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Abstract

Informality represents a pervasive feature of many emerging and developing economies, yet standard macroeconomic models often ignore its effects, potentially biasing the analysis of shocks and the design of monetary policy. This paper studies the macroeconomic and policy implications of informality using a structural VAR for Colombia and a two-agent New Keynesian model with formal and informal sectors, featuring heterogeneous households including hand-to-mouth consumers. I show that informal labor supply shocks generate sectoral reallocation: informal activity absorbs part of the shock, sustaining aggregate output while altering wages, hours, and capital allocation. In contrast, monetary policy shocks propagate more strongly when informality is present, amplifying distributional and capital-reallocation effects. Critically, the presence of informality alters equilibrium determinacy: standard Taylor rules may fail to ensure uniqueness, with stability depending on the share of Ricardian households, the size of the informal sector, and the monetary policy stance. My findings highlight that accounting for informal production is essential for understanding transmission mechanisms and designing effective policy in economies with significant informality.

Keywords: Informal economy; Tax evasion; Monetary policy transmission; Fiscal policy; Public debt; DSGE model; Capital reallocation; Colombia.

JEL Classification: E52; E62; E26; H26; O17; O54.

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1 Introduction

A growing body of research has recently examined the phenomenon of informality in both developing and advanced economies. Across all contexts, informality is widely recognized as a major challenge for economic performance, as it involves a large share of the labor market and directly affects public finances.

This paper studies how labor market informality affects inflation dynamics and the transmission of monetary policy. It develops an augmented Taylor rule in which the central bank responds to aggregate output—including output produced in the informal sector—rather than to formal output alone. The analysis further examines how monetary policy tightening propagates in economies characterized by pervasive informality and how informality modifies the diffusion of monetary policy shocks.

Informality generate deep structural distortions that influence the economy’s aggregate behavior. As shown by [Restrepo-Echavarría \(2014\)](#), when these phenomena are not accurately measured, they may lead to an overestimation of GDP and distort the indicators used by policymakers to assess the state of the economy. From this perspective, understanding how informality interacts with macroeconomic policies is crucial for the correct interpretation of business cycle dynamics and the design of effective stabilization tools.

The existing literature has predominantly approached to the informality as a fiscal issue, emphasizing its effects on government revenues, fiscal sustainability, and the transmission of fiscal policy (e.g., [Pappa et al., 2015](#); [Annicchiarico and Cesaroni, 2018](#); [Colombo et al., 2024](#)). In these frameworks, informality, and more precise tax evasion, modifies the effectiveness of fiscal policy by altering the response of output and consumption to government spending shocks. However, this strand of research largely overlooks the interaction between informality and monetary policy.

Only a limited number of contributions have explored the monetary dimension of tax evasion. Pioneering works such as [Castillo and Montoro \(2010\)](#), [Ospina and Hairault \(2021\)](#); [Ospina \(2023\)](#), and [Yepez \(2025\)](#) investigate how monetary policy affects tax evasion, yet pay little attention to the reverse channel, how informality and evasion may shape the transmission of monetary policy. These studies typically assume that the central bank reacts to fluctuations in aggregate GDP without distinguishing between the formal and informal components of output. Because informal activity is only partially reflected in official statistics, such an approach may obscure important feedback mechanisms linking the informal sector and monetary policy.

To address this gap, I conduct an empirical analysis based on a structural vector autoregression (SVAR) for Colombia, an economy characterized by a persistently high level of informality. In fact, according to data from the OECD and the ILO ([OECD, 2024](#)), approximately 60 percent of the Colombian labor force operates in the informal sector, with only the remaining share formally employed.

The empirical findings reveal that an increase in informal employment immediately boosts aggregate output, confirming that in highly informal economies the formal and informal sectors operate as a co-integrated system rather than as separate entities ([Ulyssea, 2020](#)).

However, the response of prices is delayed: inflation remains initially muted and rises only after several quarters. This apparently counterintuitive pattern can be rationalized, theoretically, by the increase in marginal costs associated with market labor adjustment and hidden

compliance risks faced by firms hiring informally.

Despite the eventual rise in prices, the nominal interest rate declines, revealing a clear *puzzle*. This result departs from standard New Keynesian predictions and aligns with the inverted aggregate demand mechanism described by [Bilbiie \(2008, 2020\)](#), where a large share of Hand-to-Mouth consumers in the informal sector causes the aggregate demand curve to slope upward. In such economies, a reduction in interest rates stimulates demand even amid rising prices, leading the central bank to accommodate inflationary pressures rather than counteract them.

Hence, the Central Bank’s response is not an anomaly but an equilibrium outcome in economies dominated by informality and liquidity-constrained households, consistent with heterogeneous-agent frameworks ([Kaplan et al., 2018](#); [McKay et al., 2016](#); [Broer et al., 2020](#)).

When a contractionary monetary policy shock materializes, the empirical evidence points to a transmission pattern consistent with the canonical tightening mechanism documented in the literature. Specifically, such shocks are associated with a decline in inflation, a reduction in output growth, and a contraction in the informal employment rate. This finding suggests that informality does not merely influence monetary policy outcomes indirectly—through its effects on inflation dynamics and GDP growth—but is itself endogenously affected by monetary policy interventions. In particular, standard transmission channels of monetary policy, extensively discussed in the existing literature, exert a non-negligible impact on the size and dynamics of the informal sector.

To assess how this mechanism differs from the benchmark case in which informality is absent, I estimate a baseline structural vector autoregression (SVAR) model that does not explicitly account for informal employment. The comparison between the two frameworks yields noteworthy results. Informality appears to act as a buffer during economic downturns, partially absorbing adverse shocks and mitigating their impact on formal economic activity. At the same time, however, the presence of informality amplifies the standard monetary transmission mechanism during expansionary phases, strengthening the response of macroeconomic variables to monetary policy shocks.

Overall, these results highlight the dual role of informality as both a stabilizing force in recessions and an amplification channel during periods of economic expansion, underscoring the importance of incorporating labor market informality into the analysis of monetary policy transmission.

To replicate the empirical dynamics, I develop a Two-Agent New Keynesian (TANK) model featuring heterogeneity in skills, sectoral allocation, and consumption behavior. The labor force is divided into a fraction of high-skilled workers, employed exclusively in the formal sector, and a remaining share of low-skilled workers, who can be employed either formally or informally. Unemployment is ruled out, implying that all low-skilled workers excluded from the formal market are absorbed by the informal economy ([Busato and Chiarini, 2004](#); [Annicchiarico and Cesaroni, 2018](#)). Wages differ across skill and sector. High-skilled workers earn a taxable wage and can smooth consumption through savings and investment in physical capital and government bonds. In contrast, low-skilled workers earn a lower wage, taxed only in the formal sector, and follow rule-of-thumb behavior, consuming their entire disposable income each period.

Firms operate two production technologies—formal and informal—using a shared capital

stock and heterogeneous labor inputs. Capital is subject to adjustment costs that reflect its joint use across both sectors, consistent with the idea that firms reallocate resources internally to optimize production (Restuccia and Rogerson, 2008; La Porta and Shleifer, 2014a). Informal production relies exclusively on low-skilled labor, while formal output employs both skill groups. Building on the framework developed by Busato and Chiarini (2004), I show formally that firms cannot optimally choose one type of labor input without simultaneously employing the other. Within this environment, the profit maximization problem admits no corner solutions with respect to labor demand: the optimal allocation requires strictly positive levels of all labor inputs. Thus, consistently with the underlying production structure, each type of labor is essential, and the joint use of all labor inputs is necessary to maximize profits in formal production. This setting captures how informality interacts with investment and labor decisions, influencing aggregate dynamics and the transmission of monetary policy.

