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Monetary Policy in the Euro Area: Active or Passive?*

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Abstract

We estimate a medium-scale DSGE model for the euro area allowing and testing for indeterminacy since the introduction of the euro until mid-2023. Our estimates suggest that the data prefer a model implying a passive monetary policy in the euro area, leading to indeterminacy and self-fulfilling dynamics. However, this preference is fragile, depending on specific technical assumptions that can allow greater degrees of freedom to the indeterminate model. Moreover, indeterminacy yields inflation responses that contradict both standard economic theory and external non-structural evidence on the effects monetary policy shocks.

Keywords: monetary policy, indeterminacy, euro area, business cycle fluctuations, inflation

JEL classification: E32, E52, C11, C13

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1 Introduction

The establishment of the European Central Bank (ECB) in 1998 unified monetary policy in the euro area. Since then, the ECB faced a series of large and diverse shocks, including the Great Financial Crisis, the sovereign debt crisis, that triggered a prolonged period of near-zero interest rates, and recent events like the pandemic and surging energy prices. Each of these events posed enormous challenges to the ECB's monetary policy, that raise the question of whether the ECB set monetary policy according to its own primary objectives in response to these challenges.

In this paper, we estimate a medium-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model for the euro area allowing and testing for (in)determinacy since the introduction of the euro until mid-2023. The focus of our analysis is to assess the ECB monetary policy related to its unique goal: the stabilization of inflation. Through the lens of the model, the central bank follows a Taylor-type monetary policy rule. In an inflation targeting framework, this necessitates an active monetary policy, reacting more than proportionally to inflation, to control it and prevent self-fulfilling unanchored inflation dynamics.

There is an extensive body of literature on US data aimed at determining whether the FED's monetary policy was active or passive. The literature on monetary policy in the US debated about indeterminacy as a possible explanation for the inflation surge in the '70s in the US (e.g., Clarida et al., 2000, Lubik and Schorfheide, 2004, Ascari et al., 2019). Contrary to the extant literature, Haque et al. (2021) find support for determinacy in the US in the pre-Volcker period. More recently, Nicolò (2023) estimates the Smets and Wouters (2007) model to investigate monetary policy stance in the US in the post-war period, allowing for indeterminacy. Similarly, Albonico et al. (2024) estimate a New Keynesian model with rule of-thumb agents and find support for indeterminacy in the pre-Volker period. Despite the extensive and influential literature on the US, there have been no studies using a mediumscale DSGE model to assess ECB monetary policy in the euro area. Hirose (2013) estimates a small-scale two country model for the US and the euro area, and for a relatively short sample period, i.e., 1983Q1-2002Q4. He finds that the data point to a passive monetary policy for the euro area during this period. Allowing for the possibility of indeterminacy of rational expectations equilibrium is, therefore, an essential aspect of an analysis to assess the nature of the conduct of ECB monetary policy. From a methodological perspective, after the seminal contribution by Lubik and Schorfheide (2003, 2004), more recently, Farmer et al. (2015) and Bianchi and Nicolò (2021) proposed new techniques to estimate a DSGE model under indeterminacy. As in the recent contributions by Nicolò (2023) and Albonico et al. (2024), we employ the Bianchi and Nicolò (2021) methodology to estimate our model and assess the possible role of equilibrium indeterminacy in the euro area using post-ECB data. To the best of our knowledge, we are the first to estimate a medium scale model for the euro area allowing for indeterminacy.

Our sample does not contain the '70s, but it contains the Great Financial Crisis, the sovereign debt crisis in the euro area and the Covid period. These are all episodes where concerns could arise about the possibility of monetary policy being characterized by a passive behavior, because of the zero lower bound constraint. Moreover, we believe that including the recent surge in inflation is crucial for understanding the behavior of ECB monetary policy, because it is the only inflationary episode in the relatively short life of the ECB. While the recent inflation was supply driven, it could have also been possibly amplified by passive monetary policy. Therefore, on top of the typical frictions of New Keynesian models, in line with Christiano et al. (2005), we incorporate energy as an input in consumption and in production, following Blanchard and Galí (2007). We believe it is important to include energy in the model to take into account the recent inflation surge in the very last part of the sample and the dynamics of different inflation measures, i.e., headline HICP, core, GDP deflator.¹

Our findings are as follows. First, we find that monetary policy in the euro area was passive in our sample. The response of the nominal interest rate is estimated to be less than proportional to headline inflation changes from target. Hence, the data prefer a specification of the model that implies indeterminacy and self-fulfilling business cycle fluctuations driven by a sunspot shock, rather than a specification that implies determinacy and a unique equilibrium. However, this finding is not robust. More specifically, it depends on: (i) which variable forecast error, together with the sunspot, enters in the auxiliary variable specification in the Bianchi and Nicolò (2021) methodology; (ii) allowing or not for correlation between the sunspot shock and the fundamental shocks. Second, sunspot shocks and self-fulling expectations significantly alter the propagation of the fundamental shocks in our model economy, and notably the inflation responses. Specifically, under the indeterminacy specification, the responses of inflation to the fundamental shocks are at odds with standard economic theory: inflation increases after a positive supply or a negative demand shock. While some papers in the literature find a counterintuitive response of inflation under indeterminacy on US data, this is usually confined to the response to monetary policy shocks (i.e., Lubik and Schorfheide, 2004, Castelnuovo and Surico, 2010, and Ascari and Bonomolo, 2019). Here, instead, we find this to occur for all shocks. Fundamental demand shocks, such as monetary policy or risk premia shocks, induce a supply-like economic response, implying negative comovement between inflation and output. Similarly, fundamental supply shocks, like technology

¹This approach aligns with very recent work on the US economy, such as Gagliardone and Gertler (2023) and Chan et al. (2024). We decided to abstract from explicitly modeling financial frictions as an explanation or amplification mechanism of the Great Financial Crisis. Financial frictions are captured implicitly by the marginal efficiency of investment (MEI) shock in our model, representing disturbances to the process by which investment goods are turned into capital ready for production. As argued by Justiniano et al. (2011), the MEI shock can be thought of as a proxy for the effectiveness with which the financial sector channels the flow of household savings into new productive capital.

or labor supply shocks, induce a demand-like economic response, implying positive comovement between inflation and output. Third, under determinacy inflation is mostly supply-driven, while under indeterminacy inflation is mostly demand-driven. Fourth, the behavior of the natural interest rate and the output gap are similar between determinacy and indeterminacy, and both specifications imply that the natural rate of interest entered in positive (restrictive) territory in the recent period characterized by the increase in inflation. Finally, these results are robust to different measures of the interest rate, different sample sizes and the possibility of active fiscal policy.

In the paper, we discuss what we can conclude, in light of these results, regarding our main question. All in all, we think the determinacy specification of the model is to be preferred. First, the superior fit of the indeterminate model holds only when the forecast error for either core or HICP inflation is used to define the sunspot shock according to Bianchi and Nicolò's (2021) methodology, that is, only in two cases out of eight possible ones. Second, the superior fit of the indeterminate model disappears if the sunspot shock is orthogonal to the other fundamental shocks. Allowing correlations between the sunspot and the fundamental shocks adds many parameters, hence inducing more degrees of freedom for the estimation to fit the model. This raises concerns about fairness in the comparison to the more constrained determinate model. Third, the impulse response functions of inflation under indeterminacy are not only in contrast with economic theory and intuition, but above all they contrast with empirical results of non-structural methodologies in the literature. To explicitly make this point, we rely on external evidence about the effects of monetary policy shocks, given that the paper is about monetary policy behavior. More specifically, we analyze local projections of inflation – as well as GDP – on the monetary policy shocks identified on high-frequency data for the euro area by Altavilla et al. (2019). In response to a contractionary monetary policy shock, output persistently declines, along with all three inflation measures: headline, core, and GDP deflator inflation. These responses are statistically significant and closely align with the determinate model outcomes, while they diverge from the positive responses of the three inflation measures observed under indeterminacy. This non-structural and "theory-free" evidence thus supports the determinate model.

The paper is organized as follows. Section 2 presents the model. Section 3 explains the estimation strategy based on Bianchi and Nicolò (2021). Section 4 provides and discusses the main results. Section 5 checks for the robustness of our findings. Finally, Section 6 concludes.

2 Model

We develop a Dynamic Stochastic General Equilibrium (DSGE) model following Smets and Wouters (2003, 2007). The model includes all the standard features and frictions which are typical of New-Keynesian medium scale models: habits in consumption, variable capital utilization, investment adjustment costs, sticky prices, indexation on past and trend inflation and real wage rigidity. We think the recent inflation surge is a key episode to be able to identify the nature of the ECB policy response. Given that the inflation surge in the euro area was initially spurred by a sharp increase in energy prices, we find it essential to incorporate the role of energy into the model. Thus, we deviate from the Smets and Wouters' framework by introducing energy both as an input in consumption and as an input in production, on the footsteps of Blanchard and Galí (2007). Note that the inclusion of energy allows us to define in the model three different inflation measures – headline, core and GDP deflator – that we can use as observables in the estimation. Moreover, we assume that the country (the euro area) is an energy importer, and that the real price of energy (in terms of domestic goods) follows an exogenous process. As a consequence, we consider a small open economy setup, in the vein of Galí and Monacelli (2005), with imperfect financial integration (Schmitt-Grohé and Uribe, 2003, Lindé et al., 2009). In what follows an asterisk (*) is attached to foreign variables.

2.1 Households

There is a continuum of households indexed by $i \in [0, 1]$. Households choose how much to consume and how much to work maximizing their utility function, which is defined as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} \left(c_t^i - b c_{t-1} \right)^{1-\sigma} - \varepsilon_t^l \frac{(h_t)^{1+\phi_l}}{1+\phi_l} \right\},\tag{1}$$

where individual and aggregate consumption (c_t^i, c_t) are adjusted by the deterministic growth trend g_z , h_t stands for individual hours worked, $0 < \beta < 1$ is the subjective discount factor, σ measures the inverse of the intertemporal elasticity of substitution and ϕ_l is the inverse of Frisch elasticity. The parameter 0 < b < 1measures the degree of external habits in consumption. ε_t^l is a shock to the labor supply.

Households allocate their resources between consumption C_t , investments I_t , domestic government-issued bonds B_t and foreign assets B_t^* . They receive income from labor services $W_t h_t$, from dividends D_t , from renting capital services $u_t K_t$ at the rate R_t^k and from holding domestic bonds and foreign assets. The budget constraint is:

$$P_{C,t} (C_t + I_t) + \frac{B_{t+1}}{\varepsilon_t^b} + E R_t \Gamma_t \left(\bar{B}_{t+1}^* \right) B_{t+1}^* = R_{t-1} B_t + E R_t R_{t-1}^* B_t^* + W_t h_t + D_t + \left[R_t^k u_t - a \left(u_t \right) P_{C,t} \right] K_t - T_t.$$
(2)

 $P_{C,t}$ is the domestic consumer price index,² R_t is the domestic gross nominal interest rate, K_t is the physical capital stock and u_t defines capital utilization. T_t are lump-sum taxes. \bar{B}_t^* are aggregate for eign assets and R_t^* is the foreign gross nominal interest rate. Then, in equilibrium $\bar{B}_t^* = B_t^*$. The term $\Gamma_t(\bar{B}_{t+1}^*)$ is a premium on foreign asset holdings, which depends on the real aggregate net foreign asset position of the domestic economy. When the domestic economy is a net borrower, households face a premium on foreign interest rates, whereas when it is a net lender, they receive reduced returns on their international savings. This translates to higher domestic interest rates relative to foreign ones in the first case (net borrower), even in the absence of anticipated exchange rate depreciation. Conversely, when the domestic economy is a net lender, domestic interest rates are lower than those abroad. Consequently, fluctuations in the net foreign asset position directly influence the interest rate differentials between domestic and foreign economies. ER_t defines the nominal exchange rate. ε_t^b is a risk premium shock that affects the intertemporal margin, creating a wedge between the interest rate controlled by the central bank and the return on assets held by the households. The capital accumulation equation is:

$$K_{t+1} = (1-\delta) K_t + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t,$$
(3)

where δ is the capital depreciation rate and ε_t^i is a shock to the marginal efficiency of investment (see Justiniano et al., 2010).

We allow for real wage rigidities, following Blanchard et al. (2010) and Blanchard and Riggi (2013). These papers show that this mechanism is an important feature in relation to oil price shocks, so it seems to be relevant to include it in a model with

²We assume here that final goods can be used either for consumption or investments, abstracting from considerations about different pricing of the two components. Thus, we are implicitly assuming that the consumer price index is the same as the investment (and hence capital) price index. This implies equal fractions of domestically produced consumption goods and investment goods and the same elasticity of substitution between home goods and imported energy goods. See Section 2.2 for details.

energy.³ Instead of the standard optimality condition, where the real wage equals the marginal rate of substituion, we use the following:

$$\frac{W_t}{g_z^t P_{C,t}} = \left(\frac{W_{t-1}}{g_z^{t-1} P_{C,t-1}}\right)^{\gamma} \left[\varepsilon_t^l \left(h_t\right)^{\phi_l} \left(c_t - bc_{t-1}\right)^{\sigma}\right]^{1-\gamma}.$$
(4)

2.2 Optimal allocation of consumption expenditures

The overall consumption basket C_t is a CES bundle of an aggregator of domestically produced goods, $C_{q,t}$ and imported energy, $C_{m,t}$:

$$C_{t} \equiv \left[\varpi_{c}^{\frac{1}{v}} \left(C_{m,t} \right)^{\frac{v-1}{v}} + \left(1 - \varpi_{c} \right)^{\frac{1}{v}} \left(C_{q,t} \right)^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}},$$
(5)

where $1 - \varpi_c$ represents the fraction of domestically produced consumption goods, v is the elasticity of substitution between home goods and imported energy goods. The optimal allocation of consumption expenditures between imported and domestically produced goods delivers:

$$C_{m,t} = \varpi_c \left(\frac{P_{m,t}}{P_{C,t}}\right)^{-\upsilon} C_t \tag{6}$$

$$C_{q,t} = (1 - \varpi_c) \left(\frac{P_{q,t}}{P_{C,t}}\right)^{-\upsilon} C_t \tag{7}$$

In turn, $C_{q,t}$ is itself a CES bundle of domestically produced goods z, so that $C_{q,t} = \left[\int_0^1 C_{q,t}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}$ and the relative domestic consumer price index is:

$$P_{C,t} = \left[\varpi_c \left(P_{m,t} \right)^{1-\upsilon} + \left(1 - \varpi_c \right) \left(P_{q,t} \right)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}, \tag{8}$$

where $P_{m,t}$ is the nominal price of energy and $P_{q,t}$ is the price index for domestic goods: $P_{q,t} = \left[\int_0^1 P_{q,t}(z)^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$.

Following Blanchard and Galí (2007), we assume that the variable $s_t = \frac{P_{m,t}}{P_{q,t}}$, ³See also Gagliardone and Gertler (2023) for more recent results. that is, the price of energy in terms of the price of domestically produced goods – i.e., equal to the terms of trade in this model – follows an exogenous AR(1) process.

The homogeneous investment good is produced in a symmetric fashion as the final consumption good. The overall investment basket Q_t^I is a CES bundle of domestically produced goods, $Q_{q,t}^I$ and imported energy goods, $Q_{m,t}^I$, thus households choose also the optimal allocation of investments expenditures between domestic and energy inputs.

2.3 Production

The final good Q_t is produced under perfect competition. A continuum of intermediate inputs Q_t^z is combined as in Kimball (1995). Intermediate firms z are monopolistically competitive and use as inputs capital services, $u_t^z K_t^z$, labor services, h_t^z , and energy, M_t^z . The production technology is a CES bundle between the energy input and domestic inputs:

$$Q_t^z = \varepsilon_t^a \left\{ (1-\mu)^{\frac{1}{\epsilon}} \left[(u_t^z K_t^z)^\alpha \left(g_z^t h_t^z \right)^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \mu^{\frac{1}{\epsilon}} \left(M_t^z \right)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}} - g_z^t \Phi, \qquad (9)$$

where Φ are fixed production costs. ϵ defines the elasticity of substitution between energy and the Cobb-Duglas bundle of capital and labor. When $\epsilon = 1$ this formulation gives the standard three inputs Cobb-Douglas production function. ε_t^a is a temporary total factor productivity shock. The term g_z is the deterministic growth rate.

Domestic prices are sticky following the Calvo (1983) mechanism. A firm z can optimally reset its price with probability $(1 - \xi_p)$. Firms that cannot re-optimize adjust the price according to the scheme $P_{q,t}^z = \pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} P_{q,t-1}^z$, where $\chi_p \in [0, 1]$ allows for any degree of combination of indexation to past or trend inflation. Intermediate goods are packed by final firms with the Kimball (1995) aggregator.⁴

Cost minimization implies that energy demand is:

$$M_t^z = \mu \left(\varepsilon_t^a\right)^{\epsilon-1} \left(\frac{MC_t^z}{P_{m,t}}\right)^{\epsilon} \left(Q_t^z + g_z^t \Phi\right),\tag{10}$$

while from the labor demand and the capital demand schedules, we obtain an expression for capital to labor services:

$$\frac{u_t^z K_t^z}{h_t^z} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}.$$

Finally, the marginal cost is constant across firms and equal to:

$$MC_{t}^{z} = (\varepsilon_{t}^{a})^{-1} \left[(1-\mu) \left(\alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \left(g_{z}^{t} \right)^{-(1-\alpha)} \left(R_{t}^{k} \right)^{\alpha} (W_{t})^{1-\alpha} \right)^{1-\epsilon} + \mu \left(P_{m,t} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$
(11)

2.4 Government

The government budget constraint is:

$$P_{q,t}G_t + R_{t-1}B_t = B_{t+1} + T_t.$$
(12)

We assume that it is balanced every period. G_t is government spending, which evolves exogenously.

The monetary authority sets the nominal interest rate according to the same Taylor rule as in Smets and Wouters (2007):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_{C,t}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{flex}}\right)^{\phi_y} \right]^{1-\phi_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^{flex}/Y_{t-1}^{flex}}\right)^{\phi_{\Delta y}} \varepsilon_t^r, \quad (13)$$

⁴See the Appendix for more details.

where R_t is the gross nominal interest rate, $\pi_{C,t}$ is the gross CPI inflation rate, Y_t is the level of GDP and Y_t^{flex} is the level of potential GDP prevailing in a flexible prices and wages environment and ε_t^r is an exogenous interest rate shock.

2.5 Foreign block

We assume that the foreign block is mostly exogenous. In particular, foreign demand Y_t^* , foreign nominal interest rates R_t^* and foreign inflation $\pi_t^* = \frac{P_{C,t}^*}{P_{C,t-1}^*}$ are exogenous processes. The model is closed assuming foreign demand for the domestically produced good is specified as:

$$EXP_t = \left(\frac{P_{q,t}}{ER_t P_{C,t}^*}\right)^{-\eta} Y_t^*.$$
(14)

Net foreign asset position evolves according to:

$$\frac{ER_t B_{t+1}^*}{P_{C,t}} = R_{t-1}^* \frac{ER_t B_t^*}{P_{C,t}} + \frac{NX_t}{P_{C,t}},\tag{15}$$

and net exports are defined as:

$$NX_t = P_{q,t}EXP_t - \underbrace{P_{m,t}\left(M_t + C_{m,t} + Q_{m,t}^I\right)}_{P_{m,t}IMP_t},\tag{16}$$

where EXP_t and IMP_t are exports and imports, respectively.

2.6 Value added and aggregate resource constraint

Value added (GDP) is defined as:

$$P_{y,t}Y_t = P_{q,t}Q_t - P_{m,t}M_t,$$
(17)

where the GDP deflator, $P_{\boldsymbol{y},t},$ is implicitly defined by:

$$P_{q,t} \equiv \left[(1-\mu) P_{y,t}^{1-\epsilon} + \mu P_{m,t}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$
 (18)

The aggregate resource constraint is:

$$P_{y,t}Y_{t} = P_{C,t} \left[C_{t} + I_{t} + a\left(u_{t}\right)K_{t}\right] + P_{q,t}G_{t} + P_{q,t}EXP_{t} - \underbrace{\left(P_{m,t}M_{t} + P_{m,t}C_{m,t} + P_{m,t}Q_{m,t}^{I}\right)}_{P_{m,t}IMP_{t}}.$$
(19)

3 Estimation strategy

3.1 Data

To estimate the model, we use Bayesian techniques and the measurement equations that relate the macroeconomic data to the endogenous variables of the model are defined as:

$$\begin{bmatrix} dlGDP_{t} \\ dlCONS_{t} \\ dlINV_{t} \\ dlWAG_{t} \\ lEMPL_{t} \\ dlP_{C,t} \\ dlP_{m,t} \\ dlGDP_{t}^{*} \\ dlP_{t}^{*} \\ INTRATE_{t}^{*} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\pi} \\ \overline$$

where dl denotes the percentage change measured as log difference, l denotes the log, and hatted variables denote log deviations from steady state. We use nine quarterly euro area macroeconomic time series. More specifically, the series considered are: growth rate in real GDP, consumption, investment and wages, log of employment (linearly detrended)⁵, the short-term interest rate, measured using Krippner's shadow rate – See Krippner (2013, 2015) –, and three inflation measures. We include headline inflation rate measured by the 'All items HICP index', energy inflation measured by 'Energy HICP index' and GDP deflator inflation. $\overline{\gamma}$ denotes a deterministic growth trend common to the real variables GDP, consumption, investment and wages ($\overline{\gamma} = 100 (g_z - 1)$), \overline{e} is the (log) steady-state employment (normalized to zero), $\overline{\pi}$ is the quarterly steady-state net inflation rate, and \overline{R} is the quarterly steadystate net nominal interest rate. In addition to data from the euro area, we use data from the United States for foreign output growth, inflation and the interest rate to

 $^{^5{\}rm The}$ Appendix provides the auxiliary equation relating observed employment to unobserved hours worked.

discipline the exogenous processes for the underlying foreign variables in the model. The respective series are: growth rate in real GDP, CPI inflation, and Krippner's shadow rate measure for the US. The sample period covers 1999Q1-2023Q2.

