DEMS WORKING PAPER SERIES

The Horizontal Geometry of Production Networks

Valerio Dionisi

No. 559 – October 2025

Department of Economics, Management and Statistics
University of Milano – Bicocca
Piazza Ateneo Nuovo 1 – 2016 Milan, Italy
http://dems.unimib.it/

THE HORIZONTAL GEOMETRY OF PRODUCTION NETWORKS

by VALERIO DIONISI*

October 2025

[click here for the latest version]

Abstract

Complementarities in intermediate inputs trigger the even transmission of asymmetric shocks in production networks. This paper linearly captures these complementarities through shared inter-sectoral trade relationships, and makes two novel contributions to the understanding of networked economies. First, it introduces a theoretical framework that distinguishes between *factor input demand* and *factor input supply* network distances, measuring economic distance between sectors based on common upstream sellers or downstream buyers. These horizontal interdependencies determine how sector-specific shocks transmit horizontally across the network, complementing and balancing the standard vertical (Leontief inverse) mechanism. Second, using sector-level U.S. employment data, the paper provides empirical evidence that positive employment shocks in closely demand- or supply-connected sectors are attenuated, whereas larger distances generate employment comovement. Together, these two contributions reveal that the horizontal geometry of a production network plays a critical role in understanding how sectoral interactions propagate micro-originated shocks in an Input-Output economy.

JEL: C67, D57, E32, F16, L14

KEYWORDS: Input-Output economy, production networks, network distance, horizontal transmission, sectoral comovement, tools for policy design

---

* Department of Economics, Management and Statistics, University of Milano-Bicocca, *email*: `valeriod.97@icloud.com`. I am deeply grateful to Francesco Zanetti for his steady feedback since the earliest stages of this paper's drafting, and to Andrea Colciago for his guidance throughout. My sincere gratitude goes also to Andrea Ferrero, Gino Gancia, Isabelle Mejean, and Daniele Siena for valuable and detailed reflections. I also thank participants to seminars held at University of Milano-Bicocca and University of Oxford. Last but not least, I am especially thankful to Davide Debortoli for suggesting me to explore the world of production networks, and to Mishel Ghassibe for his enlightening lectures. The idea for this paper occurred during a visiting research period at the economics department of Universitat Pompeu Fabra (2023), while most of the theory has been developed while I was visiting the economics department of University of Oxford (2025). I thank both faculties for their dynamic environment as exchange of ideas, and for their hospitality. *first version*: July 3, 2025.

Introduction

An hallmark of contemporary production systems is the comovement of economic activity across sectors. Recent advances in the production networks literature have demonstrated how independent, idiosyncratic shocks to a specific sector can propagate through Input-Output linkages, generating sectoral comovement in production and contributing to aggregate fluctuations. The presence of "(anti-)cascade effects" diffuses the shock along parallel supply chains via the tangle of Input-Output connections that tie sectors together.[1] In essence, a positive sectoral shock transmits positively and vertically to other linked sectors participating in the network, thereby inducing economic variables across different sectors to move together over time.

Yet, once complementarities in intermediate inputs constrain substitution possibilities, the transmission of shocks proves less straightforward.[2] One contribution of this paper is to theoretically assert that sectoral comovement depends crucially on the nature of *economic distance* between sectors. Beyond vertical linkages, sectors can be additionally connected through demand and/or supply interdependencies, contingent upon their common structure of inter-sectoral trade. Specifically, a pair of sectors exhibits *factor input demand* network distance if they are buying intermediate inputs from a similar set of upstream suppliers, whereas the pair reflects *factor input supply* network distance whenever its sectors are selling part of their production as intermediate input to a similar set of downstream buyers (*e.g.*, Conley and Dupor 2003). As sectors depend on overlapping sets of buyers/suppliers, intermediate inputs substitutability across the production network is linearly embedded in its horizontal geometry, and patterns of sectoral comovement hinge on the network economic distance between sectors: nearby sectors move in opposite directions due to shared Input-Output relations, whereas distant sectors tend to comove as the standard (vertical) propagation prevails.

Under this perspective, an Input-Output structure not only reflects *vertical* complementarities in propagating shocks (*i.e.*, the intensity of the comovement between two sectors depends on their intensity of being interconnected; refer to Shea 2002), but also *horizontal* complementarities as sectors are additionally held together by demand and supply relations from having in common a resembling Input-Output geometry, also connecting disconnected sectors (as established by Theorem 1). While vertical propagation mechanisms are broadly utilized in the literature, this paper

[1] Central insight is that a positive shock to a sector enhances its productive performance, triggering: *(i)* a *downstream effect*, as other sectors have an incentive to expand their demand from that sector; *(ii)* an *upstream effect*, as higher production in the shocked sector raises its demand of intermediate inputs, benefiting its suppliers; and *(iii) network spillovers*, where the positive effects ripple beyond immediate trading partners, benefiting indirectly connected sectors. On the verticality of Input-Output economies refer to Pasinetti (1973).

[2] Limits to the ability to adjust input combinations within sectors challenge classical comovement, as reflected in a Constant Elasticity of Substitution (CES), non-linear setting (*e.g.*, Corsetti et al. 2008, Atalay 2017, Baqaee and Farhi 2019). My network's horizontal geometry – captured by sectoral "economic" distances through shared upstream suppliers or downstream buyers – embeds this logic in reduced form, as distance-based interdependencies initiate an horizontal propagation of sector-specific shocks; refer to Subsection 1.3.

explores how horizontal interdependencies and various forms of network economic distance affect comovement across sectors participating in the production network, with theoretical insights tested empirically using sectoral U.S. employment data.

On the empirical validation, a number of studies have documented the presence of sectoral comovement in employment levels (*e.g.*, Cooper and Haltiwanger 1990, Christiano and Fitzgerald 1998, Yedid-Levi 2016).[3] Yet, much of this evidence treats sectors as isolated units and gives limited attention to the role of inter-sectoral linkages. Understanding how these Input-Output relationships shape sectoral comovement remains an open question, and there is a lack of empirical research specifically examining sectoral employment comovement within the context of a production network. Another contribution of this paper is to attempt at filling this empirical void.

From a network perspective comovement is endogenous. The aforementioned standard propagation system suggests that comovement origins from the even transmission of sectoral shocks through Input-Output linkages. Does this vertical transmission mechanism persist when different types of network distance are factored in? Does a positive shock to one sector propagate in the same way to others under horizontal demand and supply linkages? My empirical findings show that positive comovement primarily arises between sectors that are "economically distant" in the network (with few common upstream suppliers and downstream buyers), while the picture is mixed for closely demand-related sectors sharing similar suppliers: vertical and horizontal transmissions overlap, rendering it difficult to identify which one prevails. In contrast, supply-based distances produce opposite movements: a positive shock to closer sectors negatively affects other sectors that sell intermediate inputs to the same downstream buyers, thereby counteracting the expected and classical upward transmission of shocks along the production network.[4]

Indeed, building a coherent theoretical framework that emphasizes the role of demand- and supply-driven network distances in shaping sectoral comovement within an Input-Output economy, and validating its main predictions using sector-level U.S. network and employment data, constitute the two central contributions of this paper.

Section 1 introduces two complementary measures of Input-Output economic distance: *(i) factor input demand* network distance, when sectors are buying their intermediate inputs from similar sectors; and *(ii) factor input supply* network distance, when sectors are selling their intermediate outputs as intermediate inputs to similar sectors. Both distances are formalized as full matrices, derived from the supply-side with profit-maximizing behaviour, summarizing demand or supply horizontal linkages from common Input-Output relationships. Demand-based distances cap-

[3] Sectoral activity and comovement are not separate phenomena but overlapping features. Various papers suggest how business cycles are driven by micro-level shocks rather than aggregate ones (*e.g.*, Long and Plosser 1987, Foerster et al. 2011, Gabaix 2011, Moro 2012, Garin et al. 2018), and that a synchronization between changes in output and employment, either at aggregate or sectoral level, is occurring (*e.g.*, Quah and Sargent 1993, Hornstein and Praschnik 1997, Stock and Watson 1999, Rebelo 2005, Barrot, Grassi, et al. 2021).

[4] Differently, Conley and Dupor (2003) empirically find that sectors with similar upstream suppliers exhibit stronger correlations in productivity growth than those selling to similar downstream buyers.

ture how shocks to a downstream buyer trigger coordinated adjustments in input usage across sectors that share common upstream suppliers, whereas supply-based distances capture how shocks to an upstream seller drive coordinated relative price adjustments across sectors supplying the same downstream buyers. In other words, on the input side, sectors that buy from the same suppliers reveal simultaneous adjustments as they rebalance labour and intermediates; on the output side, sectors selling to the same buyers show revenue shifts when downstream demand changes. A sufficient-statistic perspective interprets these distance effects in reduced form. Under demand linkages, negative comovement arises only when intermediate inputs complementarity in downstream buyers is strong (a positive shock to one sector increases the price of a common supplier, and other downstream buyers with limited substitutability reduce demand). By contrast, in the supply-based case, negative comovement is immediate and mechanical, as a sector capturing demand from a shared downstream buyer reduces activity among competing suppliers after a positive supply shock (it effectively lowers its relative price). In this sense, demand linkages make comovement ambiguous and dependent on substitution patterns in downstream demand, while supply linkages make comovement transparent and directly tied to upstream competition for downstream markets. Network distances thus reproduce the negative comovement from non-linear intermediate inputs complementarities (*e.g.*, Baqaee and Farhi 2019), with demand- and supply-driven horizontal interdependencies linearly summarizing its propagation mechanisms.

Building on these effects, Section 2 examines their general equilibrium implications for employment comovement across interconnected sectors. Consistent with standard exogenous production network models (*e.g.*, Long and Plosser 1983, Acemoglu, Carvalho, et al. 2012), a shock to a given sector propagates vertically along its supply chains. Once network distances are introduced, the strength of vertical spillovers is partly offset or reshaped by horizontal propagation. Employment responses are ambiguous under demand-based distances, since vertical and horizontal channels overlap and may either reinforce or counteract one another. By contrast, clearer responses are identified under supply-based distances: when serving the same downstream buyers, a decline in the relative price of a nearby supplier increases its own employment while reducing that of competing suppliers. In sum, network distances govern not only the direction but also the intensity of cross-sectoral comovement, with horizontal transmission acting as a distance-dependent trigger factor which gradually fades with weaker distance-based interconnections.

In developing this perspective, a central theoretical result (Theorem 2) emerges: measures of network "economic" distances are not themselves weighted by the Input-Output structure. In other words, sectoral distances in terms of common demand or supply relationships retain an independent force in shaping sectoral comovement, distinct from the intensity of inter-sectoral trade flows. This separation strengthen the importance of horizontal complementarities between sectors.

As a validation of theoretical insights, the role of network distances on sectoral comovement is empirically tackled on U.S. employment and Input-Output data, whose distance-based production network characteristics are presented in Section 3. Section 4 implements the two-stage approach in Barattieri and Cacciatore (2023): first, sector-specific structural shocks are isolated by extracting residuals from panel Fixed Effects (FE) regressions; second, these identified shocks are used in a panel Local Projection (LP) analysis to estimate how employment changes in one sector affect others, distinguishing effects according to both demand- and supply-driven network distances. The empirical results are threefold. Firstly, an increase in the set of intermediate inputs specific to a sector leads to a comparatively smaller rise in its employment with minimal, but alternate, effects from similar changes in more distant sectors. Secondly, sectors closer under factor input supply distance exhibit opposite employment comovement: an increase in employment in nearby sectors tends to reduce employment in the sector; for closely demand-linked sectors, the responses are often ambiguous (increases for some, reductions for others), but mostly pointing to a dampening effect on the positive shock's transmission. Thirdly, positive comovement emerges among more distantly demand- and supply-related sectors, since sectoral employment rises in response to increases in more distant sectors.

Overall, these findings provide strong empirical support for the main theoretical predictions: horizontal linkages in terms of similar demand and supply relationships across sectors complement and balance the Leontief transmission of sector-specific shocks, thereby revealing the multiple channels through which the vertical and the horizontal structures of sectoral interdependencies shape the distribution and the propagation of economic activity across networked economies.

To conclude, Section 5 examines the policy implications of the network's horizontal geometry and how incorporating it can improve the design of policy interventions.

Literature.– The objective of this paper is to advance both theoretically and empirically the understanding of how Input-Output economies work. On the theoretical side, it builds on the modern and rapidly expanding literature on production networks. In the wake of Gabaix (2011)'s "granular hypothesis", the seminal observation of Long and Plosser (1983) – that comovements across sectors are not dictated by a shared disturbance but rather by sectoral interdependencies –, has been modernized through a series of papers of the early 2010s (Acemoglu, Carvalho, et al. 2012, Carvalho and Gabaix 2013, Carvalho 2014, Barrot and Sauvagnat 2016). This renewed perspective gave rise to studies on production networks related to efficient economies (*e.g.*, Baqaee and Farhi 2019, vom Lehn and Winberry 2022, Liu and Tsyvinski 2024), monetary policy and nominal rigidities (*e.g.*, La'O and Tahbaz-Salehi 2022, Rubbo 2023, Ghassibe and Nakov 2025), inefficiencies (*e.g.*, Jones 2011, Grassi 2017, Baqaee 2018, Baqaee and Farhi 2020), policy-oriented issues (*e.g.*, Liu 2019, Grassi and Sauvagnat 2019, Lane 2025), endogenous network formation (*e.g.*, Oberfield 2018, Acemoglu and Azar 2020, Ghassibe 2024, Taschereau-Dumouchel 2025), economic growth (*e.g.*, Hausmann and Hidalgo 2011, Gualdi and Mandel 2019, McNerney et al. 2022), and international contexts (*e.g.*, Caliendo et al. 2022, Qiu et al. 2025, Huo et al. 2025). A common mechanism explored in these works emphasizes the vertical propagation of sectoral variations, where shocks originating in upstream sectors transmit downstream via the Leontief (1936)'s inverse, amplifying their effects and contributing to aggregate outcomes.[5] My contribution broadens this perspective by introducing horizontal linkages between sectors, defined by their network "economic" distances as measured by common upstream (demand-related) or downstream (supply-related) Input-Output relationships. These linkages forge horizontal complementarities that bind even disconnected sectors, allowing shocks to propagate across the network in ways beyond traditional vertical supply chains.

On the empirical side, relatively few studies examine the role of sectoral production networks on macroeconomic outcomes (*e.g.*, Acemoglu, Akcigit, et al. 2015, Ghassibe 2021, Barattieri and Cacciatore 2023, Barattieri, Cacciatore, and Traum 2023, Monti and Van Keirsbilck 2025). From this standpoint, the paper contributes novel empirical evidence on the role of Input-Output linkages in shaping sectoral dynamics, particularly in explaining the comovement of employment across sectors. Synchronized employment patterns are an important characteristic of business cycles, as most sectors tend to move together over time (Christiano and Fitzgerald 1998). Several studies explore the nature and the sources of this sectoral comovement.[6] Cooper and Haltiwanger (1990) find that sectoral employment levels are positively correlated, in ways not fully explained by aggregate shocks. Cassou and Vázquez (2014) attribute high employment comovement to similar shock transmission channels. In Yedid-Levi (2016) comovement is stronger along the extensive margin (number of workers) than the intensive margin (hours per worker). Room for structural changes is in Kim (2020), where positive technology shocks in manufacturing raise employment in both manufacturing and services. While these studies provide valuable insights, they do not explicitly address how production networks shape employment comovement. By contrast, my paper focuses on short-run employment variations within a networked framework. Empirically, the paper contributes by *(i)* advancing the literature on the comprehension of Input-Output linkages and shocks' transmission via network distances, and *(ii)* providing novel evidence on sectoral employment comovement from a production network perspective.

1. Foundational Theory

Preliminaries.– The economy is populated by a finite set of sectors, each labelled as $\{s, s', s'', \ldots, S\} \in \Phi(s)$; moreover, denote by Φ_s the set of all sectors not including sector-s. Each of them produces a single good using either labour and a set of circulating intermediate inputs that each sector buys from other sectors. The whole

[5] As noted by Shea (2002), a (positive) sectoral shock has a price (decrease in its price level, and reduction in its nominal expenditure for intermediate inputs) and a quantity (its supply to other sectors increase, sector's production rises, thereby increasing its input demand) effects. These opposing forces affect upstream suppliers but, under Cobb-Douglas production function, they exactly offset each other and leaving intermediate inputs demand unchanged, thereby only affecting downstream suppliers. In general terms, only (final) demand shocks can propagate upstream (*e.g.*, Acemoglu, Akcigit, et al. 2015, Barrot and Sauvagnat 2016, Ferrari 2024).

[6] Theories beyond employment comovement are in Rogerson (1987) and Boldrin et al. (2001).

economic system is thus represented by an $S \times 1$ vector of sectoral intermediate outputs, $\boldsymbol{Y} = \left[y\left(s\right) \right]$ with general element defined as $y\left(s\right) > 0$, an $S \times 1$ vector of sectoral employment levels, $\boldsymbol{N} = \left[n\left(s\right) \right]$ with general element defined as $n\left(s\right) > 0$, each associated to its output elasticity of labour $\alpha\left(s\right)$ in order to deliver $y\left(s\right)$, and by an $S \times S$ square matrix, $\boldsymbol{X} = \left[x\left(s, s'\right) \right]$, indicative of the inter-sectoral trade of intermediate input quantities, with general element given by $x\left(s, s'\right) \geq 0$, $\forall s' \in \Phi\left(s\right)$. Finally, an $S \times S$ squared matrix, $\boldsymbol{H} = \left[\alpha\left(s, s'\right) \geq 0 \right]$, defines the intensity of the good produced by sector-s' in the total intermediate inputs used by sector-s, with $\alpha\left(s, s'\right) = 0$ indicating that sector-s does not make use of the good produced by sector-s' in producing its own intermediate good; in addition, denote its Leontief inverse matrix as $\boldsymbol{\mathcal{H}} = \left(\boldsymbol{I} - \boldsymbol{H} \right)^{-1} = \left[\ell\left(s, s'\right) \geq 0 \right]$, with $\boldsymbol{I}$ being an $S \times S$ identity matrix.

Under these specifications, a perfectly competitive final good producer combines intermediate outputs from sectors

$$Y = \prod_{s \in \Phi(s)} y\left(s\right)^{\beta(s)} \propto \boldsymbol{\beta}' \boldsymbol{Y}$$

where $\boldsymbol{\beta}$ is a $1 \times S$ vector of $\beta\left(s\right) > 0$, a parameter governing the relative importance of each sectoral intermediate good in the definition of aggregate output Y.

A representative firm characterizes each sector, producing its own good, $y\left(s\right)$, using either a set of intermediate inputs bought from other sectors, $x\left(s, s'\right)$, $\forall s' \in \Phi\left(s\right)$, and labour force, $n\left(s\right)$. Production function in sector-s is then

$$y\left(s\right) = z\left(s\right) f^{s} \Big(n\left(s\right) \; , \; \left\{ x\left(s, s'\right) \right\}_{s' \in \Phi(s)} \Big)$$

where $z\left(s\right)$ is an exogenous Hicks (1932)-neutral sectoral productivity. Under a general perspective, assume the following regularity conditions hold.

Assumption 1 (Production technology requirements) *As main characteristics, the production function: (i) it has constant returns to scale in either $x\left(s, s' \in \Phi\left(s\right)\right)$ and $n\left(s\right)$, so that production inputs shares sum to one, $\alpha\left(s\right) + \sum_{s'} \alpha\left(s, s'\right) = 1$; (ii) it is differentiable, continuous, strictly quasi-concave, homogeneous of degree one, and increasing in $z\left(s\right)$, $x\left(s, s' \in \Phi\left(s\right)\right)$, and $n\left(s\right)$; (iii) the case $f^{s}\left(0 \; , \; \cdot\right)$ is ruled out as labour input is essential to production; (iv) at least two elements of $x\left(s, s' \in \Phi\left(s\right)\right)$ has to be positive, thus $f^{s}\left(\cdot \; , \; 0\right)$ cannot exist; consequentially, (v) part of the sectoral output is directly produced by the sector, $x\left(s, s\right) > 0$, $\forall s \in \Phi\left(s\right)$, that is a production plan with a bundle of intermediate inputs as $f^{s}\left(\cdot \; , \; x\left(s, s' \in \Phi_{s}\right)\right)$ is precluded.*

The first two sub-assumptions are common in production technology;[7] the other ones ensure the production function to be consistent with the empirical exploration in Section 4. By way of *A1.iii*, workers cannot be fully substituted by any combi-

[7] Constant returns to scale, as well homogeneity of degree one, purely address the technical relation between inputs and output, ruling out any interference of external factors (*e.g.*, relative price changes associated to scalable production, non-homogeneity of degree one due to market dynamics). Continuity and differentiability guarantees well-defined (no "jumps") and smooth marginal products, while strict quasi-concavity ensures convex technologies and unique optimal input choice.

nation of intermediate inputs (so that sectoral output is finite), and changes in intermediates' quantity will determine subsequent variations in the labour force. The assumptions characterizing the intermediate inputs bundle, *A1.iv* and *A1.v*, ensure the sector not only to participate in the network, but also that part of the production process is roundabout: for sector-s delivering its intermediate good $y(s)$ requires to be at least vertically integrated (buying from one upstream sector), while imposing that a fraction of intermediate inputs must be directly produced within the sector.

Each representative firm is seeking to maximize profits in a perfectly competitive environment, and optimality conditions for both intermediate inputs and demanded labour are then continuous function as $x(s,s') = f_{(s,s')}\big(y(s), p(s), p(s')\big)$ and $n(s) = f_{(s,s')}\big(y(s), p(s), w(s)\big)$, so that their combination yields

$$x(s,s') = f^s\Big(\alpha(s,s'), p(s')\ ;\ \alpha(s), n(s), w(s)\Big) \tag{1}$$

where $w(s)$ is the sectoral wage rate, chosen as the numeraire. Sector-s's optimal demand for intermediate inputs of sector-s' is increasing in their inter-sectoral trade intensity, $\alpha(s,s')$, and in labour market variables, $w(s)$ and $n(s)$, while it is decreasing in $\alpha(s)$, and in its purchased intermediates' price, $p(s')$. Effectively, the ratio among factors of production depends on their relative intensity in production, and on their relative price.

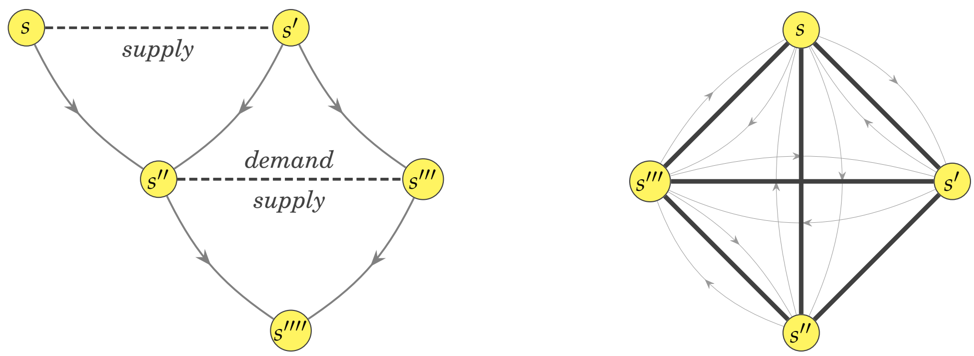
Distances.– In a networked economy, comovement of production inputs is endogenously induced: each sector both relies on and supplies to others, so a change in one sector's optimal demand spreads through the network, thus reshaping the relation among factors of production within and between sectors. A rigorous comparison of two sectors can be conducted by examining their relation in the production network, specifically in terms of their trading partnerships with the same set of sectors.

Paraphrasing Conley and Dupor (2003), two types of network "economic" distances between sectors, sketched in Panel 1a of Figure 1, can be defined:

(a) *factor input demand*, when sectors are *buying* part of their intermediate inputs *from* similar upstream sectors;

(b) *factor input supply*, when sectors are *selling* part of their intermediate outputs *to* similar downstream sectors.

Under this perspective, connections among sectors extend beyond their flows of intermediate inputs. These additional interdependencies highlight the complexity of inter-sectoral dynamics and complement the *Leontief inverse*, a reminiscent result from Leontief (1936): it captures how sectors not directly trading among each others are indirectly related – *e.g.*, a sector buying from an upstream one introjects such sector's purchases as well the other of more upstream sectors –, thus mainly emphasizing the depth of vertical linkages through layers of Input-Output relationships. Network-based economic distances, by contrast, allow to consider how participants are mapped not only vertically, but horizontally as well: sectors can be also closely related because they are relying on the same suppliers or are serving the same customers. These broader notions of sectoral linkages provide a richer understanding of



(A) DISTANCE DEFINITION

(B) DISTANCE NETWORK

FIGURE 1: HORIZONTAL INPUT-OUTPUT GEOMETRIES

Note: the figure presents a stylized production network and its corresponding network of sectoral distances. Panel 1a illustrates both network distance definitions. Panel 1b highlights the horizontal dimension of the network, where solid thick lines connecting all vertexes indicate the distance relationships between sectors; production network linkages are depicted with oriented thin lines, and represent the inter-sectoral trade flows derived from the Input-Output matrix in Example 1.

the structure of production networks and help to explain *horizontal complementarities* in demand and supply that classical Input-Output analysis overlook. They reveal how seemingly unrelated sectors can experience synchronized fluctuations, and how the structure of common upstream or downstream connections shapes the propagation of independent and idiosyncratic shocks. In other words, the notion of sectoral interdependencies is not confined to "who trades with whom", but also emerges from "who depends on whom": horizontal complementarities thereby deliver a novel perspective on how sectoral shocks reverberate across a networked economy.[8]

Panel 1b sketches a stylized economy associated to its distance-based network: distances effectively double Input-Output linkages and bound disconnected sectors.

EXAMPLE 1 (Distance in the network) *Consider an economy populated by four sectors,* $\{s, s', s'', s'''\} \in \Phi(s)$*, where some trade with all others, while some do not. The resulting production network of Panel 1b displays an Input-Output matrix,* **H**, *where some cells equal zero. Focus on the pair* $\{s', s''\}$*: they trade with all other sectors but not with each other, so* $\alpha(s', s'') = 0 = \alpha(s'', s')$*. Nevertheless, their linkages with the remaining sectors generate a common structure of inter-sectoral trade. An Input-Output architecture induces a unique geometry of sectoral distances (both in demand and supply), as depicted in Panel 1b. Yet, while the underlying trade and distance structures are unique, the horizontal metrics between sectors vary under alternative specifications of network distance, as Examples 2-3 clarify.*

[8] This conceptual distinction is further developed in the discussion of Figure 3: the drawn networks clarify how these forms of "economic" proximity can influence the transmission of shocks and the synchronization of sectoral dynamics, despite having no direct transactional relationship between sectors.

1.1. Stylized economies and horizontal geometry

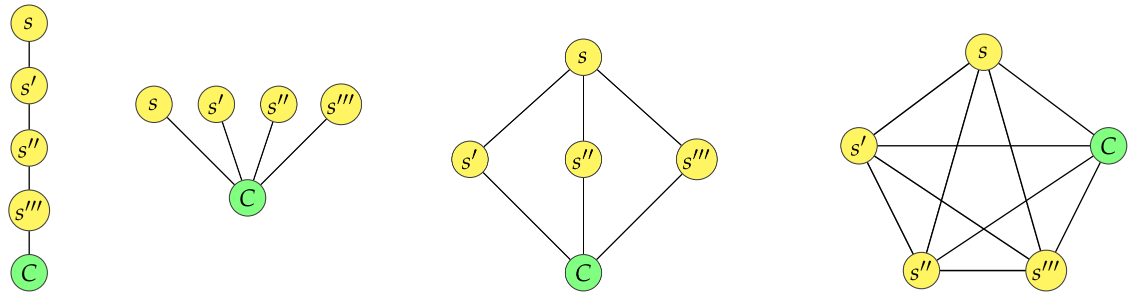
Before turning to the technical analysis, I illustrate the economic relevance of the network's horizontal geometry through a set of highly stylized production structures, sketched in Figure 2. I consider three benchmark configurations: *(i) snake economies*, in which transmission occurs purely along a vertical chain (sector by sector toward the final consumer) or, alternatively, purely horizontally (distinct sectors supplying only the final consumer); *(ii) rhomboid economies*, where sectors depend on a common upstream supplier while simultaneously selling to the final consumer; and *(iii) star economies*, with all sectors mutually connected while serving the final consumer.

Snake economies.– Consider the left graph of Figure 2a. Connections are purely vertical, and asymmetric shocks propagate sequentially along the supply chain, with no scope for horizontal interdependencies. By contrast, on the right, sectors sell directly to the final consumer without trading among themselves. This represents a purely horizontal structure, characteristic of standard multi-sector models without inter-sectoral trade. Supply-based horizontal complementarities are central: even without explicit trade linkages, sectors are tied together through their shared exposure to final demand. A shock to one sector (affecting its output or relative prices) reshapes consumers' expenditure across all others, propagating horizontally through final demand reallocation and generating comovement *absent any network structure*.

Rhomboid economies.– The slightly more complex framework of Figure 2b combines the two snake economies into a unique structure. This configuration blends vertical and horizontal propagation, as shocks travel vertically (via sector-specific supply chain) but also horizontally across sectors exposed to common supplier and final demand conditions. Rhomboid economies thus epitomize horizontal complementarities, as even sectors not directly trading with each other become tightly synchronized through their shared dependence on upstream and downstream markets.[9]

Star economies.– Figure 2c classifies three archetypes. In fully symmetric star networks, where all sectors buy and sell with identical intensity, network distances collapse to zero: propagation is only vertical, but the network structure becomes irrelevant as sectoral shocks average out (*e.g.*, Lucas 1977, Acemoglu, Carvalho, et al. 2012). In partially symmetric star networks, trade intensities in buying and selling are equal but sector-specific; vertical and horizontal dimensions overlap, though the two distance measures coincide. Finally, under asymmetric star networks (*i.e.*, different trade intensities across sectors) generate parallel vertical and horizontal transmission channels, with amplification and comovement emerging endogenously from network distances: horizontal complementarities in production and shared exposure to final demand weave a dense web of horizontal interdependencies, and sectors with

[9] The rhomboid structure closely mirrors the configuration underlying the "Keynesian transmission mechanism" (*e.g.*, Guerrieri et al. 2022; see more in Section 5). The horizontal geometry offers a tractable stylization of this logic, as it formalizes how distance-based complementarities in production and final consumption create systemic interdependencies. In doing so, it transforms the Keynesian transmission – traditionally modelled as sectoral spillovers – into a "structural network propagation mechanism", where amplification and comovement emerge endogenously from the economy's geometric horizontal interconnections.



(A) SNAKES (B) RHOMBOID (C) STAR

FIGURE 2: BENCHMARK ECONOMIES

Note: This figure illustrates three benchmark economic systems used to highlight the role of horizontal geometry in Input-Output propagation of idiosyncratic shocks. Sectors are in yellow, while the green circles with C represent the final consumer.

differing upstream and downstream connections experience unrelated effects.

These illustrative examples represent simplified and extreme cases. Yet, horizontal propagation remains a central feature of how shocks diffuse through production systems. In more realistic, highly interconnected systems, the boundaries between these benchmark economies blur: vertical supply chains coexist with horizontal interdependencies through shared buyers and suppliers, and sectors participate simultaneously in multiple propagation layers. Depending on the complex configuration of the network, the horizontal geometry is concentrated more upstream or downstream, with strong implications for idiosyncratic shocks' propagation. Nevertheless, as inter-sectoral linkages densify and the network expands, horizontal complementarities become increasingly important (with more sectors sharing common suppliers and buyers, interdependencies multiply), amplifying the complexity of shock transmission far beyond what purely vertical Input-Output structures would predict.

1.2. RECOVERING DEMAND AND SUPPLY DISTANCE MATRICES

Consider the case in which two sectors, say $\{s, s'\}$, are buying part of their own intermediate inputs from the same sector, say s^*. In this scenario, by substituting out the combined optimality conditions in eq. (1) for such sectors purchasing from the same upstream sector, then one obtains, after total *log*-differentiation, that

$$\begin{aligned} d\log \mathcal{B}_{\leftarrow s^*}(s') &= \frac{\alpha(s,s^*)}{\alpha(s',s^*)}\, d\log \mathcal{B}_{\leftarrow s^*}(s) \qquad (2)\\ \longleftrightarrow \qquad \mathcal{F}_{\leftarrow s^*}[s,s'] &= d_{\leftarrow s^*}[s,s'] \end{aligned}$$

with $\mathcal{F}_{\leftarrow s^*}[s,s'] = d\log \mathcal{B}_{\leftarrow s^*}(s') \,/\, d\log \mathcal{B}_{\leftarrow s^*}(s)$ and $d_{\leftarrow s^*}[s,s'] = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}$. Given $\gamma^{\mathcal{B}}_{(\cdot)} = \frac{1}{\log \overline{x}(\cdot,s^*)}$ and $\delta^{\mathcal{B}}_{(\cdot)} = \frac{\log \overline{n}(\cdot)}{\log \overline{x}^2(\cdot,s^*)}$ being steady-state components for production inputs, the differential elements $d\log \mathcal{B}_{\leftarrow s^*}(s) = \gamma^{\mathcal{B}}_s\, d\log n(s) - \delta^{\mathcal{B}}_s\, d\log x(s,s^*)$ and $d_{\leftarrow s^*}\log \mathcal{B}(s') = \gamma^{\mathcal{B}}_{s'}\, d\log n(s') - \delta^{\mathcal{B}}_{s'}\, d\log x(s',s^*)$ identify the *log*-difference of employment levels and stock of intermediate inputs in the two sectors buying from the same sector. Subscript "$\leftarrow s^*$" reads as "purchasing from sector-s^*" by sectors in brackets.

Suppose a shock occurs in sector-s, altering the composition of its production inputs. Propagating vertically along the supply chain, the shock modifies the optimal input supply of sector-s^*, thereby influencing its trade with sector-s' since both are connected as well. As from the first line, any induced variation in the composition of production inputs within a given sector is horizontally transmitted to another sector in proportion to their relative Input-Output trade intensity with the common upstream sector to which an idiosyncratic shock, originated elsewhere, is transmitted.

Iteratively, the above condition can be rewritten accounting for all sectors to which the pair is buying from:

$$\boldsymbol{\mathcal{F}}\left[s,s'\right] = \left(\frac{\alpha\left(s,s\right)}{\alpha\left(s',s\right)}, \frac{\alpha\left(s,s'\right)}{\alpha\left(s',s'\right)}, \frac{\alpha\left(s,s''\right)}{\alpha\left(s',s''\right)}, \ldots, \frac{\alpha\left(s,S\right)}{\alpha\left(s',S\right)}\right) = \boldsymbol{d}^{fd}\left[s,s'\right]$$

where $\boldsymbol{\mathcal{F}}\left[s,s'\right]$ is a $1 \times S$ vector of changes in the ratio of any difference in production inputs between the pair of sectors, and $\boldsymbol{d}^{fd}\left[s,s'\right]$ is the associated $1 \times S$ vector of their relative intermediate usage intensity when sectors $\{s,s'\}$ are buying from each of the other sectors in the economy, yielding a unique value, $d^{fd}\left[s,s'\right]$. Stacking such condition across all sectors, then the following Lemma holds.

LEMMA 1 (Factor input demand) *Changes in the ratio of production inputs' quantities across any pair of sectors buying from the same sector(s) is driven by their ratio of intermediate intensities with whom the pair is purchasing from:* $\boldsymbol{\mathcal{F}} = \boldsymbol{\mathcal{D}}^{fd}$.

Proof in Appendix A.1.

