



DEPARTMENT OF ECONOMICS
UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

**A Reconciliation of the Evidence about Bank
Lending with Portfolio Theory**

Enzo Dia

No. 56 – October 2002

Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
<http://dipeco.economia.unimib.it>

A reconciliation of the evidence about bank lending with portfolio theory*

Enzo Dia[†]

September 2002

Abstract

A Markowitz portfolio model is developed in this paper, in order to show how the optimal portfolio is changed if the bank forecasts a shock. It is shown that monetary and real shocks result in quite different patterns of reaction by the bank and these results do not depend on the existence of market power. The model allows to study the effect of different degrees of diversification of the portfolio and shows that the smoothing of shocks increases with the concentration of the portfolio. The shock determines a shift of the composition of the portfolio of assets: the bank increases the share of the less volatile assets that have a lower return as they charge a lower rate to the firm. This phenomenon emerges whenever a variation of the interest rates of any asset affects the cost or the volatility of the asset too, so that equiproportional variations of the rates affect net return and variance of different assets in a different way. The benefits of this kind of behaviour decline with the availability of a wider range of assets. These results are consistent with all the most recent empirical evidence.

*I would like to thank Jaques Mélitz, Andrew Hughes Hallett, Nicola Coniglio and John Lewis for many useful discussions and suggestions.

[†]Università degli studi di Pavia e University of Strathclyde. Email: enzo.dia@strath.ac.uk

1 Introduction

Relationship lending has been the subject of a variety of theoretical and empirical works that have shown its importance in determining the peculiarities of the banking sector. One of the outcomes of this literature has been the development of an implicit contract theory, which, in analogy with some studies of the labour market, has shown how banks might be willing to establish implicit contracts with their borrowers, in order to smooth intertemporally the effects of shocks. According to the theory, banks would be willing to bear part of the cost caused by a negative shock, either a monetary shock or a real shock because the long term character of the contractual relationship means they can charge their borrowers higher costs for their services in other periods. In the long run profits of the bank would be higher, owing to the reduction of default costs. This theory implies that banks have some sort of market power, otherwise they could not avoid being undercut in periods of boom and the implicit contract could not survive. In particular, it assumes a certain degree of stickiness of bank interest rates and of sluggishness of their reaction to shocks.

Most empirical studies have confirmed that the credit system, cannot be described as a perfectly competitive market. Transaction costs, uncertainty and asymmetric information are all very relevant in the market, suggesting the need for the long term relationship and the competitive structure of the industry. The stickiness of bank interest rates, of both loans and deposits, is one of the stylised facts that are generally accepted. Besides, recent works have all shown that the econometric evidence is quite consistent with all the implications of the implicit contract theory, while all alternative explanation of the empirical evidence are not entirely convincing.

The aim of this work is to develop a Markowitz Portfolio model, in order to analyse how the structure of the portfolio of the bank is changed, in response to different kinds of shocks. The model shows that assuming banks to be risk averse, the arguments of the implicit contract theory are reinforced. Our results are consistent with the most recent empirical evidence and shed some light on the driving forces determining the behaviour of banks. Section two of the work introduces the empirical studies on bank lending, illustrating in particular the results regarding the effects of shocks on the portfolio. They are quite supportive of the implicit contract theory, but show that different kinds of shocks exert different effects on the portfolio. This result cannot be easily explained by the implicit contract theory. In section three a Markovitz

type of mean variance portfolio model is derived for a banking intermediary. The main extension of the basic model is the introduction of an expected default cost function that takes into account that asymmetric information problems might determine a dependence of the expected default cost on the level of interest rates. Section four shows a portfolio model that includes transaction costs and default costs. In order to study different patterns of bank relationships, deposits are partitioned between firms and households deposits; loans are segmented in two classes, normal loans and commitment loans. The partition of deposits reflects the different sets of banking services that determine the demand for deposits and, accordingly, their cost. Loans are divided in two classes that reflect different degrees of closeness in the relation between the bank and its borrowers, according to the availability of inside information on the quality of the projects that the bank can obtain. The model is used to analyse the effect of shocks on the portfolio, both interest rate shocks and real shocks (that affect the bank as credit quality shocks). The theoretical results are in accordance with the empirical estimates conducted in the US.

2 Relationship lending and risk: the empirical evidence

The importance of long term relations between the bank and its borrowers has been largely recognised in the literature of the microeconomics of banking.¹ The consideration of the importance of personal relationships in banking contracts has lead to the development of a theoretical argument justifying the emergence of implicit contracts between the bank and its borrowers. According to the theory, banks often implicitly agree to insure borrowers against the effects of shocks, accepting to bear part of the cost produced by the shock, but charging as a compensation for this service a higher price, generally in the form of a higher interest rate.² Due to the long term character of the relation with their customers, banks can enter in a risk sharing agreement with their borrowers, sharing the risks associated with an uncertain future. In the case of monetary shocks, banks would be willing to smooth interest

¹Since the appearance of Fama [9].

²Fried and Howitt [10] showed that interest rate smoothing is part of an optimal contract.

rate shocks intertemporally, thus avoiding transmitting the variation of the rate in full.³ When a real shock hits the economy, it raises the risk of the portfolio of loans. The bank might absorb part of the shock, by refraining from contracting the portfolio of loans to the extent that the increase of non-performing loans would otherwise require. As a compensation for this service it would charge a higher average interest rate, increasing the rate when market conditions are more favourable.

A few empirical works have tested some of the implications of the theory. The results seemed to support most of the main theoretical predictions. A first, necessary condition that has been tested is that interest rates on loans are sticky. Empirical studies conducted in different periods and different markets showed that rates on loans are less volatile than other interest rates, so that it looks like being a permanent characteristic of the market for banking credit. In general, many different econometric estimations have shown that the commercial loan rate is slow to adjust to open market rate changes. Banks would smooth the effects of monetary policy, reducing the effects of monetary contraction in order to prevent too many bankruptcies, compensating the effect on their balance sheets through a slower reduction of interest rates, in periods of monetary expansion.

Petersen and Rajan [18] showed that the availability of financing increases for firms that have close ties with an institutional creditor. According to their results, "relationships are valuable and appear to operate more through quantities rather than prices".⁴ Their work is based on the analysis of a data set of small firms in the USA. Berger and Udell [3], in a similar work appeared the following year, showed that "borrowers with longer relationships pay lower interest rates and are less likely to pledge collateral".⁵ They use data for small business, and the main difference with respect with the former work is their focus on loans issued under lines of credit, a restricted set of loans, the one most likely to be affected by the nature of the relationship. A study by Cole [8] has analysed "The importance of relationship to the availability of

³*If loans were negotiated in spot auction markets then customers would then be exposed to the risk of fluctuating interest rates on loans. A bank may be willing to insure the customer against part of such risk by a policy of keeping interest rates less variable than they would be in spot auction markets, in return for which the customers may be willing to compensate the bank in the form of a higher average interest rate.* Fried and Howitt [10], p. 472.

⁴Petersen and Rajan [18], p. 3.

⁵Berger and Udell [3], p. 351.

credit”, confirming its importance for credit relations. It seems likely that a bank cannot easily and quickly increase the number of its borrowers because it has to evaluate their solvency. If it wishes to rapidly expand its activities it has to rely on already existent relationships, and this represents a severe constraint on the possibility of expanding the number of loans granted. This might lead the bank to have different kind of contractual agreements with different classes of customers.