We include eight fundamental shock processes in the estimation, several of which are the same as in Smets and Wouters (2007). In particular, we include a technology shock, a risk premium shock, an investment (MEI) shock, a monetary policy shock, a government spending shock, a price markup shock and a labor supply shock. In addition, we include an energy shock captured by an AR(1) process for the real price of energy (s_t) . Moreover, we add a measurement error, me_t , to GDP deflator inflation, and three foreign shocks: foreign demand, inflation and nominal interest rates. All shocks have an autoregressive component of order 1. The government spending shock is assumed to be correlated with the technology shock. Finally, the price markup shock also has a MA(1) component.

To address the unusual volatility in the data during the Covid-19 quarters, we scale up the shock innovation variance by common scaling factors v_t as in Lenza and Primiceri (2022). More specifically, for each quarter of the Covid-19 period (2020Q1, 2020Q2 and 2020Q3), a scaling factor is set according to $\sigma_{i,t}^2 = (v_t \sigma_i)^2$, where $v_t = 1$ except for the three quarters affected by the Covid-19 pandemic. We scale up the following shocks: technology, risk premium, investment, government spending, price markup, labor supply and foreign demand. The scaling factors are common to the shocks (*i*) but differ across the three quarters (*t*).

3.2 Calibration and Priors

We calibrate a number of parameters. In particular, the discount factor β is fixed at 0.999, corresponding to a 1.2 annual real interest rate at the prior mean. The steady-state depreciation rate δ is 0.025, corresponding to a 10% depreciation rate per year. The elasticity of the demand for goods is set at 4, which implies a 33% net price markup in steady state. We set the government spending-to-GDP ratio at 20%, in line with its sample average. The share of energy in consumption and production are both set at 10%, the average share of energy in HICP index, and the capital exponent in the Cob-Douglas bundle of capital and labor is set at 36%. Finally, the shock scaling factors for the Covid quarters are set as $v_{2020Q1} = 3.34$; $v_{2020Q2} =$ 5.26; $v_{2020Q3} = 4.71$ following the posterior mean estimates of Haderer (2023) for a medium-scale DSGE model for the euro area.

Table 1 reports the prior distributions for the structural parameters of the model and the exogenous processes that drive the dynamics of the economy, which are mostly similar to Smets and Wouters (2007). One notable difference relates to the Taylor rule coefficient associated with the response of the monetary authority to changes in the inflation rate (ϕ_{π}). Smets and Wouters (2007) specify a normal distribution truncated at 1, centered at 1.50 and with standard deviation 0.25 and impose determinacy. Instead, here, we want to allow for indeterminacy, so we consider a prior which assigns roughly equal probability of observing indeterminacy as well as determinacy. In particular, for ϕ_{π} we set a flatter normal prior distribution centered at 1 and with standard deviation 0.35 following Nicolò (2023).

The next Section explains how we deal with the determinacy/indeterminacy issue in the estimation, following Bianchi and Nicolò (2021).

3.3 Methodology

Bianchi and Nicolò (2021) develop a new method to solve and estimate linear rational expectations (LRE) models that accommodates both determinacy and indeterminacy. Their characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2003, 2004) and Farmer et al. (2015). We closely follow Bianchi and Nicolò (2021) and in the following briefly sketch their methodology while referring the readers to their paper for detailed exposition. The LRE model can be compactly

			Priors	
		shape	mean	st. dev.
TR response to inflation	ϕ_{π}	norm	1	0.35
TR response to output	ϕ_y	norm	0.1	0.05
TR response to output growth	ϕ_{gy}	norm	0.1	0.05
TR interest rate smoothing	ϕ_R	beta	0.75	0.1
inverse Frisch elasticity	ϕ_l	gamm	2	0.75
habits	b	beta	0.7	0.1
investment adjustment costs	γ_I	gamm	4	1.5
Calvo price stickiness	ξ_p	beta	0.5	0.1
real wage rigidity	γ	beta	0.5	0.2
Employment parameter	ξ_e	beta	0.5	0.1
price indexation	χ_p	beta	0.5	0.15
capital utilization elasticity	σ_u	beta	0.5	0.15
intertemporal elasticity	σ	norm	1.5	0.37
inputs elasticity	ε	gamm	0.5	0.2
nome/imported goods elast.		gamm	0.5	0.2
ss growth	g_z	norm	0.2	0.1
ss inflation		romm	05	2 0.1
Shocks pe	n reistor	gamm	0.5	0.1
risk premium		heta	0.7	0.1
investment	ρ_b	beta	0.7	0.1
monetary	ρ_i	beta	0.1	0.1
price markup	ρ_r	beta	0.7	0.1
labor supply		beta	0.7	0.1
gov spending	ρ_a	beta	0.7	0.1
technology	ρ_a	beta	0.7	0.1
energy price	ρ_s	beta	0.9	0.05
MA price markup	ρ_{ma}^p	beta	0.5	0.1
gy correlation	ρ_{gy}	norm	0.5	0.25
Shocks standa	rd dev	viations		
risk premium	σ_b	invg	0.1	2
investment	σ_i	invg	0.1	2
monetary	σ_r	invg	0.1	2
price markup	σ_p	invg	0.1	2
labor supply	σ_l	invg	0.1	2
government spending	σ_g	invg	0.1	2
technology	σ_a	invg	0.1	2
energy price	σ_s	invg	2	2
measurement error	$\sigma_{\pi_y}^{mc}$	invg	0.1	2
sunspot	σ_{ν}	unif	0.5	0.289
Shocks co	rrelati	ons	0	0.577
corr sunspot, fisk premium	$\rho_{\nu b}$	unnif	0	0.577
corr sunspot, monotory	$\rho_{\nu i}$	unif	0	0.577 0.577
corr suppot, price markup	$\rho_{\nu r}$	unif	0	0.577 0.577
corr sunspot labor supply	$\rho_{\nu p}$	unif	0	0.577
corr suppot, gov spending	$\rho_{\nu l}$	unif	0	0.577 0.577
corr sunspot, technology	$\rho \nu g$	unif	0	0.577
corr sunspot, energy price	ρνα 0	unif	0	0.577
Foreign pa	aramet	ters		
SS foreign inflation	π^*	gamm	0.6	0.1
SS foreign int rate	\bar{R}^*	gamm	0.3	0.1
foreign demand persistence	ρ_u^*	beta	0.7	0.1
foreign inflation persistence	$\rho_{\pi^{1}}^{*}$	beta	0.7	0.1
foreign rate persistence	ρ_R^{*15}	beta	0.3	0.1
foreign demand std dev	σ_{y}^{*}	invg	0.1	2
foreign inflation std dev	σ_{π}^{*}	invg	0.1	2
foreign rate std dev	σ_R^*	invg	0.1	2

Table 1: Prior distributions

written in the canonical form as:

$$\Gamma_{0}(\Theta) X_{t} = \Gamma_{1}(\Theta) X_{t-1} + \Psi(\Theta) \varepsilon_{t} + \Pi(\Theta) \eta_{t}, \qquad (21)$$

where X_t is the vector of endogenous variables, Θ is the vector of model parameters, ε_t is the vector of fundamental shocks, and η_t are one-step ahead forecast errors for the expectational variables. Bianchi and Nicolò (2021) propose to augment the original model by appending an independent process, which could be either stable or unstable. The priors are such that there is roughly a 50-50 prior probability of determinacy and one degree of indeterminacy. Following Bianchi and Nicolò (2021), we append the following autoregressive process to the original LRE model:

$$\omega_t = \varphi^* \omega_{t-1} + \nu_t - \eta_{f,t}, \qquad (22)$$

where ν_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast error vector η_t . The key insight consists of choosing this auxiliary process in a way to deliver the 'correct' solution. When the original model is determinate, the auxiliary process must be stationary so that the augmented representation also satisfies the Blanchard-Kahn condition. Accordingly, we set φ^* such that its absolute value is inside the unit circle. Then the autoregressive process for ω_t does not affect the solution for the endogenous variables X_t . On the other hand, under indeterminacy, the additional process should be explosive so that the Blanchard-Kahn condition is satisfied for the augmented system, though it is not for the original model. Hence, the absolute value of φ^* is set outside the unit circle. Under indeterminacy, we estimate the standard deviation of the sunspot shock, σ_{ν} , and so we specify a uniform distribution over the interval [0, 1] following Nicolò (2023).

We use Bayesian techniques to estimate the model parameters and to test for (in)determinacy using posterior model probabilities. First, we find the mode of the posterior distribution by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In a second step, the Metropolis-Hastings algorithm is used to simulate the posterior distribution and to evaluate the marginal likelihood of the model.⁶

Before analyzing the results of our estimations, it is important to stress that the Bianchi and Nicolò (2021) methodology we employ requires two discretionary choices by the researcher. First, we need to pick one forward-looking variable, whose forecast error determines the dynamics of the auxiliary variable in (22). When the variancecovariance matrix of shocks remains unrestricted, this specific choice is irrelevant (see Farmer et al., 2015). However, as it is standard in the literature, we assume uncorrelated fundamental shocks. It follows that in principle this discretionary choice might matter, and, as we will see, it does matter in practice in our estimations. Second, we need also to define the 'nature' of the sunspot shock, ν_t , in case of indeterminacy. If we treat it as a structural shock, then the sunspot shock should be orthogonal to all the other shocks. However, one could argue that the sunspot shock is different from the truly structural ones and it could be potentially related to the structural shocks of the model. Again, this is a researcher's discretionary choice. While the importance of these two discretionary choices have been somewhat overlooked in the literature, we will thoroughly discuss it in the next Section.

4 Results

In what follows, we begin by taking an agnostic stance on the two discretionary choices explained above. The next four subsections present the results of various estimations under different assumptions regarding the forward-looking variable in (22)

⁶All estimations are done using Dynare. The posterior distributions are based on 1000,000 draws, with the first 500,000 draws being discarded as burn-in draws. The average acceptance rate is around 25-30%.

and the correlation between the sunspot shock and the structural shocks. Finally, subsection 4.5 critically evaluates the advantages and limitations of the different specification assumptions.

4.1 Assessing the ECB policy reaction function: Determinacy vs Indeterminacy

As said, we need to pick one forward-looking variable among the eight forwardlooking variables in our model, $\pi_{C,t}$, $\pi_{q,t}$, c_t , e_t , ΔER_t , q_t^k , r_t^k and i_t for equation (22). Moreover, when we estimate the model allowing for the correlation between the sunspot shock and the structural shocks, we set a uniform prior distribution over the interval [-1, 1], as in Nicolò (2023). In addition, we also estimate a version of the indeterminacy specification where we restrict the correlations between fundamental and sunspot shocks to be zero. Table 2 shows the resulting log data densities for each different forward-looking variable the forecast error refers to in equation (22). Not surprisingly, the indeterminate model with correlations, labeled *unrestricted* correlations, exhibits a higher log data density than the model without correlations, labeled *restricted correlations*, whatever the forecast error. Moreover, it emerges that the preferred specification under indeterminacy (figures in **bold** in the Table) is the one where we include in equation (22) the forecast error associated with the core inflation rate $\eta_{\pi_{q,t}} = \pi_{q,t} - E_{t-1}(\pi_{q,t})$ as $\eta_{f,t}$ in the augmented representation.⁷ This is true both when we allow and when we do not allow for correlations between the sunspot shocks and the structural shocks.

Based on this, we provide a comparison between the best fitting indeterminacy specifications and the standard determinacy specification. Table 3 reports the parameter estimates and the log data densities. A first main result from the estimation is that, by comparing the log data densities, the data favors the indeterminate

⁷Nicolò (2023) considers the specification with the expectational error on headline inflation rate.

	Log data density					
Forecast error	Indeterminacy	Indeterminacy				
	(unrestricted correlations)	(restricted correlations)				
$\pi_{C,t}$	-1054.7	-1118.7				
$\pi_{q,t}$	-1050.5	-1073.0				
c_t	-1080.3	-1084.5				
e_t	-1090.8	-1105.7				
ΔER_t	-1079.0	-1082.3				
q_t^k	-1074.1	-1090.8				
r_t^k	-1081.9	-1085.8				
i_t	-1110.2	-1141.3				

Table 2: Model selection

model with unrestricted correlations, then, in order, the determinate model and the restricted indeterminate one. The indeterminacy result is in line with finding by Hirose (2013), who estimates a small-scale two country model for the US and the euro area over the period 1983Q1-2002Q4. He finds that the data point to a passive monetary policy for the euro area during this period. Our results suggest that monetary policy in the euro area has continued to remain passive even in the post-1999 period.

However, the superior fit of the indeterminate model rests on specific assumptions. First, it depends on whether we allow the sunspot shock to be correlated with the structural shock. If we do not allow so and instead estimate an indeterminate model imposing no correlation between the sunspot shock and all the structural shocks, then this restricted model yields a worse fit of the data than the determinate model. Second, by looking at Table 2, note that whether the unrestricted indeterminate model is preferred or not by the data depends on which forward-looking variable one picks. Specifically, the indeterminate model is preferred if the auxiliary variable is driven by the forecast errors of one of the two inflation variables, while this is not the case if one chooses one of the other forward-looking variables. Hence, indeterminacy is preferred by the data in two specifications out of eight for

		D			T 1			T 1			
		Determinacy			1110 (etermin		Indeterminacy			
			0007 1		(unrestric	cted cor	IDD:	(restrict	ed corr	(IDD: ()	
		post. mean	90% 1	APD interval	post. mean	90% F	1PD interval	post. mean	90% 1	HPD interval	
TR response to inflation	ϕ_{π}	1.59	1.07	2.07	0.58	0.27	0.90	0.78	0.58	0.99	
TR response to output	ϕ_y	0.17	0.11	0.23	0.08	0.01	0.13	0.09	0.02	0.15	
TR response to output growth	ϕ_{gy}	0.02	0.01	0.02	0.02	0.01	0.03	0.02	0.01	0.02	
TR interest rate smoothing	ϕ_R	0.89	0.86	0.92	0.92	0.89	0.96	0.91	0.87	0.95	
inverse Frisch elasticity	ϕ_l	0.23	0.10	0.35	0.23	0.11	0.35	0.31	0.15	0.46	
habits	b	0.43	0.34	0.52	0.36	0.25	0.47	0.41	0.32	0.50	
investment adjustment costs	γ_I	5.00	3.21	6.72	3.63	1.96	5.19	3.81	2.24	5.33	
Calvo price stickiness	ξ_p	0.87	0.82	0.91	0.81	0.75	0.87	0.65	0.59	0.70	
real wage rigidity	γ	0.69	0.63	0.76	0.57	0.44	0.70	0.66	0.58	0.73	
Employment parameter	ξ_e	0.46	0.33	0.59	0.41	0.30	0.53	0.40	0.30	0.50	
price indexation	χ_p	0.51	0.28	0.73	0.31	0.13	0.49	0.21	0.08	0.34	
capital utilization elasticity	σ_u	0.85	0.75	0.95	0.84	0.73	0.95	0.88	0.80	0.96	
intertemporal elasticity	σ	1.03	0.86	1.20	0.97	0.81	1.14	1.01	0.81	1.21	
inputs elasticity	ϵ	0.23	0.09	0.36	0.20	0.08	0.32	0.20	0.08	0.31	
home/imported goods elast.		0.44	0.17	0.70	0.43	0.17	0.67	0.42	0.16	0.67	
ss growth	q_z	0.25	0.22	0.28	0.24	0.21	0.28	0.24	0.21	0.28	
ss hours	Ē	1.93	0.19	3.76	1.52	-0.22	3.31	1.76	0.01	3.52	
ss inflation	$\bar{\pi}$	0.56	0.42	0.70	0.47	0.31	0.62	0.48	0.32	0.64	
			-	Shocks pers	istences						
risk premium	0h	0.92	0.89	0.95	0.92	0.87	0.97	0.90	0.84	0.97	
investment	0:	0.34	0.22	0.44	0.35	0.23	0.48	0.34	0.23	0.46	
monetary		0.36	0.22	0.47	0.39	0.20	0.51	0.01	0.20 0.27	0.53	
price markup	$ _{0}^{pr}$	0.30	0.59	0.41	0.82	0.20 0.72	0.92	0.40	0.21	0.96	
labor supply	ρ_p	0.10	0.00	0.01	0.02	0.12	0.92	0.90	0.85	0.55	
gov sponding	$ P_l $	0.87	0.85	0.95	0.31	0.80	0.90	0.90	0.80	0.95	
technology	ρ_g	0.87	0.84	0.92	0.87	0.85	0.92	0.87	0.82	0.92	
chemical price	ρ_a	0.89	0.04	0.94	0.80	0.79	0.95	0.80	0.79	0.92	
MA price morely	ρ_s	0.98	0.90	0.99	0.97	0.90	0.99	0.97	0.90	0.99	
ma price markup	ρ_{ma}	0.54	0.41	0.08	0.57	0.45	0.71	0.55	0.20	0.47	
gy correlation	ρ_{gy}	0.12	0.01	0.22		0.01	0.22	0.11	0.01	0.21	
		0.17	<u>5n</u>	OCKS Standard	1 deviations	0.14	0.00	0.10	0.10	0.00	
risk premium	σ_b	0.17	0.13	0.21	0.22	0.14	0.29	0.19	0.10	0.28	
investment	σ_i	1.10	0.93	1.25	1.12	0.96	1.29	1.11	0.95	1.27	
monetary	σ_r	0.12	0.11	0.14	0.12	0.10	0.13	0.12	0.10	0.13	
price markup	σ_p	0.11	0.08	0.13	0.09	0.07	0.12	0.14	0.11	0.17	
labor supply	σ_l	1.28	0.99	1.55	1.05	0.78	1.31	1.20	0.91	1.48	
government spending	σ_g	0.77	0.67	0.86	0.77	0.67	0.87	0.78	0.68	0.89	
technology	σ_a	0.88	0.72	1.03	0.88	0.73	1.02	0.81	0.68	0.93	
energy price	σ_s	3.14	2.77	3.52	3.16	2.77	3.53	3.15	2.79	3.53	
measurement error	$\sigma_{\pi_y}^{me}$	0.33	0.29	0.36	0.32	0.28	0.36	0.32	0.28	0.36	
sunspot	σ_{ν}	-	-	-	0.24	0.20	0.28	0.23	0.19	0.26	
				Shocks corr	elations						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.27	0.08	0.45	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.34	-0.48	-0.20	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.23	0.13	0.35	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.75	0.65	0.85	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	-0.11	-0.24	0.02	0	0	0	
corr sunspot, gov spending	$\rho_{\nu a}$	-	-	-	-0.18	-0.31	-0.05	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.16	-0.30	-0.03	0	0	0	
corr sunspot, energy price	ρ_{us}	-	-	-	0.20	0.09	0.32	0	0	0	
Foreign parameters											
SS foreign inflation	$\bar{\pi}^*$	0.61	0.48	0.73	0.60	0.48	0.72	0.60	0.47	0.73	
SS foreign int rate	\bar{R}^*	0.34	0.20	0.47	0.33	0.20	0.46	0.34	0.20	0.47	
foreign demand persistence	*	0.92	0.88	0.96	0.92	0.88	0.96	0.92	0.88	0.96	
foreign inflation persistence	$\begin{vmatrix} Py \\ o^* \end{vmatrix}$	0.52	0.41	0.63	0.52	0.00	0.63	0.53	0.42	0.65	
foreign rate persistence	$ \rho^{*}_{\pi}$	0.87	0.84	0.00	0.82	0.42	0.00	0.87	0.42	0.00	
foreign demand std dov	$\left \begin{array}{c} P_R\\ \sigma^* \end{array}\right $	0.64	0.04	0.50	0.00	0.54	0.50	0.64	0.57	0.50	
foreign inflation std dov	$\int_{\sigma^*}^{\sigma_y}$	0.04	0.07	0.12	0.05	0.57	0.12	0.04	0.57	0.12	
foreign rate std dev	σ^*	0.57	0.50	0.04	0.57	0.50	0.04	0.57	0.50	0.04	
Log data donsity		0.17	1069.2	0.20	0.17	1050 5	0.19	0.17	1072.0	0.20	
LUE UATA UCHSITY	1		-1000.9			-1000.0			-1010.0	,	

the unrestricted model, and in none of the specifications for the restricted model. Hence, the finding of a superior fit of the indeterminate model is not robust to other specifications of the indeterminate model.