An important feature of the model concerns price setting: intermediate-goods producers do not choose prices solely based on conditions in the formal sector. Instead, because firms operate simultaneously in the formal and underground economies, they take into account production that relies on informal labor. As a result, total output reflects the combination of a single good produced using two distinct technologies—formal and informal.

This modeling choice is crucial for capturing the cyclical properties of the economy and is consistent with both my empirical findings and those of the related literature. Moreover, it helps clarify why informality affects inflation dynamics and why it is essential for policymakers to explicitly account for the informal sector when designing and evaluating policy interventions.

To capture the empirically counterintuitive endogenous response of the central bank, I explore whether this behavior is driven by *Inverted aggregate demand logic*. Specifically, I examine how changes in the share of workers employed in the formal sector affect the model's equilibrium, holding all other parameters constant.

This finding constitutes a novel contribution to the literature by shifting the perspective on informality. Rather than focusing exclusively on how monetary policy shocks propagate through the informal economy, it highlights how the size and structure of the informal sector itself shape both the transmission mechanism and the conduct of monetary policy.

More importantly, it reveals a critical macroeconomic implication: when informality becomes sufficiently pervasive, the economy may enter a region of equilibrium indeterminacy. In such an environment, standard active monetary policy rules—commonly assumed to be stabilizing in New Keynesian frameworks—may fail to ensure determinacy. I show formally that there exists a threshold level of informality beyond which an active monetary policy satisfying the Taylor principle no longer guarantees equilibrium uniqueness, thereby calling for alternative policy formulations to restore determinacy.

This paper shows that introducing labor market informality substantially alters the transmission of both real and monetary shocks through sectoral reallocation mechanisms. An informal labor supply shock is absorbed primarily by the informal sector, increasing informal employment and output while inducing a contraction in formal activity via wealth effects, leaving aggregate output broadly unchanged. Informality also reshapes factor demand by allowing capital to be reallocated toward informal production, thereby buffering aggregate fluctuations while amplifying formal–informal distortions. Following a monetary policy tightening, the

qualitative transmission remains contractionary, but its magnitude and persistence are significantly affected by informality, as capital and labor reallocation dampen the aggregate response. Crucially, policy rules that respond only to formal-sector variables misrepresent the state of the economy, whereas Taylor rules defined on aggregate output and inflation deliver dynamics comparable to the benchmark economy without informality.

Related Literature A growing literature has examined how informality interacts with monetary policy, both theoretically and empirically. Early theoretical contributions emphasize that standard monetary prescriptions may not be optimal in economies with significant informal sectors. Classical studies such as Phelps (1973), Keynes (1923), and Nicolini (1998) highlight that inflation can act as an implicit, hard-to-evade tax, suggesting that the Friedman Rule may not imply a zero nominal interest rate when informal activity is widespread.

Building on this foundation, Castillo and Montoro (2010) develop a two-sector New Keynesian framework with formal and informal labor markets, incorporating labor frictions à la Diamond-Mortensen-Pissarides. They show that the informal sector acts as a buffer, absorbing part of aggregate demand shocks, dampening wage and inflation responses, and weakening the conventional interest rate channel. Batini et al. (2011) further demonstrate that stronger financial frictions in the informal sector exacerbate time-inconsistency problems under simple Taylor rules. Alberola-Ila and Urrutia (2019) extend these results, showing that informality reduces the impact of demand, financial, and sectoral shocks on wages and inflation, while amplifying the inflationary effects of aggregate productivity shocks; greater labor market flexibility can mitigate divergences in sacrifice ratios between formal and informal sectors, improving policy effectiveness. More recently, Lahcen (2020) emphasizes that search frictions and limited access to credit in the informal sector amplify the effects of unemployment and reduce money demand, consumption, profits, and job creation, reinforcing the view that informal activity fundamentally alters the transmission of monetary policy.

Empirical evidence supports these theoretical insights. Ospina and Hairault (2021) and Ospina (2023) show that informal activity amplifies inefficiencies arising from output, wage, and employment fluctuations, increasing unemployment and welfare losses, particularly when central banks follow simple Taylor-type rules. Yopez (2025) provides a pioneering contribution using Mexican firm-level data, documenting the effects of monetary policy shocks on informal firms. His results show that contractionary monetary policy reduces informal employment, indicating that informal activity is sensitive to interest rate fluctuations and that standard policy transmission is dampened. Similarly, Chikonda and Chortareas (2024) analyze small open economies with sizable informal sectors and financially constrained households, demonstrating that interest rate shocks can produce counterintuitive effects, with informal employment partially offsetting contractions in formal output.

Overall, these studies converge on the conclusion that informality systematically modifies the transmission and effectiveness of monetary policy. Larger informal sectors dampen standard channels, reshape factor allocation across formal and informal sectors, and modify inflation–output trade-offs. Ignoring informal activity in either theoretical or empirical analyses risks mismeasuring policy effectiveness and misunderstanding the macroeconomic impact of interest rate changes.¹

¹For a comprehensive overview, the reader is referred to the surveys by Chiarini et al. (2024), which document how informal activity affects fiscal multipliers, public debt sustainability, and the optimal design of fiscal and

The contribution of this paper to the existing literature is twofold. First, using an SVAR framework, I empirically examine whether and how monetary policy effectively influences the informal sector—an aspect that has remained largely unexplored in previous empirical works. This analysis not only tests the robustness of theoretical predictions but also provides a clearer understanding of the transmission channels of monetary policy in economies with pervasive informality. Differently from most existing studies, which focus mainly on how the informal sector weakens the effectiveness of monetary policy, this paper explicitly adopts a two-way perspective by also analyzing how shocks originating *within* the informal economy affect the dynamics of key macroeconomic variables, including inflation, output, and interest rates, as well as the monetary policy response itself. This approach offers policymakers valuable insights into the actual functioning of the economy, allowing them to design more effective and context-specific stabilization measures.

Second, the paper provides both theoretical and empirical support for the procyclicality of informality, showing that the informal sector cannot be treated as a separate or dual system but rather as a co-integrated component of the aggregate economy. This finding complements and extends previous research by highlighting that informality interacts actively with macroeconomic fluctuations and monetary policy, rather than passively absorbing shocks.

In addition, the model introduces an original mechanism linking informality to macroeconomic determinacy. By reconciling empirical evidence with theoretical predictions, I demonstrate that allowing the degree of informality to vary can push the economy into indeterminacy, highlighting the potential need for alternative monetary policy rules to restore stability in highly informal settings. This finding opens a compelling research frontier, emphasizing how structural characteristics of the labor market critically shape the stability conditions of New Keynesian economies.

A limitation of the present work is that it does not explicitly analyze the interaction between fiscal and monetary policies. Although the model does not explore in detail how fiscal instruments should optimally respond to different monetary regimes. Future research could address this shortcoming by studying the coordination between fiscal and monetary authorities and assessing how joint policy design can improve macroeconomic stability and welfare in economies with significant informality.

2 Data Description

To analyze whether and how monetary policy affects informal employment—and vice versa—I employ a Structural Vector Autoregressive (SVAR) model following [Yepez \(2025\)](#).

Unlike most of the existing literature, this analysis focuses on Colombia, a country characterized by a persistently high level of labor informality, averaging around 60% of total employment [Maloney \(1999, 2004\)](#); [Ulyssea \(2018, 2020\)](#). While [Yepez \(2025\)](#) centers on Mexico using a DSGE-VAR framework to study the impact of monetary policy on informality through firm-level dynamics, the present study adopts a macroeconomic perspective applied to the Colombian economy.

monetary policy in economies with large shadow sectors.

2.1 Dataset and Preprocessing

The dataset collected for this analysis includes the GDP deflator—used to compute inflation—GDP growth, the nominal interest rate, and three measures of informality.