In other words, the comovement of inputs of production for a pair of sectors is determined by their distance in the production network. In fact, in Lemma 1, each cell of the $S \times S$ matrix $\boldsymbol{\mathcal{F}}$ displays the differential variation across factors of production between the row-sector and the column-sector, while matrix $\boldsymbol{\mathcal{D}}^{fd}$ is given by

$$\underset{S\times S}{\boldsymbol{\mathcal{D}}^{fd}} = \begin{bmatrix} \left(\frac{\alpha(s,s)}{\alpha(s,s)}, \ldots, \frac{\alpha(s,S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(s,s)}{\alpha(s',s)}, \ldots, \frac{\alpha(s,S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(s,s)}{\alpha(S,s)}, \ldots, \frac{\alpha(s,S)}{\alpha(S,S)}\right) \\ \left(\frac{\alpha(s',s)}{\alpha(s,s)}, \ldots, \frac{\alpha(s',S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(s',s)}{\alpha(s',s)}, \ldots, \frac{\alpha(s',S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(s',s)}{\alpha(S,s)}, \ldots, \frac{\alpha(s',S)}{\alpha(S,S)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\alpha(S,s)}{\alpha(s,s)}, \ldots, \frac{\alpha(S,S)}{\alpha(s,S)}\right) & \left(\frac{\alpha(S,s)}{\alpha(s',s)}, \ldots, \frac{\alpha(S,S)}{\alpha(s',S)}\right) & \cdots & \left(\frac{\alpha(S,s)}{\alpha(S,s)}, \ldots, \frac{\alpha(S,S)}{\alpha(S,S)}\right) \end{bmatrix}$$

The greater the value of each cell in the matrix, the closer the considered sectors are within the production network (the stronger is the vertical effect, the stronger will be the horizontal transmission).[10] This concept of "network proximity" will be foundational in the model of Section 2, where distance relationships are further explored. In the empirics, the interpretation of each cell is inverted: as discussed in

[10] A lower trade intensity ratio in eq. (2) implies that a shock to the common sector-s^* transmits mildly to sector-s. For $\alpha\left(s,s^*\right) < \alpha\left(s',s^*\right)$, the resulting change in its relative quantities, $d\log\mathcal{B}_{\leftarrow s^*}\left(s\right)$, then affects that of sector-s', $d\log\mathcal{B}_{\leftarrow s^*}\left(s'\right)$, less than proportionally, signaling low horizontal complementarity (and sector-s' is hit relatively more by sector-s^* vertically than by sector-s horizontally). Conversely, if $\alpha\left(s,s^*\right) > \alpha\left(s',s^*\right)$, then $d_{\leftarrow s^*}\left[s,s'\right] > 1$, resulting in a more-than-proportional effect and high horizontal complementarity. Moreover, the higher the number of common linkages, the more sectors are closer. The same logic applies to eq. (3).

Section 3, the entries of matrix $\mathcal{D}^j$, for $j = \{fd, fs\}$, represent a "network distance" (how many steps are necessary for a sector to reach another one), so that larger values denote how dissimilar sectors are in terms of shared (upstream or downstream) Input-Output structure. Henceforth, each $\mathcal{D}^j$ matrix is reflecting the proper "economic" distance between sectors within the production network, whose characterizing element, $d^j[s, s'] > 0$, $\forall s, s' \in \Phi(s)$, is symmetric outside the main diagonal (the network economic distance from s to s' is the same as from s' to s), and $diag(\mathcal{D}^j) = 0$.

EXAMPLE 2 (Factor input demand metrics) *Consider the pair of sectors $\{s', s''\}$ in an Input-Output matrix mirroring Example 1, where the common sectors from which both are purchasing are $\{s, s'''\}$, and assume fictional trade intensities for sector-s' to be $\alpha(s', s) = 0.2$ and $\alpha(s', s''') = 0.1$, while that of sector-s'' are $\alpha(s'', s) = 0.1$ and $\alpha(s'', s''') = 0.2$. Accordingly, the metrics expressing their factor input demand distance relation is composed of $\boldsymbol{d}^{fd}[s', s''] = (.2, .1), (.1, .2)$ when buying from sector-s and sector-s''', respectively. Demand-based distance value is positive even in the absence of direct trade linkage between considered sectors in the pair.*

Consider now the case in which two sectors, say $\{s, s'\}$, are selling a part of their own intermediate good as intermediate input to the same sector, say s^*. In this scenario, by substituting out the combined optimality conditions in eq. (1) for such sectors trading to the same downstream sector, total *log*-differentiation leads to

$$
\begin{aligned}
d\log \mathcal{Q}_{\to s^*}[s, s'] &= \frac{\alpha(s^*, s)}{\alpha(s^*, s')}\, d\log \mathcal{P}_{\to s^*}[s', s] \\
\longleftrightarrow \qquad \mathcal{R}_{\to s^*}[s, s'] &= d_{\to s^*}[s, s']
\end{aligned}
\tag{3}
$$

with $\mathcal{R}_{\to s^*}[s, s'] = d\log \mathcal{Q}_{\to s^*}[s, s'] \,/\, d\log \mathcal{P}_{\to s^*}[s', s]$ and $d_{\to s^*}[s, s'] = \frac{\alpha(s^*, s)}{\alpha(s^*, s')}$. Given $\gamma^{\mathcal{Q}}_{(\cdot)} = \frac{1}{\log \overline{x}(s^*, \cdot)}$, $\delta^{\mathcal{Q}}_{(\cdot)} = \frac{\log \overline{x}(s^*, \neq \cdot)}{\log \overline{x}^2(s^*, \cdot)}$, $\gamma^{\mathcal{P}}_{(\cdot)} = \frac{1}{\log \overline{p}(\cdot)}$ and $\delta^{\mathcal{P}}_{(\cdot)} = \frac{\log \overline{p}(\neq \cdot)}{\log \overline{p}^2(\cdot)}$ being steady-state components for intermediate inputs and their associated price levels, then differentials $d\log \mathcal{Q}_{\to s^*}[s, s'] = \gamma^{\mathcal{Q}}_{s'}\, d\log x(s^*, s) - \delta^{\mathcal{Q}}_{s'}\, d\log x(s^*, s')$ and $d\log \mathcal{P}_{\to s^*}[s', s] = \gamma^{\mathcal{P}}_{s}\, d\log p(s') - \delta^{\mathcal{P}}_{s}\, d\log p(s)$ identify the *log*-difference of the stock of intermediate inputs and of the price levels between both sectors selling to the same downstream sector. Subscript "$\to s^*$" reads as "selling to sector-s^*" by sectors in brackets.

Suppose a shock occurs in sector-s', altering the composition of its production inputs, and thus its optimal price. Still, when propagating vertically along the supply chain, the shock modifies the optimal input demand of sector-s^*, and thereby its trade with sector-s, as well connected. As from the first line, any induced variation in the relative price, by altering the relative quantity of its circulating intermediate inputs, is horizontally transmitted to another sector in proportion to their relative Input-Output trade intensity with the common downstream sector to which an idiosyncratic shock, originated elsewhere, is transmitted.

Following the previous logic scheme, then the following Lemma must hold.

LEMMA 2 (Factor input supply) *Changes in the ratio of intermediate input profits across any pair of sectors selling to the same sector(s) is driven by their ratio of* *intermediate intensities with whom the pair is purchasing from:* $\mathcal{R} = \mathcal{D}^{fs}$.

Proof in Appendix A.1.

In other words, the comovement across (marginal) revenues of production for a pair of sectors is determined by their distance in the production network. When sectors are selling to the same downstream sector, the change in the revenues of sector-s – *i.e.*, $p(s)\,x(s^*,s)$ – given a change in those of any sector-s' – *i.e.*, $p(s')\,x(s^*,s')$ – is scaled by the ratio of their intermediate input intensities.

Example 3 (Factor input supply metrics) *Consider the pair of sectors* $\{s', s''\}$ *in an Input-Output matrix mirroring Example 1, where the common sectors to which both are selling are* $\{s, s'''\}$*, and assume fictional trade intensities for sector-*s' *to be* $\alpha(s,s') = 0.4$ *and* $\alpha(s''',s') = 0.3$*, while that of sector-*s'' *are* $\alpha(s,s'') = 0.6$ *and* $\alpha(s''',s'') = 0.6$*. Accordingly, the metrics expressing their factor input demand distance relations is composed of* $\boldsymbol{d}^{fs}[s',s''] = (.4,.3),(.6,.6)$ *when selling to sector-*s *and sector-*s'''*, respectively. Supply-based distance value is positive even in the absence of direct trade linkage between considered sectors in the pair.*

1.3. Sufficient statistics for distance-lead comovement

A shadow implication of the framework developed so far is that intermediate inputs generally function as substitutes, allowing sectors to freely adjust their input mix in response to shocks. However, the horizontal geometry of a network can embed two aspects: *(i)* vertical propagation through the supply chain may remain the dominant channel, and what may appear as a horizontal transmission can in fact be the by-product of vertical propagation; and *(ii)* intermediate inputs complementarity in production may alter the transmission of shocks. This tension between substitutability at the input level and horizontal complementarities at the network level sets the stage for a more precise characterization of how sectoral comovement occurs.

In the context of factor input supply, alleviating the tension is straightforward since sectors' competition for common downstream buyers yields a more direct and unambiguous form of horizontal interdependence. Conversely, the dual transmission in factor input demand makes the direction and the strength of induced comovement subtle, emerging the need for sufficient-statistic conditions that systematically identify when positive or negative comovement arises under input substitutability.

The structure of upstream connections, in fact, renders demand-based linkages ambiguous to interpret. Alongside to the overlap of vertical and horizontal transmission,[11] pivotal is the way in which upstream sector's price pass-through translates shocks into input reallocation across sectors, potentially inducing negative comove-

[11] A shock in sector-s that expands its demand for intermediate inputs is going to alter the input supply composition of sector-s^*; subsequent adjustments in s^* then spill over to sector-s'; and the subsequent reactions of s' may again feed back into s^*, thereby completing a cycle of vertical propagation across the interconnected sectors. Comovements across sectors may often reflect overlapping vertical transmissions rather than genuine horizontal complementarities in demand.

ment (substitute away from more expensive suppliers) even when vertical links suggest positive propagation. Considering a *single-input* case (purchasing intermediate inputs from only one upstream supplier, with no substitution possibilities), and an extended *multiple-input* case (allowing substitutability as sectors source from multiple suppliers), and assuming a Constant Elasticity of Substitution (CES) among intermediate inputs, I provide the following sufficient statistics.

COROLLARY 1 (Sufficient statistics for demand-driven comovement) *In the single-input case, the sufficient statistic for negative comovement in demand implies* $d \log x\,(s, s^*) \propto -\sigma\ \tau^{fd}$*, with the magnitude determined solely by the upstream price pass-through* τ^{fd}*. In the multi-input CES case, the sufficient statistics augments to*

$$d \log x\,(s, s^*) \propto -\sigma \left[1 - e\,(s, s^*)\right] \tau^{fd}$$

with the magnitude determined by the combination of substitution elasticity, sectoral expenditure share for downstream buyer, and upstream price response. Negative comovement occurs if $\tau^{fd} > 0$.

Proof in Appendix B.1.

Negative comovement between downstream sectors arises only under specific demand-lead complementarity effects in downstream buyers. Intuitively, when s' experiences a positive demand shock, it bids up the price of inputs from the common supplier-s^*. If this higher input cost makes production more expensive for s, its demand for s^* falls, generating negative comovement. Whether this happens hinges on the degree of complementarity between s^*'s input and the other inputs purchased by s: when $\sigma > 0$, any price increase in s^* alters s's input mix; in the limit $\sigma \to \infty$, inputs become perfect substitutes and s fully shifts away from s^*. In the more general *multi-input CES case*, the effect is further scaled by s's expenditure share on s^*, $e\,(s, s^*)$, so that sectors more reliant on s^* are strongly affected. Thus, demand-driven negative comovement reflects not automatic crowding-out, but the interplay of upstream price pass-through, input complementarities, and expenditure shares.

Considering instead supply-based distances, since shocks to a supplier directly affect its relative price against competitors for common downstream buyers, the induced reallocation of demand across suppliers creates a clear channel for comovement, with horizontal and vertical effects occurring in parallel.

COROLLARY 2 (Sufficient statistics for supply-driven comovement) *In the single-input case, no sufficient statistic exists for negative comovement in supply due to the absence of competition for downstream markets. Differently, in the multi-input CES case, the sufficient statistic is given by*

$$d \log x\,(s^*, s) \propto \sigma\ e\left(s^*, s'\right)\ \tau^{fs}$$

with the magnitude determined by the combination of substitution elasticity, expenditure share, and upstream price response. Negative comovement occurs if $\tau^{fs} < 0$.

Proof in Appendix B.2.

Negative comovement between upstream suppliers mechanically arises from competition for downstream buyers. Intuitively, when s' receives a positive supply shock, it can lower its relative price and capture a larger share of s^*'s demand from the upstream price pass-through $\tau^{fs} = \frac{\partial \log p(s')}{\partial \log x(s^*,s')}$ from competing suppliers. This revenue reallocation necessarily comes at the expense of competing supplier-s, generating negative comovement. The strength of the effect depends on two forces: the elasticity of substitution σ across s^*'s inputs, which controls how easily s^* reallocates spending, and the expenditure share $e\left(s^*, s'\right)$, which scales the importance of s' in s^*'s input mix. By contrast, in the single-input case, no downstream competition arises, and supply shocks propagating vertically cannot induce the horizontal reallocations.

Discussion.– A networked economy inherently embeds complementarities in intermediate inputs, as sectors depend on overlapping sets of buyers and suppliers. Moreover, upstream or downstream degrees of complementarity matter in magnitude but not in the direction of sectoral comovement. In the demand-driven case, negative comovement between sectors is often ambiguous: it reflects a combination of vertical propagation along the supply chain and horizontal complementarities arising from shared upstream suppliers, making it difficult to disentangle the two overshadowed channels. In the supply-driven case, negative comovement emerges when multiple upstream suppliers compete for the same downstream sectors, and horizontal transmission directly arises from the reallocation of inputs in response to adjustments in relative prices: co-movement is transparent, and the mechanism governing the interaction between vertical and horizontal dimension is mechanically determined. Importantly, the formulas are *local* elasticities: they describe marginal responses to small shocks, whereas large shocks may trigger non-linear reallocations. These results are consistent with those presented by Baqaee and Farhi (2019), who develop a “global” non-linear theory showing how complementarities can amplify or reverse the impact of idiosyncratic shocks. While their framework emphasizes non-linear amplification, mine linearizes these non-linearities into tractable sufficient statistics in reduced form: network “economic” distances naturally capture how vertical and horizontal interdependencies interact so that, even in a linear representation, the sufficient statistics summarize the same underlying complementarity forces that drive non-linear dynamics.

1.4. An inspection into the horizontal transmission

An important aspect to consider when introducing distances in the production network is that, under the same configuration of the inter-sectoral trade structure, sectors can be in a different “network economic distance relation” depending on whether they are buying from or selling to other sectors, with all sectors being simultaneously connected by network distances, that is, by horizontal complementarities in comovement of production factors stemming from common demand and supply linkages.

Theorem 1 (On the configuration of network “economic” distances) *The following properties emerge in a distance-based network: (i) all sectors are related, so* *that each network of sectoral distances is always represented by a non-negative, full matrix; (ii) a unique Input-Output structure generates distinct distance matrices but a unique distance structure; and (iii) if the Input-Output matrix is non-symmetric, then all values differ across distance matrices. Differently, for a network with $S \geq 4$ sectors, there exist at least two values that coincide across matrices of sectoral distances if $\alpha(s,s') = \alpha(s',s)$ for at least $S-2$ sectors relative to a common one, and for one additional symmetric entry in whatever pair of sectors.*

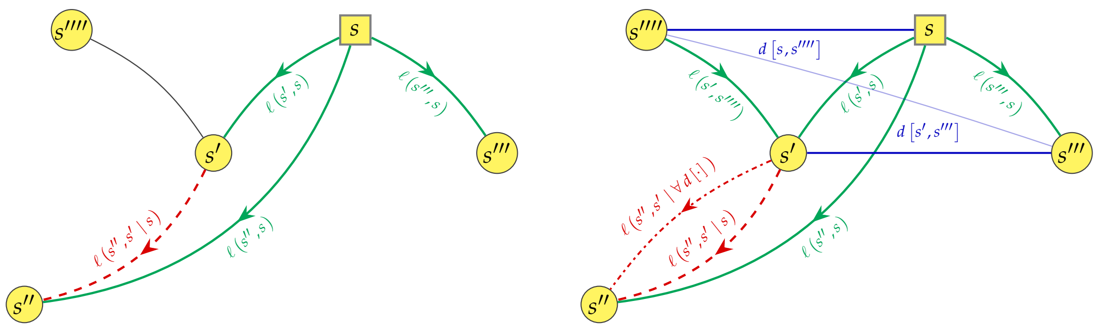
Proof in Appendix A.1.

The first two clearly emerge in Examples 2-3 and Figure 1. The final property clarifies when the distance between two sectors is uniquely determined: if each sector in a pair buys and sells the same quantity of intermediate inputs with the common third sector, then their factor input demand and factor input supply distances coincide. Actual quantities may differ between the two sectors; what matters is that, for each sector individually, its purchases and sales with the common sector are equal.

Outlined horizontal dimension raises some implications in the transmission of idiosyncratic sectoral shocks to other sectors: the magnitude of the propagation not only depends on the size of the (direct and indirect) connection among sectors, but also by their network distance. For instance, as shown in Figure 3, a shock originating in a particular sector propagates vertically in downstream sectors along different chains while, at the same time, transmitting horizontally to sectors which are characterized by such existing production network linkage – even across any pair of sectors potentially neither directly nor indirectly related – due to the simultaneous presence of demand and/or supply linkages with their common sector.[12]

EXAMPLE 4 (Leontief inverse and network economic distances) *Consider an economy populated by five sectors, $\{s,s',s'',s''',s''''\} \in \Phi(s)$, where some trade with all the others, while some others do not. All sectors except s'''' are connected to s, with s'' additionally linked to s', as well related with s''''. Suppose a (supply or final demand) shock originates in s. In the left panel of Figure 3, the shock propagates downstream: it directly affects $\{s',s'',s'''\}$ and indirectly (i.e., Leontief inverse effect) reaches s'', yielding a purely vertical transmission, with s'''' unaffected. Unknown remain the potential effects of induced demand and/or supply adjustments in s' on s'''', even though both are simultaneously connected. By contrast, the right panel illustrates both vertical and supply-driven horizontal propagation: a shock originating in s not only affects its immediate connections but also alters its interaction with s'''': changes in s' triggered by the initial shock in s generate an horizontal effect that links s and s'''' through their common connection with s'. Subsequent adjustments in s'''' then feed back into s', producing an additional Leontief inverse propagation toward s''; simi-*

[12] The example is presented as the more intuitive and visually transparent case of a supply-side sectoral shock propagating downstream through the production network, affecting sectors that depend on the intermediate output of the initially shocked sector. Yet, the same underlying logic of network "economic" distances applies also in the case of a demand-side shock, where the direction of transmission is reversed, and with the shock moving upstream as sectors increase demand for intermediate inputs from their suppliers.



(A) LEONTIEF

(B) LEONTIEF AND ECONOMIC DISTANCES

FIGURE 3: VERTICALITY AND HORIZONTALITY OF A TRANSMISSION

Note: this figure displays a stylized example of the way in which a shock occurring in a given sector propagates to other sectors along the production network. The Input-Output structure is the same in the two panels. Left-panel depicts the situation in which only the Leontief inverse is considered: a shock to sector-s will propagate vertically to other sectors. Differently, right-panel sketches the case where sectors are not just connected via the Leontief inverse, but also with their demand/supply relation with the sector in which the shock originates: in this case there is an influence on the transmission due to horizontal effects across Leontief-related sectors. In addition, also a pair of sectors not connected through the downstream propagation is as well related due to such demand/supply factor with the common sector.

larly for the horizontal effect between $\{s', s'''\}$*. In the case of an already-existing linkage between* $\{s, s''''\}$*, the Leontief transmission would first affect* s' *and subsequently extend to* s''*. With due demand-based adjustments, analogous is the logic underlying* $d[s', s''']$*. Finally, note that network distances generate a compounded (i.e., vertical and horizontal effects) indirect relation within the sectoral pair* $\{s''', s''''\}$ *operating through* s'*. All things considered, this extra "horizontal" dimension may reinforce or mitigate the strength of vertical propagation, and these horizontal complementarities (i.e., horizontal spillover effects) depend on the nature of the demand or supply relationships each sector has with sector-*s*.*

Example 4 illustrates the central message of the paper: incorporating factor input demand and factor input supply network economic distances, thereby horizontal complementarities across sectors in the network, fundamentally reshapes the vertical transmission of micro-level shocks canonically occurring through the Leontief inverse. The new propagation stems from demand- or supply-driven linkages between sectors sharing a similar Input-Output structure (*i.e.*, common upstream suppliers or downstream buyers), and this paper is mostly on the conceptualisation of the $d[\cdot]$-types of linkages. In a fully-networked economy, the horizontal nexus between already-connected nodes plays a prominent role, as it is uncommon for sectors to be entirely isolated from one another.[13,14] Stated differently, network distance complementarities that emerge along this horizontal dimension alter the traditional

[13] Mitigated isolation might be the prevalent in highly granular production networks. On the complexity of the North-American sectoral production network refer to Slater (1977), Slater (1978), Xu et al. (2011), Choi and Foerster (2017), Fed (2019), and Mungo et al. (2023).

[14] Since horizontal complementarities establish linkages even between otherwise unconnected nodes, the concept of network economic distances is of particular significance for firm-to-firm networks, typically characterised by considerable granularity and high levels of sparsity; their complexity is depicted in Atalay et al. (2011), Pichler et al. (2023), and Bacilieri et al. (2025).

Leontief transmission mechanism by either compounding or mitigating the effect of a micro-originated shock to the common (demand or supply related) sector.

Ultimately, along the vertical propagation of sectoral shocks, it is the horizontal dimension of the network structure that leads to positive or negative comovement of economic variables when intermediate inputs are circulating between sectors.

2. A Network Model with Distances

Once having rationalized network distances in the sense of horizontal demand and supply linkages across sectors in partial equilibrium, I now build a framework to analyse their general equilibrium properties in a static Input-Output economy. The model builds on the seminal contributions by Long and Plosser (1983) and Acemoglu, Carvalho, et al. (2012), characterized by a Real Business Cycle (RBC) set-up with an exogenous sector-to-sector production network structure. Given a fairly standard households side, I am going to introduce *factor input demand* and *factor input supply* network-based economic distances, as determined by Lemmas 1 and 2, upon the equilibrium conditions of the model. Then, I turn to analytically characterize how sectoral variations in production input quantities propagate along inter-sectoral linkages. In Subsection 2.1 I explore the effect of changes both in intermediate inputs and other sectors' employment levels on sector-specific employment. I include the notion of network distances to analyse their role in propagating variations in employment levels of nearby and further sectors on sectoral employment; these considerations are discussed in Subsection 2.2. Subsection 2.3 discusses the relation between the vertical and the horizontal network's dimensions (Theorem 2), and present the model's implications for aggregate fluctuations.

Within a unit mass, household-$i \in \mathcal{I}$ is getting utility from consumption of all sectoral goods and dis-utility from working in a given sector

$$\mathcal{U}_i\left(\{c_i(s)\}_{\forall s\in\Phi(s)}, n_i(s)\right) := \prod_{s\in\Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi}$$

where $\beta(s)$ identifies the weight of each sectoral good in household i's consumption basket and $\sum_{s\in\Phi(s)} \beta(s) = 1$, while the assumption on the inverse Frisch labour supply elasticity, measuring the elasticity of hours worked to the wage rate under a constant marginal utility of income, $\phi \to \infty$, rules out indivisible labour (Rogerson 1988), as Section 4 examines sectoral employment rather than hours (*e.g.*, Yedid-Levi 2016). Individual sectoral consumption and labour supplied are labelled as $c_i(s)$ and $n_i(s)$, respectively, and each is characterized by a price, $p(s)$ and $w(s)$. Utility maximization, subject to the intra-temporal budget constraint $\sum_{s\in\Phi(s)} p(s)\, c_i(s) = w(s)\, n_i(s) + \sum_{s\in\Phi(s)} D_i(s)$, with component $D_i(s)$ being the constant share of sector-specific profits rebated to household-i, reports standard conditions for consumption and labour choices $\frac{p(s)\,c(s)}{p(s')\,c(s')} = \frac{\beta(s)}{\beta(s')}$ and $w(s) = n(s)^{\phi}\, \frac{c(s)}{\beta(s)\,C}\, p(s)$, already aggregated across households. The first condition defines how the total expenditure from households for sectoral goods gives rise to the relative importance of each good in total consumption, C. This is an aggregator over sectoral consumption levels:

$$C = f\left(c(s), c(s'), c(s''), \ldots, c(S)\ ;\ \{\beta(s)\}_{s=1}^{S}\right)$$

Assumption 2 (Final consumption requirements) *As main characteristics, the final consumption aggregator: (i) it is strictly quasi-concave, non-decreasing and homogeneous of degree one in each of the sectoral consumption levels; (ii) consumption goods are normal, so that their demand increases with households' income; and (iii) the weight of sectoral goods in total consumption is driven by $\beta(s)$, which is the relative weight that households give in the consumption of the good produced by sector-s.*

Moreover, denote by $\mathbf{C} = \left[c(s)\right]$ the $S \times 1$ vector of consumption levels over sectoral goods, with common element defined as $c(s) > 0$. Under this specification and the regularities in Assumption 2, it holds that $C = \prod_{s \in \Phi(s)} c(s)^{\beta(s)} \varpropto \boldsymbol{\beta}'\mathbf{C}$, as well equivalent to the condition on the final output at the beginning of Section 1. Due to a closed economy with no investment schedule, in equilibrium $C = Y$ must be true.

The production side of the economy is made of a finite set of sectors, $\{s, s', \ldots, S\} \in \Phi(s)$, populated by a representative firm whose intermediate output is the result of a Cobb-Douglas technology satisfying Assumption 1:

$$y(s) = z(s)\left(n(s)\right)^{\alpha(s)} \prod_{s' \in \Phi(s)} \left(x(s, s')\right)^{\alpha(s,s')} \varkappa_j(s) \tag{4}$$

where $\varkappa_j(s)$, for $j = \{fd, fs\}$, is a normalization constant ruling out double counting of intermediate inputs when considering both factor input demand and supply network distances between sectors;[15] absent any of these two cases, then $\varkappa_d(s) = 1$.

Taking factor inputs prices as given, each sectoral representative firm demands labour to households, $n(s)$, and intermediate inputs from other sectors, $x(s, s')$, $\forall s' \in \Phi(s)$, to maximize total revenues from a profit function, $D(s) = p(s)\,y(s)$, net of total costs, $\mathcal{C}(s) = w(s)\,n(s) + \sum_{s' \in \Phi(s)} p(s')\,x(s, s')$. Optimal competitive factor demands of sector-s relative to sector-s' are given by $n(s) = \alpha(s)\,\frac{p(s)\,y(s)}{w(s)}$ and $x(s, s') = \alpha(s, s')\,\frac{p(s)\,y(s)}{p(s')}$. Both optimized quantities decrease the higher is their market price and the lower is its analogous in other connected sector, and their optimal combination for sector-s buying from sector-s' is equal to $x(s, s') = \alpha(s, s')\,w(s)\,n(s)\,\frac{1}{\alpha(s)\,p(s')}$.

According to the structure of the model, equilibrium conditions read as follows.

(Equilibrium) *A competitive equilibrium for this efficient economy is defined by a set of sectoral prices for labour and intermediate inputs, $\left\{w(s), p(s)\right\}_{s \in \Phi(s)}$, a set of sectoral production input quantities, $\left\{n(s), x(s, s')\right\}_{s, s' \in \Phi(s)}$, exogenous sectoral productivities, $\left\{z(s)\right\}_{s \in \Phi(s)}$, and a set of aggregate quantities, $\Omega = \left(Y, C, N, X, D\right)$, such that (i) each household satisfies its optimality conditions, (ii) the representative firm of each sector maximizes profits, and (iii) all markets clear, shaping Ω.*

---

[15] Such constant elements are mathematically defined in Appendix A.4.

In addition to the equilibrium definition, rearranging optimality conditions for labour market variables from both households and sectors, the following equilibrium condition for sectoral labour force is detected

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ y(s)\right]^{\frac{1}{1+\phi}} \tag{5}$$

which defines that employment is a positive function of both output and consumption (recall that total consumption C increases with sector-specific consumption).

Equilibrium adjustments in production networks are central to understand the origins of aggregate business cycles. Within this perspective, an important question emerges: to what extent does employment comovement shape the dynamics of aggregate fluctuations in output? In a similar theoretical set-up vom Lehn and Winberry (2022) show that, in equilibrium, the impact of a given sector-specific shock on real Gross Domestic Product (GDP) – exploiting the *Divisia Index* – can be decomposed into its propagation on aggregate Total Factor Productivity (TFP) and its effect on aggregate employment.[16] In Appendix A.2, following closely the authors, I determine that fluctuations in aggregate real GDP growth are the result of

$$d\log Y = \sum_{s\in\Phi(s)} \Big(\lambda(s)\ d\log z(s) + \nu(s)\ d\log n(s)\Big) \tag{6}$$

where $\lambda(s) = \frac{p(s)y(s)}{PY}$ is the usual *Domar weight* of sector-s, and $\nu(s) = \frac{p^Y(s)y^Y(s)}{PY}$ its *value-added Domar weight*, with $p^Y(s)\,y^Y(s)$ being the sector-specific nominal value-added. These components identify the ratio of the gross nominal and value-added intermediate output of sector-s to GDP, respectively, and both define the importance of a given sector to aggregate fluctuations and business cycle dynamics.[17]

Employment comovement thus constitutes a central phenomenon to investigate under inter-sectoral trade network: the cyclical behaviour of the economy origins from both changes in sectoral TFP, $d\log z(s)$, and in sectoral employment, $d\log n(s)$. Focusing on employment comovement offers distinct advantages. It provides a directly observable measure of sectoral activity, avoiding the interpretability (whether there is shock transmission or shocks correlation; see Huo et al. 2025) and the identification challenges associated with productivity.[18] At the same time, it allows the

[16] Upon impact, capital in their *investment network* is fixed. Moreover, fluctuations in sectoral employment arise from the household's valuation of investment goods (investment weakens the income effect on labour supply) rather than the household's valuation of sectoral intermediate goods.

[17] Note how eq. (6) does not preserve the Hulten (1978)'s theorem – that, under standard neoclassical assumptions and nearby a steady state, fluctuations in aggregate GDP are an approximated linear combination of sectoral productivity shocks, weighted by each Domar weight (*i.e.*, sector's total sales – to final demand and intermediate users – relative to aggregate GDP, reflecting its direct and indirect contribution to the economy). Shifting the focus from productivity changes (the "origins") to directly observable labour adjustments (the "causes"), eq. (6) reveals employment as a key channel through which sectoral linkages translate micro-level shocks into macroeconomic dynamics – in the spirit of the vom Lehn and Winberry (2022)'s analysis.

[18] Identifying the sources of productivity comovement is challenging since TFP changes residually absorb a wide range of measurement errors, making it difficult to isolate and interpret the precise antecedents of productivity comovement between sectors. In contrast, my work focuses on sectoral employment, a directly observable input. This choice enables a more transparent analysis of cross-sectoral comovement, since em-

analysis to remain general, abstracting from the specific nature of shocks while capturing the general-equilibrium adjustments of labour across the production network. Hence, it is possible to separate the “origins” from the “causes” of business cycles.

Consider, then, an economy subject to a supply- or demand-driven idiosyncratic, independent, and asymmetric sectoral disturbance. Section 1 has theoretically shown how different types of network distance generate differential behaviour in changes in primary factors of production. On the same line, next sections are going to argue, both theoretically and empirically, that not only employment levels are subject to comovement across sectors due to Input-Output linkages, but also that the direction of the comovement among paired sectors depend on their demand (supply) relation with their common upstream supplier (downstream buyer) sectors. As a result, besides vertical transmission, depending on the type of distance relation among horizontally-connected sectors in the network, the propagation of a sector-specific shock will generate different types of business cycles.

2.1. First-order propagation of production inputs

Starting from the response of sectoral employment to its own set of intermediate inputs, substitute the production function of eq. (4) into the labour market equilibrium of eq. (5). In such a way it is possible to express how changes in intermediate inputs usage relate to changes in sector-specific employment levels.

Proposition 1 (Propagation of variations to intermediate inputs) *Consider a market economy characterized by Input-Output linkages as in eq. (4), whose labour market equilibrium is eq. (5). Then, the response of sectoral employment to changes in sectoral intermediate inputs is a first-order (log-linear) approximation given by*

$$d\,log\,\mathbf{N} = \Theta \left\{ d\,log\,C + \underbrace{d\,log\,\mathbf{z} - d\,log\,\mathbf{C}}_{d\,log\,\boldsymbol{\mathcal{S}}} + \boldsymbol{H}\,d\,log\,\boldsymbol{X} \right\} \tag{7}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

(b) $\Theta = \frac{1}{1+\phi+\boldsymbol{\alpha}}$ *is a compound of structural parameters, and* $\boldsymbol{\alpha} = [\alpha(s), \ldots, \alpha(S)]$;

(c) C *is aggregate consumption;*

(d) $\boldsymbol{\mathcal{S}}$ *identifies sector-specific changes in productivity and consumption;*

(e) $\boldsymbol{H}$ *is the Input-Output matrix, comprising sectoral weight on other sectors, influencing the elements of* $\boldsymbol{X}$*, a matrix with intermediate inputs purchases.*

Proof in Appendix A.4.

Such proposition expresses how any change in consumption, sectoral productivities, and a given intermediate input bought by a sector-s from sector-s', $\forall s, s' \in \Phi(s)$,

ployment is not subject to the same identification and interpretability challenges as productivity, facilitating a clearer identification of demand and supply forces operating through the production network.

impacts sectoral employment, and the magnitude of the variations in the set of intermediate inputs is driven by the intensity in the Input-Output relation between these two sectors, drawn from matrix $\boldsymbol{H}$. Positive changes in $d \log \boldsymbol{X}$ work through sectoral output: an expansion in intermediates will determine an expansion in output which, as in eq. (5), will determine an increase in employment level of sector-s.

To account for the role of changes in other sectors' employment to sectoral employment level, notice that when sector-s buys intermediate inputs from sector-s', it is introjecting also the quantities of production inputs in such sector. Henceforth, each element in the intermediate inputs bundle of sector-s can be taught as $x(s,s') = \vartheta(s,s')\ y(s')$, $\forall s' \in \Phi(s)$, which states that each intermediate input is just a given share $\vartheta(s,s')$ of other sector's output. Labour market equilibrium in eq. (5) can be thus rewritten as

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ z(s) \left(n(s)\right)^{\alpha(s)} \prod_{s' \in \Phi_s} \left(\vartheta(s,s')\ y(s')\right)^{\alpha(s,s')}\right]^{\frac{1}{1+\phi}} \tag{8}$$

so that, once plugging in intermediate output of sector-s', sectoral employment levels are transparently related through the production network structure. As a result, the above labour market condition allows to study the general equilibrium comovement of employment levels across sectors.

Proposition 2 (Direct propagation of variations to sectoral employment) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (8). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by*

$$d\log \mathbf{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + \underbrace{\boldsymbol{H}\left(\Psi_{s,s'}\right)\left[d\log \mathbf{z} + \boldsymbol{\alpha}\ d\log \mathbf{N}\right] + \boldsymbol{\mathcal{E}}\ d\log \mathbf{X}}_{d\log \boldsymbol{\mathcal{N}}} \right\} \tag{9}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

...

(e) $\boldsymbol{\mathcal{N}}$ *identifies the production network effect of other sectors' changes in productivities, employment levels, and intermediate inputs usage, where:*

- $\boldsymbol{H}(\Psi_{s,s'})$ *is the Input-Output matrix, comprising the weight of each sector on other sectors, whose entries are set to 0 whenever* $s = s'$*;*
- $\boldsymbol{\mathcal{E}} = \boldsymbol{H}(\Psi_{s,s'})'\boldsymbol{H}(\Psi_{s',s})$ *is a compounded network effect, made of the inner product of the Input-Output matrix, influencing the elements of* $\mathbf{X}$*.*

Proof in Appendix A.4.

Just like Proposition 1, increases in consumption, productivities and intermediate inputs will have a positive effect on sectoral employment. In addition, Proposition 2 predicts that any increase in labour force of other sectors will positively propagate to employment level of a given sector, and the magnitude of the shock transmission would ultimately depend by the intensity of the inter-sectoral trade. This result is in line with the literature, where the impact of an idiosyncratic shock is determined by the elements of its "influence vector", whose entries are non-negative (Acemoglu, Carvalho, et al. 2012). Therefore, positive shocks will positively transmit to vertically-connected sectors, and the change in aggregate employment is just the linear combination of sectoral-originated shocks. In the next section it is shown how the results of Proposition 2 are dampened when considering factor input demand and factor input supply network distances of Section 1.

