A_t^{j^*}$, are

$$K_t^{EA,\left\{A_t^j > A_t^{j^*}\right\}} = \eta_{t-1} \frac{\alpha\, \gamma p_t^m \left\{\frac{\gamma (p_t^m)}{p_t^z}\right\}^{\frac{\gamma}{1-\gamma}} \xi (\hat{A}_{t-1}^I (1-\delta^{inc}))^{\xi}}{\left[Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t\right]} \int_{A_t^{j^*}}^{+\infty} \left(A_t^j\right)^{\frac{1}{1-\gamma} - (\xi+1)} d(A_t^j)$$

$$K_t^{EA,\left\{A_t^j > A_t^{j^*}\right\}} = \eta_{t-1} \frac{\alpha\, \gamma p_t^m \left\{\frac{\gamma (p_t^m)}{p_t^z}\right\}^{\frac{\gamma}{1-\gamma}} (\hat{A}_{t-1}^I (1-\delta^{inc}))^{\xi}}{\left[Q_{t-1} r_t^k - \Xi_t (1-\delta) Q_t\right]} \frac{\xi\,(1-\gamma)}{\xi\,(1-\gamma) - 1} \left(A_t^{j^*}\right)^{\frac{1}{1-\gamma} - \xi} \tag{41}$$

under the efficient allocation, and

$$K_t^{b^{\eta},\left\{A_t^j > A_t^{j^*}\right\}} = \eta_{t-1} (\hat{A}_{t-1}^I (1-\delta^{inc}))^{\xi} \frac{b_{t-1}^{\eta}}{Q_{t-1}} \xi \int_{A_t^{j^*}}^{+\infty} (A_t^j)^{-(\xi+1)} d(A_t^j) = \eta_{t-1} (\hat{A}_{t-1}^I (1-\delta^{inc}))^{\xi} \frac{b_{t-1}^{\eta}}{Q_{t-1}} \left(A_t^{j^*}\right)^{-\xi} \tag{42}$$

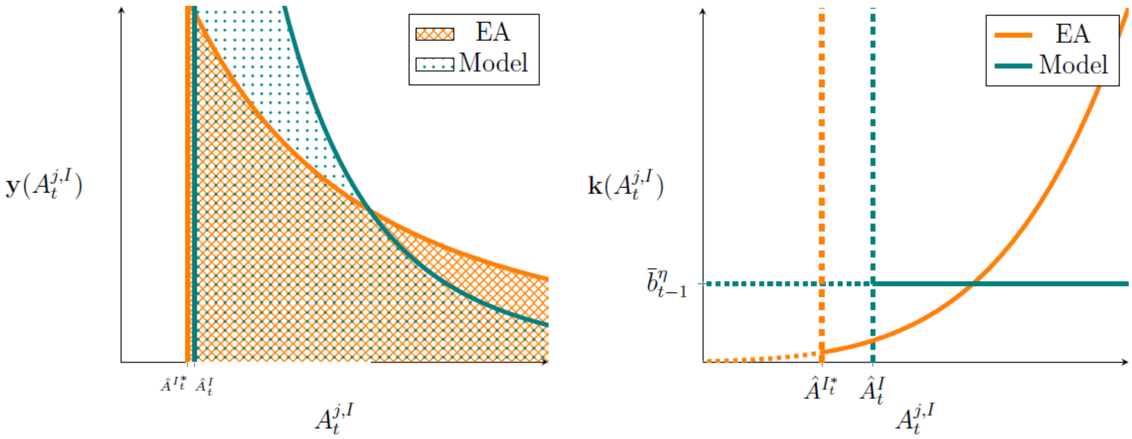
under the pre-determined capital friction.

The ratio between the two is independent from the state of the economy:

$$\frac{K_t^{EA,\left\{A_t^j > A_t^{j^*}\right\}}}{K_t^{b^{\eta},\left\{A_t^j > A_t^{j^*}\right\}}} = \frac{\xi\,(1-\gamma)}{\xi\,(1-\gamma) - 1}$$

According to our parameter choice, on average the most efficient firm loose about 25% of capital. Note that the elasticity of $\frac{K_t^{EA,\left\{A_t^j > A_t^{j^*}\right\}}}{K_t^{b^{\eta},\left\{A_t^j > A_t^{j^*}\right\}}}$ w.r. to the tail index of the pareto distribution, ξ, and to the returns to scale parameter, γ, respectively are $-\frac{1}{[\xi(1-\gamma)-1]}$ and $\frac{(1+\xi)\gamma+1}{(1-\gamma)[\xi(1-\gamma)-1]}$. Thus, the more incumbent firms are concentrated at the lower end of their support, the less important is the capital misallocation simply because very efficient firm are relatively scarse. By contrast, the capital loss for more efficient firms grows in the degree of returns to scale because under EA the elasticity of firms demand for capital is $\frac{1}{1-\gamma}$.

[7]Figure 3 panel (a) plots output distributions at the detrended deterministic steady state.



(a) Output distribution **(b)** Capital demand

Figure 3: Productivity and misallocation.

We also consider an additional measure of capital misallocation:

$$K_t^{MIS,\left\{A_t^j>A_t^{j*}\right\}} = \frac{K_t^{EA,\left\{A_t^j>A_t^{j*}\right\}} - K_t^{b^\eta,\left\{A_t^j>A_t^{j*}\right\}}}{INC_t^P \frac{b^\eta_{t-1}}{Q_{t-1}}} = \overline{K}\left\{1+\frac{\phi_t^I}{\left(\frac{b^\eta_{t-1}}{Q_{t-1}}\right)\left(Q_{t-1}r_t^k - \Xi_t(1-\delta)Q_t\right)}\right\}^{[1-(1-\alpha)\gamma]\xi} \tag{43}$$

where $\overline{K} = \frac{\left[\frac{(1-(1-\alpha)\gamma)}{\alpha\gamma}\right]^{-\xi[1-(1-\alpha)\gamma]}}{\xi(1-\gamma)-1}$. Condition (43) identifies the "capital demand" gap for the most efficient firms, normalized with the predetermined capital allocation to profitable incumbents. If $\phi_t^I = 0$, net returns from predetermined capital would identically affect $K_t^{EA,\left\{A_t^j>A_t^{j*}\right\}} - K_t^{b^\eta,\left\{A_t^j>A_t^{j*}\right\}}$ and $INC_t^P \frac{b^\eta_{t-1}}{Q_{t-1}}$ because they are the only fixed cost that profitable incumbents are confronted with. In this case the misallocation index would be a constant. By contrast, they have an asymmetric effect when $\phi_t^I > 0$. For instance, an unexpected increase in $\left(Q_{t-1}r_t^k - \Xi_t(1-\delta)Q_t\right)$, that raises both A_t^{j*} and $\hat{A}_t^{IP}$, will lower $K_t^{MIS,\left\{A_t^j>A_t^{j*}\right\}}$(see conditions (28), (31), (40)).

2.4 Financial intermediaries.

The representative banker's balance sheet is:

$$L_t^b = NW_t + D_t \tag{44}$$

where NW_t defines the banker's net wealth and

$$L_t^b = b^\eta_{t-1}\eta_{t-1} + Q_{t-1}K_t^{NE}$$

is the amount of extended loans.

Proceedings from re-negotiated loans are

$$\Pi_t^{NP} = \left[1 - (1-\alpha)\gamma\right] P_t^m Y_t^{I,NP} + INC_t^{INP}\left[\frac{(1-\delta)\,\Xi_t Q_t b_{t-1}^{\eta}}{Q_{t-1}} - \phi_t^I\right]$$

Bank returns from defaulting firms amount to $\left(r_t^k \frac{Q_t b_{t-1}^{\eta}}{Q_{t-1}} - \mu_t\right) EX_t$. The average return on loans, r_t^b, is:

$$r_t^b L_t^b = r_t^k \left[INC_t^P b_{t-1}^{\eta} + Q_{t-1} K_t^{NE}\right] + \Pi_t^{NP} + \left(r_t^k b_{t-1}^{\eta} - \mu_t\right) EX_t \tag{45}$$

In each period the banker can divert a fraction $\lambda_{b,t}$ of funds and exit the market so, for lenders to be willing to supply funds to the banker, the incentive compatibility constraint must be $V_t^b \geq \lambda_{b,t} L_t^b$, where V_t^b implicitly defines the banker's continuation value. The amount of bank deposits constrains the banks' leverage ratio, and generates a loan rate spread $r_t^b - r_{t-1}^d$ such that the representative banker's continuation value matches the incentive to divert funds. The lower V_t^b, the smaller the amount of bank deposits and the supply of loans, L_t^b. This, in turn, raises the return on loans and ensures that the incentive-compatibility constraint of the banker is satisfied.

