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Monetary Policy in the Euro Area: Active or Passive?*

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Abstract

We estimate a medium-scale DSGE model for the Euro Area allowing and testing for indeterminacy since the introduction of the euro until mid-2023. Our estimates suggest that monetary policy in the euro area was passive, leading to indeterminacy and self-fulfilling dynamics. Indeterminacy dramatically alters the transmission of fundamental shocks, particularly for inflation whose responses are inconsistent with standard economic theory. Inflation increases following a positive supply or a negative demand shock. Consequently, demand shocks look like supply shocks and vice versa, making the dynamics of the model under indeterminacy challenging to interpret. However, this finding is not robust across different assumptions on the way the sunspot shock is specified in the estimation. Both under determinacy and indeterminacy, the model estimates a natural rate of interest that turned positive after the recent inflation episode.

Keywords: monetary policy, indeterminacy, euro area, business cycle fluctuations, inflation

JEL classification: E32, E52, C11, C13

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1 Introduction

The establishment of the European Central Bank (ECB) in 1998 unified monetary policy in the Euro Area. Since then, the ECB faced a series of large and diverse shocks, including the Great Financial Crisis, sovereign debt crisis, that triggered a prolonged period of near-zero interest rates, and recent events like the pandemic and surging energy prices. Each of these events posed enormous challenges to the ECB's monetary policy, that raise the question of whether the ECB set monetary policy according to its own primary objectives in response to these challenges.

In this paper, we estimate a medium-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model for the euro area allowing and testing for (in)determinacy since the introduction of the euro until mid-2023. The model includes the typical frictions of New Keynesian models, in line with Christiano et al. (2005). On top of this, we add energy as an input in consumption and in production, following Blanchard and Galí (2007), to take into account the recent inflation surge in the very last part of the sample and the dynamics of different inflation measures, i.e., headline HICP, core, GDP deflator.[1]

The focus of our analysis is to assess the ECB monetary policy related to its unique goal: the stabilization of inflation. With the lens of the model, the central bank follows a Taylor type monetary policy rule. In an inflation targeting framework, this necessitates an active monetary policy, reacting more than proportionally to inflation, to control it and prevent self-fulfilling unanchored inflation dynamics. Hence, an essential aspect of our analysis is to allow for the possibility of indeterminacy of rational expectations equilibrium. While most studies on the Euro Area assume active monetary policy and the so-called Taylor Principle, it is well-known

[1]We decided to include a role of energy and not to include financial frictions as an explanation or amplification mechanism of the Great Financial Crisis. Generally, including financial frictions does not significantly enhance the accuracy of the widely used New Keynesian model, such as Smets and Wouters (2007), see e.g., Brzoza-Brzezina and Kolasa (2013), Suh and Walker (2016) and Lindé et al. (2016)

that monetary policy rules can lead to equilibrium indeterminacy and self-fulfilling expectations. The literature on monetary policy in the U.S. debated about indeterminacy as a possible explanation for the inflation surge in the '70s in the U.S. after the oil shocks (e.g., Clarida et al., 2000, Lubik and Schorfheide, 2004, Ascari et al., 2019) . Hirose (2013) estimates a small-scale two country model for the U.S. and the Euro Area (1983Q1-2002Q4). He finds that the data point to a passive monetary policy for the euro area during this period. Haque et al. (2021) also find support for determinacy in the pre-Volcker period, albeit for different reasons. More recently, Nicolò (2023) estimate Smets and Wouters (2007) model to investigate monetary policy stance in the U.S. in the post-war period, allowing for indeterminacy. Similarly, Albonico et al. (2023) estimate a New Keynesian model with rule of-thumb agents and find support for indeterminacy in the pre-Volker period. Our sample does not contain the '70s, but it contains the Great Financial Crisis and the sovereign debt crisis in the Euro Area where concerns could arise about the possibility of monetary policy being characterized by a passive behavior, because of the zero lower bound constraint and a dominant active fiscal policy. Moreover, the recent inflation surge in the Euro Area also could have been possibly amplified by passive monetary policy.

From a methodological perspective, after the seminal contribution by Lubik and Schorfheide (2003, 2004), more recently, Farmer et al. (2015) and Bianchi and Nicolò (2021) proposed new techniques to estimate a DSGE model under indeterminacy. As in the recent contributions by Nicolò (2023) and Albonico et al. (2023), we employ the Bianchi and Nicolò (2021) methodology to estimate our model and assess the possible role of equilibrium indeterminacy in the Euro Area using post-ECB data. To the best of our knowledge, we are the first to estimate a medium scale model for the Euro Area allowing for indeterminacy.

Our findings are as follows. First, we find that monetary policy in the Euro

Area was passive in our sample. The response of the nominal interest rate is estimated less than proportional to headline inflation changes from target.[2] Hence, the data prefer a specification of the model that implies indeterminacy and self-fulfilling business cycle fluctuations driven by a sunspot shock. Second, this finding is not robust to some technical choices, more specifically; (i) which variable forecast error, together with the sunspot, enters in the auxiliary variable specification in the Bianchi and Nicolò (2021) methodology and (ii) allowing or not for correlation between the sunspot shock and the fundamental shocks. Third, sunspot shocks and self-fulling expectations significantly alter the propagation of the fundamental shocks in our model economy, and notably the inflation responses. Specifically, under the indeterminacy specification preferred by the data, the responses of inflation to the fundamental shocks are at odds with standard economic theory: inflation increases after a positive supply or a negative demand shock. It follows that fundamental demand shocks, such as monetary policy or risk premia shocks, induce a supply-like economic response, implying negative comovement between inflation and output. Similarly, fundamental supply shocks, like technology or labor supply shocks, induce a demand-like economic response, implying positive comovement between inflation and output. Fourth, while under determinacy inflation is mostly supply-driven, under indeterminacy inflation is mostly demand-driven. Finally, the behavior of the natural interest rate and the output gap is similar between determinacy and indeterminacy, and both specification imply that the natural rate of interest entered in positive (restrictive) territory in the recent period characterized by the increase in inflation. On the one hand, these results are particularly significant, given that the majority, if not all, the models designed for monetary policy analysis in the Euro Area rely on the standard assumption of an active monetary

[2] We do not explicitly consider unconventional monetary policy. However, we use a shadow rate measure for the short run nominal interest rate to take into account the ZLB period, and thus, implicitly the effects of unconventional monetary interventions.

policy. Once one allows for indeterminacy, instead, the data select a model that implies a very different inflation dynamics, rendering it impossible for monetary policy to fulfill its inflation stabilizing role.

On the other hand, the implied responses of the main variables to the fundamental shocks, particularly the inflation ones, are rather peculiar. The specific rational expectation equilibrium selected by the estimation, among the infinitely many admissible one, is impossible to square with basic economic theory. Additionally, the preference for an indeterminacy specification by the data lacks robustness with respect to certain technical details of the employed methodology. Hence, there may be reasons to be skeptical about the indeterminacy results. We refrain from taking a stance and leave the readers to form their own opinion: determinacy lies in the eye of the beholder.

The paper is organized as follows. Section 2 presents the model. Section 3 explains the estimation strategy based on Bianchi and Nicolò (2021). Section 4 provides the main results, while Section 5 checks for the robustness of our findings. Finally, Section 6 concludes.

2 Model

We develop a Dynamic Stochastic General Equilibrium (DSGE) model following Smets and Wouters (2003, 2007). The model includes all the standard features and frictions which are typical of New-Keynesian medium scale models: habits in consumption, variable capital utilization, investment adjustment costs, sticky prices, indexation on past and trend inflation and real wage rigidity. We deviate from Smets and Wouters' framework in some respects. First, we consider a small open economy setup, in the vein of Galí and Monacelli (2005), with imperfect financial integration (Schmitt-Grohé and Uribe, 2003, Lindé et al., 2004). Second, and most importantly, we introduce energy both as an input in consumption and as an input in production, on the footsteps of Blanchard and Galí (2007). We assume that the country (the euro area) is an energy importer, and that the real price of energy (in terms of domestic goods) follows an exogenous process. In what follows an '$*$' is attached to foreign variables.

2.1 Households

There is a continuum of households indexed by $i \in [0,1]$. Households choose how much to consume and how much to work maximizing their utility function, which is defined as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} \left(c_t^i - b c_{t-1} \right)^{1-\sigma} - \varepsilon_t^l \frac{(h_t)^{1+\phi_l}}{1+\phi_l} \right\}, \tag{1}$$

where individual and aggregate consumption (c_t^i, c_t) are adjusted by the deterministic growth trend g_z, h_t stands for individual hours worked, $0 < \beta < 1$ is the subjective discount factor, σ measures the inverse of the intertemporal elasticity of substitution and ϕ_l is the inverse of Frisch elasticity. The parameter $0 < b < 1$ measures the degree of external habits in consumption. ε_t^l is a shock to the labor supply.

Households allocate their resources between consumption C_t, investments I_t, domestic government-issued bonds B_t and foreign assets B_t^*. They receive income from labor services $W_t h_t$, from dividends D_t, from renting capital services $u_t K_t$ at the rate R_t^k and from holding domestic bonds and foreign assets. The budget constraint is:

$$\begin{aligned} P_{C,t} \left(C_t + I_t \right) + \frac{B_{t+1}}{\varepsilon_t^b} + ER_t \Gamma_t \left(\bar{B}_{t+1}^* \right) B_{t+1}^* = R_{t-1} B_t + ER_t R_{t-1}^* B_t^* + W_t h_t + D_t \\ + \left[R_t^k u_t - a \left(u_t \right) P_{I,t} \right] K_t - T_t. \end{aligned} \tag{2}$$

$P_{C,t}$ is the domestic consumer price index,[3] R_t is the domestic gross nominal interest rate, K_t is the physical capital stock and u_t defines capital utilization. T_t are lump-sum taxes. $\bar{B}_t^*$ are aggregate foreign assets and R_t^* is the foreign gross nominal interest rate. Then, in equilibrium $\bar{B}_t^* = B_t^*$. The term $\Gamma_t\left(\bar{B}_{t+1}^*\right)$ is a premium on foreign asset holdings, which depends on the real aggregate net foreign asset position of the domestic economy. When the domestic economy is a net borrower, households face a premium on foreign interest rates, whereas when it is a net lender, they receive reduced returns on their international savings. This translates to higher domestic interest rates relative to foreign ones in the first case (net borrower), even in the absence of anticipated exchange rate depreciation. Conversely, when the domestic economy is a net lender, domestic interest rates are lower than those abroad. Consequently, fluctuations in the net foreign asset position directly influence the interest rate differentials between domestic and foreign economies. ER_t defines the nominal exchange rate. ε_t^b is a risk premium shock that affects the intertemporal margin, creating a wedge between the interest rate controlled by the central bank and the return on assets held by the households. The capital accumulation equation is:

$$K_{t+1} = (1-\delta)\,K_t + \varepsilon_t^i\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t, \tag{3}$$

where δ is the capital depreciation rate and ε_t^i is a shock to the marginal efficiency of investment (see Justiniano et al., 2010).

We allow for real wage rigidities, following Blanchard et al. (2010) and Blanchard and Riggi (2013). These papers show that this mechanism is an important feature in relation to oil price shocks, so it seems to be relevant to include it in a model with

[3] We assume here that final goods can be used either for consumption or investments, abstracting from considerations about different pricing of the two components. Thus, we are implicitly assuming that the consumer price index is the same as the investment price index. This implies equal fractions of domestically produced consumption goods and investment goods and the same elasticity of substitution between home goods and imported energy goods. See Section 2.2 for details.

energy.[4] Instead of the standard optimality condition, where the real wage equals the marginal rate of substituion, we use the following:

$$\frac{W_t}{g_z^t P_{C,t}} = \left(\frac{W_{t-1}}{g_z^{t-1} P_{C,t-1}}\right)^{\gamma} \left[\varepsilon_t^l \left(h_t\right)^{\phi_l} \left(c_t - bc_{t-1}\right)^{\sigma}\right]^{1-\gamma}. \tag{4}$$

2.2 Optimal allocation of consumption expenditures

The overall consumption basket C_t is a CES bundle of an aggregator of domestically produced goods, $C_{q,t}$ and imported energy, $C_{m,t}$:

$$C_t \equiv \left[\varpi_c^{\frac{1}{\upsilon}} \left(C_{m,t}\right)^{\frac{\upsilon-1}{\upsilon}} + \left(1-\varpi_c\right)^{\frac{1}{\upsilon}} \left(C_{q,t}\right)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}},$$

where $1-\varpi_c$ represents the fraction of domestically produced consumption goods, υ is the elasticity of substitution between home goods and imported energy goods. The optimal allocation of consumption expenditures between imported and domestically produced goods delivers:

$$C_{m,t} = \varpi_c \left(\frac{P_{m,t}}{P_{C,t}}\right)^{-\upsilon} C_t \tag{5}$$

$$C_{q,t} = \left(1-\varpi_c\right) \left(\frac{P_{q,t}}{P_{C,t}}\right)^{-\upsilon} C_t \tag{6}$$

In turn, $C_{q,t}$ is itself a CES bundle of domestically produced goods z, so that $C_{q,t} = \left[\int_0^1 C_{q,t}\left(z\right)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}$ and the relative domestic consumer price index is:

$$P_{C,t} = \left[\varpi_c \left(P_{m,t}\right)^{1-\upsilon} + \left(1-\varpi_c\right) \left(P_{q,t}\right)^{1-\upsilon}\right]^{\frac{1}{1-\upsilon}},$$

where $P_{m,t}$ is the nominal price of energy and $P_{q,t}$ is the price index for domestic goods: $P_{q,t} = \left[\int_0^1 P_{q,t}\left(z\right)^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$.

Following Blanchard and Galí (2007), we assume that the variable $s_t = \frac{P_{m,t}}{P_{q,t}}$,

[4]See also Gagliardone and Gertler (2023) for more recent results.

that is, the price of energy in terms of the price of domestically produced goods – i.e., equal to the terms of trade in this model – follows an exogenous AR(1) process.

2.3 Production

The final good Q_t is produced under perfect competition. A continuum of intermediate inputs Q_t^z is combined as in Kimball (1995). Intermediate firms z are monopolistically competitive and use as inputs capital services, $u_t^z K_t^z$, labor services, h_t^z, and energy, M_t^z. The production technology is a CES bundle between the energy input and domestic inputs:

$$Q_t^z = \varepsilon_t^a \left\{ (1-\mu)^{\frac{1}{\epsilon}} \left[(u_t^z K_t^z)^{\alpha} (z_t h_t^z)^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \mu^{\frac{1}{\epsilon}} (M_t^z)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}} - z_t \Phi,$$

where Φ are fixed production costs. ϵ defines the elasticity of substitution between energy and the Cobb-Duglas bundle of capital and labor. When $\epsilon = 1$ this formulation gives the standard three inputs Cobb-Douglas production function. ε_t^a is a temporary total factor productivity shock. The term z_t represents labor augmenting technological progress and grows deterministically at a rate g_z.

Domestic prices are sticky following the Calvo (1983) mechanism. Intermediate goods are packed by final firms with the Kimball (1995) aggregator.[5]

Cost minimization implies that energy demand is:

$$M_t^z = \mu \left(\varepsilon_t^a \right)^{\epsilon-1} \left(\frac{MC_t^z}{P_{m,t}} \right)^{\epsilon} \left(Q_t^z + z_t \Phi \right),$$

while from the labor demand and the capital demand schedules, we obtain an ex-

[5]See the Appendix for more details.

pression for capital to labor services:

$$\frac{u_t^z K_t^z}{h_t^z} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}.$$

Finally, the marginal cost is constant across firms and equal to:

$$MC_t^z = (\varepsilon_t^a)^{-1} \left[(1-\mu) \left(\alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} (z_t)^{-(1-\alpha)} \left(R_t^k\right)^{\alpha} (W_t)^{1-\alpha} \right)^{1-\epsilon} + \mu \left(P_{m,t}\right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

2.4 Government

The government budget constraint is:

$$P_{q,t} G_t + R_{t-1} B_t = B_{t+1} + T_t. \tag{7}$$

We assume that it is balanced every period. G_t is government spending, which evolves exogenously.

The monetary authority sets the nominal interest rate according to the same Taylor rule as in Smets and Wouters (2007):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_{C,t}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{flex}}\right)^{\phi_y} \right]^{1-\phi_R} \left(\frac{Y_t / Y_{t-1}}{Y_t^{flex} / Y_{t-1}^{flex}} \right)^{\phi_{\Delta y}} \varepsilon_t^r, \tag{8}$$

where R_t is the gross nominal interest rate, $\pi_{C,t}$ is the gross CPI inflation rate, Y_t is the level of GDP and Y_t^{flex} is the level of potential GDP prevailing in a flexible prices and wages environment and ε_t^r is an exogenous interest rate shock.

2.5 Foreign block

We assume that the foreign block is mostly exogenous. In particular, foreign demand Y_t^*, foreign nominal interest rates R_t^* and foreign inflation $\pi_t^* = \frac{P_{C,t}^*}{P_{C,t-1}^*}$ are exogenous processes. The model is closed assuming foreign demand for the domestically produced good is specified as:

$$EXP_t = \left(\frac{P_{q,t}}{ER_t P^*_{C,t}} \right)^{-\eta} Y^*_t.$$

Net foreign asset position evolves according to:

$$\frac{ER_t B^*_{t+1}}{P_{C,t}} = R^*_{t-1} \frac{ER_t B^*_t}{P_{C,t}} + \frac{NX_t}{P_{C,t}},$$

and net exports are defined as:

$$NX_t = P_{q,t} EXP_t - \underbrace{P_{m,t} \left(M_t + C_{m,t} + Q^I_{m,t} \right)}_{P_{m,t} IMP_t},$$

where EXP_t and IMP_t are exports and imports, respectively.