2.2. Comovement and network distances

Once inserting the production function of eq. (4) in the labour market equilibrium condition of eq. (8), then optimal quantities of a sector are related to that of the other ones. To incorporate *factor input demand* distance, in Appendix A.2 I show that manipulation in general equilibrium would result in

$$n\left(s\right)=\left[\alpha\left(s\right)\ \frac{C}{c(s)}\ \beta\left(s\right)\ z(s)\ n(s)^{\alpha(s)}\ \aleph^{fd}\ \boldsymbol{\Lambda}^{fd}\ \boldsymbol{\Xi}^{fd}\right]^{\frac{1}{1+\phi}} \tag{10}$$

where $\aleph^{fd}=\prod_{s'\in\Phi(s)}\left(\prod_{s\in\Phi(s)}x\left(s',s\right)^{\alpha(s',s)}\right)^{\frac{\alpha(s,s')}{1+\phi}}$ identifies the set of intermediate inputs bought by sector-s and all the other sectors connected to it, $\boldsymbol{\Lambda}^{fd}=\prod_{s'\in\Phi(s)}\left(\vartheta\left(s,s'\right)z(s')\,n(s')^{\alpha(s)}\right)^{\alpha(s,s')}$ captures the relevance of other sectors' productivities and employment levels on sector-s working through the production network, and the component $\boldsymbol{\Xi}^{fd}=\prod_{s'\in\Phi(s)}\left[\prod_{s\in\Phi(s)}\left(\frac{x(s',s)}{x(s,s)}\right)^{\frac{\alpha(s',s)}{\alpha(s,s)}}\right]$ determines the ratio among the intermediate inputs between sector-s and each of the other sectors, when these are buying from the same upstream sector. Total *log*-differentiation of eq. (10) while integrating the result in Lemma 1 allows to determine how changes in sectoral employment levels are related in the production network, both vertically (Input-Output matrix) and horizontally (distance matrix), as the following proposition shows.

Proposition 3 (Direct propagation under factor input demand distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (10). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by*

$$d\,log\,\mathbf{N}=\Theta\left\{d\,log\,C+d\,log\,\boldsymbol{\mathcal{S}}+d\,log\,\boldsymbol{\mathcal{N}}+\underbrace{\boldsymbol{\mathcal{D}}^{fd}\left[d\,log\,\mathbf{N}\Big(\Phi\left(s\right)\Big)-d\,log\,\mathbf{N}\right]}_{d\,log\,\boldsymbol{\mathcal{D}}(n)}\right\} \tag{11}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

...

(f) $\mathcal{D}^{fd}$ *identifies all the demand-based distances across any pair of sectors;*

(g.i) $\mathcal{D}(n)$ *identifies the production network distance effect of other sectors' variations in employment levels,* $d\log \mathbf{N}(\Phi(s))$*, when these are buying their intermediate inputs from the same upstream sector(s).*

Proof in Appendix A.4.

Proposition 3 extends Proposition 2 by incorporating the Input-Output network distance between sectors in terms of shared upstream suppliers. Beyond the standard effects of aggregate and sector-specific variations, it formalizes how sectoral employment responds to changes in other sectors through demand-based horizontal linkages. Importantly, vertical and horizontal effects are intertwined: employment changes in other sectors appear simultaneously in $\mathcal{N}$ and $\mathcal{D}(n)$, making it ambiguous whether observed comovement is driven primarily by supply-chain spillovers or by common upstream network linkages.[19] When sectors are far apart, positive comovement is guaranteed, as the vertical component $\mathcal{N}$ dominates since horizontal interactions in $\mathcal{D}(n)$ are negligible. Strong horizontal interdependencies materialize when sectors are close (low demand-distance), and comovement patterns depend on the sufficient-statistic conditions identified in Corollary 1: negative or positive comovement depends on how shocks to one sector affect the price of common suppliers and how easily downstream buyers can substitute between inputs. When complementarity is strong, a positive shock in a horizontally related sector raises the cost of inputs from the shared supplier, potentially dampening sector-s's input demand and generating negative comovement; when substitutability is high, this effect diminishes. Thus, the propagation of employment is shaped jointly by vertical supply-chain effects, captured in $\mathcal{N}$, and horizontal demand linkages through $\mathcal{D}(n)$, producing an overlapping pattern that can obscure the exact source of observed comovement, where intermediate inputs complementarities and substitution patterns determine whether horizontal linkages reinforce or offset the vertical propagation, yielding nuanced sectoral responses that reflect the joint effect of both channels. Overall, Proposition 3 highlights that sectoral shocks propagate not only vertically along the supply chain but also horizontally through common input relationships, with the sufficient statistics providing an interpretative precise lens to disentangle when horizontal linkages amplify or mitigate employment comovement.

Manipulation in general equilibrium to include *factor input supply* distance for pairs of sectors would result in

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ z(s)\ n(s)^{\alpha(s)}\ \ \aleph^{fs}\ \boldsymbol{\Lambda}^{fs}\ \boldsymbol{\Xi}^{fs}\right]^{\frac{1}{1+\phi}} \tag{12}$$

[19] Component $\mathcal{D}(n)$ captures the additional response due to shared suppliers, and its magnitude is increasing with the proximity in the distance matrix $\mathcal{D}^{fd}$: without specifying an exact scaling, a higher value $d^{fd}(s,s')$ indicates that sectors s and s' are closer, as in Section 1. In the empirics (Sections 3 and 4), distances are used to classify sectors along an extensive margin, with values near zero corresponding to closely related sectors.

where $\aleph^{fs} = \prod_{s' \in \Phi(s)} \left(\prod_{s' \in \Phi(s)} x\left(s', s'\right)^{\alpha(s',s')} \right)^{\frac{\alpha(s,s')}{1+\phi}}$ is the compounded set of intermediate inputs, $\boldsymbol{\Lambda}^{fs} = \prod_{s' \in \Phi(s)} \left(\vartheta\left(s, s'\right) z(s')\, n(s')^{\alpha(s)} \right)^{\alpha(s,s')}$ considers sectoral productivities and employment levels, and $\Xi^{fs} = \prod_{s' \in \Phi(s)} \left[\prod_{s \in \Phi(s)} \left(\frac{x(s',s)}{x(s',s')} \right)^{\frac{\alpha(s',s)}{\alpha(s',s')}} \right]$ determines the ratio among the intermediate inputs of paired sectors, when these are selling to the same downstream sector. Embedding Lemma 2, the following holds.

PROPOSITION 4 (Direct propagation under factor input supply distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (12). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation given by*

$$d\,log\,\mathbf{N} = \Theta \left\{ d\,log\,C + d\,log\,\boldsymbol{\mathcal{S}} + d\,log\,\boldsymbol{\mathcal{N}} + \underbrace{\boldsymbol{\mathcal{D}}^{fs}\left[d\,log\,\mathbf{P}\Big(\Phi\left(s\right)\Big) - d\,log\,\mathbf{P} \right]}_{d\,log\,\boldsymbol{\mathcal{D}}(p)} \right\} \tag{13}$$

where:

(a) *$\mathbf{N}$ identifies sectoral employment levels;*

...

(f) *$\boldsymbol{\mathcal{D}}^{fs}$ identifies all the supply-based distances across any pair of sectors;*

(g.ii) *$\boldsymbol{\mathcal{D}}(p)$ identifies the production network distance effect of other sectors' variations in employment levels, $d\,log\,\mathbf{P}\big(\Phi\left(s\right)\big)$, when these are selling part of their output to the same downstream sector(s).*

Proof in Appendix A.4.

Proposition 4 formalizes how sectoral employment responds to changes in other sectors through supply-based horizontal linkages. The role of network supply distances on sectoral employment comovement works through sectoral output $y\left(s\right)$: for instance, an increase in the price level of sector-s', captured in $\mathbf{P}\big(\Phi(s)\big)$, raises its demand for inputs, reduces its own supply of intermediate inputs, and contracts its production;[20] a decrease in intermediate output then translates into lower employment for s' while, under analogous but reverse logic, the higher relative price simultaneously increases employment in competing supplier sectors. Vertical and horizontal effects are thus intertwined: the vertical component $\boldsymbol{\mathcal{N}}$ captures standard supply-chain propagation through downstream production, while the horizontal component $\boldsymbol{\mathcal{D}}\left(p\right)$ captures reallocation effects among sectors supplying to common downstream buyers. For sectors close in the supply network (high $\boldsymbol{\mathcal{D}}^{fs}$), the horizontal mechanism dominates, generating negative comovement between nearby suppliers; for sectors far apart (low $\boldsymbol{\mathcal{D}}^{fs}$), horizontal effects are negligible, and positive comovement emerges via vertical propagation. Consistently with the sufficient-

[20] Since these demand effects cancel out, the shock only propagates downstream (*e.g.*, Shea 2002).

statistic conditions in Corollary 2, the supply-based propagation is more transparent than the demand-based case: positive or negative comovement can be clearly traced to either vertical propagation or horizontal reallocation among competing suppliers, with proximity in the supply-distance network determining the intensity and direction of horizontal effects. Overall, Proposition 4 highlights that sectoral shocks propagate both vertically along the supply chain and horizontally through competition for downstream markets, with negative comovement being stronger among nearby suppliers and fading with network distance, while more distanced sectors display positive comovement dominated by vertical effects.

2.3. Discussion

In conclusion of this theoretical exploration, the overall production network effects on sectoral comovement of employment can be summarized by the following:

$$d\log \boldsymbol{N} = \Theta\Big\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}} + d\log \boldsymbol{\mathcal{D}}(\cdot) \Big\}$$

so that changes in sectoral employment levels are function of aggregate consumption (C), both sector-specific productivity and consumption of its intermediate good ($\boldsymbol{\mathcal{S}}$), other sectors' productivities and their associated households' final consumption levels ($\boldsymbol{\mathcal{N}}$) whose magnitudes depend on the Input-Output matrix, and "economic" distances in the production network architecture ($\boldsymbol{\mathcal{D}}$) for both demand (sectors are closer if they share similar upstream suppliers) or supply (sectors are closer if they share similar downstream buyers) trade relationships among any pair of sectors.

After all the discussions, the following result can be then established.

Theorem 2 (On the weighting of network "economic" distances) *In an Input-Output economy, the horizontal dimension of a network (defined by demand- and supply-driven distance matrices) is not weighted by its vertical dimension (defined by the direct or Leontief inverse matrix).*

Proof in Appendix A.4.

The theorem postulates that, within an Input-Output economy, the horizontal dimension of the network, captured by demand- and supply-driven distance matrices, remains unweighted and independent by its vertical dimension, encoded in the Input-Output matrix. This distinction is crucial for understanding the economics of production networks: while the vertical structure reflects the intensity of inter-sectoral trade linkages, the horizontal structure identifies demand/supply complementarities across sectors related through common upstream suppliers or downstream buyers. In other words, by separating the horizontal from the vertical dimension, the theorem clarifies that whatever distance matrix is not merely a transformation of the Input-Output matrix, but a complementary object that reveals otherwise hidden propagation channels within the production network.

Finally, through the lens of eq. (6), changes in sectoral employment would play the same role on aggregate fluctuations, and idiosyncratic magnitude depends on the influence played by a sector in the network: a sector-specific shock that increases sectoral employment will determine a subsequent increase in other sectors' employment, thus leading to fluctuations in both aggregate employment and output. However, as discussed so far and in Subsections 1.2 and 2.2, for different types of network distances the component $d\log\mathcal{D}(\cdot)$ plays an important role in the way in which sectoral shocks propagate to other ones: the direction of changes in employment levels depend on whether any pair of sectors is characterized by high/low distance relations, thus differently influencing the employment component of aggregate fluctuations of GDP. In particular, it can be stated that

$$d\log Y = \begin{cases} \sum_{s\in\Phi(s)} \Big(\lambda(s)\; d\log z(s) + \nu(s)\;\Delta\,\text{comovement}\Big) & \textit{under major distance} \\ \sum_{s\in\Phi(s)} \Big(\lambda(s)\; d\log z(s) + \nu(s)\;\Delta\,\text{reallocation}\Big) & \textit{under minor distance} \end{cases}$$

As a summary in terms of sectoral employment and comovement, the following conjectures have been established: *(i)* positive changes in sectoral intermediate inputs increase the sector-specific employment level; *(ii)* if sectors are buying from the same upstream sector(s), the strength and clarity of this effect can be ambiguous due to the overlapping of vertical and horizontal propagation, and positive changes in other sectors' employment transmit positively to sectoral employment under low network distance; accordingly, *(iii)* if sectors are selling to the same downstream sector(s), changes in sectoral employment due to other sectors' changes in employment are driven by the entity of their supply distance relation. These theoretical predictions will be empirically tested in Section 4. Before doing so, however, I shall provide a description of the data and of the characteristics of the U.S. production network.

3. Data and Network Anatomy

From the United States' Bureau of Economic Analysis (BEA), available data display information, for 3-digit U.S. 2017 North American Industrial Classification System (NAICS) sectors, on value-added, employment, net exports, and inter-sectoral linkages exploiting Input-Output (I-O) matrices.[21] The (balanced) panel data built is made of 65 private sectors over the period 1998-2022.[22] Paragraphs below briefly describe the data content related to the production network and its sectoral distances, and their mapping to the outlined theory of Sections 1 and 2; for additional details and an extended discussion of the data refer to Appendix C.

To investigate the network structure, weights for upstream and downstream sectors as well as sectoral distances are pinned down using the *commodity-by-commodity*

[21] Row sectors identify supplier (upstream), and column sectors identify buyer (downstream). Hence, row sector is the origin and column sector is the destination of the circulating intermediate input.

[22] The 3-digit classification is the most granular if one wants to keep track of the evolution in the set of intermediate inputs, and 1998 is the first year in which the NAICS system has been adopted.

total direct requirements Input-Output table, whose elements are expressing the amount of intermediate inputs needed to produce the sectoral good in terms of one unit of final consumption. In principle, this object is the equivalent of the $\boldsymbol{H}$ matrix in the model of Sections 1 and 2. However, for what concerns the creation of network distance matrices only, small transactions among sectors are disregarded so that the reader should think of matrix $\boldsymbol{\mathcal{D}}^j$, for $j = \{fd, fs\}$, in all the derived propositions, as factor input demand (or factor input supply) distance matrix computed from an Input-Output structure in which each inter-sectoral trade linkage is greater than 1% of total purchases (or sales) of the *common* sector – *i.e.*, from an adjusted network matrix $\boldsymbol{H}^j = \left[\alpha^j\left(s, s'\right)\right]$ whose generic element is then $\alpha^j\left(s, s'\right) > 0.01$.[23]

It is important to emphasize that, hereafter, either network weights and distance measures are derived from the *directed* network, $\boldsymbol{H}$, rather than from its *Leontief inverse* counterpart, $\boldsymbol{\mathcal{H}}$. Whereas the latter incorporates indirect sectoral exposures as well, and thus amplifies the magnitude of interconnections beyond direct relationships, the directed network isolates the immediate intensity of trade between pairs of sectors, thereby offering a more parsimonious and transparent representation of demand/supply linkages. Employing the directed configuration to characterize the entries of matrices $\boldsymbol{\mathcal{D}}^{fd}$ and $\boldsymbol{\mathcal{D}}^{fs}$ avoids overstating distance values between sectors, preventing the inflation of horizontal complementarities and instead preserving the strictly observed interdependencies in the Input-Output structure.[24]

3.1. Characteristics of distance networks

Later on, the empirical estimation will consider network distances at their *extensive margin* for sectoral classification purposes (reminiscent from Theorem 2). To avoid notational confusion, extensive margin version of the Input-Output network distance matrices are labelled as $\boldsymbol{\mathcal{D}}^j_{ext}$, for $j = \{fd, fs\}$, identifying both factor input demand and factor input supply cases, whose generic entry is $d^j_{ext}\left[s, s'\right] = \{1, 2, \ldots, d_{max}\}$.

As a characterization of matrices $\boldsymbol{\mathcal{D}}^{fd}_{ext}$ and $\boldsymbol{\mathcal{D}}^{fs}_{ext}$, network distance-$d$ among any two sectors $\{s, s'\}$ is computed as a *shortest path problem* from the *intensive margin* distance matrices, $\boldsymbol{\mathcal{D}}^j$, with $d^j\left[s, s'\right] \in \left[0, 1\right]$ and $j = \{fd, fs\}$ (for details on their construction, refer to Appendix C), implementing an algebraic version (*i.e.*, using matrix multiplication) of the traditional Dijkstra (1959)'s or Roy (1959)-Floyd (1962)-Warshall (1962)'s types of algorithms, since it is *(i)* suitable for discrete-value adjacency matrices (as the one pre-built to account for distances at the extensive margin),

[23] I adhere to the established literature (*e.g.*, Conley and Dupor 2003, Acemoglu, Carvalho, et al. 2012, Carvalho 2014). This approach enables to compute a finite set of network distances at the extensive margin, as it removes any bias stemming from meaningless sectoral interconnections.

[24] From the perspective of Example 4, the directed network ensures that $\ell\left(s'', s' \,\middle|\, s\right) = 0$, consistently with the absence of an indirect link: the additional propagation from sector-s' to sector-s'' – arising in the Leontief inverse mechanism through indirect network effects – would not be artificially introduced to compute network distances based on common upstream and downstream sectors. Put differently, a micro-originated shock in sector-s affecting sector-s'''', conditional on adjustments in sector-s', would remain contained by avoiding indirect Leontief-based linkages. While distance *sparsity* is preserved under both representations, the directed network prevents the exaggeration of distance values that emerges under the Leontief inverse.

and *(ii)* fairly efficient for small to medium-sized sparse graphs (as a sector-level production network). In Appendix C it is reported the technically sophisticated version of the implemented *shortest path algorithm* which, in plain words, reads as follows.

(Shortest path algorithm) *Central idea is to find, for each single node, all the other nodes that can be reached in one step, then in two steps, and so on. Each time a node is reached for the first time, it records the number of steps it took to get there – this is the shortest path approach. In other words, find the shortest number of steps it takes to get from every node to every other node in a network. In particular:*

(a) identify the direct connections between any pair of nodes (those that take exactly one step to be connected);

(b) for not-connected pairs, increase the path length by 1 in each round (two steps, then three steps, etc.). Multiply the graph by itself to discover which new pairs of nodes are now connected through longer paths;

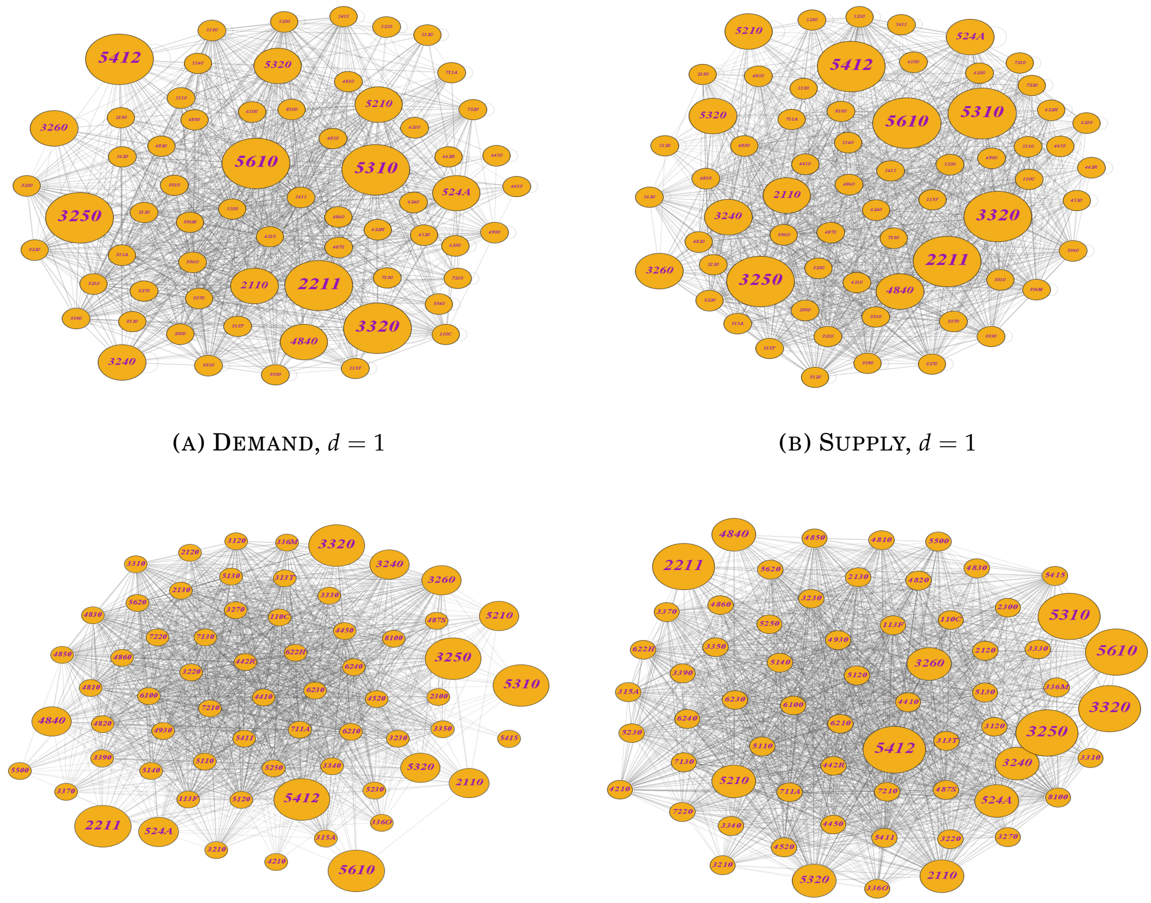
(c) if a pair of nodes becomes connected (i.e., only if it has not be already found a shorter path), record the current length as the shortest distance between them;

(d) repeat this process until any new reachable pairs of nodes is found. For any pair of nodes that are still unreachable (i.e., the distance is still zero and they are not linked in the network), set their distance to infinity.

Outside this routine set to zero all the disconnected nodes (those at ∞). Each matrix $\mathcal{D}^{j}_{ext}$, whose values are at the extensive margin and identify network distances among any pair $\{s, s'\}$, is built, and the generic element, identifying the existence of a link between node-s and node-s', is then $d^{j}_{ext}[s, s'] = \{1, 2, \ldots, d_{max}\}$ for $j = \{fd, fs\}$. Unlike in the theoretical framework, values closer to one correspond to shorter horizontal distances. From this perspective, while the elements of the distance matrices in Sections 1 and 2 reflect *network economic proximity* between pairs of sectors (indicating how closely connected pairs of sectors are), the distance matrices used from this point onward are interpreted as measures of *network economic distance* (emphasizing the degree of separation between sectors within the network).

To illustrate the constructed distance matrices Figure 4 displays, for each extensive margin value, the network distance relationships among sectors based on both factor input demand and factor input supply horizontal linkages. Importantly, each connection does not necessarily represent inter-sectoral trade between two sectors, but rather it reflects an horizontal linkage based on demand- or supply-driven network distance relationships. A few concatenated points are then worth noting.

First, the distribution of horizontal complementarities is uneven across distance levels. At very short range ($d = 1$), the demand-based network appears slightly denser than the supply-based one, suggesting that sectors are more frequently linked through common upstream suppliers than through shared downstream buyers. However, this asymmetry vanishes as the distance increases ($d = 2$), where both types of horizontal ties spread more diffusely across the network.



(A) DEMAND, $d = 1$ (B) SUPPLY, $d = 1$

(C) DEMAND, $d = 2$ (D) SUPPLY, $d = 2$

FIGURE 4: VISUALIZATION OF PRODUCTION NETWORK DISTANCES

Note: each panel of the figure represents the distance-based inter-sectoral production network corresponding to the (*commodity-by-commodity total direct requirements*) U.S. Input-Output matrix in the year 2007. *Factor input demand* and *factor input supply* network distances are computed implementing the *shortest path algorithm* on distance matrices whose values are at the intensive margin, as well derived from the *directed* configuration of the production network. Panels 4a and 4c correspond to the demand-based network distances, with $d = 1$ for low distance (closest sectors) and $d = 2$ for major distance (further sectors), among any pair of sectors; analogously it is for the supply-based network distances in Panels 4b and 4d. There are 65 3-digit U.S. 2017 NAICS private sectors, and each orange node corresponds to one of them. Two sectors connected under $d = 1$ cannot be linked under $d = 2$, and otherwise; constructed distance matrices are full (Theorem 1). Biggest nodes correspond to top 10% of mostly connected sectors (*i.e.*, major number of inter-sectoral connections), while intermediate ones the additional top 20%. Figure drawn with the software package *Gephi*, version 0.10, exploiting the *ForceAtlas2* layout algorithm. *Source*: BEA and own calculations.

Second, sectors with the highest number of Input-Output linkages tend to concentrate at the very core of the production system when considering only close connections ($d = 1$). As shown in Panels 4a and 4b, both the largest hubs (top 10% by connectivity) and the mid-sized hubs (top 20%) are similarly central, occupying short distances from the network's nucleus.

Third, and in contrast with the previous point, these same highly connected sectors tend to become relatively more peripheral once longer distances ($d = 2$) are considered. Panels 4c and 4d show that, at this range, their positions are more evenly distributed across the network, reflecting the fact that their trade structures – both upstream and downstream – are highly similar to those of other sectors. In other words, the most connected sectors are less unique at longer distances, as commonalities in demand and supply linkages become more widespread.

Altogether, these observations highlight that the structure and role of horizontal complementarities depend crucially on the notion of distance. At shorter distances, horizontal linkages cluster around the same set of key sectors, whereas at longer distances, such complementarities are more evenly distributed across the production network. This pattern confirms that sectors typically share both upstream suppliers and downstream buyers at smaller but not at a greater distance, so that the same vertical production system can give rise to distinct horizontal geometries depending on the perspective adopted, as illustrated by Examples 1 to 3, Section 1. The significance of these insights will be examined using U.S. sector-level employment and Input-Output data in the following section.

4. Empirics

In this empirical section I am going to use sector-level data for the United States (U.S.) of America economy to test the predictions about the sectoral comovement of employment in a production network characterized in Section 2. To do so, estimation relies on the two-stages procedure outlined by Barattieri and Cacciatore (2023): to identify the exogenous variation in a given sectoral series, perform sectoral panel Fixed-Effect (FE) regressions to isolate the exogenous dynamics in a given input of production not due to other (sector-specific, other sectors-specific and aggregate) factors; then, the estimated sector-specific residual is used as an identified structural shock in a Local Projection (LP), consisting in a battery of (predictive) panel regressions of the identified structural shock on the employment series for each sector, in order to asses the short-run cumulative variation in sectoral employment.

Initially, I estimate variations in sectoral employment due to changes in the set of intermediate inputs (Proposition 1); then, I turn to analyse how network distances (horizontal demand/supply linkages) affect the sectoral comovement of employment through the production network structure, as read by Propositions 2, 3, and 4.

4.1. Intermediate shares and sectoral employment

Central premise of Sections 1 and 2 is that adjustments in the stock of intermediate inputs lead to analogous shifts in employment levels. This subsection examines how changes in sector-specific employment are induced by own variations in circulating intermediate inputs within the production network, both for its set and that of nearby sectors. In a preliminary step, the building idea is to identify exogenous variations in the share of intermediate inputs that each sector purchases from others, that is changes in the share of production of sector-s bought from sector-s', $\forall s' \in \Phi_s$, which are plausibly exogenous to employment dynamics. I estimate the following panel fixed-effects regression over $s \in \Phi(s)$ sectors

$$
\begin{aligned}
x_t(s) = \beta_n\, \dot{n}_t(s) \;+\; \sum_d \beta_{n(d)}\, \dot{n}_t(\Phi_s, d) \;+\; \sum_k \beta_{n(k)}\, \dot{n}_t(\Phi_s, k) \\
+\, \beta_{z(s)}\, \mathcal{Z}_t(s) \;+\; \beta_{\dot{Z}(s)}\, \dot{\mathcal{Z}}_t(s) \;+\; \beta_z\, \mathcal{Z}_t \;+\; \beta_{\dot{Z}}\, \dot{\mathcal{Z}}_t \;+\; \psi(s) \;+\; u_t^x(s)
\end{aligned}
\tag{14}
$$

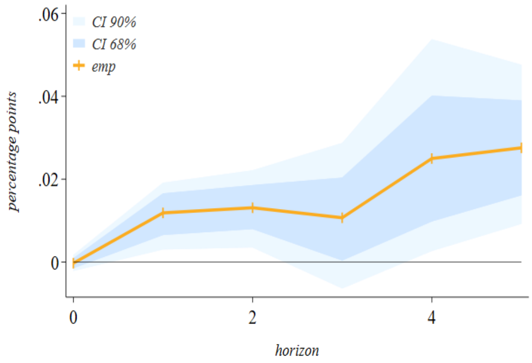
where $x_t(s)$ identifies the set of intermediate inputs bought via the inter-sectoral trade by sector-s (taken as a share of its intermediate output), $\dot{n}_t(s)$, $\dot{n}_t(\Phi_s, d)$, and $\dot{n}_t(\Phi_s, k)$ represent employment growth for sector-s, employment growth of its closer and further sectors, and employment growth of its associated upstream and downstream sectors, respectively. The latter two measures are defined as

$$
\begin{aligned}
\dot{n}_t(\Phi_s, d) &= \sum_{s' \neq s \in \Phi_s} \Delta\, n(s', d) \qquad , \quad \forall d \in \mathcal{D}^j_{ext} \quad and \quad j = \{fd, fs\} \\
\dot{n}_t(\Phi_s, k) &= \sum_{s' \neq s \in \Phi_s} \ell(s, s')\; \Delta\, n(s', k) \quad , \quad \forall k = \{up, dw\}
\end{aligned}
$$

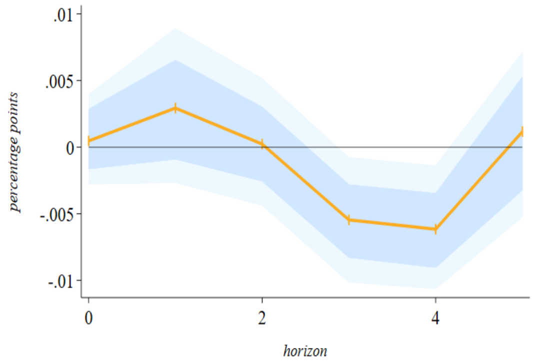
where $\Delta n(\cdot) = n_t(\cdot) - n_{t-1}(\cdot)$. As for vertical propagation, each change in employment is weighted by the Leontief inverse relation, $\ell(s, s') = \left[1 - \alpha(s, s')\right]^{-1}$, between sector-$s$ and each of the other sectors whenever these are located upstream or downstream to sector-s. Note that the Input-Output element $\ell(s, s')$ is thus capturing the *direct* and *indirect* exposure to the production network of sector-s when it is buying intermediate inputs from sector-s', so that the former sector is not impacted only by the latter one, but also by how such sector is impacted by its connected sectors. Differently, changes in employment under network distances are capturing all the horizontal relationships across sectors and, in line with the theory (refer to Theorem 2), they are not leveraged by the Leontief inverse weights. Sectors are classified according to their extensive margin distance, $d = \{1, 2, \ldots, d_{max}\}$, relative to sector-$s$ for both factor input demand and factor input supply configurations.

A set of control variables characterizes the second line of eq. (14). Sector-level controls, $\mathcal{Z}_t(s)$ and $\dot{\mathcal{Z}}_t(s)$, comprise the labour force size of the sector, both its level and growth rate of value-added, and net exports that allow to control for changes in demand and supply not due to internal factors but rather due the rest of the world. The set of aggregate controls, $\mathcal{Z}_t$ and $\dot{\mathcal{Z}}_t$, is made of aggregate value-added and aggregate employment in size as well as in growth terms. Finally, while β's are coefficients to be estimated, the element $\psi(s)$ imposes sector fixed-effect to control for unobserved heterogeneity across sectors. Standard errors are clustered at sector-level. For any sector, the estimated residual $\widehat{u}_t^x(s)$ from eq. (14) identifies the exogenous variation in the share of production of sector-s that is bought from all the other sectors at which it is linked through Input-Output relations.

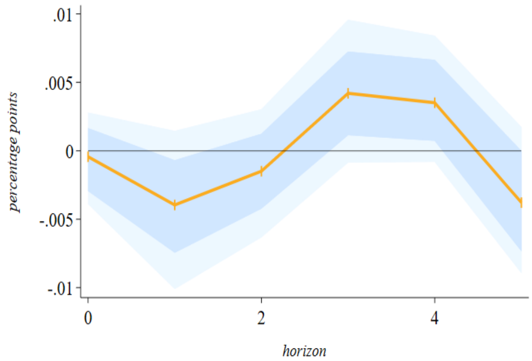
The study of the response of sectoral employment to changes in the sector-specific set of circulating intermediate inputs relies on the estimation of impulse response functions using the Local Projection method outlined in Jordà (2005), consisting in performing predictive panel regression techniques of the identified structural shock, $\widehat{u}_t^x(s)$, on the cumulative difference in sectoral employment levels at different hori-



(A) SECTOR-SPECIFIC



(B) $d = 1$



(C) $d = 2$

FIGURE 5: SECTORAL EMPLOYMENT RESPONSE TO INTERMEDIATE INPUTS

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the sector-specific set of intermediate inputs as a share of its value-added for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. In particular, Panel 5a represents the sector-level employment response to changes in its intermediate inputs, while Panel 5b-5c to changes in the set of intermediates in closer (distance equal to 1) and further (distance equal to 2) sectors. The solid-orange line corresponds to the average response of employment across sectors, while shadow-blue and shadow-light blue areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eqs. (15)-(16). *Source*: BEA and own calculations.

zons.[25] In this way, the resulting Impulse Response Functions (IRFs) are defined by a sequence of estimated coefficients on the identified structural shock (Barattieri and Cacciatore 2023), averaged across sectors.

Given $\hbar = \{1, \ldots, h\}$ time-horizons, and for all sectors $\{s, s', \ldots, S\} \in \Phi(s)$, a battery of panel FE predictive regressions of the form

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x,\hbar}\, \widehat{u}^x_t(s) \; + \; \psi(t+\hbar) \; + \; \psi_\hbar(s) \; + \; \nu^x_{t+\hbar}(s) \tag{15}$$

is performed, with $\dot{n}^{cum}_{t+\hbar}(s) = n_{t+\hbar}(s) - n_{t-1}(s)$ denoting the cumulative change

[25] Cumulative variables are very likely to be highly correlated with the error term in panel data regressions due to the fact these are a sum of past values of the given variable of interest, thus bequeathing also the past error terms; such endogeneity issue may lead to biased and inconsistent estimates. Taking the difference between cumulative values allows to soothe endogeneity concerns.

in sector-s employment at each horizon. Estimated coefficient $\beta_{\widehat{u}^x,\hbar}$ has to be interpreted as the response of the variation in the cumulative difference of sectoral employment at time $t+\hbar$ given the identified shock hitting in period-t. Prediction error term $\nu^x_{t+\hbar}(s)$ is horizon-specific, while standard errors and bootstrapped Confidence Interval (CI) of $\beta_{\widehat{u}^x,\hbar}$ are clustered by sectors. To avoid measurement errors due to unobserved heterogeneity in response, sectoral fixed-effects $\psi_\hbar(s)$, and horizon fixed-effects $\psi(t+\hbar)$ that remove common trends across sectors in the comovement of sectoral employment, are imposed.[26]

Impulse responses from eq. (15) estimate the cumulative response of employment of sector-s to changes in its set of intermediate inputs (as a share of its intermediate output). The same estimation typology is also performed to variations induced by nearby sectors, that is, considering also the d^{th}-distance between sectors:[27]

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x(d),\hbar} \sum_{s' \neq s} \widehat{u}^x_t(s',d) \; + \; \psi(t+\hbar) \; + \; \psi_\hbar(s) \; + \; \nu^x_{t+\hbar}(s,d) \tag{16}$$

Performed estimation of eqs. (15) and (16), under *Leontief inverse* network structure and *factor input demand* distances, delivers Figure 5. The cumulative (short-term) response of sector-specific employment levels to a 1% change in its set of intermediate input (as a share of its total production) is depicted in the top panel: averaging across sectors, the increase induces a statistically-significant positive change in sectoral employment, in line with Proposition 1. The rationale beyond relies on the output effect, where an increase in intermediates will drive up total production of a given sector. Whether this effect can be attributed to a complementarity between employment level and intermediate inputs is still an open question; however, the magnitude of the change on the y-axis may ratify the Cobb-Douglas specification: an increase in the set of intermediate inputs by 1% induces a positive shift in employment by less than 1% upon impact. This implies that for a given increase in one unit of intermediates it is required less than one worker so that factors of production may be at least substitute, as the Cobb-Douglas production function instructs.