The standard characterization of the banker's moral hazard problem simply parameterizes $\lambda_{b,t}$ at a time-invariant value. In the following we consider an extension which links it to the expected evolution of non-performing firms:

$$\lambda_{b,t} = \lambda_b \left[1 + \alpha_{\lambda_b}\left(\frac{INC_{t+1}^{NP}}{INC^{NP}} - 1\right)\right] \tag{46}$$

The underlying intuition here is that non-performing loans add opacity to the bank balance sheet, and that the number of non-performing firms, as opposed to the average size of the loan is better suited to capture this effect, because it is relatively more difficult for the market to monitor the specific features of an increasing number of non-performing loans (Suarez and Sánchez Serrano, 2018).

Bankers are subject to a leverage constraint which generates the following law of motion for banks equity capital, NW_t:

$$NW_t = \theta_b\left[(r_t^b - r_{t-1}^d)\Phi_{t-1}^b + r_{t-1}^d\right] NW_{t-1} + \omega \Xi_t K_{t-1} Q_t \tag{47}$$

where θ_b is the probability that an incumbent banker exits, ω is the households' transfer to new bankers, and $\Phi_{t-1}^b = \frac{Q_{t-1}K_{t-1}}{NW_{t-1}}$ is bank leverage.

2.5 Monetary Policy

Monetary policy evolves accordingly to a Taylor rule that responds to inflation and to the output gap.

$$\frac{r_t^n}{r^n} = \left(\frac{r_{t-1}^n}{r^n}\right)^{\rho^i} \left[(\pi_t)^{\kappa^\pi} \left(Y_t^{GAP}\right)^{\kappa^y}\right]^{1-\rho^i} exp(\varepsilon_t^r);\ \varepsilon_t^r \sim \mu\,(0,1)\,. \tag{48}$$

where Y_t^{GAP} is the output gap, and ε_t^r is an interest rate shock. Condition $\frac{r_t^n}{\pi_t} = r_t^d$ links the policy nominal rate to the rate on bank deposits. Following GK, we also consider a policy scenario where the central bank engages in Quantitative Easing (QE) policies. Essentially we assume that the Central Bank can sell government-backed securities that households treat as perfect substitutes for bank deposits. The proceedings are then used to supply loans, L_t^{CB}, to intermediate goods producers. In this framework the total value of loans amounts to

$$\overline{b}_{t-1}^{\eta}\eta_{t-1} + Q_{t-1}K_t^{NE} = L_t^{CB} + L_t^b$$

The Central bank sets her loans as a fraction of total intermediate assets:

$$L_t^{CB} = \Psi_t \left[\overline{b}_{t-1}^{\eta}\eta_{t-1} + Q_{t-1}K_t^{NE}\right].$$

The quantitative easing policy rule is[8]

$$ln\,(\Psi_t) = \rho^{\Psi} ln\,(\Psi_{t-1}) + \nu\left[\ln(r_t^b - r_{t-1}^d) - \ln(r^b - r^d)\right]. \tag{49}$$

The intuition here is that a crisis lowers the bankers' continuation value, causing a reduction in deposits and an increase in the spread on loans returns. The QE policy raises the supply of loans, strengthening the relative price of capital goods and lowering the cost of capital for intermediate goods producers.

2.6 Calibration

All parameters and shock processes are reported in Table 1. Firms returns to scale, $\gamma = 0.8$, are set at the lower bound of Basu and Fernald (1997) estimates, and the tail index of the Pareto distribution, $\xi = 6.1$ is set as in Asturias et al. (2017). As shown in section (3) below, the deterministic quarterly growth rate, $g = (g^e)^{\frac{1}{1-\alpha\gamma}}$ drives the economy along the BGP. We therefore calibrate the deterministic growth rate of the technology frontier at a level such that $g = 1.0025$, implying a yearly growth rate at 1%. We calibrate the discount factor $\beta = 0.9925$ in order to obtain a steady state value of the risk-less rate of deposits $r^d = 1.0101$. The investment adjustment cost is set to $\gamma_i = 3.14$, following Justiniano et al.

[8]See Foerster (2015) for a detailed derivation of QE policies.

(2010).

We set the detrended support of the NEs distribution, e, the depreciation rate of firms efficiency, δ^{inc}, the detrended fixed production costs, ϕ^{I} and ϕ^{NE}, and the detrended liquidation cost, μ, to calibrate the values of some variables that characterize firms distribution in steady state. The firm exit rate, $\frac{EX}{\eta} = \frac{NE}{\eta}$, is set at 10% on annual basis (Bilbiie et al., 2012); the steady state number of firms, η, is normalized at 1, the total fixed costs of production amount to 5% of total GDP (Bilbiie et al., 2012; Etro and Colciago, 2010). Following Banerjee and Hofmann (2018), the share of non-performing firms, $\frac{INC^{NP}}{\eta}$, is set at 8%.[9] We also obtain that the relative size of new entrant firms is about 60%, as Clementi and Palazzo (2016).

The agency problem in the banking sector is parameterized as in GK. The fraction of funds that can be diverted in steady state, λ_b, is set at 0.338, the probability survival of an incumbent banker, θ_b, is 0.9725, the proportional transfer to new bankers, ω, is 0.002. These values allow to obtain in steady state a return spread, $r^b - r^d$, of one hundred basis points per annum, a leverage ratio $\Phi^b = 4$ and 10 years as the average horizon for bankers. Further, we set α_{λ_b} to a value such that our financial crisis experiment replicates the spread observed by Gilchrist and Mojon (2018) for the Euro Area during the financial crisis.

The stochastic process for μ_t is only suggestive and meant to gauge the potential consequences of liquidation costs congestion. We set $\sigma^{\mu} = 0.5$, and fix at 0.9 both the autoregressive parameter ρ^{μ} and the *corr* $\left(\varepsilon_t^{\mu}, \varepsilon_t^{\Xi}\right)$ parameter. The persistence of capital quality and liquidity shocks are respectively $\rho^{\Xi} = 0.9$ and $\rho^{\varsigma} = 0.9$. All the remaining parameters are borrowed from GK.

3 The deterministic steady state

In the deterministic steady state, the standard features of the Solow/Ramsey BGP must hold for our model, *i.e.* C_t, I_t, Y_t, K_t, w_t, must grow at the same rate g. This, in turn, implies that Y_t^{NE}, Y_t^{P} and Y_t^{NP} also grow at the g rate.

To simplify the model, we posit that the number of firms is constant along the BGP. From (20) we must have that $g^{\phi^{NE}} = g$. Using (19), a constant mass of new entrants obtains only if $\frac{\hat{A}_t^{NE}}{\hat{A}_{t-1}^{NE}} = g^{\hat{A}^{NE}} = g^{e}$ and from (18) we must have

$$g^{\hat{A}^{NE}} = \left(g^{\phi^{NE}}\right)^{1-\gamma} g^{(1-\alpha)\gamma} = g^{1-\alpha\gamma}, \tag{50}$$

[9]The empirical literature on non-performing firms focuses on firms that are not "young" and that remain insolvent for several quarters. The total amount of firms unable to service interest payments, our definition of non-performing firms, is certainly larger. Our results would not change even if we tripled $\frac{INC^{NP}}{\eta}$.

Table 1: Calibrated Parameters.

Households		
φ	0.276	Inverse Frisch elasticity of labor supply
β	0.9925	Discount rate
δ	0.025	Depreciation rate
ψ	2.6	Relative utility weight of labor
h	0.815	Habit parameter
g	1.0025	Gross BGP rate
Intermediate Good Firms		
ϕ^{NE}	0.05	Entry cost
ϕ^{I}	0.11	Fixed production cost for incumbents
γ	0.8	Decreasing return index
α	0.33	Capital share
ξ	6.1	Pareto distribution shape parameter
e	0.8614	Technology frontier initial value
H	0.025	Share of NEs over total firms in steady state
H_{NP}	0.08	Share of INC^{NP}s over total firms in steady state
Financial Intermediaries		
λ_b	0.338	Fraction of capital that can be diverted
θ_b	0.9725	Survival probability of banks
ω	0.002	Proportional transfer to the entering bankers
μ	0.0208	Repossession cost for defaulting firms' debt
α_λ	1.4	Moral hazard parameter
Capital Producing Firms		
γ_i	3.14	Inverse elasticity of net investment to the price of capital
Retail Firms		
Γ	0.779	Probability of keeping prices fixed
ε	5	Elasticity of substitution
Central Bank		
κ_y	0	Output gap coefficient of the Taylor rule
κ_y	3.1	Inflation coefficient of the Taylor rule
ρ^i	0.8	Smoothing parameter of the Taylor rule
ν	100	Quantitative Easing Parameters
Exogenous Processes		
ρ^Ξ	0.66	Persistence of capital quality shock
ρ^μ	0.9	Persistence of μ shock
ρ^ς	0.9	Persistence of liquidity shock
$\rho^{\Xi,\mu}$	0.9	Correlation μ and Ξ shocks

therefore it must be that

$$g = (g^e)^{\frac{1}{1-\alpha\gamma}},$$

From (35) and (37) it must also be that $g^{\phi^I} = g^{\mu^*} = g$. From (28) and (30), we obtain that $g^{\hat{A}^I} = g^e$. Therefore, using (21) and (31), the steady state entry rate amounts to

$$\frac{NE}{\eta} = 1 - \frac{INC}{\eta} = 1 - \left(\frac{1-\delta^{inc}}{g^e}\right)^{\xi}. \tag{51}$$