The estimates for most of the structural and shock parameters are largely similar under determinacy and indeterminacy with some differences. In particular, the degree of habits (b), investment adjustment cost (γ_I), the degree of price rigidity (ξ_p) and indexation (χ_p) , and real wage rigidity (γ) turn out to be smaller under indeterminacy. On the one hand, this is not surprising since under indeterminacy, a rational expectations model displays endogenous inertia (see, e.g., Beyer and Farmer, 2007). On the other hand, only for the price stickiness parameter the posterior mean under both the indeterminate cases is not within the 90% credible bands of the determinate model. The persistences and standard deviations of the structural shocks are virtually identical across the three estimated specifications in Table 3. In the indeterminacy case, the estimation also delivers the posterior distribution of the standard deviation of the sunspot shock, which are tightly estimated in both cases. In the unrestricted case, Table 3 reports also the correlations between the sunspot shock and the fundamental exogenous shocks. The sunspot shock turns out to be positively correlated with the risk premium, monetary, price markup and energy price shock, while being negatively correlated with the investment, labor supply, government spending and technology shock.

4.2 Shock propagations

This section analyzes and compares the transmission of shocks under both determinacy and indeterminacy. For the latter, the propagation of the fundamental shocks is altered due to self-fulfilling inflationary or deflationary expectations in response to the shocks. Here we look at the propagation of five shocks that play a key role in driving inflation – i.e., risk premium and monetary policy – and output fluctuations - i.e., technology, energy, and labor supply – under indeterminacy, in terms of the forecast error variance decomposition (discussed below).



Figure 1: Impulse responses to a one standard deviation sunspot shock from the baseline estimation. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

First, however, we consider the effects of the sunspot shock, which are not only interesting in itself, but also instrumental to interpret the impulse response functions (IRFs) to the fundamental shocks when the sunspot is correlated with the fundamental shocks under unrestricted indeterminacy (allowing correlations). Figure 1 shows the IRFs of selected variables to the sunspot shock for restricted – solid black lines – and unrestricted – solid blue lines – indeterminacy . In both cases, we also plot the posterior density (HPD) regions – dashed lines for restricted (no correlations) indeterminacy and shaded blue regions for unrestricted indeterminacy.⁸ This shock looks like a positive demand shock: output and inflation measures increase, as well as the marginal cost. Hence, in our estimation a sunspot shock has the flavor of a positive sentiment shock in the spirit of Angeletos et al. (2018). Note that the

⁸Given that the fundamental shocks are correlated with the sunspot shock under indeterminacy, the shocks need to be orthogonalized in order to look at their transmission mechanism. The orthogonalization is such that fundamental shocks in the economy trigger a sunspot shock, but not the other way round, i.e., sunspot shocks are ordered last in the Cholesky ordering.

responses are smaller in the case of unrestricted indeterminacy despite the estimated shock standard deviations being similar in size.

Figures 2-6 plot the IRFs to the structural shocks mentioned above. The solid lines are posterior means and the areas are highest posterior density (HPD) regions in the three cases of determinacy (red lines and shaded red regions), unrestricted indeterminacy (blue lines and shaded blue regions), and restricted indeterminacy, i.e., no correlations (black lines and dashed lines to delimitate the regions).⁹

Under unrestricted indeterminacy, there are two channels at work which are absent in the determinacy case: (i) self-fulfilling expectations on inflation due to passive monetary policy; and (ii) the sunspot shock, as an additional extrinsic nonfundamental disturbance to the economy. Thus: (i) the propagation of structural shocks is different because self-fulfilling expectations alter the transmission and generate additional persistence, and (ii) non-fundamental sunspot disturbances produce an additional source of volatility, adding, depending on the correlation, either a positive or a negative demand shock, as just seen. In the case of restricted indeterminacy, only channel (i) is present. Essentially, in each panel, one can interpret the difference between the red lines and black ones as illustrating (i), that is, the different estimated propagation mechanism between determinacy and indeterminacy due to different parameters and monetary policy reaction, while the difference between the black lines and the blue ones mainly as illustrating (ii), that is, the estimated effect of the marginal contribution of the sunspot shock, due to the correlation between sunspot and fundamental shocks.¹⁰

⁹We use dashed lines, rather than shaded areas, for the third region for readability. The IRFs for the remaining shocks, namely, investment, price markup, and government spending, are shown in the Appendix.

¹⁰This interpretation should consider that the restricted and unrestricted indeterminacy versions are both estimated, and thus they feature different parameters. In other words, the unrestricted model could use both (i) and (ii) to fit the data, while the restricted model only (i). However, if we plot the IRFs of the unrestricted model while shutting down the correlations, we would obtain similar lines as the black ones. This is because the parameter estimates are similar across the two estimations under indeterminacy, as shown in Table 3.



Figure 2: Impulse responses to a one standard deviation technology shock. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

Figure 2 displays the IRFs to a technology shock. The difference between the black line and red one reveals that the IRFs of output and the marginal cost exhibit similar dynamics, but the ones for the three inflation measures in the panels in the second row exhibit a different propagation mechanism. The technology shock generates deflationary pressures because of the decrease in marginal costs. Under determinacy this yield a reduction of inflation, as standard economic reasoning would predict. Under restricted indeterminacy, instead, the shock triggers self-fulfilling inflationary expectations that lead to a persistent hump-shaped increase in inflation.¹¹ It follows that the reaction of monetary policy is different in the two cases. Under determinacy, the monetary authority responds by lowering the policy rate. In contrast, under restricted indeterminacy, monetary authority increases the nominal rate, but the response is gradual and not aggressive enough to stabilize the inflation rate, so that the inflationary expectations are accommodated by the passive mon-

¹¹Note that this does not need to be, but it is the results of the estimation. In theory, agents choose one among the infinitely many paths and that could entail inflationary or deflationary expectations.

etary authority, yielding a sizeable and persistent rise in inflation. The difference between the black line and the blue one, instead, reveals the negative correlation between the technology and the sunspot shock, as from Table 3. The technology shock triggers a negative demand-like sunspot shock so that both the response of output and inflation is lowered.



Figure 3: Impulse responses to a one standard deviation risk premium shock. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

Figure 3 shows the impulse responses to a risk premium shock. A risk premium shock creates a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive realization of the shock has negative effects on the economy. Under determinacy, all households reduce consumption because households anticipate a prolonged real interest rate decline, in line with the previous estimates for the euro area (Smets and Wouters, 2005, Albonico et al., 2019). With an active monetary authority, there is deflation and the nominal interest rate decreases, responding more than one-to-one to inflation. Under indeterminacy, again as in the previous case, the responses of the real variables – output and marginal cost – is only quantitatively different, while the responses of the

three inflation measures are qualitatively different. Again, agents form self-fulfilling inflationary expectations – look at the black line – that are partly accommodated by the passive monetary policy, such that inflation persistently increases.



Figure 4: Impulse responses to a one standard deviation monetary policy shock. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

Figure 4 shows the impulse responses to a contractionary monetary policy shock, that, under determinacy, produces the expected outcome: a negative response of inflation and negative effects on aggregate demand and economic activity. In contrast, again, the presence of a passive monetary policy flips the sign of the response of inflation, due to self-fulfilling inflationary expectations, in line with the empirical findings of Lubik and Schorfheide (2004) and Ascari and Bonomolo (2019) for the US. The IRFs under the two indeterminate models are extremely close, the only difference being the initial response of inflation. The latter is restricted to be zero under the no-correlation model – see the discussion below in subsection 4.5 – while it jumps upward given the positive correlation of the monetary policy with the sunspot in the unrestricted determinacy case.

Figure 5 displays the responses to a labor supply shock. A positive realization



Figure 5: Impulse responses to a one standard deviation labor supply shock. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

of this shock implies that individuals dislike working relatively more, thus the labor supply curve shifts in. This generates an upward pressure on real wages (and thus marginal costs), while decreasing hours worked, which results in inflationary pressure and a decrease of production. Under active monetary policy, the contractionary effect is exacerbated by the reaction of monetary policy. Conversely, under indeterminacy, the shock generates self-fulfilling disinflationary expectations, so that the inflation measures decrease despite the increase in the marginal cost – see the black line. The nominal interest rate decreases as a reflection of subdued inflationary pressures. This shock is negatively correlated to the sunspot shock, so that the sunspot deepens the reduction of output, inflation and interest rate.

Figure 6 shows the responses to an exogenous increase in energy prices. Since this is a shock to a price, the effects on inflation are quite short-lived under determinacy. Headline inflation jumps up, while the positive effect on core is muted. Monetary policy reacts forcefully to contain inflation, causing a persistent decrease in output and the negative response of the GDP deflator on impact. Under restricted inde-



Figure 6: Impulse responses to a one standard deviation energy price shock. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.

terminacy, the black responses of the real variables are qualitatively similar, even if the reaction of monetary policy is milder, but again the reaction of the inflation variables is quite puzzling being persistently negative.. When we allow for correlations, the energy shock is positively correlated with the sunspot, so the blue lines lie above the black ones. The reaction of core inflation – as well as the other two inflation measures – becomes positive and persistent, despite output contracting.

The analysis of the propagation of the fundamental shocks is quite puzzling in the case of indeterminacy. The auxiliary variable approach of Bianchi and Nicolò (2021) allows the estimation to pick one of the infinitely many possible rational expectations solutions under indeterminacy. The responses of the endogenous variables to the fundamental shocks are then twisted by self-fulfilling expectations. Under indeterminacy, however, the responses to most fundamental shocks contrast simple economic theory in both cases – restricted and unrestricted. The blue and black lines following a labor supply shock in Figure 5, for instance, imply a positive comovement between inflation and output. In this case, the negative correlation with

the sunspot shock amplifies the negative responses of both output and inflation. The relationship between the marginal cost and inflation is even more puzzling, because the marginal cost reacts in accordance with theory – qualitatively, by moving in opposite direction with respect to output after a supply shock – but the dynamics of inflation is de-linked from the marginal cost due to self-fulfilling (disinflationary) expectations. A positive technology shock in Figure 2 induces a similar dynamics, exhibiting a positive comovement between output and inflation. The strong and positive response of inflation (and output) in the restricted indeterminate model is dampened by the negative correlation with the sunspot shock in the unrestricted model, and the positive reaction of inflation is not significant. Also for a technology shock under indeterminacy, the dynamics of inflation is completely de-linked from the ones of the marginal costs. These self-fulfilling dynamic equilibrium paths are not grounded in theory, but they are simply data-driven, in the sense that they are picked by the estimation methodology. In these two cases – technology and labor supply shock – indeterminacy alters the propagation of the shocks so much that it makes these fundamental supply shocks look like demand shocks instead. Similarly, demand shocks propagate like supply shocks under indeterminacy. The black and blue lines of output and inflation move in *opposite* directions following either a risk premium shock (Figure 3) or a monetary policy shock (Figure 4). Also in this case, there is a disconnection between the dynamics of inflation and the marginal cost, where the latter reacts according to theory.

To conclude, it seems that under the two indeterminacy specifications – associated with the forecast error with respect to core inflation rate in (22) – the responses of inflation to the fundamental shocks are at odds with standard economic theory: inflation increases after a positive supply or a negative demand shock.

	y	h	i	С	w	R	π_c	π_q	π_y
Determinacy									
ε^{b}	16.3	20.5	15.9	16.4	18.3	71.9	20.4	30.8	22.3
ε^{i}	13.2	17.6	24.5	5.4	5.0	1.1	0.5	0.8	0.6
ε^r	5.9	8.4	3.5	3.9	4.6	11.3	4.8	7.2	5.2
ε^p	3.2	4.2	2.9	1.3	2.0	5.8	34.5	55.2	38.1
ε^l	12.1	16.0	9.2	8.0	3.3	2.5	0.9	1.4	1.0
ε^{a}	25.0	9.8	16.2	12.7	11.9	3.2	1.1	1.7	1.2
ε^{g}	4.3	8.4	0.3	0.6	0.2	0.2	0.1	0.1	0.1
ε^{s}	19.5	14.3	27.5	51.7	54.8	3.9	37.7	2.8	31.5
Indeterminacy (unrestricted correlations)									
								,	
ε^{b}	8.7	11.6	9.0	8.0	10.1	32.9	49.6	52.5	51.1
ε^{i}	4.0	11.0	15.0	3.0	2.1	5.0	6.7	7.2	7.0
ε^r	4.1	4.4	2.9	2.3	3.3	51.7	24.1	25.6	24.9
ε^p	11.7	12.8	10.2	5.0	7.7	4.0	5.5	5.8	5.7
ε^l	23.5	29.1	16.3	14.6	3.6	1.5	2.0	2.1	2.1
ε^{a}	25.1	11.0	16.0	11.0	11.3	0.5	0.8	0.9	0.9
ε^{g}	1.9	4.2	3.7	3.8	3.3	2.0	2.7	2.8	2.8
ε^s	19.1	13.3	25.9	51.6	57.5	1.3	7.4	1.6	4.4
ε^{ν}	1.4	1.7	1.0	0.8	1.1	1.0	1.3	1.4	1.3
		Indet	ermina	cy (res	stricted	d corre	lations	3)	
ε^{b}	1.0	2.0	1.5	2.6	2.4	25.2	32.1	33.0	32.4
ε^{i}	7.7	11.2	17.7	4.1	3.2	0.2	0.6	0.6	0.6
ε^r	0.8	1.4	0.5	0.6	0.8	30.0	18.5	19.0	18.7
ε^p	32.7	39.0	28.5	19.7	27.8	34.1	30.3	31.0	30.6
ε^l	14.4	19.9	11.0	10.3	2.4	1.0	1.1	1.1	1.1
ε^{a}	21.5	4.4	14.4	11.6	10.8	2.9	4.3	4.5	4.4
ε^{g}	2.4	5.9	0.8	1.1	0.4	0.3	0.4	0.4	0.4
ε^s	17.4	12.8	24.7	48.5	50.4	0.7	3.4	0.9	2.5
ε^{ν}	1.8	2.9	0.9	1.6	1.9	5.6	9.2	9.5	9.3

Table 4: Forecast error variance decompositions

4.3 Variance decomposition

The different propagation of the shocks under determinacy versus indeterminacy is reflected in the relative importance of the fundamental shocks for the volatility of the endogenous variables, and especially so for inflation. Table 4 reports the mean of the forecast error variance decomposition based on the posterior distribution of the parameter estimates.¹² Under determinacy, output appears to be relatively more supply-driven (about 60% of total variance), still demand shocks explains 40% of its volatility. Under unrestricted indeterminacy, technology and energy shocks are still the main determinants of the volatility of output, but now the labor supply and price markup shocks gain in importance in contrast to a reduced importance of the risk premium and the investment specific shocks. Overall this makes demand shocks contribute to only roughly 20% to the volatility of output. Under restricted indeterminacy, the relative importance of supply shocks is even more pronounced, particularly of the price markup shock. However, the main differences between the two specifications arise when we look at inflation. As it is standard in the literature, inflation appears to be mostly supply-driven when monetary policy is active. In contrast, under unrestricted indeterminacy the risk premium shock and monetary policy shocks turn out to be the main drivers of the fluctuations of the three measures of inflation, explaining around 70% of total volatility. Similarly for the restricted indeterminate model, these two shocks explain 50% of total inflation variability and a further 10% is explained by the sunspots shock. The latter is instead immaterial, when allowing for correlation in the case of unrestricted indeterminacy.¹³ In the case of restricted indeterminacy, the price markup shock explains a large part of

¹²We do not report the contributions of the three foreign shocks and the measurement error to the forecast error variance decompositions as they play a negligible role.

¹³For correlated shocks, the variance decomposition depends upon the order of the variables and is computed as in the VAR literature through a Cholesky decomposition of the covariance matrix of the exogenous variables. Fundamental shocks are ordered first and the sunspot shock is ordered last, meaning that a fundamental shock triggers a sunspot shock and not the other way round.

the variability of all the endogenous variable in Table 4, and around a 30% of the variability of both output and inflation. Figure A.2 in the Appendix shows that the IRFs after a price markup shock. Under restricted indeterminacy, output and inflation both decrease substantially and persistently after a positive price markup shock, so that, again, self-fulfilling expectations transform a negative supply shock into a negative demand shock. Notably, under unrestricted indeterminacy, instead, the responses of output and inflation are close to the ones of a determinate model, because of the very large positive correlation (0.75, see Table 3) between the sunspot and the price markup shocks.

To sum up, the variability of inflation is mainly explained by supply shocks in the case of determinacy while mainly by demand shocks in (both) case of indeterminacy. This is hardly surprising given the analysis in the previous subsection: indeterminacy turns upside down the effect on inflation of demand and supply shocks, hence inflation, that is supply-driven under determinacy, becomes demand-driven under indeterminacy.

4.4 Natural rate and output gap estimates

Our framework allows us to study the behavior of the natural rate of interest and the output gap. As standard in the DSGE literature, we define 'natural' variables as the ones implied by a flexible prices and wages version of the model. Figure 7 shows the dynamics of the natural interest rate, r^* , both under determinacy and under the two indeterminacy specifications. The dynamics is very similar across specifications, the determinacy estimate diverges only during the effective lower bound and quantitative easing period. The natural rate is defined as the one implied by the model with flexible prices and wages, so in theory should not be affected either by the conduct of monetary policy or by the sunspot shock. However, in an estimated model the behavior of r^* is dictated by the time-series of the estimated shocks hitting the



Figure 7: The natural rate of interest in the determinate and indeterminate models model. The three measures, hence, differ because of the different estimated paths of some fundamental shocks. -To some extent, it is actually not obvious, at least to us, that the natural rate dynamics ought to be similar across determinacy and indeterminacy specifications.

 r^* is positive until the Great Financial Crisis when it abruptly turns negative. Then, it continues to drive deeper into negative territory as the sovereign debt crisis unfolds, and finally it starts to increase after 2016 with the exception of the two waves of the pandemic. The recent inflation surge causes a rapid increase in r^* that crosses into positive territory after almost 15 years. This could be caused by global supply chain disruptions and the increase in energy price that should be associated with an increase in r^* to keep demand in line with these constraints in aggregate supply. The very last quarters show a decrease in r^* , consistent with improvements in the supply conditions. This behavior is quantitatively in line with other estimates
from the literature for the euro area as Neri and Gerali (2019) or the update of this estimate in the ECB Economic Bulletin, Issue 1/2024 (see Box 7 by Brand et al.).

Recall that r^* in the DSGE-New Keynesian framework is a cyclical concept that exhibits temporary fluctuations in response to fundamental shocks. It serves as a guidance for monetary policy because, from the point of view of the model, r^* is the real interest rate that neither stimulates nor restricts economic activity. Thus, targeting r^* would eliminate the inefficiencies caused by the nominal rigidities and stabilize inflation. In this sense, a higher r^* for the euro area in the recent postpandemic inflationary episode is coherent with the reaction of the ECB monetary policy.¹⁴

Next, the model output gap is measured as the log difference between actual output and the natural level of output implied by the flexible model counterpart. Panel (a) in Figure 8 shows the smoothed estimate of the output gap for both determinacy (dashed line) and two indeterminacy cases (solid lines). The three series exhibit the same fluctuations and are almost perfectly correlated. However, there is a difference in the level: the output gap is always lower under determinacy, and the restricted indeterminacy estimate of the output gap stays in between the unrestricted indeterminacy and determinacy one. Moreover, the difference between unrestricted indeterminacy and determinacy widens over time being equal to roughly 1% at the beginning of the sample and to roughly 3% at the end of the

¹⁴This short-term measure is different from the slow-moving measures of r^* anchored to long-run economic trends in demographics, productivity or risk aversion, as for example the well-known one in Holston et al. (2017). See also the discussion in Del Negro et al. (2017). Obstfeld (2023) distinguishes between two types of real rate of interest. He defines as natural rate, the real interest rate prevailing over a long-run equilibrium, and as neutral rate, the short-run real interest that that eliminates inflationary pressures. Of course, our interpretation is model dependent and we acknowledge the many measures of r^* in the literature due to different model specifications and assumptions. Slow-moving long-run equilibrium r^* might be better measured by term-structure models (see, e.g., Brand et al., 2021; Christensen and Mouabbi, 2024) or by semi-structural models (see, e.g., Holston et al., 2017). Consistent with our results, Brand et al. (Box 7, ECB Economic Bulletin 1/2024) find that cyclical measures of r^* may have risen post-pandemic due to increased economic activity and supply constraints. However, long-term r^* measures remain largely unchanged, anchored by structural factors like low productivity growth and demographic shifts.



Figure 8: Panel (a): Output gap: percentage deviation of output, Y_t , from the natural level of output, Y_t^{flex} . Panel (b): $\hat{y}_t^{flex} =$ percentage deviation of the natural level of output, Y_t^{flex} from the deterministic trend. Panel (c): $\hat{y}_t =$ percentage deviation of output, Y_t , from the deterministic trend. In all the three panels the dashed line is used for the determinate model and the solid line for the indeterminate model

sample.