Results for changes in intermediate inputs of other distance-related sectors are almost always not statistically significant at 90% except when the impact of the shock is fading out, but with a negligible magnitude. However, besides statistical significance which may be due to external market dynamics factors not accounted for my purposes, it is interesting to note the different response between Panel 5b and Panel 5c: for intermediate inputs changes in closer sectors there is an average negative effect, while changes in further sectors are positively related with varia-

[26] Differently from the first-stage, where I control common trends over time using aggregate variables, $\psi_\hbar(t+\hbar)$ removes the possibility of common trend in the *response* of sector-specific employment to idiosyncratic shocks; the possibility that employment responses of sector-s and sector-s' to sector-specific shocks are influenced by a common trend between the two is ruled out.

[27] The analysis does not leverage every distance value but instead categorizes sectors by their relative closeness in the network (in line with Theorem 2). Indeed, the *shortest path algorithm* constructs extensive margin distance matrices, to be then exploited to classify sectors accordingly.

tions in sectoral employment.[28] Under *factor input supply* network distances, analogous results are obtained (decrease for closer sectors, increase for further ones).[29] Such observations might suggest that certain types of intermediate inputs are more essential than others in production (*e.g.*, Carvalho and Voigtländer 2015), but it is actually the entity of economic distance between sectors what determines their importance both in the production of sectoral intermediate output and in their role of being amplifiers of idiosyncratic and sector-specific shocks.

4.2. Sectoral comovement of employment

This section analyses how a sector's employment shifts in response to employment changes in other sectors, depending on their network distance. To this aim, the previous identification and estimation scheme is developed, but now purging the dynamics of sector-specific employment with changes in its and other sectors' value-added, since employment is not exogenous to growth rates of sectoral output.[30]

Identification.– To identify exogenous variations in sectoral employment, $n_t(s)$, a panel fixed-effects regression over each sector $s \in \Phi(s)$ is estimated:

$$\begin{aligned} n_t(s) = \beta_y\, \dot{y}_t(s) \;+\; \sum_d \beta_{y(d)}\, \dot{y}_t(\Phi_s, d) \;+\; \sum_k \beta_{y(j)}\, \dot{y}_t(\Phi_s, k) \\ +\; \beta_{z(s)}\, \mathcal{Z}_t(s) \;+\; \beta_{\dot{Z}(s)}\, \dot{\mathcal{Z}}_t(s) \;+\; \beta_z\, \mathcal{Z}_t \;+\; \beta_{\dot{Z}}\, \dot{\mathcal{Z}}_t \;+\; \psi(s) \;+\; u_t^n(s) \end{aligned} \tag{17}$$

where $\dot{y}_t(s)$ is the value-added growth for sector-s, $\dot{y}_t(\Phi_s, d)$ identifies the value-added growth of its related sectors according to network distance, and $\dot{y}_t(\Phi_s, k)$ represents the value-added growth of its upstream and downstream sectors. Each measure is defined as that of eq. (14) but using $y(s)$ instead of employment. Moreover, for this specification, sector-level controls, $\mathcal{Z}_t(s)$ and $\dot{\mathcal{Z}}_t(s)$, comprise the labour force size of the sector and its value-added level, while aggregate controls, $\mathcal{Z}_t$ and $\dot{\mathcal{Z}}_t$, are the same of eq. (14). Just like the previous specification (Subsection 4.1), the coefficients to be estimated are the β's, and I only impose sector fixed-effects, $\psi(s)$.[31]

[28] These results align with the demand-based sufficient statistics in Corollary 1: for closely related sectors, changes in their intermediate inputs tend to generate an average negative effect on a given sector. This arises because a positive shock in one sector might raise the prices of shared upstream suppliers, which, through upstream price pass-through (τ^{fd}), dampens the input demand of other downstream sectors, illustrating how horizontal complementarities interact with vertical spillovers to shape sectoral responses.

[29] In Appendix D, Panel D.1a of Figure D.1, I show how that the same impact of Panel 5a still holds when network distances in eqs. (15)-(16) are identified under *factor input supply* distance relations. Results for distances' effects (Panels D.1b and D.1c) identify whiter but analogous results.

[30] Hornstein and Praschnik (1997) highlight positive comovement of output and employment across sectors (as strong cross-sectoral employment correlations support the observed comovement in value-added), driven by intermediate inputs in durable goods production. Similarly, Acemoglu, Akcigit, et al. (2015) examine how sectoral shocks propagate through production networks while resulting in output and employment comovement, and Sandqvist (2017) demonstrates that the strength of these linkages on comovement varies over time and intensifies in downturns. More recently, Barattieri, Cacciatore, and Traum (2023) find that targeted government spending benefits recipient and upstream sectors but reduces output and employment downstream.

[31] In both identifications I exclude time fixed-effect while controlling for variations in aggregate variables. Comovement between aggregate and sectoral variables is well established in the literature. Stock and Watson

Sector-specific estimated residual $\widehat{u}_t^n(s)$ from eq. (17) identifies the exogenous variation in sectoral employment. Standard errors are clustered at sector-level.

Local Projection analysis.– Sectoral comovement is defined as the response of employment in a given sector to changes in employment of other sectors. This issue can be addressed using Local Projections with the following form:

$$\dot{n}_{t+\hbar}^{cum}(s) = \beta_{\widehat{n}(d),\hbar} \sum_{s' \neq s} \widehat{u}_t^n(s',d) \; + \; \psi(t+\hbar) \; + \; \psi_\hbar(s) \; + \; \nu_{t+\hbar}^n(s,d) \tag{18}$$

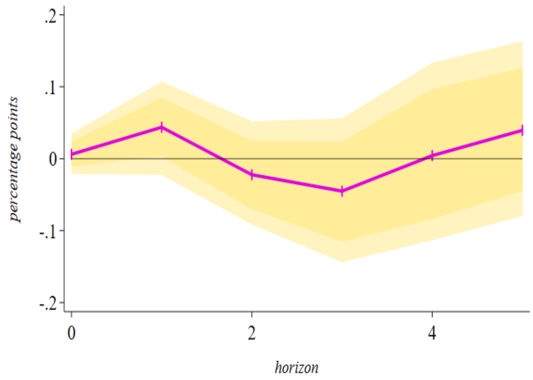
with $\dot{n}_{t+\hbar}^{cum}(s) = n_{t+\hbar}(s) - n_{t-1}(s)$ denoting the cumulative change in sector-s employment at each horizon given the identified employment shock in closer or further sectors at the d^{th}-distance. Coefficient $\beta_{\widehat{n}(d),\hbar}$ is the $\hbar$-step-ahed response of sector-specific employment cumulative difference due to identified shock $\widehat{u}_t^n(s',d)$. Still, sector and horizon fixed effects, $\psi_\hbar(s)$ and $\psi(t+\hbar)$ respectively, are imposed to remove unobserved heterogeneity and common trends across sectors in the comovement of sectoral employment. Prediction error term $\nu_{t+\hbar}^n(s,d)$ is horizon- and distance-specific in each predictive panel regression, and standard errors and bootstrapped Confidence Interval (CI) of $\beta_{\widehat{u}^x,\hbar}$ are again clustered by sectors.

Impulse responses from eq. (18) estimate employment comovement as the average cumulative response of sectoral employment to changes in employment in other sectors, distinguishing between both types of *factor input demand* and *factor input supply* distances. As a preview of the results, the outlined estimation ratifies the theoretical predictions of Proposition 3, as sectors purchasing intermediate inputs from common upstream suppliers do not necessarily exhibit positive comovement of employment levels: responses of closely linked sectors are muted or ambiguous (increase for some, decrease for some others), whereas more distance sectors tend to comove. Results also corroborate Proposition 4, in particular for sectors with the major number of Input-Output connections in the production network: when the distance is low, sectors selling their intermediate output as intermediate inputs to the same downstream buyers are characterized by an opposite comovement in employment levels while, when the distance is substantial, supply linkages among sectors will determine a positive comovement of sectoral employment. Henceforth, at short network distances horizontal propagation takes over, offsetting any positive comovement effect, and eclipsing the standard vertical transmission of idiosyncratic shocks.

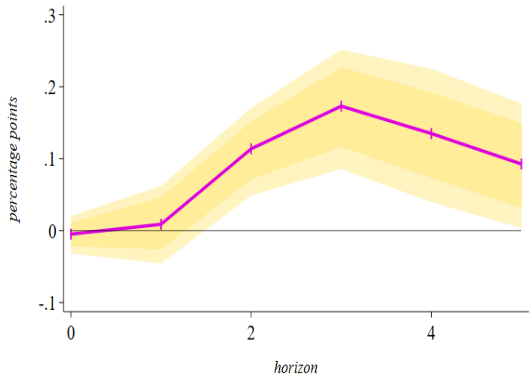
Results for factor input demand distances

Considering sectors whose distance is defined in terms of buying from the same set of upstream suppliers, estimating eq. (18) yields Figure 6. As already emphasized in Subsection 1.3, demand-based complementarities are inherently ambiguous to

(1999) use quarterly data over 1953:I-1996:IV period showing the positive association of the cyclical component of real GDP with aggregate employment and hours worked, and sectoral employment. Strong pro-cyclicality characterizes sectoral hours worked and employment, positively correlated with aggregate output in Huffman and Wynne (1999). As from Rebelo (2005), the correlation between aggregate and sectoral hours worked is .68 to .80. DiCecio (2009) emphasizes the comovement to be driven by sticky wages.



(A) $d = 1$



(B) $d = 2$

FIGURE 6: EMPLOYMENT COMOVEMENT UNDER DEMAND LINKAGES

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 6a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 6b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.

interpret, since they depend on the joint action of vertical propagation and horizontal spillovers. The sufficient-statistic conditions derived in Corollary 1 clarify this ambiguity: positive comovement arises only when the combined effect of upstream price pass-through (transmitted through the common supplier) and downstream substitutability across intermediate inputs works in the same direction. When downstream buyers can easily substitute inputs, an increase in employment in one sector strengthens demand for the shared supplier without crowding out the other sector, producing positive comovement. By contrast, when inputs are strong complements, a positive shock in one sector raises the supplier's price, eroding the competitiveness of the other sector and dampening comovement. Where these forces conflict or remain weak, observed correlations are muted or ambiguous.

These mechanisms echo in observed employment responses. Panels 6a and 6b show that a 1% increase in employment in closely linked sectors ($d = 1$) produces an ambiguous (statistically not significant) response, whereas more distant sectors ($d = 2$) consistently show positive comovement, reflecting horizontal propagation of demand interdependencies where sufficient conditions are satisfied.[32] Although a formal computation of the sufficient statistics is out of scope, interpreting the results through their lens helps clarify the role of horizontal linkages, disentangle them from overlapping vertical effects, and understand why sectors sharing common suppliers tend to move together in response to shocks.

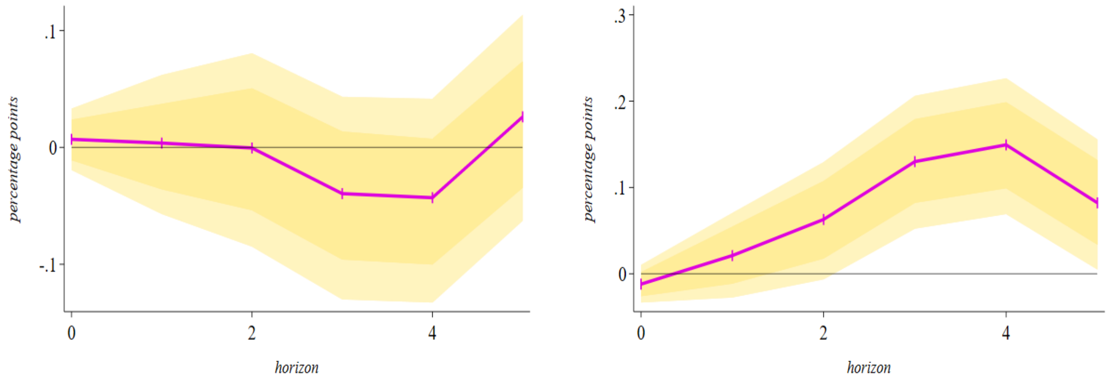
[32] Analogous conclusions emerge when sectors are partitioned into smaller subsets defined by their number of Input-Output linkages, whose responses (Panels D.2a and D.2b of Figure D.2, Appendix D) confirm that horizontal complementarities operate in tandem with the vertical propagation, overshadowing one another.

Observed comovement broadly aligns with the theory developed in Subsection 2.2 for factor input demand distance. Positive comovement is most likely to emerge when sectors are "economically" distant, since vertical effects dominate and the conflicting role of horizontal spillovers is attenuated. For closely connected sectors, however, the balance between vertical and horizontal channels is more delicate, producing the mixed responses observed in the data. This suggests that observed employment comovement mirrors an interplay of vertical and horizontal propagation (potentially moderated by factors such as price markups, sectoral technologies, and the endogenous configuration of upstream connections), thereby highlighting the inherent complexity in interpreting demand-based complementarities. In this sense, factor input demand distances reveal how horizontal geometry can either reinforce or challenge the standard vertical logic of propagation, thereby adding an essential layer of complexity to the interpretation of comovement in production networks.

Results for factor input supply distances

Considering now sectors whose distance is defined in terms of selling to the same set of downstream buyers, Figure 7 plots the responses from the estimation of eq. (18). Interpreting the effect of further ($d = 2$) sectors' changes in employment yields the same results of the factor input demand case: from Panel 7b, a 1% increase in their employment increases sectoral employment as well, thus determining positive comovement and a positive transmission of the sector-idiosyncratic shock. Different is the case of closer ($d = 1$) sectors, where it appears a positive and then a negative cumulative effect on sector-specific employment under a small impact magnitude.

However, sectoral shocks to employment levels shown in Panel 7a do not produce any statistically significant effects, as the estimated responses fail to reach significance at both the 90% and 68% confidence levels. Two complementary interpretations: *(i)* changes in employment in nearby sectors have no measurable impact on others when the sectors are linked through a common set of downstream buyers; and *(ii)* the factor input supply network distance seems to dampen the vertical propagation of shocks across the production network, thereby weakening the expected comovement in production inputs across sectors. As discussed in Subsection 2.2, this attenuation effect arises due to what can be interpreted as an intermediate output effect, $y(s)$. Specifically, when sectors share the same downstream buyers, a change in the relative revenues received from the common buyer leads to opposite changes in their relative supply of intermediate inputs (Lemma 2). This generates an inverse comovement in intermediate outputs and, consequently, an opposite comovement in sectoral employment levels. In the context of Panel 7a, these opposing forces roughly cancel each other out when averaged across all sectors, resulting in the non-significant response. However, it is possible that sectors with fewer inter-sectoral linkages introduce noise or bias to the estimation, masking the true magnitude. In other words, the lack of a statistically significant effect in this panel may be driven by including all sectors indiscriminately, which dilutes the measurable comovement



(A) $d = 1$ (B) $d = 2$

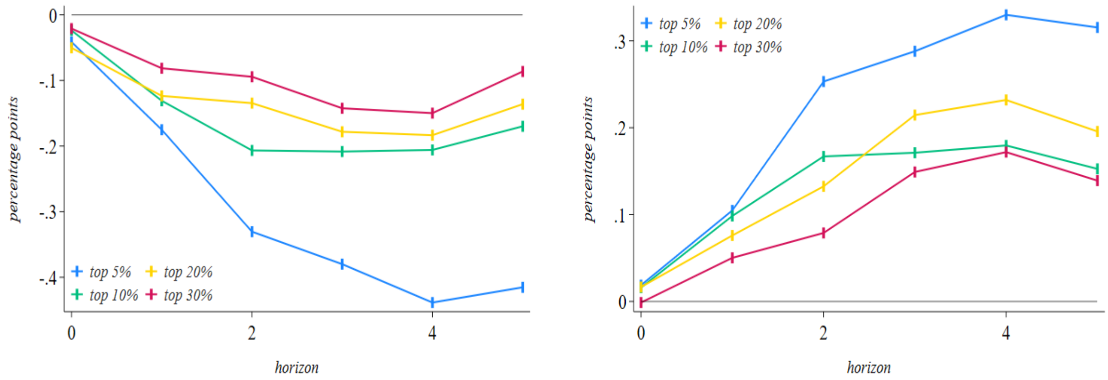
FIGURE 7: EMPLOYMENT COMOVEMENT UNDER SUPPLY LINKAGES

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 7a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 7b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.

in sectoral employment: restricting the sample to sectors with most linkages could reveal more meaningful propagation effects, in line with Subsection 1.1.

Given these considerations, the performed analysis is also conducted focusing exclusively on highly interlinked sectors – *i.e.*, those sectors with the highest number of Input-Output linkages within the considered production network. By concentrating on this subset, I aim to better capture the dynamics of employment propagation where inter-sectoral connections are strongest and thus where network effects are more pronounced. Impulse responses for this analysis are presented in Figure 8. The results for major network distances, shown in Panel 8b, largely mirror those found using the full sample, but with a notably higher magnitude in the point estimates responding to the structural employment shock. This suggests that shocks in highly interconnected sectors generate stronger propagation effects through the production network, amplifying the employment response among these sectors.

More strikingly, the results under minor supply distances, depicted in Panel 8a, fully confirm Proposition 4: any positive increase in employment levels within closely linked sectors induces a negative transmission effect on employment in other nearby sectors, resulting in an opposite comovement in sectoral employment levels. This inverse relationship highlights the counter-intuitive dynamics of factor input supply distance networks, where shocks do not simply propagate in a uniform positive manner but can generate divergent employment responses depending on network proximity and the structure of interdependencies. Importantly, this negative comovement effect grows stronger as the analysis narrows to an even smaller group of sectors with increasingly dense linkages: the closer the sectors are in the pro-



(A) $d = 1$ (B) $d = 2$

FIGURE 8: SUPPLY LINKAGES AND COMOVEMENT IN INTERLINKED SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 8a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 8b plots the response to employment changes in further (distance equal to 2) ones. Each line corresponds to the average response of employment across sectors with different numbers of linkages, robust to 68% and 90% significance levels of bootstrapped Confidence Interval (CI) computed from eq. (18). *Source*: BEA and own calculations.

duction network (under supply-based horizontal relationships) the more pronounced these opposing employment responses become. In sum, as in Subsection 1.1, restricting the analysis to highly interlinked sectors provides a clearer picture of how employment shocks travel through the production network, revealing that network structure and horizontal proximity fundamentally condition either the direction and the strength of sectoral comovement in employment levels.

Interpreting these results through the sufficient-statistic lens of Corollary 2 clarifies when negative comovement arises: a positive shock to a sector reduces intermediate output in other closely linked sectors via shared downstream buyers. Yet, the data show that such negative comovement can occur even when the formal sufficient conditions are only partially satisfied, reflecting the combined influence of vertical propagation and supply-driven horizontal interactions.

Summary.– The theoretical insights of Sections 1 and 2 find empirical support. Sectoral employment shocks propagate differently depending on the types of "economic" distance in the production network. For factor input demand distances, sector-specific changes in employment generate positive comovement in other sectors' employment when far apart, though responses for closely linked sectors remain ambiguous and muted due to the overlapping interplay of vertical and horizontal propagation through shared upstream suppliers. By contrast, supply-driven distances tend to produce negative comovement among closely linked sectors, effectively counteracting the standard vertical propagation mechanism and illustrating how competition for common downstream buyers can dampen or reverse the expected positive transmission. While minor demand-based distances attenuate vertical supply chain mechanisms, these findings underscore how it is the factor input supply distance that majorly disrupts standard vertical propagation: shocks in closely linked sectors often produce opposite employment responses, as competition for shared downstream buyers counteracts the usual (and even) transmission along the production network. Moreover, they ratifies the notion that as sectors are increasingly connected, horizontal complementarities matter the most.

4.3. Robustness

Results on comovement hold along different "economically-based" robustness checks, sequentially presented in Appendix D. Initially, I shift the focus towards peripheral sectors within the production network. Much of the existing literature holds on the role of central sectors, typically measured using Bonacich-Katz centrality (*e.g.*, Carvalho 2014). The underlying premise is that a positive shock in a central sector transmits outward, potentially inducing broader economic expansions – *i.e.*, microeconomic shocks can scale up to macroeconomic consequences through network centrality. From a comovement perspective, periods of economic expansion (contraction) are expected only when a positive (negative) shock in a central sector effectively propagates to more peripheral sectors: fluctuating aggregate activity is not merely a direct realization of shocks to central nodes, but rather it stems from the manner in which such asymmetric shocks ripple across the network to other, more peripheral sectors.[33] If network distances were irrelevant, comovement would be expected universally. However, should demand- and supply-side linkages prove significant, the comovement patterns ought to remain consistent with those documented thus far.

I then test the robustness of the presented findings by employing alternative base years for the U.S. production network. My main analysis is anchored at the 2007 Input-Output structure, which conveniently divides the sample period into two equal sub-periods. Theoretically, the results should remain valid provided that the production network structure exhibits limited evolution over time.[34] Empirically, I replicate the full set of analyses using network data from other benchmark years.

Centrality scores.– Figures D.7 and D.8 present the impulse responses of employment to factor input demand and supply distances, respectively, considering only the peripheral sectors of the U.S. production network. Results are broadly consistent with those discussed in earlier sections: opposite comovement under supply distances, and positive comovement for shocks originated in further sectors. The only noteworthy exception concerns the response of employment to changes in closer sectors under demand-based distances: rather than exhibiting the ambiguity observed for all sectors (overlapping of vertical and horizontal dimensions), inspect-

[33] Stated differently, it is not the initial shock itself that generates business cycle fluctuations, but the subsequent consequences it produces as it transmits across the network to less central nodes.

[34] A condition broadly supported in the empirical literature. For instance, Carvalho (2014) and Acemoglu, Ozdaglar, et al. (2016) document relative stability of U.S. sectoral production network and its role on shock propagation. Classical Input-Output analysis (Leontief 1986, Miller and Blair 2009) also emphasises the persistence of production linkages in the short to medium run.

ing horizontal complementarities in demand for more peripheral sectors generate effects that resemble those observed under supply-based distances, with negative comovement occurring across sectors sharing the same network structure in terms of buying from common suppliers. This refines the observed pattern under direct demand-based measures of Figure 6, suggesting that negative employment comovement among sectors purchasing from similar upstream suppliers largely occurs in sectors not playing a key role in the propagation of shocks.[35]

Different base years.– All the documented results, derived using the 2007 U.S. tables, remain robust if replicated for alternative benchmark years (2002 and 2012), as depicted in Figures D.9, D.10, and D.11. This temporal consistency suggests that the structural features of the U.S. production network have remained remarkably stable over time, in line with the academic debate. Such outcomes document the enduring nature of Input-Output linkages and the slow-moving evolution of production networks, thus lending further credibility to the proposed horizontal mechanisms.

5. Future Policy Perspectives

The Input-Output structure of the economy shapes how shocks propagate and contributes to business cycle dynamics. This paper has explored how the structure of production networks shapes sectoral comovement, moving beyond the traditional focus on vertical transmission to emphasize the role of horizontal economic relationships. By analysing both the demand- and supply-driven dimensions of sectoral interconnections, the analysis reveals that shocks do not propagate solely along upstream or downstream chains. Instead, sectors that share common suppliers or buyers experience intertwined vertical and horizontal effects. Overall, sectoral comovement arises not only from direct or indirect vertical production network linkages, but also from horizontal distances defined by shared inter-sectoral trade structure. By highlighting this additional dimension, the paper offers a new perspective on how idiosyncratic shocks propagate in networked economies: it is not merely the existence of Input-Output connections, but rather the demand and supply geometry of these linkages that fundamentally governs the transmission of micro-originated shocks and the resulting macroeconomic comovement patterns. Several are the contributions that horizontal complementarities can provide to the ongoing literature exploiting a production network perspective to form and deliver policy prescriptions.[36]

[35] In line with the discussion in Baqaee and Farhi (2019): within a sector, complementarities ("horizontal" in my framework) across intermediate inputs attenuate the aggregate effects of a positive idiosyncratic shock.

[36] On the theory side, horizontal complementarities can provide valuable insights into: *(i)* the functioning of global production networks (*e.g.*, Caliendo et al. 2022, Huo et al. 2025), extending beyond merely vertical supply chains; *(ii)* the complexity of firm-to-firm production networks (*e.g.*, di Giovanni et al. 2018, Boehm et al. 2019), as demand- and supply-based distance networks help to reveal relationships between nodes that would otherwise appear unconnected; *(iii)* the endogenous formation of production networks, since "economically" closer sectors tend to share similar Input-Output structures and might rely on common intermediate inputs that are more essential than others (*e.g.*, Carvalho and Voigtländer 2015); and *(iv)* the contribution of sectoral shocks to aggregate outcomes (*e.g.*, Acemoglu, Carvalho, et al. 2012, Atalay 2017, Baqaee and Farhi 2019).

Fiscal policy.– A positive government-spending shock works through an increase in its demand of goods and services; affected sectors thus face an increase in their supply, stimulating an expansion of sectoral output. Enlarged production that meets the increased demand from the government then spread through Input-Output linkages as sectors rely on intermediate inputs sourced from their suppliers. In this regard, a micro-originated shock generates upstream effects (*e.g.*, Acemoglu, Akcigit, et al. 2015, Barattieri, Cacciatore, and Traum 2023). It is thus essential the horizontal dimension of a network since changes in demand and supply relationships – neglected in standard analysis – are at the core of mechanism triggered by an increased government demand. Moreover, horizontal effects play a critical role when considering the specific composition of public expenditure since certain sectors may be disproportionately exposed to changes in demand:[37] while the vertical transmission mechanism only depends on the intensity of inter-sectoral trade, incorporating measures of demand- and supply-driven network distances allows for a more accurate weighting in the propagation of sectoral shocks. Tariff policies operate through similar network mechanisms, not by raising demand directly, but rather altering relative prices (*e.g.*, Barattieri and Cacciatore 2023, Antonova et al. 2025, Clausing and Obstfeld 2025), effectively acting as revenue reallocation shocks, effectively lending further perspectives from network's horizontal geometries.

Industrial policy.– Two contemporaneous papers analyse how production networks amplify the impact of industrial policies supporting strategic sectors to promote sustained economic growth: essentials are the notions of "distortionary effects" (market imperfections accumulate through backward demand linkages; Liu 2019) and "downstream spillovers" (positive effects on buyers of targeted sectors; Lane 2025), and both perspectives exploit the verticality of sectoral connections.[38] Yet, how do industrial policies affect untargeted sectors exhibiting similar Input-Output structures to the targeted ones? This is where the horizontal dimension matters.

Transmission mechanism.– Complementarities (either in consumption or production) might turn an asymmetric supply shock, affecting a subset of sectors and reducing their demand for other sectors, into a demand-like shock at the aggregate level (the "Keynesian supply shock"; see Guerrieri et al. 2022). Production network is a natural field to study this mechanism (*e.g.*, Cesa-Bianchi and Ferrero 2021) and, reminiscent from Subsection 1.1 (Figure 2b), horizontal complementarities among sectors can help to clearly, easily and linearly identify demand and supply forces within the network that guide the Keynesian transmission. This extended mechanism is important for interventions aimed at stabilizing business cycles, acting as

[37] For example, a large-scale infrastructure program would boost demand for construction, cement, and steel sectors, in turn strongly engaging with closely connected suppliers (such as mining, heavy machinery manufacturing, and architectural or engineering services). By contrast, weakly connected sectors with core infrastructure-related sectors (such as apparel manufacturing, publishing, residential cleaning services, or artisanal food production) would be only marginally affected.

[38] Explanations related to cross-country income differences, Input-Output economies and industrialization are presented in Ciccone (2002) and Fadinger et al. (2022). Conley and Ligon (2002) study how cross-country dependence and GDP growth rates are shaped by "economic distance".

a suitable extension of the production network framework for monetary policy (*e.g.*, Pasten et al. 2020, Kalemli-Özcan et al. 2025).

Finally, by embedding sectoral similarities through common structures of Input-Output linkages, the horizontal dimension transcends the confines of production networks rendering it applicable to a broader class of economic interdependencies, including studies analysing investment (*e.g.*, vom Lehn and Winberry 2022) or financial (*e.g.*, Acemoglu, Ozdaglar, et al. 2015, Huremovic et al. 2025) networks.

It is along these avenues that I intend to orient my future research agenda.

Concluding Remarks

Initial inquiries into production networks chiefly centred on how disaggregation induced by inter-sectoral trade generates comovement of macroeconomic variables. Reviving this foundational perspective, this paper moves beyond traditional vertical linkages by theoretically introducing and empirically validating the horizontal dimension of Input-Output economies, measured through network "economic" distances reflecting similarities between sectors in terms of common upstream suppliers or downstream buyers. Constitutive premise is that sectors are densely interconnected not only by their direct trade flows but also through shared inter-sectoral relationships, providing a richer understanding of the complex architecture underlying any capitalist economy. Demand-based distances promote an overlap of horizontal and vertical propagation, as shocks transmitted through shared upstream suppliers interact with vertical supply-chain effects. By contrast, supply-based distances directly threaten the design of vertical transmission, as revenue reallocation among sectors selling to the same downstream buyers counteracts the standard vertical propagation of shocks. These mechanisms operate independently of inter-sectoral trade intensities, offering a powerful refinement to prevailing network analysis. By conceptualising horizontal complementarities, which linearly encode non-linear intermediate input complementarities, the paper enriches the understanding of how independent and idiosyncratic shocks propagate from one sector to another. Ultimately, it reveals that the demand/supply geometry of an Input-Output structure and its horizontal dimension critically shape sectoral shocks' transmission and comovement, offering novel insights into the networked nature of economic activity.

References

Acemoglu, D., U. Akcigit, and W. Kerr, "Networks and the macroeconomy: an empirical exploration", *NBER Macroeconomics Annual*, vol. 30 (2015), pp. 273-335. 5, 35, 43.

Acemoglu, D. and P. D. Azar, "Endogenous production networks", *Econometrica*, vol. 88, no. 1 (2020), pp. 33-82. 4.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi, "The network origins of aggregate fluctuations", *Econometrica*, vol. 80, no. 5 (2012), pp. 1977-2016. 3, 4, 9, 18, 23, 28, 42.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi, "Networks, shocks, and systemic risk", in: *The Oxford Handbook of the Economics of Networks*, pp. 569-608, edited by Yann Bramoullé, Andrea Galeotti, Brian W. Rogers, Oxford University Press 2016. 41.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi, "Systemic risk and stability in financial networks", *American Economic Review*, vol. 105, no. 2 (2015), pp. 564-608. 44.

Antonova, A., L. Huxel, M. Matvieiev, and G. Müller, "The propagation of tariff shocks via production networks", *CEPR Discussion Paper No. 20305* (2025). 43.

Atalay, E., "How important are sectoral shocks?", *American Economic Journal: Macroeconomics*, vol. 9, no. 4 (2017), pp. 254-280. 1, 42.

Atalay, E., A. Hortaçsu, J. Roberts, and C. Syverson, "Network structure of production", *Proceedings of the National Academy of Sciences (PNAS)*, vol. 108, no. 13 (2011), pp. 5199-5202. 17.

Bacilieri, A., A. Borsos, P. Astudlo-Estevez, M. Hoefer, and F. Lafond, "Firm-level production networks: what do we (really) know?", *INET Oxford Working Paper Series No. 2025-14* (2025). 17.

Baqaee, D. R., "Cascading failures in production networks", *Econometrica*, vol. 86, no. 5 (2018), pp. 1819-1838. 4.

Baqaee, D. R. and E. Farhi, "The macroeconomic impact of microeconomic shocks: beyond Hulten's Theorem", *Econometrica*, vol. 87, no. 4 (2019), pp. 1155-1203. 1, 3, 4, 15, 42.

— "Productivity and misallocation in general equilibrium", *Quarterly Journal of Economics*, vol. 135, no. 1 (2020), pp. 105-163. 4.

Barattieri, A. and M. Cacciatore, "Self-harming trade policy? Protectionism and production networks", *American Economic Journal: Macroeconomics*, vol. 15, no. 2 (2023), pp. 97-128. 4, 5, 31, 33, 43.

Barattieri, A., M. Cacciatore, and N. Traum, "Estimating the effects of government spending through the production network", *NBER Working Paper n. 31680* (2023). 5, 35, 43.

BARROT, J.-N., B. GRASSI, and J. SAUVAGNAT, "Sectoral effects of social distancing", *American Economic Association: Papers & Proceedings*, vol. 111 (2021), pp. 277-281. 2.

BARROT, J.-N. and J. SAUVAGNAT, "Input specificity and the propagation of idiosyncratic shocks in production networks", *Quarterly Journal of Economics*, vol. 131, no. 3 (2016), pp. 1543-1592. 4, 5.

BOEHM, C. E., A. FLAAEN, and N. PANDALAI-NAYAR, "Input linkages and the transmission of shocks: firm-level evidence from the 2011 Tōhoku earthquake", *Review of Economics and Statistics*, vol. 101, no. 1 (2019), pp. 60-75. 42.

BOLDRIN, M., L. J. CHRISTIANO, and J. D. M. FISHER, "Habit persistence, asset returns, and the business cycle", *American Economic Review*, vol. 91, no. 1 (2001), pp. 149-166. 5.

CALIENDO, L., F. PARRO, and A. TSYVINSKI, "Distortions and the structure of the world economy", *American Economic Journal: Macroeconomics*, vol. 14, no. 4 (2022), pp. 274-308. 4, 42.

CARVALHO, V. M., "From micro to macro via production networks", *Journal of Economic Perspectives*, vol. 28, no. 4 (2014), pp. 23-48. 4, 28, 41.

CARVALHO, V. M. and X. GABAIX, "The Great Diversification and its undoing", *American Economic Review*, vol. 103, no. 5 (2013), pp. 1697-1727. 4.

CARVALHO, V. M. and N. VOIGTLÄNDER, "Input diffusion and the evolution of production networks", *NBER Working Paper n. 20025* (2015). 35, 42.

CASSOU, S. P. and J. VÁZQUEZ, "Employment comovements at the sectoral level over the business cycle", *Empirical Economics*, vol. 46 (2014), pp. 1301-1323. 5.

CESA-BIANCHI, A. and A. FERRERO, "The transmission of Keynesian supply shocks", *CEPR Discussion Paper n. 16430* (2021). 43.

CHOI, J. and A. T. FOERSTER, "The changing Input-Output network structure of the U.S. economy", *Economic Review QII, Federal Reserve Bank of Kansas City* (2017), pp. 23-49. 17.

CHRISTIANO, L. J. and T. J. FITZGERALD, "The business cycle: it's still a puzzle", *Economic Perspectives*, vol. 22, no. 4 (1998), pp. 56-83. 2, 5.

CICCONE, A., "Input chains and industrialization", *Review of Economic Studies*, vol. 69, no. 3 (2002), pp. 565-587. 43.

CLAUSING, K. and M. OBSTFELD, "Tariffs as fiscal policy", *CEPR Discussion Paper No. 20677* (2025). 43.

CONLEY, T. G. and B. DUPOR, "A spatial analysis of sectoral complementarity", *Journal of Political Economy*, vol. 11, no. 2 (2003), pp. 311-352. 1, 2, 7, 28.

CONLEY, T. G. and E. LIGON, "Economic distance and cross-country spillovers", *Journal of Economic Growth*, vol. 7 (2002), pp. 157-187. 43.

COOPER, R. and J. HALTIWANGER, "Inventories and the propagation of sectoral shocks", *American Economic Review*, vol. 80, no. 1 (1990), pp. 170-190. 2, 5.

CORSETTI, G., L. DEDOLA, and S. LEDUC, "International risk sharing and the transmission of productivity shocks", *Review of Economic Studies*, vol. 75, no. 2 (2008), pp. 443-473. 1.