Thus, only technology developments matter for the distribution between incumbents and new entrants. The term $\frac{1-\delta^{inc}}{g^e}$, defines the $t-$period of the average efficiency depreciation of the η_{t-1} potential incumbents relative to the efficient improvement of the $t-$period potential new entrants. The larger g^e the greater the competitive pressure from new entrants and the larger is firm turnover. The fraction of non-performing incumbents is:

$$\frac{INC^{NP}}{\eta} = \frac{INC}{\eta}\frac{INC^{NP}}{INC} = \left(\frac{1-\delta^{inc}}{g^e}\right)^{\xi}\left[1-\left(\frac{\widetilde{\hat{A}^I}}{\widetilde{\hat{A}^{I,P}}}\right)^{\xi}\right] \tag{52}$$

where[10]

$$\frac{\widetilde{\widehat{A}^I}}{\widetilde{\hat{A}^{I,P}}} = \left\{1 - \frac{\mu}{\phi^I + \left[r^k - (1-\delta)\right]\widetilde{b}^{\eta}}\right\}^{1-(1-\alpha)\gamma} \tag{53}$$

The fraction of non-performing firms $\frac{INC^{NP}}{\eta}$ increases in the ratio between μ and the true fixed cost that performing firms must bear, $\phi^I + \left[r^k - (1-\delta)\right]\widetilde{b}^{\eta}$. Payments on predetermined capital

$$\left[r^k - (1-\delta)\right]\widetilde{b}^{\eta} = \frac{\alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\ \phi^I\left(g^e\right)^{\frac{1}{1-\alpha\gamma}}}{\left\{1 - \left[(1-\alpha)\gamma\right] - \alpha\gamma\left(\frac{\xi}{\xi-1}\right)^{\frac{1}{1-\gamma+\alpha\gamma}}\right\}} \tag{54}$$

are independent from the distribution of non-performing firms and, more generally, from the financial frictions that the model incorporates.[11] Note that the growth driver g^e has a twofold negative effect on the fraction of non-performing incumbents $\frac{INC^{NP}}{\eta}$: it lowers the fraction of incumbent firms and, by inducing incumbents to raise demand for pre-determined capital, it limits banks' incentive to renegotiate because it raises the default cutoff $\widetilde{\widehat{A}^I}$ relative to the efficiency cutoff for profitable incumbents, $\widetilde{\hat{A}^{I,P}}$. This essentially happens because faster productivity growth of NEs lowers the price of intermediate goods. The rate of return on bank deposits is $r^d = \frac{g}{\beta}$ and, the rate of return on bank loans is independent from the pre-

[10]Note that $\widetilde{x}$ identifies the de-trended steady state value of variable x_t. See the Online Appendix.

[11]To obtain condition (54), we simoultaneously solved for (24) and (28) after detrending.

determined capital friction and from the bank's incentive to renegotiate loans.[12] The rental price of capital solves

$$r^k - r^b = \frac{\mu - g\frac{\widetilde{\Pi}^{NP}}{1-INC}}{\left[\frac{\alpha\gamma p^m \phi^{NE}}{r^k-(1-\delta)} + \frac{INC}{1-INC}\widetilde{b}^{\eta}\right]}, \tag{55}$$

where

$$\widetilde{\Pi}^{I,NP} = (1-\alpha)\gamma p^m \frac{\xi\left[1-(1-\alpha)\gamma\right]}{\xi\left[1-(1-\alpha)\gamma\right]-1}\left[INC - INC^P\left(\frac{\widetilde{\widehat{A}^{I,P}}}{\widetilde{\widehat{A}^{I}}}\right)^{\frac{1}{1-(1-\alpha)\gamma}}\right]$$

$$\left\{\frac{\phi^I + \frac{\left[r^{k,\eta}-(1-\delta)\right]\widetilde{b}^{\eta}}{g}}{\left[1-(1-\alpha)\gamma\right]}\right\} + INC^{NP}\frac{\widetilde{b}^{\eta}(1-\delta)}{g} - INC^{NP}\phi^I.$$

The large the share of non-performing incunmbents the greater $r^k - r^b$. Based on our calibrations, we obtain that, on a quarterly basis, $r^k - r^b = 0.09\%$

We now investigate the specific role of the pre-determined capital assumption, which is crucial to generate situations where commercial banks are induced to renegotiate debt contracts. Based on the set of parameters presented in Table 1, we essentially compare three scenarios. The first one is our baseline model, characterized by pre-determined capital and debt renegotiations. The second one is the efficient capital allocation. The third one, labelled efficient re-allocation (ERA) model, maintains the assumption of pre-determined capital, but it allows the opening of a secondary capital market after shocks have been observed, thus allowing the efficient reallocation of capital.[13]

In Table 2 we report selected variables as ratios to the corresponding values obtained for the baseline model. Right from the outset, note that the ERA model is characterized by outcomes that are quite close to the ones obtained in the EA model, showing that the predetermined capital friction would be almost irrelevant if a capital reallocation scheme could be properly designed. By contrast, our baseline model predicts lower aggregate output, consumption and investment.

The EA model is characterized by an allocation where firms optimally select their capital stock, whereas predetermined capital is inefficiently concentrated in the hands of the less efficient firms. This has several far-reaching implications. First, the productivity cutoff for surviving incumbents is substantially lower in the EA model because in the baseline model the predetermined capital stock generates a "fixed cost" effect that can be borne only by firms characterized by a relatively high idiosyncratic productivity. Second, capital misallocation limits the scale of production that can be attained by the more efficient firms. In fact, average

[12] See the Online Appendix for a proof.

[13] The full derivation of these models is in the Online Appendix.

firm size is much larger in the EA model. Third, the larger amount of output observed in the EA model is associated with higher salaries. This, in turn, worsens the relative position of NE firms, that efficiently choose their size even in our baseline model. As a result, the EA model is characterized by a higher productivity cutoff for NE firms, whose number falls causing an equivalent reduction in the total mass of firms.

Using (43), we are able to show that

$$K^{EA,\left\{A^j>A^{j^*}\right\}} - K^{b^\eta,\left\{A^j>A^{j^*}\right\}} = \frac{INC^P b^\eta \left\{\left(\frac{\xi-1}{\xi}\right)^\xi\right\}}{\xi\left(1-\gamma\right)-1} \approx 1.52 INC^P b^\eta.$$

The pre-determined capital friction therefore causes a substantial misallocation of capital.

Var	Y	C	I	NE	$\frac{Y}{\eta}$	$\hat{A}^{NE}$	$\hat{A}^{I}$	$\frac{NE}{\eta}$
SS^{EA}	2.57%	2.63%	1.99%	−8.21%	11.75%	1.41%	−7.54%	1%
SS^{ERA}	2.62%	2.85%	2.13%	−8.16%	11.75%	1.41%	−7.54%	1%

Table 2: Steady state percentage change with perfect capital allocation hypothesis SS^{EA} and ex-post capital reallocation hypotesis SS^{ERA}

3.1 Long Run Effects of Structural Changes

We implement here a simple comparative statics exercise to assess the effects of a change in some key parameters that determine the share of non-performing incumbents, $\frac{INC^{NP}}{\eta}$ in our model. Straightforward manipulations allow to obtain the elasticity of $\frac{INC^{\ NP}}{\eta}$ to the technology growth rate, g^e:

$$\frac{\partial\left(\frac{INC^{NP}}{\eta}\right)}{\partial g}\frac{\partial g}{\partial g^e}\frac{g^e}{\left(\frac{INC^{NP}}{\eta}\right)} = -\xi(1-\alpha\gamma)$$

where $(1-\alpha\gamma)$ defines the elasticity of the BGP rate g to the growth rate of the efficiency frontier. To grasp intuition, note that a reduction in g^e unambiguously reduces the flow of NEs. Correspondingly, the share of incumbents must rise. The share of profitable firms also increases, but to a lesser extent because $\frac{\widetilde{\hat{A}^I}}{\widetilde{\hat{A}^{I,P}}} < 1$. The fraction of non-performing incumbents must therefore be larger. A fall in the long-run growth rate raises the share on non-performing incumbents by a factor of $\xi(1-\alpha\gamma) = 4.49$, suggesting that the growth slowdown of the last twenty years might have had an important role in raising the share of non-performing incumbents. Thus our model provides a rationale for the inverse correlation between the natural interest rate and the share of non-performing incumbents .

By contrast, an interest rate fall caused by an increase in the subjective discount rate,

β, has no effect on firms turnover (Table 2) and on the share of non-performing incumbents, just like changes in the loan-deposits spread. These changes do not matter because they act symmetrically across all incumbent firms and do not affect the relative demand for loans from non-performing incumbents.