Panels (b) and (c) disentangle the dynamics of the output gap into its components by showing the dynamics of the (flexible-price) natural output and of actual output, respectively, both expressed in deviations from the deterministic growth trend. A first thing to note is the dramatic drop of around 10% in the natural output during the Great Financial Crisis for both determinacy and indeterminacy, that

explains the increase in the output gap between 2007 and 2009 in Panel (a). Hence, mimicking the flexible price allocation would have required an even larger drop in output than the one experienced during the Great Recession. In contrast, during the Covid shock output fell much more than the natural level of output causing a large and sudden drop in the output gap in Panel (a). Comparing Panels (b) and (c) explains also where the difference in the level of the output gap between determinacy and indeterminacy is coming from. Detrended actual output is always lower under determinacy, while the natural level is always higher so that both components contribute in the same direction in making the output gap lower under determinacy. However, the difference in the natural level is marginal, so that most of the difference in the output gap between the two specifications comes from detrended output. This means that the difference we see in Panel (a) is mostly due to the different trend estimated in the two specifications. The trend is estimated to start from a higher value under determinacy – so that detredend output is roughly 1% lower – and then the difference between the two output gaps widens because the estimated rate of growth of output is slightly larger under determinacy (see g_z Table 3).

The same considerations for the natural rate of interest apply to the measure of the flexible-price natural output – and thus, to the measured output gap. The natural output should not be affected either by the conduct of monetary policy or by the sunspot shock, but these measures can indeed differ across the *estimated* determinacy and indeterminacy specifications, because the estimated fundamental shocks driving the flexible price allocation – as well as parameters – differ. For this reason, the fact that the natural output dynamics is similar across determinacy and indeterminacy specifications is not granted and it should be seen as an outcome of our estimation procedure.

Finally, we can compare the output gap estimates from our model with different alternative measures of output gap for the Euro area. We consider: i) the AMECO



Figure 9: Actual output in deviation from the deterministic trend in the determinate and indeterminate model, cycle component of GDP obtained with Hodrick-Prescott filter, output gap measure from AMECO and output gap measure from Holston-Laubach-Williams

output gap estimates, where AMECO is the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs; ii) the Holston-Laubach-Williams (HLW) measure available on the New York Fed's website; iii) a simple statistical measure derived fitting an Hodrick-Prescott filter (HP cycle) to our data for real GDP. Figure 9 shows that AMECO, HLW, and HP cycle are very similar in terms of cyclical fluctuations.¹⁵ However, they are very different from our measures of output gap in Panel (a) of Figure 8. Not surprisingly, Figure 9 shows that these measures are, instead, similar to our \hat{y}_t in Panel (c) of Figure 8, i.e., the log-deviation of actual output from the linear deterministic trend estimated by the model. Hence the difference between the output gaps comes from the conceptual definition of 'output gap' in a New Keynesian model. The DSGE uses a Woodfordian 'normative' or prescriptive measure that implies the estimate of the unobserved flexible-price natural level of output. As shown above, this variable is subject to cyclical fluctuations which creates the difference with respect to 'positive' measures such as AMECO, HLW or HP cycle, which are based on some statistical procedure to filter out the low frequency component of output. Our model does exactly that in order to compute the variable \hat{y}_t . Note that while the cyclical behavior of \hat{y}_t correlates almost perfectly with these three statistical measures of output gap, a difference opens up after the Great Financial Crisis and the Sovereign Debt Crisis. After 2011 our measure of detrended output is substantially lower than the others because of the different specification of the trend. While a nonlinear specification gives to the trend the flexibility to adjust downwards following these deep crisis, our linear specification does not allow for that, thus our estimated trend is higher after 2011.

¹⁵Note that the HLW and HP cycle are calculated at a quarterly frequency (in line with our data), while AMECO is available only on an annual base. In the plot the annual AMECO data have been interpolated.

4.5 Discussion of the Results and External Validation

In this section, we begin by critically assessing the results presented above, focusing on the advantages and limitations of the different specification assumptions. Next, we present the findings from non-structural local projection exercises, using these as a form of external validation to help distinguish among the various outcomes.

4.5.1 Pros and Cons of Different Assumptions

Let us summarize the findings thus far. First, the specification most preferred by the data is unrestricted indeterminacy, with the relevant forecast error being for core inflation. Second, this finding is fragile because determinacy becomes the preferred specification if (i) the forecast error pertains to any variable other than (core or headline) inflation, or (ii) the sunspot shock is not correlated with fundamental shocks (restricted indeterminacy). Third, there are significant differences between determinacy and indeterminacy in terms of IRFs and variance decomposition. Notably, under indeterminacy, the IRFs are difficult to reconcile with standard economic theory, as supply shocks induce dynamics typically associated with demand shocks, and vice versa.

On one hand, allowing correlations between the sunspot and fundamental shocks grants the model significantly more degrees of freedom to fit the data. Unrestricted indeterminacy features not only an extra shock, that chooses one of the infinitely many self-fulfilling equilibrium paths, and it cannot be directly linked to anything, but it also leads to the estimation of eight additional parameters, i.e., the correlations. This raises the question of whether the comparison with the determinacy case is fair. The fact that the unrestricted indeterminate model fits the data better might indicate that the DSGE model under determinacy imposes too much structure compared to indeterminacy, with the latter mimicking a VAR-type model which allows for correlations of non-structural shocks and so may not have much to do with monetary policy behavior after all. Hence, indeterminacy could be preferred by the data because it offers more flexibility, not necessarily because monetary policy was passive. Moreover, recall that this finding is not robust: it holds only in two out of eight cases in Table 2.

On the other hand, it is crucial to emphasize the main implication of assuming no correlation between the sunspot shock and the fundamental shock in the Bianchi and Nicolò (2021) methodology, as this aspect appears to have been underappreciated in the literature. Under indeterminacy, equation (22) becomes explosive, which immediately implies: $\nu_t = \eta_{f,t}$, $\forall t$. Consequently, the forecast error of the endogenous variable $\eta_{f,t}$ must equal the sunspot shock and must be zero in the absence of a sunspot shock. In other words, the forecast error must be zero on impact following any fundamental shock, indicating that the variable is predetermined in response to fundamental shocks. Indeed, core inflation does not change upon impact in response to fundamental shocks in the IRFs presented in Figures 2-6. This restriction imposed on one of the endogenous variables stems directly from the methodology and equation (22). However, indeterminacy introduces a significant degree of freedom by adding the sunspot shock (one additional shock) to fit the data relative to determinacy, so that, as previously argued, imposing some constraints on the indeterminate model allows for a fairer comparison.

All in all, we are inclined to prefer the determinacy specification of the model because: (i) the preference for unrestricted indeterminacy is fragile; (ii) the comparison may not be on an even playing field; and (iii) restricted indeterminacy performs worse than determinacy in all cases. To support our judgment, we then rely on external evidence about the effects of monetary policy shocks, given that the paper is about monetary policy behavior.



Figure 10: Local Projections using monthly data and high-frequency identified monetary policy shocks. Sample: 1999m1-2023m6. Solid lines are point estimates and shaded areas are 68% and 90% confidence bands.

4.5.2 Local Projections

Given the discussion above about the pros and cons of admitting correlations between the sunspot and the structural shocks, we resort to external validation as an indication about whether determinacy or indeterminacy should be preferred. To achieve this, we utilize monetary policy shocks derived from a distinct procedure and run a series of local projections. Specifically, we use the monetary policy shocks identified by Altavilla et al. (2019) based on high-frequency data, that is, the surprises in the 3-month Overnight Index Swap (OIS) during Monetary Event Windows. To remove "information effects" from the shock measure, we follow the sign restriction approach suggested by Jarocinski and Karadi (2020).¹⁶ Given the

¹⁶They identify a pure monetary policy shock when the surprise changes in the 3-month OIS are accompanied by a negative co-movement of the EUROSTOXX50 index. Note that we get similar results (available upon request) if we directly use the monetary policy shock series from Jarocinski and Karadi (2020).

high frequency nature of the shocks, we run local projections at a monthly frequency for each dependent variable on the monetary policy shock. We use 12 lags of the monetary policy shock, CPI, GDP growth, shadow rate, unemployment rate, and the log of the EUROSTOXX index as controls. To interpolate the quarterly series of GDP and the GDP deflator to a monthly frequency, we follow the procedure outlined by Chow and Lin (1971).¹⁷ Figure 10 presents the results. Solid lines are point estimates and the dark and light shaded areas are 68% and 90% confidence bands, respectively. In response to a contractionary monetary policy shock, output persistently decreases, as well as the three inflation measures: headline, core and GDP deflator inflation. These responses are statistically significant and well align with the responses under determinacy in Figure 4.¹⁸ In contrast, they conflict with the positive responses of the three inflation measure under indeterminacy in the same figure. This non-structural and "theory-free" evidence, therefore, supports the model with determinacy. The Appendix shows that the local projection results in Figure 10 are robust to considering different samples, as excluding the recent interest rate hike period (see Figure A.4) or considering only the period before quantitative easing (see Figure A.5).

5 Robustness

The presence of the effective lower bound on nominal interest rate and the implementation of unconventional monetary policy have posed significant challenges to modeling a DSGE model. To circumvent this issue, we used Knipper's shadow rate. We acknowledge that the shadow rate serves as a summary statistic for various un-

¹⁷This is a standard procedure in the literature. We use the industrial production index and the unemployment rate as monthly indicators to interpolate GDP, and the consumer price index and the producer price index as monthly indicators to interpolate the GDP deflator. The local projections employ GDP growth as dependent variable, and as for the IRFs in the DSGE model, we plot the impulse response for GDP, computed by cumulating the GDP growth one.

¹⁸The response of the nominal interest rate is much less persistent in the local projections.

conventional policy tools, but a negative shadow rate is not the actual borrowing or lending rate that firms and households face, as assumed by the model. To avoid the problem, many studies in the literature stop the sample before the Great Financial Crisis, as e.g., Nicolò (2023) and Albonico et al. (2024), that consider the period 1955Q4-2007Q4 for studies on the US economy. However, the history of the ECB is much shorter and the sample starts in 1999. In what follows, therefore, we check the robustness of our findings for different measures of the interest rate and different sample sizes. Moreover, we also run estimations with a different specification for fiscal policy. Table 5 reports the log-data densities.

]	Determinacy	Indeterminacy	Indeterminacy		
			(unrestricted correlations)	(restricted correlations)		
Shadow Rate						
Wu & Xia Shadow Rate (1999Q1-2023Q2)		-1085.1	-1065.6	-1087.7		
1-year OIS (1999Q1-2023Q2)		-1192.6	-1175.7	-1201.2		
Sample						
pre-lift off, Krippner Shadow Rate (1999Q1-2022Q2)		-1020.5	-1003.6	-1025.3		
pre-Covid, Krippner Shadow Rate (1999Q1-2019Q4)		-709.1	-694.8	-699.9		
pre-QE, 3-month Euribor (1999Q1-2014Q4)		-478.1	-470.9	-474.0		
pre-GFC, 3-month Euribor (1999Q1-2007Q4)		-246.4	-248.3	-246.7		
Fiscal Policy						
	(AMPF)	Determinacy FTPL (PMAF)	Indeterminacy (PMPF) (unrestricted correlations)	Indeterminacy (PMPF) (restricted correlations)		
Krippner Shadow Rate (1999Q1-2023Q2)	-1071.0	-1076.1	-1051.5	-1074.7		

Table 5: Determinacy versus Indeterminacy - Robustness

Shadow rate. We run estimations replacing Krippner's shadow rate with Wu and Xia (2017, 2020)'s measure and with the 1-year OIS, as an observable for the nominal interest rate. The ranking among the log-data densities of three specifications is unchanged, as well as parameter estimates, reported in Tables A.1 and A.2, respectively.

We run the baseline estimation using four different samples: (i) Sample. 1999Q1-2022Q2, so dropping the recent hike period; (ii) 1999Q1-2019Q4, so dropping data from the post-Covid period; (iii) 1999Q1-2014Q4, so considering only the period before quantitative easing; (iv) 1999Q1-2007Q4, so considering only the pre-GFC (Great Financial Crisis) period. For (iii) and (iv), we use the 3-month Euribor as the observable for the nominal interest rate as the samples pertain to pre-ZLB periods. Tables A.3, A.4, A.5 and A.6 report the parameter estimates for these four estimations. Dropping the recent hike from the ECB does not change the ranking across the three specifications, which is reassuring because it means that the ranking of determinacy above restricted indeterminacy is not due to the recent hike in the interest rate. However, for both (ii) and (iii) restricted indeterminacy is preferred by the data over determinacy. Once again, this result is fragile, because the restricted indeterminacy case exhibits a lower log-data density than determinacy if the forecast error is on headline inflation -(-737.3) for (ii) and (-507) for (iii) rather than on core inflation - and the same holds if the forecast error is on consumption, i.e., a real variable – (-735.7) for (ii) and (-495.9) for (iii). For (iv), even if the sample is quite short, we check the pre-GFC (1999Q1-2007Q4) period, because it excludes significant economic episodes such as the GFC, the sovereign debt crisis in the Euro area, and the COVID-19 pandemic. All these major events might have generated structural breaks, or adjustments in the conduct of monetary policy, due to the effective lower bound and the high reliance on unconventional monetary policies. Therefore, the potential sensitivity of the results to this sample period is particularly compelling. We obtain a log-data density of -246.4 under determinacy, while the values for unrestricted and restricted indeterminacy are -248.3 and -246.7, respectively. Consequently, determinacy fits the data almost as well as the indeterminacy specification, which further supports our preference for determinacy.

Fiscal Policy. Finally, we add a more detailed fiscal block to the model by

incorporating distortionary taxes and relaxing the assumption of a balanced budget by allowing for government bonds. Specifically, we introduce a consumption tax and a labor income tax, along with feedback tax rules where, as commonly assumed in the literature, the tax rates, i.e., τ_c and τ_l , respond to the value of real debt. In log-deviation terms:

$$\hat{\tau}_c = \phi_b^{\tau c} \tilde{b}_{t-1}, \qquad \hat{\tau}_l = \phi_b^{\tau l} \tilde{b}_{t-1}, \qquad (23)$$

where $\hat{\tau}_c$ ($\hat{\tau}_l$) is the log-deviation of the consumption (labor) tax rate from the steady state value, $\phi_b^{\tau c}$ ($\phi_b^{\tau l}$) is a parameter that measure the elasticity of the tax rates to the real debt level, and \hat{b} is the deviation of the real debt from its steady state value as a percentage of GDP (y). As well known from Leeper's (1991) seminal contribution, monetary and fiscal policy interaction yields novel combinations of equilibrium dynamics. Like monetary policy, fiscal policy could be either 'passive' or 'active', depending on how strongly tax rates respond to the real debt, which is determined by the values of $\phi_b^{\tau c}$ and $\phi_b^{\tau l}$. Fiscal policy is passive when the tax rates respond sufficiently strongly, ensuring the sustainability of real debt dynamics and the stability of the government budget constraint. Conversely, fiscal policy is active when the tax rates do not respond sufficiently strongly, meaning the government's reaction does not guarantee the sustainability of real debt dynamics. Implicitly, by assuming a balanced budget so far, we considered a passive fiscal policy. This assumption appears most natural given the EU's institutional framework, particularly the Stability and Growth Pact, which constrains member states' government behavior to ensure public finance sustainability. In this case, as highlighted throughout the paper, the model can be determinate or indeterminate depending on whether monetary policy is active – AMPF, i.e., Active Monetary and Passive Fiscal policymix – or passive — PMPF, i.e., Passive Monetary and Passive Fiscal policy-mix –, respectively. However, when fiscal policy is active, the model could be either explosive or determinate depending on whether monetary policy is active – AMAF, i.e., Active Monetary and Active Fiscal policy-mix – or passive — PMAF, i.e., Passive Monetary and Active Fiscal policy-mix. The latter case represents the so-called Fiscal Theory of the Price Level (FTPL), where monetary policy loses control of inflation because the price level dynamics adjust to ensure the sustainability of the government budget constraint.

The last line in Table 5 reports the log-data densities for the three cases where a solution exists, i.e., those that are not explosive. Table A.7 reports the parameter estimates. Our previous results are robust to these changes. First, the active fiscal policy case, i.e., the FTPL case, provides the worst fit, supporting the view that fiscal policy in the EU is passive, consistent with our original assumption. Second, the rankings of the log-data densities between the three passive fiscal policy specifications remain unchanged. Third, the log-data densities of the three specifications are substantially unchanged — they actually worsen slightly — indicating that adding distortionary taxes does not improve the model's fit, so again supporting our baseline specification.

6 Conclusion

This paper aims to evaluate the European Central Bank's (ECB) monetary policy concerning its unique objective of stabilizing inflation. In the model, this translates into examining whether the central bank adheres to a monetary policy rule that enables to control inflation and prevents self-fulfilling unanchored dynamics in inflation. While most papers on the euro area data rely on models where monetary policy is active and the so-called Taylor Principle holds, we estimate a DSGE model on euro area data for the sample period 1999Q1-2023Q2 allowing for the possibility of indeterminacy of rational expectations equilibrium, using the methodology proposed by Bianchi and Nicolò (2021). Our sample covers the entire existence of the ECB. Both the zero lower bound period following the Sovereign Debt Crisis, possibly characterized by fiscal dominance, and the recent surge in inflation, reminiscent of the 1970s to some extent, call for an examination of the possibility of monetary policy exhibiting passive behavior. To the best of our knowledge, we are the first to estimate a medium scale model for the euro area allowing for indeterminacy.

Our analysis provide several results. First, if the sunspot shock is allowed to be correlated with the fundamental shocks, then the indeterminacy specification is preferred by the data. This finding, however, is not robust to the choice of the forecast error used to define the sunspot shock in the Bianchi and Nicolò (2021)methodology. Moreover, this finding rests on the assumption of correlation between the sunspot shock and the fundamental shocks; without this assumption, the determinacy specification is preferred by the data. Introducing correlations between the sunspot shock and the fundamental shocks introduces numerous extra parameters, thereby increasing the degrees of freedom in the model estimation, which raises concerns about the fairness of comparing it to the more constrained determinate model. Second, sunspot shocks and self-fulfilling expectations significantly influence the propagation of fundamental shocks, particularly affecting inflation responses. Under indeterminacy, the responses of inflation to fundamental shocks contradict standard economic theory. For instance, positive supply shocks or negative demand shocks lead to higher inflation. Third, all the above findings are robust across various measures of interest rates, different sample sizes, and the consideration of active fiscal policy.

In light of these findings, we conclude that the determinacy specification of the model is preferable. The superior fit of the indeterminate model is limited to specific assumptions. Moreover, the indeterminate model's inflation responses starkly contrast with both economic theory and empirical evidence from non-structural methodologies. Local projections of inflation and GDP in response to monetary policy shocks, as identified in high-frequency data, consistently align with the determinate model and diverge from the indeterminate outcomes.

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A Appendix

A.1 The model

A.1.1 Functional forms

In line with Christiano at al. (2005) we define the following functional forms. The capital utilization cost function is:

$$a(u_t) = \gamma_{u1}(u_t - 1) + \frac{\gamma_{u2}}{2}(u_t - 1)^2$$

The investment adjustment costs function defined as:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2} \left(\frac{I_t}{I_{t-1}} - g_z\right)^2,\tag{A.1}$$

where γ_I is a parameter measuring the degree of investment adjustment costs. In line with Lindé et al. (2009), the risk premium function is defined as:

$$\Gamma_t \left(\bar{B}_{t+1}^* \right) = \exp\left\{ \Gamma^b \frac{ER_t \bar{B}_{t+1}^*}{yg_z^t P_{C,t}} \right\}.$$
(A.2)

A.1.2 Production

Price setting Intermediate goods prices are sticky à la Calvo (1983). A firm z can optimally reset its price with probability $(1 - \xi_p)$. Firms that cannot re-optimize adjust the price according to the scheme $P_{q,t}^z = \pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} P_{q,t-1}^z$, where $\chi_p \in [0, 1]$ allows for any degree of combination of indexation to past or trend inflation.

The aggregate price index is:

$$P_{q,t} = (1 - \xi_p) \tilde{P}_{q,t}^z G'^{-1} \left(\frac{\tilde{P}_{q,t}^z \iota_t}{P_{q,t}} \right) + \xi_p \pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} P_{q,t-1} G'^{-1} \left(\frac{\pi_{q,t-1}^{\chi_p} \pi^{1-\chi_p} P_{q,t-1} \iota_t}{P_{q,t}} \right),$$
(A.3)

where $\iota_t = \int_0^1 G'\left(\frac{Q_t^z}{Q_t}\right) \frac{Q_t^z}{Q_t} dz.$

The representative firm chooses the optimal price $\tilde{P}_{q,t}^{z}$ that maximizes expected profits subject to the demand schedule. The resulting first order condition is:

$$E_{t} \sum_{s=0}^{\infty} \xi_{p}^{s} \frac{\Xi_{t,t+s}}{P_{q,t+s}} Q_{t+s}^{z} \left[\begin{array}{c} \tilde{P}_{q,t}^{z} \pi_{q,t,t+s-1}^{\chi_{p}} \pi^{s(1-\chi_{p})} \\ + \left(\tilde{P}_{q,t}^{z} \pi_{qt,t+s-1}^{\chi_{p}} \pi^{s(1-\chi_{p})} - MC_{t+s}^{z} \right) \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \end{array} \right] = 0,$$

where $\omega_t = \frac{\tilde{P}_{q,t}^z}{P_{q,t}} \iota_t$ and $x_t = G'^{-1}(\omega_t)$.