Before discussing the methodological aspects, it is important to highlight several considerations regarding the data. GDP growth is taken from the World Bank dataset, which defines it as the value of all goods and services produced in Colombia’s economy. At first glance, one might think that this measure implicitly incorporates some degree of informality, at least when informality is interpreted along the extensive margin². However, this is only one possible dimension of informality. The situation changes when the intensive margin is considered—specifically, the number of inputs or hours that agents allocate across sectors. As well established in the literature [Bosch and Maloney \(2008\)](#); [Ulyssea \(2018\)](#); [Restrepo-Echavarría \(2014\)](#); [Ulyssea \(2020\)](#); [Leyva and Urrutia \(2020\)](#), GDP growth cannot fully capture the dynamics of informality when both margins are taken into account. Particular caution is therefore required when interpreting this series.

To investigate whether this limitation may also apply to the Colombian economy, I examined the methodological notes of the World Bank dataset. A clear and relevant statement appears:

“Informal economic activities pose particular measurement problems, especially in developing countries, where much economic activity is unrecorded. A complete picture of the economy requires estimating household output produced for home use, sales in informal markets, barter exchanges, and illicit or deliberately unreported activities.”

This reinforces the idea that GDP growth does not necessarily reflect the full extent of informal economic activity. Given the complexity of measuring informality, choosing the appropriate indicator is crucial. The literature typically relies on two main approaches. The first involves using a Dynamic Stochastic General Equilibrium (DSGE) model, such as the framework developed by [Orsi et al. \(2012\)](#). The second is the Multiple Indicators Multiple Causes (MIMIC) model, which treats informality as a latent variable influenced by various observable factors.

An alternative and widely used measurement strategy relies on data from the International Labour Organization (ILO). The ILO defines informality through the Informal Employment Rate, measured as the share of individuals in informal employment relative to total employment. In statistical terms, the informal sector consists of economic units that produce goods and services primarily for the market but are not formally recognized by government authorities and therefore are not subject to formal regulatory arrangements.

A complementary proxy often used to capture a narrower aspect of informality is the share of workers employed outside the formal sector who are not actively seeking employment. Although not a complete measure, it provides additional insight into labor market dynamics.

In this analysis, I adopt the ILO’s definition of informality and use the Informal Employment Rate as the primary measure³. This indicator captures the informal sector comprehensively,

²That is, the possibility of workers or firms moving between sectors.

³Informal employment comprises all informal jobs carried out, during a given reference period, in formal sector enterprises, informal sector enterprises, or households. According to the definition adopted by the International Labour Organization (ILO), informal employment includes several categories of employment status. Specifically, it encompasses: (i) own-account workers and employers operating in informal sector enterprises; (ii) contributing family workers, regardless of whether they are engaged in formal or informal sector enterprises; (iii) members

as it is constructed from surveys covering both firms and households. As a robustness check, I also employ the previously mentioned proxy—the share of individuals employed outside the formal sector—which allows me to verify the consistency of the main findings.

The dataset consists of quarterly observations spanning from the second quarter of 2007 to the fourth quarter of 2024, yielding a total of 71 observations. Prior to estimating the SVAR, the GDP deflator was transformed into an inflation rate following standard practice in the literature.

2.2 Model Specifications

In the model specification, informal employment is introduced at the aggregate level. The model is defined as follows:

$$A_0 X_t = B + \sum_{i=1}^p A_i X_{t-i} + \varepsilon_t$$

where

$$\mathbf{X}_t = \begin{bmatrix} \text{Inflation}_t \\ \text{GDP Growth}_t \\ \text{Informal Employment}_t \\ \text{Nominal Interest Rate}_t \end{bmatrix} \Rightarrow \begin{bmatrix} \pi_t \\ g_t \\ \Gamma_t \\ i_t \end{bmatrix}.$$

B is the constant term, A_i is coefficient matrix and ε_t is the matrix of structural shocks. The economic rationale for this identification is that aggregate informal employment responds contemporaneously to innovations in both inflation and GDP growth. This assumption stems from the central research question: whether informality plays a role in shaping the nominal interest rate, and, if so, how the central bank should adjust its policy. By allowing informal employment to respond contemporaneously to key macroeconomic variables, the model captures the potential interaction between labor market informality and monetary policy.

Before estimation, it is useful to frame informality as a structural and persistent component of the economy, rather than as a residual or cyclical phenomenon [La Porta and Shleifer \(2014b\)](#); [Gerxhani \(2004\)](#). Empirical evidence supporting this view remains limited, yet it offers an important conceptual foundation for interpreting the results.

of informal producers' cooperatives, namely cooperatives that are not formally established as legal entities and that meet the criteria for informal sector enterprises established by the ILO; (iv) employees holding informal jobs, whether employed in formal sector enterprises, informal sector enterprises, or as paid domestic workers employed by households; and (v) own-account workers engaged in the production of goods exclusively for own final use by their household, provided that such activities are classified as employment according to international statistical standards. Employees are considered to hold informal jobs when their employment relationship is not, in law or in practice, subject to national labour legislation, income taxation, or social protection systems, nor does it entitle them to employment-related benefits such as advance notice of dismissal, severance pay, or paid annual or sick leave. This may result from, among other factors, the non-declaration of jobs or workers, the casual or short-term nature of employment, working hours or earnings below statutory thresholds, employment by unincorporated enterprises or households, or the non-application, non-enforcement, or non-compliance with labour regulations. The operational criteria used to identify informal jobs are determined in accordance with national institutional contexts and data availability.

2.3 Empirical Results

Figure 1 and 2 show the Impulse Response Function of the empirical analysis. In this section I'm going to discuss the result of a shock to informal employment and a monetary policy shock. The shocked variable is highlight in orange color while the response of others aggregate variable is indicated in blue.

When I speak about Informal Shock, a central challenge in the macroeconomic literature concerns the definition and interpretation of what can be termed an “aggregate informality shock.” Unlike canonical shocks to demand or supply, whose mechanisms are relatively well understood and widely accepted, the dynamics of informality remain far less transparent. This ambiguity is not a weakness, but rather a defining feature of the phenomenon: informality can manifest simultaneously through channels that resemble demand-side pressures and others that operate on the supply side.

On the demand side, one could imagine scenarios where higher consumer confidence, shifts in household preferences, or saturation in the formal labor market lead individuals to seek opportunities in the informal sector. Workers may decide to engage in secondary or part-time jobs, or even substitute formal employment with informal activities in response to changes in expected utility. In such cases, informality operates much like a demand shock: an expansion of labor supply outside the formal sector feeds into aggregate consumption and, through this channel, influences broader economic dynamics.

Yet, this is only part of the story. Informality can also emerge from supply-side disruptions. A negative productivity shock, for instance, may reduce firms' output capacity, forcing them to downsize their formal workforce and unintentionally pushing displaced workers into informal employment. Conversely, a positive productivity shock may stimulate production, with firms simultaneously relying on informal labor to cut costs and maximize profits. In this perspective, informality becomes a reallocation mechanism that amplifies or mitigates supply disturbances, directly shaping the structure of employment and production.

The implication is clear: an aggregate informality shock cannot be neatly classified within the conventional taxonomy of macroeconomic disturbances. It may be demand-driven, supply-driven, or—most plausibly—a hybrid of the two, depending on the structural and institutional conditions of the economy. This duality is precisely what makes informality such a compelling and yet elusive object of study: it resists simple categorization and challenges economists to rethink standard transmission channels.

