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On the welfare effects of compatibility with Hotelling competition

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Abstract

In a setting with Hotelling differentiation and (weak) network effects, when the market is already covered duopoly adoption of compatibility leads to anti-competitive effects and tends to be welfare excessive. We show that the latter result is reversed if the market is not assumed to be covered even under incompatibility (a condition which depends on the value of the intrinsic/stand-alone benefits).

1 Introduction

Network benefits (a positive externality) arise in many settings, deeply affect the demand for services (which depends on the expected adoptions and it may not behave well) and then their market provision and its welfare properties: the related and by now very large literature is summarized by Belleflamme & Peitz (2021). Compatibility (namely, interconnection across networks) issues have also been extensively studied, starting from the literature on *the adoption of standard* (see e.g. Katz and Shapiro, 1985 and Farrell and Saloner, 1985).

The choice of compatibility, that has been sometimes enforced by regulators (e.g. in the telecommunications networks), directly increases welfare both by *powering network effects* and by *encouraging user participation.* By affecting users' demand, it can also change pricing, profitability and the returns from adopting it, which is usually costly. In a competitive environment, the last effects seem to depend crucially on the nature of competition. In particular, earlier results (see e.g. Katz and Shapiro, 1985 and Crémer *et al.*, 2000) suggest

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that under quantity competition *larger (dominant) firms may prefers incompatibility even when it is inefficient*.[2] On the contrary, more recent results suggest that price competition with *horizontally differentiated products* makes the option for compatibility anti-competitive, and produces an unambiguous excess of (costly) compatibility: see Doganoglu and Wright (2006).

In this note we reconsider the latter result, derived in a Hotelling setting, which appears (perhaps surprisingly) rather robust to the introduction of a number of variants (in particular of asymmetries across competitors), and show that nevertheless it crucially depends on the assumption that the market is covered even under incompatibility. Intuitively, the latter assumption removes a crucial role of compatibility (to encourage participation), and matters for its welfare assessment.

2 The setting

The Hotelling model is one of the work-horses of present-day industrial organization (see e.g. Belleflamme and Peitz, 2015), but the literature focuses almost exclusively on the case in which the intrinsic/stand-alone benefit of the service is sufficiently large to make the market covered *and* to provide all users with a positive surplus. However, it is known (see e.g. Mas-Colell *et al.*, 1995: exercise 12.C.14)[3] that the Hotelling setting delivers equilibria in which the market is not covered and sellers are "isolated monopolists" whenever the intrinsic benefit is sufficiently low, and also equilibria in which the market is "just covered", meaning that the equilibrium prices are those that leave the "indifferent" user with no surplus, whenever the value of the intrinsic benefit is intermediate.[4]

As discussed in this note, the implicit assumption that the market would be covered even under compatibility removes an "extensive margin" to the impact of compatibility (namely, it cannot encourage participation), and biases its welfare assessment. In particular, within the Hotelling setting, whenever compatibility increases users' participation it also raises their surplus (even though it also increases the price they pay for the service) and thus delivers a result of insufficient adoption.

To see, consider the following symmetric Hotelling duopoly with network effects: a unit mass of potential users are uniformly distributed on the interval $[0, 1]$. We assume that users singlehome (namely, each user is served at most by a single firm),[5] and for the sake of simplicity also that providing costs are null and "transport" costs (sustained by users) are linear with respect to the

[2] For a similar result in a different setting see the case of the so-called "legacy platform": Belleflamme and Peitz (2021: section 3.3.1).

[3] According to Bacchiega *et al.* (2023), the first to discuss the alternative market equilibria was Salop (1979), although in a slightly different setting.

[4] In the last case, as noted by Rey and Tirole (2019), the *symmetric* market allocation is the same that would be chosen by a two-product monopolist.