Var	β $(+0.5\%)$	μ (-90%)	$\phi^{NE} = \phi^{I}$ (-1%)
Y	4.86%	−0.16%	0.43%
C	2.51%	0.72%	0.44%
I	1.73%	−3.76%	0.43%
η	4.86%	−2.8%	10.72%
$\frac{Y}{\eta}$	$\sim 0\%$	2.71%	−10.31%
NE	4.86%	−2.8%	10.72%
INC^{NP}	4.86%	−90%	20.45%
$\frac{NE}{\eta}$	$\sim 0\%$	$\sim 0\%$	$\sim 0\%$
$\frac{INC^{NP}}{\eta}$	$\sim 0\%$	−89%	9.72%

Table 3: Long Run Effects

Andrews et al. (2017) emphasize the importance of poorly designed insolvency regimes as a con-cause that raised the share of non-performing loans. In our model, this effect is captured by the liquidation cost μ. From equation (53) it is easy to see that a fall in μ reduces the share of incumbents. According to our numerical calculation a 90% fall in μ lowers $\frac{INC^{NP}}{\eta}$ by approximately 0.89 percentage points. The reduction in μ implies an initial saving in bankruptcy costs which is less than 0.2%, but the effect on firms efficiency is very strong, in the new steady state the total number of firms falls by 2.8%, and output per firm increases by 2.71%. Furthermore, investment falls by 3.76%, implying that firms efficiency on average increases. Total output falls, but consumption increases.

Let us now consider the effect of a fall in the fixed cost ϕ^{I}. Generalized reductions in fixed production costs are often interpreted as the consequence of market deregulations (Égert and Gal, 2018). Here we show that 1% fall in ϕ^{I} raises output, consumption and investment by about 43%. The firm turnover rate is not affected, but the number of firms increases by 10%. This, in turn, entails a fall in output per firm of approximately the same size. This should be hardly surprising, in our model intermediate goods producers are fully competitive, and the reduction in fixed costs implies that less efficient firms will survive in the market. What is perhaps less obvious is that the fall in ϕ^{I} is associated with a large increase in the share of non-performing incumbents, +9.72%.

From conditions (52), (51) and (53) we know that the reduction in ϕ^{I} has no effect on the incumbents share but, by lowering the cutoff $(\frac{\widetilde{\widehat{A}^{I}}}{\widetilde{\hat{A}^{I,P}}})$, it does increase the share of non-performing incumbents. This latter effect requires some discussion. The variation in ϕ^{I} has a twofold positive effect on both $\widetilde{\hat{A}^{I,P}}$ and $\widetilde{\hat{A}^{I}}$. First, there is a direct effect on profitability. Second, as shown in (54), the lower ϕ^I reduces borrowing costs and this contributes to pushing down the productivity cut-offs $\widetilde{\hat{A}^{I}}$ and $\widetilde{\hat{A}^{I,P}}$, but the response of $\widetilde{\hat{A}^{I}}$ is unambiguously stronger.

4 Numerical simulations

In this section we discuss a set of numerical simulations. First, we analyse IRFs to the different shocks discussed above, also considering the effectiveness of alternative monetary policy regimes. Then we consider the potential effects of making the banks moral hazard problem endogenous to the number of non-performing incumbents.

4.1 Financial Crisis Experiment

Our model differs from GK in several dimensions, i.e. firms heterogeneity and entry/exit flows, inefficient allocation of capital across incumbents, and debt renegotiation between commercial banks and a subset of firms. To shed light on the specific role that debt forbearance plays in determining our results, we compare our results (NPL model) with GK and with those obtained in two alternative models where: i) incumbent firms can optimally choose their capital stock after shock realizations are observed, so that the allocation of capital is efficient and there is no incentive to debt re-negotiation (EA); ii) bank loans are predetermined to shock realizations, but debt re-negotiation is artificially forbidden, so all non-profitable firms are forced to exit the market (No_NPL model). We consider a negative capital quality shock (equation (9)) and maintain that the Taylor rule is based on pure inflation targeting ($\kappa^y = 0$). From Figure 4, it is easy to see that the NPL and GK models exhibit a similar pattern in the output and investment dynamics, but in the NPL model the amplification effect is unambiguously stronger and more persistent. In addition, we observe a stronger fall in inflation. The different dynamic patterns observed for the price of intermediate goods suggest that this latter result is essentially due to the dampening effect that decreasing returns to scale have on marginal costs.

From equation (10), it is clear that the fall in the price of intermediate goods reduces profitability for all intermediate goods producers. The flow of new entrants immediately shrinks (Figure 5), but we do not observe a symmetrical adjustment in exit rates. In fact, exit rates immediately contract. In our model defaults are driven by condition (30). The fall in p_t^m and in the market valuation of capital, Q_t, unambiguously push up the cut-off $\hat{A}_t^I$, potentially raising exit rates, but these effects are dominated by the fall in factor prices that works in the opposite direction. Note that the same effects determine the productivity cutoff for profitable incumbents, $\hat{A}_t^{I,P}$, and the co-movements between the cut-offs $\hat{A}_t^I$ and $\hat{A}_t^{I,P}$ in equation (33) are crucial to identify dynamics in the number of non-performing

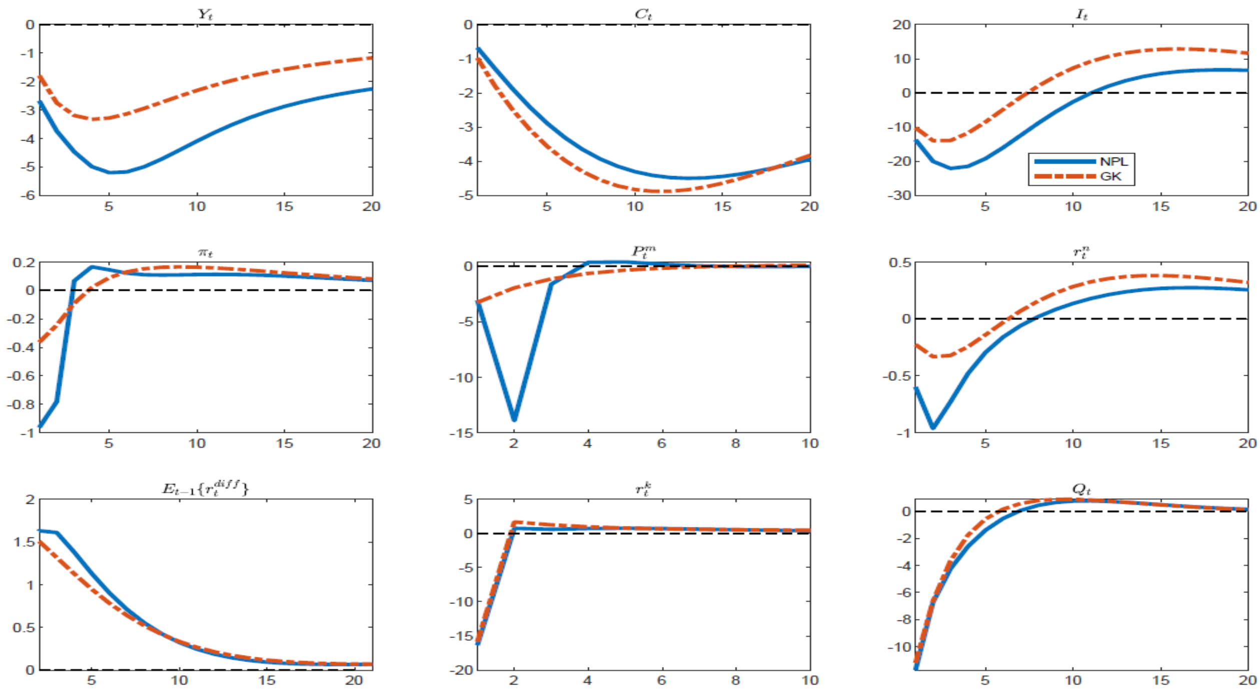


Figure 4: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state.

firms, INC_t^{NP}. In fact the fraction of non-performing firms exhibits a sharp and prolonged increase.

Figure 5 allows us to draw a comparison between the NPL, No_NPL and EA. The output contraction is deeper in the NPL and the No_NPL models, mainly due to the stronger fall in consumption. This latter effect is driven by the differences in the real rates on deposits which, due to the interest rate rule, closely follow inflation differentials. The NPL-No_NPL models are initially characterized by a severely deflationary outcome, then inflation quickly rebounds and remains for several quarters above the level predicted by the EA model. Note that the output contraction is marginally stronger in the No_NPL model, suggesting that debt renegotiations provide a minimal recession-dampening effect.

To rationalize the specific pattern of the EA model, one should focus on the Incumbent firms cut-offs $\hat{A}^I$. The η_{t-1} firms are not saddled with predetermined capital when the shock hits the economy, and the cut-off $\hat{A}_t^I$ immediately increases because profitability conditions deteriorate with the fall in p_t^m. In subsequent periods, $\hat{A}^I$ falls as both p^m and the prices of factor inputs return to steady state equilibrium.