2.4 Informal Labor Supply Shock

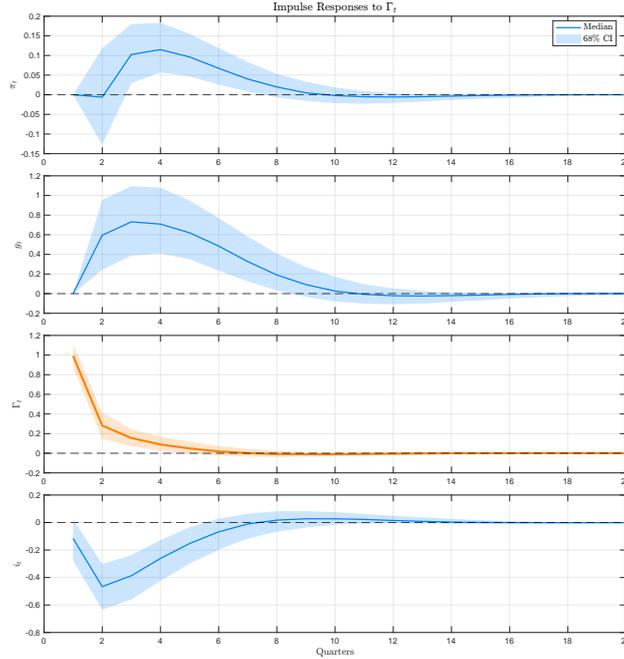


Figure 1: Informal labor supply shock

Figure 1 reports the impulse responses to an Informal Labor Supply Shock, defined as a negative preference shock that increases the incentive for individuals to work in the informal sector. The blue line depicts the response of aggregate GDP, while the red line shows the response of inflation.

The empirical results indicate that a one-standard-deviation increase in the shock generates a statistically significant rise in informal employment, with the effect peaking approximately one quarter after the shock. Aggregate GDP also responds positively, although the magnitude is smaller than that observed for informal employment. In contrast, inflation exhibits no statistically significant response during the initial quarters, with a delayed and modest increase appearing only after approximately three quarters.

These patterns are consistent across alternative specifications of the SVAR and robust to different identification schemes. Notably, the dynamics suggest that informal labor supply shocks have a measurable impact on macroeconomic activity, even when controlling for standard demand and supply factors.

These empirical findings align with previous studies documenting the role of the informal sector in shaping macroeconomic outcomes [Bosch and Maloney \(2008\)](#), [Bosch and Esteban-Pretel \(2012\)](#) [Maloney \(2004\)](#) [Maloney \(1999\)](#) [Ulyseas \(2018\)](#), [Ulyseas \(2020\)](#).⁴

⁴The underlying mechanisms generating these responses—such as wage adjustments, factor reallocation, and interactions with monetary policy—are examined in detail in the theoretical model in Section

2.5 Monetary Policy Shock

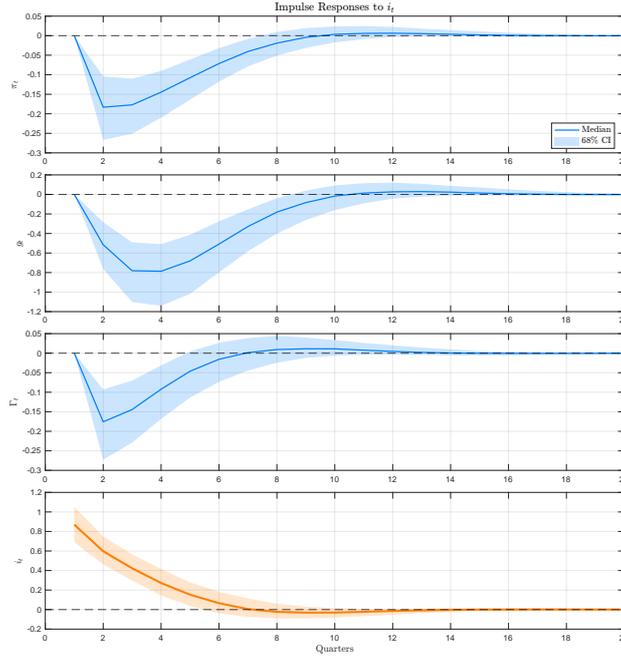


Figure 2: Monetary policy shock

Figure 2 displays the impulse response functions to a monetary policy shock, identified as a one-standard-deviation increase in the nominal policy interest rate. The blue line reports the response of informal employment, while the red and green lines show the responses of aggregate GDP and inflation, respectively.

The empirical results indicate that a monetary tightening leads to a statistically significant reduction in informal employment. Aggregate GDP also declines following the shock, whereas inflation exhibits a smaller and delayed response, with effects becoming noticeable after two to three quarters. The peak impact on informal employment occurs within the first two quarters, while the contraction in output is more persistent over time.

These findings suggest that monetary policy shocks propagate to the informal sector, challenging the notion that informal employment simply acts as a residual or countercyclical buffer. Instead, informal and formal labor markets appear to be closely interconnected, with changes in aggregate conditions affecting both simultaneously⁵.

3 Theoretical model

To match the empirical findings, I propose a two agent new Keynesian model. In the proposed model, the labor force is divided into two categories: a fraction ζ consists of highly skilled workers (*high-skilled*), while the remaining share $(1 - \zeta)$ is composed of low-skilled workers (*low-skilled*).

⁵The results are robust to alternative specifications of the SVAR and identification schemes. They align with prior empirical evidence highlighting the sensitivity of informal employment to macroeconomic conditions and monetary interventions Maloney (1999); Bosch and Maloney (2008); Maloney (1999); Ulyssea (2020); Yepez (2025); The theoretical mechanisms through which these responses occur—such as wage adjustments, factor reallocation, and interactions with aggregate demand—are discussed in Section

High-skilled workers are employed exclusively in the formal sector, where they supply labor in the amount of $n_{H,t}$ and receive a wage $w_{H,t}$ subject to taxation at rate τ_t^H . It is assumed that these workers face no idiosyncratic risk of being reallocated to the informal sector.

With regard to low-skilled workers, and in line with the existing literature, it is assumed that each individual supplies one unit of labor, i.e., $n_{L,t} = 1$. However, due to their lower qualifications, only a fraction θ_t is employed in the formal sector, while the remaining share $(1 - \theta_t)$ works in the informal sector. The allocation between sectors is not determined by the workers themselves, but rather by the firms. Low-skilled workers only decide how many hours to work, taking into account the probability of being allocated to either sector.

The model abstracts from unemployment: any low-skilled worker not employed in the formal sector will be employed in the informal sector. Therefore, in their utility function, low-skilled workers consider the possibility of being employed informally, as highlighted in the reference literature [Busato and Chiarini \(2004\)](#); [Annicchiarico and Cesaroni \(2018\)](#).

In terms of compensation, low-skilled workers earn a lower wage compared to their high-skilled counterparts. The base wage for low-skilled workers is homogeneous, but taxation differs by sector. Specifically, low-skilled workers in the formal sector receive a net wage of $w_{L,f,t} = (1 - \tau_t^H)w_{L,t}$, while those in the informal sector receive the full wage: $w_{L,i,t} = w_{L,t}$.

This wage structure leads to different consumption behaviors. Low-skilled workers, being unable to smooth consumption, spend all their income in the period in which it is earned, following a rule-of-thumb (R.O.T.) behavior. By contrast, high-skilled workers, due to their higher earnings, are able to save and invest in both physical capital and government bonds.