2.6 Value added and aggregate resource constraint

Value added (GDP) is defined as:

$$P_{y,t} Y_t = P_{q,t} Q_t - P_{m,t} M_t,$$

where the GDP deflator, $P_{y,t}$, is implicitly defined by:

$$P_{q,t} \equiv \left[(1-\mu) P^{1-\epsilon}_{y,t} + \mu P^{1-\epsilon}_{m,t} \right]^{\frac{1}{1-\epsilon}}.$$

The aggregate resource constraint is:

$$P_{y,t} Y_t = P_{C,t} \left[C_t + I_t + a(u_t) K_t \right] + P_{q,t} G_t + P_{q,t} EXP_t - \underbrace{\left(P_{m,t} M_t + P_{m,t} C_{m,t} + P_{m,t} Q^I_{m,t} \right)}_{P_{m,t} IMP_t}.$$

3 Estimation strategy

3.1 Data

To estimate the model, we use Bayesian techniques and the measurement equations that relate the macroeconomic data to the endogenous variables of the model are defined as:

$$\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lEMPL_t \\ INTRATE_t \\ dlP_{C,t} \\ dlP_{m,t} \\ dlP_{y,t} \\ dlGDP_t^* \\ dlP_t^* \\ INTRATE_t^* \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{e} \\ \overline{R} \\ \overline{\pi} \\ \overline{\pi} \\ \overline{\pi} \\ \overline{\gamma} \\ \overline{\pi}^* \\ \overline{R}^* \end{bmatrix} + \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{c}_t - \widehat{c}_{t-1} \\ \widehat{i}_t - \widehat{i}_{t-1} \\ \widehat{w}_t - \widehat{w}_{t-1} \\ \widehat{e}_t \\ \widehat{R}_t \\ \widehat{\pi}_{C,t} \\ \widehat{\pi}_{q,t} + \widehat{s}_t - \widehat{s}_{t-1} \\ \widehat{\pi}_{y,t} + me_t \\ \widehat{y}_t^* - \widehat{y}_{t-1}^* \\ \widehat{\pi}_t^* \\ \widehat{R}_t^* \end{bmatrix} \quad (9)$$

where dl denotes the percentage change measured as log difference, l denotes the log, and hatted variables denote log deviations from steady state. We use nine quarterly euro area macroeconomic time series. More specifically, the series considered are: growth rate in real GDP, consumption, investment and wages, log of employment (linearly detrended)[6], the short-term interest rate, measured using Krippner's shadow rate – See Krippner (2013, 2015) –, and 3 three inflation measures. We include headline inflation rate measured by the 'All items HICP index', energy in-

[6]The Appendix provides the auxiliary equation relating observed employment to unobserved hours worked.

flation measured by ‘Energy HICP index’ and GDP deflator inflation. $\overline{\gamma}$ denotes a deterministic growth trend common to the real variables GDP, consumption, investment and wages ($\overline{\gamma} = 100\,(g_z - 1)$), $\overline{e}$ is the (log) steady-state employment (normalized to zero), $\overline{\pi}$ is the quarterly steady-state net inflation rate, and $\overline{R}$ is the quarterly steady-state net nominal interest rate. In addition to data from the euro area, we use data from the United States for foreign output growth, inflation and the interest rate to discipline the exogenous processes for the underlying foreign variables in the model. The respective series are: growth rate in real GDP, CPI inflation, and Krippner's shadow rate measure for the US. The sample period covers 1999Q1-2023Q2.

We include eight fundamental shock processes in the estimation, several of which are the same as in Smets and Wouters (2007). In particular, we include technology shock, risk premium shock, investment shock, monetary policy shock, government spending shock, price markup shock and labor supply shock. In addition, we include an energy shock captured by an $AR(1)$ process for the real price of energy (s_t). Moreover, we add a measurement error, me_t, to GDP deflator inflation. All shocks have an autoregressive component of order 1. The government spending shock is assumed to be correlated with the technology shock. Finally, the price markup shock also has a MA(1) component. Moreover, we add three foreign shocks, to foreign demand, inflation and nominal interest rates.

3.2 Calibration and Priors

We calibrate a number of parameters. In particular, the discount factor β is fixed at 0.999, corresponding to a 1.2 annual real interest rate at the prior mean. The steady-state depreciation rate δ is 0.025, corresponding to a 10% depreciation rate per year. The elasticity of the demand for goods is set at 4, which implies a 33% net price markup in steady state. We set the government spending-to-GDP ratio at

20%, in line with its sample average. Finally, the share of energy in consumption and production are both set at 10% and the capital share in production is set at 36%.

Table 1 reports the prior distributions for the structural parameters of the model and the exogenous processes that drive the dynamics of the economy, which are mostly similar to Smets and Wouters (2007). One notable difference relates to the Taylor rule coefficient associated with the response of the monetary authority to changes in the inflation rate (ϕ_π). Smets and Wouters (2007) specify a normal distribution truncated at 1, centered at 1.50 and with standard deviation 0.25 and impose determinacy. Instead, here, we want to deal with the possibility of indeterminacy.

The next Section explains how we deal with the determinacy/indeterminacy issue in the estimation, following Bianchi and Nicolò (2021). Regarding priors, we consider a prior which assigns roughly equal probability of observing indeterminacy as well as a unique solution. In particular, for ϕ_π we set a flatter normal prior distribution centered at 1 and with standard deviation 0.35 following Nicolò (2023).

3.3 Methodology

Bianchi and Nicolò (2021) develop a new method to solve and estimate linear rational expectations (LRE) models that accommodates both determinacy and indeterminacy. Their characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2003, 2004) and Farmer et al. (2015). We closely follow Bianchi and Nicolò (2021) and in the following briefly sketch their methodology while referring the readers to their paper for detailed exposition. The LRE model can be compactly written in the canonical form as:

$$\Gamma_0 (\Theta) X_t = \Gamma_1 (\Theta) X_{t-1} + \Psi (\Theta) \varepsilon_t + \Pi (\Theta) \eta_t,$$

where X_t is the vector of endogenous variables, Θ is the vector of model parameters, ε_t is the vector of fundamental shocks, and η_t are one-step ahead forecast errors for the expectational variables. Bianchi and Nicolò (2021) propose to augment the original model by appending an independent process, which could be either stable or unstable. The priors are such that there is roughly a 50-50 prior probability of determinacy and one degree of indeterminacy. Following Bianchi and Nicolò (2019), we append the following autoregressive process to the original LRE model:

$$\omega_t = \varphi^* \omega_{t-1} + \nu_t - \eta_{f,t}, \tag{10}$$

where ν_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast error vector η_t. The key insight consists of choosing this auxiliary process in a way to deliver the 'correct' solution. When the original model is determinate, the auxiliary process must be stationary so that the augmented representation also satisfies the Blanchard-Kahn condition. Accordingly, we set φ^* such that its absolute value is inside the unit circle. Then the autoregressive process for ω_t does not affect the solution for the endogenous variables X_t. On the other hand, under indeterminacy, the additional process should be explosive so that the Blanchard-Kahn condition is satisfied for the augmented system, though it is not for the original model. Hence, the absolute value of φ^* is set outside the unit circle. Under indeterminacy, we estimate the standard deviation of the sunspot shock, σ_ν, and so we specify a uniform distribution over the interval $[0, 1]$ following Nicolò (2023). In addition, the newly defined sunspot shock, ν_t, is potentially related to the structural shocks of the model. For the correlation between the sunspot shock and the structural shocks, we set a uniform prior distribution over the interval $[-1, 1]$, as in Nicolò (2023).

We use Bayesian techniques to estimate the model parameters and to test for (in)determinacy using posterior model probabilities. First, we find the mode of the posterior distribution by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In a second step, the Metropolis-Hastings algorithm is used to simulate the posterior distribution and to evaluate the marginal likelihood of the model.[7]

4 Results

4.1 Assessing the ECB policy reaction function: Determinacy vs Indeterminacy

Following Bianchi and Nicolò (2021), we need to pick one forward-looking variable, whose forecast error determines the dynamics of the auxiliary variable in (10). In our model, we consider eight forward looking variables, $\pi_{C,t}$, $\pi_{q,t}$, c_t, e_t, ΔER_t, q_t^k, r_t^k and i_t. When the variance-covariance matrix of shocks remains unrestricted, this specific choice is irrelevant (see Farmer et al., 2015). However, as it is standard in the literature, we assume uncorrelated fundamental shocks. Consequently, the selection produces different results. Thus, we examine which specification yields a superior model fit to the data through a comparison of marginal data densities. Table 2 shows the resulting log data densities for each forecast error. It emerges that the preferred specification under indeterminacy is the one where we include the forecast error associated with the core inflation rate $\eta_{\pi_q,t} = \pi_{q,t} - E_{t-1}(\pi_{q,t})$ as $\eta_{f,t}$ in the augmented representation.[8]

Based on this, we provide a comparison between the best fitting indeterminacy specification and the standard determinacy specification. Table 1 reports the parameter estimates and the log data densities. A first main result from the estimation

[7]All estimations are done using Dynare. The posterior distributions are based on 1000,000 draws, with the first 500,000 draws being discarded as burn-in draws. The average acceptance rate is around 25-30%.

[8]Nicolò (2023) considers the specification with the expectational error on headline inflation rate.

Table 1: Parameter estimates

		Priors			Determinacy			Indeterminacy		
		shape	mean	st. dev.	post. mean	90% HPD interval		post. mean	90% HPD interval	
TR response to inflation	ϕ_π	norm	1	0.35	1.590	1.067	2.066	0.577	0.267	0.902
TR response to output	ϕ_y	norm	0.1	0.05	0.166	0.108	0.225	0.080	0.010	0.134
TR response to output growth	ϕ_{gy}	norm	0.1	0.05	0.015	0.010	0.020	0.018	0.010	0.026
TR interest rate smoothing	ϕ_R	beta	0.75	0.1	0.892	0.864	0.921	0.920	0.886	0.958
inverse Frisch elasticity	ϕ_l	gamm	2	0.75	0.226	0.104	0.346	0.231	0.106	0.347
habits	b	beta	0.7	0.1	0.427	0.338	0.517	0.361	0.251	0.466
investment adjustment costs	γ_I	gamm	4	1.5	5.002	3.211	6.718	3.629	1.964	5.194
Calvo price stickiness	ξ_p	beta	0.5	0.1	0.865	0.818	0.912	0.806	0.749	0.865
real wage rigidity	γ	beta	0.5	0.2	0.694	0.633	0.755	0.574	0.438	0.697
Employment parameter	ξ_e	beta	0.5	0.1	0.457	0.331	0.585	0.411	0.300	0.527
price indexation	χ_p	beta	0.5	0.15	0.506	0.279	0.733	0.311	0.130	0.486
capital utilization elasticity	σ_u	beta	0.5	0.15	0.849	0.751	0.948	0.836	0.731	0.949
intertemporal elasticity	σ	norm	1.5	0.37	1.031	0.855	1.197	0.973	0.805	1.141
inputs elasticity	ϵ	gamm	0.5	0.2	0.227	0.090	0.355	0.202	0.083	0.318
home/imported goods elast.	υ	gamm	0.5	0.2	0.441	0.172	0.704	0.427	0.169	0.673
ss growth	g_z	norm	0.2	0.1	0.249	0.217	0.281	0.244	0.214	0.275
ss hours	$\bar{E}$	norm	0	2	1.932	0.187	3.756	1.519	-0.222	3.306
ss inflation	$\bar{\pi}$	gamm	0.5	0.1	0.556	0.416	0.698	0.472	0.314	0.622
Shocks persistences										
risk premium	ρ_b	beta	0.7	0.1	0.921	0.890	0.953	0.920	0.871	0.970
investment	ρ_i	beta	0.7	0.1	0.335	0.223	0.444	0.353	0.234	0.476
monetary	ρ_r	beta	0.3	0.1	0.358	0.246	0.471	0.388	0.263	0.510
price markup	ρ_p	beta	0.7	0.1	0.696	0.588	0.809	0.816	0.716	0.919
labor supply	ρ_l	beta	0.7	0.1	0.886	0.826	0.949	0.907	0.859	0.956
gov spending	ρ_g	beta	0.7	0.1	0.866	0.816	0.917	0.874	0.830	0.919
technology	ρ_a	beta	0.7	0.1	0.886	0.835	0.937	0.856	0.788	0.925
energy price	ρ_s	beta	0.9	0.05	0.975	0.962	0.989	0.974	0.960	0.989
MA price markup	ρ^p_{ma}	beta	0.5	0.1	0.542	0.406	0.680	0.567	0.426	0.712
gy correlation	ρ_{gy}	norm	0.5	0.25	0.123	0.010	0.223	0.124	0.010	0.224
Shocks standard deviations										
risk premium	σ_b	invg	0.1	2	0.168	0.125	0.212	0.217	0.139	0.291
investment	σ_i	invg	0.1	2	1.097	0.931	1.254	1.120	0.956	1.287
monetary	σ_r	invg	0.1	2	0.121	0.105	0.137	0.119	0.104	0.133
price markup	σ_p	invg	0.1	2	0.105	0.082	0.127	0.094	0.069	0.117
labor supply	σ_l	invg	0.1	2	1.275	0.988	1.546	1.046	0.779	1.305
government spending	σ_g	invg	0.1	2	0.769	0.673	0.864	0.769	0.669	0.870
technology	σ_a	invg	0.1	2	0.878	0.724	1.032	0.877	0.730	1.024
energy price	σ_s	invg	2	2	3.144	2.771	3.517	3.160	2.768	3.528
measurement error	$\sigma^{me}_{\pi_y}$	invg	0.1	2	0.325	0.286	0.363	0.323	0.284	0.362
sunspot	σ_ν	unif	0.5	0.289	-	-	-	0.238	0.200	0.275
Shocks correlations										
corr sunspot, risk premium	$\rho_{\nu b}$	unif	0	0.577	-	-	-	0.268	0.081	0.445
corr sunspot, investment	$\rho_{\nu i}$	unif	0	0.577	-	-	-	-0.339	-0.475	-0.204
corr sunspot, monetary	$\rho_{\nu r}$	unif	0	0.577	-	-	-	0.233	0.125	0.346
corr sunspot, price markup	$\rho_{\nu p}$	unif	0	0.577	-	-	-	0.751	0.651	0.854
corr sunspot, labor supply	$\rho_{\nu l}$	unif	0	0.577	-	-	-	-0.106	-0.241	0.021
corr sunspot, gov spending	$\rho_{\nu g}$	unif	0	0.577	-	-	-	-0.182	-0.312	-0.049
corr sunspot, technology	$\rho_{\nu a}$	unif	0	0.577	-	-	-	-0.159	-0.297	-0.030
corr sunspot, energy price	$\rho_{\nu s}$	unif	0	0.577	-	-	-	0.203	0.088	0.324
Foreign parameters										
SS foreign inflation	$\bar{\pi}^*$	gamm	0.6	0.1	0.605	0.476	0.728	0.601	0.479	0.721
SS foreign int rate	$\bar{R}^*$	gamm	0.3	0.1	0.337	0.202	0.471	0.334	0.204	0.460
foreign demand persistence	ρ^*_y	beta	0.7	0.1	0.916	0.875	0.958	0.917	0.876	0.960
foreign inflation persistence	ρ^*_π	beta	0.7	0.1	0.521	0.412	0.628	0.519	0.416	0.627
foreign rate persistence	ρ^*_R	beta	0.3	0.1	0.867	0.843	0.897	0.864	0.838	0.895
foreign demand std dev	σ^*_y	invg	0.1	2	0.643	0.566	0.720	0.645	0.567	0.723
foreign inflation std dev	σ^*_π	invg	0.1	2	0.572	0.504	0.637	0.572	0.504	0.636
foreign rate std dev	σ^*_R	invg	0.1	2	0.174	0.151	0.195	0.173	0.151	0.194
Log data density						-1068.3			-1050.5	

Table 2: Model selection

Forecast error	Log data density
$\pi_{C,t}$	-1054.7
$\pi_{q,t}$	-1050.5
c_t	-1080.3
e_t	-1090.8
ΔER_t	-1079.0
q_t^k	-1074.1
r_t^k	-1081.9
i_t	-1110.2

is that, by comparing the log-likelihoods, the data favors the indeterminate model, therefore rejecting equilibrium uniqueness. The indeterminacy result is in line with finding by Hirose (2013), who estimates a small-scale two country model for the US and the euro area over the period 1983Q1-2002Q4. He finds that the data point to a passive monetary policy for the euro area during this period. Our results suggest that monetary policy in the euro area has continued to remain passive even in the post-1999 period. However, by looking at Table 2, note that whether the indeterminate model is preferred or not by the data depends on which forward-looking variable one picks. Specifically, the indeterminate model is preferred if the auxiliary variable is driven by the forecast errors of one of the two inflation variables, while this is not the case if one chooses one of the other forward-looking variables. Moreover, estimating an indeterminate model imposing no correlation between the sunspot shock and all the structural shocks yields a worse fit of the data than the determinate model. Hence, while indeterminacy is preferred by the data in two specifications, this result is not robust to other specifications of the indeterminate model.

The estimates for most of the structural and shock parameters are mostly similar under determinacy and indeterminacy with some differences. In particular, the degree of habits (b), investment adjustment cost (γ_I), the degree of price rigidity

(ξ_p) and indexation (χ_p), and real wage rigidity (γ) turn out to be smaller under indeterminacy. In the indeterminacy case, the estimation also delivers the posterior distribution of the standard deviation of the sunspot shock and its correlation with the fundamental exogenous shocks. The standard deviation of the sunspot shock turns out to be tightly estimated. In addition, the sunspot shock turns out to be positively correlated with the risk premium, monetary, price markup and energy price shock, while being negatively correlated with the investment, labor supply, government spending and technology shock.

4.2 Shock propagation and the ECB's monetary policy response

This section analyzes and compares the transmission of shocks under both determinacy and indeterminacy. For the latter, the propagation of the fundamental shocks is altered due to self-fulfilling inflationary or deflationary expectations in response to the shocks. Here we look at the propagation of five shocks that play a key role in driving inflation and output fluctuations under indeterminacy – according to the forecast error variance decomposition results (discussed below) – namely, technology, risk premium, monetary policy, energy, and labor supply shock.

To help the interpretation of the impulse response functions (IRFs) to these fundamental shocks under indeterminacy, however, it is instructive first to consider what are the effects of the sunspot shock, because the sunspot is correlated with the fundamental shocks. Figure 1 shows the IRFs of selected variables to the sunspot shock.[9] This shock looks like a positive demand shock: output and inflation measures increase, as well as the marginal cost. Hence, in our estimation a sunspot

[9]Given that the fundamental shocks are correlated with the sunspot shock under indeterminacy, the shocks need to be orthogonalized in order to look at their transmission mechanism. The orthogonalization is such that fundamental shocks in the economy trigger a sunspot shock, but not the other way round, i.e., sunspot shocks are ordered last in the Cholesky ordering.