Berlin and Mester [4] [5], provided further evidence in favour of the hypothesis of intertemporal smoothing through the exploitation of long term relationships. These works are important as well because they are the first to test empirically the reaction to shocks. The first of the two works was based on a panel of data consisting of information regarding the terms of each loan of a set of US banks. This permitted the authors to estimate the degree of smoothing and to run a regression of smoothing on profits. They considered a positive relation to be a proof of the optimality of a risk sharing implicit contract. The relation they found was significantly positive in the case of an *interest rate shock*, significantly negative in the case of a *credit risk shock*. According to their results ”in general loan rate smoothing in response to a *credit risk* shock is not part of an optimal long-term contract between a bank and its borrower, while loan rate smoothing in response to an *interest-rate shock* is”.⁶ Only for the smallest subset of banks a significant positive relation could be found for a credit risk shock. After testing for the alternative hypothesis that the relation could be driven by inefficient pricing, which they rejected, they concluded that interest rates smoothing is part of an efficient contract between banks and borrowers for all classes of banks, only in the case of interest rate shocks. The second work is based on the same dataset as the previous one, and considers the relation between smoothing and core deposits that are deposits whose interest rates are particularly inelastic. They found that ”banks more heavily funded with core deposits provide borrowers with smoother loan rates in response to changes in aggregate credit risk.”⁷ The reason is that core deposits insulate the bank’s cost of funds from shocks, so that banks that rely mainly on core deposits can provide more loan rates smoothing in response to shocks. These findings are probably not at odds with the findings of the former work, as it is widely recognised that small banks are the ones that rely more heavily

⁶Berlin and Mester [4], p. 873. Author’s italic.

⁷Berlin and Mester [5] p. 580.

on core deposits as a source of finance. A higher dependence on deposits is normally peculiar to small banks, and this work seems to justify the widely held opinion that small local banks are in general more willing to smooth shocks.

3 The portfolio problem of the bank

Markovitz type of portfolio models are based on the assumption that returns and risks are exogenous and the quantities of each security held is the choice variable of the bank. In order to take into account the effect of information costs and default costs in the portfolio choice, we describe the problem of the bank as a two-stage process. We assume that the bank classifies different available groups of assets or liabilities in categories, on the base of institutional properties and of the sector of activity of the counterpart. In the first stage the bank evaluates the optimal amount of investment in information that is worth taking for every class of security that it might choose to hold. External information is available to any participant in the market at a relatively small cost that we consider to be part of the transaction cost necessary to purchase the security. Certain categories of assets and liabilities, such as deposits and many classes of loans, are available to banks only. Relationship lending allows the bank to obtain internal information, that is not available to the market in general, but in order to establish the relationship the bank has to bear an information cost. Assuming the existence of a competitive market where rates are set exogenously, the problem can be formulated as the optimal choice of loan quantity and investment in information.⁸ Information affects the returns of the bank reducing expected default costs. The default probability can be formalised as a function of the information regarding the single loan and the quantity of the loan itself. The expected return of a loan is:

$$E[\pi] = r_L L - p(1 + r_L)L - FC - sz, \quad (1)$$

where r_L is the interest rate on loans, L is the quantity of the loan, FC is a fixed cost, sz is the cost of information and p is the subjective evaluation on part of the lender of the probability of default of the single loan. p is defined as

$$p = f(q, l) = f(z + a, L). \quad (2)$$

⁸As in Aigner and Sprengle [1].

The parameter a represents the available stock of information, accumulated in previous periods through relationship lending, z is the amount of information that the bank can obtain in the short run, incurring in a cost that for simplicity is assumed to be linear and equal to sz .

The investment in information is worthwhile as long as the reduction of default cost that it produces is larger than the cost of the information. Under standard assumptions regarding the cost functions there is an optimal amount of investment in information z , and an optimal quantity of the single loan L , which minimise the default cost. It is important to observe that the level of the default cost crucially depends on the value of the stock a , that is the result of the investment entertained in the past. The investment in information reduces the default cost of every single loan and the average default cost of a class of loans, given the exogenous possibility of diversification inside the particular class. It is the availability of internal information, obtained through relationship lending, that allows the bank to reduce the average cost of default, while the market cannot reduce default costs by means of the available external information. The bank accordingly classifies the securities under consideration in the portfolio problem, calculating an expected return for every security (and the corresponding expected variance) composed of three terms: the basic interest rate component, the transaction cost (that includes mainly the cost of obtaining and processing the information), and the default cost. Every class is composed of many different securities, but we will assume that the choice of the quantity of every security inside the class is fixed in the first stage. The degree of diversification inside a class is very important, but we will assume that it depends on the peculiarities of the sector itself. For example we can imagine that as a policy the bank chooses to issue mortgages of amounts lying within a predefined range. The degree of diversification of the whole portfolio of mortgages will be constant and will depend on market conditions at the moment of the first-stage decision. Mortgages are a class of loans that are diversified to a certain degree, which is higher than the one for other types of loans, for example industrial loans, because the mortgages are issued in larger numbers of smaller loans than are loans to industrial firms. We will study the portfolio choice among different classes.

3.1 Portfolio theory and banks

The Markovitz portfolio approach has been applied to banks and other financial intermediaries, assuming banks to be risk averse agents that maximise a concave utility function where profits are the argument. Assuming all returns to be exogenous, the optimal portfolio of assets and liabilities can be jointly obtained. We base our work on the model developed by Hart and Jaffe [11] and on the extensions of the theory that Szegö [22] has introduced. The main result of the model is a separation theorem that shows that the choice of the optimal composition of the portfolio (in terms of the relative size of asset shares) is independent from the choice of the optimal size of the portfolio itself. The other major conclusion is a comparative static theorem that we use largely in this work. In order to establish the comparative static results, only a fairly general assumption regarding the utility function is required. The requirement is that the utility function exhibit non-increasing absolute risk aversion. This restriction is often considered to be generally acceptable, and seems very reasonable for a financial intermediary. The main flaw of the model is that under the assumption that interest rates on assets are higher than on liabilities, the bank might be willing to expand its portfolio indefinitely. This problem though can be solved, introducing the innovations proposed by Szegö [22].

The model of Hart and Jaffe assumes that there are no risk free assets and that net worth of the intermediary is a small part of its liabilities and can be set to 0 or considered as another liability. The bank chooses simultaneously the optimal quantity of both assets and liabilities; liabilities have a negative sign, assets a positive one. A portfolio is represented by the vector \mathbf{x} , with

$$\sum_{l=1}^{j+i} x_l = 0, \quad (3)$$

where the index j represent liabilities and i assets and the first j components are non positive while the last i are non negative. Assuming R_i to be a random variable that measures the real return of the security, the expected value of the real wealth of the bank is

$$E(W) = \sum_{l=1}^{j+i} x_l R_l. \quad (4)$$

The average return on every assets is assumed to exceed that on liabilities.

The standard deviation of W is given by $\sigma = (\mathbf{x}'S\mathbf{x})^{\frac{1}{2}}$ where

$$\mathbf{S} = \begin{bmatrix} \sigma_{11} & \cdot & \cdot & \cdot & \sigma_{1,j+i} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{j+i,1} & \cdot & \cdot & \cdot & \sigma_{j+i,j+i} \end{bmatrix}$$

Imposing a restriction on the utility function, the Arrow-Pratt hypothesis of non-increasing absolute risk aversion, some interesting comparative static result can be obtained.⁹ The main results of their comparative static analysis are synthesised in the following theorems¹⁰:

1. If the expected return on one security rises, *ceteris paribus*, the quantity of the security held does not fall. If the quantity held is not zero, it will rise.
2. If the covariance of two securities rises, *ceteris paribus*, the product of the two quantities does not rise. If the product of two is different from 0 (if both securities are held in the portfolio) the product itself falls.
3. If the variance of any security rises, *ceteris paribus*, the absolute value of the quantity held will not rise. If the quantity held is not zero, it will fall.

The second result holds either if the two securities are both assets or both liabilities or if one of is an asset, the other a liability. The interpretation of the result is opposite, in the two cases. When the two assets have the same sign, the theorem states that an increase in the covariance determine a reduction in the quantity held of both, or if one of the two is increased,

⁹In order to rule out the possibility of the existence of riskless non-zero portfolios some assumption regarding S is required. The simplest possible is that S is positive definite, assumption that implies that no riskless security is part of the portfolio. A weaker assumption that implies that no two securities can be riskless is sufficient:

$$\mathbf{x}'S\mathbf{x} > 0. \tag{5}$$

The other important assumption of the model is that there is not a positive and fixed net worth component of liabilities.