3.1 High Skills Consumer

Starting from the optimization problem of *high-skilled* workers, I assume that they maximize their utility function which is the following:

$$\begin{aligned} & \max_{\{c_{H,t}, n_{H,t}, k_t, B_t, i_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{H,t}^{1-\sigma}}{1-\sigma} - h_h \frac{n_{H,t}^{1+\gamma}}{1+\gamma} \right) \quad (1) \\ \text{s.t.} \quad & \begin{cases} p_t c_{H,t} + p_t i_t + p_t B_t = r_t^k p_t k_{t-1} + p_t r_{t-1} B_{t-1} + p_t (1 - \tau_t^H) w_{H,t} n_{H,t} + p_t \Gamma_t^F \\ k_t = (1 - \delta) k_{t-1} + \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \end{cases} \end{aligned}$$

where c_t^H is the consumption, n_t^H is hours worked by high skills, i_t is the investment, B_t is the risk-less Government bonds, r_t^k is the rent rate of capital, k_t is the amount of capital, r_{t-1} is the rent rate of risk-less bonds, Γ_t^F are the dividends and δ is the depreciation rate.

From the maximization problem in equation 26, I derive the Euler equation that governs the consumption path of high-skilled agents, the labor supply condition—which follows the standard formulation in the literature given the absence of idiosyncratic risk—the shadow value of capital, and the marginal utility of consumption. ⁶

⁶Derivations are available in the Appendix upon request.

3.2 Low skilled Consumers

As defined above, I follow the framework of [Busato and Chiarini \(2004\)](#). The authors assume the existence of a continuum of consumers, uniformly distributed over the unit interval $[0, 1]$, each supplying labor to both the market and the underground sectors.

Each consumer has preferences over sequences of consumption and labor effort, and maximizes expected lifetime utility, given by the utility function:

$$u(C_{L,t}, \theta_t) = \frac{(C_{L,t})^{1-q} - 1}{1-q} - h \frac{(\theta_t)^{1+\gamma}}{1+\gamma} (1 - \theta_t) - f \frac{(1 - \theta_t)^{1-\eta}}{1-\eta}$$

This function is separable in consumption and labor, allowing for an analysis of how households allocate labor between the market and the underground sectors. To ensure that the utility function is well-behaved, the authors assume $h, f \geq 0$, $\gamma > -1$, and $\eta \in (0, 1)$, and that all components of the utility function are twice continuously differentiable.

The second term represents the total disutility of labor, while the third term reflects the idiosyncratic cost of working in the underground sector. This cost can be interpreted as capturing the absence of social and health insurance coverage associated with informal employment.⁷

According to [Busato and Chiarini \(2004\)](#), I assume that a varying share of hours worked, denoted by θ_t , is employed in the formal sector, while the remaining share of hours worked $1 - \theta_t$ works in the informal sector. Both types of workers receive the same gross wage; however, formal workers are subject to a labor income tax τ_t^h , so their net wage is $w_{L,f,t} = w_t^L (1 - \tau_t^h)$, while informal workers receive the untaxed wage $w_{L,i,t} = w_t^L$.

In this setting, and at this stage of the model, I assume that workers cannot invest in capital or purchase debt instruments. This is motivated by the assumption that their wage income is too low to allow them to accumulate capital or participate in financial markets—essentially, they are credit constrained.

Their budget constraint is

$$\bar{C}_{L,t} = w_{L,t} (1 + \theta_t \tau_t^h) \quad (2)$$

From the maximization problem, I obtained the following labor supply:

$$w_{L,t} \tau_t^h = \bar{C}_t^L \left(h (\theta_t - 1) (\theta_t)^\gamma + f (1 - \theta_t)^{-\eta} + h \frac{(\theta_t)^{(1+\gamma)}}{1+\gamma} \right) \quad (3)$$

The labor supply equation describes the optimal allocation of time between formal and informal activities. The after-tax formal wage, $w_{L,t} \tau_t^h$, is balanced against the marginal utility of consumption and the overall disutility of labor, which depends on the share of hours allocated to each sector.

⁷To study shocks to informal labor supply, I depart from the standard labor supply shock commonly analyzed in the literature. In conventional frameworks, labor supply shocks are typically modeled as stochastic components, ε , entering the utility function through the disutility of labor. Under this representation, such shocks capture fluctuations in the marginal disutility of working. In contrast, the informal labor supply shock considered in this paper is conceptually distinct. It does not operate through changes in the intrinsic disutility of labor. Instead, it captures variation in the idiosyncratic costs and risks associated with participation in the informal sector. In the model, these costs are explicitly represented by a separate term in the utility function—namely, the third additive component—allowing the shock to affect informal labor supply independently of preferences over hours worked.

The parameter θ_t represents the fraction of time devoted to formal work, while $(1 - \theta_t)$ denotes the share of informal labor. Since total available time is fixed, the two components move in opposite directions: an increase in formal labor necessarily reduces time spent in informal activities, and vice versa. However, the relationship between the two is not purely substitutive.

In this framework, a positive productivity shock may lead to an *increase in informal employment*. This occurs because higher productivity raises aggregate demand and household income, inducing individuals to supply more labor overall, including in the informal sector. Hence, informality acts as a *cyclical adjustment margin* of the labor market—expanding during booms and absorbing part of the adjustment in employment and output fluctuations.

The disutility terms inside the brackets capture the heterogeneous costs associated with the two types of work. The first component, $h(\theta_t - 1)(\theta_t)^\gamma$, reflects the increasing disutility from participating in the formal sector, whereas $f(1 - \theta_t)^{-\eta}$ represents the cost of informal work, which rises when the share of informal activity becomes too small. The final term, $h \frac{(\theta_t)^{1+\gamma}}{1+\gamma}$, normalizes total labor disutility.

In equilibrium, individuals choose the optimal composition between formal and informal labor (θ_t) by equating the marginal benefit of working an additional unit in the formal sector with the marginal cost arising from reduced informal participation and the higher total disutility of labor.

3.3 Informal Labor Supply shock

Within the framework described above, an informal labor supply shock differs fundamentally from a standard labor supply disturbance. In particular, a negative informal labor supply shock does not merely reduce the disutility of labor for low-skilled workers, as in conventional models. Rather, it simultaneously lowers the perceived disutility from supplying labor and the idiosyncratic cost or risk associated with participation in informal activities.

The first channel operates through a reduction in the marginal disutility of work in general, captured by the term

$$h \frac{\theta_t^{1+\gamma}}{1+\gamma} (1 - \theta_t),$$

while the second channel reflects a decrease in the perceived risk of informality, represented by

$$f \frac{(1 - \theta_t)^{1-\eta}}{1 - \eta}.$$

Accordingly, the period utility function is specified as

$$u(C_{L,t}, \theta_t) = \frac{C_{L,t}^{1-q} - 1}{1 - q} - \iota_t h \frac{\theta_t^{1+\gamma}}{1+\gamma} (1 - \theta_t) - \iota_t f \frac{(1 - \theta_t)^{1-\eta}}{1 - \eta},$$

where θ_t denotes the share of labor supplied in the formal sector, and $(1 - \theta_t)$ corresponds to informal labor. The stochastic process ι_t governs informal labor supply conditions and follows an AR(1) process, capturing persistent fluctuations in both the disutility of work and the perceived cost of informality.

Proposition 1 (Steady-state existence and economic admissibility). *Given the parameter values reported in Table 1, the steady state of the economy exists and is economically admissible if and only if*

$$\theta \in \Theta, \quad \Theta = [\underline{\theta}_1, \bar{\theta}_1],$$

where

$$[\underline{\theta}_1, \bar{\theta}_1] = [0.2, 0.76].$$

For values of θ outside the set Θ , at least one steady-state allocation becomes negative or undefined, rendering the steady state economically inadmissible.

Proof. See Appendix 6.3 □

3.4 Intermediate goods producers

In line with [Annicchiarico and Cesaroni \(2018\)](#), I consider an economy characterized by monopolistic competition, where a continuum of firms index i , $i \in (0, 1)$ can partially evade taxes and social security contributions by employing undeclared (informal) labor. Firms face nominal rigidities in price-setting behavior, modeled à la Rotemberg (1982).