The increase in incumbent firms efficiency, and the efficient capital allocation, unambiguously reduce inflationary pressures relative to what we observe for the NPL-No_NPL models, where the η_{t-1} firms cannot adjust their capital stock. In this case, capital reallocation to more productive incumbent firms does not occur and the immediate outcome is that the

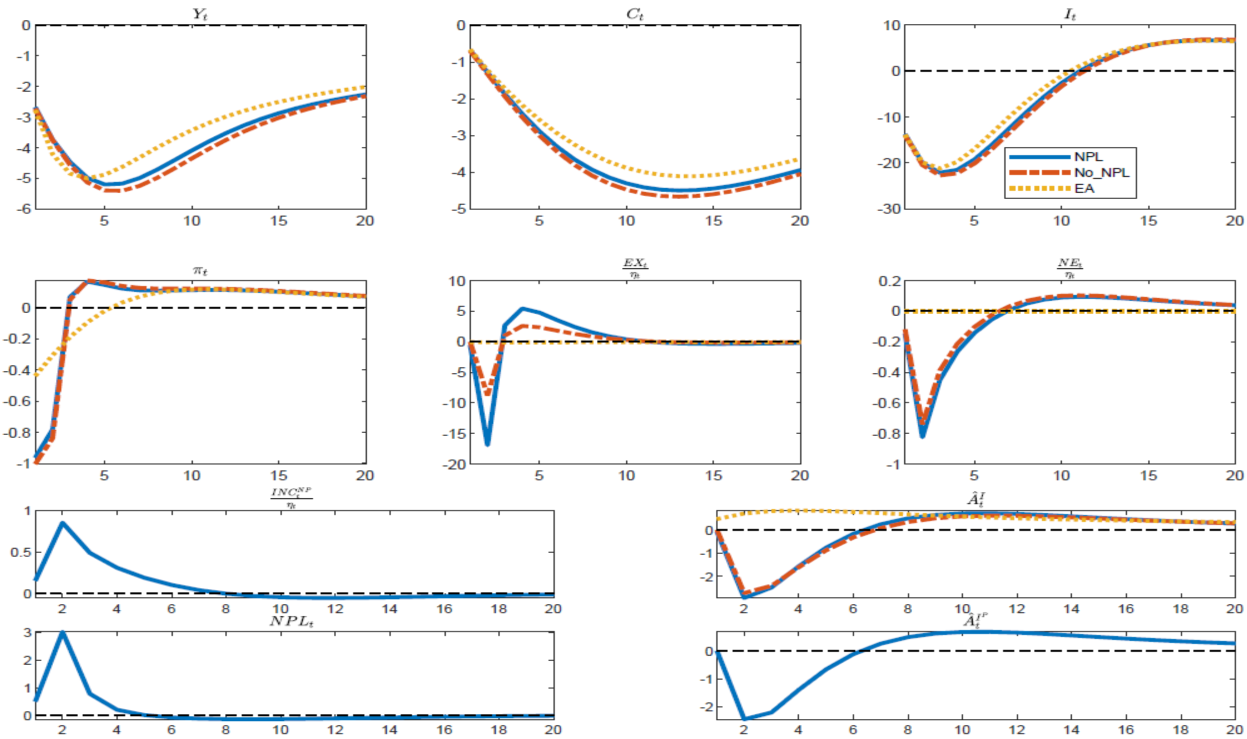


Figure 5: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state.

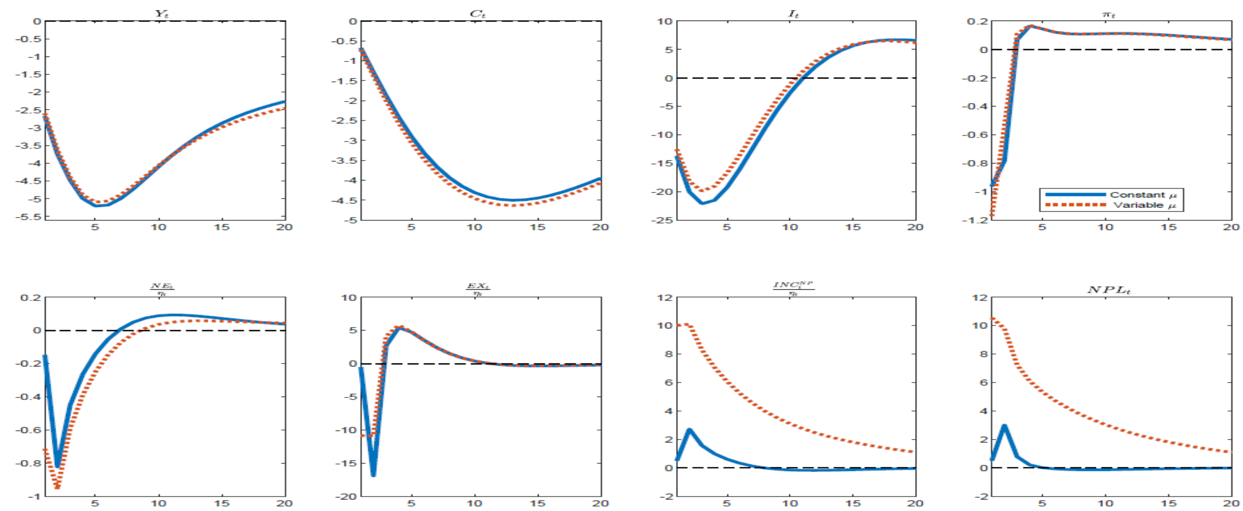


Figure 6: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state.

rental price r_t^k collapses. This, in turn, facilitates the survival of less productive firms. In the first few periods, total production is similar across the different models, but in the EA model production is allocated to fewer, more productive firms. Over time, the inefficient allocation of factor inputs raises inflationary pressures in the NPL-No_NPL models, triggering a more contractionary monetary stance that causes the slack of demand (and production) relative to the EA model.

The EA model is characterized by complete stability of entry and exit flows in percentage of total firms. This happens because, in the absence of pre-determined capital, the choice of factor inputs is symmetrical across incumbents and new entrants.[14] The NPL and the No_NPL models are characterized by an initial contraction in exit rates, which is stronger for the NPL model. After a few quarters, exit rates rebound and become positive, an effect which is larger for the NPL model. The different exit rate patterns detected for the NPL and No_NPL models are obviously due to the initial surge in the number of non-performing incumbents.

In the NPL model, debt renegotiations do not seem to cause any significant congestion effect on firm entry relative to the No_NPL model. By contrast, allowing for congestion effects in liquidation costs does have important implications (Figure 6). We observe a large and persistent increase in the share on non-performing incumbents (and loans). This is associated with milder contractions in output and investment, followed by less favourable paths during the recovery. The stronger initial fall in inflation suggests that the large increase in the number of INC_t^{NP} firms does cause a supply congestion that deters entry, especially in the initial phase of the crisis.

4.2 Technology Shocks

The technology shock is modelled as a rightward shift of the potential *NE*s' pdf due to the impact of ε_t^z on e_t(see conditions (11) and (12)).

For any given entry threshold, $\hat{A}_t^{NE}$, this causes an inflow of a larger mass of more productive *NE*s in the market (see condition (19)). The shock has been normalized to generate a 7.14% permanent increase in the de-trended value of Y. In Figure 7 we show that the economy is characterized by an initial consumption boom and by a contraction in investment. Consumption, output and investment follow an inverse hump-shaped pattern, which is driven by the interest rate response to the surge in inflation. In spite of the increase in demand, the persistent surge in the flow of *NE*s triggers a process of creative destruction. In fact, incumbent firms are confronted with a price of intermediate goods that falls relative to the cost of the production bundle Z_t. As a result, they are forced to restore competitiveness by scaling down production. The number of profitable incumbents inevitably falls, and both defaults and non-performing incumbents increase.

[14] See the Online Appendix for a proof.