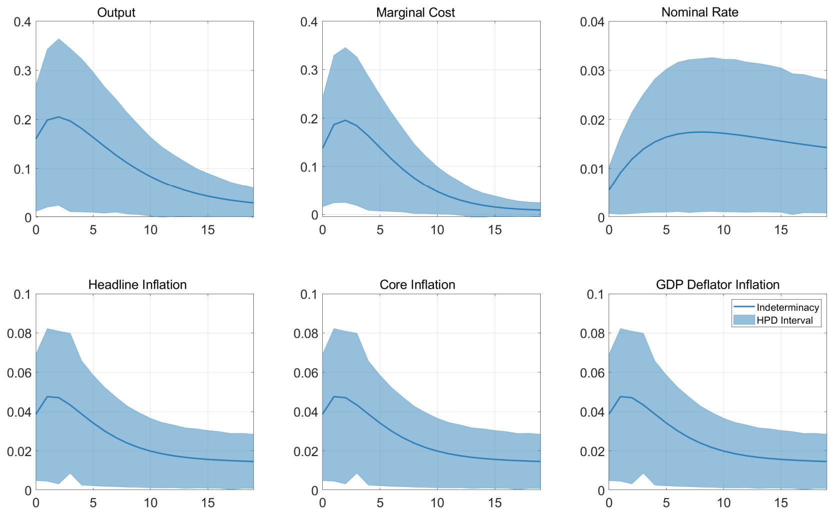


Figure 1: Impulse responses to a one standard deviation sunspot shock from the baseline estimation. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

shock has the flavor of a positive sentiment shock in the spirit of Angeletos et al. (2018).

Figures 2-5 plot the IRFs to the structural shocks mentioned above. The solid lines are posterior means and the shaded areas are highest posterior density (HPD) regions in the two cases of determinacy – red lines and regions – and indeterminacy – blue lines and regions.[10] The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy where the correlations of the structural shock with the sunspot shock is set to zero. Recall that under indeterminacy, there are two channels at work which are absent in the determinacy case: (i) self-fulfilling expectations on inflation due to passive monetary policy; and (ii) the sunspot shock, as an additional extrinsic non-fundamental disturbance to the economy. Thus: (i) the propagation of structural shocks is different because self-fulfilling expectations alter the transmission and generate additional persistence, and (ii) non-fundamental

[10] The IRFs for the remaining shocks, namely, investment, price markup, government spending, and sunspot, are shown in the Appendix.

sunspot disturbances produce an additional source of volatility, adding, depending on the correlation, either a positive or a negative demand shock, as just seen. Essentially, in each panel, the difference between the dashed black line and red one illustrates (i), that is, the different propagation mechanism between determinacy and indeterminacy due to different parameters and monetary policy reaction, while the difference between the dashed black line and the blue one illustrate (ii), that is, the marginal contribution of the sunspot shock, due to the correlation between sunspot and fundamental shocks.

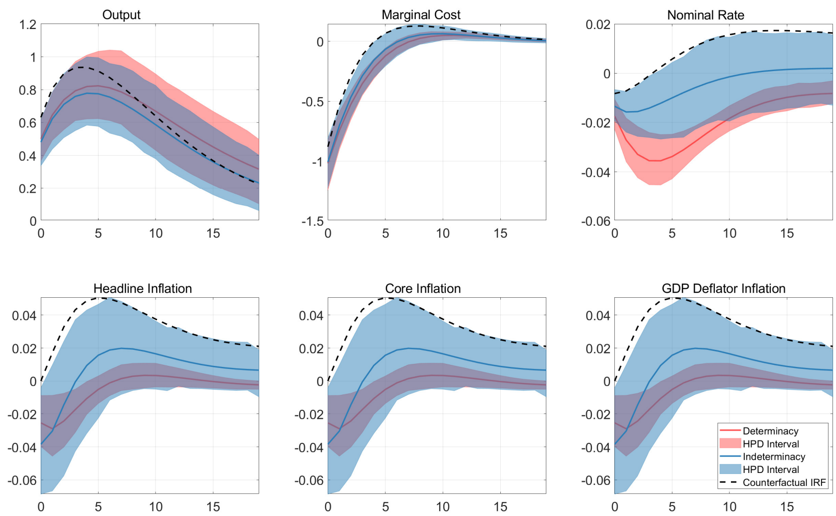


Figure 2: Impulse responses to a one standard deviation technology shock. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy setting to zero the correlation with the sunspot shock.

Figure 2 displays the IRFs to a technology shock. The difference between the dashed black line and red one reveals that the IRFs of output and the marginal cost are very similar, but the ones for the three inflation measures in the panels in the second row exhibit a different propagation mechanism. The technology shock generates deflationary pressures because of the decrease in marginal costs. Under determinacy this yield a reduction of inflation, as standard economic reasoning would predict. Under indeterminacy, instead, the shock triggers self-fulfilling inflationary expectations that lead to a persistent hump-shaped increase in inflation.[11] It follows that the reaction of monetary policy is different in the two cases. Under determinacy, the monetary authority responds by lowering the policy rate. In contrast, under indeterminacy, monetary authority increases the nominal rate, but the response is gradual and not aggressive enough to stabilize the inflation rate, so that the inflationary expectations are accommodated by the passive monetary authority. The difference between the dashed black line and the blue one, instead, reveals the negative correlation between the technology and the sunspot shock, as from Table 1. The technology shock triggers a negative demand-like sunspot shock so that both the response of output and inflation is lowered.

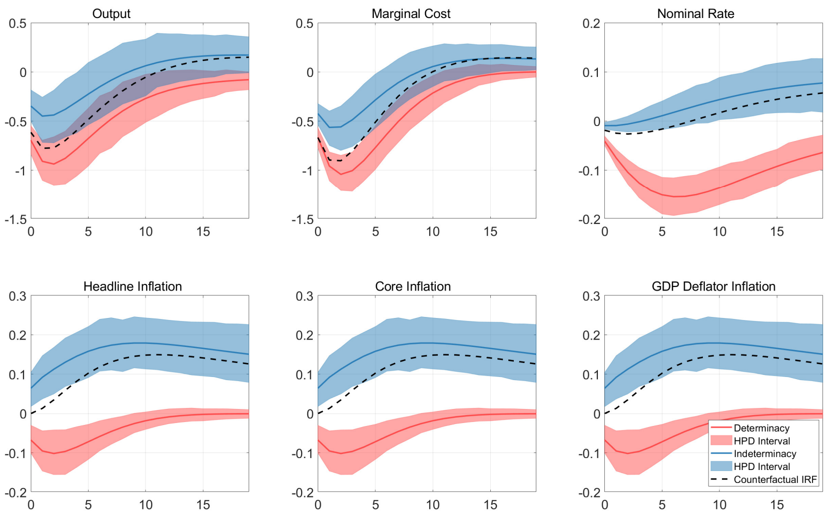


Figure 3: Impulse responses to a one standard deviation risk premium shock. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy setting to zero the correlation with the sunspot shock.

Figure 3 shows the impulse responses to a risk premium shock. A risk premium

[11]Note that this does not need to be, but it is the results of the estimation. In theory, agents choose one among the infinitely many paths and that could entail inflationary or deflationary expectations.

shock creates a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive realization of the shock has negative effects on the economy. Under determinacy, all households reduce consumption because households anticipate a prolonged real interest rate decline, in line with the previous estimates for the euro area (Smets and Wouters, 2005, Albonico et al., 2019). With an active monetary authority, there is deflation and the nominal interest rate decreases, responding more than one-to-one to inflation. Under indeterminacy, again as in the previous case, the responses of the real variables – output and marginal cost – is only quantitatively different, while the responses of the three inflation measures are qualitatively different. Again, agents form self-fulfilling inflationary expectations – look at the dashed black line – that are partly accommodated by the passive monetary policy, such that inflation persistently increases. In addition, the risk premium shock is positively correlated with the sunspot, thereby generating additional inflationary pressures and a higher level of output.

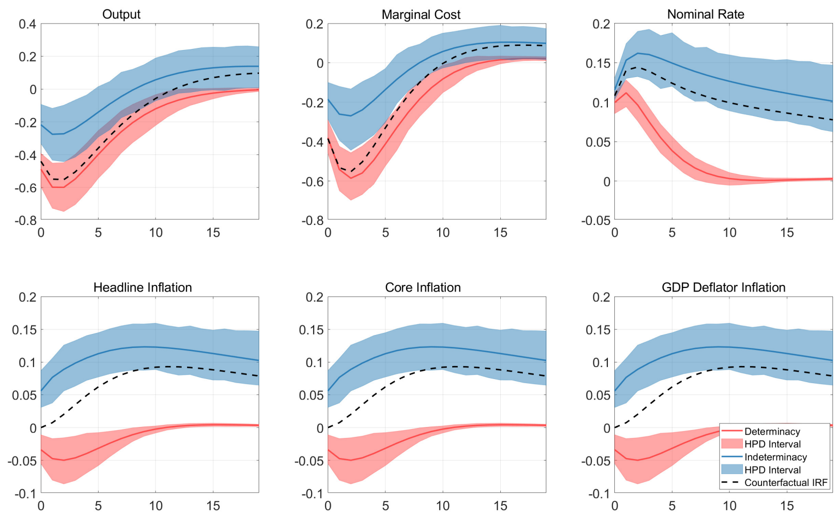


Figure 4: Impulse responses to a one standard deviation monetary policy shock. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy setting to zero the correlation with the sunspot shock.

Figure 4 shows the impulse responses to a contractionary monetary policy shock, that, under determinacy, produces the expected outcome: a negative response of inflation and negative effects on aggregate demand and economic activity. In contrast, again, the presence of a passive monetary policy flips the sign of the response of inflation, due to self-fulfilling inflationary expectations, in line with the empirical findings of Lubik and Schorfheide (2004) and Ascari and Bonomolo (2019) for the U.S.. The monetary policy shock is positively correlated with the sunspot, so the inflation and the output paths are shifted upward from the dashed black line to the blue one.

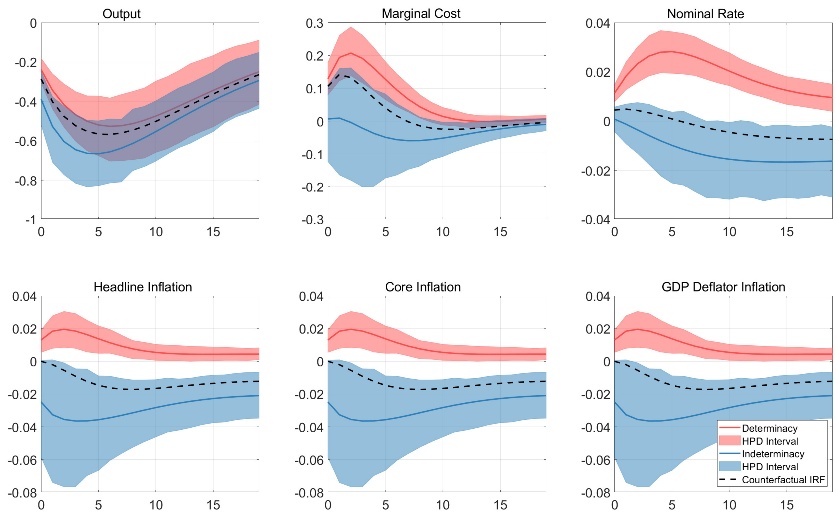


Figure 5: Impulse responses to a one standard deviation labor supply shock. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy where the correlation with the sunspot shock is shut down.

Figure 5 displays the responses to a labor supply shock. A positive realization of this shock implies that individuals dislike working relatively more, thus the labor supply curve shifts in. This generates an upward pressure on real wages (and thus marginal costs), while decreasing hours worked, which results in a decrease of production. Under active monetary policy, the contractionary effect is exacerbated by the reaction of monetary policy. Conversely, under indeterminacy, the shock generates self-fulffiling disinflationary expectations, so that the inflation measures decrease despite the increase in the marginal cost – see again the dashed black line. The nominal interest rate decreases as a reflection of subdued inflationary pressures. This shock is negatively correlated to the sunspot shock, so that the sunspot deepens the reduction of output, inflation and interest rate.

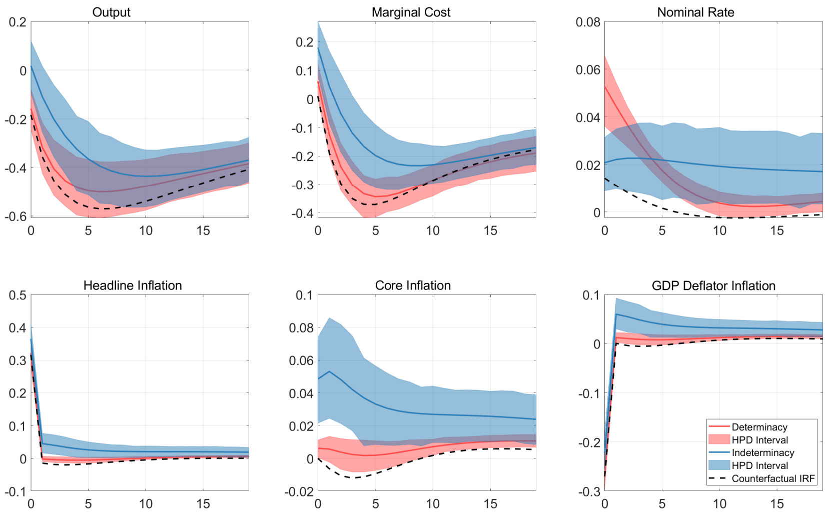


Figure 6: Impulse responses to a one standard deviation energy price shock. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy setting to zero the correlation with the sunspot shock.

Figure 6 shows the responses to an exogenous increase in energy prices. Since this is a shock to a price, the effects on inflation are quite short-lived under determinacy. Headline inflation jumps up, while the positive effect on core is muted. Monetary policy reacts forcefully to contain inflation, causing a persistent decrease in output and the negative response of the GDP deflator on impact. Under indeterminacy, the dashed black responses of all the variables are now qualitatively similar, even if the reaction of monetary policy is milder. The energy shock is positively correlated with the sunspot, so the blue lines lie above the dashed black ones. The reaction of core inflation – as well as the other two inflation measures – becomes positive and quite persistent, despite output contracting.

The analysis of the propagation of the fundamental shocks is quite puzzling in the case of indeterminacy. The auxiliary variable approach of Bianchi and Nicolò (2021) allows the estimation to pick one of the infinitely many possible rational expectations solutions under indeterminacy. The responses of the endogenous variables to the fundamental shocks are then twisted by self-fulfilling expectations. The preferred solution by the data, however, implies reactions to most fundamental shocks that contrast simple economic theory. The dashed black lines following a positive technology shock in Figure 2, for example, imply a positive response of output and a *positive* response of inflation. The positive response of inflation (and output) is then dampened by the negative correlation with the sunspot shock. Also the labor supply shock, induces a positive comovement between inflation and output, see dashed black lines in Figure 5. In this case, the negative correlation with the sunspot shock amplifies the negative responses of both output and inflation. The relationship between the marginal cost and inflation for both these supply shocks is even more puzzling, because the marginal cost reacts in accordance with theory – i.e., moving in opposite direction with respect to output after a supply shock – but the dynamics of inflation is de-linked from the marginal cost due to self-fulfilling (inflationary or disinflationary) expectations. These expectations are not grounded in the theory, but they are simply data-driven, in the sense that they are picked by the estimation methodology. In these two cases – technology and labor supply shock – indeterminacy alters the propagation of the shocks so much that it makes these fundamental supply shocks look like demand shocks instead. Similarly, demand shocks propagate like supply shocks under indeterminacy. The dashed black lines of output and inflation move in *opposite* directions following either a risk premium shock (Figure 3) or a monetary policy shock (Figure 4). Also in this case, there is a disconnection between the dynamics of inflation and the marginal cost, where the latter reacts according to the theory. Moreover, the correlation with the sunspot shock, which is positive for both these demand shocks, amplifies the positive response of inflation to a contractionary demand shock.

To conclude, it seems that under the preferred indeterminacy specification – associated with the forecast error with respect to core inflation rate in (10) – the responses of inflation to the fundamental shocks are at odds with standard economic theory: inflation increases after a positive supply or a negative demand shock.

4.3 Variance decomposition

Table 3: Forecast error variance decomposition

	y	h	i	c	w	R	π_c	π_q	π_y
Determinacy									
ε^b	16.3	20.5	15.9	16.4	18.3	71.9	20.4	30.8	22.3
ε^i	13.2	17.6	24.5	5.4	5.0	1.1	0.5	0.8	0.6
ε^r	5.9	8.4	3.5	3.9	4.6	11.3	4.8	7.2	5.2
ε^p	3.2	4.2	2.9	1.3	2.0	5.8	34.5	55.2	38.1
ε^l	12.1	16.0	9.2	8.0	3.3	2.5	0.9	1.4	1.0
ε^a	25.0	9.8	16.2	12.7	11.9	3.2	1.1	1.7	1.2
ε^g	4.3	8.4	0.3	0.6	0.2	0.2	0.1	0.1	0.1
ε^s	19.5	14.3	27.5	51.7	54.8	3.9	37.7	2.8	31.5
Indeterminacy									
ε^b	8.7	11.6	9.0	8.0	10.1	32.9	49.6	52.5	51.1
ε^i	4.0	11.0	15.0	3.0	2.1	5.0	6.7	7.2	7.0
ε^r	4.1	4.4	2.9	2.3	3.3	51.7	24.1	25.6	24.9
ε^p	11.7	12.8	10.2	5.0	7.7	4.0	5.5	5.8	5.7
ε^l	23.5	29.1	16.3	14.6	3.6	1.5	2.0	2.1	2.1
ε^a	25.1	11.0	16.0	11.0	11.3	0.5	0.8	0.9	0.9
ε^g	1.9	4.2	3.7	3.8	3.3	2.0	2.7	2.8	2.8
ε^s	19.1	13.3	25.9	51.6	57.5	1.3	7.4	1.6	4.4
ε^ν	1.4	1.7	1.0	0.8	1.1	1.0	1.3	1.4	1.3

The different propagation of the shocks under determinacy versus indeterminacy is reflected in the relative importance of the fundamental shocks for the volatility of the endogenous variables, especially so for inflation. Table 3 reports the mean of the forecast error variance decomposition based on the posterior distribution of the parameter estimates.[12] Under determinacy, output appears to be relatively more supply-driven (about 60% of total variance), still demand shocks explains 40% of its volatility. Under indeterminacy, technology and energy shocks are still the main determinants of the volatility of output, but the reduced importance of the risk premium and the investment specific shock makes demand shocks contribute to only roughly 20% to the volatility of output. However, the main differences between the two specifications arise when we look at inflation. As it is standard in the literature, inflation appears to be mostly supply-driven when monetary policy is active. In contrast, under indeterminacy the risk premium shock turns out to be the main driver of inflation fluctuations followed by monetary policy shocks (summing to about 70% of total volatility). This is hardly surprising given the analysis in the previous subsection: indeterminacy turns upside down the effect on inflation of demand and supply shocks, hence inflation, that is supply-driven under determinacy, becomes demand-driven under indeterminacy.