Each representative firm produces a differentiated intermediate good using three inputs supplied by households: high-skilled labor, low-skilled labor, and physical capital. Specifically, the firm hires both high- and low-skilled labor for production in the formal (market) sector, while a fraction of the low-skilled labor force can also be allocated to the informal (underground) sector. Capital, on the other hand, is rented exclusively from high-skilled households.

To prevent excessive informality, I impose the restriction that firms operating in the underground economy cannot scale production without limit, as doing so would increase their visibility and thus the likelihood of detection by tax authorities. This assumption aligns with the frameworks adopted in [Busato and Chiarini \(2004\)](#); [Annicchiarico and Cesaroni \(2018\)](#); [Restrepo-Echavarría \(2014\)](#); [Orsi et al. \(2012\)](#), which introduce technological or institutional constraints on informal production to ensure internal consistency and empirical plausibility.

At this stage, I assume that intermediate goods can be produced using two distinct technologies: one for the formal sector and one for the informal sector. In the formal sector, production combines high- and low-skilled labor using a constant elasticity of substitution (CES) production function, reflecting the complementary nature of these inputs. This approach captures the realistic scenario in which firms cannot operate solely with either high-skilled or low-skilled workers. Instead, both types of labor are essential. For instance, one may think of a large firm comprising both a research and development (R&D) division, predominantly staffed by high-skilled workers, and a manufacturing unit, largely reliant on low-skilled labor. The classification of worker types depends on the specific definition adopted within the model.

Importantly, in the formal sector, I assume that there is no intensive margin of substitution between worker types: firms must employ both high- and low-skilled labor in fixed proportions determined by the CES structure. In contrast, in the informal sector, firms face both intensive and extensive margins of adjustment. That is, they not only decide how many low-skilled workers to employ in total, but also how to allocate them between the formal and informal sectors.

The market sector production function is specified as a CES-type aggregator that combines capital with an effective labor composite, itself composed of high-skilled and low-skilled labor. Specifically, output in the market sector at time t is given by:

$$Y_t^m(i) = M_t \left\{ k_{t-1}^\alpha \left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1-\alpha}{\xi}} \right\}, \quad (4)$$

where Y_t^m denotes output, M_t represents a time-varying productivity (or efficiency) shifter, and k_{t-1} is the predetermined stock of physical capital. The terms $n_{h,t}$ and θ_t represent high-skilled and formal low-skilled labor, respectively. Parameters B and G are share parameters satisfying $B, G > 0$, and $\alpha \in (0, 1)$ denotes the capital share in production.

The elasticity of substitution between high-skilled and low-skilled labor is governed by the parameter ξ . The inner CES aggregator,

$$\left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1}{\xi}},$$

determines the degree of substitutability between the two types of labor. As is standard in the CES specification, the associated elasticity of substitution is given by:

$$\varrho = \frac{1}{1 - \xi}.$$

This implies the following relationships between the value of ξ and the nature of labor input interactions:

- When $\xi < 0$, the implied elasticity of substitution satisfies $0 < \varrho < 1$. In this case, high-skilled and low-skilled labor are **complements**, consistent with the notion of capital-skill or labor-skill complementarity [Dolado et al. \(2021\)](#).
- When $\xi = 0$, the aggregator reduces to the **Cobb–Douglas** form, implying a unitary elasticity of substitution, i.e., $\varrho = 1$.
- When $0 < \xi < 1$, the elasticity satisfies $\varrho > 1$, indicating that the two labor inputs are **imperfect substitutes**, with a degree of substitutability that increases in ϱ [Blanchard and Kiyotaki \(1987\)](#); [Galí \(2008\)](#); [Dolado et al. \(2021\)](#).

This functional form provides a flexible framework to model varying degrees of substitutability or complementarity between capital, high-skilled labor, and low-skilled labor. It captures the empirically relevant feature that skilled labor and capital may be complements, while unskilled labor can be more easily substituted, reflecting the dual nature of modern production processes observed in firm-level and macroeconomic data .

Concerning the informal (underground) sector, following the approach of [Busato and Chiarini \(2004\)](#); [Pappa et al. \(2015\)](#); [Restrepo-Echavarria \(2014\)](#); [Annicchiarico and Cesaroni \(2018\)](#); [Orsi et al. \(2012\)](#), I assume a distinct production technology in which firms exclusively employ informal labor. In the model setup, informal workers are identified as low-skilled laborers who are unable to secure employment in the formal sector and are consequently compelled to participate in the informal sector. This assumption also implies that, at the current stage of the model, there is no unemployment; rather, the total labor force is given by

$$\bar{N}_t = \zeta n_{h,t} + (1 - \zeta)(\theta_t n_{l,t} + (1 - \theta_t)n_{l,t}),$$

where $n_{h,t}$ and $n_{l,t}$ denote high-skilled and low-skilled labor inputs, respectively, ζ represents the share of high-skilled workers in the labor force, and θ_t captures the allocation of low-skilled labor between formal and informal sectors. Assuming $n_{l,t} = 1$, the expression simplifies to

$$\bar{N}_t = \zeta n_{h,t} + (1 - \zeta)n_{l,t}.$$

The informal (underground) technologies is

$$Y_t^u(i) = Z_t(1 - \theta_t) \quad (5)$$

where M_t, Z_t follow an AR(1)

$$\log\left(\frac{M_t}{M}\right) = \rho_m \log\left(\frac{M_{t-1}}{M}\right) + \epsilon_m \quad (6)$$

$$\log\left(\frac{Z_t}{Z}\right) = \rho_z \log\left(\frac{Z_{t-1}}{Z}\right) + \epsilon_z \quad (7)$$

Then, according to the literature I have one output $\bar{Y}_t(i)$

$$\bar{Y}_t(i) = Y_t^m(i) + Y_t^u(i)$$

The total cost of the representative firms is given by the rent rate of capital r_t^k , the wage paid to high skilled labor $w_{H,t}$ plus the the tax τ_t^f , the wage paid to low skilled workers employed into formal sector $w_{L,f,t}(1 + \tau_t^f)$, the wage paid to low skilled workers employed into informal (underground) sector $w_{L,i,t}$ and the quadratic adjustment cost setting the price AC_t . The last one is in the nominal terms à la Rotemberg (1982)

$$AC_t(i) = \frac{K_P}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi} \right) \bar{Y}_t$$

In the model, capital adjustment costs are not confined solely to the formal sector, as is commonly assumed in standard macroeconomic models. Instead, given the inclusion of informality, I consider capital as a firm-level resource that is shared across both formal (market) and informal (underground) production technologies. The representative firm produces a single final good using two distinct production technologies: one operating in the formal sector and the other in the informal sector. While capital costs are typically associated with the formal sector due to the observability and traceability of investment decisions, in this framework, the total adjustment cost of capital reflects the joint use of capital across both sectors.

This modeling choice is grounded in the idea that firms may reallocate inputs internally across sectors to optimize production, consistent with the literature on multi-technology firms and sectoral reallocation (Restuccia and Rogerson, 2008; La Porta and Shleifer, 2014a). Specifically, I assume that informal production relies exclusively on low-skilled labor, which is excluded from formal employment due to regulatory or institutional constraints. However, the capital stock is shared across both technologies, implying that investment decisions affect the entire

productive structure of the firm. A potential extension of this research could explore how capital depreciation rates vary across formal and informal uses. It would be especially interesting to investigate whether capital employed in informal activities depreciates more rapidly, possibly due to inefficiencies, lack of maintenance, or less environmentally sustainable practices.

This line of inquiry is particularly relevant in the context of the ongoing green transition, as accelerated capital degradation in informal sectors may pose additional challenges for sustainable development and carbon-neutral investment strategies (Acemoglu et al., 2012; Dabla-Norris et al., 2023).