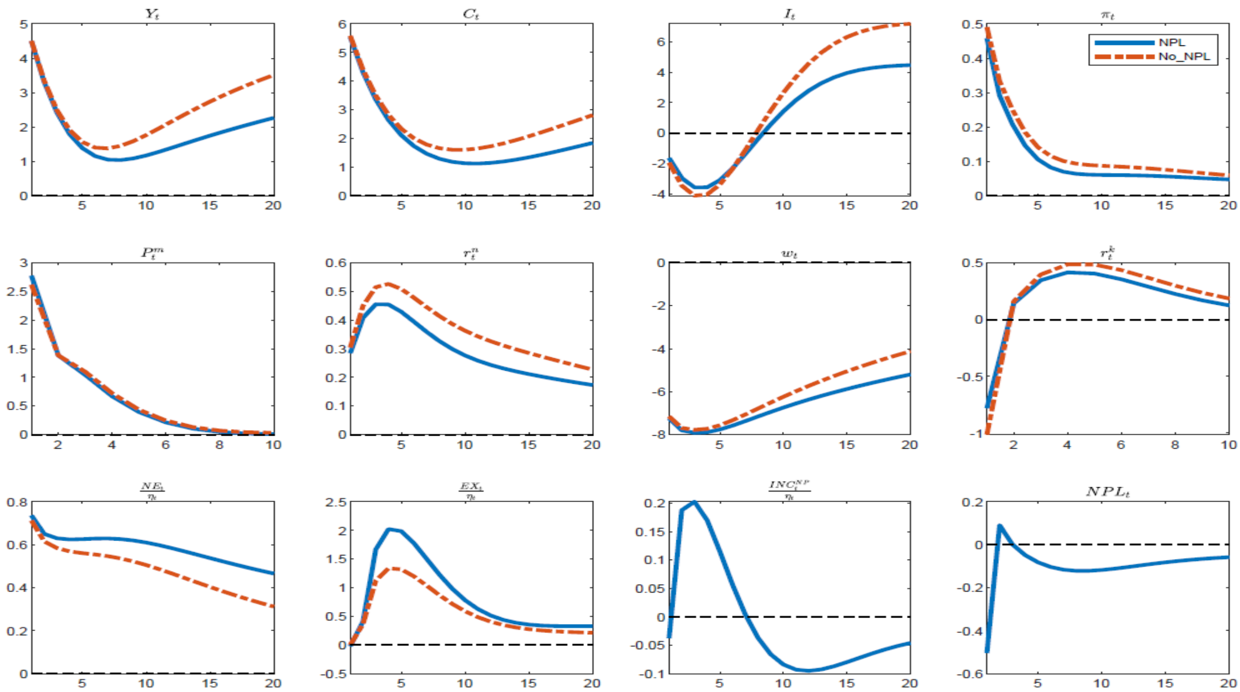


Figure 7: Quarterly responses to an expansionary 5% permanent technology innovation shock in percentage deviations from steady state.

The No_NPL model is characterized by a faster convergence to the new steady state. Furthermore, we observe a milder increase in the number of exits. This happens for a simple reason. The two models in steady state are characterized by identical calibrations for the rental price of capital and for the exit rate but in the No_NPL model the incumbents' productivity cutoff is unambiguously higher because all incumbents are able to service their debt. This in turn implies that, relative to the benchmark model, the incumbent firms cutoff is less sensitive to the adverse cost dynamics described above.

These latter effects dominate, and for this reason, entry and exit rates increase, and we observe a sharp contraction in the share of both INC^{NP} and NPLs. These latter effects obtain because the cutoff associated with the total number of incumbents outperforms the increase in the cutoff of profitable firms.

4.3 Liquidity shocks

Shocks to discount rates do temporarily affect firms' profitability and the price of capital, which could have effects on the firm distribution. The issue is important because influential studies such as (Del Negro et al., 2017) found that a sequence of liquidity shocks caused the persistent decline in the natural and in the policy rates.[15] We mimick their liquidity shock with the risk-premium shock ς_t in condition (6). Figure 8 reports IRFs to a 0.1% shock. The

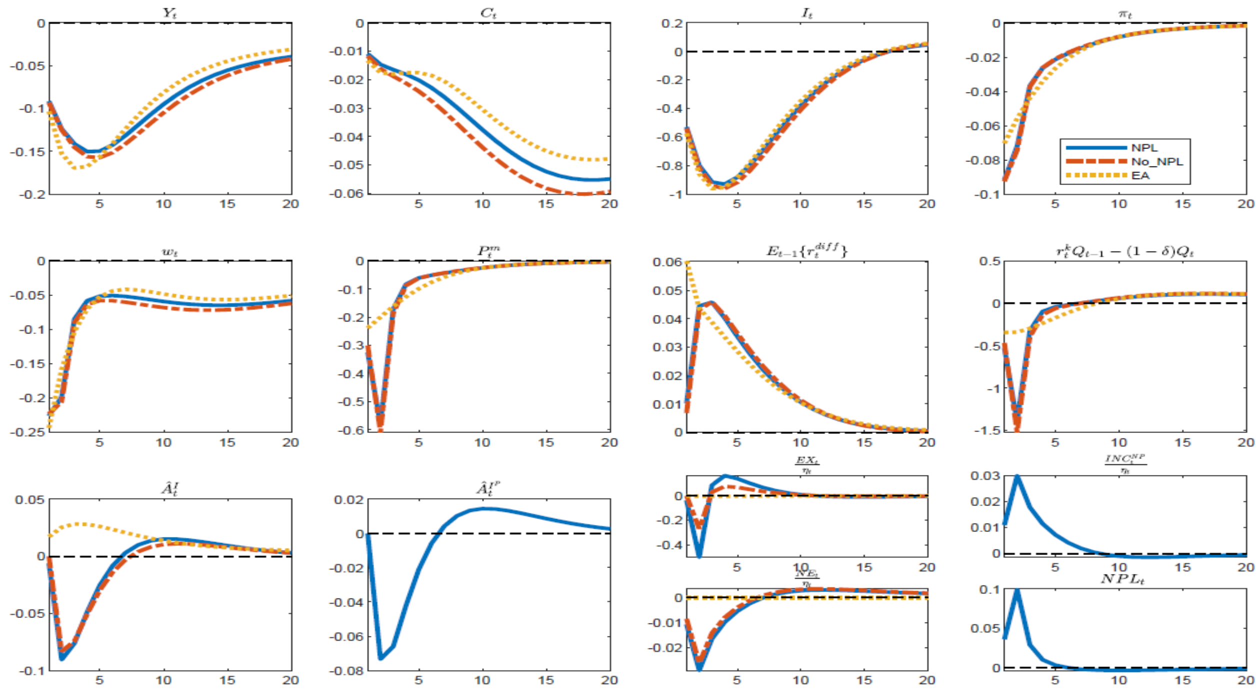


Figure 8: Quarterly responses to a negative 0.1% liquidity shock in percentage deviations from steady state.

shock has standard contractionary effects on both consumption and investment. The banker's moral hazard friction causes an increase in the loan-deposit return spread, dampening the stabilizing role of the countercyclical monetary policy. In spite of the output contraction, the efficiency threshold $\hat{A}^{I^P}_t$ falls because the reduction in the intermediate goods price p^m_t has a smaller impact than the corresponding reductions in the cost of predetermined capital and in the real wage (see condition (28)). The sharp increase in the production value of the liquidation cost, $\frac{\mu_t}{p^m_t}$, explains why banks are incentivized to renegotiate their loans, so that $\hat{A}^I_t$ falls with respect to $\hat{A}^{I^P}_t$. The share of non-performing incumbents increases, and the exit rate falls. The entry rate falls as well and comparison with the entry rate dynamics observed under the EA provides illuminating evidence of the congestion effect associated with the predetermined capital friction. In fact, under EA the thresholds $\hat{A}^I_t$ and $\hat{A}^{NE}_t$ increase symmetrically and neither the entry nor the exit rate react to the shock. By contrast, the predetermined capital friction reverses the response of $\hat{A}^I_t$, causing a more severe contraction in p^m_t and a fall in the entry rate. Relative to the ERA model, the possibility of loans

15. Barsky et al. (2014) and Neri and Gerali (2019) also find that risk premium shocks caused large fluctuations in the natural rate.

renegotiations has a marginal effect on the entry rate but significantly increases the fall in the exit rate.

4.4 Monetary Policies

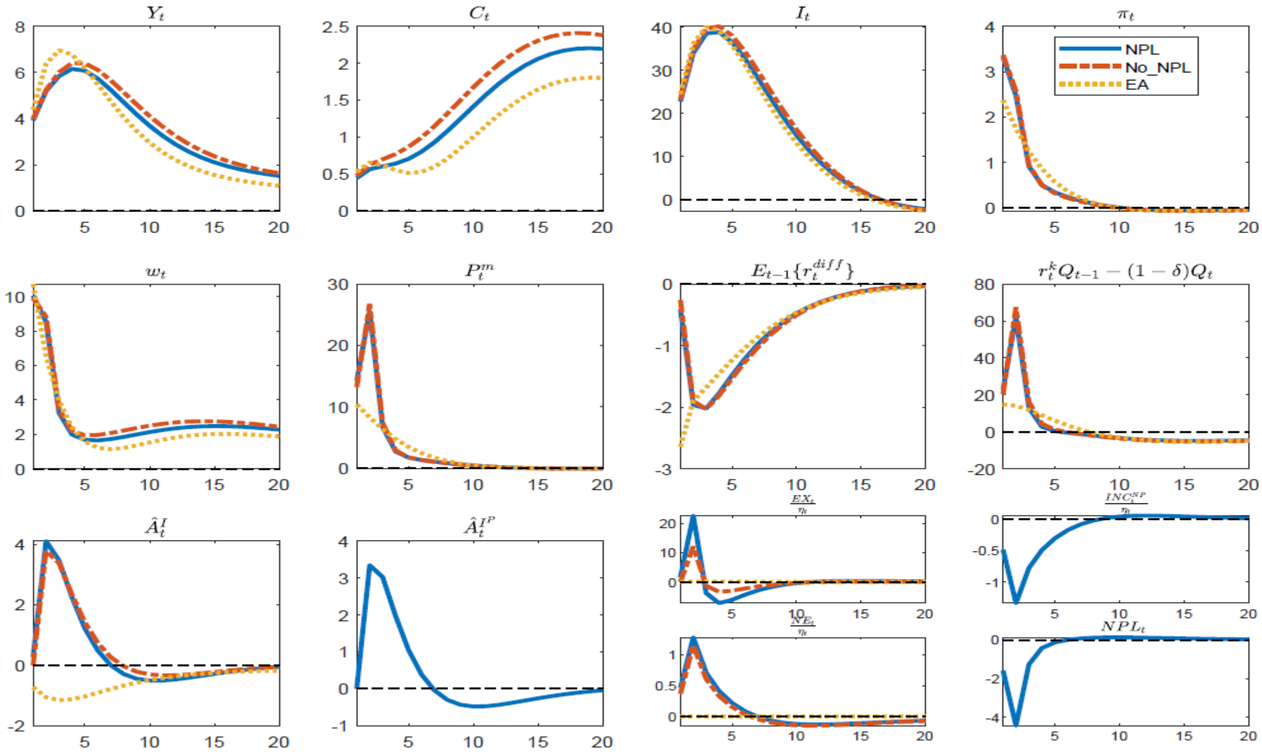


Figure 9: Quarterly responses to an expansionary 5% monetary policy shock in percentage deviations from steady state.