¹⁰Hart and Jaffe, 1973,[11] pp. 140.

the reduction in the other one has be larger. On the other hand, when one the securities is a liability, the other an asset, since they have opposite sign, the effect of the increase of the covariance on the quantities held is opposite. If the covariance increases, the quantity held of both securities is increased, or the quantity of one of the two is reduced, the quantity of the other is increased more than proportionally. The reason is that since the two securities have opposite sign, a stronger covariance is analogous to a higher negative correlation between two assets in the standard portfolio problem. So the bank is willing to increase the holding of the two, because it can hedge the asset buying the correlated liability or the other way round.

3.2 Assets returns

3.2.1 Interest rate component

The definition of different risk categories is necessary in order to model the portfolio choice of a bank and its behaviour towards risk. Our main aim is to evaluate different diversification policies. Accordingly we will concentrate on the factors affecting single banks, leaving aside systemic risk due to the possibility of a bank run, which we will consider to be dealt with by public authorities. We will not consider a second factor that could be relevant. Different arrangements of the capital structure could possibly guarantee different levels of protection from the exposure to exogenous factors, but to focus on the main problem, we will consider the capital structure of the bank to be irrelevant, at least in the first instance. The set of risk factors affecting the balance sheet of the bank is assumed to be exogenous. We will consider returns from the portfolio to depend on three distinct sets of factors, interest rates, default costs and transaction costs. Interest rates are assumed to be exogenous and to depend on a rate set by the central bank, whose variations determine a shift of the return of all assets. Different securities available to the banking firm are characterised by a different mark up (normally positive, negative in the case of deposits) with respect to the interest rate set by the central bank:

$$r_i = r_{cb}[1 + f(i)]; \quad (6)$$

the rate set by the central bank is assumed to behave as a random walk

$$r_{t+1}^{cb,e} = r_t^{cb} + \epsilon_t^{cb}. \quad (7)$$

Every asset return is subject as well to an idiosyncratic shock that we assume to be white noise, so that

$$r_{t+1}^{i,e} = r_t^{cb}[1 + f(i)] + \epsilon_t^{cb} + \epsilon_t^i, \quad (8)$$

where the subscript e indicates the expected values. The securities we are considering could be both assets and liabilities. The main difference in the problem of the bank is that they enter in the portfolio with an opposite sign. Since the basic portfolio model requires returns to be set exogenously, what we are implicitly assuming is that banks divide loans and deposits in classes, whose interest rates are set according to a fixed rule, as a mark up on the interest rate set by the central bank.

3.2.2 An expected default cost function

In every period, the bank has to forecast return and variance of its portfolio of loans. The problem we are considering here is the evaluation of those factors of risk that affect the probability of default of the loans that compose the portfolio. Our assumption is that on the basis of the stock of its private information, due to past and current investment, the bank makes its evaluation of the risk factors affecting every single loan. The probability of default of single individual loans can be considered to be affected by idiosyncratic shocks that we assume to behave as random walk with a drift. Formally

$$U_{it} = U_{it-1} + \mu_i + \epsilon_{it} = U_{i0} + \mu_i t + \sum_{k=1}^t \epsilon_k, \quad (9)$$

where U_{it} is the specific risk factor of the i^{th} loan, with $i = 1...n$; μ is the drift factor, and ϵ is the term. The expected value is

$$E[U_{it}] = U_{i0} + \mu_i t, \quad (10)$$

the variance

$$Var[U_{it}] = t\sigma_\epsilon^2. \quad (11)$$

The risk for the portfolio of loans that define every class is a weighted average, with weights represented by the share of the i^{th} loan in the entire portfolio that we measure as $h_i = \frac{L_i}{L}$. The share of every loan inside a class

is exogenous, because the optimal quantity of every loan is established in the first-stage process, together with the optimal investment in information.

$$U_t = \sum_{i=1}^n h_i U_{it} = \sum_{i=1}^n h_i (U_{i0} + \mu_i t + \sum_{k=1}^t \epsilon_k), \quad (12)$$

its expected value is given by

$$E[U_t] = \sum_{i=1}^n h_i (U_{i0} + \mu_i t), \quad (13)$$

its variance by

$$Var[U_t] = \sum_{i=1}^n h_i^2 t \sigma_\epsilon^2. \quad (14)$$

The last equation shows how the variance of the risk is affected by diversification. The value $\sum_{i=1}^n h_i^2$ is in fact a standard measure of concentration and it is equal to 1 only when the entire portfolio is concentrated on a single loan.

Because of the existence of asymmetric information and uncertainty, variation of interest rates affect the probability of default of borrowers. In order to show the effects of interest rates on the default cost function of the bank, we introduce a function that shows the link between default probabilities and interest rates variations.¹¹ The function $\alpha(LR)$ shows how the probability of default of the single borrower is increasing in the amount of the interest factor LR . It measures the sensibility of the default cost to variation in the interest rate factor, and depends on the stock of information. The higher the information, the lower the asymmetry in the information, the lower is the value of α_i . The function is stochastic, because the bank estimates the sensibility of defaults to interest rates shocks with an error. Since we have to assume that the correlation $\rho_{\alpha r_i}$ between the error in the estimate of the sensibility and the random error in forecasting default costs is constant, the function α is linear with respect to the interest rate factor. Since we want to apply the function to the entire loans portfolio of the bank, we have to derive a similar function defined on the entire portfolio of loans. We define ϕ to be the weighted average of all the single individual functions α_i , with weights given by the ratio $h_i = \frac{L_i}{L}$ that measures the weight of every single

¹¹ derived from Jaffee and Russell [15].

loan with respect to the entire portfolio. (From now on L will represent the entire portfolio of loans while L_i indicates the single individual loan.)

$$\phi = \sum_{i=1}^n h_i \alpha_i. \quad (15)$$

The function ϕ is defined in the interval $[0, 1]$. The value of ϕ increases with α_i that is a positive function of the interest rate factor, and with the value of the concentration h_i . Its value decreases with the current investment z in information and the level of the stock previously accumulated a .

$$\begin{aligned} \frac{\partial \phi}{\partial LR} &= \alpha'_i(LR) h_i > 0 \\ \frac{\partial \phi^2}{\partial LR^2} &= \alpha''_i(LR) h_i < 0 \\ \frac{\partial \phi}{\partial h_i} &= \alpha_i > 0 \\ \frac{\partial \phi}{\partial z + a} &< 0, \end{aligned} \quad (16)$$

where $R = 1 + r_i$ is the interest factor.