DI GIOVANNI, J., A. A. LEVCHENKO, and I. MEJEAN, "The micro origins of international business-cycle comovement", *American Economic Review*, vol. 108, no. 1 (2018), pp. 82-108. 42.

DICECIO, R., "Sticky wages and sectoral labor comovement", *Journal of Economic Dynamics and Control*, vol. 33 (2009), pp. 538-553. 36.

DIJKSTRA, E. W., "A note on two problems in connexion with graphs", *Numerische Mathematik*, vol. 1 (1959), pp. 269-271. 28.

FADINGER, H., C. GHIGLINO, and M. TETERYATNIKOVA, "Income differences, productivity, and Input-Output networks", *American Economic Journal: Macroeconomics*, vol. 14, no. 2 (2022), pp. 367-415. 43.

FED, "The interconnectivity of production networks", *Podcast, Timely Topics*, no. Q2 (2019). 17.

FERRARI, A., "Inventories, demand shocks propagation and amplification in supply chains", *CEPR Discussion Paper 19716* (2024). 5.

FLOYD, R. W., "Algorithm 97: shortest path", *Communications of the ACM*, vol. 5, no. 6 (1962), p. 345. 28.

FOERSTER, A. T., P.-D. G. SARTE, and M. W. WATSON, "Sectoral versus aggregate shocks: a structural factor analysis of industrial production", *Journal of Political Economy*, vol. 119, no. 1 (2011), pp. 1-38. 2.

GABAIX, X., "The granular origins of aggregate fluctuations", *Econometrica*, vol. 79, no. 3 (2011), pp. 733-772. 2, 4.

GARIN, J., M. J. PRIES, and E. R. SIMS, "The relative importance of aggregate and sectoral shocks and the changing nature of economic fluctuations", *American Economic Journal: Macroeconomics*, vol. 10, no. 1 (2018), pp. 119-148. 2.

GHASSIBE, M., "Monetary policy and production networks: an empirical investigation", *Journal of Monetary Economics*, vol. 119 (2021), pp. 21-39. 5.

— "Endogenous production networks and non-linear monetary transmission", *Working Paper* (2024). 4.

GHASSIBE, M. and A. NAKOV, "Business cycles with pricing cascades", *CEPR Discussion Paper No. 20605* (2025). 4.

GRASSI, B., "IO in I-O: size, industrial organization, and the Input-Output network make a firm structurally important", *Working Papers 619, IGIER, Bocconi University* (2017). 4.

GRASSI, B. and J. SAUVAGNAT, "Production networks and economic policy", *Oxford Review of Economic Policy*, vol. 35, no. 4 (2019), pp. 638-677. 4.

GUALDI, S. and A. MANDEL, "Endogenous growth in production networks", *Journal of Evolutionary Economics*, vol. 29 (2019), pp. 91-117. 4.

GUERRIERI, V., G. LORENZONI, L. STRAUB, and I. WERNING, “Macroeconomic implications of COVID-19: can negative supply shocks cause demand shortages?”, *American Economic Review*, vol. 112, no. 5 (2022), pp. 1437-1474. 9, 43.

HAUSMANN, R. and C. A. HIDALGO, “The network structure of economic output”, *Journal of Economic Growth*, vol. 16 (2011), pp. 309-342. 4.

HICKS, J. R., *The theory of wages*, Palgrave Macmillan, London 1932. 6.

HORNSTEIN, A. and J. PRASCHNIK, “Intermediate inputs and sectoral comovement in the business cycle”, *Journal of Monetary Economics*, vol. 40, no. 3 (1997), pp. 573-595. 2, 35.

HUFFMAN, G. W. and M. A. WYNNE, “The role of intratemporal adjustment costs in a multisector economy”, *Journal of Monetary Economics*, vol. 43, no. 2 (1999), pp. 317-350. 36.

HULTEN, C. R., “Growth accounting with intermediate inputs”, *Review of Economic Studies*, vol. 45, no. 3 (1978), pp. 511-518. 20.

HUO, Z., A. A. LEVCHENKO, and N. PANDALAI-NAYAR, “International comovement in the global production network”, *Review of Economic Studies*, vol. 92, no. 1 (2025), pp. 365-403. 4, 20, 42.

HUREMOVIC, K., G. JIMENEZ, E. MORAL-BENITO, J.-L. PEYDRO, and F. VEGA-REDONDO, “Production and financial networks in interplay”, *Working Paper* (2025). 44.

JONES, C. I., “Intermediate goods and weak links in the theory of economic development”, *American Economic Journal: Macroeconomics*, vol. 3, no. 2 (2011), pp. 1-28. 4.

JORDÀ, Ò., “Estimation and inference of impulse responses by Local Projections”, *American Economic Review*, vol. 95, no. 1 (2005), pp. 161-182. 32.

KALEMLI-ÖZCAN, C. SOYLU, and M. A. YILDIRIM, “Global networks, monetary policy and trade”, *NBER Working Paper n. 33686* (2025). 44.

KIM, B.-G., “Sectoral shifts and comovements in employment”, *Economics Letters*, vol. 192 (2020), p. 109208. 5.

LA’O, J. and A. TAHBAZ-SALEHI, “Optimal monetary policy in production networks”, *Econometrica*, vol. 90, no. 3 (2022), pp. 1295-1336. 4.

LANE, N., “Manufacturing revolutions: industrial policy and industrialization in South Korea”, *Quarterly Journal of Economics*, vol. 140, no. 3 (2025), pp. 1683-1741. 4, 43.

LEONTIEF, W. W., “Quantitative input and output relations in the economic systems of the United States”, *Review of Economics and Statistics*, vol. 18, no. 3 (1936), pp. 105-125. 5, 7.

— *Input-Output Economics*, Oxford University Press, 2nd edition 1986. 41.

LIU, E., “Industrial policies in production networks”, *Quarterly Journal of Economics*, vol. 134, no. 4 (2019), pp. 1883-1948. 4, 43.

LIU, E. and A. TSYVINSKI, “A dynamic model of input-output networks”, *Review of Economic Studies*, vol. 91, no. 6 (2024), pp. 3608-3644. 4.

LONG, J. B. and C. I. PLOSSER, “Real business cycles”, *Journal of Political Economy*, vol. 91, no. 1 (1983), pp. 39-69. 3, 4, 18.

— “Sectoral vs. aggregate shocks in the business cycle”, *American Economic Association: Papers & Proceedings*, vol. 77, no. 2 (1987), pp. 333-336. 2.

LUCAS, R. E., “Understanding business cycles”, *Carnegie-Rochester Conference Series on Public Policy*, vol. 5 (1977), pp. 7-29. 9.

MCNERNEY, J., C. SAVOIE, F. CARAVELLI, V. M. CARVALHO, and D. J. FARMER, “How production networks amplify economic growth”, *Proceedings of the National Academy of Sciences (PNAS)*, vol. 119, no. 1 (2022), e2106031118. 4.

MILLER, R. E. and P. D. BLAIR, *Input-Output Analysis: Foundations and Extensions*, Cambridge University Press, 2nd edition 2009. 41.

MONTI, F. and L. VAN KEIRSBILCK, “The transmission of shocks across sectors and the dynamics of sectoral prices”, *CEPR Discussion Paper No. 20621* (2025). 5.

MORO, A., “The structural transformation between manufacturing and services and the decline in the US GDP volatility”, *Review of Economic Dynamics*, vol. 15, no. 3 (2012), pp. 402-415. 2.

MUNGO, L., F. LAFOND, P. ASTUDILLO-ESTÉVEZ, and D. J. FARMER, “Reconstructing production networks using machine learning”, *Journal of Economic Dynamics and Control*, vol. 148 (2023), p. 104607. 17.

OBERFIELD, E., “A theory of Input-Output architecture”, *Econometrica*, vol. 86, no. 2 (2018), pp. 559-589. 4.

PASINETTI, L. L., “The notion of vertical integration in economic analysis”, *Metroeconomica*, vol. 25, no. 1 (1973), pp. 1-29. 1.

PASTEN, E., R. SCHOENLE, and M. WEBER, “The propagation of monetary policy shocks in a heterogeneous production economy”, *Journal of Monetary Economics*, vol. 116 (2020), pp. 1-22. 44.

PICHLER, A., C. DIEM, A. BRINTRUP, F. LAFOND, G. MAGERMAN, G. BUITEN, T. CHOI, V. M. CARVALHO, J. D. FARMER, and S. THURNER, “Building an alliance to map global supply networks”, *Science*, vol. 382, no. 6668 (2023), pp. 270-272. 17.

QIU, Z., Y. W. WANG, L. XU, and F. ZANETTI, “Monetary policy in open economies with production networks”, *CEPR Discussion Paper n. 19828* (2025). 4.

QUAH, D. and T. J. SARGENT, “A dynamic index model for large cross sections”, IN: *Business Cycles, Indicators and Forecasting*, pp. 285-310, edited by James H. Stock and Mark W. Watson, University of Chicago Press 1993. 2.

REBELO, S., “Real Business Cycle models: past, present and future”, *The Scandinavian Journal of Economics*, vol. 107, no. 2 (2005), pp. 217-238. 2, 36.

ROGERSON, R., “An equilibrium model of sectoral reallocations”, *Journal of Politcal Economy*, vol. 95, no. 4 (1987), pp. 824-834. 5.

ROGERSON, R., "Indivisible labor, lotteries and equilibrium", *Journal of Monetary Economics*, vol. 21, no. 1 (1988), pp. 3-16. 18.

ROY, B., "Transitivité et connexité", *Comptes rendus de l'Académie des Sciences*, vol. 249 (1959), pp. 216-218. 28.

RUBBO, E., "Networks, Phillips curves, and monetary policy", *Econometrica*, vol. 91, no. 4 (2023), pp. 1417-1455. 4.

SANDQVIST, A. P., "Dynamics of sectoral business cycle comovement", *Applied Economics*, vol. 49, no. 47 (2017), pp. 4742-4759. 35.

SHEA, J., "Complementarities and comovements", *Journal of Money, Credit and Banking*, vol. 34, no. 2 (2002), pp. 412-433. 1, 5, 25.

SLATER, P. B., "The determination of groups of functionally integrated industries in the United States using a 1967 interindustry flow table", *Empirical Economics*, vol. 2, no. 1 (1977), pp. 1-9. 17.

— "The network structure of the United States input-output table", *Empirical Economics*, vol. 3, no. 1 (1978), pp. 49-70. 17.

STOCK, J. H. and M. W. WATSON, "Business cycle fluctuations in us macroeconomic time series", IN: *Handbook of Macroeconomics*, vol. 1, pt. A, pp. 3-64, edited by John B. Taylor and Michael Woodford, North-Holland 1999. 2, 35.

TASCHEREAU-DUMOUCHEL, M., "Cascades and fluctuations in an economy with an endogenous production network", *Review of Economic Studies*, vol. forthcoming (2025). 4.

VOM LEHN, C. and T. WINBERRY, "The investment network, sectoral comovement, and the changing U.S. business cycle", *Quarterly Journal of Economics*, vol. 137, no. 1 (2022), pp. 387-433. 4, 20, 44.

WARSHALL, S., "A theorem on Boolean matrices", *Journal of the ACM*, vol. 9, no. 1 (1962), pp. 11-12. 28.

XU, M., B. R. ALLENBY, and J. C. CRITTENDEN, "Interconnectedness and resilience of the U.S. economy", *Advances in Complex Systems*, vol. 14, no. 5 (2011). 17.

YEDID-LEVI, Y., "Why does employment in all major sectors move together over the business cycle?", *Review of Economic Dynamics*, vol. 22 (2016), pp. 131-156. 2, 5, 18.

Appendix

(Outline) *In this appendix I report all the material complementary to the main text; it is made of additional tables and figures, and of discussions on further analytical results. Section A deepens further all the theoretical findings; in particular, Subsection A.1 derives all the salient elements of the partial equilibrium results on production network distances of Section 1; Subsection A.2 derives all the characterizing features of the Input-Output general equilibrium model outlined in Section 2, whose propositions are extended in Subsection A.3, and proved in Subsection A.4. Section B proves and provides intuition on the sufficient statistics for sectoral comovement in Subsection 1.3. Section C consolidates and further discusses the data presented in Section 3. Finally, Section D complements and enriches the exposition of the empirical findings in Section 4. Part of this appendix is not intended for publication purposes.*

A. Theoretical results and model derivation

A.1. Theoretical results for network distances

(Proof of Lemma 1) *Consider a Cobb-Douglas economy as stated in Section 2. The combination of optimal demand for both labour and intermediate inputs would yield*

$$x\left(s,s'\right)=\alpha\left(s,s'\right)w\left(s\right)n\left(s\right)\ \alpha\left(s\right)^{-1}\frac{1}{p\left(s'\right)}$$

Then, consider the case in which two sectors $\{s,s'\}$ are buying their own intermediate inputs from the same sector-s^*. Associated system of equations is

$$\begin{cases} x\left(s,s^*\right)=\alpha\left(s,s^*\right)w\left(s\right)n\left(s\right)\ \alpha\left(s\right)^{-1}\dfrac{1}{p\left(s^*\right)} \\ x\left(s',s^*\right)=\alpha\left(s',s^*\right)w\left(s'\right)n\left(s'\right)\ \alpha\left(s'\right)^{-1}\dfrac{1}{p\left(s^*\right)} \end{cases}$$

$$\longrightarrow\quad \begin{cases} x\left(s,s^*\right)=\alpha\left(s,s^*\right)\left[x\left(s',s^*\right)\dfrac{1}{\alpha\left(s',s^*\right)n\left(s'\right)}\alpha\, p\left(s^*\right)\right]n\left(s\right)\ \alpha^{-1}\dfrac{1}{p\left(s^*\right)} \\ w=x\left(s',s^*\right)\dfrac{1}{\alpha\left(s',s^*\right)n\left(s'\right)}\alpha\, p\left(s^*\right) \end{cases}$$

where I exploit the fact that wage is the numeraire of the economy, so that $w\left(s\right)=w$, $\forall s\in\Phi\left(s\right)$, and, without any loss of generality (to simplify notation and build better intuition), $\alpha\left(s\right)=\alpha$, $\forall s\in\Phi\left(s\right)$ as well. The above substituted equation will result in

$$\frac{x\left(s,s^*\right)}{x\left(s',s^*\right)}=\frac{\alpha\left(s,s^*\right)}{\alpha\left(s',s^*\right)}\frac{n\left(s\right)}{n\left(s'\right)}\quad\equiv\quad\frac{n\left(s'\right)}{x\left(s',s^*\right)}=\frac{\alpha\left(s,s^*\right)}{\alpha\left(s',s^*\right)}\frac{n\left(s\right)}{x\left(s,s^*\right)} \tag{A.1}$$

Now, note that solving the representative firm's problem with log-quantities will report exactly the same optimality conditions. Henceforth, it is not necessary to take logs in total log-differentiation. In other words, consider the following expression, in *which both* $n(\cdot)$ *and* $x(\cdot)$ *are already expressed in logarithmic form:*

$$\frac{\log n(s')}{\log x(s',s^*)} = \frac{\alpha(s,s^*)}{\alpha(s',s^*)} \frac{\log n(s)}{\log x(s,s^*)}$$

Let me define the following for notational convenience: $N(s) := \log n(s)$, $X(s,s^*) := \log x(s,s^*)$, *and* $d_{\leftarrow s^*}[s,s'] := \frac{\alpha(s,s^*)}{\alpha(s',s^*)}$. *The expression becomes*

$$\frac{N(s')}{X(s',s^*)} = d_{\leftarrow s^*}[s,s'] \frac{N(s)}{X(s,s^*)}$$

I proceed by performing a first-order Taylor expansion around the steady state. Let $\overline{N}(s)$ *and* $\overline{X}(s,s^*)$ *denote the steady-state values of* $N(s)$ *and* $X(s,s^*)$*, respectively, and let* $d\,N(s)$ *and* $d\,X(s,s^*)$ *denote their log-deviations. Using the total differential of a ratio, it is possible to obtain*

$$d\left(\frac{N(s)}{X(s,s^*)}\right) = \frac{1}{\overline{X}(s,s^*)} d\,N(s) - \frac{\overline{N}(s)}{\overline{X}^2(s,s^*)} d\,X(s,s^*)$$

and similarly for sector-s', $d\left(\frac{N(s')}{X(s',s^*)}\right) = \frac{1}{\overline{X}(s',s^*)} d\,N(s') - \frac{\overline{N}(s')}{\overline{X}^2(s',s^*)} d\,X(s',s^*)$. *Since* $d_{\leftarrow s^*}[s,s']$ *is a constant parameter, it is possible to differentiate both sides of the original equation,* $\frac{N(s')}{X(s',s^*)} = d_{\leftarrow s^*}[s,s'] \frac{N(s)}{X(s,s^*)}$*:*

$$d\left(\frac{N(s')}{X(s',s^*)}\right) = d_{\leftarrow s^*}[s,s']\, d\left(\frac{N(s)}{X(s,s^*)}\right)$$

Inserting differentials, I derive the following first-order log-linearized expression

$$\frac{1}{\overline{X}(s',s^*)} d\,N(s') - \frac{\overline{N}(s')}{\overline{X}^2(s',s^*)} d\,X(s',s^*) = d_{\leftarrow s^*}[s,s'] \left(\frac{1}{\overline{X}(s,s^*)} d\,N(s) - \frac{\overline{N}(s)}{\overline{X}^2(s,s^*)} d\,X(s,s^*)\right)$$

which, given $d_{\leftarrow s^*}[s,s'] = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}$*, will deliver the two-sector condition in Subsection 1.2. This result illustrates that the log-linearized response of the ratio between logged employment and logged intermediate inputs in sector-*s' *is proportional to that of sector-*s*, with the scaling factor given by their relative intensity of intermediate input usage,* $\alpha(s,s^*)\,/\,\alpha(s',s^*)$. *Steady-state values* $\overline{N}(\cdot)$ *and* $\overline{X}(\cdot,s^*)$ *cannot be avoided in this log-linearization due to the non-linear nature of the ratio between logarithms.*

Stacked across all sectors, the above condition yields Lemma 1. Note that the same outcome would have been obtained under Constant Elasticity of Substitution (CES) technology, and for any production function satisfying Assumption 1.

□

(Proof of Lemma 2) *Consider a Cobb-Douglas economy as stated in Section 2. The combination of optimal demand for both labour and intermediate inputs would yield*

$$x(s,s') = \alpha(s,s')\, w(s)\, n(s)\, \alpha(s)^{-1} \frac{1}{p(s')}$$

Then, consider the case in which two sectors $\{s, s'\}$ are selling their own intermediate inputs to the same sector-s^*. Associated system of equations is

$$\begin{cases} x(s^*, s) = \alpha(s^*, s)\, w(s^*)\, n(s^*)\ \alpha(s^*)^{-1} \dfrac{1}{p(s)} \\ x(s^*, s') = \alpha(s^*, s')\, w(s^*)\, n(s^*)\ \alpha(s^*)^{-1} \dfrac{1}{p(s')} \end{cases}$$

$$\longrightarrow \quad \begin{cases} x(s^*, s) = \alpha(s^*, s) \left[x(s^*, s') \dfrac{1}{\alpha(s^*, s')\, n(s^*)} \alpha(s^*)\ p(s') \right] n(s^*)\ \alpha(s^*)^{-1} \dfrac{1}{p(s)} \\ w(s^*) = x(s^*, s') \dfrac{1}{\alpha(s^*, s')\, n(s^*)} \alpha(s^*)\ p(s') \end{cases}$$

The above substituted equation will result in

$$\frac{x(s^*, s)}{x(s^*, s')} = \frac{\alpha(s^*, s)}{\alpha(s^*, s')} \frac{p(s')}{p(s)} \tag{A.2}$$

which, if log-linearized around its steady state (note that solving the representative firm's problem with log-quantities will report exactly the same optimality conditions; thus, not necessary to take logs in total log-differentiation), closely following the proof for Lemma 1, will deliver the two-sector condition in Subsection 1.2 which, stacked across all sectors, delivers the result in Lemma 2. Note that the same result would have been obtained under Constant Elasticity of Substitution (CES) technology, and for any production function satisfying Assumption 1.

For completeness, consider the following expression

$$\frac{\log x(s^*, s)}{\log x(s^*, s')} = \frac{\alpha(s^*, s)}{\alpha(s^*, s')} \frac{\log p(s')}{\log p(s)}$$

and define the following for notational convenience: $P(s) := \log p(s)$, $X(s^*, s) := \log x(s^*, s)$, and $d_{\to s^*}[s, s'] := \frac{\alpha(s^*, s)}{\alpha(s^*, s')}$. Thus, the expression becomes

$$\frac{X(s^*, s)}{X(s^*, s')} = d_{\to s^*}[s, s'] \frac{P(s')}{P(s)}$$

Proceed by performing a first-order Taylor expansion around the steady state. Let $\overline{X}(s^*, s)$ and $\overline{P}(s)$ denote the steady-state values of $X(s^*, s)$ and $P(s)$, respectively, and let $d\,X(s)$ and $d\,P(s)$ to denote their log-deviations. Using the total differential of a ratio, then

$$d\left(\frac{X(s^*, s)}{X(s^*, s')}\right) = \frac{1}{\overline{X}(s^*, s')} d\,X(s^*, s) - \frac{\overline{X}(s^*, s)}{\overline{X}(s^*, s')^2} d\,X(s^*, s')$$

In the same manner the right-hand side can be manipulated, $d\left(d(s^* \mid s, s') \frac{P(s')}{P(s)}\right) = d(s^* \mid s, s') \left(\frac{1}{\overline{P}(s)} d\,P(s') - \frac{\overline{P}(s')}{\overline{P}^2(s)} d\,P(s)\right)$, since $d_{\to s^*}[s, s']$ is not changing. Differentiat-

ing both sides of the original equation, $\frac{X(s^*,s)}{X(s^*,s')} = d_{\to s^*}[s,s'] \frac{P(s')}{P(s)}$*, one obtains*

$$\frac{1}{\overline{X}(s^*,s')} d\,X(s^*,s) - \frac{\overline{X}(s^*,s)}{\overline{X}(s^*,s')^2} d\,X(s^*,s') = d_{\to s^*}[s,s'] \left(\frac{1}{\overline{P}(s)} d\,P(s') - \frac{\overline{P}(s')}{\overline{P}(s)^2} d\,P(s) \right)$$

This result demonstrates how the log-linearized relationship between price and quantity ratios in sectors $\{s,s'\}$ *depends on their relative intensity* $d_{\to s^*}[s,s'] = \frac{\alpha(s^*,s)}{\alpha(s^*,s')}$*. Similar to the previous proof, the steady-state values* $\overline{X}(s^*,\cdot)$ *and* $\overline{P}(\cdot)$ *appear due to the non-linear nature (i.e., the ratio of logarithms) of the initial expression.*

□

(Proof of Theorem 1) *To prove the theorem, I begin by referring to Examples 1, 2, and 3 in Section 1 of the main text to illustrate the first two results. For the third result, I introduce a related example with a partially symmetric Input-Output matrix.*

First, characterize the Input-Output matrix of Example 1, sectors $\{s',s''\}$*. Consider again an economy populated by four sectors,* $\{s,s',s'',s'''\} \in \Phi(s)$*, where some trade with all others, while some do not. The resulting production network of Panel 1b displays an Input-Output matrix,* ***H****,*

$$\underset{4\times 4}{\boldsymbol{H}} = \begin{bmatrix} 1 & 0.2 & 0.1 & 0.5 \\ 0.4 & 1 & 0 & 0.3 \\ 0.7 & 0 & 1 & 0.6 \\ 0.3 & 0.1 & 0.2 & 1 \end{bmatrix}$$

Cells set to one are irrelevant in this context and helps to shift attention to the off-diagonal entries, necessary to compute network-based distances.

In this sectoral production network, sectors $\{s',s''\}$ *are not connected: cell* α_{23} *indicates that sector-*s' *does not purchase intermediate inputs from sector-*s''*; conversely, cell* α_{32} *indicates that sector-*s'' *does not purchase from sector-*s'*. Consequently, the production network designed in Panel 1b of Figure 1 shows no linkage between these two sectors. However, when considering network distances in terms of a shared Input-Output structure, sectors* $\{s',s''\}$ *appear connected, as both simultaneously buy from and sell to the common sector-*s*. This horizontal linkage, reflecting the combined demand and supply relationships, effectively connects the sectors and is depicted as the crosswise line in the bottom-right of Panel 1b, linking sectors that were otherwise disconnected in the production network. This proves part one.*

Constructing a distance matrix requires defining relationships between pairs of sectors that buy from or sell to the same sector. Consider the pair $\{s',s''\}$*. For the factor input demand distance, the common sector from which both sectors purchase is sector-*s*. Specifically, the inter-sectoral trade intensity for sector-*s' *is* $\alpha(s',s) = 0.2$*, while for sector-*s'' *it is* $\alpha(s'',s) = 0.1$*. For the factor input supply distance, the common sector to which both sectors sell is still sector-*s*. Here, the inter-sectoral trade intensity for sector-*s' *is* $\alpha(s,s') = 0.4$*, and for sector-*s'' *it is* $\alpha(s,s'') = 0.7$*. Accordingly,* *their distance relations can be expressed as* $d_{\leftarrow s}[s',s''] = (0.2, 0.1)$ *and* $d_{\rightarrow s}[s',s''] = (0.4, 0.7)$*, which clearly yield dissimilar distance metrics. Additionally, considering also their intensities with the other common sector-*s'''*, then* $d_{\leftarrow s'''}[s',s''] = (0.1, 0.2)$ *and* $d_{\rightarrow s'''}[s',s''] = (0.3, 0.6)$*. Applying the same procedure to all other sector pairs allows to examine the full network-based distance structures.*

Reporting the related inter-sectoral trade intensity associated to the Input-Output matrix $\mathbf{H}$*, the factor input demand distance matrix* $\mathcal{D}^{fd}$ *is then*

$$\underset{4\times4}{\mathcal{D}^{fd}} = \begin{bmatrix} 0 & (.7,.0),(.3,.1) & (.4,.0),(.3,.2) & (.4,.3),(.7,.6) \\ (.7,.0),(.3,.1) & 0 & (.2,.1),(.1,.2) & (.2,.5),(.0,.6) \\ (.4,.0),(.3,.2) & (.2,.1),(.1,.2) & 0 & (.1,.5),(.0,.3) \\ (.4,.3),(.7,.6) & (.2,.5),(.0,.6) & (.1,.5),(.0,.3) & 0 \end{bmatrix}$$

which is symmetric with zeros along the main diagonal (no distance between a sector and itself). For instance, element $d^{fd}_{14} = (.4,.3), (.7,.6) = d^{fd}_{41}$ *is the distance relation between sectors* $\{s, s'''\}$ *when buying from sector-*s' *and sector-*s''*, respectively.*

Furthermore, reporting the related inter-sectoral trade intensity associated to the Input-Output matrix $\mathbf{H}$*, the factor input supply distance matrix* $\mathcal{D}^{fs}$ *is then*

$$\underset{4\times4}{\mathcal{D}^{fs}} = \begin{bmatrix} 0 & (.1,.0),(.5,.3) & (.2,.0),(.5,.6) & (.2,.1),(.1,.2) \\ (.1,.0),(.5,.3) & 0 & (.4,.7),(.3,.6) & (.4,.3),(.0,.2) \\ (.2,.0),(.5,.6) & (.4,.7),(.3,.6) & 0 & (.7,.3),(.0,.1) \\ (.2,.1),(.1,.2) & (.4,.3),(.0,.2) & (.7,.3),(.0,.1) & 0 \end{bmatrix}$$

which is symmetric with zeros along the main diagonal (no distance between a sector and itself). For instance, element $d^{fs}_{23} = (.4,.7), (.3,.6) = d^{fs}_{32}$ *is the distance relation between sectors* $\{s', s''\}$ *when selling to sector-*s *and sector-*s'''*, respectively.*

Henceforth, the resulting matrices $\mathcal{D}^{fd}$ *and* $\mathcal{D}^{fs}$ *display different values everywhere, thereby establishing part two of the theorem.*

Finally, I turn to the the third result. Consider an economy populated by four sectors, $\{s, s', s'', s'''\} \in \Phi(s)$*, where some trade with all the others, while some others do not. The resulting production network, illustrated in Panel 1b would display an Input-Output matrix,* $\mathbf{H}$*, in which some cells are set to zero. Cells set to one helps to concentrate on values outside the main diagonal, necessary to compute network-based distances, while cells in bold are imposed to be symmetric. Hence:*

$$\underset{4\times4}{\boldsymbol{H}} = \begin{bmatrix} 1 & \mathbf{0.4} & 0.1 & \mathbf{0.5} \\ \mathbf{0.4} & 1 & 0 & 0.3 \\ 0.7 & 0 & 1 & \mathbf{0.6} \\ \mathbf{0.5} & 0.1 & \mathbf{0.6} & 1 \end{bmatrix}$$

In this production network, sector-s' *is purchasing intermediate inputs from sector-*s *and, simultaneously, selling back the same amount of intermediates,* $\alpha(s', s) = 0.4 = \alpha(s, s')$*. Relative to sectors* $\{s, s''\}$*, analogous is the situation for sector-*s'''*:* $\alpha(s''', s) = 0.5 = \alpha(s, s''')$*, and* $\alpha(s''', s'') = 0.6 = \alpha(s'', s''')$*. The procedure to construct factor* *input demand and factor input supply distance matrices is the usual:*

$$\underset{4\times 4}{\boldsymbol{\mathcal{D}}^{fd}} = \begin{bmatrix} 0 & (.7,.0),(.5,.1) & \mathbf{(.4,.0),(.5,.6)} & (.4,.3),(.7,.6) \\ (.7,.0),(.5,.1) & 0 & (.4,.1),(.1,.6) & \mathbf{(.4,.5),(.0,.6)} \\ \mathbf{(.4,.0),(.5,.6)} & (.4,.1),(.1,.6) & 0 & (.1,.5),(.0,.3) \\ (.4,.3),(.7,.6) & \mathbf{(.4,.5),(.0,.6)} & (.1,.5),(.0,.3) & 0 \end{bmatrix}$$

and

$$\underset{4\times 4}{\boldsymbol{\mathcal{D}}^{fs}} = \begin{bmatrix} 0 & (.1,.0),(.5,.3) & \mathbf{(.4,.0),(.5,.6)} & (.4,.1),(.1,.6) \\ (.1,.0),(.5,.3) & 0 & (.4,.7),(.3,.6) & \mathbf{(.4,.5),(.0,.6)} \\ \mathbf{(.4,.0),(.5,.6)} & (.4,.7),(.3,.6) & 0 & (.7,.5),(.0,.1) \\ (.4,.1),(.1,.6) & \mathbf{(.4,.5),(.0,.6)} & (.7,.5),(.0,.1) & 0 \end{bmatrix}$$

In both matrices, the bolded cells correspond to their shared, equal counterpart. Consequently, in a network with at least four sectors, whenever there are two sectors that each buy from and sell to a common sector-s (as examples, imagine sector-s' buys and sells 10 units with sector-s, while sector-s'' buys and sells 5 units with sector-s), there exists at least one distance metric that is identical under both the factor input demand and factor input supply network "economic" distances.

□

A.2. Model derivation

(Households problem) *The household-i's utility problem is to maximize utility function subject to its budget constraint:*

$$\max_{c_i(s),\, n_i(s)} \quad \mathcal{U}_i\left(\left\{c_i(s)\right\}_{\forall s\in\Phi(s)};\, n_i(s)\right) := \prod_{s\in\Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi}$$

$$\text{s.t.} \qquad \sum_{s\in\Phi(s)} p(s)\, c_i(s) = w(s)\, n_i(s) + \sum_{s\in\Phi(s)} D_i(s)$$

where sectoral consumption and labour supplied are $c_i(s)$ and $n_i(s)$, respectively, and each is determined by a price, $p(s)$ and $w(s)$. Parameter $\beta(s)$ identifies the weight of each sectoral good in household i's consumption basket and $\sum_{s\in\Phi(s)} \beta(s) = 1$, ϕ is the inverse Frisch labour supply elasticity, measuring the elasticity of hours worked to the wage rate under a constant marginal utility of income. Finally, $D_i(s)$ is the constant share of sector-specific profits flowing from sector-s to household-i.

Utility maximization implies the Lagrangian function to be

$$\underset{c_i(s),\, n_i(s)}{\mathcal{L}} := \prod_{s\in\Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi} + \\ - \psi^{\mathcal{L}}\left[\sum_{s\in\Phi(s)} p(s)\, c_i(s) - \left(w(s)\, n_i(s) + \sum_{s\in\Phi(s)} D_i(s)\right)\right]$$

with $\psi^{\mathcal{L}}$ being the penalty multiplier. Optimality conditions are in order

$$\frac{p(s)\,c_i(s)}{\beta(s)} = \frac{p(s')\,c_i(s')}{\beta(s')}$$
$$w(s) = n_i(s)^{\phi}\,p(s)\,c_i(s)\left[\beta(s)\ \ c_i\right]^{-1}$$

which, once aggregated across all households, $\frac{p(s)\int_i c_i(s)\mathrm{d}\,i}{\beta(s)} = \frac{p(s')\int_i c_i(s')\mathrm{d}\,i}{\beta(s')}$ and $w(s) = \int_i n_i(s)^{\phi}\,\mathrm{d}\,i\,p(s)\int_i c_i(s)\,\mathrm{d}\,i\left[\beta(s)\ \ \int_i c_i \mathrm{d}\,i\right]^{-1}$, would report the optimality conditions for consumption and labour supply in Section 2.

□

(Sectoral optimization) *A perfectly competitive representative firm in sector-s maximizes total revenues from production net of costs of inputs of production:*

$$\max_{n(s),\ x(s,s')} \quad y(s) = z(s)\left(n(s)\right)^{\alpha(s)} \prod_{s'\in\Phi(s)} \left(x(s,s')\right)^{\alpha(s,s')} \varkappa_j(s)$$
$$\text{s.t.} \qquad \mathcal{C}(s) := w(s)\,n(s) + \sum_{s'\in\Phi(s)} p(s')\,x(s,s')$$

where $\alpha(s) + \sum_{s'}\alpha(s,s') = 1$, and $\varkappa_d(s) = 1$ for $j = \{fd, fs\}$. Profit maximization then implies

$$\max_{n(s),\ x(s,s')} D(s) :=\ p(s)\,y(s) - \mathcal{C}(s)$$

Optimality conditions for demand of intermediate inputs and labour demand, and their combination, are those in Section 2.

□

(Equilibrium characterization) *In equilibrium, the model should specify the clearing conditions of labour, circulating intermediate inputs, and goods markets. Starting from the labour market, equating labour demand and labour supply would deliver the labour market equilibrium condition, $n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ y(s)\right]^{\frac{1}{1+\phi}}$, i.e., eq. (5) in Section 2. Moreover, total labour demand is found by aggregating labour optimality condition of sectors, $N^{dem} = \sum_s n^{dem}(s)$, and total labour supply is found by aggregating labour optimality condition of households, $N^{sup} = \sum_s n^{sup}(s)$. Labour market clearing then requires that $N^{dem} = N^{sup}$.*

For what concerns equilibrium in the circulating intermediate inputs market, total demand of intermediates from sectors is $x^{dem}(s) = \sum_{s'} x(s,s')$. Analogously, its total supply of intermediate inputs is $x^{sup}(s) = \sum_{s'} x(s',s)$ so that, in equilibrium it must be true that $x^{dem}(s) = x^{sup}(s)$ which, in aggregate, simply states that $\sum_s x^{dem}(s) = \sum_s x^{sup}(s)\ \equiv\ X^{sup} = X^{dem}$.

The equilibrium in the good market is given by summing over all sectors the sectoral equilibrium in which total production must equal its total final consumption (by households) and its circulating intermediate input bought by other sectors: $\sum_s y(s) = \sum_s \left(\int_i c_i(s)\,\mathrm{d}\,i \right) + \sum_s \left(\sum_{s'} x(s,s') \right)$.

Finally, by aggregating the households' budget constraints over households and sectors, and imposing the clearing conditions so far, the aggregate resource constraint for this economy reads as

$$P^c C = WN + D$$

which equals total output, defined by $Y \propto \boldsymbol{\beta}_s \boldsymbol{Y}$ *of Section 1. Equilibrium conditions are described in Section 2. Throughout the proof, all summations are over* $\Phi(s)$.