A.1.3 System of non-linear equations

After deriving the first order conditions of the model, we adjust variables to guarantee that the model has a balanced growth. Lower case letters stand for detrended variables, for example, $y_t = \frac{Y_t}{g_z^t}$, $w_t = \frac{W_t}{P_{C,t}g_z^t}$, $r_t^k = \frac{R_t^k}{P_{C,t}}$, $\lambda_t = \Lambda_t g_z^t$. Then using the definition $\frac{P_{m,t}}{P_{q,t}} = s_t$, we define all model equations in terms of relative prices. Given that the model is then log-linearized, we omit price and wage dispersion variables. We add exogenous shock processes for the following variables: ε_t^a , ε_t^b , ε_t^i , ε_t^r , λ_t^p , ε_t^l , g_t , s_t , $\pi_{C,t}^*$, R_t^* , y_t^* . Given that the government budget constraint is balanced every period, we can omit this equation.

(

$$\left(c_t - bc_{t-1}\right)^{-\sigma} = \lambda_t \tag{A.4}$$

$$R_t = \pi_{C,t+1} g_z \frac{\lambda_t}{\beta \varepsilon_t^b \lambda_{t+1}} \tag{A.5}$$

$$1 = Q_{t}^{k} \varepsilon_{t}^{i} \left\{ 1 - \gamma_{I} \left(g_{z} \frac{i_{t}}{i_{t-1}} - g_{z} \right) g_{z} \frac{i_{t}}{i_{t-1}} - \frac{\gamma_{I}}{2} \left(g_{z} \frac{i_{t}}{i_{t-1}} - g_{z} \right)^{2} \right\}$$
(A.6)
$$+ \frac{1}{g_{z}} \frac{\lambda_{t+1}}{\lambda_{t}} Q_{t+1}^{k} \varepsilon_{t+1}^{i} \beta \gamma_{I} \left(g_{z} \frac{i_{t+1}}{i_{t}} - g_{z} \right) \left(g_{z} \frac{i_{t+1}}{i_{t}} \right)^{2}$$

$$\frac{1}{g_z} \frac{\lambda_{t+1}}{\lambda_t} \beta \left\{ \left[r_{t+1}^k u_{t+1} - a \left(u_{t+1} \right) \right] + Q_{t+1}^k \left(1 - \delta \right) \right\} = Q_t^k$$
(A.7)

$$r_t^k = [\gamma_{u1} + \gamma_{u2} (u_t - 1)]$$
(A.8)

$$k_{t+1} = (1-\delta) \frac{k_t}{g_z} + \varepsilon_t^i \left[1 - \frac{\gamma_I}{2} \left(g_z \frac{i_t}{i_{t-1}} - g_z \right)^2 \right] i_t$$
(A.9)

$$\frac{\left(\frac{1-\mu s_t^{1-\epsilon}}{1-\mu}\right)^{\frac{1}{1-\epsilon}}}{\left(\varpi_c s_t^{1-\nu}+1-\varpi_c\right)^{\frac{1}{1-\nu}} y_t} = c_t + \left(\varpi_c s_t^{1-\nu}+1-\varpi_c\right)^{-\frac{1}{1-\nu}} g_t \qquad (A.10)$$

$$+ \left(\frac{\varpi_i s_t^{1-\nu}+1-\varpi_i}{\varpi_c s_t^{1-\nu}+1-\varpi_c}\right)^{\frac{1}{1-\nu}} \left[i_t + \frac{a\left(u_t\right)k_t}{g_{z,t}}\right]$$

$$+ \left(\varpi_c s_t^{1-\nu}+1-\varpi_c\right)^{-\frac{1}{1-\nu}} ex_t - s_t \left(\varpi_c s_t^{1-\nu}+1-\varpi_c\right)^{-\frac{1}{1-\nu}} im_t$$

$$w_t = (w_{t-1})^{\gamma} (MRS_t)^{1-\gamma}$$
 (A.11)

$$MRS_t = (c_t - bc_{t-1})^{\sigma} \varepsilon_t^l (h_t)^{\phi_l}$$
(A.12)

$$\frac{u_t k_t}{h_t g_z} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t^k} \tag{A.13}$$

$$mc_{t} = (\varepsilon_{t}^{a})^{-1} \begin{bmatrix} (1-\mu)\alpha^{-\alpha(1-\epsilon)} (1-\alpha)^{-(1-\alpha)(1-\epsilon)} (r_{t}^{k})^{\alpha(1-\epsilon)} (w_{t})^{(1-\alpha)(1-\epsilon)} \\ +\mu \frac{s_{t}^{1-\epsilon}}{(\varpi_{c}s_{t}^{1-\nu}+1-\varpi_{c})^{\frac{1-\epsilon}{1-\nu}}} \end{bmatrix}^{\frac{1}{1-\epsilon}}$$
(A.14)

$$q_t = \varepsilon_t^a \left\{ (1-\mu)^{\frac{1}{\epsilon}} \left[\left(u_t \frac{k_t}{g_{z,t}} \right)^{\alpha} (h_t)^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \mu^{\frac{1}{\epsilon}} (m_t)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}} - \Phi$$
(A.15)

$$E_{t} \sum_{s=0}^{\infty} \left(\xi_{p}\beta\right)^{s} \varepsilon_{t}^{b} \frac{\lambda_{t+s}^{o}}{\lambda_{t}^{o}} q_{t+s}^{z} \left[\begin{array}{c} \tilde{p}_{q,t}^{z} \frac{\pi_{q,t,t+s-1}^{\chi_{p}} \pi^{1-\chi_{p}}}{\pi_{q,t,t+s}} \left(1 + \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})}\right) \\ -mc_{t+s} \left(\varpi_{c} s_{t+s}^{1-\upsilon} + 1 - \varpi_{c}\right)^{\frac{1}{1-\upsilon}} \frac{1}{G'^{-1}(\omega_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \end{array} \right] = 0$$
(A.16)

$$1 = (1 - \xi_p) \tilde{p}_{q,t}^z G'^{-1} \left(\tilde{p}_{q,t}^z \int_0^1 G' \left(\frac{q_{t+s}^z}{q_{t+s}} \right) \frac{q_{t+s}^z}{q_{t+s}} dz \right)$$

$$+ \xi_p \pi_{q,t-1}^{\chi_p} \pi^{-1} \chi_p \pi_{q,t}^{-1} G'^{-1} \left(\pi_{q,t-1}^{\chi_p} \pi^{-1} \chi_p \pi_{q,t}^{-1} \int_0^1 G' \left(\frac{q_{t+s}^z}{q_{t+s}} \right) \frac{q_{t+s}^z}{q_{t+s}} dz \right)$$
(A.17)

$$m_t = \mu \left(\varepsilon_t^a\right)^{\epsilon-1} m c_t^{\epsilon} \left(\frac{s_t}{\left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c\right)^{\frac{1}{1-\upsilon}}}\right)^{-\epsilon} (q_t + \Phi)$$
(A.18)

$$c_{m,t} = \varpi_c s_t^{-\upsilon} \left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c \right)^{\frac{\upsilon}{1-\upsilon}} c_t$$
(A.19)

$$c_{q,t} = (1 - \varpi_c) \left(\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c \right)^{\frac{\upsilon}{1-\upsilon}} c_t$$
 (A.20)

$$q_{m,t}^{I} = \varpi_i s_t^{-\upsilon} \left(\varpi_i s_t^{1-\upsilon} + 1 - \varpi_i \right)^{\frac{\upsilon}{1-\upsilon}} q_t^{I}$$
(A.21)

$$q_{q,t}^{I} = (1 - \varpi_{i}) \left(\varpi_{i} s_{t}^{1-\upsilon} + 1 - \varpi_{i} \right)^{\frac{\upsilon}{1-\upsilon}} q_{t}^{I}$$
(A.22)

$$\left(\frac{1-\mu s_t^{1-\epsilon}}{1-\mu}\right)^{\frac{1}{1-\epsilon}} y_t = q_t - s_t m_t \tag{A.23}$$

$$q_t^I = i_t + a\left(u_t\right) \frac{k_t}{g_{z,t}} \tag{A.24}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_{C,t}}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{y_t}{y_t^{flex}}\right)^{\phi_y} \right]^{1-\phi_R} \left(\frac{y_t}{y_t^{flex}} \frac{y_{t-1}^{flex}}{y_{t-1}}\right)^{\phi_{\Delta y}} \varepsilon_t^r \tag{A.25}$$

$$\frac{\pi_{C,t}}{\pi_{q,t}} = \left(\frac{\varpi_c s_t^{1-\upsilon} + 1 - \varpi_c}{\varpi_c s_{t-1}^{1-\upsilon} + 1 - \varpi_c}\right)^{\frac{1}{1-\upsilon}}$$
(A.26)

$$\frac{\pi_{y,t}}{\pi_{q,t}} = \left(\frac{1 - \mu s_t^{1-\epsilon}}{1 - \mu s_{t-1}^{1-\epsilon}}\right)^{\frac{1}{1-\epsilon}}$$
(A.27)

$$nx_{t} = \left(\varpi_{c}s_{t}^{1-\upsilon} + 1 - \varpi_{c}\right)^{-\frac{1}{1-\upsilon}} ex_{t} - s_{t} \left(\varpi_{c}s_{t}^{1-\upsilon} + 1 - \varpi_{c}\right)^{-\frac{1}{1-\upsilon}} im_{t}$$
(A.28)

$$a_t = \frac{R_{t-1}^*}{g_{z,t}\pi_{C,t}^*} a_{t-1} + \frac{nx_t}{RER_t}$$
(A.29)

$$im_t = m_t + c_{m,t} + q_{m,t}^I$$
 (A.30)

$$\frac{ER_{t+1}R_t^*}{ER_t\Gamma_t\varepsilon_t^bR_t} = 1 \tag{A.31}$$

$$\Gamma_t = \exp\left\{\Gamma^b \frac{a_t}{y} RER_t\right\} \tag{A.32}$$

$$ex_{t} = \left(\varpi_{c}s_{t}^{1-\upsilon} + 1 - \varpi_{c}\right)^{\frac{\eta}{1-\upsilon}} RER_{t}^{\eta}y_{t}^{*}$$
(A.33)

$$\frac{RER_t}{RER_{t-1}} = \frac{ER_t}{ER_{t-1}} \frac{\pi_{C,t}^*}{\pi_{C,t}}$$
(A.34)

A.1.4 System of log-linearized equations

The above equations are log-linearized. Hatted variables are in log-deviation from their steady state. Some variables are expressed in deviation from steady state output, i.e. $\tilde{x}_t = \frac{x_t - x}{y}$. We define $A = \frac{1}{\lambda^p \alpha^{p+1}}$, where α^p is elasticity of substitution between goods. We assume s = 1 in steady state, so that all relative prices are equal to 1 at steady state. The steady state of all domestic inflation measures is the same and corresponds to π . It is implicit that the system below is completed with flexible prices and wages equilibrium conditions which are not reported here.

$$-\sigma \frac{1}{1-b}\hat{c}_t + \sigma \frac{b}{1-b}\hat{c}_{t-1} = \hat{\lambda}_t \tag{A.35}$$

$$\hat{R}_t = -\hat{\varepsilon}_t^b + \hat{\pi}_{C,t+1} + \hat{\lambda}_t - \hat{\lambda}_{t+1}$$
(A.36)

$$\hat{i}_{t} = \frac{1}{\gamma_{I} g_{z}^{2} (1+\beta)} \left(\hat{Q}_{t}^{k} + \hat{\varepsilon}_{t}^{i} \right) + \frac{1}{1+\beta} \hat{i}_{t-1} + \frac{\beta}{1+\beta} \hat{i}_{t+1}$$
(A.37)

$$\hat{\lambda}_{t+1} - \hat{\lambda}_t + \frac{\beta}{g_z} r^k \hat{r}_{t+1}^k + \frac{\beta}{g_z} \left(1 - \delta\right) \hat{Q}_{t+1}^k = \hat{Q}_t^k \tag{A.38}$$

$$\hat{r}_t^k = \frac{\gamma_{u2}}{r^k} \hat{u}_t \tag{A.39}$$

$$\hat{k}_{t+1} = \frac{(1-\delta)}{g_z}\hat{k}_t + \frac{i}{k}\hat{\imath}_t + \frac{i}{k}\hat{\varepsilon}_t^i$$
(A.40)

$$0 = \frac{c}{y}\hat{c}_t + \tilde{g}_t + \frac{i}{y}\hat{i}_t - \hat{y}_t + \frac{\gamma_{u1}k}{yg_z}\hat{u}_t + \frac{ex}{y}\widehat{ex}_t - \frac{im}{y}\widehat{im}_t + \left[\left(\varpi_i - \varpi_c\right)\frac{i}{y} + \frac{\mu}{1 - \mu} + \varpi_c - \frac{ex}{y} - \varpi_c\frac{g}{y}\right]\hat{s}_t$$
(A.41)

$$\hat{m}_t = (\epsilon - 1)\hat{\varepsilon}_t^a + \epsilon \widehat{mc}_t + \frac{q}{q + \Phi}\hat{q}_t - \epsilon (1 - \varpi_c)\hat{s}_t$$
(A.42)

$$\frac{y}{q}\left(\hat{y}_t - \frac{\mu}{1-\mu}\hat{s}_t\right) = \hat{q}_t - \frac{m}{q}\left(\hat{s}_t + \hat{m}_t\right) \tag{A.43}$$

$$\hat{q}_t^I = \hat{\imath}_t + \frac{\gamma_{u1}}{g_z} \frac{k}{i} \hat{u}_t \tag{A.44}$$

$$(1 + \beta \chi_p) \hat{\pi}_{q,t} = \chi_p \hat{\pi}_{q,t-1} + \beta \hat{\pi}_{q,t+1} - \beta (1 - \chi_p) \widehat{\pi}_{t+1} + (1 - \chi_p) \widehat{\pi}_t + A \frac{(1 - \beta \xi_p) (1 - \xi_p)}{\xi_p} \left(\widehat{mc}_t + \hat{\lambda}_t^p + \varpi_c \hat{s}_t \right)$$
(A.45)

$$\hat{w}_t = \gamma \hat{w}_{t-1} + (1-\gamma) \widehat{MRS}_t \tag{A.46}$$

$$\widehat{MRS}_t = \frac{\sigma}{1-b}\hat{c}_t - \frac{b\sigma}{1-b}\hat{c}_{t-1} + \phi_l\hat{h}_t + \hat{\varepsilon}_t^l \tag{A.47}$$

$$\hat{u}_t + \hat{k}_t - \hat{h}_t - \hat{g}_{z,t} = \hat{w}_t - \hat{r}_t^k \tag{A.48}$$

$$\widehat{mc}_{t} = -\widehat{\varepsilon}_{t}^{a} + \alpha \left(1 - \mu m c^{\epsilon-1}\right) \widehat{r}_{t}^{k} + (1 - \alpha) \left(1 - \mu m c^{\epsilon-1}\right) \widehat{w}_{t} + \mu m c^{\epsilon-1} \left(1 - \varpi_{c}\right) \widehat{s}_{t}$$
(A.49)

$$\hat{q}_t = \frac{q+\Phi}{q} \left\{ \hat{\varepsilon}_t^a + \left(1 - \mu m c^{\epsilon-1}\right) \left[\alpha \left(\hat{k}_t + \hat{u}_t - \hat{g}_{z,t} \right) + \left(1 - \alpha\right) \hat{h}_t \right] + \mu m c^{\epsilon-1} \hat{m}_t \right\}$$
(A.50)

$$\hat{R}_{t} = \phi_{R}\hat{R}_{t-1} + (1 - \phi_{R})\left(\phi_{\pi}\left(\hat{\pi}_{t} - \hat{\bar{\pi}}_{t}\right) + \phi_{y}\left(\hat{y}_{t} - \hat{y}_{t}^{flex}\right)\right) + \phi_{\Delta y}\left(\hat{y}_{t} - \hat{y}_{t-1} - \left(\hat{y}_{t}^{flex} - \hat{y}_{t-1}^{flex}\right)\right) + \hat{\varepsilon}_{t}^{r}$$
(A.51)

$$\hat{c}_{m,t} = \hat{c}_t - \upsilon \left(1 - \varpi_c\right) \hat{s}_t \tag{A.52}$$

$$\hat{c}_{q,t} = \hat{c}_t + \upsilon \varpi_c \hat{s}_t \tag{A.53}$$

$$\hat{q}_{m,t}^{I} = \hat{q}_{t}^{I} - \upsilon \left(1 - \varpi_{i}\right) \hat{s}_{t}$$
(A.54)

$$\hat{q}_{q,t}^{I} = \hat{q}_{t}^{I} + \upsilon \varpi_{i} \hat{s}_{t} \tag{A.55}$$

$$\hat{\pi}_{C,t} - \hat{\pi}_{q,t} = \varpi_c \left(\hat{s}_t - \hat{s}_{t-1} \right)$$
 (A.56)

$$\hat{\pi}_{y,t} - \hat{\pi}_{q,t} = -\frac{\mu}{1-\mu} \left(\hat{s}_t - \hat{s}_{t-1} \right) \tag{A.57}$$

$$\widetilde{nx}_t = \frac{ex}{y}\widehat{ex}_t - \frac{im}{y}\widehat{im}_t - \frac{ex}{y}\widehat{s}_t$$
(A.58)

$$\tilde{a}_t = \frac{R^*}{g_z \pi} \tilde{a}_{t-1} + \widetilde{nx}_t \tag{A.59}$$

$$\widehat{im}_t = \frac{m}{y} \frac{y}{im} \hat{m}_t + \varpi_c \frac{c}{y} \frac{y}{im} \hat{c}_{m,t} + \varpi_i \frac{i}{y} \frac{y}{im} \hat{q}_{m,t}^I$$
(A.60)

$$\hat{R}_t = \widehat{\Delta ER}_{t+1} + \hat{R}_t^* - \hat{\Gamma}_t - \hat{\varepsilon}_t^b \tag{A.61}$$

$$\hat{\Gamma}_t = \Gamma^b \tilde{a}_t \tag{A.62}$$

$$\widehat{ex}_t = \eta \varpi_c \hat{s}_t + \eta \widehat{RER}_t + \hat{y}_t^* \tag{A.63}$$

$$\widehat{RER}_t - \widehat{RER}_{t-1} = \widehat{\Delta ER}_t + \hat{\pi}^*_{C,t} - \hat{\pi}_{C,t}$$
(A.64)

Finally, following Christoffel et al. (2008) and Albonico et al. (2019), the auxiliary equation relating observed employment to unobserved hours worked is given by:

$$\widehat{e}_t = \frac{\beta}{1+\beta}\widehat{e}_{t+1} + \frac{1}{1+\beta}\widehat{e}_{t-1} + \frac{(1-\xi_e)\left(1-\beta\xi_e\right)}{(1+\beta)\xi_e}\left(\widehat{h}_t - \widehat{e}_t\right)$$
(A.65)

A.2 Additional Results

- Figures A.1-A.3 plot the impulse response functions (IRFs) to the additional shocks from the baseline estimation.
- Figures A.4 and A.5 plot the local projections as specified in Section 4.5.2

for the pre-lift off sample (1999m1-2022m6) and the pre-QE sample (1999m1-2014m12), respectively.

- Table A.1 reports the parameter estimates when using Wu and Xia shadow rate as an observable for the nominal interest rate; Sample: 1999Q1-2023Q2.
- Table A.2 reports the parameter estimates when using 1-year OIS as an observable for the nominal interest rate; Sample: 1999Q1-2023Q2.
- Table A.3 reports the parameter estimates when using Krippner shadow rate as an observable for the nominal interest rate; Sample: 1999Q1-2022Q2.
- Table A.4 reports the parameter estimates when using Krippner shadow rate as an observable for the nominal interest rate; Sample: 1999Q1-2019Q4.
- Table A.5 reports the parameter estimates when using 3-month Euribor as an observable for the nominal interest rate; Sample: 1999Q1-2014Q4.
- Table A.6 reports the parameter estimates when using 3-month Euribor as an observable for the nominal interest rate; Sample: 1999Q1-2007Q4.
- Table A.7 reports the parameter estimates for the extended model with taxes and fiscal feedback rules. Samples: 1999Q1-2023Q2.



Figure A.1: Impulse responses to a one standard deviation investment shock from the baseline estimation. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.



Figure A.2: Impulse responses to a one standard deviation price markup shock from the baseline estimation. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.



Figure A.3: Impulse responses to a one standard deviation government spending shock from the baseline estimation. The solid lines are posterior means while the shaded and dashed areas are highest posterior density (HPD) regions.