It is now possible to summarise the risk factors in an expected default cost function. We express it as a negative return in per cent terms that can be simply subtracted from the exogenously given expected returns. In general, this function is given by

$$D = E[LRp(D)], \quad (17)$$

where L is the amount of loans of the entire portfolio, R is the interest factor, $p(D)$ is the subjective evaluation on part of the banker of probability of default. This equation simply states that a share $p(D)$ of the loan portfolio will not be repaid. Considering the fact that in case of default both principal and interest are not repaid, the expected default cost function is

$$\begin{aligned} E[D_{t+1}] &= E[LRp(D)] = \\ LE[R]E\left[\sum_{i=1}^n h_i(\alpha_i + U_{i0} + \mu_i t + \sum_{k=1}^t \epsilon_k)\right] &= \\ = L(1 + r_i)\left[\sum_{i=1}^n h_i(\alpha_i + U_{i0} + \mu_i t)\right]. \end{aligned} \quad (18)$$

As a consequence, we will consider the expected per cent cost of default to be

$$d = (1 + r_i) \left[\sum_{i=1}^n h_i (\alpha_i + U_{i0} + \mu_i t) \right], \quad (19)$$

with a variance of

$$\begin{aligned} \sigma_d^2 = & (1 + r_i)^2 \left\{ \sum_{i=1}^n h_i^2 (\sigma_\alpha^2 + t \sigma_\epsilon^2) \right\} + \\ & + \sigma_r^2 \left[\sum_{i=1}^n h_i \alpha_i + \sum_{i=1}^n h_i (U_{i0} + \mu_i t) \right] + 2 \rho_{r'_i \alpha} \sigma_{r'_i} \sigma_\alpha. \end{aligned} \quad (20)$$

3.2.3 Transaction costs

Transaction costs are assumed to be different for any security. The term transaction cost is used in the wide sense and its meaning includes in particular the cost of the investment in information (search cost, such as advertising, promotions plus the cost of the evaluation of the reliability of the depositor or the borrower), z_i . The only fundamental assumption, in order to comply with the requirement of exogenous returns, is that the optimal transaction costs are proportional to the amount of the security acquired $\frac{\partial z_i}{\partial x_i} = 0$. This assumption is quite restrictive, since it rules out any economy of scale in the management of information. The meaning of the assumption and its acceptability varies considerably with the level of aggregation that we are considering. In the case of complete disaggregation of the portfolio of assets, it means that the cost of the optimal information for a loan of a dollar and for a loan of a million dollar is the same fixed proportion of the loan, even if it is issued to the same person. And this seems hardly acceptable. But if we assume that every asset represents a particular contract issued to a definite set of customers pooled on the base of some common characteristic, the story seems more reasonable. What we are now assuming is that, on average, the bank estimates the cost of information for that kind of loans, to be a fixed percentage of the loans. The assumption is now more tenable, because the range of the eventual economies or diseconomies of scale of the whole set of loans pooled together will be much smaller than on the single loan. Considering, as it seems reasonable, that the cost of information depends on the evaluation of the borrower, not just on the evaluation of the single project. From another perspective the model allows us to take into

account the existence of economies of scope. Economies of scope would cause transaction costs of different assets to be correlated with each other. There might be two relevant sets of economies of scope: economies in acquisition of the information regarding the quality of loans that would determine a correlation between certain categories of assets; and economies between some sets of assets and some sets of securities, as for example loans and deposits of firms.

3.3 Expected net returns and variance

We are now able to show expected net returns of the securities and their respective variance. Throughout this work we assume that financial markets are highly competitive and banks have to behave competitively in order to attract households' deposits and to provide loans. Accordingly banks are considered to be price takers. This implies that they can choose the quantity of the liability that these services represent in their balance sheet, at a given cost that is represented by the market interest rate plus a transaction cost. The expected return (cost) of any liability is composed of two terms, the interest rate component and the transaction cost component.

$$r'_j = r_j + c_j, \quad (21)$$

where

$$r_j = r_{t+1}^{j,e} = r_t^{cb} + f(j) + \epsilon_t^{cb} + \epsilon_t^j, \quad (22)$$

and

$$c_j = c_{t+1}^{j,e} = c_t^j + \epsilon_t^{cj}. \quad (23)$$

Transaction costs are assumed to be forecasted with an error, in order to capture the effect of technological shocks that affect the financial markets. The respective variances, assuming different errors to be uncorrelated, are

$$\sigma_j^2 = \sigma_{\epsilon_{cb}}^2 + \sigma_{\epsilon_j}^2 + \sigma_{\epsilon_{cj}}^2. \quad (24)$$

The return on the asset represented by loans has three components, a positive component represented by the expected interest rate earned, minus the transaction cost expressed in percent terms, and minus the default cost of the loan. Formally ¹²

$$r'_i = r_i - c_i - d_i, \quad (25)$$

¹²As in Kane and Malkiel [16].

where

$$r_i = r_{t+1}^{i,e} = r_t^{cb} + f(i) + \epsilon_t^{cb} + \epsilon_t^i, \quad (26)$$

$$d_i = (1 + r_i) \left[\sum_{i=1}^n h_i (\alpha_i + U_{i0} + \mu_i t) \right], \quad (27)$$

and

$$c_i = c_{t+1}^{i,e} = c_t^i + \epsilon_t^{ci}. \quad (28)$$

The forecasted variance is

$$\sigma_i^2 = \sigma_{\epsilon_{cb}}^2 + \sigma_{\epsilon_i}^2 + \sigma_{\epsilon_{ci}}^2 + \sigma_{d_i}^2 + 2\rho_{cbdi}\sigma_{cb}\sigma_{di} + 2\rho_{idi}\sigma_i\sigma_{di}, \quad (29)$$

where

$$\begin{aligned} \sigma_{d_i}^2 &= (1 + r_i)^2 \left\{ \sum_{i=1}^n h_i^2 (\sigma_\alpha^2 + t\sigma_\epsilon^2) \right\} + \\ &+ (\sigma_{cb}^2 + \sigma_i^2) \left[\sum_{i=1}^n h_i \alpha_i + \sum_{i=1}^n h_i (U_{i0} + \mu_i t) \right] + 2\rho_{cb\alpha}\sigma_{cb}\sigma_\alpha + 2\rho_{i\alpha}\sigma_i\sigma_\alpha. \end{aligned} \quad (30)$$

It is important to remark that the degree of concentration is exogenously given for every class of security. That degree is the outcome of a prior choice of a standard composition for every class of securities. Every class is formally analogous to a mutual fund, whose degree of diversification depends on the peculiarities of the securities that compose it. For example we might assume a higher degree of diversification in the case of consumer credit than in the case of loans issued to firms.

4 The loans portfolio model

The model is an application of the model of Hart and Jaffe [11]. We use the theorems they developed in order to study the choice of the bank among commitment and normal loans and among the different sources of deposits. Liabilities enter in the portfolio problem of the bank with a negative sign, and the expected return (measuring the cost) of each of them is the sum of the interest rate and the transaction cost component. Both component are be different for each liability. We consider the existence of three different kind of liabilities: deposits of households, deposits of firms and money market instruments. Money market instruments are assumed to have negligible

transaction costs. The expected returns of liabilities are represented in the following vector:

$$\mathbf{x}_j = \begin{bmatrix} r_m \\ r_h + c_h \\ r_f + c_f \end{bmatrix}$$

The model is based on a bipartition of deposits services that banks provide in two different categories: deposits of households and of firms. Households make use of deposit services mainly to invest their savings. To attract these customers, banks have to pay an interest rate that has to be competitive with the returns offered by other financial instruments and institutions.

The relation between the bank and the firm is of a different kind. Firms need the bank in order to obtain a wide range of credit facilities. They use the bank to deposit their liquidity because it is in the mutual interest of both the bank and the firm itself. The bank can increase in this way its liabilities and extend accordingly its revenues (this represents the implicit payment that Sprenkle [20] was considering as the reason for the firm to hold the apparently irrational amount of deposits that they normally hold); on the other hand the firm using the bank to administer its payment increases enormously its potential liquidity. The bank in fact furnishes implicitly a screening and certification service of the firm, for everyone that receives a payment from the firm. As a consequence, we have to assume that *both the transaction costs and the idiosyncratic component of the interest rates of loans are highly correlated with the cost component of firm deposits*. Recent developments in the microeconomic theory of banking have shown the importance of the economies of scope that the bank can exploit providing jointly services of deposit and credit.^{13 14}

We divide the assets of the bank in three categories: normal loans, commitment loans and bonds. Commitment loans are a class of loans for which information problems of the bank are less severe. Commitment loans are normally issued to the best and most reliable clients. Accordingly it seems natural to assume that the adverse selection and moral hazard problem as interest rates rise will be more severe for the borrowers with normal loans.