□

(Derive aggregate fluctuations result of eq. (6)) *Following exactly the same operating steps in vom Lehn and Winberry (2022), to characterize the dynamical changes in aggregate Gross Domestic Product (GDP), first it is necessary to determine the existing relation between sectoral intermediate output,* $y(s)$, *and the Divisia Index.*[39] *Notice that constant returns to scale and homogeneity of degree one in Assumption 1 and in the production function in eq. (4) implies the following zero-profit condition*

$$p(s)\,y(s) = w(s)\,n(s) + \sum_{s' \in \Phi(s)} p(s')\,x(s,s')$$

where $p(s)\,y(s)$ *is the (gross) nominal output of sector-*s*, and from where the national accounting definition of nominal value added is just* $p(s)\,y(s) - p^s x^s = w(s)\,n(s)$*, where I define* $p^s x^s = \sum_{s' \in \Phi(s)} p(s')\,x(s,s')$ *the total expenditure of sector-*s *for intermediate inputs from other connected sectors. Define the left-hand side of the previous equation as* $p^Y(s)\,y^Y(s) \equiv p(s)\,y(s) - p^s x^s$*, i.e., the nominal value-added.*

In order to construct a single, aggregate measure of real output (or value added) across multiple sectors, I use a Divisia index. This index combines the growth rates of individual sectoral outputs into an overall growth rate, but does so in a way that accounts for each sector's economic importance – measured by its share in total nominal value added. Specifically, the Divisia index computes a weighted average of the growth rates of sectoral real outputs, where the weights are each sector's share in total nominal value added. This allows us to track how the economy's output is changing over time, adjusting dynamically as the composition of output across sectors evolves.

Henceforth, the Divisia Index requires to differentiate the nominal value added under constant price levels associated to variables:

[39] This indicator is a composite index measuring changes in certain aggregate quantity of a given variable, weighting all its defining components according to their relevance on that variable. It is particularly suited to analyse variables made of several and differentiated elements. Refer to Oulton (2022) for further details.

$$
\begin{aligned}
& p^Y(s)\ d\,log\,y^Y(s) = p(s)\ d\,log\,y(s) - p^s\,d\,log\,x^s \\
\rightarrow\ & p^Y(s)\,y^Y(s)\ d\log y^Y(s) = p(s)\,y(s)\ d\log y(s) - p^s x^s\,d\log x^s \\
\rightarrow\ & \alpha(s)\ d\log y^Y(s) = d\log y(s) - \alpha^s\,d\log x^s \\
\rightarrow\ & \alpha(s)\ d\log y^Y(s) = \left(d\log z(s) + \alpha(s)\ d\log n(s) + \alpha^s\,d\log x^s\right) - \alpha^s\,d\log x^s \\
\longrightarrow\ & d\log y^Y(s) = \frac{1}{\alpha(s)}\,d\log z(s) + d\log n(s)
\end{aligned}
$$

where $\alpha^s = \sum_{s\in\Phi(s)} \alpha(s,s')$. Plug the above sector-level nominal value added growth, $d\log y^Y(s)$, into the Divisia Index, $d\log Y = \sum_{s\in\Phi(s)} \left(\frac{p^Y(s)y^Y(s)}{P^Y Y}\right) d\log y^Y(s)$, so that it is possible to obtain a non-complete equation for real GDP growth:

$$
d\log Y = \sum_{s\in\Phi(s)} \left(\frac{p^Y(s)\,y^Y(s)}{P^Y Y}\right)\left[\frac{1}{\alpha(s)}\,d\log z(s) + d\log n(s)\right] \tag{A.3}
$$

Summing over all sectors the sectoral optimality conditions for intermediate inputs, $\sum_{s'\in\Phi(s)} x(s,s') = \sum_{s'\in\Phi(s)} \alpha(s,s')\left[p(s)\ \ y(s)\right]\left(p(s')\right)^{-1}$, noticing that $\alpha(s) = 1-\alpha^s$, and inserting the zero-profit condition would yield $\alpha(s) = \frac{p^Y(s)y^Y(s)}{p(s)y(s)}$, which is the ratio of sectoral value-added to sectoral intermediate (gross) output.

By plugging into the eq. (A.3) the condition for $\alpha(s)$, and making all the adjustments to simplify the elements multiplying the sectoral productivity it is then possible to get eq. (6), which determines the drivers of real aggregate GDP fluctuations.

□

(Derive labour market equilibrium of eqs. (10) and (12)) *Start from the sector-level equilibrium definition of the labour market, $n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ y(s)\right]^{\frac{1}{1+\phi}}$. Including the condition $x(s,s') = \vartheta(s,s')\ \ y(s')$ would imply eq. (8), that is*

$$
n(s) = \left[\alpha(s)\ \ \frac{C}{c(s)}\ \beta(s)\ \ z(s)\left(n(s)\right)^{\alpha(s)}\prod_{s'\neq s}\left(\vartheta(s,s')\ \ y(s')\right)^{\alpha(s,s')}\ \varkappa_j(s)\right]^{\frac{1}{1+\phi}} \tag{A.4}
$$

where it should be inserted the production function of eq. (4) for sector-s', and $\varkappa_j(s) \neq 1$, with $j = \{fd, fs\}$, for the purpose of this derivation. The resulting equation can be simplified by imposing an infinite inverse Frisch elasticity of labour, $\phi \to \infty$. In fact, in the model of Section 2, only the extensive margin of employment (i.e., total number of workers) is important, while it is not the intensive margin (i.e., total hours worked), which is ruled out under $\phi \to \infty$; refer to Rogerson (1988). However, for the sake of generality, I will proceed with $\phi \in [0,\infty)$.

Factor input demand.– *Assume the non-common (constant) element in the pro-*

duction function to be given by[40]

$$\varkappa_{fd}(s) = \tau_{fd}(s) \prod_{s'} \left[\frac{\prod_s x(s',s)^{\frac{1}{\alpha(s,s)}+\alpha(s',s)}}{\prod_s x(s,s)^{\frac{\alpha(s',s)}{\alpha(s,s)}}} \right]^{\frac{\alpha(s,s')}{1+\phi}} \tag{A.5}$$

Then, multiply and divide both sides of the above labour market equilibrium, including the production function, by $\prod_{s'} \left\{ \left[\prod_s x(s',s)^{\frac{1}{\alpha(s,s)}} \right] \Big/ \left[\prod_s x(s,s)^{\frac{\alpha(s',s)}{\alpha(s,s)}} \right] \right\}^{\frac{\alpha(s,s')}{1+\phi}}$. *Finally, making all the required adjustments once including* $\varkappa_{fd}(s)$*, the resulting condition will deliver eq. (10). All the multiplications are over* $\Phi(s)$.

Factor input supply.– *Assume now the non-common (constant) element in the production function to be given by*

$$\varkappa_{fs}(s) = \tau_{fs}(s) \prod_{s'} \left[\frac{\prod_s x(s',s)^{\frac{1}{\alpha(s,s)}}}{\prod_{s'} x(s',s')^{\frac{\alpha(s',s)}{\alpha(s',s')}-\alpha(s',s')}} \right]^{\alpha(s,s')} \tag{A.6}$$

Multiply and divide both sides of the above labour market equilibrium, including the production function, by $\prod_{s'} \left\{ \left[\prod_s x(s',s)^{\frac{1}{\alpha(s',s')}} \right] \Big/ \left[\prod_{s'} x(s',s')^{\frac{\alpha(s',s)}{\alpha(s',s')}} \right] \right\}^{\frac{\alpha(s,s')}{1+\phi}}$.

[40] In eq. (A.5), the component $\tau_{fd}(s) = \prod_{s'\in\Phi(s)} \left[\prod_{s\in\Phi(s)} \left(\frac{x(s',s)}{x(s,s)} \right)^{\frac{\alpha(s',s)}{\alpha(s,s)}} \right]^{\frac{1}{\alpha(s,s')}}$ allows to ensure that, in Section 2.2, the distance effects are not influenced by the Leontief inverse weight. Specifically, incorporating the condition $x(s,s') = \vartheta(s,s')\ y(s')$ in the labour market equilibrium of eq. (A.4) would result in a double counting of the values from the Input-Output matrix: one of them is used to capture sectoral distances, while the other would have been multiplied by distance matrix $\mathcal{D}^{fd}$ when analysing the effect of sectoral propagation in Proposition 3. Within this context, it is essential to highlight that, starting from the baseline labour market equilibrium in eq. (5), and deriving the results presented in this section and in Proposition 3 without including the exogenous condition $x(s,s') = \vartheta(s,s')\ y(s')$ would produce an outcome reflecting the effect of network distances *without* the influence of the Input-Output Leontief inverse weights. In other words, as discussed in the main text (Theorem 2), the effect of the network "economic" distance across pairs of sectors is independent of inter-sectoral trade intensities characterizing the production network; imposing the $\tau_{fd}(s)$ term due to the condition $x(s,s') = \vartheta(s,s')\ y(s')$ allows to avoid that such concept is violated. Otherwise, without $\tau_{fd}(s)$, the component Ξ^{fd} in eq. (10) would be equal to

$$\Xi^{fd} = \prod_{s'\in\Phi(s)} \left[\prod_{s\in\Phi(s)} \left(\frac{x(s',s)}{x(s,s)} \right)^{\frac{\alpha(s',s)}{\alpha(s,s)}} \right]^{\alpha(s,s')}$$

Analogous is the rationale beyond component $\tau_{fs}(s) = \prod_{s'\in\Phi(s)} \left[\prod_{s\in\Phi(s)} \left(\frac{x(s',s)}{x(s',s')} \right)^{\frac{\alpha(s',s)}{\alpha(s',s')}} \right]^{\frac{1}{\alpha(s,s')}}$, proper of eq. (A.6) that delivers Proposition 4. Avoiding to consider it, the component Ξ^{fs} in eq. (12) is

$$\Xi^{fs} = \prod_{s'\in\Phi(s)} \left[\prod_{s\in\Phi(s)} \left(\frac{x(s',s)}{x(s',s')} \right)^{\frac{\alpha(s',s)}{\alpha(s',s')}} \right]^{\alpha(s,s')}.$$

Making all the required adjustments once including $\varkappa_{fs}(s)$, the resulting condition will deliver eq. (12). All the multiplications are over $\Phi(s)$.

$\square$

A.3. Additional theoretical results on propagation

PROPOSITION 5 (Leontief propagation of changes to sectoral employment) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (8). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by*

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + \underbrace{\boldsymbol{H}\left(\Upsilon_{s,s'}\right) d\log \boldsymbol{z} + \boldsymbol{\mathcal{E}}\ d\log \boldsymbol{X}}_{d\log \boldsymbol{\mathcal{N}_{\mathcal{H}}}} \right\}' \boldsymbol{\mathcal{H}}\left(\Upsilon_{s,s'}\right) \tag{A.7}$$

where:

(a) $\boldsymbol{N}$ identifies sectoral employment levels;

(b) Θ identifies a compound of structural parameters;

(c) C is aggregate consumption;

(d) $\boldsymbol{\mathcal{S}}$ identifies sector-specific changes in productivity and consumption;

(e) $\boldsymbol{\mathcal{N}_{\mathcal{H}}}$ identifies the production network effect of other sectors' changes in productivities and intermediate inputs usage impacting sector-s where:

- *$\boldsymbol{H}(\Upsilon_{s,s'})$ is the Input-Output matrix, comprising the weight of each sector on other sectors, whose entries are set to 0 whenever $s = s'$;*
- *$\boldsymbol{\mathcal{E}} = \boldsymbol{H}(\Upsilon_{s,s'})' \boldsymbol{H}(\Upsilon_{s',s})$ is a compounded network effect, made of the inner product of the Input-Output matrix, influencing the elements of $\boldsymbol{X}$, a matrix with intermediate inputs purchases.*

(f) $\boldsymbol{\mathcal{H}}(\Upsilon_{s,s'}) = \left[\boldsymbol{I} - \Theta_{|\alpha}\ \boldsymbol{H}(\Upsilon_{s,s'})\right]^{-1}$ is the Leontief inverse of the Input-Output matrix, which is as well adjusted by a compound of structural parameters now defined as $\Theta_{|\alpha} = \frac{1}{1+\phi-\boldsymbol{\alpha}}\boldsymbol{\alpha}$.

Proof in Appendix A.4.

PROPOSITION 6 (Leontief propagation under factor input demand distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (10). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by*

$$d\log \boldsymbol{N} = \Theta \left\{ d\log C + d\log \boldsymbol{\mathcal{S}} + d\log \boldsymbol{\mathcal{N}_{\mathcal{H}}} + \underbrace{\boldsymbol{\mathcal{D}}^{fd}\left[d\log \boldsymbol{N}\left(\Phi(s)\right) - d\log \boldsymbol{N}\right]}_{d\log \boldsymbol{\mathcal{D}}(n)} \right\}' \boldsymbol{\mathcal{H}}\left(\Upsilon_{s,s'}\right) \tag{A.8}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

(b) Θ *identifies a compound of structural parameters;*

(c) C *is aggregate consumption;*

(d) $\mathcal{S}$ *identifies sector-specific changes in productivity and consumption;*

(e) $\mathcal{N}_{\mathcal{H}}$ *identifies the production network effect of other sectors' changes in productivities and intermediate inputs usage impacting sector-*s.

(f) $\mathcal{H}(\Psi_{s,s'}) = \left[\boldsymbol{I} - \Theta_{|\alpha}\ \boldsymbol{H}(\Psi_{s,s'})\right]^{-1}$ *is the Leontief inverse of the Input-Output matrix, which is as well adjusted by a compound of structural parameters now defined as* $\Theta_{|\alpha} = \frac{1}{1+\phi-\boldsymbol{\alpha}}\boldsymbol{\alpha}$*;*

(g) $\mathcal{D}^{fd}$ *identifies all the network distances across any pair of sectors;*

(h.i) $\mathcal{D}(n)$ *identifies the production network distance effect of other sectors' variations in employment levels,* $d\log\mathbf{N}(\Phi(s))$*, when these are buying their intermediate inputs from the same upstream sector(s).*

Proof in Appendix A.4.

Proposition 7 (Leontief propagation under factor input supply distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (12). Then, the direct and indirect network effects governing the response of sectoral employment is a first-order (log-linear) approximation given by*

$$d\log\boldsymbol{N} = \Theta\left\{d\log C + d\log\mathcal{S} + d\log\mathcal{N}_{\mathcal{H}} + \underbrace{\mathcal{D}^{fs}\left[d\log\boldsymbol{P}\Big(\Phi(s)\Big) - d\log\boldsymbol{P}\right]}_{d\log\mathcal{D}(p)}\right\}'\mathcal{H}\Big(\Psi_{s,s'}\Big) \tag{A.9}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

(b) Θ *identifies a compound of structural parameters;*

(c) C *is aggregate consumption;*

(d) $\mathcal{S}$ *identifies sector-specific changes in productivity and consumption;*

(e) $\mathcal{N}_{\mathcal{H}}$ *identifies the production network effect of other sectors' changes in productivities and intermediate inputs usage impacting sector-*s.

(f) $\mathcal{H}(\Psi_{s,s'}) = \left[\boldsymbol{I} - \Theta_{|\alpha}\ \boldsymbol{H}(\Psi_{s,s'})\right]^{-1}$ *is the Leontief inverse of the Input-Output matrix, which is as well adjusted by a compound of structural parameters now defined as* $\Theta_{|\alpha} = \frac{1}{1+\phi-\boldsymbol{\alpha}}\boldsymbol{\alpha}$*;*

(g) $\mathcal{D}^{fs}$ *identifies all the network distances across any pair of sectors;*

(h.ii) $\mathcal{D}(p)$ *identifies the production network distance effect of other sectors' variations in employment levels,* $d\log\boldsymbol{P}(\Phi(s))$*, when these are selling part of their output to the same downstream sector(s).*

Proof in Appendix A.4.

A.4. Proofs of theoretical results on propagation

(Proof of Proposition 1) *The interest is in characterizing the way in which sectoral employment changes in response to variations in sector-specific intermediate inputs usage. In order to inspect these changes, it is necessary to express the equilibrium conditions in terms of log-linear conditions. Starting from the sector-s labour market equilibrium of eq. (5), i.e.,*

$$n\left(s\right)=\left[\alpha\left(s\right)\ \frac{C}{c(s)}\ \beta\left(s\right)\ y\left(s\right)\right]^{\frac{1}{1+\phi}}$$

the associated log-linear form is $\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{y}\left(s\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$, *with tilded variables,* $\widetilde{\cdot}$, *expressing the deviation from their respective steady state. Such log-linearized equation contains the log-linearized output, obtained by log-differentiating the production function in eq. (4):* $\widetilde{y}\left(s\right)=\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\sum_{s'\in\Phi(s)}\alpha\left(s,s'\right)\widetilde{x}\left(s,s'\right)$. *Substituting out these two equations, one easily gets that log-linearized sectoral employment is a function of*

$$\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\sum_{s'\in\Phi(s)}\alpha\left(s,s'\right)\widetilde{x}\left(s,s'\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$$

expressing the elements whose variations induce changes in employment level of sector-s. In vectorial notation, one can rewrite the summation as

$$\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\boldsymbol{h}\left(s\right)\widetilde{\boldsymbol{x}}\left(s\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$$

which, stacked over all sectors can be written in matrix form:

$$\widetilde{\mathbf{N}}=\frac{1}{1+\phi}\left\{\widetilde{C}-\widetilde{\mathbf{C}}+\widetilde{\mathbf{z}}+\boldsymbol{\alpha}\widetilde{\mathbf{N}}+\boldsymbol{H}'\widetilde{\mathbf{X}}\right\}$$

where $\widetilde{\mathbf{N}}$ is an $S\times 1$ vector of changes in sector-specific employment levels, identifies changes in aggregate consumption, $\widetilde{\mathbf{C}}$ is an $S\times 1$ vector of changes in sector-specific final consumption by households, $\widetilde{\mathbf{z}}$ is an $S\times 1$ vector of changes in sectoral productivities, $\mathbf{H}$ is an $S\times S$ squared Input-Output matrix identifying the structure of inter-sectoral trade, and $\widetilde{\mathbf{X}}$ is an $S\times 1$ vector of changes in sector-specific set of intermediate inputs bought within the production network. Finally, ϕ is a scalar for aggregate inverse Frisch elasticity of labour supply to wage level, and $\boldsymbol{\alpha}=\left[\alpha\left(s\right),\alpha\left(s'\right),\ldots,\alpha\left(S\right)\right]$ comprises sector-specific labour force as a share of its intermediate output.

Bringing the vector of sectoral employments, $\widetilde{\mathbf{N}}$, on the left-hand side, then rewriting $\widetilde{\cdot}=d\log\left(\cdot\right)$, and rearranging terms, then one obtains eq. (7) in Proposition 1.

□

(Proof of Proposition 2) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels. In order to inspect these changes, it is necessary to express the equilibrium con-*

ditions in terms of log-linear conditions. Starting from the sector-s labour market equilibrium of eq. (5), i.e.,

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ y(s)\right]^{\frac{1}{1+\phi}}$$

the associated log-linear form is $\widetilde{n}(s) = \frac{1}{1+\phi}\left\{\widetilde{y}(s) + \widetilde{C} - \widetilde{c}(s)\right\}$, *with tilded variables,* $\widetilde{\cdot}$, *expressing the deviation from their respective steady state. Then, use the condition* $x(s,s') = \vartheta(s,s')\ y(s')$, *which states that each intermediate input is just a given share* $\vartheta(s,s')$ *of other sector's output. Inserting its log-linearized form* $\widetilde{x}(s,s') = \widetilde{y}(s')$, *together with the log-differentiation of the production function in eq. (4),* $\widetilde{y}(s) = \widetilde{z}(s) + \alpha(s)\widetilde{n}(s) + \sum_{s' \in \Phi(s)} \alpha(s,s')\widetilde{x}(s,s')$, *into the log-linear condition for labour market equilibrium one gets*

$$\widetilde{n}(s) = \frac{1}{1+\phi}\left\{\begin{array}{l} \widetilde{C} - \widetilde{c}(s) + \widetilde{z}(s) + \alpha(s)\widetilde{n}(s) \quad + \\ + \sum\limits_{s' \in \Phi_s} \alpha(s,s')\left[\widetilde{z}(s') + \alpha(s')\widetilde{n}(s') + \sum\limits_{s \in \Phi_{s'}} \alpha(s',s)\widetilde{x}(s',s)\right] \end{array}\right\}$$

expressing the elements whose variations induce changes in employment level of sector-s. Note that using the above condition $x(s,s') = \vartheta(s,s')\ y(s')$ *implies that summations are not over the whole set of sectors, but rather it should be excluded the sector whose one is summing for. To this aim, denote* $\Phi(s)$ *the set of all sectors; then, both* Φ_s *and* $\Phi_{s'}$ *denotes improper subsets of* $\Phi(s)$, *since they are excluding sector-s and sector-s', respectively. In other words, all the elements in* $\{\Phi_s, \Phi_{s'}\}$ *are contained in* $\Phi(s)$ *but* $\{\Phi_s, \Phi_{s'}\}$ *and* $\Phi(s)$ *are not equal,* $\Phi_s \subseteq \Phi(s)$ *with* $\Phi_s \nsubseteq \Phi(s)$, *and* $\Phi_{s'} \subseteq \Phi(s)$ *with* $\Phi_{s'} \nsubseteq \Phi(s)$. *This implies that, when stacking the above condition over all sectors expressing it in matrix notation, the Input-Output matrix from* $\alpha(s,s')$ *and* $\alpha(s',s)$ *are not full but rather are set to zero whenever* $s = s'$, *i.e., using the* $\Psi_{s,s'}$ *as a matrix indicator.*

Using such notation, and expressing the above log-linearized labour market equation in vectorial form and then in matrix form (as in the Proof of Proposition 1), and solving for $d\log n(s)$, *one obtains eq. (9) of Proposition 2.*

□

(Proof of Proposition 3) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels according to their factor input demand network distance relationships. Taking the logarithm of eq. (10) would result in*

$$\log n(s) = \frac{1}{1+\phi} \left\{ \begin{array}{l} \log\alpha(s) + \log C + \log z(s) + \alpha(s)\,\log n(s) - \log c(s) \quad + \\ + \displaystyle\sum_{s' \in \Phi(s)} \alpha(s,s') \left[\log\vartheta(s,s') + \log z(s') + \alpha(s')\,\log n(s') \right] \quad + \\ + \displaystyle\sum_{s' \in \Phi(s)} \alpha(s,s') \left\{ \sum_{s \in \Phi(s)} \alpha(s',s)\log x(s',s) \right\} + \log\beta(s) \quad + \\ + \displaystyle\sum_{s' \in \Phi(s)} \left\{ \frac{\sum_{s\in\Phi(s)} \alpha(s',s)}{\sum_{s\in\Phi(s)} \alpha(s,s)} \log\left(\frac{\sum_{s\in\Phi(s)} x(s',s)}{\sum_{s\in\Phi(s)} x(s,s)} \right) \right\} \end{array} \right\}$$

Inserting the condition in eq. (A.1) used to derive Lemma 1 in the $\log[\cdot]$ *component in the last row, compute the associate total differentiation, following closely the steps to derive Proposition 2, and using the notation associated to network distance matrix* $\mathcal{D}^{fd}$*, then one obtains eq. (11) characterizing Proposition 3.*

∎

(Proof of Proposition 4) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels according to their factor input supply network distance relationships. Taking the logarithm of eq. (12) would result in*

$$\log n(s) = \frac{1}{1+\phi} \left\{ \begin{array}{l} \log\alpha(s) + \log C + \log z(s) + \alpha(s)\,\log n(s) - \log c(s) \quad + \\ + \displaystyle\sum_{s' \in \Phi(s)} \alpha(s,s') \left[\log\vartheta(s,s') + \log z(s') + \alpha(s')\,\log n(s') \right] \quad + \\ + \displaystyle\sum_{s' \in \Phi(s)} \alpha(s,s') \left\{ \sum_{s' \in \Phi(s)} \alpha(s',s')\log x(s',s') \right\} + \log\beta(s) \quad + \\ + \displaystyle\sum_{s \in \Phi(s)} \left\{ \frac{\sum_{s'\in\Phi(s)} \alpha(s',s)}{\sum_{s'\in\Phi(s)} \alpha(s',s')} \log\left[\frac{\sum_{s'\in\Phi(s)} x(s',s)}{\sum_{s'\in\Phi(s)} x(s',s')} \right] \right\} \end{array} \right\}$$

Inserting the condition in eq. (A.2) used to derive Lemma 2 in the $\log[\cdot]$ *component in the last row, compute the associate total differentiation, following closely the steps to derive Proposition 2, and using the notation associated to network distance matrix* $\mathcal{D}^{fs}$*, then one obtains eq. (13) characterizing Proposition 4.*

∎

(Proof of Proposition 5) *Proof follows that of Proposition 2.* ∎

(Proof of Proposition 6) *Proof follows that of Proposition 3.* ∎

(Proof of Proposition 7) *Proof follows that of Proposition 4.* ∎

(Proof of Theorem 2) *The proof is organised in two parts. The first proceeds directly along the lines of proving Propositions 3 and 4; the second rests entirely on the economic intuition underlying the theorem.*

Technicalities. *To begin with, recall that both cited proofs start by manipulating the labour market equilibrium in such a way as to introduce factor input demand and factor input supply production network distances between sectors (see eqs. 10 and 12, respectively). As noted in Footnote 40 the term $\tau_j(s)$, with $j = \{fd, fs\}$, simplifies the calculations by eliminating the Input-Output component $\alpha(s, s')$, thereby rendering the effect of the network "economic" distance across sectors independent of their inter-sectoral trade intensities that characterise the production network; this component is required when working under the condition $x(s, s') = \vartheta(s, s')\ y(s')$. Put differently, assuming this condition in conjunction with $\tau_j(s)$ would implicitly rule out any influence of the Input-Output matrix on the distance matrix. The purpose of the subsequent discussion is to establish that the argument holds even without imposing this condition, thereby showing that, in all cases, the distance matrices, $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$, remain independent of the Input-Output matrix (whether in its direct – $\mathbf{H}$ – or Leontief inverse – $\boldsymbol{\mathcal{H}}$ – configuration).*

Imagine to start from the labour market equilibrium condition

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ z(s)\ \Big(n(s)\Big)^{\alpha(s)} \prod_{s' \in \Phi(s)} \Big(x(s, s')\Big)^{\alpha(s,s')} \varkappa_j(s)\right]^{\frac{1}{1+\phi}} \tag{A.10}$$

where it should be inserted the production function of eq. (4) for sector-s, and $\varkappa_j(s) \neq 1$, with $j = \{fd, fs\}$, for the purpose of this proof. Avoid to insert the condition $x(s, s') = \vartheta(s, s')\ y(s')$, as in the proof of Proposition 1. Then, proceed under the same logic when proving Propositions 3 and 4: for both factor input demand and factor input supply, multiply both sides of the equilibrium equation to insert distances while preserving, at the same time, the component for intermediate inputs.[41] *For the case of demand-based distance, premultiply both sides of eq. (A.10) by* $\left(\frac{\prod_{s' \in \Phi(s)} x(s,s')^{\frac{1}{\alpha(s',s')}}}{\prod_{s' \in \Phi(s)} x(s',s')^{\frac{\alpha(s,s')}{\alpha(s',s')}}}\right)^{\frac{1}{1+\phi}}$ *and set* $\varkappa_{fd}(s) = \frac{\prod_{s' \in \Phi(s)} x(s,s')^{\frac{1}{\alpha(s',s')}}}{\prod_{s' \in \Phi(s)} x(s',s')^{\frac{\alpha(s,s')}{\alpha(s',s')} - \alpha(s',s')}}$, *while supply-based distance features* $\left(\frac{\prod_{s' \in \Phi(s)} x(s,s')^{\frac{1}{\alpha(s,s)}}}{\prod_{s \in \Phi(s)} x(s,s)^{\frac{\alpha(s,s')}{\alpha(s,s)}}}\right)^{\frac{1}{1+\phi}}$ *with* $\varkappa_{fs}(s) = \frac{\prod_{s' \in \Phi(s)} x(s,s')^{\frac{1}{\alpha(s,s)}}}{\prod_{s \in \Phi(s)} x(s,s)^{\frac{\alpha(s,s')}{\alpha(s,s)} - \alpha(s,s)}}$. *Rearranging all terms will lead to*

$$n(s) = \left[(\dots) \prod_{s' \in \Phi(s)} \Big(x(s', s')\Big)^{\alpha(s',s')} \underbrace{\prod_{s' \in \Phi(s)} \left(\frac{x(s, s')}{x(s', s')}\right)^{\frac{\alpha(s,s')}{\alpha(s',s')}}}_{\Xi^{fd}}\right]^{\frac{1}{1+\phi}}$$

under demand-driven network distance, and to

[41] In other words, the aim is to maintain the same components of eq. (A.10) but $\varkappa_j(s)$ while including an additional term indicating the network distance.

$$n(s) = \left[(\dots) \prod_{s \in \Phi(s)} \left(x(s,s) \right)^{\alpha(s,s)} \underbrace{\left(\frac{\prod_{s' \in \Phi(s)} x(s,s')^{\frac{\alpha(s,s')}{\alpha(s,s)}}}{\prod_{s \in \Phi(s)} x(s,s)^{\frac{\alpha(s,s')}{\alpha(s,s)}}} \right)}_{\Xi^{fs}} \right]^{\frac{1}{1+\phi}}$$

under supply-driven network distance. Both distance terms, Ξ^{fd} *and* Ξ^{fs}*, are not subject to the generic element of the Input-Output matrix,* $\boldsymbol{H} = \left[\alpha(s,s') \geq 0\right]$*, so that, when totally differentiating and log-linearizing, the resulting factor input demand and factor input supply distance matrices,* $\boldsymbol{\mathcal{D}}^{fd}$ *and* $\boldsymbol{\mathcal{D}}^{fs}$*, will not be multiplied by the (directed or Leontief inverse) Input-Output matrix. This result leads to Theorem 2.*

Characterizing intuition*. Turning to the conceptual part of the proof, it is essential to recall that the horizontal dimension of the network, as captured by its "economic" distances, is not confined to sectors already connected – directly or indirectly – through inter-sectoral trade. Rather, it also encompasses sectors without such direct or indirect linkages, which are nonetheless related through similar demand or supply relationships arising from common upstream suppliers or downstream buyers, as discussed in Subsection 1.4 and elaborated throughout the main text. This reasoning implies that certain sectors may exhibit a strictly positive distance linkage despite the absence of any Input-Output connection. In other words, considers two sectors, say* $\{s, s'\}$*, with zero (direct and Leontief inverse) inter-sectoral trade intensity –* $\alpha(s,s') = \alpha(s',s) = 0$ *and* $\ell(s,s') = \ell(s',s) = 0$ *–, held together by their demand / supply relationship with a common sector, say* s^**, so that* $d_{\leftarrow s^*}\left[s', s''\right] = d_{\rightarrow s^*}\left[s', s''\right] > 0$*. In this case, the Input-Output matrix and the distance matrix would be given by* $\boldsymbol{H} \equiv \boldsymbol{\mathcal{H}} = \begin{bmatrix} \cdot & 0 \\ 0 & \cdot \end{bmatrix}$ *and* $\boldsymbol{\mathcal{D}}^{j} = \begin{bmatrix} \cdot & > 0 \\ > 0 & \cdot \end{bmatrix}$*, for* $j = \{fd, fs\}$*. Accordingly, multiplying the distance matrix by the production network matrix would effectively neutralise the role of network distances. This is the rationale underpinning Theorem 2.*

$\square$

B. SUFFICIENT STATISTICS FOR SECTORAL COMOVEMENT

B.1. *Factor Input Demand network distance*

Analytical conditions for sufficient statistics under which demand-based network distances between sectors generate *negative comovement* highlight as key driver the interaction between *(i)* the elasticity of substitution across intermediate inputs, and *(ii)* the upstream sector's price response to demand shocks. The derivation covers both the single-input case and the more general setting in which buyer sectors aggregate multiple intermediate inputs via a Constant Elasticity of Substitution (CES) function, illustrating how substitution elasticities and upstream price pass-through jointly determine the sign and magnitude of the induced responses to downstream demand shocks.[42] Formal proofs and economic intuitions are provided.

Derivation of the Sufficient Statistic

Consider a buyer sector-s that demands an intermediate input supplied by an upstream sector-s^*. The buyer's technology follows a CES aggregator with elasticity of substitution $\sigma > 0$. From cost minimization, the *log*-demand elasticity of the intermediate with respect to its own price is given by $\frac{\partial \log x(s,s^*)}{\partial \log p(s^*)} = -\sigma$. Suppose a shock in sector-s' increases demand for the input produced by s^*. Let the upstream price respond according to $\frac{\partial \log p(s^*)}{\partial \log x(s',s^*)} = \tau^{fd}$. By the chain rule, the effect of the demand shock on sector-s's input demand is then

$$\frac{\partial \log x\left(s, s^*\right)}{\partial \log x\left(s', s^*\right)} = (-\sigma) \cdot \tau^{fd} \tag{B.1}$$

Thus, the sign of the comovement depends on the product $\sigma \cdot \tau^{fd}$. If $\sigma \cdot \tau^{fd} > 0$, the comovement is negative. This condition serves as a ***sufficient statistic*** for identifying negative comovement in demand-based distances, and the mechanism operates through two key channels: *(i)* the *price channel*, where a shock to sector-s' increases demand for inputs from s^* and, if upstream supply is inelastic, the price $p\left(s^*\right)$ rises, so that $\tau^{fd} > 0$; and *(ii)* the *substitution channel*, where the buyer sector-s responds by substituting away from the now more expensive input, and the magnitude of this effect depends on the elasticity of substitution σ.

The above expression captures a *marginal* elasticity, *i.e.*, the infinitesimal *log*-response of sector-s's conditional demand for the input of s^* with respect to a small perturbation transmitted via the upstream pass-through τ^{fd}. It does not describe

[42] The *single-input case* considers each sector as purchasing intermediate inputs from only one upstream supplier, with no substitution possibilities. In this benchmark, a shock in one sector that affects the upstream supplier propagates vertically, and any resulting negative comovement in downstream sectors depends solely on the upstream price pass-through. The multi-input CES case generalizes this setup by allowing sectors to source from multiple suppliers, with the elasticity of substitution σ determining the magnitude of reallocation away from the affected input. Here, the negative comovement emerges only when substitution and expenditure shares align with the upstream price effect, thereby illustrating the interplay and potential ambiguity between vertical and horizontal propagation channels in production networks.

the global reallocation of demand after a finite shock. In particular, $\sigma = 1$ corresponds to the Cobb-Douglas case, not to a corner solution: the effect remains finite and negative, reflecting that sector-s marginally substitutes away from s^*, but never shifts *all* of its purchases. Complete reallocation can only occur in extreme cases (*e.g.*, $\sigma \to \infty$, where intermediate inputs are perfect substitutes) or when analysing large shocks beyond the range of the linearization, in which case the *log*-derivative approximation breaks down.[43] Hence, for finite σ the formula correctly implies a finite marginal adjustment, while corner solutions require either infinite elasticity or a fully non-linear analysis of the exact CES demand system.

When both effects are active (positive τ^{fd} and sizeable σ), sector-s reduces demand, creating negative comovement with the shocked sector. This sufficient statistic captures the interplay between substitution and upstream constraints, providing a tractable criterion to guide both theoretical and empirical work.

Extending the set-up: multi-input CES aggregator

Consider a downstream sector-s that buys a finite set of intermediate inputs supplied by upstream sectors. Let the composite intermediate input be a CES aggregator:

$$X(s) = \left(\sum_{s^* \in \Phi(s)} \lambda(s, s^*)^{\frac{1}{\sigma}} x(s, s^*)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where $x(s, s^*)$ is the quantity of the intermediate supplied by upstream sector-s^* used by downstream buyer sector-s, while $\lambda(s, s^*) \in (0, 1)$ are share parameters, and $\sigma > 0$ is the (constant) elasticity of substitution across intermediate inputs.

Define the associated (unit) price index for the composite intermediate by the usual CES formula, $P(s) = \left(\sum_{s^* \in \Phi(s)} \lambda(s, s^*) p(s^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$, where $p(s^*)$ is the price of the intermediate supplied by upstream sector-s^*.