Figure A.4: LP pre-lift off sample: 1999m1-2022m6



Figure A.5: LP pre-QE sample: 1999m1-2014m12

Image Image <th< th=""><th></th><th></th><th colspan="3">Determinacy</th><th>Ind</th><th>eterminac</th><th>y</th><th colspan="4">Indeterminacy</th></th<>			Determinacy			Ind	eterminac	y	Indeterminacy			
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$			-			(unrestricted correlations)			(restricted correlations)			
$\begin{array}{c c c c c c c c c c c c c c c c c c c $			post. mean	90% H	PD interval	post. mean	90% HP	D interval	post. mean	90% HI	PD interval	
TR response to output growth φ ₀ φ ₀ 0.208 0.152 0.206 0.101 0.021 0.167 0.104 0.023 0.005 TR interser rate smoothing φ ₀ 0.808 0.876 0.922 0.0238 0.910 0.966 0.932 0.933 0.933 0.935 Interser Frick elastity φ 0.226 0.354 0.234 0.314 0.312 0.534 0.541 0.333 0.618 investmint adjustment cost ζ_{μ} 0.453 0.542 0.333 0.669 0.579 0.630 0.733 0.651 0.669 0.599 0.739 0.631 0.333 0.652 0.333 0.698 0.909 0.739 0.631 0.333 0.652 0.588 0.484 0.749 0.523 0.384 0.221 0.848 0.848 0.828 0.834 0.220 0.848 0.848 0.431 0.221 0.848 0.849 0.432 0.171 0.271 0.171 0.271 0.231 0.344 0.202	TR response to inflation	ϕ_{π}	1.743	1.21	2.262	0.575	0.251	0.941	0.753	0.527	0.993	
TR response to out[put growth φ _p 0.016 0.012 0.024 0.014 0.039 0.017 0.010 0.025 Inverse Firsch elasticity φ 0.206 0.034 0.316 0.216 0.010 0.323 0.331 0.517 0.468 babits φ 0.433 0.878 0.222 0.311 0.448 0.424 0.333 0.518 Calvo price stickiness γ 4.333 2.878 6.145 3.337 1.067 5.036 3.770 2.224 5.244 Calvo price stickiness γ 0.461 0.430 0.733 0.635 0.669 0.599 0.738 Employment parameter γ 0.467 0.233 0.698 0.312 0.120 0.494 0.228 0.883 0.808 0.935 inputs clasticity σ 0.350 0.173 0.727 0.425 0.767 0.684 0.322 0.666 0.894 0.351 0.351 0.371 0.021 0.322 0.328	TR response to output	φ.,	0.208	0.152	0.266	0.101	0.021	0.167	0.104	0.024	0.173	
TR incres open 0.988 0.976 0.924 0.938 0.910 0.966 0.932 0.031 0.0135 0.043 habits b 0.435 0.344 0.522 0.341 0.213 0.448 0.424 0.333 0.518 investment adjustment costs γ_1 4.533 2.878 6.145 3.537 1.067 5.038 0.370 0.524 0.533 0.534 0.535 0.538 0.535 0.534 0.539 0.538 0.569 0.539 0.538 0.465 0.440 0.529 0.238 0.898 0.321 0.212 0.494 0.218 0.048 0.465 0.446 0.446 0.446 0.446 0.571 0.11 1.014 0.117 1.209 inprits elasticity ϵ 0.238 0.138 0.212 0.038 0.334 0.221 0.437 0.239 0.431 0.431 0.431 0.431 0.431 0.431 0.431 0.431 0.431 0.431 </td <td>TR response to output growth</td> <td>ϕ_{au}</td> <td>0.016</td> <td>0.01</td> <td>0.022</td> <td>0.024</td> <td>0.01</td> <td>0.039</td> <td>0.017</td> <td>0.010</td> <td>0.025</td>	TR response to output growth	ϕ_{au}	0.016	0.01	0.022	0.024	0.01	0.039	0.017	0.010	0.025	
	TR interest rate smoothing	ϕ_R	0.898	0.876	0.922	0.938	0.910	0.966	0.932	0.903	0.963	
babis b b 0.435 0.341 0.521 0.441 0.231 0.448 0.424 0.333 0.518 Calvo price stickines ζ 0.87 0.828 0.913 0.774 0.733 0.587 0.678 0.688 0.730 0.534 0.733 0.630 0.733 0.534 0.733 0.544 0.733 0.646 0.400 0.733 0.534 0.238 0.669 0.530 0.734 0.221 0.212 0.444 0.218 0.081 0.311 capital utilization elasticity σ_{i} 0.490 0.728 0.212 0.085 0.341 0.717 0.425 0.170 0.637 1.101 1.014 0.117 1.209 inputs clasticity ϵ 0.435 0.146 0.346 0.777 0.427 0.780 0.171 0.203 0.427 0.780 0.171 0.203 0.427 0.331 0.455 sa hours σ_{i} 0.217 0.427 0.780 0.474	inverse Frisch elasticity	φ	0.206	0.094	0.316	0.216	0.104	0.323	0.314	0.152	0.468	
investment adjustment costs γ1 4.333 2.878 6.145 3.337 1.967 5.038 3.70 2.224 5.254 Calvo price sitchines 0. 0.828 0.913 0.734 0.733 0.655 0.665 0.758 0.685 price indexation χ_{s} 0.661 0.534 0.733 0.054 0.733 0.605 0.669 0.799 0.739 price indexation χ_{s} 0.811 0.742 0.946 0.840 0.769 0.739 0.739 0.741 0.938 0.088 0.668 0.812 0.122 0.781 0.888 0.688 0.688 0.688 0.688 0.688 0.688 0.688 0.688 0.688 0.688 0.687 0.528 0.248 0.288 0.248 0.288 0.248 0.288 0.248 0.288 0.248 0.288 0.248 0.287 0.401 0.339 0.227 0.448 0.329 0.440 0.323 0.690 0.637 0.550 <td< td=""><td>habits</td><td>b</td><td>0.435</td><td>0.344</td><td>0.522</td><td>0.341</td><td>0.231</td><td>0.448</td><td>0.424</td><td>0.333</td><td>0.518</td></td<>	habits	b	0.435	0.344	0.522	0.341	0.231	0.448	0.424	0.333	0.518	
	investment adjustment costs	γ_I	4.533	2.878	6.145	3.537	1.967	5.036	3.770	2.224	5.254	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Calvo price stickiness	ξ_n	0.87	0.828	0.913	0.794	0.733	0.857	0.635	0.578	0.688	
	real wage rigidity	γ^{sp}	0.691	0.630	0.753	0.534	0.373	0.695	0.669	0.599	0.739	
price indexation capital utilization elasticity γ _x σ 0.476 0.238 0.068 0.312 0.129 0.494 0.218 0.081 0.363 intertemporal elasticity σ 0.990 0.855 1.146 0.946 0.779 0.157 1.011 0.877 1.218 0.083 0.083 0.090 0.352 0.218 0.051 0.334 0.202 0.087 0.939 0.632 0.017 0.677 0.121 0.0450 0.023 0.223 0.029 0.223 0.686 0.947 3.289 0.430 0.322 0.647 3.289 0.430 0.323 0.643 0.432 0.235 0.646 0.541 0.532 0.646 0.541 0.532 0.646 0.541 0.532 0.646 0.541 0.532 0.646 0.541 0.533 0.542 0.244 0.323 0.645 0.533 0.523 0.640 0.344 0.533 0.545 0.541 0.556 0.411 0.556 0.4131 0.733 0.556	Employment parameter	É	0.465	0.340	0.592	0.415	0.303	0.523	0.398	0.293	0.496	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	price indexation	χ_n	0.476	0.253	0.698	0.312	0.129	0.494	0.218	0.081	0.351	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	capital utilization elasticity	σ_{u}	0.841	0.742	0.946	0.849	0.749	0.952	0.883	0.808	0.963	
	intertemporal elasticity	σ	0.990	0.835	1.146	0.946	0.787	1.101	1.014	0.817	1.209	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	inputs elasticity	F	0.238	0.098	0.372	0.212	0.085	0.334	0.202	0.080	0.319	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	home/imported goods elast.	v v	0.459	0.173	0.727	0.425	0.170	0.674	0.432	0.170	0.687	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	ss growth	a.	0.256	0.224	0.288	0.248	0.218	0.279	0.243	0.209	0.277	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	ss hours	Ē	2.020	0.272	3.778	1.414	-0.323	3.129	1.559	-0.247	3.289	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	ss inflation		0.571	0.427	0.708	0.474	0.320	0.623	0.490	0.332	0.646	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $			0.012		Shocks pers	istences	0.020	0.020	0.100		0.0.00	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	risk premium	01	0.927	0.902	0.952	0.937	0.903	0.972	0.914	0.866	0.964	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	investment	P0 0:	0.339	0.227	0.448	0.342	0.225	0.460	0.348	0.233	0.465	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	monetary		0.301	0.190	0.414	0.393	0.220	0.510	0.426	0.307	0.550	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	price markup	Pr	0.710	0.603	0.816	0.813	0.703	0.924	0.899	0.840	0.962	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	labor supply	Pp	0.875	0.812	0.940	0.907	0.860	0.958	0.896	0.843	0.950	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	gov spending		0.861	0.811	0.919	0.867	0.800	0.914	0.866	0.040	0.900	
$\begin{array}{c} \mbox{termarkup} & \rho_a & 0.393 & 0.393 & 0.393 & 0.393 & 0.393 & 0.393 & 0.393 & 0.391 & 0.183 & 0.314 & 0.183 & 0.314 \\ \mbox{markup} & \rho_{rm}^{a} & 0.545 & 0.411 & 0.687 & 0.556 & 0.411 & 0.709 & 0.340 & 0.205 & 0.474 \\ \mbox{gy correlation} & \rho_a & 0.123 & 0.01 & 0.223 & 0.137 & 0.01 & 0.248 & 0.117 & 0.010 & 0.220 \\ \mbox{mometary} & \sigma_t & 1.102 & 0.937 & 1.267 & 1.123 & 0.959 & 1.285 & 1.118 & 0.954 & 1.283 \\ \mbox{mometary} & \sigma_t & 0.143 & 0.123 & 0.163 & 0.142 & 0.161 & 0.139 & 0.122 & 0.157 \\ \mbox{price markup} & \sigma_t & 0.143 & 0.123 & 0.163 & 0.142 & 0.124 & 0.161 & 0.139 & 0.122 & 0.157 \\ \mbox{price markup} & \sigma_t & 0.143 & 0.123 & 0.163 & 0.142 & 0.098 & 0.752 & 1.217 & 1.236 & 0.941 & 1.528 \\ \mbox{government spending} & \sigma_g & 0.782 & 0.684 & 0.876 & 0.773 & 0.672 & 0.870 & 0.772 & 0.666 & 0.879 \\ \mbox{technology} & \sigma_a & 0.897 & 0.733 & 1.053 & 0.363 & 0.709 & 1.012 & 0.813 & 0.681 & 0.936 \\ \mbox{sunpot} & \sigma_v & - & - & 0.245 & 0.207 & 0.281 & 0.234 & 0.197 & 0.271 \\ \mbox{corr sunspot, investment} & \rho_{rh} & 0.325 & 0.286 & 0.362 & 0.323 & 0.284 & 0.360 \\ \mbox{sunpot} & \sigma_v & - & - & - & 0.245 & 0.207 & 0.281 & 0.234 & 0.197 & 0.271 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.319 & 0.134 & 0.504 & 0 & 0 & 0 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.245 & 0.438 & -0.124 & 0 & 0 & 0 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.245 & 0.292 & 0.045 & 0 & 0 & 0 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.131 & -0.207 & 0.432 & 0 & 0 & 0 & 0 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.245 & 0.292 & 0.045 & 0 & 0 & 0 & 0 \\ \mbox{corr sunspot, investment} & \rho_{rh} & - & - & - & 0.123 & -0.292 & 0.045 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	tochnology	P_g	0.885	0.833	0.032	0.864	0.021	0.033	0.851	0.020	0.014	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	oporgy price	ρ_a	0.005	0.000	0.958	0.004	0.155	0.955	0.075	0.765	0.914	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	MA price markup	ρ_s	0.545	0.303	0.585	0.574	0.300	0.388	0.340	0.901	0.333	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	ma price markup	P_{ma}	0.122	0.411	0.007	0.127	0.411	0.705	0.117	0.200	0.474	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	gy correlation	ρ_{gy}	0.123	0.01 Sho	0.225	0.157	0.01	0.240	0.117	0.010	0.220	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	rial promium		0.101	0.147	0.222		0.161	0.202	0.210	0.120	0.200	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	risk premium	06	0.191	0.147	0.252	0.232	0.101	0.502	0.210	0.129	0.290	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	investment	01	0.142	0.957	1.207	1.125	0.959	1.260	1.110	0.904	1.265	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	monetary	σ_r	0.143	0.123	0.103	0.142	0.124	0.101	0.139	0.122	0.157	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	price markup	σ_p	0.101	0.078	0.124	0.098	0.072	0.124	0.141	0.108	0.173	
government spending σ_g 0.762 0.876 0.773 0.672 0.870 0.772 0.080 0.772 0.080 0.772 0.080 0.772 0.080 0.772 0.080 0.772 0.080 0.072 0.870 0.772 0.570 0.012 0.813 0.681 0.936 energy price σ_s 3.149 2.779 3.514 3.163 2.782 3.525 3.143 2.782 3.511 measurement error $\sigma_{\pi_v}^{\pi_v}$ 0.325 0.286 0.362 0.323 0.281 0.234 0.197 0.271 Shocks correlations Corr sunspot, risk premium $\rho_{\nu \mu}$ $ 0.217$ 0.432 0 0 0 0 0 0 0 0.07 corr sunspot, risk premium $\rho_{\nu \mu}$ $ 0.233$ -0.243 0.432 0 0 0 0 0	labor supply	σ_l	1.249	0.974	1.524	0.989	0.752	1.217	1.230	0.941	1.528	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	government spending	σ_g	0.782	0.084	0.870	0.773	0.072	0.870	0.772	0.000	0.879	
energy price $\sigma_{\pi_y}^{me}$ 3.149 $2.7/9$ 3.514 3.163 2.782 3.511 measurement error $\sigma_{\pi_y}^{me}$ 0.325 0.286 0.323 0.285 0.362 0.323 0.284 0.360 sunspot σ_{ν} - - 0.245 0.207 0.281 0.234 0.197 0.271 corr sunspot, risk premium $\rho_{\nu t}$ - - -0.283 -0.438 -0.124 0 0 0 corr sunspot, investment $\rho_{\nu t}$ - - -0.283 -0.438 -0.124 0 0 0 0 corr sunspot, monetary $\rho_{\nu t}$ - - 0.317 0.207 0.432 0 <td>technology</td> <td>σ_a</td> <td>0.897</td> <td>0.733</td> <td>1.053</td> <td>0.803</td> <td>0.709</td> <td>1.012</td> <td>0.813</td> <td>0.081</td> <td>0.936</td>	technology	σ_a	0.897	0.733	1.053	0.803	0.709	1.012	0.813	0.081	0.936	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	energy price	σ_s	3.149	2.779	3.514	3.163	2.782	3.525	3.143	2.782	3.511	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	measurement error	$\sigma_{\pi_y}^{me}$	0.325	0.286	0.362	0.323	0.285	0.362	0.323	0.284	0.360	
Shocks correlationscorr sunspot, risk premium ρ_{lvl} 0.3190.1340.504000corr sunspot, investment ρ_{lvl} 0.283-0.438-0.124000corr sunspot, monetary $\rho_{\nu p}$ 0.3170.2070.432000corr sunspot, price markup $\rho_{\mu p}$ 0.7250.6210.831000corr sunspot, labor supply $\rho_{\mu l}$ 0.131-0.2800.022000corr sunspot, technology $\rho_{\mu a}$ 0.112-0.2570.026000corr sunspot, technology $\rho_{\mu a}$ 0.2090.0910.355000corr sunspot, technology $\rho_{\mu a}$ 0.2090.0150.3630.2200.494corr sunspot, technology $\rho_{\mu a}$ 0.6050.4790.7290.6660.4800.735<	sunspot	σ_{ν}	-	-	-	0.245	0.207	0.281	0.234	0.197	0.271	
corr sunspot, risk premium $\rho_{\nu b}$ - - - 0.319 0.134 0.504 0 0 0 corr sunspot, investment $\rho_{\nu i}$ - - - -0.283 -0.438 -0.124 0 0 0 0 corr sunspot, investment $\rho_{\nu p}$ - - 0.317 0.207 0.432 0 0 0 0 corr sunspot, investment $\rho_{\nu p}$ - - 0.725 0.621 0.831 0 0 0 0 corr sunspot, labor supply $\rho_{\nu a}$ - - - 0.112 -0.257 0.026 0 0 0 0 corr sunspot, technology $\rho_{\nu a}$ - - - 0.209 0.010 0.325 0 0 0 0 corr sunspot, technology $\rho_{\nu a}$ - - 0.209 0.011 0.325 0 0 0 0 0 0 0 0 0 </td <td></td> <td></td> <td></td> <td></td> <td>Shocks corre</td> <td>elations</td> <td></td> <td></td> <td></td> <td></td> <td></td>					Shocks corre	elations						
corr sunspot, investment $\rho_{\nu i}$ - 0.317 0.207 0.432 0 0 0 0 corr sunspot, price markup $\rho_{\nu p}$ - - - 0.131 0.207 0.432 0	corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.319	0.134	0.504	0	0	0	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.283	-0.438	-0.124	0	0	0	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.317	0.207	0.432	0	0	0	
corr sunspot, labor supply corr sunspot, gov spending corr sunspot, gov spending corr sunspot, technology $\rho_{\nu g}$ - - - -0.131 -0.280 0.022 0 0 0 corr sunspot, gov spending corr sunspot, technology $\rho_{\nu g}$ - - - -0.112 -0.257 0.026 0 0 0 0 corr sunspot, technology $\rho_{\nu a}$ - - - 0.209 0.091 0.325 0 0 0 0 corr sunspot, energy price $\rho_{\nu a}$ - - 0.209 0.091 0.325 0 0 0 0 corr sunspot, energy price $\rho_{\nu a}$ - - 0.209 0.091 0.325 0 0 0 corr sunspot, energy price $\rho_{\nu a}$ - - 0.209 0.091 0.325 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.725	0.621	0.831	0	0	0	
$\begin{array}{c} \mbox{corr sunspot, gov spending} & \rho_{\nu g} & - & - & - & -0.112 & -0.257 & 0.026 & 0 & 0 & 0 & 0 \\ \mbox{corr sunspot, technology} & \rho_{\nu a} & - & - & - & -0.168 & -0.292 & -0.045 & 0 & 0 & 0 & 0 \\ \mbox{corr sunspot, energy price} & \rho_{\nu s} & - & - & - & 0.209 & 0.091 & 0.325 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & $	corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	-0.131	-0.280	0.022	0	0	0	
$\begin{array}{c} \mbox{corr sunspot, technology} & \rho_{\nu a} & - & - & - & -0.168 & -0.292 & -0.045 & 0 & 0 & 0 & 0 \\ \mbox{corr sunspot, energy price} & \rho_{\nu s} & - & - & 0.209 & 0.01 & 0.325 & 0 & 0 & 0 & 0 \\ \hline & & & & & & & & & & & & & & & & & &$	corr sunspot, gov spending	$\rho_{\nu g}$	-	-	-	-0.112	-0.257	0.026	0	0	0	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.168	-0.292	-0.045	0	0	0	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.209	0.091	0.325	0	0	0	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$					Foreign para	ameters						
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	SS foreign inflation	$\bar{\pi}^*$	0.605	0.479	0.730	0.605	0.479	0.729	0.606	0.480	0.735	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	SS foreign int rate	$ \bar{R}^* $	0.363	0.223	0.495	0.363	0.226	0.493	0.360	0.220	0.494	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	foreign demand persistence	ρ_y^*	0.915	0.875	0.957	0.917	0.877	0.958	0.918	0.877	0.962	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	foreign inflation persistence	$\rho_{\pi}^{\check{*}}$	0.516	0.408	0.624	0.515	0.408	0.620	0.521	0.413	0.629	
	foreign rate persistence	ρ_R^*	0.869	0.846	0.898	0.865	0.840	0.896	0.868	0.845	0.897	
foreign inflation std dev σ_{π}^{*} 0.571 0.504 0.638 0.570 0.504 0.635 0.572 0.504 0.638 foreign rate std dev σ_{R}^{*} 0.159 0.138 0.179 0.160 0.139 0.180 0.159 0.138 0.178 Log data density -1085.1 -1065.6 -1087.7	foreign demand std dev	σ_u^*	0.642	0.563	0.716	0.643	0.565	0.718	0.643	0.567	0.719	
foreign rate std dev σ_R^* 0.159 0.138 0.179 0.160 0.139 0.180 0.159 0.138 0.178 Log data density -1085.1 -1065.6 -1087.7	foreign inflation std dev	σ_{π}^{*}	0.571	0.504	0.638	0.570	0.504	0.635	0.572	0.504	0.638	
Log data density -1085.1 -1065.6 -1087.7	foreign rate std dev	σ_R^*	0.159	0.138	0.179	0.160	0.139	0.180	0.159	0.138	0.178	
	Log data density			-1085.1			-1065.6			-1087.7		

Table A.1: Parameter estimates - Wu & Xia Shadow Rate (1999Q1-2023Q2)