4.4 Natural rate and output gap estimates

Our framework allows us to study the behavior of the natural rate of interest and the output gap. As standard, we define 'natural' variables as the ones implied by a flexible prices and wages version of the model. Figure 7 shows the dynamics of the natural interest rate, r^*, both under determinacy and under indeterminacy. The dynamics is very similar in both specifications – r^* is positive until the Great Financial Crisis when it abruptly turns negative. Then, it continues to drive deeper

[12] We do not report the contributions of the three foreign shocks and the measurement error to the forecast error variance decompositions as they play a negligible role.

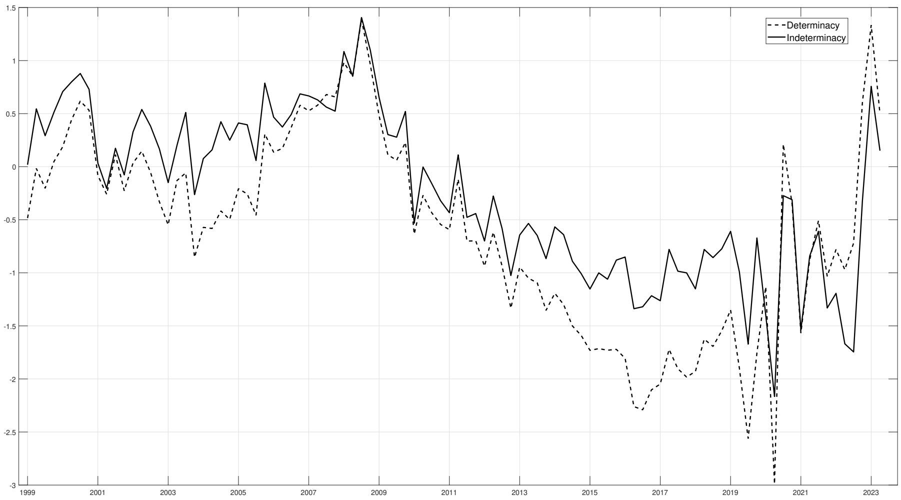
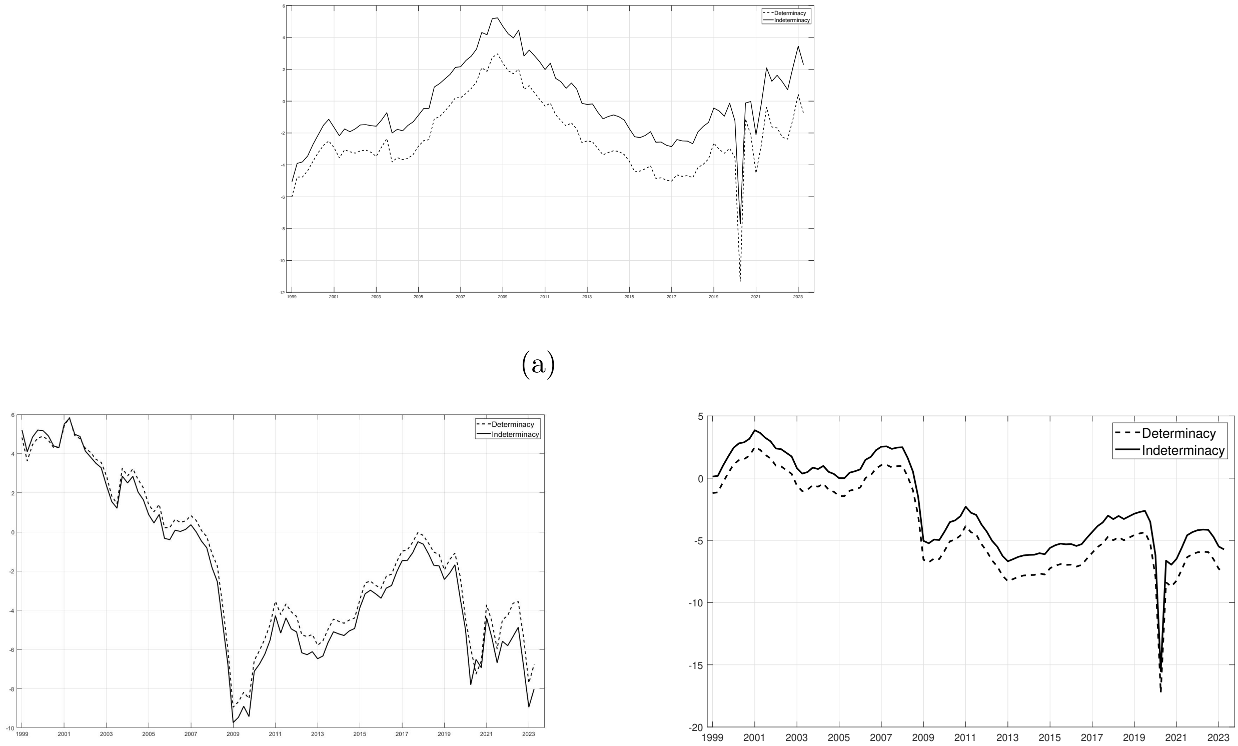


Figure 7: The natural rate of interest in the determinate and indeterminate model

into negative territory as the sovereign debt crisis unfolds, and finally it starts to increase after 2016 with the exception of the two waves of the pandemic. The recent inflation surge causes a rapid increase in r^* that crosses to positive territory after almost 15 years. This could be caused by global supply chain disruptions and the increase in energy price that should be associated with an increase in r^* to keep demand in line with these constraints in aggregate supply. The very last quarters show a decrease in r^*, consistent with improvements in the supply conditions. This behavior is quantitatively in line with other estimates from the literature for the Euro Area as Neri and Gerali (2019) or the update of this estimate in the ECB Economic Bulletin, Issue 1/2024 (see Box 7 by Brand et al.).

Recall that r^* in the DSGE-New Keynesian framework is a cyclical concept that exhibits temporary fluctuations in response to fundamental shocks. It serves as a guidance for monetary policy because targeting r^* would eliminate the inefficiencies caused by the nominal rigidities and stabilize inflation. In this sense, a higher r^* for the Euro Area in the recent post-pandemic inflationary episode is coherent with the



(a)

(b)

(c)

Figure 8: Panel (a): Output gap: percentage deviation of output, Y_t, from the natural rate of output, Y_t^{flex}. Panel (b): $\hat{y}_t^{flex}$ = percentage deviation of the natural rate of output, Y_t^{flex} from the deterministic trend. Panel (c): $\hat{y}_t$ = percentage deviation of output, Y_t, from the deterministic trend. In all the three panels the dashed line is used for the determinate model and the solid line for the indeterminate model

reaction of the ECB monetary policy.[13]

The model output gap is measured as the log difference between actual output and the natural level of output implied by the flexible model counterpart. Panel (a) in Figure 8 shows the smoothed estimate of the output gap for both determinacy (dashed line) and indeterminacy (solid line). The two series exhibit the same fluc-

[13]This short-term measure is different from the slow-moving measures of r^* anchored to long-run economic trends in demographics, productivity or risk aversion, as for example the well-known one in Holston et al. (2017). See also the discussion in Del Negro et al. (2017). Obstfeld (2023) distinguishes between two types of real rate of interest. He defines as natural rate, the real interest rate prevailing over a long-run equilibrium, and as neutral rate, the short-run real interest that that eliminates inflationary pressures.

tuations and are almost perfectly correlated, but there is a difference in the level: the output gap is always lower under determinacy. Moreover, this difference widens over time being equal to roughly 1% at the beginning of the sample and to roughly 3% at the end of the sample. Panels (b) and (c) disentangle the dynamics of the output gap into its components by showing the dynamics of the (flexible-price) natural output and of actual output, respectively, both expressed in deviations from the deterministic growth trend. A first thing to note is the dramatic drop of around 10% in the natural output during the Great Financial Crisis for both determinacy and indeterminacy, that explains the increase in the output gap between 2007 and 2009 in Panel (a). Hence, mimicking the flexible price allocation would have required an even larger drop in output than the one experienced during the Great Recession. In contrast, during the Covid shock output fell much more than the natural level of output causing a large and sudden drop in the output gap in Panel (a). Comparing Panels (b) and (c) explains also where the difference in the level of the output gap between determinacy and indeterminacy is coming from. Detrended actual output is always lower under determinacy, while the natural rate is always higher so that both components contribute in the same direction in making the output gap lower under determinacy. However, the difference in the natural rate is marginal, so that most of the difference in the output gap between the two specifications comes from detrended output. That means that the difference we see in Panel (a) is mostly due to the different trend estimated in the two specifications. The trend is estimated to start from a higher value under determinacy – so that detredend output is roughly 1% lower – and then the difference between the two output gaps widens because the estimated rate of growth of output is slightly larger under determinacy (see g_z Table 1).

We can compare the output gap estimate from our model with different alternative measures of output gap for the Euro area. We consider: i) the AMECO output

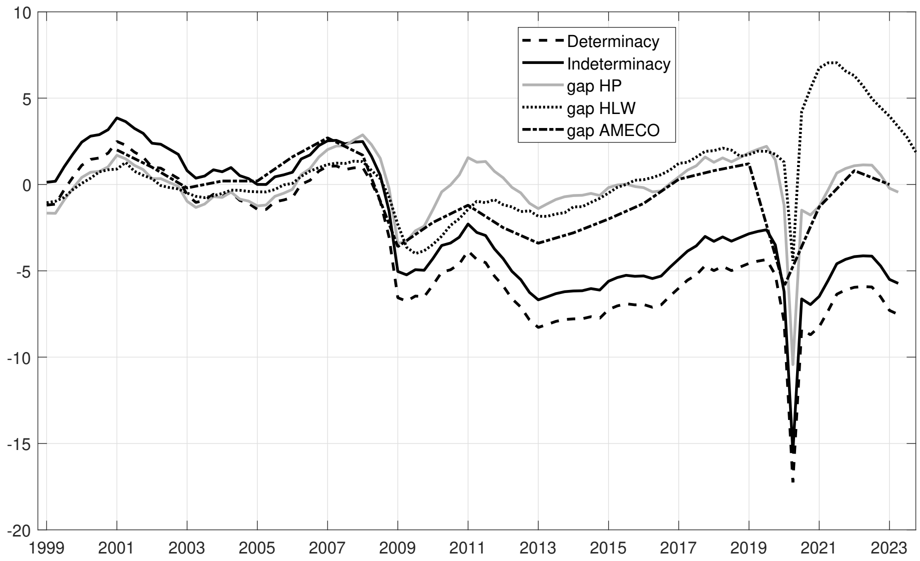


Figure 9: Actual output in deviation from the deterministic trend in the determinate and indeterminate model, cycle component of GDP obtained with Hodrick-Prescott filter, output gap measure from AMECO and output gap measure from Holston-Laubach-Williams

gap estimates, where AMECO is the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs; ii) the Holston-Laubach-Williams (HLW) measure available on the New York fed website; iii) a simple statistical measure derived fitting an Hodrick-Prescott (HP cycle) to our data for real GDP. Figure 9 shows that AMECO, HLW, and HP cycle are very similar in terms of cyclical fluctuations.[14] However, they are very different from our measure of output gap in Panel (a) of Figure 8. Not surprisingly, Figure 9 shows that these measures are, instead, very similar to our $\hat{y}_t$ in Panel (c) of Figure 8, i.e., the log-deviation of actual output from the linear deterministic trend estimated by the model. Hence the difference between the output gaps comes from the conceptual definition of 'output gap' in a New Keynesian model. The DSGE uses a Woodfor-

[14]Note that the HLW and HP cycle are calculated at a quarterly frequency (in line with our data), while AMECO is available only on an annual base. In the plot the annual AMECO data have been interpolated.

dian ‘normative’ or prescriptive measure that implies the estimate of the unobserved natural/flexible price level of output. As shown above, this variable is subject to cyclical fluctuations which create the difference with respect to ‘positive’ measures such as AMECO, HLW or HP cycle, which are based on some statistical procedure to filter out the low frequency component of output. Our model does exactly that, in order to compute the variable $\hat{y}_t$. Note that while the cyclical behavior of $\hat{y}_t$ correlates almost perfectly with these three statistical measures of output gap, a difference opens up after the Great Financial Crisis and the Sovereign Debt Crisis. After 2011 our measure of detrended output is substantially lower than the others because of the different specification of the trend. While a non-linear specification gives to the trend the flexibility to adjust downwards following these deep crisis, our linear specification does not allow for that, thus our estimated trend is higher after 2011.

5 Robustness

Our results point to the ECB’s monetary policy response to inflation to be passive, which, in turn, implies equilibrium indeterminacy and self-fulfilling inflationary (or disinflationary) expectations driving inflation fluctuations in the Euro Area. We have discussed how the resulting self-fulfilling expectations and sunspot shocks have altered the propagation of various structural shocks in the economy and affected business cycle fluctuations. In what follows we check the sensitivity of our findings along two main dimensions: (i) measure of the shadow rate; (ii) data from the post-Covid period. For (i), we replace Krippner’s shadow rate with Wu and Xia (2017, 2020)’s measure.[15] For (ii), we re-do the baseline estimation using Krippner’s shadow rate but dropping data from the post-Covid period and estimating over the

[15] We do this both for the Euro area and the United States.

sample 1999Q1-2019Q2. Table 4 reports the log-data densities and the posterior probabilities of (in)determinacy, where we can see that indeterminacy continues to unambiguously fit better in both cases.

Table 4: Determinacy versus Indeterminacy - Robustness

	Log-data density		Probability	
	Determinacy	Indeterminacy	Determinacy	Indeterminacy
Alternative Shadow Rate: Wu and Xia	-1085.1	-1065.6	0	1
Alternative Sample: 1999Q1-2019Q4	-709.1	-694.8	0	1

Table A1 and Figures A5-A13 in the Appendix report the parameter estimates and the IRFs to the various shocks, respectively, under both determinacy and indeterminacy when using Wu and Xia's shadow rate measure. Both the parameter estimates and the dynamic responses to the shocks (including the altered propagation under indeterminacy) are essentially indistinguishable from our baseline results, despite the divergence between the two shadow rate measures for the Euro area since around 2015.

Table A2 and Figures A14-A22 report the parameter estimates and the IRFs when estimating the baseline model over the period 1999Q1-2019Q4, i.e., excluding Covid-period data. The monetary authority's response to inflation, which is a key parameter in our study, is very similar to our baseline estimate, which shows that our indeterminacy result is not driven by the data from the Covid period. Most structural parameter estimates remain similar over the two sub-samples with some exceptions. For instance, the inverse Frisch elasticity and the degree of habit formation and real wage rigidity turns out to be higher. In terms of the shocks, the persistence remains quite similar while the standard deviation differs somewhat for the risk-premium, labor supply and the energy shock. This is not unusual given, e.g., labor hoarding during Covid and the surge in energy prices in the post-Covid period. In terms of the altered propagation of key shocks driving business cycles under indeterminacy, the findings also remain robust. Unlike the determinacy specification, technology, risk-premium and monetary policy shocks turn out to be inflationary while the labor supply shock is deflationary. For the energy shock, headline inflation and nominal interest rate go up as expected, but the response of the nominal rate under indeterminacy is muted as before. One notable exception is that while core inflation still rises on impact, it goes down after a few quarters.

Table A3 in the Appendix reports the forecast error variance decompositions (FEVDs) for the two robustness checks, where we also include the baseline results for comparison. For ease of exposition, here we only report the FEVDs under indeterminacy. As seen in Table A3, the key drivers of business cycles remain mostly the same. For instance, output is mainly driven by technology, labor supply and energy shocks, while inflation and nominal interest rate are mainly driven by risk-premium and monetary policy shocks. As before and unlike the determinacy specification, inflation in the economy is mostly demand-driven.

6 Conclusion

This paper aims to evaluate the European Central Bank's (ECB) monetary policy concerning its unique objective of stabilizing inflation. In the model, this translates into examining whether the central bank adheres to a monetary policy rule that enables to control inflation and prevents self-fulfilling unanchored dynamics in inflation. While most papers on the Euro Area data rely on models where monetary policy is active and the so-called Taylor Principle holds, we therefore estimate a DSGE model on Euro area data for the sample period 1999Q1-2023Q2 allowing for the possibility of indeterminacy of rational expectation equilibrium, using the methodology proposed by Bianchi and Nicolò (2021). Our sample covers the entire existence of the ECB. Both the zero lower bound period following the Sovereign Debt

Crisis, possibly characterized by fiscal dominance, and the recent surge in inflation, reminiscent of the 1970s to some extent, call for an examination of the possibility of monetary policy exhibiting passive behavior. To the best of our knowledge, we are the first to estimate a medium scale model for the Euro Area allowing for indeterminacy.

The data favor a specification of the model where monetary policy in the euro area was passive, i.e., the response of the nominal interest rate is estimated less than proportional to headline inflation changes from target. This specification implies indeterminacy and self-fulfilling business cycle fluctuations driven by sunspot shocks, which alter the propagation of the fundamental shocks in our model economy, notably affecting the inflation responses. Specifically, the responses of inflation to the fundamental shocks are inconsistent with standard economic theory: inflation increases following a positive supply or a negative demand shock. Consequently, while under determinacy inflation is mostly supply-driven, under indeterminacy inflation is mostly demand-driven. Finally, the behavior of the natural interest rate and the output gap is similar between determinacy and indeterminacy, and both specifications imply that the natural rate of interest entered into positive (restrictive) territory in the recent period characterized by the increase in inflation. However, our main finding regarding the ECB's monetary policy being passive is somewhat sensitive to certain technical choices in implementing the Bianchi and Nicolò (2021) methodology.