¹³See for example Vale [23], "The Dual Role of Demand Deposits under Asymmetric Information" and Kashyap, Rajan and Stein [17], "Banks as Liquidity Providers: An Explanation for the Co-Existence of Lending and Deposit Taking".

¹⁴In the model with limited liability issuance, the correlation between rates is explicitly modelled and depends in particular on the feedback coefficient k .

Commitment loans are issued to the most reliable borrowers and must produce a lower expected return than normal loans, because they are less risky. If this was not the case, than they would have both a higher return and a lower variance, dominating the other class of loans and the bank would never choose to hold normal loans in its portfolio. Since there is no reason to assume default cost or transaction costs to be higher for the most reliable class of borrowers (they will most likely be lower), the mark up must be substantially lower. In other words, competition in the credit market would transmit the benefits of the increased information to the borrowers in terms of a lower interest rate on loans.

Commitment loans are issued to the most reliable customers and their maturities are spanned over a longer period of time. The lower mark up implies that an increase in the common interest rate factor causes a lower increase of the rate of this class than of the rate of normal loans, implying a higher degree of smoothing of the shocks. We will assume for simplicity that bonds have no default and transaction costs. Transaction costs of bonds are clearly much lower than transaction costs of loans for the bank and accordingly they can safely be approximated to zero. Default cost of bonds are clearly not null since there are bonds of every class of risk. But banks have a lower cost of evaluation of the information of projects¹⁵ than the market. So in normal conditions they should not be willing to buy bonds more risky than the loans they can issue, because high yield (and high risk) bonds would normally be dominated by loans. We can assume that the relevant alternative for the portfolio choice of a bank are bonds with a lower risk and return, such as state bonds. The expected returns can be shown by the following vector:

$$\mathbf{x}_i = \begin{bmatrix} r_b \\ r_l - c_l - d_l \\ r_c - c_c - d_c \end{bmatrix}$$

We can now define the matrix of variance and covariance, assuming the following:

The costs of loans and deposits are positively correlated. This assumption is largely justified by the literature on banking and is the effect of the economies of scope in the banking sector.

¹⁵See Fama [9].

The correlation between loans and firm deposits is much higher than the correlation between loans and household deposits.

$$\rho_{fl} = \rho_{fc} > \rho_{hl} = \rho_{hc} \quad (31)$$

The costs of different types of loans are positively correlated with each other and the same holds for different types of deposits.

Interest rates are positively correlated with default costs.¹⁶

The correlation between interest rates and cost of loan is higher for normal loan rather than commitment loans. Formally

$$\rho_{d_l r_l} > \rho_{d_c r_c}. \quad (32)$$

The variance of the returns on normal loans will be higher than that of commitment loans, the variance of the returns on bonds will be even lower. On the other way the expected returns from commitment loans are lower than for normal loans and the expected returns of bonds are even lower.

$$r'_b < r'_c < r'_l \quad \sigma_b^2 < \sigma_c^2 < \sigma_l^2. \quad (33)$$

The variance of household deposits will be lower than that of firm deposits, the variance of money market funds will be higher than both. Expected returns (costs) for household deposits are higher than for firm deposits and expected returns of money market funds are higher than both.

$$r'_m < r'_f < r'_h \quad \sigma_m^2 < \sigma_f^2 < \sigma_h^2. \quad (34)$$

All the shocks are independent white noise.

The resulting variance and covariance matrix is

$$\mathbf{V} = \begin{bmatrix} \sigma_m^2 & 0 & 0 & \sigma_{mb} & 0 & 0 \\ 0 & \sigma_h^2 & \sigma_{hf} & 0 & \sigma_{hl} & \sigma_{hc} \\ 0 & \sigma_{hf} & \sigma_f^2 & 0 & \sigma_{fl} & \sigma_{fc} \\ \sigma_{mb} & 0 & 0 & \sigma_b^2 & 0 & 0 \\ 0 & \sigma_{hl} & \sigma_{fl} & 0 & \sigma_l^2 & \sigma_{lc} \\ 0 & \sigma_{hc} & \sigma_{fc} & 0 & \sigma_{fc} & \sigma_c^2 \end{bmatrix}$$

¹⁶As in Jaffee and Russel [15], or Stiglitz and Weiss [21].

The variance of the return of an asset, for example normal loans will be:

$$\sigma_l^2 = \sigma_{\epsilon_{cb}}^2 + \sigma_{\epsilon_l}^2 + \sigma_{\epsilon_{cl}}^2 + \sigma_{dl}^2 + 2\rho_{cbdl}\sigma_{cb}\sigma_{dl} + 2\rho_{ldl}\sigma_l\sigma_{dl}. \quad (35)$$

The variance of the return of a liability, for example firm deposits, is

$$\sigma_f^2 = \sigma_{\epsilon_{cb}}^2 + \sigma_{\epsilon_f}^2 + \sigma_{\epsilon_{cf}}^2. \quad (36)$$

In the same way all the covariances, are as in the following example that shows the covariance between loans and commitment loans. It must be remembered that we do not take into account the direct effect of r_{cb} that is present in every return and we rule out its effect from the analysis of the correlations, assuming its direct effect to be symmetric in every covariance term and setting it for simplicity to zero.

$$\sigma_{lc} = \rho_{lc}\sigma_l\sigma_c, \quad (37)$$

the correlation terms of the covariances can be expressed, as

$$\rho_{lc} = \rho_{dl}d_c + \rho_{dl}r_c + \rho_{dcr_l} + \rho_{c_l c_c}. \quad (38)$$

The matrix can be reordered as

$$\mathbf{V} = \begin{bmatrix} \sigma_m^2 & \sigma_{mb} & 0 & 0 & 0 & 0 \\ \sigma_{mb} & \sigma_b^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_h^2 & \sigma_{hf} & \sigma_{hl} & \sigma_{hc} \\ 0 & 0 & \sigma_{hf} & \sigma_f^2 & \sigma_{fl} & \sigma_{fc} \\ 0 & 0 & \sigma_{hl} & \sigma_{fl} & \sigma_l^2 & \sigma_{lc} \\ 0 & 0 & \sigma_{hc} & \sigma_{fc} & \sigma_{lc} & \sigma_c^2 \end{bmatrix}$$

It is now apparent that the matrix is composed of two independent blocks and the system can be decomposed. In this case we can apply a separation theorem proved by Szegö [22] that states that the choice of the bank is analogous to a choice between two mutual funds, one composed of "market" securities, the other one of deposits and loans.

4.1 The effect of shocks

In order to study the effect shocks, we can make use of the theorems that we showed in section (3.1). Our results are an application of this theorem to the model we have developed. It is worth noticing the importance of our assumptions regarding the correlation. Since

$$\sigma_{ji} = \sigma_j \rho_{ji} \sigma_i, \quad (39)$$

we can note that the degree of correlation, ρ , plays a key role.

4.1.1 A monetary shock

We consider the effect of a perfectly anticipated monetary policy shock first that affects positively all the returns, applying parts 1) and 3) of the theorem. Under our assumptions, the variation of interest rates *per se*, being proportional to each security, leaves the portfolio problem unchanged. But because of asymmetric information, default costs increase as the interest rates rise, partially offsetting the positive effect on the rates.¹⁷ The key point to note, is that default cost do not increase symmetrically, because commitment loans are extended exclusively to the best customers. As customers are screened and divided into two classes, the problems of asymmetric information affect normal loans to a much higher degree. The same kind of situation affects the variance matrix. The interest rate shock in itself increases proportionally the variance of different assets, shown in equation (35). But the effect of the shock is not symmetric: the correlation between interest rates and default costs clearly causes an asymmetric impact on different classes of assets. This implies a proportional shift of the portfolio from loans to bonds. Since the effect is stronger for normal loans, the contraction is not proportional: the quantity of normal loans held is reduced more than the quantity of commitment loans. The conclusion seems inescapable: the increase of interest rate shifts the portfolio of the bank. The bank substitutes bonds for loans, and commitment loans for normal loans.