		De	terminacy	7	Ind	eterminad	ey .	Indeterminacy			
		-			(unrestric	cted corre	lations)	(restricted correlations)			
		post. mean 90% HPD interval		post. mean 90% HPD interval			post. mean 90% HPD interval				
TR response to inflation	ϕ_{π}	2.152	1.570	2.731	0.457	0.054	0.818	0.673	0.376	0.985	
TR response to output	ϕ_{u}	0.174	0.084	0.260	0.074	0.010	0.133	0.116	0.030	0.195	
TR response to output growth	ϕ_{ay}	0.033	0.010	0.056	0.021	0.010	0.035	0.040	0.010	0.067	
TR interest rate smoothing	ϕ_R	0.782	0.737	0.827	0.874	0.818	0.934	0.834	0.765	0.905	
inverse Frisch elasticity	ϕ_l	0.432	0.163	0.687	0.712	0.139	1.434	0.530	0.215	0.822	
habits	b	0.584	0.490	0.680	0.614	0.447	0.826	0.554	0.451	0.653	
investment adjustment costs	γ_I	4.878	3.139	6.514	4.331	1.789	6.990	4.265	2.573	5.874	
Calvo price stickiness	ξ_p	0.821	0.757	0.891	0.796	0.735	0.854	0.652	0.565	0.739	
real wage rigidity	γ	0.794	0.734	0.856	0.821	0.709	0.943	0.776	0.706	0.843	
Employment parameter	ξe	0.449	0.320	0.577	0.416	0.304	0.526	0.403	0.300	0.506	
price indexation	χ_p	0.672	0.453	0.897	0.745	0.532	0.948	0.266	0.069	0.469	
capital utilization elasticity	σ_u	0.815	0.697	0.935	0.784	0.642	0.930	0.856	0.761	0.956	
intertemporal elasticity	σ	1.026	0.820	1.230	1.148	0.832	1.466	1.080	0.822	1.335	
inputs elasticity	ϵ	0.229	0.094	0.365	0.216	0.087	0.340	0.203	0.085	0.322	
home/imported good elast.	ν	0.449	0.176	0.717	0.451	0.175	0.721	0.432	0.162	0.684	
ss growth	g_z	0.254	0.223	0.286	0.251	0.222	0.281	0.250	0.218	0.283	
ss hours	Ē	1.271	-0.477	3.025	1.545	-0.207	3.332	1.380	-0.296	3.096	
ss inflation	$\bar{\pi}$	0.441	0.315	0.565	0.522	0.352	0.691	0.515	0.349	0.684	
			Sh	ocks persis	tences						
risk premium	ρ_b	0.835	0.780	0.895	0.763	0.498	0.923	0.840	0.772	0.914	
investment	ρ_i	0.363	0.240	0.486	0.491	0.333	0.650	0.371	0.244	0.498	
monetary	ρ_r	0.423	0.313	0.530	0.474	0.355	0.595	0.453	0.332	0.577	
price markup	ρ_{p}	0.717	0.594	0.844	0.740	0.610	0.870	0.867	0.769	0.971	
labor supply	ρ_l	0.855	0.779	0.931	0.854	0.775	0.939	0.881	0.813	0.949	
gov spending	ρ_a	0.834	0.772	0.897	0.868	0.809	0.926	0.842	0.782	0.903	
technology	ρ_a	0.842	0.777	0.905	0.835	0.759	0.919	0.837	0.769	0.907	
energy price	ρ_s	0.978	0.965	0.991	0.976	0.960	0.991	0.975	0.961	0.989	
MA price markup	ρ_{ma}^p	0.558	0.421	0.696	0.565	0.426	0.706	0.389	0.223	0.550	
gy correlation	ρ_{ay}	0.085	0.010	0.165	0.144	0.010	0.275	0.092	0.010	0.183	
Shocks standard deviations											
risk premium	σ_b	0.347	0.274	0.416	0.466	0.321	0.653	0.370	0.273	0.461	
investment	σ_i	1.071	0.904	1.231	1.101	0.910	1.286	1.104	0.928	1.269	
monetary	σ_r	0.383	0.326	0.439	0.341	0.297	0.385	0.360	0.306	0.412	
price markup	σ_p	0.105	0.083	0.127	0.109	0.086	0.132	0.138	0.096	0.181	
labor supply	σ_l	1.915	1.306	2.518	2.712	1.036	5.200	1.827	1.205	2.417	
gov spending	σ_q	0.832	0.729	0.935	0.736	0.614	0.859	0.805	0.683	0.925	
technology	σ_a	0.935	0.765	1.102	0.876	0.684	1.068	0.851	0.692	1.000	
energy price	σ_s	3.130	2.776	3.497	3.155	2.779	3.523	3.137	2.769	3.495	
measurement error	$\sigma \pi_{u}^{me}$	0.325	0.285	0.363	0.326	0.286	0.364	0.323	0.284	0.360	
sunspot	σ_{ν}	-	-	-	0.313	0.254	0.372	0.238	0.188	0.288	
*		1	Sh	ocks correl	ations			1			
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.227	0.052	0.408	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.387	-0.486	-0.288	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.359	0.240	0.478	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.692	0.591	0.796	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	0.073	-0.049	0.189	0	0	0	
corr sunspot, gov spending	$\rho_{\nu q}$	-	-	-	-0.216	-0.335	-0.093	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.242	-0.370	-0.116	0	0	0	
corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.119	0.023	0.214	0	0	0	
Foreign parameters											
SS foreign inflation	$\bar{\pi}^*$	0.607	0.479	0.731	0.609	0.490	0.730	0.606	0.477	0.728	
SS foreign int rate	\bar{R}^*	0.329	0.194	0.463	0.332	0.197	0.465	0.329	0.193	0.466	
foreign demand persistence	$\rho_{}^{*}$	0.914	0.874	0.956	0.913	0.869	0.956	0.915	0.874	0.958	
foreign inflation persistence	ρ_{π}^{*}	0.516	0.409	0.626	0.495	0.396	0.593	0.518	0.409	0.626	
foreign rate persistence	ρ_{R}^{*}	0.873	0.851	0.898	0.869	0.847	0.897	0.873	0.851	0.898	
foreign demand std dev	σ_{u}^{*}	0.642	0.566	0.718	0.641	0.564	0.715	0.642	0.567	0.718	
foreign inflation std dev	σ_{π}^{y}	0.571	0.504	0.636	0.569	0.503	0.634	0.571	0.502	0.637	
foreign rate std dev	σ_{R}^{*}	0.172	0.150	0.193	0.173	0.152	0.195	0.172	0.150	0.193	
Log data density			-1192.6		1	-1175.7		1	-1201.2		

Table A.2: Parameter estimates - 1-year OIS (1999Q1-2023Q2)

		Determinacy			Indeterminacy			Indeterminacy			
					(unrestricted correlations)			(restricted correlations)			
		post. mean	90% HPD interval		post. mean	90% HPD interval		post. mean 90% HPD i		D interval	
TR response to inflation	φ_	1.530	1.012	1.997	0.597	0.285	0.929	0.750	0.529	0.991	
TR response to output	ϕ_{n}	0.172	0.113	0.231	0.074	0.010	0.126	0.086	0.010	0.144	
TR response to output growth	ϕ_{au}	0.014	0.010	0.020	0.022	0.010	0.037	0.015	0.010	0.022	
TR interest rate smoothing	φp	0.896	0.869	0.923	0.937	0.905	0.972	0.927	0.889	0.964	
inverse Frisch elasticity	φn dn	0.207	0.091	0.318	0.231	0.106	0.351	0.279	0.128	0.425	
habits	b^{φ_l}	0.407	0.320	0.495	0.347	0.223	0.458	0.389	0.297	0.481	
investment adjustment costs	γ_r	4 810	3 108	6 490	3 856	2.094	5 577	3 915	2.280	5 497	
Calvo price stickiness	É	0.877	0.835	0.924	0.818	0.762	0.875	0.654	0.595	0.715	
real wage rigidity	γ^{p}	0.700	0.641	0.761	0.575	0.388	0.734	0.666	0.594	0.739	
Employment parameter	É	0.465	0.338	0.593	0.419	0.306	0.534	0.402	0.297	0.507	
price indexation	v Se	0.468	0.227	0.702	0.335	0.125	0.534	0.203	0.072	0.329	
capital utilization elasticity	$\sigma^{\lambda p}$	0.851	0.221	0.950	0.839	0.734	0.051	0.883	0.808	0.964	
intertemporal electicity	σ^{u}	1.093	0.104	1.280	1.061	0.154	1.252	1.081	0.857	1 293	
inputs electicity		0.234	0.004	0.367	0.208	0.045	0.331	0.204	0.070	0.322	
home/imported good elect	с 1/	0.234	0.055	0.715	0.203	0.075	0.551	0.204	0.075	0.522	
se growth		0.445	0.175	0.283	0.440	0.101	0.701	0.420	0.105	0.030	
ss growth	$\frac{g_z}{\overline{r}}$	2.008	0.210	2 9 9 9	1 452	0.210	9 207	1.651	0.204	2 522	
ss nours		2.008	0.126	0.670	0.476	-0.001	0.624	0.482	-0.200	0.620	
ss innation	л	0.559	0.405	Chocks perc	0.470	0.320	0.034	0.462	0.322	0.059	
nial- manairma		0.026	0.807	o of 7	Istences	0.000	0.001	0.024	0.000	0.080	
risk premium	ρ_b	0.926	0.897	0.957	0.934	0.892	0.981	0.924	0.808	0.980	
investment	ρ_i	0.328	0.217	0.437	0.353	0.235	0.473	0.341	0.224	0.451	
monetary	ρ_r	0.355	0.243	0.409	0.350	0.220	0.473	0.378	0.248	0.505	
price markup	ρ_p	0.073	0.000	0.793	0.782	0.062	0.890	0.883	0.813	0.957	
labor supply	ρ_l	0.892	0.836	0.950	0.914	0.868	0.966	0.905	0.857	0.957	
gov spending	ρ_g	0.862	0.809	0.916	0.869	0.819	0.919	0.862	0.812	0.916	
technology	ρ_a	0.894	0.844	0.947	0.885	0.819	0.955	0.877	0.819	0.937	
energy price	ρ_s	0.977	0.963	0.990	0.975	0.960	0.990	0.975	0.963	0.989	
MA price markup	ρ_{ma}^p	0.550	0.412	0.689	0.571	0.426	0.724	0.342	0.201	0.477	
gy correlation	ρ_{gy}	0.161	0.011	0.278	0.177	0.010	0.302	0.153	0.010	0.277	
Shocks standard deviations											
risk premium	σ_b	0.163	0.118	0.205	0.185	0.105	0.260	0.168	0.083	0.250	
investment	σ_i	1.123	0.955	1.288	1.138	0.967	1.302	1.134	0.971	1.301	
monetary	σ_r	0.123	0.106	0.140	0.120	0.104	0.136	0.118	0.102	0.132	
price markup	σ_p	0.110	0.086	0.134	0.101	0.076	0.126	0.139	0.107	0.169	
labor supply	σ_l	1.273	0.993	1.547	1.091	0.800	1.372	1.205	0.913	1.488	
gov spending	σ_g	0.767	0.669	0.863	0.752	0.645	0.853	0.774	0.658	0.884	
technology	σ_a	0.838	0.686	0.982	0.811	0.664	0.954	0.752	0.632	0.870	
energy price	σ_s	3.050	2.679	3.405	3.076	2.696	3.436	3.050	2.686	3.420	
measurement error	$\sigma_{\pi_y}^{me}$	0.314	0.275	0.351	0.312	0.276	0.349	0.314	0.276	0.352	
sunspot	σ_{ν}	-	-	-	0.231	0.191	0.272	0.223	0.188	0.259	
				Shocks corre	elations						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.252	0.068	0.447	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.330	-0.485	-0.173	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.237	0.120	0.359	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.765	0.667	0.871	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	-0.147	-0.305	0.001	0	0	0	
corr sunspot, gov spending	$\rho_{\nu g}$	-	-	-	-0.138	-0.288	0.006	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.137	-0.278	-0.001	0	0	0	
corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.173	0.059	0.288	0	0	0	
				Foreign para	ameters				-		
SS foreign inflation	$\bar{\pi}^*$	0.606	0.473	0.734	0.603	0.471	0.736	0.606	0.472	0.735	
SS foreign int rate	\bar{R}^*	0.307	0.177	0.433	0.308	0.179	0.433	0.307	0.180	0.434	
foreign demand persistence	ρ^*_{*}	0.913	0.869	0.956	0.914	0.869	0.960	0.915	0.870	0.959	
foreign inflation persistence	ρ_{-}^{*}	0.533	0.422	0.646	0.529	0.421	0.635	0.547	0.435	0.663	
foreign rate persistence	ρ_R^*	0.865	0.838	0.895	0.863	0.836	0.894	0.864	0.837	0.895	
foreign demand std dev	σ_{*}^{*}	0.655	0.576	0.735	0.656	0.578	0.736	0.658	0.579	0.737	
foreign inflation std dev	σ_{-}^{*}	0.585	0.515	0.654	0.584	0.511	0.653	0.587	0.513	0.657	
foreign rate std dev	σ_{P}^{π}	0.168	0.146	0.190	0.168	0.146	0.190	0.168	0.146	0.190	
Log data density	K		-1020.5		1	-1003.6			-1025.3		
5 ····· · · · · · · · · · · · · · · · ·		l									

Table A.3: Parameter estimates - Krippner Shadow Rate (1999Q1-2022Q2)
		Determinacy			Ind	eterminac	у	Indeterminacy			
					(unrestric	ted correl	ations)	(restricted correlations)			
		post. mean	n 90% HPD interval		post. mean	t. mean 90% HPD interval		post. mean	90% HPD interval		
TR response to inflation	ϕ_{π}	1.451	0.962	1.845	0.660	0.367	0.974	0.756	0.533	0.993	
TR response to output	<i>φ</i>	0.205	0.148	0.263	0.099	0.021	0.167	0.104	0.023	0.177	
TR response to output growth	φ _{au}	0.077	0.018	0.133	0.092	0.031	0.148	0.066	0.012	0.109	
TR interest rate smoothing	ϕ_{R}	0.848	0.807	0.891	0.918	0.871	0.966	0.910	0.861	0.961	
inverse Frisch elasticity	ϕ_n	0.446	0.223	0.664	0.758	0.431	1 084	1 125	0.682	1 553	
habits	h^{φ_l}	0.707	0.628	0.790	0.700	0.625	0.780	0.747	0.683	0.815	
investment adjustment costs	γ_{r}	4 088	2 361	5.819	4 440	2 561	6 255	4 340	2 657	5 980	
Calvo price stickiness	É	0.904	0.873	0.936	0.655	0.577	0.730	0.558	0.502	0.615	
real wage rigidity	γ^{p}	0.789	0.718	0.863	0.723	0.640	0.812	0.792	0.737	0.849	
Employment parameter	É	0.544	0.404	0.682	0.428	0.310	0.546	0.416	0.303	0.529	
price indexation	v Se	0.353	0.133	0.575	0.265	0.093	0.427	0.179	0.062	0.289	
capital utilization elasticity	$\int_{\sigma}^{\lambda p}$	0.787	0.649	0.930	0.807	0.688	0.933	0.805	0.684	0.935	
intertemporal elasticity	σ^{u}	0.998	0.784	1 212	1 162	0.810	1 499	1 238	0.870	1.590	
inpute electicity	6	0.263	0.101	0.415	0.226	0.088	0.359	0.222	0.088	0.354	
home/imported goods elect		0.483	0.100	0.772	0.462	0.174	0.335	0.471	0.000	0.304	
ss growth		0.460	0.100	0.200	0.228	0.114	0.765	0.227	0.110	0.261	
se houre	$\frac{g_z}{\bar{F}}$	1.257	0.221	3 150	0.226	1 180	0.200	0.660	1 157	2 481	
ss inflation		0.440	0.349	0.531	0.700	-1.100	0.522	0.000	0.224	0.532	
ss initation	1	0.440	0.042	books pore	0.575	0.220	0.322	0.561	0.224	0.002	
nial- manairma		0.010	0.070	0 062	0.856	0.749	0.061	0.969	0.799	0.069	
risk premium	ρ_b	0.919	0.070	0.902	0.850	0.748	0.901	0.808	0.762	0.962	
investment	ρ_i	0.280	0.100	0.402	0.360	0.221	0.495	0.405	0.257	0.559	
monetary	ρ_r	0.307	0.233	0.501	0.340	0.205	0.470	0.304	0.217	0.503	
price markup	ρ_p	0.640	0.515	0.766	0.882	0.811	0.959	0.896	0.830	0.960	
labor supply	ρ_l	0.882	0.816	0.951	0.934	0.894	0.974	0.928	0.887	0.971	
gov spending	ρ_g	0.854	0.795	0.915	0.878	0.829	0.932	0.874	0.816	0.935	
technology	ρ_a	0.932	0.885	0.979	0.893	0.834	0.954	0.891	0.839	0.947	
energy price	ρ_s	0.975	0.960	0.991	0.975	0.961	0.989	0.977	0.963	0.991	
MA price markup	ρ_{ma}^p	0.569	0.358	0.740	0.468	0.318	0.617	0.375	0.238	0.509	
gy correlation	ρ_{gy}	0.168	0.01	0.295	0.102	0.01	0.204	0.078	0.010	0.159	
			Shoc	ks standarc	deviations						
risk premium	σ_b	0.069	0.049	0.089	0.070	0.042	0.098	0.057	0.034	0.079	
investment	σ_i	1.170	0.996	1.346	1.149	0.961	1.326	1.145	0.954	1.332	
monetary	σ_r	0.129	0.107	0.149	0.123	0.105	0.141	0.123	0.105	0.140	
price markup	σ_p	0.094	0.071	0.118	0.109	0.081	0.135	0.161	0.125	0.196	
labor supply	σ_l	1.714	1.144	2.262	1.578	1.110	2.033	2.048	1.517	2.562	
government spending	σ_g	0.799	0.692	0.905	0.762	0.648	0.873	0.765	0.646	0.889	
technology	σ_a	0.865	0.659	1.064	0.802	0.661	0.936	0.781	0.649	0.908	
energy price	σ_s	2.462	2.148	2.778	2.477	2.165	2.791	2.462	2.153	2.780	
measurement error	$\sigma_{\pi_y}^{me}$	0.255	0.222	0.287	0.254	0.221	0.286	0.254	0.220	0.286	
sunspot	σ_{ν}	-	-	-	0.147	0.115	0.180	0.137	0.109	0.164	
			ç	Shocks corre	elations						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.111	-0.164	0.390	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.178	-0.421	0.064	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.122	-0.084	0.334	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.772	0.629	0.919	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	-0.042	-0.238	0.151	0	0	0	
corr sunspot, gov spending	$\rho_{\nu g}$	-	-	-	0.134	-0.134	0.397	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.138	-0.357	0.084	0	0	0	
corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.148	-0.027	0.323	0	0	0	
	1		Ι	Foreign para	ameters						
SS foreign inflation	$\bar{\pi}^*$	0.567	0.449	0.681	0.566	0.450	0.683	0.565	0.445	0.678	
SS foreign int rate	\bar{R}^*	0.325	0.193	0.457	0.329	0.188	0.459	0.328	0.194	0.462	
foreign demand persistence	ρ_{u}^{*}	0.924	0.885	0.964	0.920	0.879	0.961	0.919	0.877	0.961	
foreign inflation persistence	ρ_{π}^{*}	0.457	0.336	0.580	0.456	0.332	0.578	0.454	0.335	0.577	
foreign rate persistence	ρ_{P}^{*}	0.873	0.851	0.897	0.871	0.848	0.897	0.872	0.850	0.898	
foreign demand std dev	$\sigma^*_{}$	0.588	0.511	0.662	0.592	0.512	0.667	0.595	0.517	0.670	
foreign inflation std dev	σ^*	0.528	0.459	0.595	0.528	0.459	0.595	0.528	0.458	0.595	
foreign rate std dev	σ_{p}^{π}	0.154	0.1331	0.175	0.155	0.133	0.176	0.155	0.134	0.177	
Log data density	- <u>R</u>		-709.1		0.100	-694.8		0.200	-699.9		
		L									

Table A.4: Parameter estimates - Krippner Shadow Rate (1999Q1-2019Q4)