Our findings are particularly noteworthy because, once indeterminacy is permitted, the data select a model that implies significantly different inflation dynamics compared to the models that rely on the standard assumption of active monetary policy, which is the case for most – if not all – the models estimated on Euro Area data. Nonetheless, the implied reactions of primary variables to core shocks, especially those related to inflation, exhibit notable peculiarities. The specific equilibrium of rational expectations identified through estimation, from an infinite admissible set, appears to contradict fundamental economic principles. Furthermore, the data's preference for an indeterminate specification lacks robustness under certain technical methodological considerations, raising doubts about the indeterminacy findings. Ultimately, determinacy seems subjective and open to interpretation.

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A Appendix

A.1 The model

A.1.1 Functional forms

In line with Christiano at al. (2005) we define the following functional forms. The capital utilization cost function is:

$$a\left(u_t\right) = \gamma_{u1}\left(u_t - 1\right) + \frac{\gamma_{u2}}{2}\left(u_t - 1\right)^2$$

The investment adjustment costs function defined as:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - g_z\right)^2, \tag{11}$$

where γ_I is a parameter measuring the degree of investment adjustment costs. In line with Lindé et al. (2004), the risk premium function is defined as:

$$\Gamma_t\left(\bar{B}^*_{t+1}\right) = \exp\left\{\Gamma^b \frac{ER_t \bar{B}^*_{t+1}}{y z_t P_{C,t}}\right\}. \tag{12}$$

A.1.2 Production

Price setting Intermediate goods prices are sticky à la Calvo (1983). A firm z can optimally reset its price with probability $(1 - \xi_p)$. Firms that cannot re-optimize adjust the price according to the scheme $P^z_{q,t} = \pi^{\chi_p}_{q,t-1}\pi^{1-\chi_p}P^z_{q,t-1}$, where $\chi_p \in [0, 1]$ allows for any degree of combination of indexation to past or trend inflation.

The aggregate price index is:

$$P_{q,t} = \left(1 - \xi_p\right)\tilde{P}^z_{q,t} G'^{-1}\left(\frac{\tilde{P}^z_{q,t}\iota_t}{P_{q,t}}\right) + \xi_p \pi^{\chi_p}_{q,t-1}\pi^{1-\chi_p}P_{q,t-1}G'^{-1}\left(\frac{\pi^{\chi_p}_{q,t-1}\pi^{1-\chi_p}P_{q,t-1}\iota_t}{P_{q,t}}\right), \tag{13}$$

where $\iota_t = \int_0^1 G'\left(\frac{Q_t^z}{Q_t}\right)\frac{Q_t^z}{Q_t}dz$.

The representative firm chooses the optimal price $\tilde{P}_{q,t}^z$ that maximizes expected profits subject to the demand schedule. The resulting first order condition is:

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\Xi_{t,t+s}}{P_{q,t+s}} Q_{t+s}^z \left[\begin{array}{c} \tilde{P}_{q,t}^z \pi_{q,t,t+s-1}^{\chi_p} \pi^{s(1-\chi_p)} \\ + \left(\tilde{P}_{q,t}^z \pi_{qt,t+s-1}^{\chi_p} \pi^{s(1-\chi_p)} - MC_{t+s}^z\right) \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \end{array} \right] = 0,$$

where $\omega_t = \frac{\tilde{P}_{q,t}^z}{P_{q,t}} \iota_t$ and $x_t = G'^{-1}(\omega_t)$.

A.1.3 System of non-linear equations

After deriving the first order conditions of the model, we adjust variables to guarantee that the model has a balanced growth. Lower case letters stand for detrended variables, for example, $y_t = \frac{Y_t}{g_z^t}$, $w_t = \frac{W_t}{P_{C,t} g_z^t}$, $r_t^k = \frac{R_t^k}{P_{C,t}}$, $\lambda_t = \Lambda_t g_z^t$. Then using the definition $\frac{P_{m,t}}{P_{q,t}} = s_t$, we define all model equations in terms of relative prices. Given that the model is then log-linearized, we omit price and wage dispersion variables. We add exogenous shock processes for the following variables: ε_t^a, ε_t^b, ε_t^i, ε_t^r, λ_t^p, ε_t^l, g_t, s_t, $\pi_{C,t}^*$, R_t^*, y_t^*. Given that the government budget constraint is balanced every period, we can omit this equation.

$$(c_t - bc_{t-1})^{-\sigma} = \lambda_t \tag{14}$$

$$R_t = \pi_{C,t+1} g_z \frac{\lambda_t}{\beta \varepsilon_t^b \lambda_{t+1}} \tag{15}$$

$$\begin{aligned} 1 \; = \; & Q_t^k \varepsilon_t^i \left\{ 1 - \gamma_I \left(g_z \frac{i_t}{i_{t-1}} - g_z \right) g_z \frac{i_t}{i_{t-1}} - \frac{\gamma_I}{2} \left(g_z \frac{i_t}{i_{t-1}} - g_z \right)^2 \right\} \\ & + \frac{1}{g_z} \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^k \varepsilon_{t+1}^i \beta \gamma_I \left(g_z \frac{i_{t+1}}{i_t} - g_z \right) \left(g_z \frac{i_{t+1}}{i_t} \right)^2 \end{aligned} \tag{16}$$

$$\frac{1}{g_z}\frac{\lambda_{t+1}}{\lambda_t}\beta\left\{\left[r_{t+1}^k u_{t+1}-a\left(u_{t+1}\right)\right]+Q_{t+1}^k\left(1-\delta\right)\right\}=Q_t^k \tag{17}$$

$$r_t^k=\left[\gamma_{u1}+\gamma_{u2}\left(u_t-1\right)\right] \tag{18}$$

$$k_{t+1}=\left(1-\delta\right)\frac{k_t}{g_z}+\varepsilon_t^i\left[1-\frac{\gamma_I}{2}\left(g_z\frac{i_t}{i_{t-1}}-g_z\right)^2\right]i_t \tag{19}$$

$$\begin{aligned}\frac{\left(\frac{1-\mu s_t^{1-\epsilon}}{1-\mu}\right)^{\frac{1}{1-\epsilon}}}{\left(\varpi_c s_t^{1-\upsilon}+1-\varpi_c\right)^{\frac{1}{1-\upsilon}}}y_t &= c_t+\left(\varpi_c s_t^{1-\upsilon}+1-\varpi_c\right)^{-\frac{1}{1-\upsilon}}g_t \\ &+\left(\frac{\varpi_i s_t^{1-\upsilon}+1-\varpi_i}{\varpi_c s_t^{1-\upsilon}+1-\varpi_c}\right)^{\frac{1}{1-\upsilon}}\left[i_t+\frac{a\left(u_t\right)k_t}{g_{z,t}}\right] \\ &+\left(\varpi_c s_t^{1-\upsilon}+1-\varpi_c\right)^{-\frac{1}{1-\upsilon}}ex_t-s_t\left(\varpi_c s_t^{1-\upsilon}+1-\varpi_c\right)^{-\frac{1}{1-\upsilon}}im_t\end{aligned} \tag{20}$$

$$w_t=\left(w_{t-1}\right)^{\gamma}\left(MRS_t\right)^{1-\gamma} \tag{21}$$

$$MRS_t=\left(c_t^o-bc_{t-1}\right)^{\sigma}\left(h_t^o\right)^{\phi_l} \tag{22}$$

$$MRS_t^{rt}=\left(c_t-bc_{t-1}\right)^{\sigma}\varepsilon_t^l\left(h_t\right)^{\phi_l} \tag{23}$$

$$\frac{u_t k_t}{h_t g_z}=\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{24}$$

$$mc_t = (\varepsilon_t^a)^{-1} \left[\begin{array}{c} (1-\mu)\alpha^{-\alpha(1-\epsilon)} (1-\alpha)^{-(1-\alpha)(1-\epsilon)} \left(r_t^k\right)^{\alpha(1-\epsilon)} (w_t)^{(1-\alpha)(1-\epsilon)} \\ +\mu \frac{s_t^{1-\epsilon}}{\left(\varpi_c s_t^{1-\upsilon}+1-\varpi_c\right)^{\frac{1-\epsilon}{1-\upsilon}}} \end{array} \right]^{\frac{1}{1-\epsilon}} \tag{25}$$

$$q_t = \varepsilon_t^a \left\{ (1-\mu)^{\frac{1}{\epsilon}} \left[\left(u_t \frac{k_t}{g_{z,t}} \right)^{\alpha} (h_t)^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \mu^{\frac{1}{\epsilon}} (m_t)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}} - \Phi \tag{26}$$

$$E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \, \varepsilon_t^b \frac{\lambda_{t+s}^o}{\lambda_t^o} q_{t+s}^z \left[\begin{array}{c} \tilde{p}_{q,t}^z \frac{\pi_{q,t,t+s-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_{q,t,t+s}} \left(1 + \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})}\right) \\ -mc_{t+s} \left(\varpi_c s_{t+s}^{1-\upsilon} + 1 - \varpi_c\right)^{\frac{1}{1-\upsilon}} \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \end{array} \right] = 0 \tag{27}$$

$$\begin{aligned} 1 \;=\;& (1-\xi_p)\, \tilde{p}_{q,t}^z G'^{-1} \left(\tilde{p}_{q,t}^z \int_0^1 G' \left(\frac{q_{t+s}^z}{q_{t+s}} \right) \frac{q_{t+s}^z}{q_{t+s}} dz \right) \\ & + \xi_p \pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} \pi_{q,t}^{-1} G'^{-1} \left(\pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} \pi_{q,t}^{-1} \int_0^1 G' \left(\frac{q_{t+s}^z}{q_{t+s}} \right) \frac{q_{t+s}^z}{q_{t+s}} dz \right) \end{aligned} \tag{28}$$

$$m_t = \mu \, (\varepsilon_t^a)^{\epsilon-1} \, mc_t^{\epsilon} \left(\frac{s_t}{\left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{\frac{1}{1-\upsilon}}} \right)^{-\epsilon} (q_t + \Phi) \tag{29}$$

$$c_{m,t} = \varpi_c s_t^{-\upsilon} \left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{\frac{\upsilon}{1-\upsilon}} c_t \tag{30}$$

$$c_{q,t} = (1-\varpi_c) \left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{\frac{\upsilon}{1-\upsilon}} c_t \tag{31}$$

$$q_{m,t}^I = \varpi_i s_t^{-\upsilon} \left(\varpi_i s_t^{1-\upsilon} + 1 - \varpi_i\right)^{\frac{\upsilon}{1-\upsilon}} q_t^I \tag{32}$$

$$q_{q,t}^{I} = (1-\varpi_i)\left(\varpi_i s_t^{1-\upsilon} + 1 - \varpi_i\right)^{\frac{\upsilon}{1-\upsilon}} q_t^{I} \tag{33}$$

$$\left(\frac{1-\mu s_t^{1-\epsilon}}{1-\mu}\right)^{\frac{1}{1-\epsilon}} y_t = q_t - s_t m_t \tag{34}$$

$$q_t^{I} = i_t + a\left(u_t\right)\frac{k_t}{g_{z,t}} \tag{35}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R}\left[\left(\frac{\pi_{C,t}}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{y_t^{flex}}\right)^{\phi_y}\right]^{1-\phi_R}\left(\frac{y_t}{y_t^{flex}}\frac{y_{t-1}^{flex}}{y_{t-1}}\right)^{\phi_{\Delta y}} \varepsilon_t^{r} \tag{36}$$

$$\frac{\pi_{C,t}}{\pi_{q,t}} = \left(\frac{\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c}{\varpi_c s_{t-1}^{1-\upsilon} + 1 - \varpi_c}\right)^{\frac{1}{1-\upsilon}} \tag{37}$$

$$\frac{\pi_{y,t}}{\pi_{q,t}} = \left(\frac{1-\mu s_t^{1-\epsilon}}{1-\mu s_{t-1}^{1-\epsilon}}\right)^{\frac{1}{1-\epsilon}} \tag{38}$$

$$nx_t = \left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{-\frac{1}{1-\upsilon}} ex_t - s_t\left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{-\frac{1}{1-\upsilon}} im_t \tag{39}$$

$$a_t = \frac{R_{t-1}^{*}}{g_{z,t}\pi_{C,t}^{*}} a_{t-1} + \frac{nx_t}{RER_t} \tag{40}$$

$$im_t = m_t + c_{m,t} + q_{m,t}^{I} \tag{41}$$

$$\frac{ER_{t+1}R_t^{*}}{ER_t\Gamma_t\varepsilon_t^{b}R_t} = 1 \tag{42}$$

$$\Gamma_t = \exp\left\{\Gamma^{b}\frac{a_t}{y}RER_t\right\} \tag{43}$$

$$ex_t = \left(\varpi_c s_t^{1-v} + 1 - \varpi_c\right)^{\frac{\eta}{1-v}} RER_t^{\eta} y_t^* \tag{44}$$

$$\frac{RER_t}{RER_{t-1}} = \frac{ER_t}{ER_{t-1}} \frac{\pi_{C,t}^*}{\pi_{C,t}} \tag{45}$$

A.1.4 System of log-linearized equations

The above equations are log-linearized. Hatted variables are in log-deviation from their steady state. Some variables are expressed in deviation from steady state output, i.e. $\tilde{x}_t = \frac{x_t - x}{y}$. We define $A = \frac{1}{\lambda^p \alpha^p + 1}$, where α^p is elasticity of substitution between goods. We assume $s = 1$ in steady state, so that all relative prices are equal to 1 at steady state. The steady state of all domestic inflation measures is the same and corresponds to π. It is implicit that the system below is completed with flexible prices and wages equilibrium conditions which are not reported here.

$$-\sigma \frac{1}{1-b} \hat{c}_t + \sigma \frac{b}{1-b} \hat{c}_{t-1} = \hat{\lambda}_t \tag{46}$$

$$\hat{R}_t = -\hat{\varepsilon}_t^b + \hat{\pi}_{C,t+1} + \hat{\lambda}_t - \hat{\lambda}_{t+1} \tag{47}$$

$$\hat{\imath}_t = \frac{1}{\gamma_I g_z^2 (1+\beta)} \left(\hat{Q}_t^k + \hat{\varepsilon}_t^i\right) + \frac{1}{1+\beta} \hat{\imath}_{t-1} + \frac{\beta}{1+\beta} \hat{\imath}_{t+1} \tag{48}$$

$$\hat{\lambda}_{t+1} - \hat{\lambda}_t + \frac{\beta}{g_z} r^k \hat{r}_{t+1}^k + \frac{\beta}{g_z} (1-\delta) \hat{Q}_{t+1}^k = \hat{Q}_t^k \tag{49}$$

$$\hat{r}_t^k = \frac{\gamma_{u2}}{r^k} \hat{u}_t \tag{50}$$

$$\hat{k}_{t+1} = \frac{(1-\delta)}{g_z}\hat{k}_t + \frac{i}{k}\hat{\imath}_t + \frac{i}{k}\hat{\varepsilon}_t^i \tag{51}$$

$$\begin{aligned} 0 =& \frac{c}{y}\hat{c}_t + \tilde{g}_t + \frac{i}{y}\hat{\imath}_t - \hat{y}_t + \frac{\gamma_{u1}k}{yg_z}\hat{u}_t + \frac{ex}{y}\widehat{ex}_t - \frac{im}{y}\widehat{im}_t \\ &+ \left[(\varpi_i - \varpi_c)\frac{i}{y} + \frac{\mu}{1-\mu} + \varpi_c - \frac{ex}{y} - \varpi_c\frac{g}{y}\right]\hat{s}_t \end{aligned} \tag{equ.8}$$

$$\hat{m}_t = (\epsilon - 1)\hat{\varepsilon}_t^a + \epsilon\widehat{mc}_t + \frac{q}{q+\Phi}\hat{q}_t - \epsilon(1-\varpi_c)\hat{s}_t \tag{equ.10}$$

$$\frac{y}{q}\left(\hat{y}_t - \frac{\mu}{1-\mu}\hat{s}_t\right) = \hat{q}_t - \frac{m}{q}(\hat{s}_t + \hat{m}_t) \tag{52}$$

$$\hat{q}_t^I = \hat{\imath}_t + \frac{\gamma_{u1}}{g_z}\frac{k}{i}\hat{u}_t \tag{53}$$

$$\begin{aligned} (1+\beta\chi_p)\hat{\pi}_{q,t} \quad = \quad & \chi_p\hat{\pi}_{q,t-1} + \beta\hat{\pi}_{q,t+1} - \beta(1-\chi_p)\hat{\overline{\pi}}_{t+1} + (1-\chi_p)\hat{\overline{\pi}}_t \\ & + A\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left(\widehat{mc}_t + \hat{\lambda}_t^p + \varpi_c\hat{s}_t\right) \end{aligned} \tag{54}$$

$$\hat{w}_t = \gamma\hat{w}_{t-1} + (1-\gamma)\widehat{MRS}_t \tag{55}$$

$$\widehat{MRS}_t = \frac{\sigma}{1-b}\hat{c}_t - \frac{b\sigma}{1-b}\hat{c}_{t-1} + \phi_l\hat{h}_t + \hat{\varepsilon}_t^l \tag{56}$$

$$\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t^k \tag{57}$$

$$\widehat{mc}_t = -\hat{\varepsilon}_t^a + \alpha \left(1 - \mu mc^{\epsilon - 1}\right) \hat{r}_t^k + (1-\alpha)\left(1 - \mu mc^{\epsilon-1}\right) \hat{w}_t + \mu mc^{\epsilon-1} \left(1 - \varpi_c\right) \hat{s}_t \quad (58)$$

$$\hat{q}_t = \frac{q + \Phi}{q} \left\{ \hat{\varepsilon}_t^a + \left(1 - \mu mc^{\epsilon-1}\right) \left[\alpha \left(\hat{k}_t + \hat{u}_t - \hat{g}_{z,t} \right) + (1-\alpha) \hat{h}_t \right] + \mu mc^{\epsilon-1} \hat{m}_t \right\} \quad (59)$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1-\phi_R) \left(\phi_\pi \left(\hat{\pi}_t - \hat{\overline{\pi}}_t \right) + \phi_y \left(\hat{y}_t - \hat{y}_t^{flex} \right) \right) + \phi_{\Delta y} \left(\hat{y}_t - \hat{y}_{t-1} - \left(\hat{y}_t^{flex} - \hat{y}_{t-1}^{flex} \right) \right) + \hat{\varepsilon}_t^r \quad (60)$$

$$\hat{c}_{m,t} = \hat{c}_t - \upsilon \left(1 - \varpi_c\right) \hat{s}_t \quad (61)$$

$$\hat{c}_{q,t} = \hat{c}_t + \upsilon \varpi_c \hat{s}_t \quad (62)$$

$$\hat{q}_{m,t}^I = \hat{q}_t^I - \upsilon \left(1 - \varpi_i\right) \hat{s}_t \quad (63)$$

$$\hat{q}_{q,t}^I = \hat{q}_t^I + \upsilon \varpi_i \hat{s}_t \quad (64)$$

$$\hat{\pi}_{C,t} - \hat{\pi}_{q,t} = \varpi_c \left(\hat{s}_t - \hat{s}_{t-1} \right) \quad (65)$$

$$\hat{\pi}_{y,t} - \hat{\pi}_{q,t} = -\frac{\mu}{1-\mu} \left(\hat{s}_t - \hat{s}_{t-1} \right) \quad (66)$$

$$\widetilde{nx}_t = \frac{ex}{y}\widehat{ex}_t - \frac{im}{y}\widehat{im}_t - \frac{ex}{y}\hat{s}_t \tag{67}$$

$$\tilde{a}_t = \frac{R^*}{g_z\pi}\tilde{a}_{t-1} + \widetilde{nx}_t \tag{68}$$

$$\widehat{im}_t = \frac{m}{y}\frac{y}{im}\hat{m}_t + \varpi_c\frac{c}{y}\frac{y}{im}\hat{c}_{m,t} + \varpi_i\frac{i}{y}\frac{y}{im}\hat{q}^I_{m,t} \tag{69}$$

$$\hat{R}_t = \widehat{\Delta ER}_{t+1} + \hat{R}^*_t - \hat{\Gamma}_t - \hat{\varepsilon}^b_t \tag{70}$$

$$\hat{\Gamma}_t = \Gamma^b\tilde{a}_t \tag{71}$$

$$\widehat{ex}_t = \eta\varpi_c\hat{s}_t + \eta\widehat{RER}_t + \hat{y}^*_t \tag{72}$$

$$\widehat{RER}_t - \widehat{RER}_{t-1} = \widehat{\Delta ER}_t + \hat{\pi}^*_{C,t} - \hat{\pi}_{C,t} \tag{73}$$