[5] Doganoglu and Wright (2006) discuss the case of multihoming and its interplay with compatibility, assuming that the latter is *per se* welfare deproving.

distance. Under incompatibility, the indirect utility of user $x \in [0,1]$ while subscribing to firm i (i, $j = 1, 2$, $i \neq j$) is thus given by

$$V_i(x) = v + \beta n_i - \tau |x - l_i| - p_i, \tag{1}$$

where n_i is the expected dimension of firm i, whose location is $l_i = i - 1$, p_i is its subscription price, v is the (common) intrinsic benefit of the service, τ is the unit transport cost and β the marginal evaluation of the network size (v, τ, $\beta > 0$). Under compatibility the previous expression becomes

$$V_i(x) = v + \beta (n_i + n_j) - \tau |x - l_i| - p_i. \tag{2}$$

Following Doganoglu and Wright (2006), we assume: (i) that network effects are *weak*, namely $\beta < \tau$ (to ensure that demands are well-behaved); (ii) that participation expectations are affected by prices (*responsive expectations*); and focus on *fulfilled-expectation equilibria* (namely, n_i is also the number of users ex-post served by firm i). Accordingly, $\pi_i = p_i n_i$ is the equilibrium revenue/profit of firm i.

2.1 Incompatibility

As indicated above, we need to distinguish 3 cases, according to the value of the stand-alone utility v (for given values of the parameters τ and β).[6]

If $v \geq \frac{3}{2}(\tau - \beta)$ the relevant linear demand is given by $q_i(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2(\tau - \beta)}$ (where $q_i = n_i$). Notice that both the choke price ($p_j + (\tau - \beta)$) and the (inverse) demand slope ($2(\tau - \beta)$) increase with respect to ($\tau - \beta$): accordingly the reaction function of firm i is given by $p_i(p_j) = \frac{p_j + \tau - \beta}{2}$. In the unique equilibrium we get

$$p_i = \tau - \beta,\ n_i = \frac{1}{2},\ \pi_i = \frac{\tau - \beta}{2},$$

$$S = v + \frac{3\beta}{2} - \frac{5\tau}{4},\ W = v + \frac{\beta}{2} - \frac{\tau}{4},$$

where $S = \sum_{i=1}^{2} n_i \left[v - p_i + \left(\beta - \frac{\tau}{2}\right) n_i\right]$ is user surplus and $W = S + \pi_1 + \pi_2$ total welfare.

If $\frac{3}{2}(\tau - \beta) > v \geq \tau - \beta$ the relevant demand curve has a kink at the equilibrium, and accordingly multiple asymmetric equilibria might arise (see Bacchiega *et al.*, 2023 for details). However, we focus on the *unique* symmetric equilibrium in which

$$p_i = v - \frac{\tau - \beta}{2},\ n_i = \frac{1}{2},\ \pi_i = \frac{2v - \tau + \beta}{4},$$

$$S = \frac{\tau}{4},\ W = v + \frac{\beta}{2} - \frac{\tau}{4}.$$

[6] Notice that the equilibrium values below vary continuously with respect to v.

Notice the market is "just covered", namely, $n_1 + n_2 = 1$ but the "indifferent" user located at $\frac{1}{2}$ gets a null surplus, equilibrium price and profit values are decreasing with respect to $(\tau - \beta)$, the equilibrium surplus value increases with respect to τ and the welfare expression is the same than in the previous case.

If $\tau - \beta > v$ firms are "isolated monopolists" and the market is not covered. Since demand is given by $q_i(p_i) = \frac{v - p_i}{\tau - \beta}$, in the unique equilibrium:

$$p_i = \frac{v}{2},\ n_i = \frac{v}{2(\tau - \beta)},\ \pi_i = \frac{v^2}{4(\tau - \beta)},$$

$$S = \frac{\tau v^2}{4(\tau - \beta)^2},\ W = \frac{(3\tau - 2\beta)v^2}{4(\tau - \beta)^2}.$$

2.2 Compatibility

Again we need to distinguish 3 cases (in the following expressions $\pi_i^c = p_i^c n_i^c$ and $W^c = S^c + \pi_1^c + \pi_2^c$ are *gross* of the possible costs of adopting compatibility) .