An important feature of the model concerns price setting. Intermediate-goods producers do not choose prices solely based on conditions in the formal sector. Instead, because firms operate simultaneously in the formal and underground economies, they take into account production that relies on informal labor. As a result, total output reflects the combination of a single good produced using two distinct technologies—formal and informal.

This modeling choice is crucial for capturing the cyclical properties of the economy and is consistent with both my empirical findings and those of the related literature. Moreover, it helps clarify why informality affects inflation dynamics and why it is essential for policymakers to explicitly account for the informal sector when designing and evaluating policy interventions.⁸

The total cost are given by

$$\begin{aligned} \mathcal{CO}(k_{t-1}, \theta_t, n_{h,t}, p_t) &= r_t^k k_{t-1} + w_{H,t}(1 + \tau_t^f) n_{H,t} + w_{L,f,t}(1 + \tau_t^f) \theta_t \\ &\quad + w_{L,i,t}(1 - \theta_t) + \frac{K_P}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi} \right) \bar{Y}_t. \end{aligned}$$

Proposition 2 (Interior Solution of Labor Choice). *Consider the firm's problem of choosing high-skilled/formal labor $n_{h,t}$ and the fraction of low-skilled/informal labor θ_t to maximize net profits given the production technology and tax schedule. Under the model assumptions, the optimal choice of θ_t is strictly interior:*

$$0 < \theta_t^* < 1.$$

Consequently, the firm always employs a positive amount of both high- and low-skilled labor, and no corner solution arises.

Proof. See Appendix 6.3 □

The firms' profit maximization problem takes place under uncertainty due to the possibility of a government audit. At this stage, the probability of an audit occurring, denoted by $p \in (0, 1)$, is a parameter. If a firm is detected evading taxes, the government imposes a penalty not only on the formal tax rate τ^f but also on the firm's declared income. This penalty is captured by a surcharge factor s , which is also determined endogenously within the model. In this setting, the firms' profit is

⁸As shown in the Appendix, introducing adjustment costs that depend only on formal-sector production leads to markedly different outcomes, including countercyclical movements between the formal and informal economies.

$$\Pi_t(i) = \begin{cases} (1 - \tau_t^f)Y_t^m(i) + (1 - ps\tau_t^f)Y_t^u(i), & \text{if } ps\tau_t^f < 0 \\ (1 - \tau_t^f)Y_t^m(i), & \text{otherwise} \end{cases} \quad (8)$$

So, the problem become

$$\max_{k_{t-1}, n_{H,t}, \theta_t, p_t(i)} \Pi_t(i) : \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t^h} \left[\begin{array}{l} \frac{p_t(i)}{p_t} \bar{Y}_t(i) - r_t^k k_{t-1} - w_{H,t}(1 + \tau_t^f)n_{H,t} \\ - w_{L,t}(1 + \theta_t \tau_t^f) - \frac{K_P}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi} \right)^2 \bar{Y}_t \end{array} \right] \right\} \quad (9)$$

subject to

$$\begin{cases} Y_t^m(i) = M_t \left\{ k_{t-1}^\alpha \left(Bn_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}} \right\}, \\ Y_t^u(i) = Z_t(1 - \theta_t), \\ \bar{Y}_t = (1 - \tau_t^f)Y_t^m(i) + (1 - ps\tau_t^f)Y_t^u(i), \\ \bar{Y}_t(i) = \bar{Y}_t \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon}, \end{cases}$$

From the optimization problem, I derive the demand for capital, the demand for high- and low-skilled labor, and the optimal price-setting condition.

The main question for this problem are the low skilled labor demand, the New Keynesian Philips Curve and the marginal cost. ⁹

$$w_{L,t}\tau_t^f = MC_t(i) \left\{ (1 - \tau_t^f) \frac{1-\alpha}{\xi} M_t k_{t-1}^\alpha \left(Bn_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} \xi G\theta_t^{\xi-1} - (1 - ps\tau_t^f)Z_t \right\} \quad (10)$$

$$(\pi_t - \bar{\pi})\pi_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[(\pi_{t+1} - \bar{\pi}) \frac{\bar{Y}_{t+1}}{\bar{Y}_t} \pi_{t+1} \right] + \frac{\varepsilon}{k_p} \left(MC_t(i) - \frac{\varepsilon - 1}{\varepsilon} \right) \quad (11)$$

$$MC_t(i) = \frac{w_{L,t}\tau_t^f}{(1 - \tau_t^f)(1 - \alpha)M_t \left[\frac{\alpha}{1-\alpha} \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} (B + G\Phi_t) \right]^\alpha (B + G\Phi_t)^{\frac{1-\alpha}{\xi}-1} G\Phi_t^{\frac{\xi-1}{\xi}} - (1 - ps\tau_t^f)Z_t} \quad (12)$$

where

$$\Phi_t := \left(\frac{B}{G} \cdot \frac{w_{L,t}\tau_t^f + (1 - ps\tau_t^f)Z_t}{w_{h,t}(1 + \tau_t^f)} \right)^{\frac{\xi}{\xi-1}}.$$

From the optimization problem, once I derive the final expression for the marginal cost, I can analyze the impact of shocks in the presence of informality.

Beyond mathematics I first consider the economy without informality, i.e., $Z_t \equiv 0$. In this case, from equation (82), the *labor supply effect* depends solely on wages, consistent with the standard findings in the literature (Dolado et al., 2021). Increases in the wages of low- or high-skilled workers translate directly into higher hours worked for the corresponding group. Taxation plays a key role: a rise in the corporate tax rate τ_t^f reduces the labor ratio, reflecting

⁹Derivations are available in the Appendix upon request

the higher cost of employing high-skilled labor.

I then consider the role of informality. From equation (82), the *labor supply effect* (LSE) influences the *capital-to-skilled labor ratio* (CSC). When underground technology becomes fully accessible ($Z_t \equiv 1$), higher productivity in the informal sector increases the labor ratio. This reflects a positive labor supply effect: firms allocate a larger share of labor to informal activities. Nevertheless, the effect is relatively modest and requires numerical simulations for precise quantification.

Attracting informal low-skilled workers requires wages that compensate for the absence of formal-sector benefits, such as social security and insurance coverage. The model assumes full employment, so low-skilled individuals not employed formally participate in the informal sector. Informal employment should therefore be interpreted not as a separate subsystem, but as an integral component of the overall economy (Ulyssea, 2018, 2020; Maloney, 1999, 2004)¹⁰.

3.5 Final Goods Producers

The representative final goods producer is modeled in a standard way. The final good Y_t is produced by aggregating a continuum of intermediate goods $\bar{Y}_t(i)$ using a CES aggregator:

$$\bar{Y}_t = \left(\int_0^1 \bar{Y}_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (13)$$

where $\bar{Y}_t(i)$ is the quantity of the intermediate good produced by firm i , sold at price $p_t(i)$.

From this CES structure, the well-known Dixit-Stiglitz demand function (?) for the intermediate goods follows:

$$\bar{Y}_t(i) = \bar{Y}_t \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon}, \quad (14)$$

where p_t denotes the price index for the aggregate intermediate good, defined as

$$p_t = \left(\int_0^1 p_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \quad (15)$$

Under the assumptions of perfect competition among final goods producers, profits are driven to zero, implying:

$$\Pi_t^f = 0.$$

¹⁰Simulation results indicate that following a positive shock to underground total factor productivity (TFP), total low-skilled labor demand is split between formal and informal sectors: the share of formal low-skilled labor rises, while the informal share declines. High-skilled labor demand increases, reflecting reallocation across skill types. These results suggest that informal employment does not absorb technological improvements in the same way as the formal sector and that expanding informal hiring is not optimal for firms. These dynamics are also reflected in marginal costs. Examining equation (82), I found that even with higher underground productivity, the increase in marginal cost is insufficient to offset the reduction in labor demand, which dominates and leads to a net decline in marginal cost.