In this section, we address one fundamental question, concerning the effects of monetary policies on endogenous firm dynamics. Are expansionary/accommodative policies a hindrance to growth because they limit creative destruction stemming from *NE*s flows?

To begin with, consider the implications of a negative interest rate shock (Figure 9). The output expansion is associated with a sharp increase in the incumbent firms cutoff, $\hat{A}_t^I$. Note that the output expansion is associated with an increase both in the price of intermediate goods and in the shadow price of capital. These two variations would bring down the $\hat{A}_t^I$, allowing survival of relatively less productive firms. By contrast, both the real wage and the rental price of capital increase. These latter effects dominate[16], and for this reason, we observe the increase in $\hat{A}_t^I$ (Figure 9). Entry and exit rates increase, and we observe a sharp

[16] We also experimented with sticky nominal wages, whose initial response to the shock is milder. At standard values of the wage Calvo parameter our results are fully confirmed.

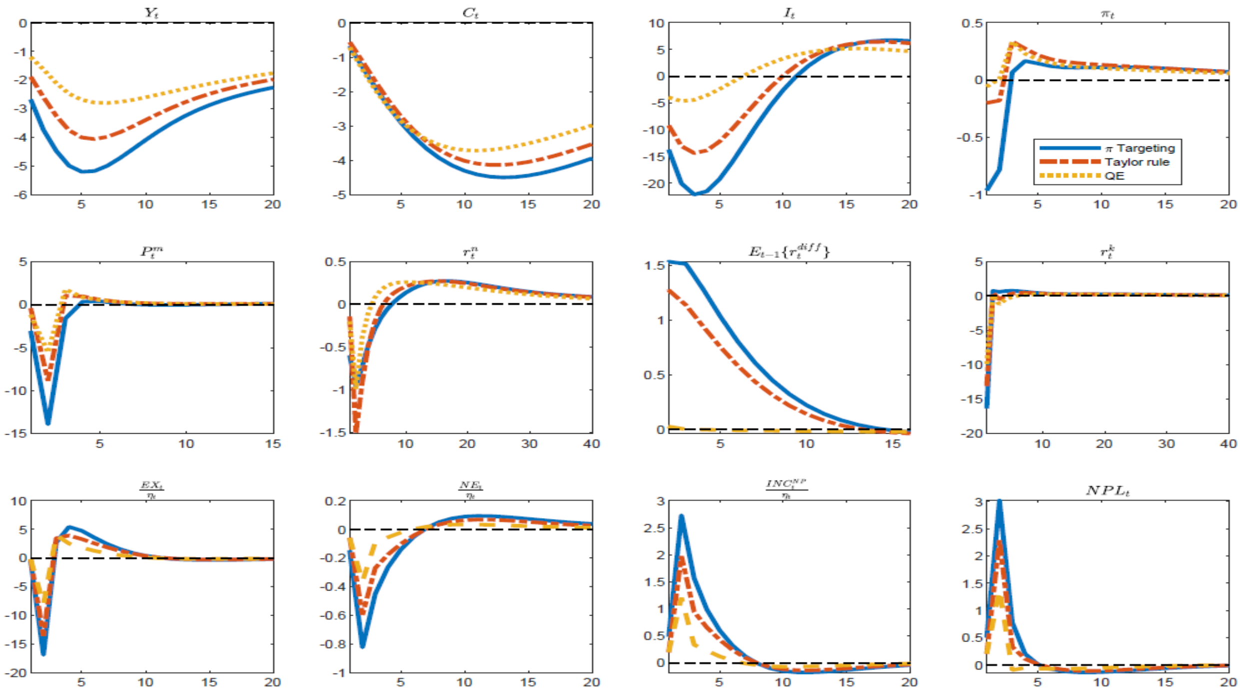


Figure 10: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state with different monetary policy rules.

contraction in the share of both INC^{NP} and NPLs. These latter effects obtain because the cutoff associated with the total number of incumbents outperforms the increase in the cutoff of profitable firms.

The shock has stronger expansionary effects in the No_NPL version of the model.[17] The two models exhibit identical entry flows, but the exit rate is unambiguously stronger in the NPL model.

The next step is the comparison of three policy regimes under a common capital quality shock: i) pure inflation targeting; ii) a standard Taylor rule as in (48); iii) A Taylor rule supplemented by the quantitative easing policy rule (49). Results are depicted in Figure 10. The more accommodative the monetary stance, the milder the contraction in output, consumption and investment. Similar results obtain for entry and exit flows. The increases in the number of INC_t^{NP} and in the share of non-performing loans is also inversely related to the strength of the monetary accommodation. Our results therefore contradict Banerjee and Hofmann (2018) and Acharya et al. (2019) because (temporarily) looser monetary policy that boosts demand actually puts more pressure on incumbents.

[17]The magnitude of IRFs responses is almost identical to those obtained under a standard GK model. Results available upon request.

4.5 Capital misallocation

Our purpose here is to assess how business cycle fluctuations affect the allocation of capital. We plot the IRFs for condition (43) in response to the shocks that hit the economy (Figure 11). Resuts have a simple and straightforward interpretation: our index of misallocation increases in contractionary episodes and *vice versa*. This happens because demand for predetermined capital is correlated with the expected production of the profitable incumbent (see condition (24)). Loans renegotiations and the variation of non-performing loans are unconsequential for the capital misallocation response to shocks. Finally, it is interesting to note that the volatility of misallocation is substantially reduced if the Central Bank implements QE policies.

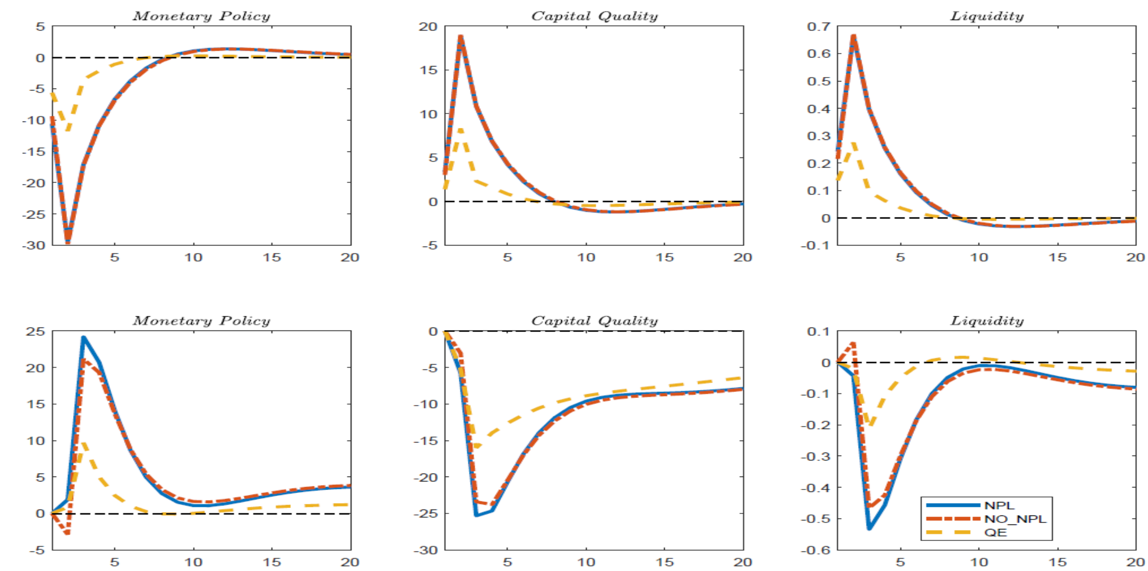


Figure 11: Quarterly responses of capital misallocation and predetermined capital demand to selected shocks in percentage deviations from steady state.

4.6 Non-Performing Loans and the Bankers' Moral Hazard Problem

In this section, we run a financial crisis experiment under the assumption that the increase in the number of non-performing firms worsens the moral hazard problem of banks, as discussed in section 2.4.