If $v \geq \frac{3}{2}\tau - \beta$ the relevant demand is given by $q_i(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2\tau}$ (assuming $n_1 + n_2 = 1$): note that compatibility removes the competitive role of the network effects. At the unique equilibrium given by

$$p_i^c = \tau,\ n_i^c = \frac{1}{2},\ \pi_i^c = \frac{\tau}{2},$$

$$S^c = v + \beta - \frac{5\tau}{4},\ W^c = v + \beta - \frac{\tau}{4}$$

(where $S^c = \sum_{i=1}^{2} n_i^c \left[v - p_i^c + \beta(n_1^c + n_2^c) - \frac{\tau}{2} n_i^c\right]$) the market is indeed covered. This is the case implicitly considered by Doganoglu and Wright (2006).[7]

If $\frac{3}{2}\tau - \beta > v \geq \tau - \frac{3}{2}\beta$ the market is just covered under compatibility, and the indifferent user gets a null surplus. At the symmetric equilibrium we get:

$$p_i^c = v + \beta - \frac{\tau}{2},\ n_i^c = \frac{1}{2},\ \pi_i^c = \frac{2(v + \beta) - \tau}{4},$$

$$S^c = \frac{\tau}{4},\ W^c = v + \beta - \frac{\tau}{4}.$$

Finally, if $\tau - \frac{3}{2}\beta > v$ (which arises only if $\tau > \frac{3}{2}\beta$, namely if the network effects are sufficiently weak) the market remains uncovered also under compatibility and firms are isolated monopolists with demand given by $q_i(p_i) = \frac{v + \beta n_j - p_i}{\tau - \beta}$. In the unique equilibrium $(n_1 = n_2)$ we get:

$$p_i^c = \frac{v(\tau - \beta)}{2\tau - 3\beta},\ n_i^c = \frac{v}{2\tau - 3\beta},\ \pi_i^c = \frac{v^2(\tau - \beta)}{(2\tau - 3\beta)^2},$$

[7] Doganoglu and Wright (2006: footnote 6) write that they assume that v is sufficiently high to cover the market (namely, $n_1 + n_2 = 1$), but actually they also assume that all users get a positive surplus under compatibility.

$$S^c = \frac{\tau v^2}{(2\tau - 3\beta)^2}, \; W^c = \frac{(3\tau - 2\beta)\, v^2}{(2\tau - 3\beta)^2}.$$

3 Results

While we should distinguish many sub-cases accordingly to the value of v, a comparison of the equilibrium expressions of sections 2.1-2.2 shows that compatibility *unambiguously* raises prices and profits (a clear-cut anti-competitive effect).

However, the welfare effect of adopting compatibility also depends on its cost and on the impact on the number of users who get served and on their surpluses. Following Doganoglu and Wright (2006), let us assume that the adoption of compatibility involve a fixed cost $F > 0$ to be shared by firms. It is useful to distinguish 3 cases:

1. Case *a.* $v \in (0, \tau - \beta)$: without compatibility the market is not covered. In this case the adoption of compatibility would increase the equilibrium market coverage and raise user surplus. Accordingly, the market adoption of compatibility is actually *insufficient* whenever $F \in (\Delta W, 2\Delta\pi_i)$ (where $\Delta z = z^c - z$).

2. Case *b.* $v \in \left[\tau - \beta, \frac{3}{2}(\tau - \beta)\right]$: without compatibility the market is just covered. In this case the adoption of compatibility would not increase market coverage but would leave unchanged user surplus. Accordingly, $\Delta W = 2\Delta\pi_i$ and the market adoption of compatibility is *welfare optimal*.

3. Case *c.* $v \in \left(\frac{3}{2}(\tau - \beta), \infty\right)$: even without compatibility the market is covered (this case encompasses that considered by Doganoglu and Wright, 2006). In this case the adoption of compatibility would reduce consumer surplus. Accordingly, market compatibility adoption is actually *excessive* whenever $F \in (2\Delta\pi_i, \Delta W)$.

4 Conclusion

In this note we have reconsidered the result of Doganoglu and Wright (2006) of an excessive adoption of (costly) compatibility in a setting with Hotelling differentiation and (weak) network effects. It was derived by assuming that the market was covered even under incompatibility, thus removing the possibility that compatibility encourages users' participation. We have shown that, on the contrary, while the adoption of compatibility has always a clear anti-competitive effect, when it also increases the overall user participation the welfare effects are reversed, leading to an insufficient market adoption. Our result formally depends on the value of the stand-alone utility provided by the service, which empirically varies hugely (from almost zero in the case of social media to a large amount in the case of some marketplaces), and it suggests that the possible variation of market coverage induced by compatibility should be carefully taken into account.

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