Finally, following Christoffel et al. (2008) and Albonico et al. (2019), the auxiliary equation relating observed employment to unobserved hours worked is given by:

$$\widehat{e}_t = \frac{\beta}{1+\beta}\widehat{e}_{t+1} + \frac{1}{1+\beta}\widehat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e}\left(\widehat{h}_t - \widehat{e}_t\right) \tag{74}$$

A.2 Additional Results

- Figures A1-A4 plot the impulse response functions (IRFs) to the additional shocks from the baseline estimation.

- Table A1 reports the parameter estimates when using alternative shadow rate measure (Wu and Xia) as an observable.

- Figures A5-A13 plot the IRFs to the shocks when using alternative shadow rate measure (Wu and Xia) as an observable.
- Table A2 reports the parameter estimates for the alternative sample period (1999Q1-2019Q4).
- Figures A14-A22 plot the IRFs for the alternative sample period (1999Q1-2019Q4).
- Table A3 reports the forecast error variance decomposition under indeterminacy for the baseline estimation and the robustness checks.

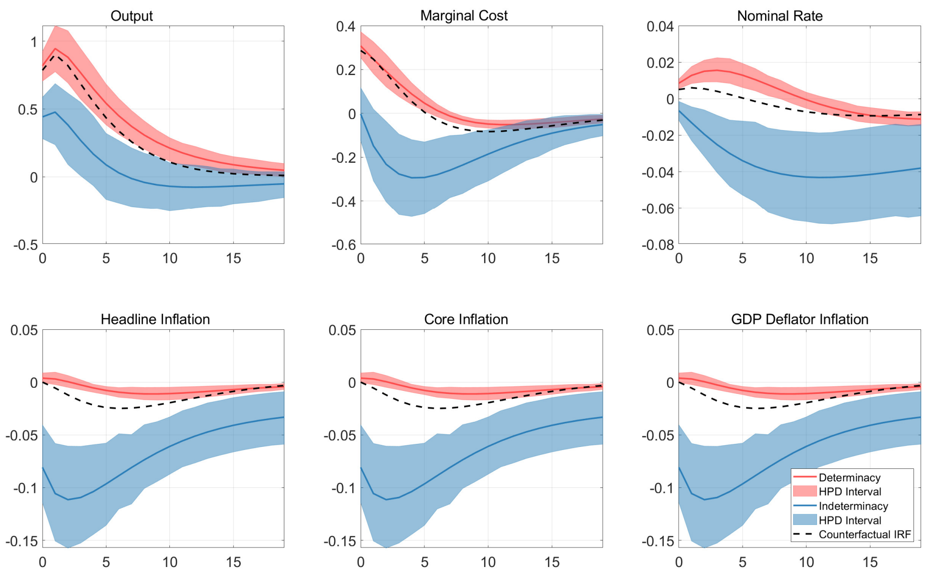

Figure A1. Impulse responses to a one standard deviation investment shock from the baseline estimation. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy where the correlation with the sunspot shock is shut down.

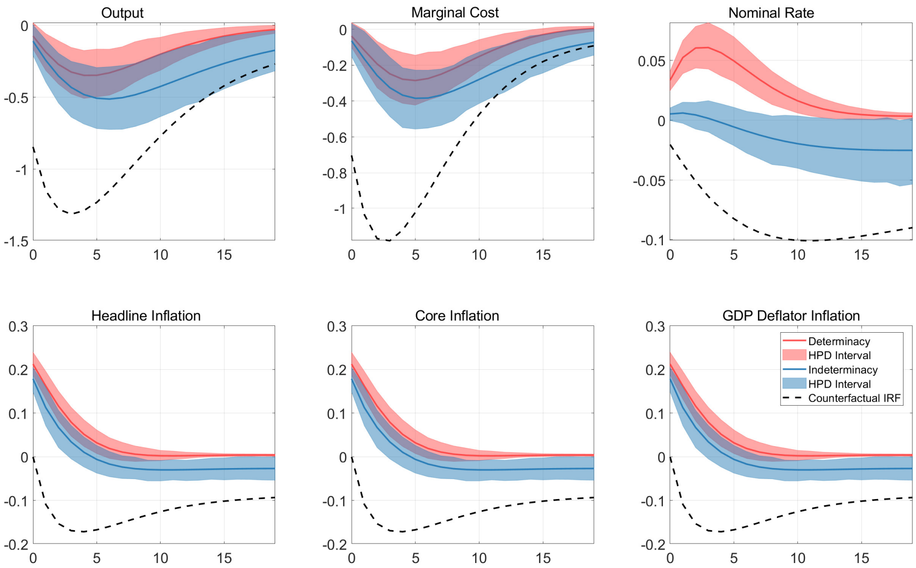

Figure A2. Impulse responses to a one standard deviation price markup shock from the baseline estimation. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy where the correlation with the sunspot shock is shut down.

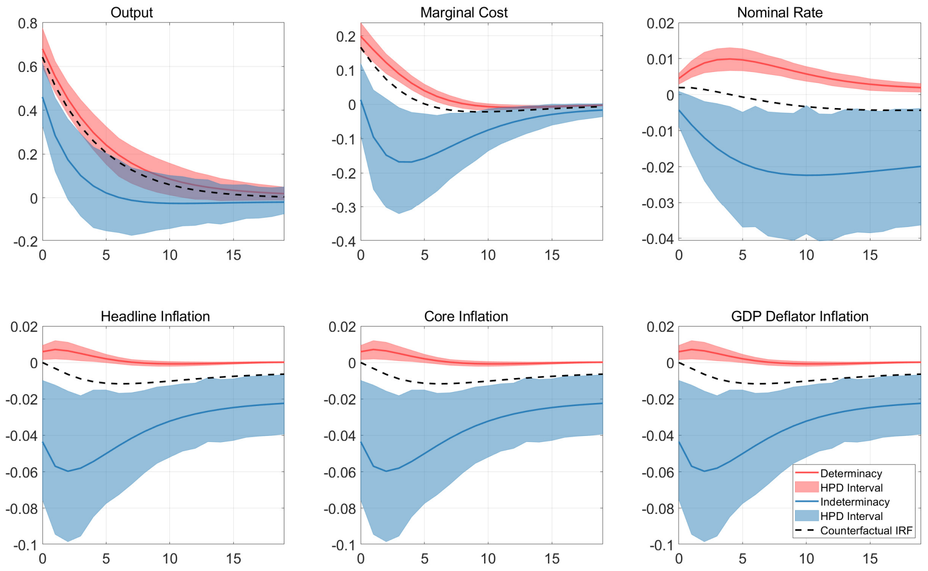

Figure A3. Impulse responses to a one standard deviation government spending shock from the baseline estimation. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions. The dashed black line is the counterfactual IRF computed at the posterior mean under indeterminacy where the correlation with the sunspot shock is shut down.

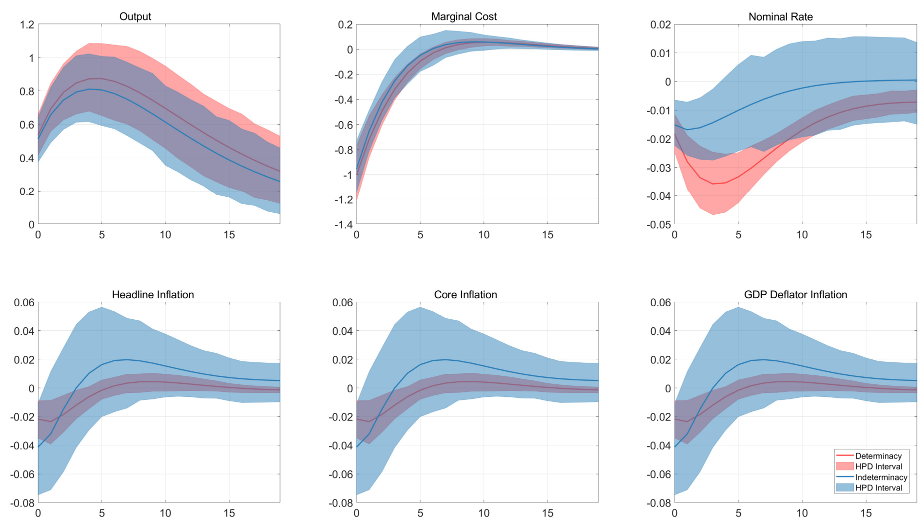

Figure A5. Impulse responses to a one standard deviation technology shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

Table A1. Parameter estimates - Wu and Xia's shadow rate

		Priors			Determinacy			Indeterminacy		
		shape	mean	st. dev.	post. mean	90% HPD interval		post. mean	90% HPD interval	
TR response to inflation	ϕ_π	norm	1	0.35	1.743	1.21	2.262	0.575	0.251	0.941
TR response to output	ϕ_y	norm	0.1	0.05	0.208	0.152	0.266	0.101	0.021	0.167
TR response to output growth	ϕ_{gy}	norm	0.1	0.05	0.016	0.01	0.022	0.024	0.01	0.039
TR interest rate smoothing	ϕ_R	beta	0.75	0.1	0.898	0.876	0.922	0.938	0.910	0.966
inverse Frisch elasticity	ϕ_l	gamm	2	0.75	0.206	0.094	0.316	0.216	0.104	0.323
habits	b	beta	0.7	0.1	0.435	0.344	0.522	0.341	0.231	0.448
investment adjustment costs	γ_I	gamm	4	1.5	4.533	2.878	6.145	3.537	1.967	5.036
Calvo price stickiness	ξ_p	beta	0.5	0.1	0.87	0.828	0.913	0.794	0.733	0.857
real wage rigidity	γ	beta	0.5	0.2	0.691	0.630	0.753	0.534	0.373	0.695
Employment parameter	ξ_e	beta	0.5	0.1	0.465	0.340	0.592	0.415	0.303	0.523
price indexation	χ_p	beta	0.5	0.15	0.476	0.253	0.698	0.312	0.129	0.494
capital utilization elasticity	σ_u	beta	0.5	0.15	0.841	0.742	0.946	0.849	0.749	0.952
intertemporal elasticity	σ	norm	1.5	0.37	0.990	0.835	1.146	0.946	0.787	1.101
inputs elasticity	ϵ	gamm	0.5	0.2	0.238	0.098	0.372	0.212	0.085	0.334
home/imported goods elast.	υ	gamm	0.5	0.2	0.459	0.173	0.727	0.425	0.170	0.674
ss growth	g_z	norm	0.2	0.1	0.256	0.224	0.288	0.248	0.218	0.279
ss hours	$\bar{E}$	norm	0	2	2.020	0.272	3.778	1.414	-0.323	3.129
ss inflation	$\bar{\pi}$	gamm	0.5	0.1	0.571	0.427	0.708	0.474	0.320	0.623
Shocks persistences										
risk premium	ρ_b	beta	0.7	0.1	0.927	0.902	0.952	0.937	0.903	0.972
investment	ρ_i	beta	0.7	0.1	0.339	0.227	0.448	0.342	0.225	0.460
monetary	ρ_r	beta	0.3	0.1	0.301	0.190	0.414	0.393	0.274	0.510
price markup	ρ_p	beta	0.7	0.1	0.710	0.603	0.816	0.813	0.703	0.924
labor supply	ρ_l	beta	0.7	0.1	0.875	0.812	0.940	0.907	0.860	0.958
gov spending	ρ_g	beta	0.7	0.1	0.861	0.811	0.912	0.867	0.821	0.914
technology	ρ_a	beta	0.7	0.1	0.885	0.833	0.938	0.864	0.795	0.933
energy price	ρ_s	beta	0.9	0.05	0.976	0.963	0.989	0.974	0.960	0.988
MA price markup	ρ^p_{ma}	beta	0.5	0.1	0.545	0.411	0.687	0.556	0.411	0.709
gy correlation	ρ_{gy}	norm	0.5	0.25	0.123	0.01	0.223	0.137	0.01	0.248
Shocks standard deviations										
risk premium	σ_b	invg	0.1	2	0.191	0.147	0.232	0.232	0.161	0.302
investment	σ_i	invg	0.1	2	1.102	0.937	1.267	1.123	0.959	1.285
monetary	σ_r	invg	0.1	2	0.143	0.123	0.163	0.142	0.124	0.161
price markup	σ_p	invg	0.1	2	0.101	0.078	0.124	0.098	0.072	0.124
labor supply	σ_l	invg	0.1	2	1.249	0.974	1.524	0.989	0.752	1.217
government spending	σ_g	invg	0.1	2	0.782	0.684	0.876	0.773	0.672	0.870
technology	σ_a	invg	0.1	2	0.897	0.733	1.053	0.863	0.709	1.012
energy price	σ_s	invg	2	2	3.149	2.779	3.514	3.163	2.782	3.525
measurement error	$\sigma^{me}_{\pi_y}$	invg	0.1	2	0.325	0.286	0.362	0.323	0.285	0.362
sunspot	σ_ν	unif	0.5	0.289	-	-	-	0.245	0.207	0.281
Shocks correlations										
corr sunspot, risk premium	$\rho_{\nu b}$	unif	0	0.577	-	-	-	0.319	0.134	0.504
corr sunspot, investment	$\rho_{\nu i}$	unif	0	0.577	-	-	-	-0.283	-0.438	-0.124
corr sunspot, monetary	$\rho_{\nu r}$	unif	0	0.577	-	-	-	0.317	0.207	0.432
corr sunspot, price markup	$\rho_{\nu p}$	unif	0	0.577	-	-	-	0.725	0.621	0.831
corr sunspot, labor supply	$\rho_{\nu l}$	unif	0	0.577	-	-	-	-0.131	-0.280	0.022
corr sunspot, gov spending	$\rho_{\nu g}$	unif	0	0.577	-	-	-	-0.112	-0.257	0.026
corr sunspot, technology	$\rho_{\nu a}$	unif	0	0.577	-	-	-	-0.168	-0.292	-0.045
corr sunspot, energy price	$\rho_{\nu s}$	unif	0	0.577	-	-	-	0.209	0.091	0.325
Foreign parameters										
SS foreign inflation	$\bar{\pi}^*$	gamm	0.6	0.1	0.605	0.479	0.730	0.605	0.479	0.729
SS foreign int rate	$\bar{R}^*$	gamm	0.3	0.1	0.363	0.223	0.495	0.363	0.226	0.493
foreign demand persistence	ρ^*_y	beta	0.7	0.1	0.915	0.875	0.957	0.917	0.877	0.958
foreign inflation persistence	ρ^*_π	beta	0.7	0.1	0.516	0.408	0.624	0.515	0.408	0.620
foreign rate persistence	ρ^*_R	beta	0.3	0.1	0.869	0.846	0.898	0.865	0.840	0.896
foreign demand std dev	σ^*_y	invg	0.1	2	0.642	0.563	0.716	0.643	0.565	0.718
foreign inflation std dev	σ^*_π	invg	0.1	2	0.571	0.504	0.638	0.570	0.504	0.635
foreign rate std dev	σ^*_R	invg	0.1	2	0.159	0.138	0.179	0.160	0.139	0.180
Log data density						-1085.6			-1065.3	