Since deposits and loans are positively correlated, we can apply part 2) of the theorem that states that the product of the deposit liabilities and the correlated loan assets have to decrease, because of the shock. This means that the absolute values of the holdings both securities cannot fall. Since we know that the quantity of loans decreases, this means that the quantity of the correlated deposits has to grow at least proportionally. The intuition behind this result is that since deposits and loans have opposite signs in the portfolio, the bank can exploit the correlation, in order to hedge its position. As a consequence, the bank has an interest in increasing the amount of the correlated deposit, because it can use the liability, deposits, to hedge its position against the volatility of the asset, loans. The higher the correlation, the higher is the tendency for the optimal amount held in the portfolio of both the liability and the asset to move together.

¹⁷When interest rates are sufficiently high and the increase in interest rates is of significant magnitude, the effect on default costs might overcome the effect on rates, as Stiglitz pointed out on many occasions. But we will consider the "normal" case here.

When a positive monetary shock affects the economy, banks tend, *ceteris paribus* to increase the holdings of deposits rather than other liabilities, such as money market funds. Besides they tend to increase the holding of deposits of firms more than that of households, because deposits of firms have a higher correlation with loans. And vice versa. In other words there is a tendency for the relationship between firms and banks to strengthen as interest rates rise. And the bank tries to rely more on "core" deposits as its main liability. We have assumed for simplicity that the increase in interest rates is symmetrical, which is not very realistic. If we relax this assumption, the results hold *a fortiori*. Competition in the credit sector is much weaker than in the stock market, where bonds are traded, because of the importance of inside information. Banks charge a mark up to their borrowers, but implicitly provide an insurance, not raising the rates proportionally when they go up, and not reducing them proportionally when they go down.¹⁸ The most reliable customers benefit from a lower mark up, as a consequence the impact of the rise of the rate would be lower on the rates of commitment loans.¹⁹

4.1.2 A real shock

A perfectly anticipated negative real shock that hits the borrowers affects the cost structure of the bank in two different ways. The first and most obvious effect is an increase of the probability of default that reduces the expected return of loans. This time there is no reason to believe that the shock hits the two kinds of borrowers in a different way. Costs of defaults are different for the two kind of loans, and we take into account the different evaluation of the reliability of different borrowers. As the shock hits the economy both kinds of debtors are affected and costs are expected to increase in the same proportion. At the same time the variance of both assets increases, reinforcing the negative effect of the shock on the holding of the loans. The higher correlation between default costs and interest rates of normal loans determines a tendency for a more than proportional reduction of the holding of normal loans rather than commitment loans. But this time the effect is much smaller than in the case of a monetary shock, because the asymmetry affects only the covariance terms, of all the components of the variance of each asset. And, as we have seen, returns are affected in a symmetric way.

¹⁸See Fried and Howitt [10].

¹⁹This can easily be seen considering the model where a limit on liability issuance is introduced.

Accordingly, the first order effect is for the bank to reduce proportionally the holdings of both types of assets, as the appeal of bonds increases. The substitution between the two kinds of liabilities will depend this time on a second order effect and in many instances it may turn out to be insignificant. Its importance depends on the segmentation of the market between the two categories of loans and it is likely to be significant only in the case of a very high correlation between the idiosyncratic component of the return and the default cost for normal loans and a very low one in the case of commitment loans.

The shock produces a second effect that goes in the opposite direction. A negative shock reduces the cash flow of firms and has a negative impact on their credit rating, as a consequence firms find increasingly difficult to obtain finance in the stock market. The demand for loans goes up and the higher demand produces an increase in the expected return on loans.²⁰ Because of the higher expected return the bank is willing to increase the share of loans in its portfolio.

A third effect regards deposits. Since a negative shock reduces the cash flow of firms, their liquidity is reduced, and they are less willing to hold deposits. In our model the bank chooses arbitrarily the quantity of deposits, but we can consider this effect as an increase in the cost of firms deposits liabilities. As firms are less willing to hold deposits the bank has to offer a higher interest rate, or offer more valuable services. Since we are considering the bank as being price taker, we assume that only the cost of raising deposits rises. As the cost of the liability rises, the bank is willing to reduce the quantity held. And the increase of the variance reinforces the effect.

We can finally consider the impact of the shock on the covariance between deposits and loans that is clearly positive. We have seen that the increase in the covariance causes the quantity measured by the product of both the asset and the liability held in the portfolio to rise. This means that this effect of the shock will generate a tendency for the quantity of the liability to grow more than proportionally if the quantity of the corresponding asset is reduced or vice-versa. And this effect is stronger the stronger the correlation, so it is stronger for firm deposits than household and for commitment loan rather than normal ones.

²⁰ Adopting the model with limited liabilities that includes the demand functions for banking services, it can be proven that the effect of a variation of the intercept of the demand equation is formally analogous to a variation of rates in the basic model, so that the comparative static results hold still.

The final effect depends on the relative strength of the two opposite effects of the shock on variance and returns of loans. If the negative effect on the return on loans prevails, the quantity of the correlated liability must rise, and this implies, as in the case of an interest rates shock, a tendency for the bank intermediation to increase and for strong relationships to be developed. If, on the other hand, the positive effect on the return on loans and on the cost of deposits prevails, banks would be willing to charge higher rates to borrowers, in order to exploit as much as possible their market power (this case will arise when the competition in the banking sector is limited). The tendency would be in this case to relax the ties between banks and firms.

4.2 Geographical diversification

A more diversified portfolio benefits the bank, because it reduces the variance at no cost on the expected returns. This is true for every mean variance portfolio model unless different assets are perfectly correlated. Our model shows that in the case of banks, the importance of diversification is even higher, because the degree of concentration of the portfolio has a strong influence on both components of the default cost, the idiosyncratic and the interest rate. Formally the effect arises because the concentration coefficients are quadratic while the expected returns are linear in the market share of each asset. Since the relative impact of defaults cost on the portfolio is the main factor affecting the profitability of banks, it is evident that diversification of the portfolio has for bank an even higher importance than for a mutual fund, for example.

The diversification of the portfolio that is possible to achieve, is fixed *a priori* by the relevant set of assets and liabilities. But the availability of a wide range of securities depends crucially on the technology to obtain and process the information and on the stock of information accumulated in the past. If relevant economies of scale are present in the information technology, large banks might have a competitive advantage with respect to the small ones. The technology would in this case grant the availability of a higher degree of diversification. In this framework the dimension is relevant only to the extent that large banks have exclusive access to certain assets (as loans to large corporations) that are precluded to the small ones. Large banks have a wider possibility of choice than small ones, because they can grant loans

to large corporations.²¹ But a local bank that has developed across time a strong network of personal relationships can issue loans with a risk- return profile that is not available to the large one. Large banks have a competitive advantage only if they are highly capitalised and at the same time they have invested heavily in the past to establish personal relationships at the local level. And it might be possible for diseconomies of scale to emerge in the management of the available information. (It can be very difficult for employees of large banking corporations that are part of a bureaucratic system, to develop the mental attitude and the skills of the entrepreneur of a local bank.)