Under cost minimization, for a given level of composite intermediate $X(s)$, the conditional demand for intermediate input s^* is given by the standard CES demand system. Define the expenditure share of variety s^* in the composite intermediate by $e(s, s^*) = \frac{\lambda(s,s^*) p(s^*)^{1-\sigma}}{\sum_{s^* \in \Phi(s)} \lambda(s,s^*) p(s^*)^{1-\sigma}}$, with $\sum_{s^* \in \Phi(s)} e(s, s^*) = 1$. A well-known property of the CES demand system is the (*log*) price elasticities for conditional demands:

$$\frac{\partial \log x(s, s^*)}{\partial \log p(s^*)} = -\sigma \left[1 - e(s, s') \right] \tag{B.2}$$

$$\frac{\partial \log x(s, s^*)}{\partial \log p(s^{**})} = \sigma\, e(s, s^{**}), \quad \forall s^{**} \neq s^* \tag{B.3}$$

Conditions (B.2)-(B.3) are standard results for CES demand systems: the former, own-price elasticity depends on the substitution parameter and on the expenditure

[43] Below, in the *multi-input CES system*, the presence of expenditure shares $e(s, s^*)$ attenuates the response: the own-price elasticity is $-\sigma[1 - e(s, s^*)]$, so that large shares dampen substitution.

share, while the latter, cross-price elasticities are proportional to the share of the other intermediate input considered.

Shock propagation through a common supplier. Focus on a particular upstream sector-s^* that supplies an intermediate used by several downstream buyers. Consider an idiosyncratic positive demand shock in some sector-s' that increases aggregate demand for the good produced by s^*. Let $x\left(s', s^*\right)$ denote the demand shifter (or demand level) associated with the shocked sector-s' from sector-s^*.

Assume the upstream price of the good produced by s^* responds to the shock according to a (*log*) pass-through parameter $\tau^{fd} = \frac{\partial \log p(s^*)}{\partial \log x(s', s^*)}$ of demand changes into prices. If $\tau^{fd} > 0$, the common supplier's price rises when downstream demand increases; if $\tau^{fd} < 0$, demand shocks lower prices; when $\tau^{fd} \approx 0$, upstream supply is effectively perfectly elastic at the relevant margin.

The interest is in the *log*-response of buyer sector-s's demand for the intermediate supplied by s^*, *i.e.*, $\partial \log x\left(s, s^*\right) / \partial \log x\left(s', s^*\right)$. By the chain rule, and noting that only $p\left(s^*\right)$ changes in response to the shock (holding other upstream prices fixed in this partial equilibrium exercise), one obtains

$$\begin{aligned}\frac{\partial \log x\left(s, s^*\right)}{\partial \log x\left(s', s^*\right)} &= \frac{\partial \log x\left(s, s^*\right)}{\partial \log p\left(s^*\right)} \cdot \frac{\partial \log p\left(s^*\right)}{\partial \log x\left(s', s^*\right)} \\ &= \frac{\partial \log x\left(s, s^*\right)}{\partial \log p\left(s^*\right)} \cdot \tau^{fd}\end{aligned}$$

Using the CES own-price elasticity in eq. (B.2) for sector-s^*, whose expenditure share is $e\left(s, s^*\right)$, I obtain

$$\frac{\partial \log x\left(s, s^*\right)}{\partial \log x\left(s', s^*\right)} = -\sigma\left[1 - e\left(s, s^*\right)\right] \tau^{fd} \tag{B.4}$$

Equation (B.4) makes three points: *(i)* the *elasticity of substitution*, σ, magnifies the response $-$ the larger σ, the stronger buyer s substitutes away from an input whose price rises; *(ii)* the *expenditure share*, $e\left(s, s^*\right)$, attenuates the own-price effect $-$ if buyer-s spends only a small fraction on the input from s^*, then $\left[1 - e\left(s, s^*\right)\right] \approx 1$ and the full $-\sigma$ factor operates, while if the share is large, the own-price sensitivity is mechanically smaller in magnitude; and *(iii)* the *upstream pass-through*, τ^{fd}, transmits the original downstream demand shock into a price change at the supplier $-$ without such pass-through ($\tau^{fd} = 0$) the substitution channel vanishes.

Hence the sign of the *log*-response in (B.4) is the sign of $-\sigma\left[1 - e\left(s, s^*\right)\right]\tau^{fd}$. Because $\sigma > 0$ and $1 - e\left(s, s^*\right) > 0$ (for an interior share), the sign is governed by τ^{fd}: if $\tau^{fd} > 0$ (upstream price rises) and $\sigma > 0$, buyer s reduces its demand for s^*'s input, thereby delivering *negative* comovement; by contrast, if $\tau^{fd} \approx 0$ (perfectly elastic upstream supply), the price channel is absent and do not expect negative comovement via this substitution mechanism.

Sufficient-statistic condition (multi-input case). Combining the above, a convenient sufficient-statistic condition for *negative* comovement of buyer s's usage of the intermediate produced by s^* (and plausibly of buyer s's employment if that input matters for labour demand) following a positive shock in sector-s' is

$$-\underbrace{\sigma}_{elasticity} \times \underbrace{\Big[1 - e\,(s, s^*)\Big]}_{residual\ expenditure\ share} \times \underbrace{\tau^{fd}}_{upstream\ pass\text{-}through} < 0 \tag{B.5}$$

Equivalently, because $\sigma(1 - e\,(s, s^*) > 0$ for interior shares, negative comovement arises whenever $\tau^{fd} > 0$. In words: whenever the common supplier's price rises in response to the shock and the downstream buyer is substitution-prone (positive σ), the result is a fall in the buyer's demand for that input and hence potential negative co-movement between the shocked sector and other buyers of the same supplier.

Summary and discussion

Single-input case. Suppose a shock occurs in sector-s', altering its demand for intermediate inputs. If both s and s' rely on the same upstream supplier s^*, then the increased demand from s' modifies the optimal input supply of s^*. This, in turn, raises the marginal cost of production in s^*, transmitting vertically along the supply chain. Sector-s, facing higher input prices, reduces its own demand, generating a potential *negative comovement* in employment. Formally, letting $\tau^{fd} > 0$ denote the price impact of the shock transmitted from s' to s^*, the reaction of sector-s is unambiguously negative. The sufficient statistic in this case is the pass-through parameter τ^{fd}: whenever $\tau^{fd} > 0$, demand-based distances generate negative comovement. This establishes a simple benchmark: competition for a unique common input induces substitution effects that overturn positive complementarities.

Multi-input CES case. The single-input setting is restrictive, since most sectors source from multiple suppliers. By contrast, considering a CES aggregator of intermediate inputs for sector-s, with elasticity of substitution σ and expenditure share on the common input s_{ss^*}, a shock from s' still propagates through s^*, raising its price by τ^{fd}. Sector-s reacts by reallocating its input mix, reducing demand for s^* and increasing demand for substitutes. The change in demand for the common input is given by $\Delta \log x\,(s, s^*) = -\sigma\big[1 - e\,(s, s^*)\big]\ \tau^{fd}$, which generalises the single-input benchmark. This expression highlights two key forces: *(i)* a higher elasticity of substitution ($\sigma \gg 1$) amplifies the negative response, since sector-s can more easily substitute away from the expensive input; and *(ii)* a larger expenditure share $e\,(s, s^*)$ attenuates the response, because the scope for substitution is smaller when the common input dominates the production structure. When $\sigma \to 0$ (Leontief case), substitution is impossible and the negative effect vanishes, aligning horizontal transmission with standard vertical propagation. Conversely, when $e\,(s, s^*) \to 0$, the sufficient statistic reduces to $-\sigma\ \tau^{fd}$, recovering the single-input case as a limiting outcome.

Interpretation and ambiguity. The analysis reveals that negative comovement under demand-based distances is not a universal feature, but a contingent one: it crucially depends on the interaction between substitution elasticity and input shares. The ambiguity arises because the same empirical pattern of sectoral co-movement may be driven either by a *(i) vertical* propagation, where shocks pass through suppliers along the Input-Output structure, or by a *(ii) horizontal* substitution effects, where sectors sharing the same upstream input reallocate their demand in opposite directions. This overlapping between vertical and horizontal mechanisms complicates the identification of genuine demand complementarities. What may appear as a horizontal spillover between sectors s and s' could, in fact, be the indirect by-product of vertical adjustments working through s^*.

Taking stocks. The distinction between the single-input and multi-input CES frameworks clarifies the mechanisms through which demand-based distances can induce negative comovement. In the single-input benchmark, any positive shock to a common supplier transmits negatively to other connected sectors via higher costs. In the CES setting, the sufficient statistic $\Delta \log x\left(s, s^*\right) = -\sigma\left[1 - e\left(s, s^*\right)\right]\ \tau^{fd}$ captures how substitution elasticity and expenditure shares modulate this effect. Strong substitutability magnifies negative comovement, while heavy reliance on the common input mutes it. Ultimately, these results underscore that observed sectoral comovement may reflect a complex interplay of vertical propagation and substitution-driven horizontal effects. This inherent ambiguity, especially acute under demand-based distances, contrasts with the clearer patterns observed in supply-based distances, where competition for downstream markets yields more direct forms of negative interdependence. As such, disentangling these two channels remains a central challenge in interpreting the propagation of shocks across production networks.

B.2. Factor Input Supply network distance

Analytical conditions for sufficient statistics under which factor input supply-based network distances generate *negative comovement* highlight the interaction between *(i)* the elasticity of substitution across factor inputs in the production technology of a common downstream buyer, and *(ii)* the response of that downstream buyer's output to a shock in one of its suppliers. As in the demand-based case, the derivation proceeds separately for the single-input case and the multi-input CES case.[44]

Derivation of the Sufficient Statistic

Consider an upstream supplier s that sells a finite set of intermediate inputs to a downstream sector-s^*. Let the composite intermediate input be a CES aggregator:

$$X\left(s^*\right) = \left(\sum_{s \in \Phi(s)} \lambda\left(s^*, s\right)^{\frac{1}{\sigma}}\ x\left(s^*, s\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{B.6}$$

where $x\left(s^*, s\right)$ is the quantity of the intermediate supplied by upstream sector-s

[44] The *single-input case* assumes that the downstream sector employs only one factor input from each upstream supplier, so that a shock to one supplier transmits mechanically to all others through the downstream buyer's production. Negative comovement is then governed solely by the elasticity of substitution between factors within the downstream production function. The multi-input CES case generalizes this to allow multiple factors, so that the effect of a shock in one supplier on the demand for another supplier's input depends on substitution elasticity and expenditure shares.

used by downstream buyer sector-s^*, while $\lambda(s, s^*) \in (0, 1)$ are share parameters, and $\sigma > 0$ is the (constant) elasticity of substitution across intermediate inputs.

Define the associated (unit) price index for the composite intermediate by the usual CES formula, $P(s^*) = \left(\sum_{s\in\Phi(s)} \lambda(s^*, s)\ p(s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, with $p(s)$ denoting the price of input from upstream supplier s.

Under cost minimization, for a given level of composite intermediate $X(s^*)$, the conditional demand for intermediate input s is given by the standard CES demand system. Define the expenditure share of variety s in the composite intermediate by $e(s^*, s) = \frac{\lambda(s^*,s)\, p(s^*)^{1-\sigma}}{\sum_{s\in\Phi(s)} \lambda(s^*,s)\, p(s)^{1-\sigma}}$, with $\sum_{s\in\Phi(s)} e(s^*, s) = 1$.

A well-known property of the CES demand system is the (*log*) price elasticities for conditional demands:

$$\frac{\partial \log x(s^*, s)}{\partial \log p(s)} = -\sigma\left[1 - e(s^*, s)\right] \tag{B.7}$$

$$\frac{\partial \log x(s^*, s)}{\partial \log p(s')} = \sigma\, e(s^*, s'), \quad \forall s' \neq s \tag{B.8}$$

Conditions (B.7)-(B.8) are standard results for CES demand systems: the former, own-factor demand elasticity is decreasing in own price, scaled by substitution elasticity and expenditure share, while the latter, cross-price elasticities says demand for s rises if another input s' becomes more expensive, proportional to s''s share.

Shock propagation through a common buyer. Focus on a particular downstream sector-s^* that buys an intermediate used by several upstream sellers. Consider an idiosyncratic positive demand shock in some sector-s' that increases aggregate supply for the good purchased by s^*. Let $x(s^*, s')$ denote the supply shifter (or supply level) associated with the shocked sector-s' to sector-s^*.

Assume the upstream price of the good produced by s' responds to the shock according to a (*log*) pass-through parameter $\tau^{fs} = \frac{\partial \log p(s')}{\partial \log x(s^*,s')}$ of supply changes into prices. If $\tau^{fs} > 0$, the supplier's price rises when downstream demand increases; if $\tau^{fs} < 0$, supply shocks lower prices; when $\tau^{fs} \approx 0$, upstream supply is effectively perfectly elastic at the relevant margin.

The interest is in the *log*-response of buyer sector-s^*'s demand for the intermediate supplied by s, *i.e.*, $\partial \log x(s^*, s) / \partial \log x(s^*, s')$. By the chain rule, and noting that only $p(s)$ changes in response to the shock (holding other upstream prices fixed in this partial equilibrium exercise), obtaining

$$\begin{aligned}\frac{\partial \log x(s^*, s)}{\partial \log x(s^*, s')} &= \frac{\partial \log x(s^*, s)}{\partial \log p(s')} \frac{\partial \log p(s')}{\partial \log x(s^*, s')} \\ &= \frac{\partial \log x(s^*, s)}{\partial \log p(s')} \cdot \tau^{fs}\end{aligned}$$

Using the CES own-price elasticity in eq. (B.7) for sector-s, whose expenditure share is $e(s^*, s)$, I obtain

$$d \log x(s^*, s) = \left[\sigma\, e(s^*, s')\right] \tau^{fs} \tag{B.9}$$

Equation (B.9) makes three points transparent: *(i)* the *elasticity of substitution*, σ, magnifies the response – a larger σ amplifies the substitution away from s and toward s' when $p(s')$ falls; *(ii)* the *expenditure share*, $e(s^*, s')$, attenuates the own-price effect – the cheaper input s' represents a bigger share of s^*'s bundle; and *(iii)* the *downstream pass-through*, τ^{fs}, transmits the original upstream supply shock into a price change at the buyer – without such pass-through ($\tau^{fs} = 0$) the substitution channel vanishes.[45]

Hence the sign of the *log*-response in (B.9) is the sign of $\sigma\, e(s^*, s')\, \tau^{fs}$. Because $\sigma > 0$ and $e(s^*, s') > 0$ (for an interior share), the sign is governed by τ^{fs}: if $\tau^{fs} < 0$, the product $\sigma\, e(s^*, s')\, \tau^{fs}$ is negative, implying *negative* comovement between s and s' through their common buyer; by contrast, if $\tau^{fs} \approx 0$ (perfectly elastic downstream demand), the price channel is absent and do not expect negative comovement via this substitution mechanism.

Sufficient-statistic condition (multi-input supply case). Consider two upstream sectors, s and s', both selling intermediate inputs to the same downstream buyer s^*. Combining the derivation above, a convenient sufficient-statistic condition for *negative* comovement of seller s's supply to s^* (and plausibly of s's employment if labour is proportional to output) following a positive shock in sector-s' is

$$\underbrace{\sigma}_{\textit{elasticity}} \quad \times \quad \underbrace{e\left(s^*, s'\right)}_{\textit{expenditure share}} \quad \times \quad \underbrace{\tau^{fs}}_{\textit{downstream pass-through}} \quad < \; 0 \tag{B.10}$$

Equivalently, because $\sigma e(s^*, s') > 0$ for interior shares, negative comovement arises whenever $\tau^{fs} < 0$. In words: whenever the common buyer's willingness-to-pay rises in response to the shock from s' and the upstream seller is substitution-prone across buyers (positive σ), the result is a fall in the other seller's marginal supply or allocation to that buyer, generating potential negative comovement between the upstream sectors competing for the same downstream market.

Downgrading the set-up: single-input case

If the downstream sector s^* uses only a *single input* from an upstream supplier (*i.e.* no possibility of substitutability across inputs), the elasticity of substitution is effectively zero ($\sigma = 0$). In this case, there is no scope for reallocation of expenditures across inputs: a price change in s' does not alter the demand for s:

---

[45] Again, this condition represents a *marginal* elasticity: it captures the infinitesimal *log*-change in revenues from sales to s^* in response to a small perturbation in relative prices, mediated by the supply-side pass-through τ^{fs}. It should not be interpreted as implying a complete diversion of sales under $\sigma = 1$. In fact, $\sigma = 1$ corresponds to Cobb-Douglas preferences of the downstream buyer, in which expenditure shares remain constant, and the negative effect merely reflects marginal substitution toward competing suppliers. Full reallocation of demand away from s occurs only in the limiting case $\sigma \to \infty$ (perfect substitutes) or for large shocks where the *log*-linear approximation ceases to hold. In the multi-input CES case, sectoral sales are scaled by expenditure shares $e(s, s^*)$, so that the effective elasticity is $\sigma e(s, s^*)$, ensuring finite adjustments for all finite σ. Thus, the sufficient statistic provides a valid local description of marginal reallocation, but corner outcomes require infinite substitution or an exact analysis of the non-linear CES demand system.

$$\frac{\partial \log x(s^*, s)}{\partial \log p(s')} = 0$$

Hence, unlike the demand-driven case, the *single-input supply case does not yield a sufficient statistic*, as there is no mechanism generating comovement.

Summary and discussion

Single-input case. If the common downstream sector s^* purchases only a single unit of input from each upstream supplier (no substitution, $\sigma = 0$), then a positive shock to s' increases s^*'s demand for s''s input, transmitting vertically along the supply chain. However, because there is no substitution possibility, sector s cannot reallocate its supply away from s^*'s demand, and hence no negative comovement arises. A negative supply-driven channel appears only when $\sigma > 0$: the upstream sectors can reallocate their supply across buyers, so that an increase in demand for s''s input induces a reduction in s's allocation to s^*, generating negative comovement.

Multi-input CES case. Consider an upstream sector-s that supplies multiple inputs to a downstream sector-s^*, which aggregates these inputs via a CES function with elasticity of substitution $\sigma > 0$. By the CES demand system, sector-s can reallocate its supply across buyers in response to relative price changes. The *log*-change in s's supply to s^* induced by the shock to s' is $\Delta \log x(s, s^*) = \sigma\, e(s^*, s')\ \tau^{fs}$, where $e(s^*, s')$ is the share of s''s input in the composite demand of s^*. This expression provides a *sufficient statistic* for negative comovement: a positive shock to sector-s' changes the demand for its input, generating a price response in the upstream allocation, captured by the pass-through parameter $\tau^{fs} = \partial \log p(s)\ /\ \partial \log x(s', s^*)$. If $\tau^{fs} < 0$ (*i.e.* the price of s''s input falls), the upstream sector reallocates supply away from s towards the cheaper s', generating negative comovement mechanically. The magnitude of this effect is amplified by a larger elasticity of substitution σ and a higher expenditure share $e(s^*, s')$. Conversely, if $\sigma = 0$ (Leontief aggregation), no substitution is possible and negative comovement does not arise.

Intuition. Negative comovement under factor-input supply distances is driven by downstream substitution: upstream suppliers respond strategically to changes in downstream demand across multiple buyers: cheaper inputs from s' attract more allocation, crowding out other suppliers. Both the ability to substitute across buyers (σ) and the relative importance of the shocked buyer in the downstream expenditure share (e) determine the strength of the effect. This mechanism highlights that supply-driven distances produce negative comovement only when upstream allocation is flexible and responsive, contrasting with demand-driven distances where even single-input scenarios can generate negative co-movement via price pass-through. In contrast to the demand-driven case, the sign depends on whether supplier's shocks lower prices ($\tau^{fs} < 0$). Observed co-movements thus reflect the extent of substitution possibilities and the composition of the downstream buyer's input bundle.

C. Data, figures and tables

(Data details) *In order to build the main dataset with a panel of 65 private 3-digit U.S. 2017 North American Industry Classification System (NAICS) sectors used in Sections 3-4, I rely on different data sources from the Bureau of Economic Analysis (BEA).*[46] *For years from 1998 to 2022, the features of these information are:*

(a) employment (Table 6.8D. - Persons Engaged in Production by Industry, in thousands. Last revised on: September 29, 2023);

(b) value-added (Value-Added by Industry, in billions of dollars. Last revised on: December 19, 2024);

(c) inter-sectoral trade and Input-Output linkages, in particular:

- *sectoral share of own production, and thus the sectoral share of intermediate inputs from other sectors (Industry-by-Commodity Market Share Matrix, After Redefinitions - Summary, in producers' prices);*
- *total inputs required (directly and indirectly) in order to deliver one dollar of output to final users (Commodity-by-Commodity Total Requirements, After Redefinitions - Summary, in producers' prices);*
- *sector-specific imports and exports of goods and services (The Use of Commodities by Industries, Before Redefinitions - Summary, in producers' prices and in millions of dollars).*

Note that "Summary" defines the level of sectoral disaggregation considered in the analysis (3-digit). The BEA also provides information on Input-Output matrices to a more detailed level (6-digit), but these are on a five-yearly basis (thus it would impossible to run the analysis in Subsection 4.1), and data on other variables are not collected. Moreover, the sample begins in 1998 so to have all sectors classified given the NAICS system, and ends in 2022, the last year available when I started this project.[47] *Last time I accessed this online data was in March 2025.*

Turn to the manipulation of Input-Output matrices. To recover the production share of intermediate inputs of a given sector coming from the production of other sectors, each cell of the yearly squared matrix contains the intermediate output share from a given sector to another and, on the main diagonal, the share of intermediate output of a given sector directly produced by that sector. Henceforth, labelling by $\boldsymbol{X} = \left[x\left(s, s'\right) \right]$, *with* $x\left(s, s'\right) \geq 0$, *this Input-Output matrix I compute, for each sector,* $x\left(s\right) = 1 - x\left(s, s\right)$. *In other words, to extract the total share of intermediate output that sector-s buys from other sectors, I compute* $\boldsymbol{x} = \mathbf{1} - diag(\boldsymbol{X})$, *where* $\mathbf{1}$ *an* $S \times 1$ *vector of ones; the resulting* $S \times 1$ *vector* $\boldsymbol{x}$ *reports, for every considered year, the total*

[46] Note that I refer to 3-digit, but four sectors are at 2-digit ("construction", "management of companies and enterprises", "educational services", and "other services, except government"), while five sectors related to finance and insurance are at 4-digit level.

[47] Before 1998, classification follows the U.S.Standard Industrial Classification (SIC) system.

production of a sectoral good which is due to intermediate inputs bought through Input-Output relationships.

For what concerns the production network structure, I investigate its characteristics through the main Input-Output matrix (Commodity-by-Commodity Total Requirements, After Redefinitions) which, consistently with the main text, I label as $\mathbf{H} = \left[\alpha\left(s,s'\right) \geq 0\right]$. *Leontief inverse matrix (labelled with* $\boldsymbol{\mathcal{H}}$ *in the main text), upstream and downstream sectors, and the weight associated to each of them (as in Subsections 4.1-4.2) are identified through it.*[48]

Differently, in order to compute sectoral distances, I follow the literature (e.g., Acemoglu et al. 2012, Carvalho 2014) and disregard small transactions across sectors: this allows me to compute a finite set of production network distances at the extensive margin due to the removal of bias from meaningless inter-sectoral trade intensity. Such adjusted matrix is labelled as $\mathbf{H}^j = \left[\alpha^j\left(s,s'\right)\right]$ *whose generic element is then* $\alpha^j\left(s,s'\right) > 0.01$ *for* $j = \{fd, fs\}$ *identifying whether the matrix allows to compute factor input demand or factor input supply network distances. In particular, I set to 0 all the linkages which are below 1% of sector's total purchases (in case of factor input demand distance) and total sales (in case of factor input supply distance). Disregarding small transactions implies that*

$$\alpha^j\left(s,s'\right) = \frac{\alpha\left(s,s'\right)}{\widetilde{v}\left(s\right)} \quad \textit{is set to} \quad 0 \quad \textit{if} \quad \alpha^j\left(s,s'\right) \leq 0.01$$

where $\widetilde{v}\left(s\right) = \frac{\alpha(s,s')}{\sum_{s' \in column} \alpha(s,s')}$ *is sector-s's total input purchases (i.e., column sum of matrix* **H***) for factor input demand distance, or* $\widetilde{v}\left(s\right) = \frac{\alpha(s,s')}{\sum_{s' \in row} \alpha(s,s')}$ *is sector-s's total input sales (i.e., row sum of matrix* **H***) for factor input supply distance. It follows that, by defining* $\widetilde{V} = \left[\widetilde{v}(1), \ldots, \widetilde{v}\left(s\right), \ldots, \widetilde{v}\left(S\right)\right]'$, *the considered Input-Output structure is*

$$\underset{S\times S}{\mathbf{H}^j} = \underset{S\times S}{\mathbf{H}}{}' \underset{S\times 1}{\widetilde{V}}$$

in which each cell, $\alpha^j\left(s,s'\right)$, *is greater than 0.01, i.e., larger than 1%.*

(Shortest path algorithm, technical but suitable for coding) *Given element* $\boldsymbol{\mathcal{G}}$ *being an* $S \times S$ *adjacency matrix with discrete values larger than 1, the idea is to compute a matrix* $\boldsymbol{\mathcal{D}}$ *where each cell* $d\left[s,s'\right]$ *is the length of the shortest path between node-s and node-s'. The numerical algorithm is as follows:*

(a) from the Input-Output matrix **H***, build an associated identity matrix,* $\boldsymbol{\mathcal{I}}$*;*

(b) set the initial length of the path, $n = 1$*, and build a length matrix* $\boldsymbol{\mathcal{G}_n} = \boldsymbol{\mathcal{G}}$*;*

(c) build a Boolean matrix, $\boldsymbol{\mathcal{L}}$*, whose elements are* $l\left(s,s'\right) = 1$ *if* $\boldsymbol{\mathcal{G}_n} \neq 0$*;*

[48] Row sectors identify (upstream) suppliers, and column sectors identify (downstream) buyers. Basically, in matrix $\boldsymbol{H}$ each entry in the upper-triangular part corresponds to the dollar value of one unit of sector-s that is bought by sector-s', while each entry in the lower-triangular part identifies the dollar value of sector-s in order to buy one unit from sector-s'.

(d) *as long as matrix $\boldsymbol{\mathcal{L}}$ contains a value different from zero (i.e., there are connections among nodes not considered):*

- *update identity matrix $\boldsymbol{\mathcal{I}}$ by adding n whenever the condition on $\boldsymbol{\mathcal{L}}$ is true, that is, $i(s,s') = i(s,s') + n$ if $l(s,s') = 1$;*
- *update lengths, $n = n + 1$;*
- *algebraic matrix calculation: $\boldsymbol{\mathcal{G}_n} = \boldsymbol{\mathcal{G}_n}'\boldsymbol{\mathcal{G}}$;*
- *update matrix $\boldsymbol{\mathcal{L}}$ with new paths: set $l(s,s') = 1$ whenever $\mathcal{G}_n(s,s') \neq 0$ and $d[s,s'] = 0$;*

(e) *once this loop concludes, set to infinite all the disconnected nodes, that is $d(s,s') = \infty$ if $d[s,s'] = 0$;*

(g) *set to the minimum distance the elements on the main diagonal, $\boldsymbol{\mathcal{D}} = \boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{I}}$.*

Outside this routine set to zero all the disconnected nodes (those at ∞). Matrix $\boldsymbol{\mathcal{D}}$, whose values are at the extensive margin and identify network distances among any pair (s,s'), is built. The generic element identifying the existence of a link between node-s and node-s' in the distance matrix $\boldsymbol{\mathcal{D}}$ is then $d[s,s'] = \{1, 2, \ldots, d_{max}\}$.

TABLE C.1: LIST OF CONSIDERED SECTORS AND THEIR CODES

denomination	***codes***	
	NAICS 2017	*BEA*
Farms	111, 112	111CA
Forestry, Fishing and Related Activities	113, 114, 115	113FF
Oil and Gas Extraction	211	211
Mining, except Oil and Gas	212	212
Support Activities for Mining	213	213
Utilities	22	22
Construction	23	23
Wood Products	321	321
Nonmetallic Mineral Products	327	327
Primary Metals	331	331
Fabricated Metal Products	332	332
Machinery	333	333
Computer and Electronic Products	334	334
Electrical Equipment, Appliances, and Components	335	335
Motor Vehicle, Bodies and Trailers, and Parts	3361-3	3361MV
Other Transportation Equipment	3364-9	3364OT
Furniture and Related Products	337	337
Miscellaneous Manufacturing	339	339
Food and Beverage and Tobacco Products	311, 312	311FT
Textile Mills and Textile Product Mills	313, 314	313TT
Apparel and Leather and Allied Products	315, 316	315AL
Paper Products	322	322
Printing and Related Support Activities	323	323
Petroleum and Coal Products	324	324
Chemical Products	325	325
Plastics and Rubber Products	326	326
Wholesale Trade	42	42
Motor Vehicle and Parts Dealers	441	441
Food and Beverage Stores	445	445
General Merchandise Stores	452	452
Other Retail	442-4, 446-8, 45 ex. 452	4AO
Air Transportation	481	481
Rail Transportation	482	482
Water Transportation	483	483
Truck Transportation	484	484
Transit and Ground Passenger Transportation	485	485

...

Sectors are defined according to the nomenclature used by the U.S. Bureau of Economic Analysis (BEA).

TABLE C.1: LIST OF CONSIDERED SECTORS AND THEIR CODES (*continued*)

	codes	
denomination	*NAICS 2017*	*BEA*
...		
Pipeline Transportation	486	486
Other Transportation and Support Activities	487, 488, 492	487OS
Warehousing and Storage	493	493
Publishing Industries, except Internet (includes Software)	511	511
Motion Picture and Sound Recording industries	512	512
Broadcasting and Telecommunications	515, 517	513
Data Processing, Internet Publishing, and Other Information Service	518, 519	514
Federal Reserve Banks, Credit Intermediation, and Related Activities	521, 5221, 5222, 5223	521CI
Securities, Commodity Contracts, and Investments	523	523
Insurance Carriers and Related Activities	5241, 5242	524
Funds, Trusts, and Other Financial Vehicles	525	525
Housing	·	HS
Other Real Estate	531	ORE
Rental and Leasing Services and Lessors of Intangible Assets	532, 533	532RL
Legal Services	5411	5411
Computer Systems Design and Related Services	5415	5415
Miscellaneous Professional, Scientific, and Technical Services	541 ex. 5411, 5415	5412OP
Management of Companies and Enterprises	55	55
Administrative and Support Services	561	561
Waste Management and Remediation Services	562	562
Educational Services	61	61
Ambulatory Health Care Services	621	621
Hospitals	622	622
Nursing and Residential Care Facilities	623	623
Social Assistance	624	624
Performing Arts, Spectator Sports, Museums, and Related Activities	711, 712	711AS
Amusement, Gambling, and Recreation industries	713	713
Accommodation	721	721
Food Services and Drinking Places	722	722
Other Services, except Government	81	81

Sectors are defined according to the nomenclature used by the U.S. Bureau of Economic Analysis (BEA).

(Network distances at the intensive margin) *In Section 3 of the main text I characterize distance matrices at their "extensive margin", $\boldsymbol{\mathcal{D}}^{fd}_{ext}$ and $\boldsymbol{\mathcal{D}}^{fs}_{ext}$ whose values are $d^{j}_{ext}\left[s,s'\right] = \left\{1,2,\ldots,d_{max}\right\}$ for $j = \left\{fd, fs\right\}$, considering the directed production network, $\boldsymbol{\mathcal{H}}$, that accounts for direct sectoral connections only.*

Instead, to characterize the "intensive margin" matrix distances $\boldsymbol{\mathcal{D}}^{fd}$ and $\boldsymbol{\mathcal{D}}^{fs}$ I follow Conley and Dupor (2003). Given $\alpha^{fd}\left(s,s'\right) = \frac{\alpha(s,s')}{\sum_{s' \in row}\alpha(s,s')}$, the generic element of factor input demand distance matrix is computed as follows:

$$d^{fd}\left[s,s'\right] = \left\{\sum_{k \in \Phi(s)}\left[\alpha^{fd}\left(s,k\right) - \alpha^{fd}\left(s',k\right)\right]^{2}\right\}^{\frac{1}{2}} \tag{C.1}$$

which defines the length of the line segment (i.e., the Euclidean distance) connecting sectors s and s' when both are buying from the same other sector-k, $\forall k \in \Phi\left(s\right)$, so that $\boldsymbol{\mathcal{D}}^{fd} = \left[d^{fd}\left[s,s'\right]\right]$ with generic element $d^{fd}\left[s,s'\right] > 0$.[49] *Demand distance is constructed by dividing each cell by its row sum. Why? A row sum represents the total sales of the sector in question, so dividing each entry by its associated row sum yields the share of that (row) sector's output purchased by each of the column sectors. This normalization allows for a meaningful comparison across dyads of sectors: by examining these shares, one can assess the relative distance between two (column) sectors in terms of their purchases from common upstream suppliers.*

In an analogous way, given $\alpha^{fs}\left(s,s'\right) = \frac{\alpha(s,s')}{\sum_{s' \in column}\alpha(s,s')}$, the generic element of factor input supply distance matrix is shaped by

$$d^{fs}\left[s,s'\right] = \left\{\sum_{k \in \Phi(s)}\left[\alpha^{fs}\left(k,s\right) - \alpha^{fs}\left(k,s'\right)\right]^{2}\right\}^{\frac{1}{2}} \tag{C.2}$$

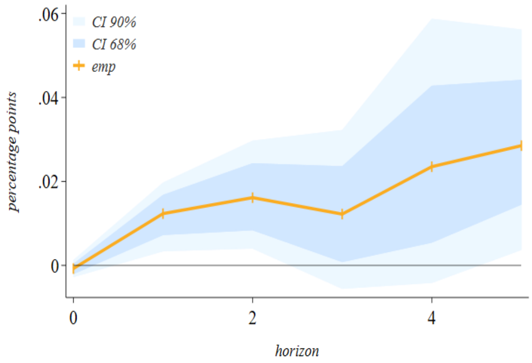
tracing a Euclidean distance between sectors $\left\{s,s'\right\}$ when both are selling to the same sector-k, $\forall k \in \Phi\left(s\right)$, so that $\boldsymbol{\mathcal{D}}^{fs} = \left[d^{fs}\left[s,s'\right]\right]$ with $d^{fs}\left[s,s'\right] > 0$. Supply distance is constructed by dividing each cell by its column sum. Why? A column sum represents the total purchases of the sector in question, so dividing each entry by its associated column sum yields the share of that (column) sector's input purchased from each of the row sectors. This normalization allows for a meaningful comparison across dyads of sectors: by examining these shares, one can assess the relative distance between two (row) sectors in terms of their traded quantities to common downstream buyers.

Both the distance matrices at the intensive margin are developed from the directed production network, $\boldsymbol{H} = \left[\alpha\left(s,s'\right) \geq 0\right]$.[50]

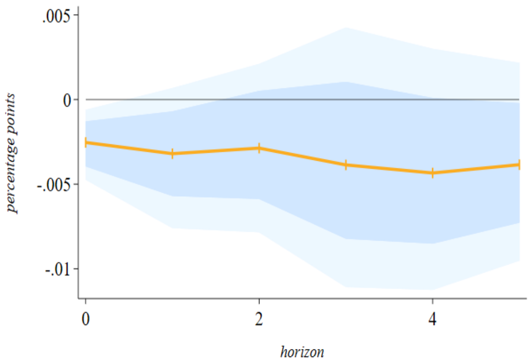
[49] Note how, to be consistent with the main text's notation, in eqs. (C.1)-(C.2) I am flipping the way sectors are defined when writing $d^{j}\left[\cdot\right]$, for $j = \left\{fd, fs\right\}$, compared to that in Conley and Dupor (2003), yet maintaining unaltered the main theoretical and empirical predictions.

[50] Recall that distance matrices are computed from an Input-Output structure in which each inter-sectoral trade linkage is greater than 1% of total purchases (or sales) of a sector.