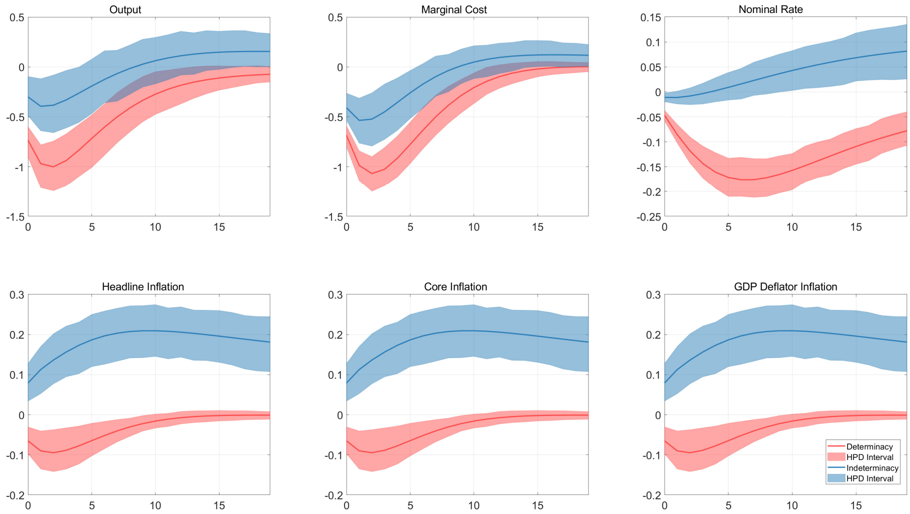

Figure A6. Impulse responses to a one standard deviation risk premium shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

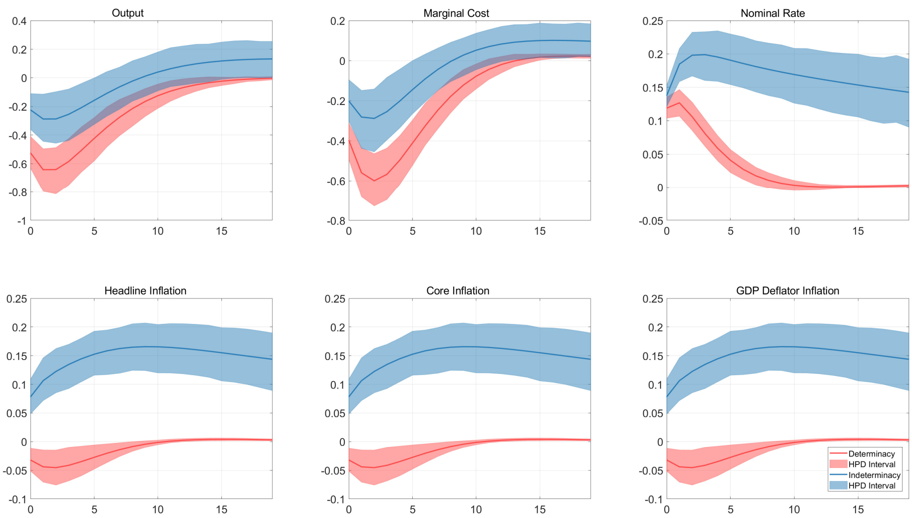

Figure A7. Impulse responses to a one standard deviation monetary policy shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

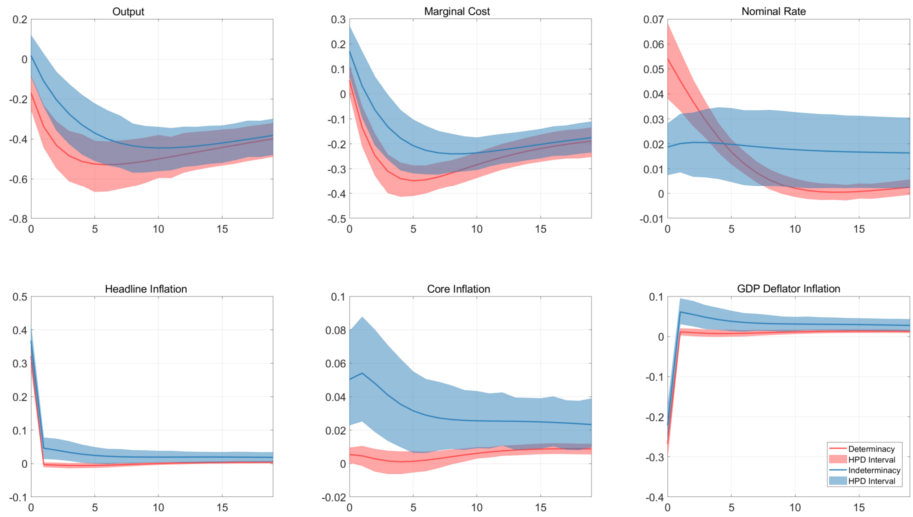

Figure A8. Impulse responses to a one standard deviation energy price shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

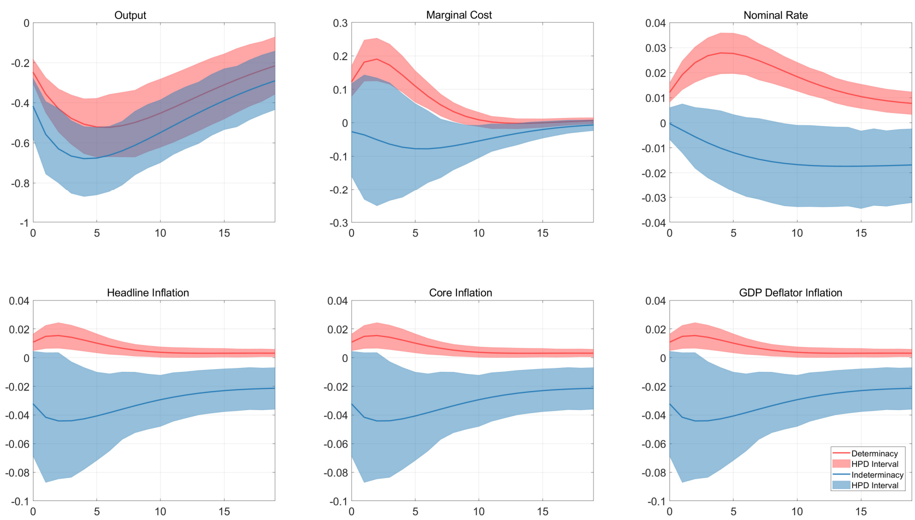

Figure A9. Impulse responses to a one standard deviation labor supply shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

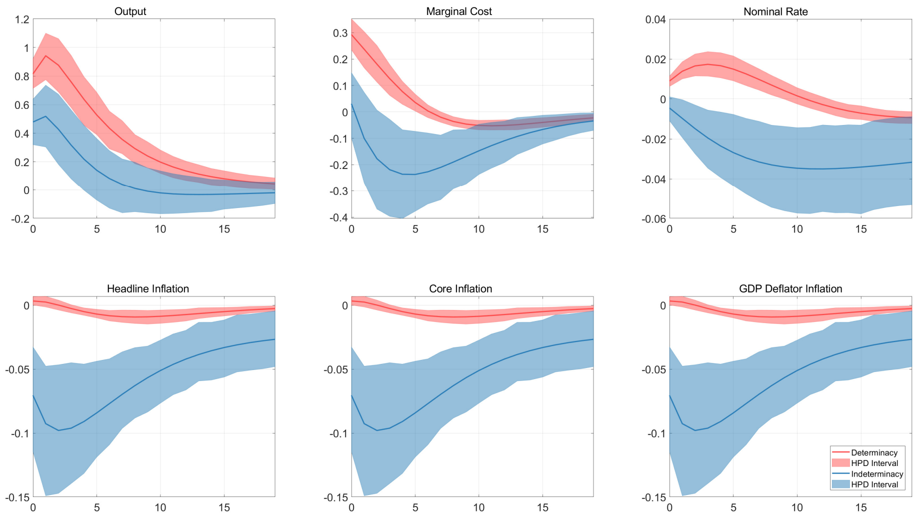

Figure A10. Impulse responses to a one standard deviation investment shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

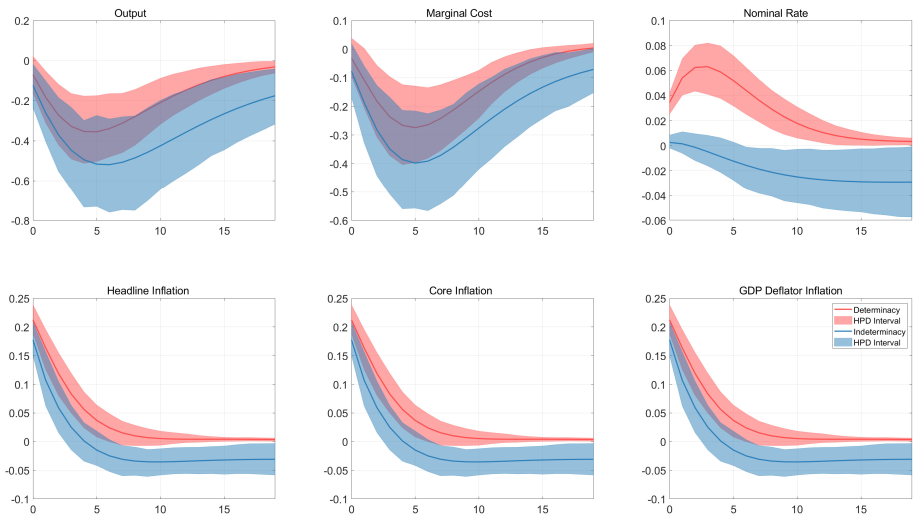

Figure A11. Impulse responses to a one standard deviation price markup shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

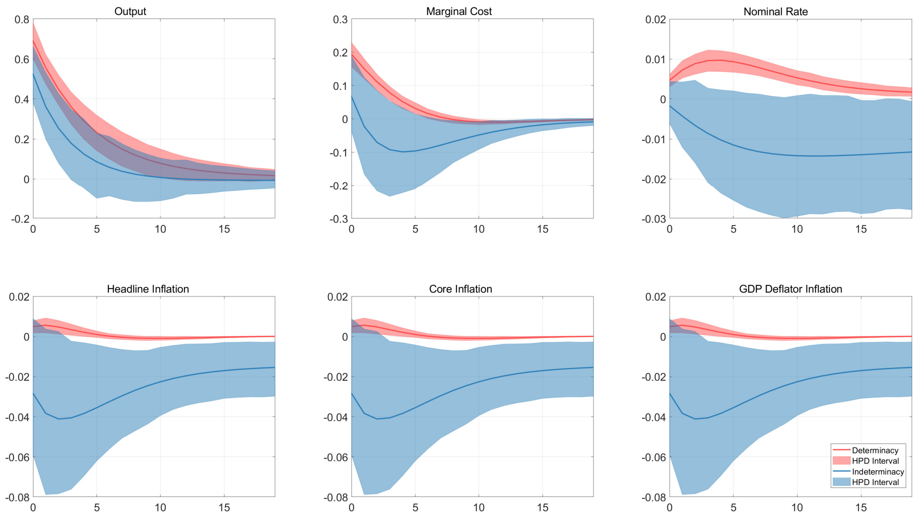

Figure A12. Impulse responses to a one standard deviation government spending shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

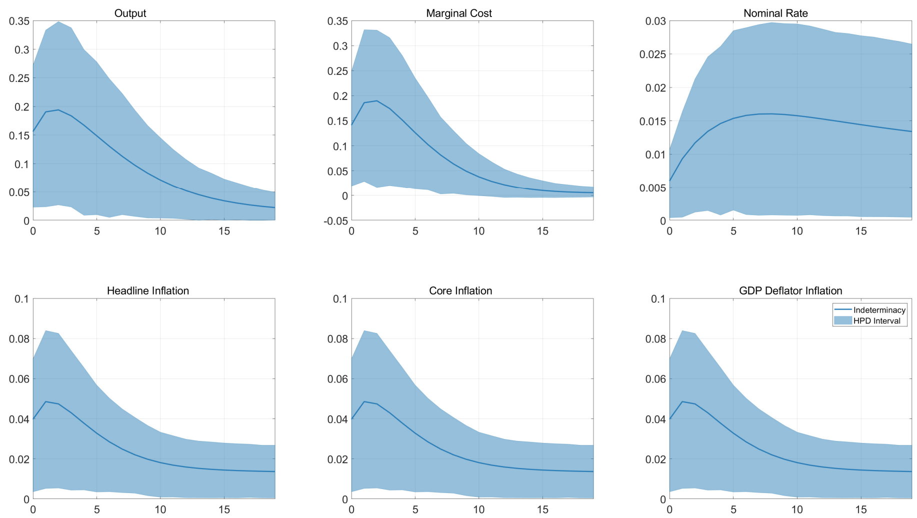

Figure A13. Impulse responses to a one standard deviation sunspot shock using Wu and Xia's shadow rate as observable. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

Table A2. Parameter estimates - 1999Q1-2019Q4

		Priors			Determinacy			Indeterminacy		
		shape	mean	st. dev.	post. mean	90% HPD interval		post. mean	90% HPD interval	
TR response to inflation	ϕ_π	norm	1	0.35	1.451	0.962	1.845	0.660	0.367	0.974
TR response to output	ϕ_y	norm	0.1	0.05	0.205	0.148	0.263	0.099	0.021	0.167
TR response to output growth	ϕ_{gy}	norm	0.1	0.05	0.077	0.018	0.133	0.092	0.031	0.148
TR interest rate smoothing	ϕ_R	beta	0.75	0.1	0.848	0.807	0.891	0.918	0.871	0.966
inverse Frisch elasticity	ϕ_l	gamm	2	0.75	0.446	0.223	0.664	0.758	0.431	1.084
habits	b	beta	0.7	0.1	0.707	0.628	0.790	0.700	0.625	0.780
investment adjustment costs	γ_I	gamm	4	1.5	4.088	2.361	5.819	4.440	2.561	6.255
Calvo price stickiness	ξ_p	beta	0.5	0.1	0.904	0.873	0.936	0.655	0.577	0.730
real wage rigidity	γ	beta	0.5	0.2	0.789	0.718	0.863	0.723	0.640	0.812
Employment parameter	ξ_e	beta	0.5	0.1	0.544	0.404	0.682	0.428	0.310	0.546
price indexation	χ_p	beta	0.5	0.15	0.353	0.133	0.575	0.265	0.093	0.427
capital utilization elasticity	σ_u	beta	0.5	0.15	0.787	0.649	0.930	0.807	0.688	0.933
intertemporal elasticity	σ	norm	1.5	0.37	0.998	0.784	1.212	1.162	0.810	1.499
inputs elasticity	ϵ	gamm	0.5	0.2	0.263	0.106	0.415	0.226	0.088	0.359
home/imported goods elast.	υ	gamm	0.5	0.2	0.483	0.186	0.772	0.462	0.174	0.739
ss growth	g_z	norm	0.2	0.1	0.260	0.221	0.299	0.228	0.192	0.265
ss hours	$\bar{E}$	norm	0	2	1.257	-0.614	3.159	0.706	-1.180	2.572
ss inflation	$\bar{\pi}$	gamm	0.5	0.1	0.440	0.342	0.531	0.373	0.220	0.522
Shocks persistences										
risk premium	ρ_b	beta	0.7	0.1	0.919	0.878	0.962	0.856	0.748	0.961
investment	ρ_i	beta	0.7	0.1	0.286	0.166	0.402	0.360	0.221	0.495
monetary	ρ_r	beta	0.3	0.1	0.367	0.233	0.501	0.340	0.205	0.476
price markup	ρ_p	beta	0.7	0.1	0.640	0.515	0.766	0.882	0.811	0.959
labor supply	ρ_l	beta	0.7	0.1	0.882	0.816	0.951	0.934	0.894	0.974
gov spending	ρ_g	beta	0.7	0.1	0.854	0.795	0.915	0.878	0.829	0.932
technology	ρ_a	beta	0.7	0.1	0.932	0.885	0.979	0.893	0.834	0.954
energy price	ρ_s	beta	0.9	0.05	0.975	0.960	0.991	0.975	0.961	0.989
MA price markup	ρ^p_{ma}	beta	0.5	0.1	0.569	0.358	0.740	0.468	0.318	0.617
gy correlation	ρ_{gy}	norm	0.5	0.25	0.168	0.01	0.295	0.102	0.01	0.204
Shocks standard deviations										
risk premium	σ_b	invg	0.1	2	0.069	0.049	0.089	0.070	0.042	0.098
investment	σ_i	invg	0.1	2	1.170	0.996	1.346	1.149	0.961	1.326
monetary	σ_r	invg	0.1	2	0.129	0.107	0.149	0.123	0.105	0.141
price markup	σ_p	invg	0.1	2	0.094	0.071	0.118	0.109	0.081	0.135
labor supply	σ_l	invg	0.1	2	1.714	1.144	2.262	1.578	1.110	2.033
government spending	σ_g	invg	0.1	2	0.799	0.692	0.905	0.762	0.648	0.873
technology	σ_a	invg	0.1	2	0.865	0.659	1.064	0.802	0.661	0.936
energy price	σ_s	invg	2	2	2.462	2.148	2.778	2.477	2.165	2.791
measurement error	$\sigma^{me}_{\pi_y}$	invg	0.1	2	0.255	0.222	0.287	0.254	0.221	0.286
sunspot	σ_ν	unif	0.5	0.289	-	-	-	0.147	0.115	0.180
Shocks correlations										
corr sunspot, risk premium	$\rho_{\nu b}$	unif	0	0.577	-	-	-	0.111	-0.164	0.390
corr sunspot, investment	$\rho_{\nu i}$	unif	0	0.577	-	-	-	-0.178	-0.421	0.064
corr sunspot, monetary	$\rho_{\nu r}$	unif	0	0.577	-	-	-	0.122	-0.084	0.334
corr sunspot, price markup	$\rho_{\nu p}$	unif	0	0.577	-	-	-	0.772	0.629	0.919
corr sunspot, labor supply	$\rho_{\nu l}$	unif	0	0.577	-	-	-	-0.042	-0.238	0.151
corr sunspot, gov spending	$\rho_{\nu g}$	unif	0	0.577	-	-	-	0.134	-0.134	0.397
corr sunspot, technology	$\rho_{\nu a}$	unif	0	0.577	-	-	-	-0.138	-0.357	0.084
corr sunspot, energy price	$\rho_{\nu s}$	unif	0	0.577	-	-	-	0.148	-0.027	0.323
Foreign parameters										
SS foreign inflation	$\bar{\pi}^*$	gamm	0.6	0.1	0.567	0.449	0.681	0.566	0.450	0.683
SS foreign int rate	$\bar{R}^*$	gamm	0.3	0.1	0.325	0.193	0.457	0.329	0.188	0.459
foreign demand persistence	ρ^*_y	beta	0.7	0.1	0.924	0.885	0.964	0.920	0.879	0.961
foreign inflation persistence	ρ^*_π	beta	0.7	0.1	0.457	0.336	0.580	0.456	0.332	0.578
foreign rate persistence	ρ^*_R	beta	0.3	0.1	0.873	0.851	0.897	0.871	0.848	0.897
foreign demand std dev	σ^*_y	invg	0.1	2	0.588	0.511	0.662	0.592	0.512	0.667
foreign inflation std dev	σ^*_π	invg	0.1	2	0.528	0.459	0.595	0.528	0.459	0.595
foreign rate std dev	σ^*_R	invg	0.1	2	0.154	0.1331	0.175	0.155	0.133	0.176
Log data density						-709.1			-694.8	