The crucial aspect of the problem is represented by the availability of alternative assets that are poorly correlated among each other. From this point of view a major factor seems to be represented by the range of possible geographical diversification. Different regions are in general characterised, both in Europe and in the US, by different productive structure and different business cycles. The extent of the possible diversification might crucially depend on the extent of the regulatory barriers that are imposed to the location of credit intermediaries. The empirical evidence available, as in Hughes et alia [14] shows that a larger geographical diversification improves expected returns and efficiency, while they report no evidence of a reduction of the insolvency risk of banks. The availability of a wider range of risk-return combinations, while is certainly beneficial for the bank, it does not necessarily imply a reduction of the risk undertaken, as the authors stress. The equilibrium in the new possibility frontier might in fact involve higher expected returns and higher variance, the final outcome being dependent on the structure of preferences. Our model suggests that the availability of a wider range of alternative assets tends to reduce the need of a close relationship between bank and firm. Large interregional banks have a lower incentive to rely on the information intensive relationship lending. Geographical and intertemporal diversification could be both complements and substitutes. Banks that have a large stock of internal information can benefit from both type of diversification. But banks that have not invested enough in the past, whose available portfolio is of poor quality, can choose only to diversify geographically as much as they can and to use different types of securities. Bonds, and even junk bonds that other bank would never hold, might provide a cheap (and risky) instrument to increase the geographical diversification of these

²¹Lines of credit for very large sums are too risky for small, less capitalised, banks.

banks. The existence of market power that is confirmed by most empirical works, allows the banks two possible sets of action. They can rely on the relationships, smoothing real shocks in order to get higher average intertemporal returns and especially lower risk. This is the only choice available to the small local bank. But the large bank might choose to exploit its market power in order to charge higher rates to liquidity constrained firms. This choice might become more profitable the larger the possibility of diversification of the portfolio, since the higher diversification reduces the variance due to default cost and idiosyncratic shocks. Larger banks might choose to create an internal capital market in order to exploit the continuous variations of the risk-return profile of assets of different regions.²² This choice might allow them to exploit their market power to extract all the rents in the different local markets.

The model with limits on liability issuance shows that a crucial role is played by the coefficient k that determines the share of loans issued by the bank that feedback in its own deposits. The higher k the higher would be the correlation between loans and firms and accordingly the importance for banks to rely on deposits as their main source of liabilities. The value of k depends on different technological and institutional factors, but a crucial role is played by the market power. In the case of monopoly its value would tend to 1, implying a strong role for liabilities to hedge against the variance of loans, as the correlation would be very high. For this reason, in the case of a merger between two regional banks, the higher the value of k , the lower is the incentive for the merged bank, to concentrate the liabilities in one region and the assets in another. This analysis shows that the effect of market power is dual, because it gives banks the possibility to exploit liquidity constrained firms, but, on the other hand, it increases the benefits from a close relationships with the borrower, since through relationship lending firm deposits might hedge the portfolio against the volatility of loans.

5 Conclusion

The main result of this work was to show that the optimal portfolio allocation of banks is affected by monetary and real shocks in a significantly different way. Interest rate shocks tend to push bank towards more informational

²²Houston and James [11] showed that this is an empirically relevant phenomenon in the US.

intensive assets, such as commitment loans, strengthening the relationship between bank and firm. This effect is likely affect all kinds of banks, as long as the increase in the rates charged causes default costs to increase.

The reaction to real shocks that bank perceives as credit risk shocks, depends on the relative strength of two effects that work in opposite directions. A closer relationship is the optimal response only if the increase in default cost induced by the shock is larger than the benefit that the bank derives from an increased demand for loans. Small banks that rely more heavily on deposits as their main source of finance are, in general, more willing to smooth credit shocks. Our model shows that this kind of behaviour implies that the impact of the shock on default cost is heavier than the benefits due to the increased demand for loans. This is likely to be the case for small banks. First of all they face a limit to the possibility of diversification of their loans portfolio, caused by the small size itself. Besides, since they traditionally rely more heavily on information intensive relationship lending, because they specialise in the evaluation of local economic conditions, it is unlikely that they can exploit their market power in order to charge firms more heavily when they are hit by the shocks. It is more likely that the eventual existence of market power will be used in order to compensate in normal times the reduction in earnings caused by the smoothing of the negative shocks. Benefits from an increased demand for loans might often overcome higher default cost in the case of large banks. Thanks to the higher diversification of their portfolio, they need less to rely on inside information and they can promptly react to shocks, exploiting their market power in order to maximise their expected returns, being less worried by the volatility of the portfolio.

The possibility of a geographical diversification of the portfolio is certainly beneficial for any bank. But this work highlights the possibility of a trade off between intertemporal smoothing of real shocks and efficient allocation of capital, whose impact on regional and local economies should worry the regulatory authorities of the credit market. In particular it seems to suggest the need of a policy of free entry for new local banks, whenever processes of concentration among local and regional banks take place.

6 Appendix

6.1 Appendix I: Limits on liability issuance.

The main problem in the application of the basic model is that banks would be willing to expand the amount of liabilities ad infinitum. They could in fact always increase profits buying extra liabilities and assets in the same proportion, since the cost (return) of liabilities is always less than the return on assets. Introducing imperfect competition in the market for loans and deposits it is possible to introduce an endogenous limit to the dimension of the portfolio. We adopt a model developed by Szegö [22] on the basis of the work of Bertoni, Mazzoleni and Szegö [6].

The main limitation of the previous model is that interest rates were assumed to be set exogenously and the bank was free to choose the desired quantity of the securities it desired. This implies that no mechanism limiting the size of loans was specified. This time we will introduce in the problem two demand equations, for deposits and loan services. The bank is no longer price taker in the market for loans and deposits and, together with the optimum amount of securities, it sets the interest rates at the level necessary to obtain the desired quantity. Modifying the demand equations we will obtain two equations that define a mapping from the choice of the quantity to the choice of the rates.

The key feature of this section is represented by a peculiar difference among household's deposits and firm deposits. In the former model we did not take into account an important phenomenon, the feedback of loans in deposits or the autogenesis of deposits through the granting of loans. The importance of this phenomenon is normally recognised mainly at the macroeconomic level, while it is considered in few microeconomic works on banking.²³ In a portfolio problem of the bank, the autogenesis of deposits has been considered by Szegö [22], and we will adopt his formulation even if our formulation is marginally different. The main feature of his model is the introduction of two demand equations for deposit and loan services on part of households and firms, in order to determine a cost function that is increasing with the quantities of the securities held.²⁴ The two equations are linked by

²³The microeconomic determinants of the feedback have been considered only by Brunner [7] that studied the supply of funds of the bank in relation with its "available reserves".

²⁴In this section we will consider demand for loans and deposit services on part of households and firms and the bank as supplier of those services. When considering the

the parameter k , the multiplier connecting deposits to loans, and it is peculiar of every bank. We will follow Szegő's formulation of the portfolio model, and our model will be different only for a small variation in the notation. His formulation of the cost of deposits depends positively on the factor $1 - k$ times the quantity of loans, where the factor k is a drain coefficient. The coefficient represents the share of every pound lent by the bank that does not return in the bank in the form of deposits. We introduce a factor k that is exactly equal to Szegő's $1 - k$, and it represents that share of every pound lent that comes back as a deposit in the liability side of the balance sheet of the bank.

On the basis of our description of the market for firms deposits, based on Sprenkle's work, we need to assume that the cost of deposits is negatively affected by the quantity of loans and the value of k . The reason is that in the feedback process loans generate the demand of deposits; the increase in the quantity of deposit liabilities in banks balance sheets will not be acquired increasing the interest rate that the bank is willing to pay, because loans generate liquidity for the firms that they will be willing to deposit. As a consequence the cost of deposits will be reduced by the supply of loans of the bank. With a high enough k the bank can possibly substitute entirely the households deposit component, pushing the interest rate down to zero. The magnitude of the multiplier depends on both institutional factors and market conditions. The main ones are probably market concentration (k is higher as the market is less competitive), the velocity of circulation of money (the more "dynamic" the area in which the bank operates, the lower k is) and the degree of openness of the region in which the bank operates (the more closed the economy of the region is, the higher k).