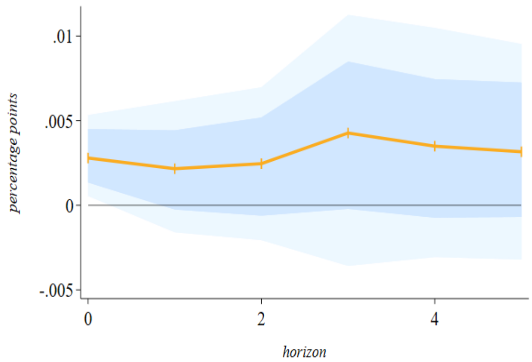
D. Complementary empirical results



(A) Sector-specific



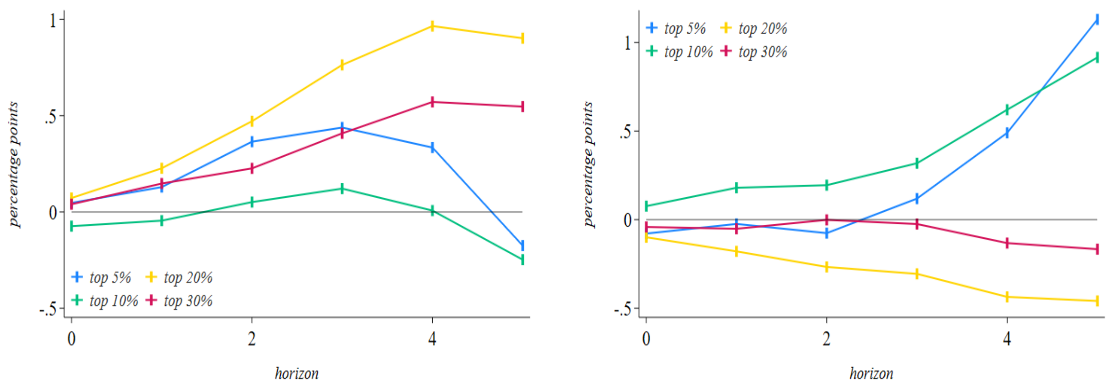
(B) $d = 1$



(C) $d = 2$

Figure D.1: Sectoral employment response to intermediate inputs

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the sector-specific set of intermediate inputs as a share of its value-added for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. In particular, Panel D.1a represents the sector-level employment response to changes in its intermediate inputs, while Panel D.1b-D.1c to changes in the set of intermediates in closer (distance equal to 1) and further (distance equal to 2) sectors. The solid-orange line corresponds to the average response of employment across sectors, while shadow-blue and shadow-light blue areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eqs. (15)-(16). *Source*: BEA and own calculations.

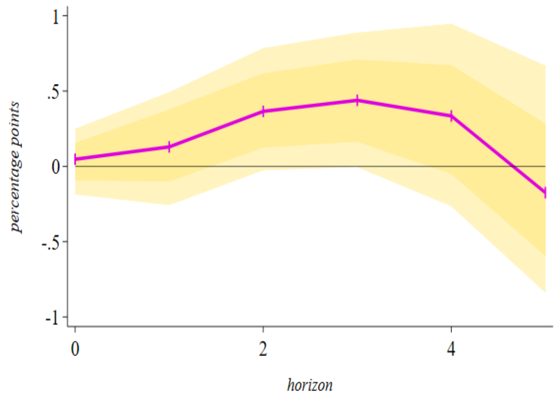


(A) $d = 1$ (B) $d = 2$

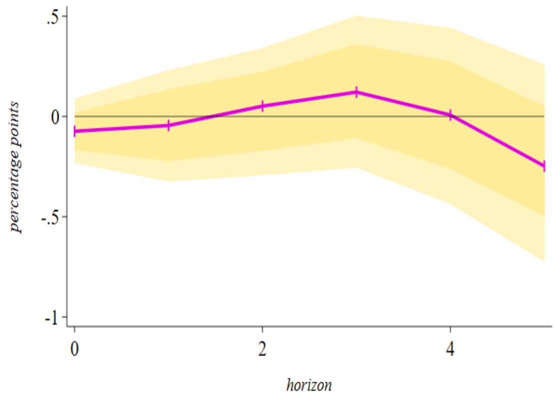
FIGURE D.2: DEMAND LINKAGES AND COMOVEMENT IN INTERLINKED SECTORS

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel D.2a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel D.2b plots the response to employment changes in further (distance equal to 2) ones. Each line corresponds to the average response of employment across sectors with different numbers of linkages, robust to 68% and 90% significance levels of bootstrapped Confidence Interval (CI) computed from eq. (18). If a plot is not appearing, it means there is no response for the specified distance value. *Source*: BEA and own calculations.

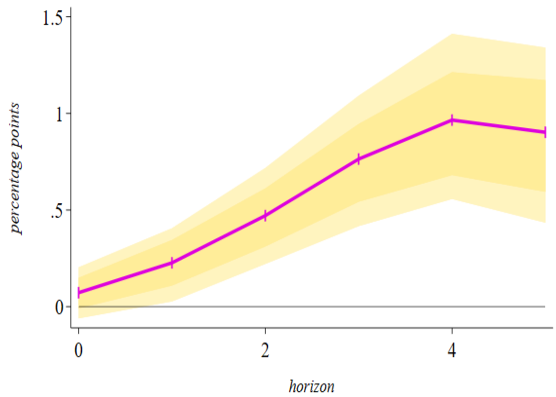
(Plotting separated results for highly interlinked sectors) *Unpacking all the combined plots, below I report the estimated Local Projection (LP) dynamics for top 5%, top 10%, top 20%, and top 30% of more connected sectors in the production network (i.e., major number of Input-Output linkages) separately. The purpose is to show the significance of the responses appearing in Panels 8a-8b of Figure 8 in Subsection 4.2 of the main text, and in Panels D.2a-D.2b of Figure D.2 in Appendix D.*



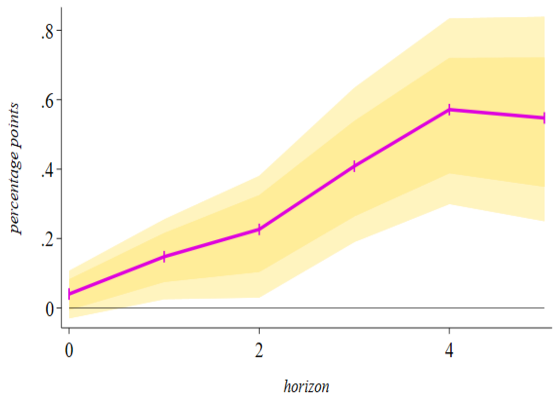
(A) TOP 5%



(B) TOP 10%



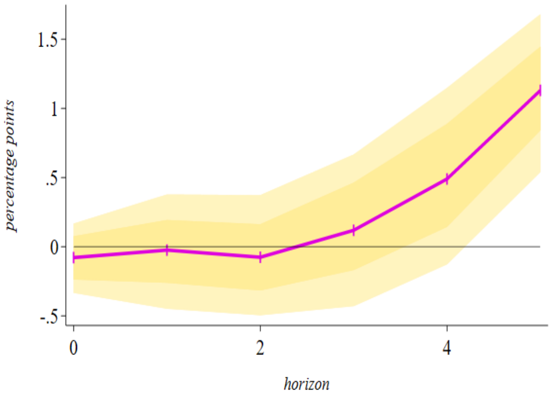
(C) TOP 20%



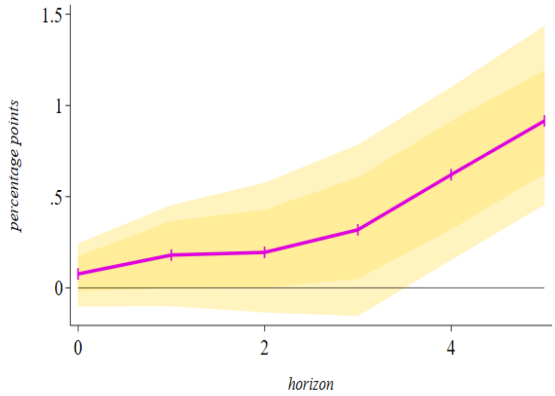
(D) TOP 30%

FIGURE D.3: DEMAND ($d = 1$) EFFECTS FOR INTERLINKED SECTORS

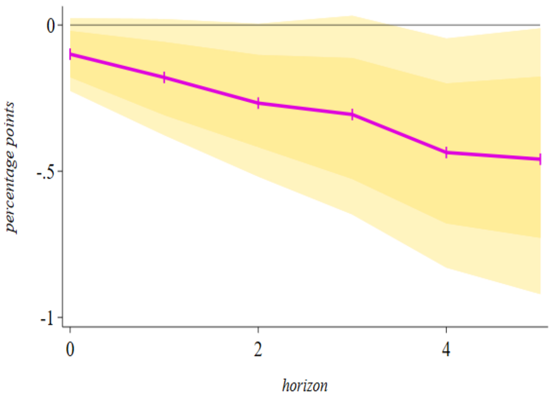
Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel D.2a of Figure D.2, each panel of this figure shows the employment response of top 5% (Panel D.3a), 10% (Panel D.3b), 20% (Panel D.3c), and 30% (Panel D.3d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.



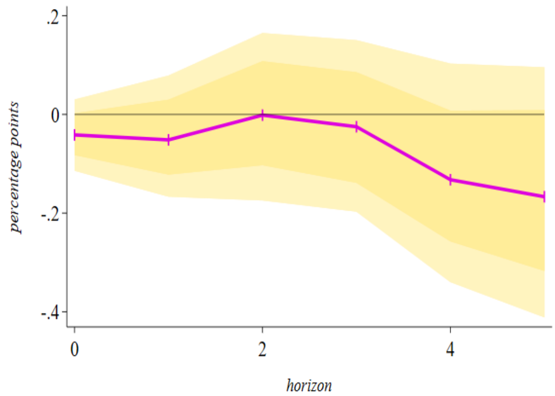
(A) TOP 5%



(B) TOP 10%



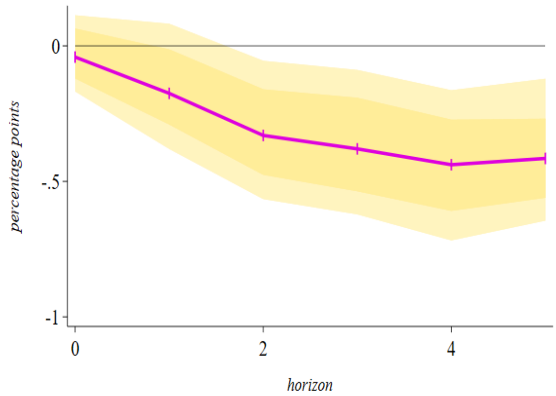
(C) TOP 20%



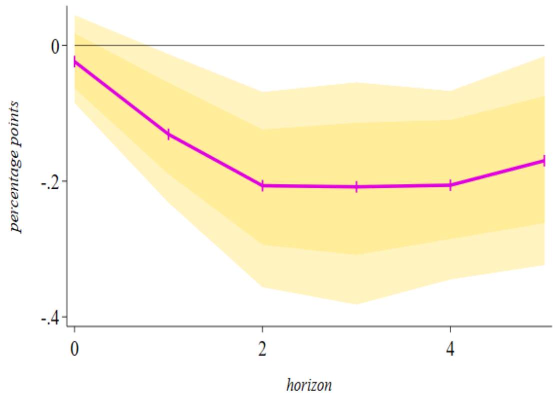
(D) TOP 30%

FIGURE D.4: DEMAND ($d = 2$) EFFECTS FOR INTERLINKED SECTORS

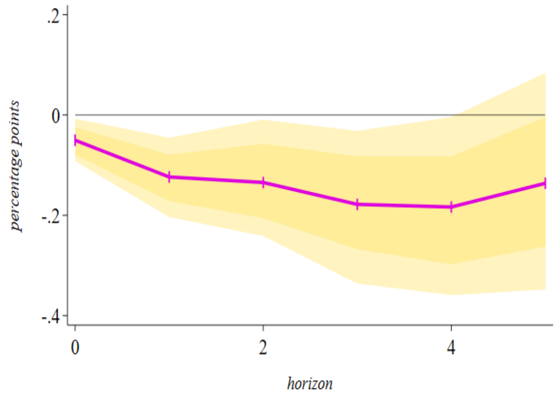
Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel D.2b of Figure D.2, each panel of this figure shows the employment response of top 20% (Panel D.4c) and 30% (Panel D.4d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) to employment changes in their further (distance equal to 2) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.



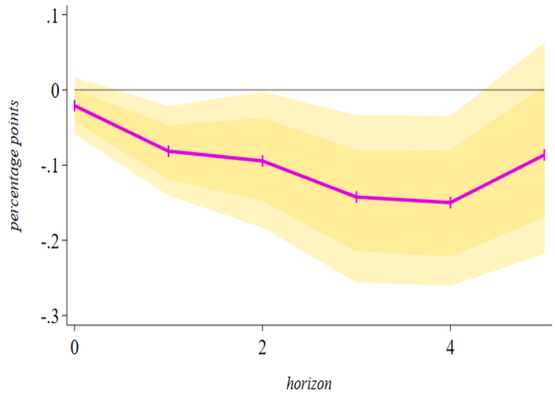
(A) TOP 5%



(B) TOP 10%



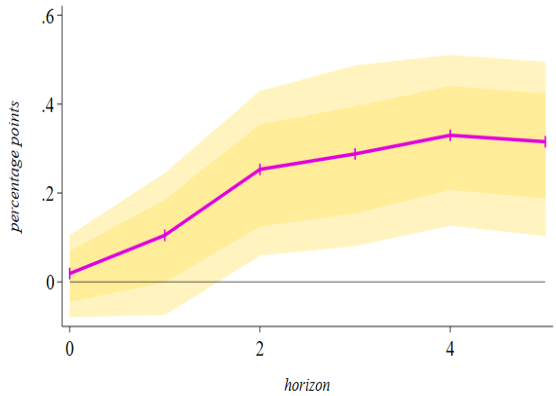
(C) TOP 20%



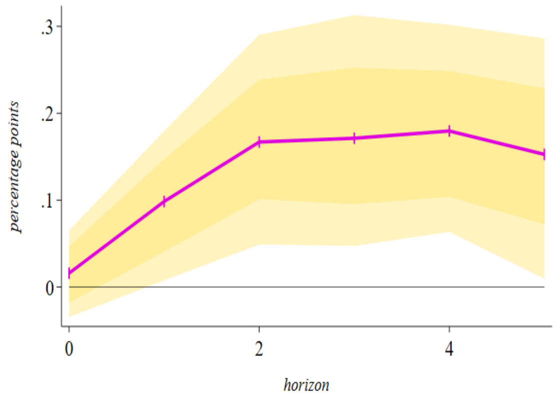
(D) TOP 30%

FIGURE D.5: SUPPLY ($d = 1$) EFFECTS FOR INTERLINKED SECTORS

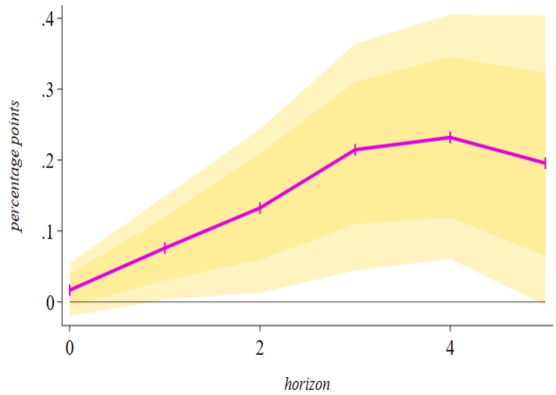
Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 8a of Figure 8, each panel of this figure shows the employment response of top 5% (Panel D.5a), 10% (Panel D.5b), 20% (Panel D.5c), and 30% (Panel D.5d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.



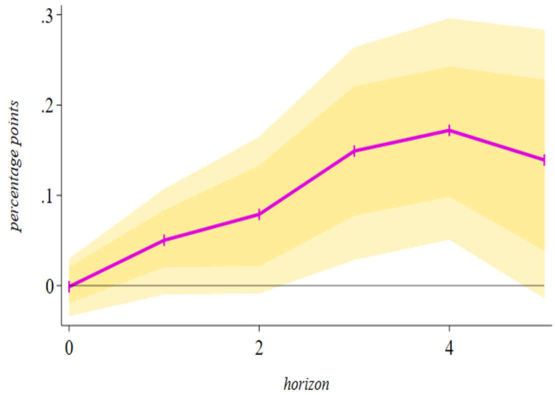
(A) TOP 5%



(B) TOP 10%



(C) TOP 20%



(D) TOP 30%

FIGURE D.6: SUPPLY ($d = 2$) EFFECTS FOR INTERLINKED SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 8b of Figure 8, each panel of this figure shows the employment response of top 5% (Panel D.6a), 10% (Panel D.6b), 20% (Panel D.6c), and 30% (Panel D.6d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their further (distance equal to 2) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.

(Robustness with centrality measures) *To identify the centrality degree of each sector, I follow the approach outlined in Carvalho (2014) to measure the sector-level Bonacich-Katz centrality.*[51] *Using the Leontief-inverse matrix $\boldsymbol{\mathcal{H}}$ for the U.S. production network as of 2007, the Bonacich-Katz centrality vector,* ***bk****, is defined as:*

$$\boldsymbol{bk} = \sum_{p=0}^{\infty} \phi^p \boldsymbol{\mathcal{H}}^p \mathbf{1} = \left(\boldsymbol{I} - \omega\,\boldsymbol{\mathcal{H}}\right)^{-1} \mathbb{1}, \tag{D.1}$$

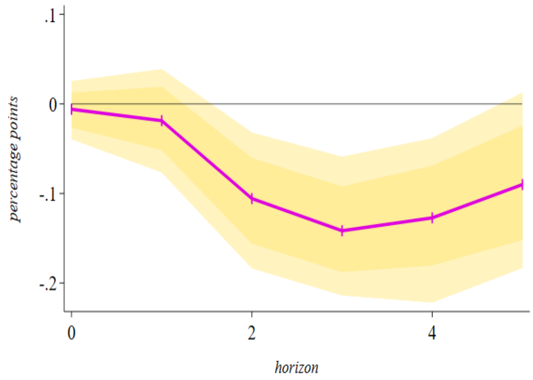
where index-p refers to the length of the paths in the production network – that is, the number of steps (or network "hops") connecting one sector to another – and, given a finite set of sectors $\{s, s', s'', \ldots, S\} \in \Phi(s)$:

- *$\mathbb{1}$ is an $S \times 1$ vector of ones, reflecting the base influence;*
- *$\omega \in \left(0, \frac{1}{\lambda_{max}}\right)$ is a dampening parameter, with λ_{max} being the largest eigenvalue of matrix $\boldsymbol{\mathcal{H}}$;*
- *$\boldsymbol{I}$ is the $S \times S$ identity matrix.*

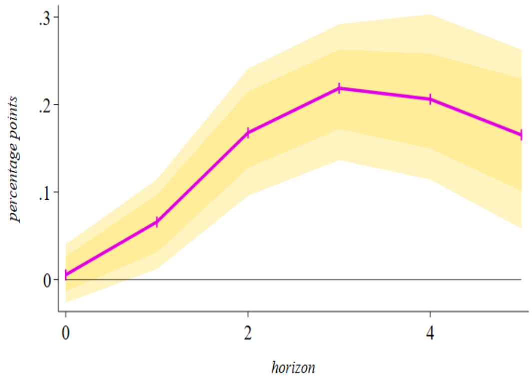
The value of ω determines the weight assigned to indirect linkages: smaller values place more emphasis on direct connections, while larger values account for deeper propagation within the network. Following Carvalho (ibid.), a value of $\omega = 0.5$ is typically used, which lies safely below the inverse of the largest eigenvalue of most empirically observed Input-Output matrices, ensuring convergence of the series. This measure reflects the idea that a sector is central not only if it is directly connected to many others, but also if it is connected to sectors that are themselves highly central.

This formulation ensures that the centrality score of each sector captures both its direct connections and its indirect influence through other sectors due to the use of matrix $\boldsymbol{\mathcal{H}} = \left(\boldsymbol{I} - \boldsymbol{H}\right)^{-1} = \left[\ell(s, s') \geq 0\right]$, where $\boldsymbol{H} = \left[\alpha(s, s') \geq 0\right]$ is the directed production network matrix (i.e., that in the BEA Input-Output tables), defining the intensity of good produced by sector-s' in the total intermediate inputs used by sector-s, with $\alpha(s, s') = 0$ indicating that sector-s does not make use of the good produced by sector-s' in producing its own intermediate good.

[51] Outlined by Bonacich (1987), it is a measure of a sector's overall importance within an Input-Output system, capturing not only its direct connections to other sectors but also the centrality of those it is connected to. In essence, a sector is considered central if it is linked to other central sectors, creating a recursive structure of influence. This measure goes beyond simple counts of linkages by incorporating indirect connections – weighted by a dampening factor – to reflect the diminishing influence of more distant sectors in the network. In the context of production network, a sector with high Bonacich-Katz centrality is one that plays a key role in the propagation of shocks, as its position allows it to influence (or be influenced by) large portions of the economy through both direct and indirect channels. Compared to other centrality measures, Bonacich-Katz is particularly well-suited for economic applications, as it accounts for the intensity of inter-sectoral relationships and the structure of the network in a realistic and nuanced way.



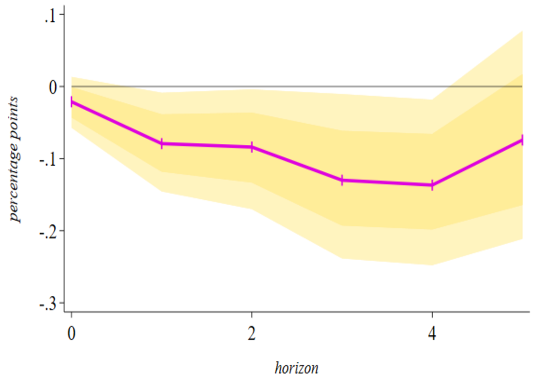
(A) $d = 1$



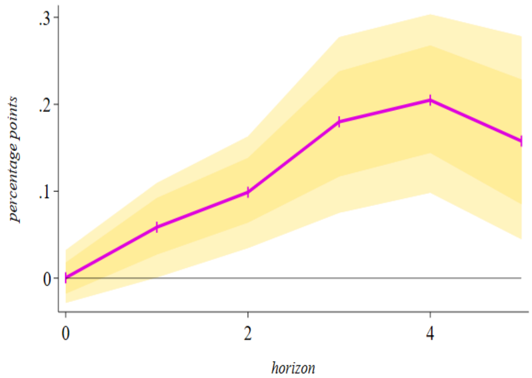
(B) $d = 2$

FIGURE D.7: DEMAND LINKAGES AND COMOVEMENT IN PERIPHERAL SECTORS

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of most central sectors (with *centrality* defined according to sector's relevance in final consumption) to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel D.7a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel D.7b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.



(A) $d = 1$

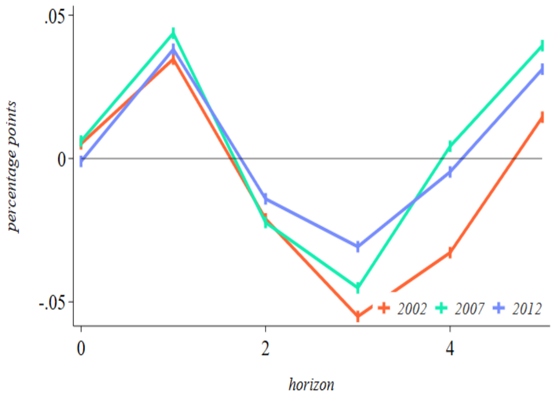


(B) $d = 2$

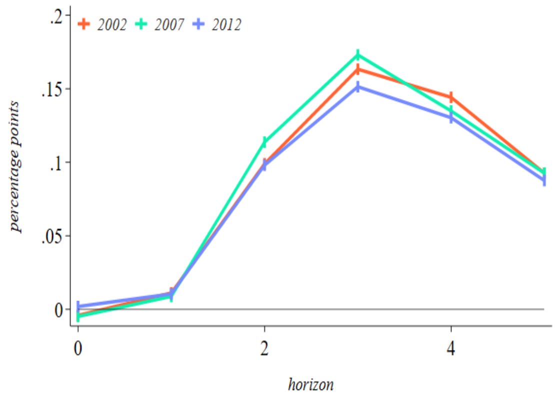
FIGURE D.8: SUPPLY LINKAGES AND COMOVEMENT IN PERIPHERAL SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of most central sectors (with *centrality* defined according to sector's relevance in final consumption) to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel D.8a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel D.8b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (18). *Source*: BEA and own calculations.

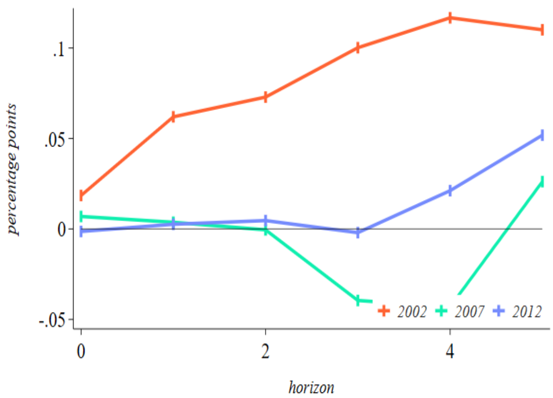
(Robustness with production network at different base years) *Responses for comovement in sectoral employment levels in the main analysis of Section 4 are centred on the implementation of a sectoral production network represented by an Input-Output matrix in the baseline year 2007. Differently, Figures D.9, D.10 and D.11 perform again all the main analysis by referring, instead, to a production network structure five years before (2002) and five years after (2012) the baseline year. Note that Input-Output matrices always correspond to their Leontief inverse configuration.*



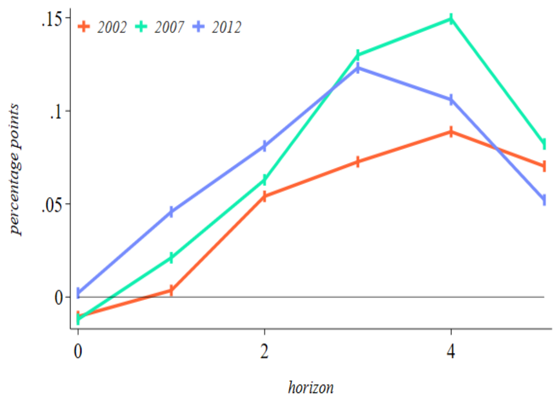
(A) DEMAND LINKAGES, $d = 1$



(B) DEMAND LINKAGES, $d = 2$



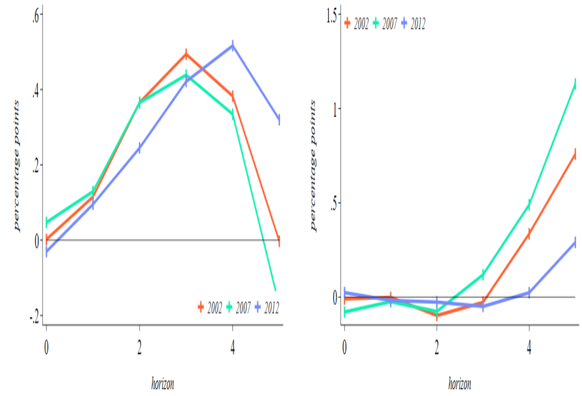
(C) SUPPLY LINKAGES, $d = 1$



(D) SUPPLY LINKAGES, $d = 2$

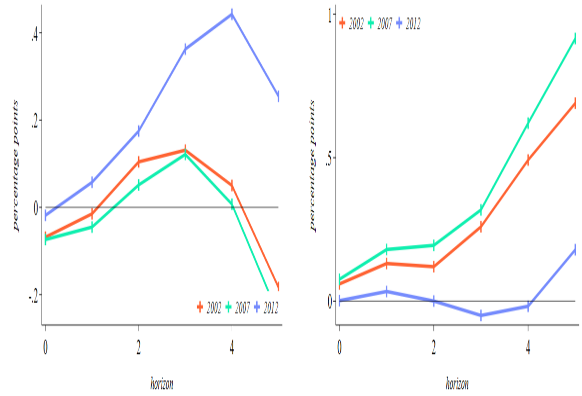
FIGURE D.9: COMOVEMENT OVER DIFFERENT BASELINE YEARS

Note: given both the *factor input demand* (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *factor input supply* (*i.e.*, supply linkages across sectors given their common downstream buyers) network distances, and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the results in the main text: Panels D.9a-D.9b expand their relatives of Figure 6, while Panels D.9c-D.9d do so for their relatives in Figure 7. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (18), roughly correspond to the associated figures in the main text. Responses are referred to employment changes in closer (distance equal to 1) and further (distance equal to 2) sectors. *Source*: BEA and own calculations.



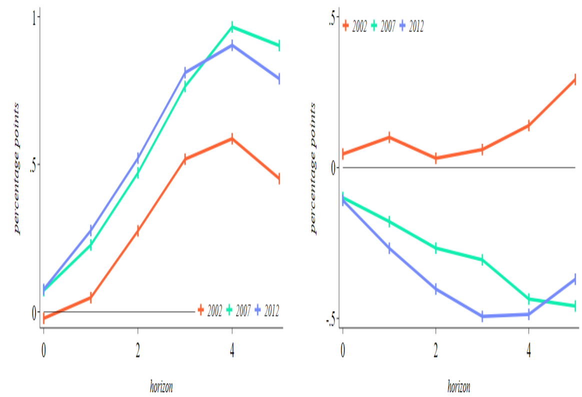
$d = 1$ $d = 2$

(A) TOP 5%



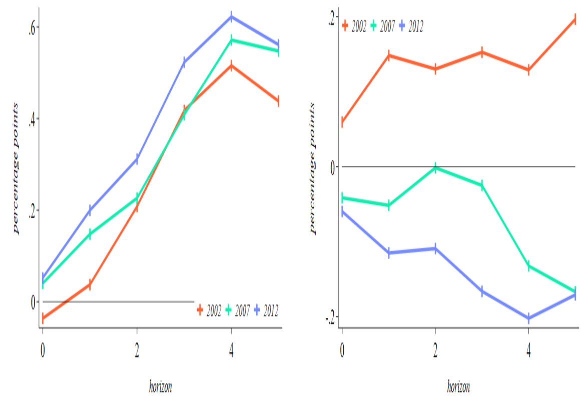
$d = 1$ $d = 2$

(B) TOP 10%



$d = 1$ $d = 2$

(C) TOP 20%

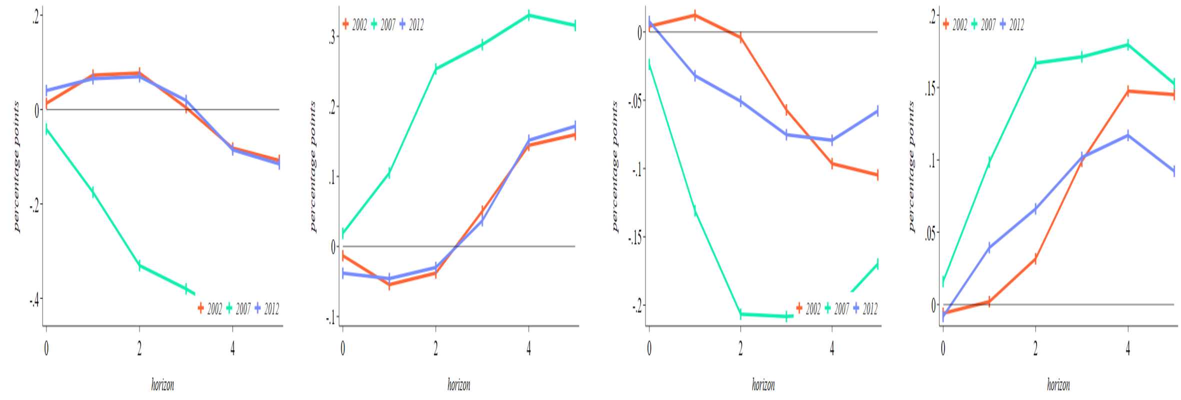


$d = 1$ $d = 2$

(D) TOP 30%

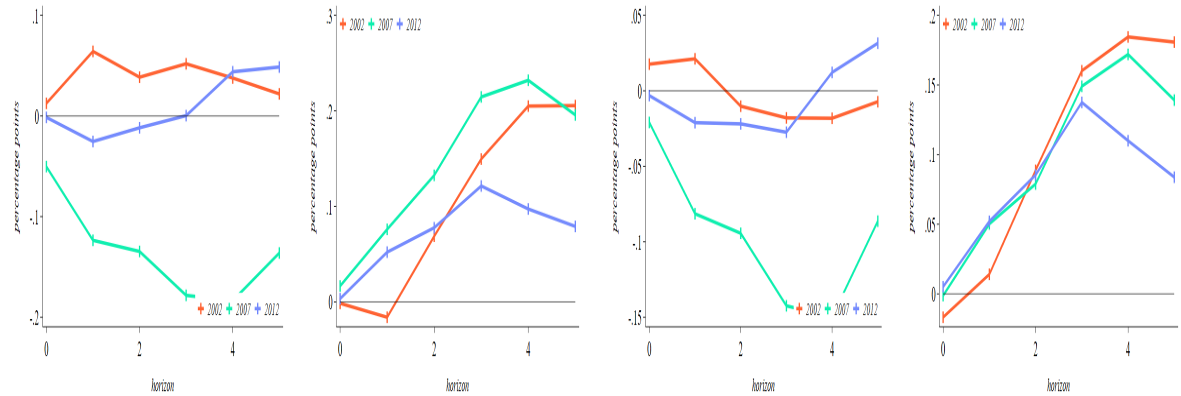
FIGURE D.10: DEMAND LINKAGES AND INTERLINKED SECTORS OVER THE YEARS

Note: given both the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the responses of Panels D.2a-D.2b of Figure D.2, each panel of this figure shows the employment response of top 5% (Panel D.10a), 10% (Panel D.10b), 20% (Panel D.10c), and 30% (Panel D.10d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) or further (distance equal to 2) sectors. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (18), roughly correspond to the associated figure. If a plot is not appearing, it means there is no response for the specified distance value. *Source*: BEA and own calculations.



$d = 1$ $d = 2$ $d = 1$ $d = 2$

(A) TOP 5% (B) TOP 10%



$d = 1$ $d = 2$ $d = 1$ $d = 2$

(C) TOP 20% (D) TOP 30%

FIGURE D.11: SUPPLY LINKAGES AND INTERLINKED SECTORS OVER THE YEARS

Note: given both the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the responses of Panels 8a-8b of Figure 8, each panel of this figure shows the employment response of top 5% (Panel D.11a), 10% (Panel D.11b), 20% (Panel D.11c), and 30% (Panel D.11d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) or further (distance equal to 2) sectors. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (18), roughly correspond to the associated figure. *Source*: BEA and own calculations.

References in appendix

ACEMOGLU, D., V. M. CARVALHO, A. OZDAGLAR, and A. TAHBAZ-SALEHI, "The network origins of aggregate fluctuations", *Econometrica*, vol. 80, no. 5 (2012), pp. 1977-2016. 77.

BONACICH, P., "Power and centrality: a family of measures", *American Journal of Sociology*, vol. 92, no. 5 (1987), pp. 1170-1182. 88.

CARVALHO, V. M., "From micro to macro via production networks", *Journal of Economic Perspectives*, vol. 28, no. 4 (2014), pp. 23-48. 77, 88.

CONLEY, T. G. and B. DUPOR, "A spatial analysis of sectoral complementarity", *Journal of Political Economy*, vol. 11, no. 2 (2003), pp. 311-352. 81.

OULTON, N., "The Divisia approach to measuring output and productivity: with an application to the BEA-BLS integrated industry-level production account, 1987-2020", *Discussion Paper 2022-17, Centre For Macroeconomics* (2022). 58.

ROGERSON, R., "Indivisible labor, lotteries and equilibrium", *Journal of Monetary Economics*, vol. 21, no. 1 (1988), pp. 3-16. 59.

VOM LEHN, C. and T. WINBERRY, "The investment network, sectoral comovement, and the changing U.S. business cycle", *Quarterly Journal of Economics*, vol. 137, no. 1 (2022), pp. 387-433. 58.