4.6.1 Financial Crisis

We run a financial crisis experiment using a negative capital quality shock and compare this new endogenous moral hazard model with the NPL model (Figure 12). Endogenous moral hazard clearly amplifies the effect of a negative shock to capital quality. We observe a more severe GDP contraction and a decrease in investment which is 50% larger. The magnitude

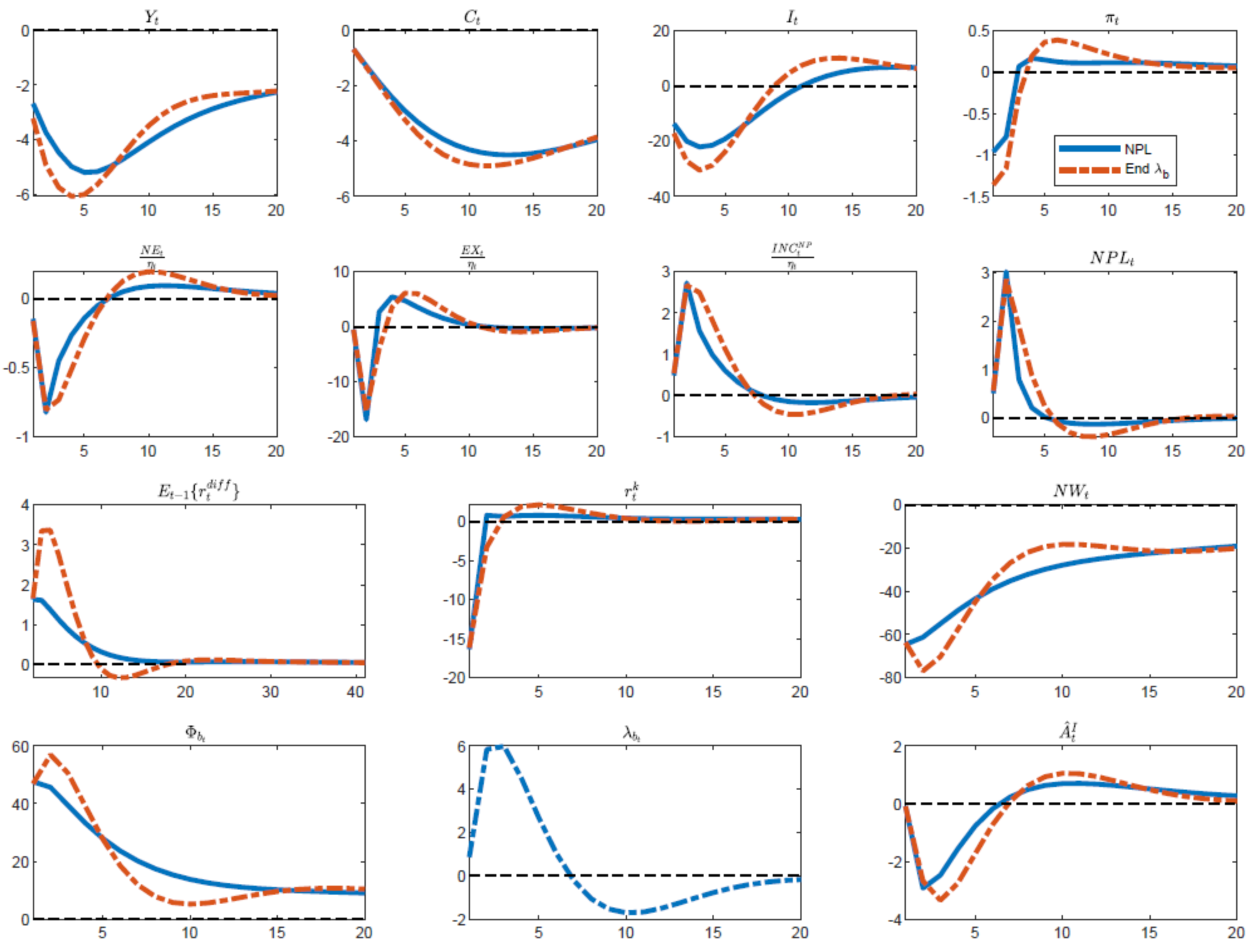


Figure 12: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state.

of the recession is driven by the spread between loans and deposit rates increases, whose increase doubles the one obtained in the benchmark NPL model.

4.6.2 Monetary Policy and Moral Hazard

Given the relevance of the endogenous moral hazard effect, we want to understand the effectiveness of less restrictive monetary policy (i.e. Quantitative easing) in this scenario. As shown in Figure 10, accommodating monetary policies can help to reduce the losses of a financial crisis if compared with inflation targeting monetary policies.

The simulation in Figure 13 shows that the unconventional monetary policy can drastically reduce output losses. In fact, the Central Bank is very effective in limiting the loan rate spread and the response of the model is almost comparable to the one obtained with the one obtained in the benchmark NPL model.

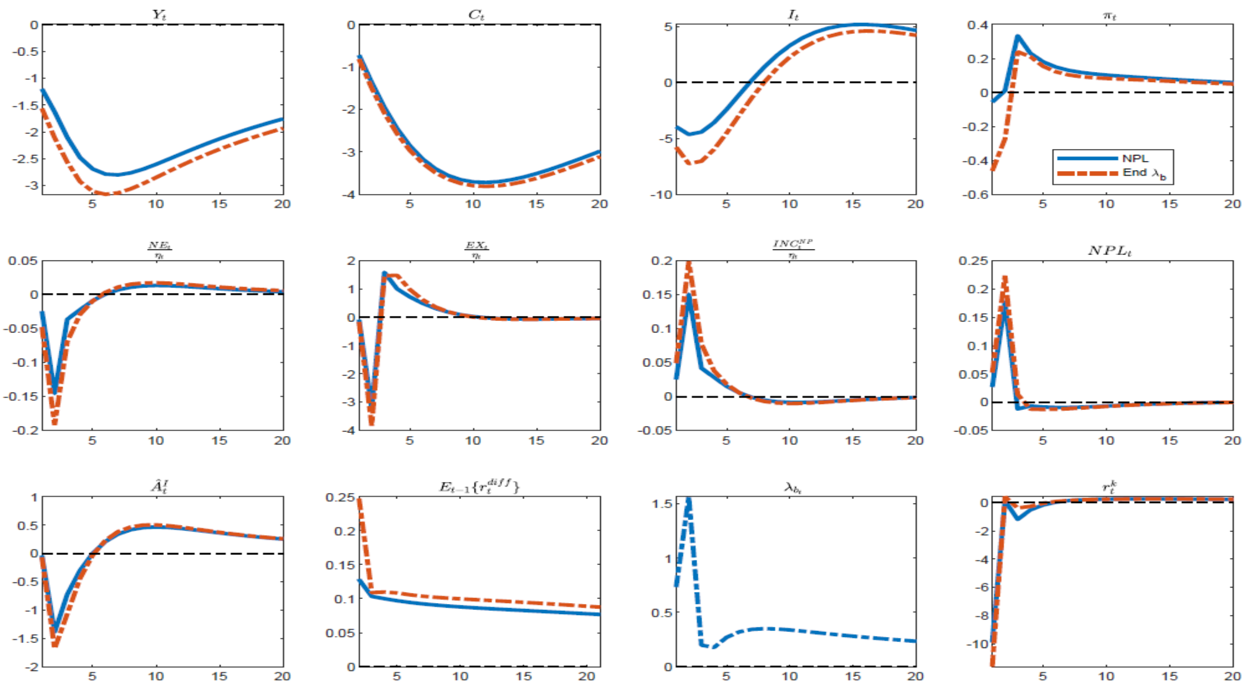


Figure 13: Quarterly responses to a negative 5% capital quality shock in percentage deviations from steady state under Quantitative Easing monetary policy.

5 Conclusions

Our models provide new insights on the causes and consequences of non-performing loans in a DSGE model. We essentially downplay the specific role of debt re-negotiations in specific financial crisis episodes, but our models predict that economies characterized by a larger share of non-performing loans find it more difficult to reap the benefits of technology shocks.

Debt renegotiations and non-performing loans can lead to a dramatically worse macroeconomic performance if they add opacity to the balance sheet of commercial banks, worsening their moral hazard problem. Accommodative monetary policies should not be blamed for encouraging excessive bank leniency on less efficient firms. Quantitative easing policies are shown to have a strong and favourable impact on macroeconomic performance, on entry rates, on the profitability of commercial banks in the aftermath of a financial crisis.

We cannot confirm the view that relates looser financial pressures to the conspicuous increase in the share of non-performing loans observed over the last three decades. According to our results, the phenomenon could be explained by the pace of technological innovation, which causes firms exit rates, by market deregulations that lower fixed production costs, and liquidation costs. Future empirical research could put our findings at test.

Our results concerning the relatively favourable outcomes obtained in the ERA model suggest that future research should also investigate the design of efficient risk-sharing schemes among intermediate goods producers, which could alleviate the adverse effects caused by predetermined capital allocation.

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