The demand equations for loans and deposits are:

$$x_d = H + a_1 r'_d + k x_l + q_d, \quad (40)$$

$$x_l = K - a_2 r'_l + q_l. \quad (41)$$

Where q_d and q_l are error terms that we will assume to be white noise. The following inverse demand functions show the relation between the rates that bank charges and the estimated level of demand.

$$r'_l = \frac{1}{a_2} (K - x_l + q_l), \quad (42)$$

entire portfolio of the bank though, we will continue to consider the supply of deposit and loan services of the bank as a demand for the particular classes of securities represented by loans and deposits.

and

$$r'_d = \frac{1}{a_1}(-H + x_d - kx_l - q_d). \quad (43)$$

In order to obtain the desired levels of the security, x , we can now define the rates that the bank has to apply as differences between the expected value of the interest rate necessary to obtain the desired quantity and the initial value of the rate.

$$\tilde{r}'_l = -\frac{1}{a_2}x_l, \quad (44)$$

and

$$\tilde{r}'_d = \frac{1}{a_1}(x_d - kx_l). \quad (45)$$

The error terms q_d and q_l are assumed to have zero mean, but their variance is not zero. Equation (44) shows the effect of k on expected returns gross of loan and deposit costs. As its value increases, the cost of deposits decreases. When its value is not 0, the cost of deposits is decreasing as well in the quantity of loans. But the value of k is one of the main factors determining the degree of correlation between loans returns and deposits costs that this time is not linked just through the costs component, but through the interest rates component too. The covariance of loans and deposits increases with k . Formally the equations (44 and 45) give a second cost component that has to be added in the vector of returns of the assets.

Interest rates and interest cost are correlated because of the coefficients of the demand functions (44 and 45), which we assume to be constant. Loan interest costs and general loan cost might be assumed to be uncorrelated. Deposit interest cost and loan cost are correlated, because of the presence of the quantity of loans in the demand for deposits. A higher loan cost reduces the willingness of the bank to supply loan services, reducing the holding of the "loan" asset; a reduced quantity of loans increases the cost of deposits. For a higher value of k , the effect of the costs of loans (both the cost we defined in the previous section and the interest cost) is stronger; the correlation is a linear function of k . Deposits general costs and deposit interest costs are not linked in an obvious way so we do not consider this eventuality. Holding the assumption of independence between bonds, money market, and banking services, the matrix of variance and covariance has the same structure than the former. The main difference with respect to the previous case is the presence of an additional variance term, the variance of interest rate cost that depends on the quantities chosen. Quantities are not

stochastic variables in themselves, as they are chosen by the bank. But since the bank has to make an estimate of the quantities demanded, the error of the estimation produces the variance of the cost. It is important to note that the variance of the interest cost of deposits c_{rd} depends through k from the variance of interest cost of loans.

$$x_l = -a_2 \tilde{r}'_l + K + q_l, \quad (46)$$

and

$$\tilde{r}'_d = \frac{1}{a_1} [-H + x_d + k(-a_2 \tilde{r}'_l + K + q_l) - q_d], \quad (47)$$

the variance of r'_l is not null because demand is estimated with an error:

$$\sigma_{r'_l} = Var\left[\frac{1}{a_2}(q_l)\right] = \left(\frac{1}{a_2}\right)^2 E[q_l^2]. \quad (48)$$

As the variance of loans interest cost increases, the variance of deposits interest costs goes up.

$$\begin{aligned} \sigma_{r'_d} &= Var\left\{\frac{1}{a_1}[k(-a_2 \tilde{r}'_l + q_l) - q_d]\right\} = \\ &= \left(\frac{1}{a_1}\right)^2 \{E[q_d^2] + k^2 E[q_l^2] - 2k E\{[q_l][q_d]\}\}. \end{aligned} \quad (49)$$

References

- [1] AIGNER, D. J. and SPRENKLE, C. M., "A Simple Model of Information and Lending Behavior", *The Journal of Finance*, vol. 23(1), 1968, pp. 151-166.
- [2] BERGER, Allen N. and UDELL, Gregory F. (1992), "Some Evidence on the Empirical Significance of Credit Rationing", *Journal of Political Economy*, vol. 100, 1992, p. 1047-1077.
- [3] BERGER, Allen N. and UDELL, Gregory F., "Relationship Lending and Lines of Credit in Small Firm Finance", *The Journal of Business*, vol. 68, 1995, n. 3, p.
- [4] BERLIN, Mitchell and MESTER, Loretta, "On the profitability and cost of relationship lending", *Journal of Banking & Finance* 22, 1998, pp. 873-879.
- [5] BERLIN, Mitchell and MESTER, Loretta, "Deposits and Relationship Lending", *The Review of Financial Studies* vol. 12, n. 3, 1999, pp. 579-607.
- [6] BERTONI, A., MAZZOLENI, P. and SZEGÖ, G., "On the analytical identification of the optimal loan and deposits interest rates of a commercial bank", *Metroeconomica* 27, 1975, p. 44-67.
- [7] BRUNNER, Karl, "A Schema for the Supply of Money", *International Economic Review*, vol. 2, n. 1, January 1961, p. 79-107.
- [8] COLE, Rebel A., "The importance of relationship to the availability of credit", *Journal of Banking & Finance* vol. 22, 1998, p. 959-977.
- [9] FAMA, Eugene, "What's Different About Banks" *Journal of Monetary Economics* 15, January 1985, p. 39-57.
- [10] FRIED, Joel and HOWITT, Peter, "Credit Rationing and Implicit Contract Theory", *Journal of Money, Credit and Banking* 12, August 1980, p. 471-487.
- [11] HART, Oliver D. and JAFFEE, Dwight M., "On the Application of Portfolio Theory to Depository Financial Intermediaries", *Review of Economic Studies*, April 1973, p. 129-147.

- [12] HESS, Alan C. "The Effects of Transaction Costs on Households' Financial Asset Demand", *Journal of Money, Credit and Banking*, Vol. 23, 1991, p. 383-409.
- [13] HESS, Alan C. "Portfolio Theory, Transaction Costs, and the Demand for Time Deposits", *Journal of Money, Credit and Banking*, Vol. 27, 1995, p. 1015-1032.
- [14] HUGHES, Joseph P., LANG, William, MESTER, Loretta J. and MOON, Choon-Geol, "Safety in numbers? Geographical diversification and bank insolvency risk", , *The Federal Reserve Bank of Philadelphia Working paper* n. 14, 1996.
- [15] JAFFE, Dwight and RUSSELL, Thomas, "Imperfect Information, Uncertainty and Credit Rationing", *Quarterly Journal of Economics*, Nov. 1976, No. 90, p. 651-666.
- [16] KANE, Edward J. and MALKIEL, Burton J., "Bank Portfolio Allocation, Deposit Variability, and the Availability Doctrine", *Quarterly Journal of Economics*,
- [17] KASHYAP, Anil K., RAJAN, Raguran and STEIN, Jeremy C., "Banks as Liquidity Providers: An Explanation for the Co-Existence of Lending and Deposit Taking" *NBER Working Papers* No. 6962, February 1999.
- [18] PETERSEN, Mitchell A. and RAJAN, Raghuram G., (1995) "The Benefits of Lending Relationships: Evidence from Small Business Data", *The Journal of Finance*, vol. XLIX no.1 March 1994, p. 3-37
- [19] PETERSEN, Mitchell A. and RAJAN, Raghuram G., (1995) "The Effect of Credit Market Competition on Lending Relationships", *Quarterly Journal of Economics* 110, p. 407-443.
- [20] SPRENKLE, Case M., "The Uselessness of transaction Demand Models", *Journal of Finance*, December 1969, p. 835-848.
- [21] STIGLITZ, Joseph and WEISS, Andrew, "Asymmetric Information in Credit Markets and Its Implications for Macroeconomics" *Oxford Economic Papers* 44, 1992 p. 694-724.

- [22] SZEGÖ, Giorgio P., *Portfolio Theory With Application to Bank Asset Managment*, Academic Press, London, 1980.
- [23] VALE, Bent, "The Dual Role of Demand Deposits under Asymmetric Information", *Scandinavian Journal of Economics* vol. 95. (1), 1993, p. 77-95.