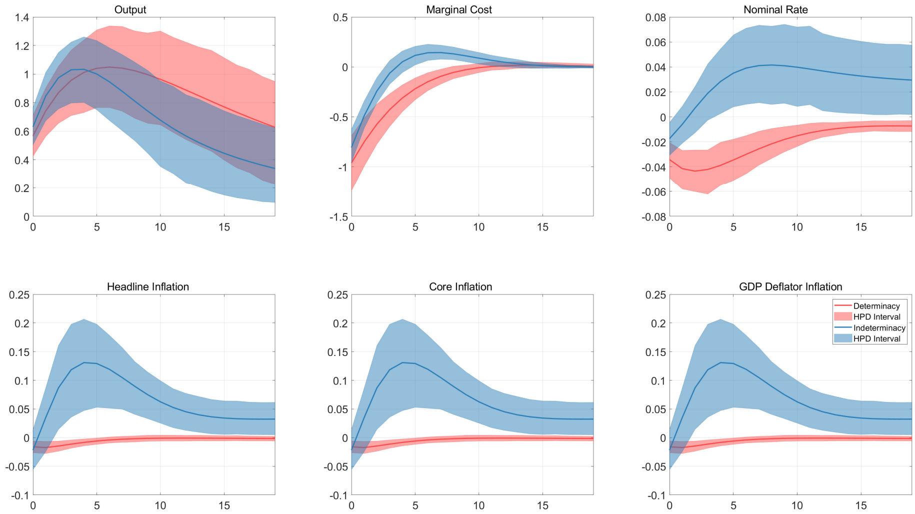

Figure A14. Impulse responses to a one standard deviation technology shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

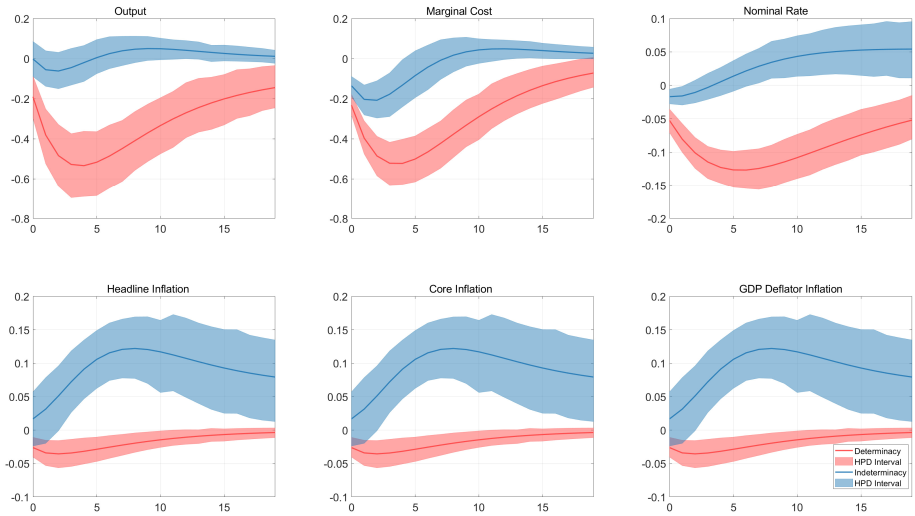

Figure A15. Impulse responses to a one standard deviation risk premium shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

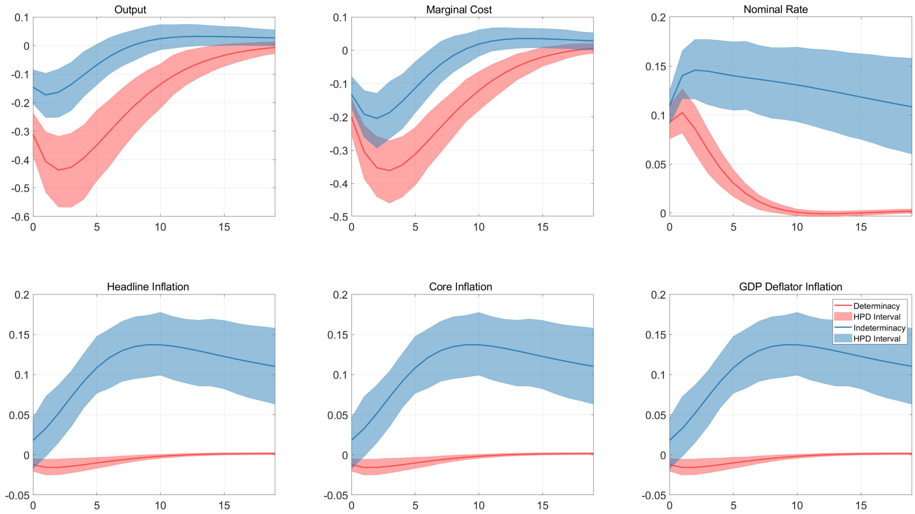

Figure A16. Impulse responses to a one standard deviation monetary policy shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

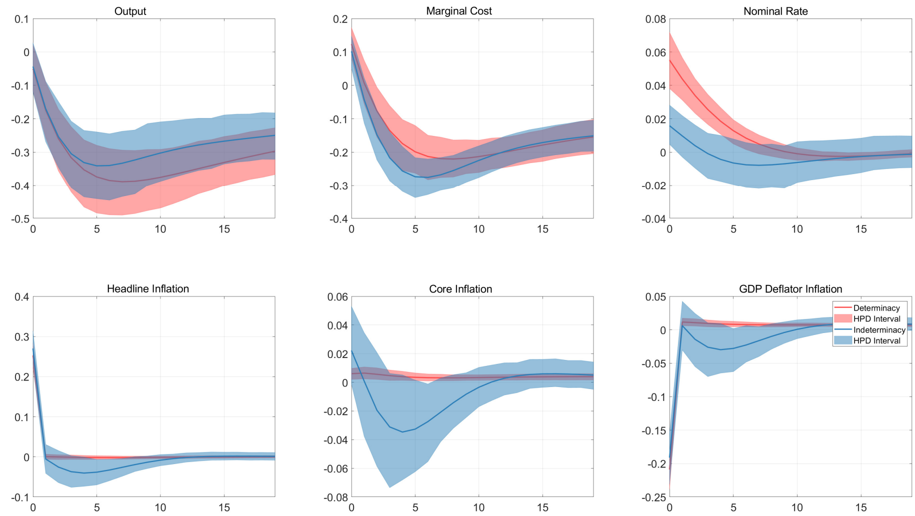

Figure A17. Impulse responses to a one standard deviation energy price shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

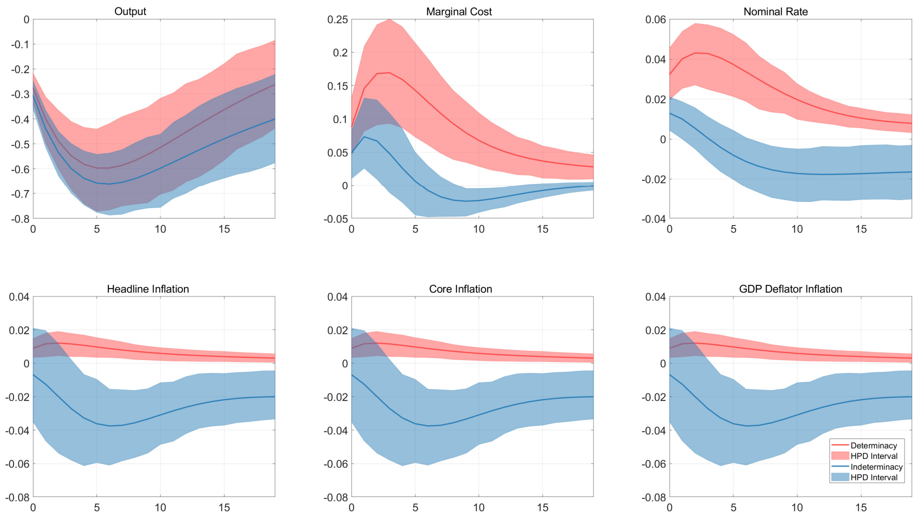

Figure A18. Impulse responses to a one standard deviation labor supply shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

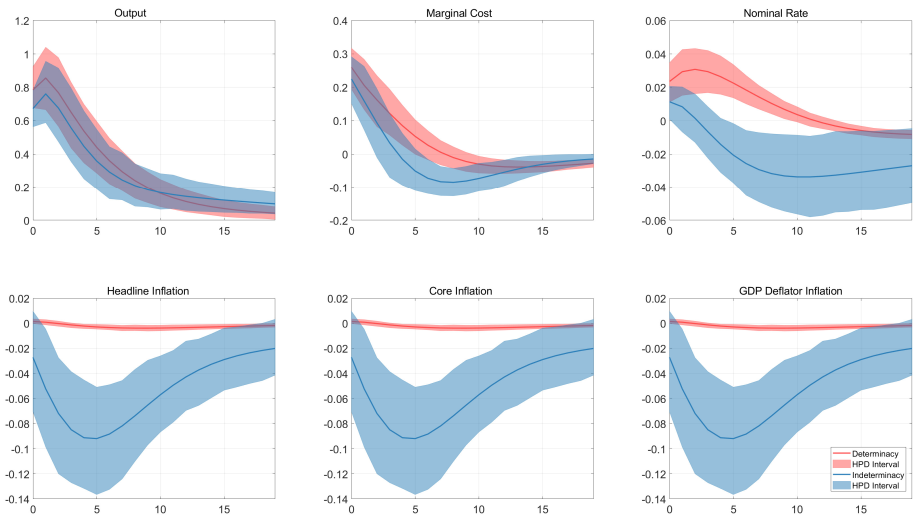

Figure A19. Impulse responses to a one standard deviation investment shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

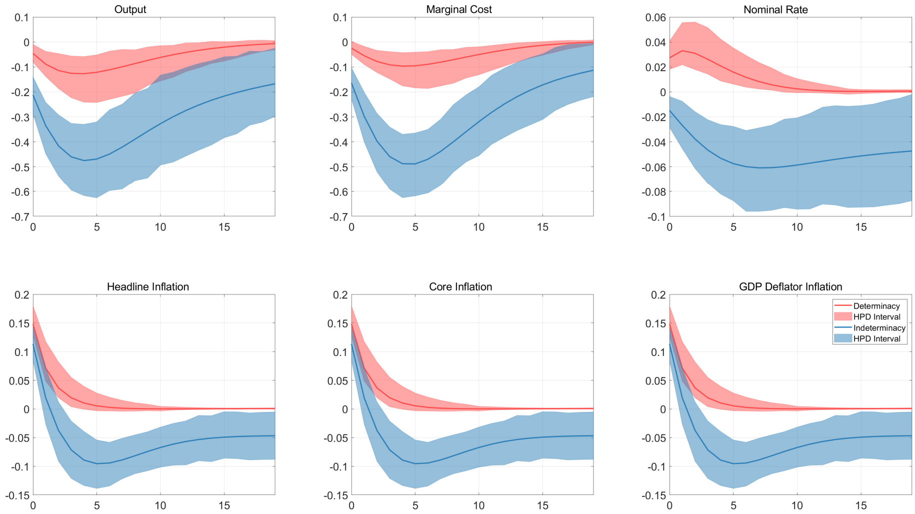

Figure A20. Impulse responses to a one standard deviation price markup shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

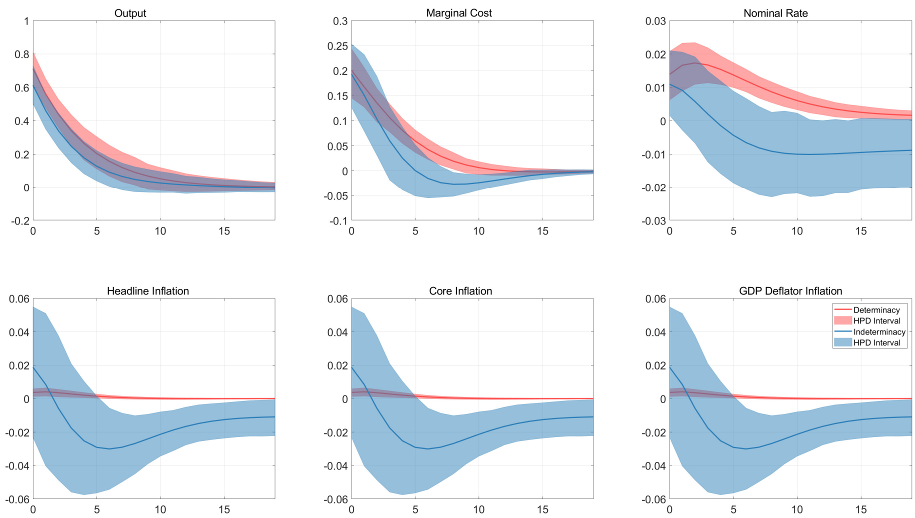

Figure A21. Impulse responses to a one standard deviation government spending shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

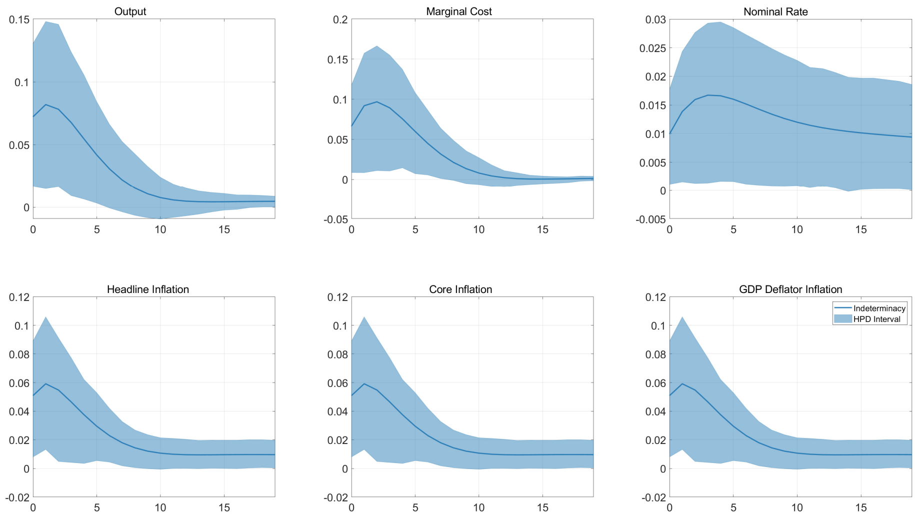

Figure A22. Impulse responses to a one standard deviation sunspot shock; Sample: 1999Q1-2019Q4. The solid lines are posterior means while the shaded areas are highest posterior density (HPD) regions.

Table A3. Variance decomposition under indeterminacy

	y	h	i	c	w	R	π_c	π_q	π_y
Baseline									
ε^b	8.7	11.6	9.0	8.0	10.1	32.9	49.6	52.5	51.1
ε^i	4.0	11.0	15.0	3.0	2.1	5.0	6.7	7.2	7.0
ε^r	4.1	4.4	2.9	2.3	3.3	51.7	24.1	25.6	24.9
ε^p	11.7	12.8	10.2	5.0	7.7	4.0	5.5	5.8	5.7
ε^l	23.5	29.1	16.3	14.6	3.6	1.5	2.0	2.1	2.1
ε^a	25.1	11.0	16.0	11.0	11.3	0.5	0.8	0.9	0.9
ε^g	1.9	4.2	3.7	3.8	3.3	2.0	2.7	2.8	2.8
ε^s	19.1	13.3	25.9	51.6	57.5	1.3	7.4	1.6	4.4
ε^ν	1.4	1.7	1.0	0.8	1.1	1.0	1.3	1.4	1.3
Alternative shadow rate: Wu and Xia									
ε^b	6.9	10.0	7.5	7.3	9.0	29.0	49.9	52.0	51.0
ε^i	4.1	10.3	15.2	2.9	2.1	2.5	3.6	3.8	3.7
ε^r	4.8	5.1	3.5	3.0	4.2	61.6	33.0	34.5	33.8
ε^p	11.8	13.6	10.6	5.2	8.0	3.4	4.3	4.5	4.4
ε^l	22.4	29.5	16.1	14.6	3.3	1.1	1.6	1.6	1.6
ε^a	26.6	10.4	17.4	12.3	12.8	0.3	0.5	0.6	0.5
ε^g	2.4	5.6	2.2	2.4	1.8	0.7	1.0	1.0	1.0
ε^s	19.4	13.2	26.6	51.6	57.7	0.8	5.2	1.1	3.0
ε^ν	1.2	1.6	0.9	0.7	1.0	0.7	0.9	0.9	0.9
Alternative sample period: 1999Q1-2019Q4									
ε^b	0.3	0.8	0.9	1.0	1.3	13.7	22.2	23.4	22.8
ε^i	9.3	16.0	23.7	7.2	5.1	2.2	5.0	5.3	5.2
ε^r	0.6	1.1	0.4	0.3	1.0	60.5	36.3	38.2	37.3
ε^p	9.4	12.0	8.6	4.7	13.0	13.8	13.3	14.0	13.7
ε^l	27.3	41.7	19.3	22.7	4.3	2.3	3.1	3.3	3.2
ε^a	38.6	9.5	26.7	22.5	26.5	5.7	11.4	12.0	11.7
ε^g	3.0	8.8	1.9	2.2	0.7	0.6	1.1	1.1	1.1
ε^s	11.2	9.4	18.2	39.2	48.0	0.6	6.3	1.2	3.6
ε^ν	0.1	0.2	0.1	0.1	0.2	0.6	1.4	1.4	1.4