		De	terminac	v	Ind	eterminac	y	Indeterminacy			
					(unrestric	ted correl	ations)	(restricted correlations)			
		post. mean	an 90% HPD interval		post. mean	ost. mean 90% HPD interval			post. mean 90% HPD ir		
TB response to inflation	φ	1 565	1.021	2.038	0.614	0.299	0.962	0.792	0.595	0.993	
TB response to output	ϕ_{π}	0.150	0.077	0.224	0.075	0.010	0.130	0.093	0.020	0.158	
TB response to output growth	ϕ_y	0.051	0.010	0.086	0.084	0.010	0.135	0.072	0.020	0.117	
TB interest rate smoothing	φ_{gy}	0.861	0.823	0.000	0.868	0.808	0.100	0.864	0.811	0.916	
inverse Frisch electicity	ϕ_R	0.870	0.387	1 332	0.801	0.000	1 168	1.067	0.615	1 506	
habite	h^{φ_l}	0.721	0.635	0.816	0.656	0.400	0.745	0.686	0.010	0.764	
invoctment adjustment costs	~	6.871	4 341	0.280	5 803	3 747	8.000	6.438	4 990	8 682	
Calvo prico stickinoss	c	0.872	0.810	0.034	0.711	0.630	0.701	0.438	4.229	0.703	
voal word vigidity	ς_p	0.860	0.010	0.934	0.711	0.050	0.731	0.075	0.749	0.155	
Employment peremeter	c	0.517	0.150	0.527	0.420	0.074	0.546	0.307	0.740	0.805	
price indevation	Se	0.317	0.375	0.052	0.425	0.062	0.340	0.430	0.046	0.047	
capital utilization electicity	χ_p	0.558	0.530	0.500	0.100	0.002	0.007	0.157	0.651	0.220	
intertemporal electicity		1 194	0.000	1.450	1 1 4 0	0.000	1.475	1.969	0.001	1 500	
inputs electicity		0.268	0.000	0.491	0.251	0.000	0.206	0.250	0.912	0.411	
home/imported good elect	с 1/	0.208	0.109	0.421	0.231	0.102	0.330	0.255	0.105	0.411	
se growth		0.452	0.109	0.725	0.441	0.109	0.704	0.402	0.177	0.755	
ss growth	$\frac{g_z}{\overline{r}}$	0.225	1.024	0.276	0.201	1.042	0.242	0.203	1 1 25	0.244	
ss nours		0.011	-1.034	2.735	0.770	-1.045	2.025	0.747	-1.155	2.023	
ss innation	л	0.401	0.304	0.002	0.471	0.318	0.017	0.465	0.325	0.059	
		0.011	0.700	Shocks pers	Istences	0.500	0.005	0.000	0.717	0.004	
risk premium	ρ_b	0.811	0.709	0.918	0.743	0.592	0.895	0.820	0.717	0.924	
investment	ρ_i	0.000	0.508	0.794	0.042	0.507	0.781	0.042	0.007	0.783	
monetary	ρ_r	0.285	0.149	0.419	0.348	0.208	0.487	0.303	0.227	0.497	
price markup	ρ_p	0.713	0.380	0.844	0.833	0.728	0.938	0.762	0.030	0.937	
labor supply	ρ_l	0.877	0.804	0.955	0.927	0.887	0.967	0.925	0.880	0.964	
gov spending	ρ_g	0.927	0.891	0.966	0.935	0.905	0.966	0.932	0.898	0.967	
technology	ρ_a	0.904	0.847	0.964	0.899	0.850	0.949	0.883	0.830	0.939	
energy price	ρ_s	0.978	0.964	0.992	0.972	0.958	0.987	0.975	0.961	0.989	
MA price markup	ρ_{ma}^{p}	0.547	0.407	0.691	0.498	0.353	0.642	0.494	0.317	0.674	
gy correlation	ρ_{gy}	0.078	0.010	0.149	0.056	0.010	0.110	0.053	0.010	0.105	
			Shoo	ks standard	deviations						
risk premium	σ_b	0.100	0.059	0.140	0.089	0.047	0.132	0.067	0.037	0.095	
investment	σ_i	0.241	0.177	0.304	0.251	0.188	0.313	0.251	0.188	0.313	
monetary	σ_r	0.102	0.084	0.120	0.096	0.080	0.112	0.094	0.078	0.109	
price markup	σ_p	0.090	0.066	0.114	0.115	0.083	0.146	0.158	0.123	0.193	
labor supply	σ_l	2.879	1.578	4.121	1.798	1.225	2.342	2.281	1.592	2.941	
gov spending	σ_g	0.436	0.367	0.509	0.485	0.399	0.567	0.492	0.403	0.582	
technology	σ_a	0.812	0.625	0.987	0.741	0.611	0.867	0.748	0.611	0.881	
energy price	σ_s	2.497	2.132	2.858	2.533	2.157	2.901	2.521	2.155	2.894	
measurement error	$\sigma_{\pi_y}^{me}$	0.271	0.231	0.310	0.271	0.231	0.310	0.272	0.230	0.311	
sunspot	σ_{ν}	-	-	-	0.135	0.104	0.166	0.126	0.076	0.174	
				Shocks corre	elations						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.039	-0.248	0.316	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	0.190	-0.006	0.392	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	-0.011	-0.265	0.241	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.759	0.602	0.924	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	-0.034	-0.281	0.197	0	0	0	
corr sunspot, gov spending	$\rho_{\nu g}$	-	-	-	0.234	0.000	0.469	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	0.008	-0.225	0.238	0	0	0	
corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.165	-0.032	0.369	0	0	0	
]	Foreign para	ameters				-		
SS foreign inflation	$\bar{\pi}^*$	0.584	0.453	0.715	0.581	0.453	0.705	0.582	0.456	0.708	
SS foreign int rate	\bar{R}^*	0.313	0.178	0.442	0.313	0.175	0.445	0.316	0.176	0.449	
foreign demand persistence	ρ_u^*	0.906	0.856	0.959	0.901	0.849	0.954	0.898	0.848	0.954	
foreign inflation persistence	ρ_{-}^{*}	0.471	0.361	0.582	0.468	0.354	0.582	0.461	0.345	0.575	
foreign rate persistence	ρ_R^*	0.828	0.788	0.869	0.835	0.796	0.874	0.839	0.801	0.878	
foreign demand std dev	σ_{u}^{*}	0.651	0.555	0.744	0.652	0.556	0.744	0.650	0.556	0.743	
foreign inflation std dev	σ_{π}^{g}	0.571	0.487	0.656	0.570	0.485	0.651	0.569	0.487	0.651	
foreign rate std dev	σ_{D}^{*}	0.194	0.160	0.228	0.191	0.158	0.223	0.190	0.157	0.222	
Log data density	n		-478.1			-470.9			-474.0		
v											

Table A.5: Parameter estimates - 3-month Euribor (1999Q1-2014Q4)

		De	terminac	7	Ind	eterminac	y	Indeterminacy			
					(unrestric	ted correl	ations)	(restricted correlations)			
		post. mean	an 90% HPD interval		post. mean	mean 90% HPD interval		post. mean	90% HPD interval		
TB response to inflation	φ_	1 315	0.984	1.636	0.517	0 155	0.880	0.645	0.350	0.971	
TB response to output	ϕ^{π}	0.168	0.099	0.238	0.125	0.049	0.198	0.133	0.055	0.207	
TB response to output growth	ϕ_y	0.035	0.010	0.061	0.059	0.010	0.099	0.052	0.010	0.089	
TB interest rate smoothing	ϕgy	0.869	0.827	0.912	0.854	0.778	0.933	0.849	0.780	0.921	
inverse Frisch elasticity	ϕ_n	1 043	0.421	1.651	1 043	0.460	1 584	1 261	0.594	1 916	
habite	$\begin{bmatrix} \varphi_l \\ h \end{bmatrix}$	0.639	0.530	0.751	0.585	0.400	0.706	0.627	0.523	0.732	
invoctment adjustment costs	~	6 338	3 753	8 8 4 0	5.042	3 471	8 940	6.287	3 750	8 803	
Calvo prico stickinoss	c	0.558	0.781	0.049	0.725	0.637	0.249	0.207	0.610	0.700	
roal wago rigidity	Sp	0.838	0.764	0.900	0.725	0.001	0.815	0.703	0.010	0.135	
Employment peremeter		0.656	0.704	0.913	0.728	0.009	0.640	0.110	0.095	0.800	
price indevation	Se	0.302	0.331	0.030	0.400	0.326	0.392	0.459	0.323	0.350	
capital utilization electicity	χ_p	0.471	0.212	0.725	0.515	0.117	0.430	0.225	0.005	0.009	
intertemporal electicity	0 u	1.461	0.405	1 0 2 2	1 421	0.028	1 997	1 500	1.026	1.081	
intertemporar elasticity		0.000	0.969	1.923	0.925	0.904	1.007	1.309	0.000	0.286	
home/imported good elect	e .	0.226	0.091	0.500	0.235	0.094	0.500	0.240	0.099	0.505	
as growth		0.334	0.134	0.301	0.370	0.130	0.390	0.375	0.140	0.393	
ss growth	g_z	1 200	0.237	2.002	1 599	0.235	2.050	1.474	0.249	0.304	
ss nours		1.388	-0.233	3.003	1.588	-0.003	3.238	1.4/4	-0.213	3.131 0.662	
ss inflation	π	0.407	0.342	0.592	0.513	0.352	0.071	0.506	0.343	0.003	
• 1 •		0.770	0.070	shocks pers	istences	0.001	0.001	0.021	0 700	0.005	
risk premium	ρ_b	0.779	0.672	0.890	0.759	0.621	0.901	0.821	0.720	0.925	
investment	ρ_i	0.784	0.660	0.916	0.766	0.622	0.914	0.780	0.644	0.922	
monetary	ρ_r	0.274	0.143	0.401	0.340	0.190	0.488	0.342	0.192	0.488	
price markup	ρ_p	0.671	0.528	0.814	0.783	0.649	0.921	0.685	0.495	0.889	
labor supply	ρ_l	0.864	0.760	0.959	0.902	0.833	0.969	0.901	0.834	0.968	
gov spending	ρ_g	0.890	0.836	0.945	0.888	0.830	0.947	0.881	0.818	0.945	
technology	ρ_a	0.812	0.705	0.919	0.837	0.735	0.942	0.821	0.719	0.926	
energy price	ρ_s	0.938	0.901	0.976	0.944	0.911	0.979	0.943	0.907	0.980	
MA price markup	ρ_{ma}^p	0.527	0.377	0.676	0.478	0.325	0.627	0.530	0.357	0.701	
gy correlation	ρ_{gy}	0.164	0.010	0.281	0.145	0.010	0.274	0.143	0.010	0.267	
			Shoo	ks standard	deviations			0.050			
risk premium	σ_b	0.095	0.056	0.134	0.076	0.037	0.112	0.070	0.040	0.099	
investment	σ_i	0.163	0.112	0.213	0.183	0.126	0.238	0.176	0.122	0.229	
monetary	σ_r	0.079	0.061	0.096	0.074	0.058	0.089	0.074	0.058	0.089	
price markup	σ_p	0.086	0.059	0.114	0.107	0.068	0.145	0.142	0.099	0.184	
labor supply	σ_l	2.561	1.402	3.682	1.750	1.063	2.430	2.099	1.345	2.856	
gov spending	σ_g	0.362	0.282	0.442	0.422	0.324	0.517	0.407	0.305	0.507	
technology	σ_a	0.656	0.469	0.836	0.569	0.432	0.701	0.593	0.439	0.740	
energy price	σ_s	2.578	2.050	3.086	2.596	2.063	3.106	2.564	2.033	3.066	
measurement error	$\sigma_{\pi_y}^{me}$	0.291	0.234	0.344	0.292	0.237	0.346	0.292	0.236	0.348	
sunspot	σ_{ν}	-	-	-	0.133	0.089	0.178	0.111	0.059	0.165	
			5	Shocks corre	elations						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	-0.026	-0.371	0.320	0	0	0	
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	0.127	-0.129	0.388	0	0	0	
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	-0.025	-0.287	0.230	0	0	0	
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.721	0.498	0.945	0	0	0	
corr sunspot, labor supply	$\rho_{\nu l}$	-	-	-	0.113	-0.162	0.408	0	0	0	
corr sunspot, gov spending	$\rho_{\nu g}$	-	-	-	-0.165	-0.455	0.123	0	0	0	
corr sunspot, technology	$\rho_{\nu a}$	-	-	-	-0.034	-0.305	0.238	0	0	0	
corr sunspot, energy price	$\rho_{\nu s}$	-	-	-	0.035	-0.219	0.281	0	0	0	
]	Foreign para	ameters				-		
SS foreign inflation	$\bar{\pi}^*$	0.631	0.496	0.764	0.629	0.492	0.762	0.633	0.496	0.765	
SS foreign int rate	\bar{R}^*	0.642	0.451	0.835	0.642	0.454	0.826	0.628	0.434	0.823	
foreign demand persistence	ρ^*_{*}	0.830	0.737	0.929	0.828	0.735	0.927	0.825	0.732	0.922	
foreign inflation persistence	ρ_{-}^{*}	0.479	0.350	0.617	0.479	0.344	0.617	0.483	0.344	0.623	
foreign rate persistence	ρ_{P}^{*}	0.758	0.682	0.837	0.763	0.687	0.841	0.775	0.702	0.850	
foreign demand std dev	$\sigma^*_{}$	0.542	0.434	0.648	0.540	0.435	0.645	0.538	0.432	0.641	
foreign inflation std dev	σ^*	0.421	0.338	0.501	0.421	0.336	0.500	0.422	0.339	0.504	
foreign rate std dev	σ_{p}^{π}	0.182	0.137	0.226	0.180	0.136	0.223	0.177	0.135	0.219	
Log data density	- <u>R</u>		-246.4			-248.3			-246.7	0.210	
	I	l									

Table A.6: Parameter estimates - 3-month Euribor (1999Q1-2007Q4)

		Determ	Indeterr	ninacy (P	MPF)	Indeterminacy (PMPF)			FTPL (PMAF)				
		and many 00% HDD : to the		(unrestrie	cted corre	lations)	(restricted correlations)						
		post. mean	90% HP	D interval	post. mean	90% HP	D interval	post. mean	90% HI	'D interval	post. mean	90% HF	'D interval
TR response to inflation	ϕ_{π}	1.272	0.949	1.639	0.640	0.379	0.929	0.775	0.571	0.989	0.610	0.308	0.952
TR response to output	ϕ_y	0.161	0.112	0.210	0.080	0.016	0.131	0.088	0.019	0.145	0.061	0.010	0.112
TR response to output growth	ϕ_{gy}	0.015	0.010	0.021	0.016	0.010	0.024	0.015	0.010	0.022	0.012	0.010	0.015
TR interest rate smoothing	ϕ_R	0.881	0.848	0.915	0.907	0.861	0.952	0.908	0.865	0.951	0.954	0.929	0.979
Cons tax response to debt	$\phi_b^{\tau c}$	0.157	0.043	0.276	0.262	0.081	0.454	0.226	0.033	0.416	0.159	-0.136	0.450
Labor tax response to debt	$\phi_b^{\tau l}$	0.005	-0.031	0.041	-0.075	-0.150	-0.010	-0.033	-0.092	0.024	-0.557	-0.697	-0.411
inverse Frisch elasticity	ϕ_l	0.205	0.097	0.313	0.242	0.113	0.368	0.298	0.142	0.447	0.217	0.090	0.342
habits	b	0.448	0.346	0.548	0.382	0.274	0.483	0.392	0.294	0.487	0.356	0.264	0.449
investment adjustment costs	γ_I	5.278	3.483	7.030	3.929	2.198	5.628	3.837	2.274	5.336	5.801	3.901	7.654
Calvo price stickiness	ξ_p	0.877	0.840	0.915	0.824	0.772	0.878	0.649	0.587	0.711	0.829	0.785	0.873
real wage rigidity	γ	0.692	0.629	0.753	0.638	0.529	0.746	0.640	0.563	0.719	0.728	0.678	0.777
Employment parameter	ξ_e	0.448	0.314	0.584	0.417	0.301	0.536	0.402	0.300	0.504	0.446	0.318	0.573
price indexation	χ_p	0.449	0.207	0.686	0.267	0.099	0.428	0.203	0.075	0.330	0.626	0.434	0.823
capital utilization elasticity	σ_u	0.868	0.784	0.956	0.817	0.697	0.941	0.883	0.807	0.961	0.896	0.825	0.969
intertemporal elasticity	σ	0.956	0.777	1.137	1.017	0.815	1.217	0.981	0.787	1.178	1.498	1.182	1.795
inputs elasticity	e	0.232	0.097	0.366	0.200	0.080	0.315	0.189	0.077	0.299	0.237	0.100	0.371
home/imported good elast.	ν	0.415	0.157	0.659	0.406	0.159	0.646	0.403	0.159	0.645	0.458	0.184	0.728
ss growth	a.	0.263	0.230	0.296	0.250	0.221	0.281	0.249	0.216	0.282	0.256	0.225	0.289
ss hours	Ē	1.577	-0.206	3.463	1.355	-1.002	3.751	1.526	-0.455	3.476	0.976	-1.473	3.432
ss inflation	$\bar{\pi}$	0.489	0.327	0.650	0.484	0.327	0.641	0.484	0.323	0.640	0.483	0.326	0.636
		0.100	0.021	0.000	Shocks I	ersistenc	0.011	0.101	0.020	0.010	0.100	0.020	0.000
risk premium	0	0.930	0.897	0.962	0.921	0.874	0.968	0.911	0.855	0.971	0.932	0.901	0.965
investment	P0 0:	0.321	0.209	0.428	0.360	0.232	0.488	0.332	0.216	0.447	0.323	0.195	0.447
monotary	ρ_i	0.321	0.205	0.420	0.300	0.232	0.400	0.302	0.210	0.515	0.323	0.135	0.447
price markup	ρ_r	0.555	0.556	0.474	0.374	0.240	0.000	0.896	0.201	0.910	0.334	0.234	0.838
labor gupply	P_p	0.000	0.550	0.025	0.904	0.000	0.048	0.805	0.840	0.052	0.058	0.025	0.081
abor suppry	p_l	0.854	0.110	0.935	0.094	0.842	0.946	0.895	0.840	0.952	0.958	0.955	0.981
technology	ρ_g	0.874	0.824	0.920	0.910	0.871	0.905	0.895	0.840	0.940	0.003	0.834	0.925
chongy price	ρ_a	0.076	0.024	0.955	0.014	0.757	0.094	0.040	0.112	0.908	0.905	0.040	0.900
MA arrive menhan	ρ_s	0.570	0.905	0.969	0.570	0.354	0.960	0.371	0.350	0.980	0.500	0.949	0.965
MA price markup	ρ_{ma}^r	0.521	0.388	0.002	0.570	0.420	0.709	0.330	0.190	0.470	0.507	0.359	0.052
gy correlation	ρ_{gy}	0.098	0.010	0.189	0.086	0.010	0.169	0.101	0.010	0.198	0.178	0.043	0.303
·		0.150	0.100	0.000	Shocks stand	ard devia	utions	0.105	0.101	0.051	0.100	0.100	0.011
risk premium	σ_b	0.176	0.123	0.228	0.205	0.137	0.269	0.187	0.101	0.271	0.192	0.139	0.244
investment	σ_i	1.118	0.935	1.302	1.063	0.899	1.221	1.087	0.928	1.248	1.115	0.954	1.275
monetary	σ_r	0.124	0.105	0.141	0.120	0.104	0.135	0.116	0.101	0.130	0.118	0.104	0.132
price markup	σ_p	0.101	0.076	0.125	0.093	0.064	0.119	0.136	0.104	0.166	0.098	0.075	0.122
labor supply	σ_l	1.244	0.968	1.512	1.148	0.830	1.465	1.133	0.852	1.406	1.563	1.272	1.846
gov spending	σ_g	0.778	0.682	0.870	0.794	0.679	0.908	0.815	0.697	0.928	0.584	0.502	0.666
technology	σ_a	0.890	0.720	1.061	0.935	0.763	1.102	0.807	0.673	0.939	0.823	0.670	0.967
energy price	σ_s	3.150	2.779	3.524	3.187	2.814	3.563	3.159	2.781	3.518	3.173	2.797	3.548
measurement error	$\sigma_{\pi_y}^{me}$	0.327	0.288	0.365	0.323	0.285	0.360	0.322	0.283	0.360	0.324	0.286	0.362
sunspot	σ_{ν}	-	-	-	0.232	0.195	0.268	0.226	0.190	0.261	-	-	-
					Shocks of	correlation	IS						
corr sunspot, risk premium	$\rho_{\nu b}$	-	-	-	0.238	0.063	0.415	0	0	0	-	-	-
corr sunspot, investment	$\rho_{\nu i}$	-	-	-	-0.375	-0.518	-0.239	0	0	0		-	-
corr sunspot, monetary	$\rho_{\nu r}$	-	-	-	0.191	0.067	0.316	0	0	0	-	-	-
corr sunspot, price markup	$\rho_{\nu p}$	-	-	-	0.747	0.649	0.848	0	0	0	-	-	-
corr sunspot, labor supply	Pul	-	-	-	-0.054	-0.194	0.083	0	0	0	-	-	-
corr sunspot, gov spending	$\rho_{\nu a}$	-	-	-	-0.224	-0.354	-0.095	0	0	0	-	-	-
corr sunspot, technology	ρ _{ua}	-		-	-0.108	-0.251	0.044	0	0	0	-	-	-
corr sunspot, energy price	Due.	-		-	0.239	0.101	0.375	0	0	0	-	-	-
1 , 0, 1	100				Foreign	paramete	s			-			
SS foreign inflation	$\bar{\pi}^*$	0.586	0.462	0.710	0.605	0.479	0.731	0.606	0.478	0.734	0.605	0.473	0.731
SS foreign int rate	\bar{R}^*	0.336	0.193	0.475	0.334	0.196	0.466	0.338	0.202	0.473	0.328	0.196	0.459
foreign demand persistence	0*	0.015	0.135	0.958	0.004	0.879	0.957	0.015	0.202	0.958	0.020	0.870	0.409
foreign inflation persistence	P_y	0.531	0.497	0.635	0.514	0.306	0.610	0.515	0.013	0.555	0.520	0.431	0.505
foreign rate persistence	ρ_{π}	0.001	0.427	0.055	0.505	0.590	0.010	0.000	0.424	0.045	0.556	0.401	0.047
foreign domand std dor	$\rho_R = \sigma^*$	0.809	0.640	0.097	0.800	0.641	0.890	0.608	0.640	0.097	0.629	0.662	0.091
foreign inflation at 1	0 y	0.000	0.571	0.731	0.044	0.505	0.622	0.044	0.500	0.721	0.038	0.505	0.710
foreign mination Std dev	σ _π _*	0.374	0.301	0.045	0.308	0.301	0.033	0.373	0.003	0.039	0.3/3	0.000	0.039
Toreign rate std dev	σ_R	0.173	1071.0	0.195	0.175	0.152	0.190	0.173	0.101	0.194	0.170	0.103	0.198
Log data density			-1071.0			-1051.5			-1074.7			-1076.1	

Table A.7: Parameter estimates - Fiscal Policy (1999Q1-2023Q2)