3.6 Monetary policy

In the current model, I assume that the central bank sets the nominal interest rate according to a Taylor rule. In the next stage of this research, I will adopt a forward-looking Taylor rule following the approach of Galí (2008).

The Taylor rule implemented in this initial stage is given by

$$\frac{r_t}{r} = \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{\bar{Y}_t}{\bar{Y}} \right)^{\phi_{\bar{Y}}} \right]^{1-\rho_r} \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \epsilon_r \quad (16)$$

where: r_t is the nominal interest rate at time t , r is the steady-state nominal interest rate, π_t and π denote the inflation rate and its steady-state level, respectively, \bar{Y}_t and \bar{Y} represent the level of aggregate output and its steady-state value, respectively, $\epsilon_r \sim \mathcal{N}(0, 1)$ is a shock to the nominal interest rate, ρ_r is the interest rate smoothing parameter, ϕ_π and $\phi_{\bar{Y}}$ are the weights assigned to deviations of inflation and output from their steady-state values, respectively.

In the Taylor rule specification, I depart from the conventional formulation that includes the output gap as in Annicchiarico and Cesaroni (2018); Ospina and Hairault (2021); Batini et al. (2010, 2011). Instead, following an approach closer to Ospina (2023), I assume that the central bank sets the nominal interest rate taking into account the overall state of the economy, including the informal sector. This broader informational set reflects the idea that informality plays a non-negligible role in shaping macroeconomic conditions and, consequently, monetary policy decisions.

3.7 Fiscal Policy

In the present model, the government finances public spending through a combination of distortive taxes levied on corporate entities and households, and a residual lump-sum tax. The lump-sum tax is introduced to ensure that the government budget constraint is balanced at all times. Public spending is assumed to be constant and equal to its steady-state level, $G_t = \bar{G}$.

The government budget constraint is given by

$$G_t = T_t^{dist} + \mathcal{T}_t, \quad (17)$$

where T_t^{dist} denotes total distortive tax revenues and \mathcal{T}_t is a lump-sum tax (or transfer) levied on households.

Distorsive tax revenues are defined as

$$T_t^{dist} = \tau_t^f Y_t^m + ps \tau_t^f Y_t^u + \zeta w_{H,t} \tau_t^h + (1 - \zeta) w_{L,t} \theta_t \tau_t^h, \quad (18)$$

where τ_t^f is the corporate tax rate at time t , Y_t^m and Y_t^u denote output in the market and underground sectors, respectively, and $ps \in (0, 1)$ captures the effective taxation of underground production. Moreover, τ_t^h is the household tax rate, $w_{H,t}$ and $w_{L,t}$ are the high-skill and low-skill wage rates, ζ denotes the share of high-skilled workers, and θ_t is a productivity scaling factor for low-skilled labor. The lump-sum tax is determined residually to satisfy the government budget constraint:

$$\mathcal{T}_t = \bar{G} - T_t^{dist}. \quad (19)$$

By construction, the lump-sum tax does not distort private agents' decisions and absorbs all fluctuations in tax revenues generated by shocks to distortive tax rates and economic activity.

Following [Busato and Chiarini \(2004\)](#), distortive tax rates are assumed to follow exogenous stochastic processes. In particular, corporate and household tax rates evolve according to AR(1) processes:

$$\log\left(\frac{\tau_t^f}{\tau^f}\right) = \rho_{\tau^f} \log\left(\frac{\tau_{t-1}^f}{\tau^f}\right) + \varepsilon_t^{\tau^f}, \quad (20)$$

$$\log\left(\frac{\tau_t^h}{\tau^h}\right) = \rho_{\tau^h} \log\left(\frac{\tau_{t-1}^h}{\tau^h}\right) + \varepsilon_t^{\tau^h}, \quad (21)$$

where $\rho_{\tau^f}, \rho_{\tau^h} \in (0, 1)$ and $\varepsilon_t^{\tau^f}, \varepsilon_t^{\tau^h}$ are i.i.d. shocks with zero mean.

3.8 Closing the model

To derive the aggregate resource constraint in the model, I begin by specifying the budget constraints for the three household types:

$$c_{H,t} + k_t + \frac{B_t}{p_t} = (r_t^k + 1 - \delta)k_{t-1} + r_{t-1} \frac{B_{t-1}}{p_t} + (1 - \tau_t^H)w_{H,t}n_{H,t} + \Gamma_t^I, \quad (22)$$

$$c_{L.f,t} = w_{L,t}(1 - \tau_t^h)\theta_t, \quad (23)$$

$$c_{L.i,t} = w_{L,t}(1 - \theta_t). \quad (24)$$

Let ζ denote the share of high-skilled households in the economy, and $1 - \zeta$ the share of low-skilled households.

By Walras' Law, if all but one market in the economy are in equilibrium, then the remaining market must also be in equilibrium. Hence, equilibrium in the labor, capital, and goods markets implies that this aggregate budget constraint holds.

Using the aggregate definitions derived above, total absorption in the economy satisfies:

$$\bar{C}_t + I_t = \bar{Y}_t \left(1 - \frac{K_P}{2}(\pi_t - \bar{\pi})^2\right). \quad (25)$$

Before turning to the quantitative calibration, I use the model to conduct a set of theoretical impulse response exercises designed to clarify the transmission mechanisms associated with informality. These exercises are not intended to deliver quantitative results, but rather to provide intuition on how the presence of an informal sector alters standard labor market and macroeconomic adjustment channels.

The analysis considers two classes of shocks. First, I examine an informal labor supply shock, and compare the responses of three economies: a benchmark model without informality, and two economies with an informal sector that differ in the specification of monetary policy. In one case, the central bank responds exclusively to developments in the formal sector, while in the other monetary policy is formulated with reference to aggregate economic activity, including informal production. This comparison isolates the role of informality in shaping sectoral labor reallocation, wage dynamics, and production responses, as well as the interaction between informal employment and alternative policy rules.

Second, I analyze a contractionary monetary policy shock, modeled as an exogenous increase in the nominal interest rate. As in the labor supply exercise, the responses of an economy without informality are contrasted with those of economies featuring an informal sector under alternative Taylor rule specifications. This framework allows me to assess how informality modifies the propagation of interest rate shocks through labor markets, capital allocation, price dynamics, and fiscal adjustments, relative to standard DSGE environments.

Taken together, these theoretical impulse response exercises provide a coherent benchmark for interpreting the quantitative results obtained from the calibrated model in the following section.

4 Calibration

Symbol	Description	Value	References
α	Capital share	0.44	Vollrath (2024)
δ	Depreciation rate	0.025	Galí (2008)
f	Idiosyncratic cost	0.3781	Author's calibration
η	Inverse Frisch elasticity (underground)	0.40	Author's calibration
γ	Inverse Frisch elasticity (market)	1.00	Busato and Chiarini (2004)
h_L	Disutility of labor for low-skilled workers	0.4810	Author's calibration
h_H	Disutility of labor for high-skilled workers	0.1938	Author's calibration
p	Detection probability	0.03	Busato and Chiarini (2004)
ϕ_d	Tax response to debt	2	Pappa et al. (2015)
ϕ_g	Tax response to public expenditure	0.10	Pappa et al. (2015)
ϕ_π	Inflation coefficient (Taylor rule)	1.50	Galí (2008)
$\phi_{\bar{Y}}$	Output gap coefficient (Taylor rule)	1.00	—
ρ_g	Public expenditure share of GDP	0.20	Author's calibration
ρ_m	Persistence of market TFP	0.90	—
ρ_u	Persistence of underground TFP	0.90	—
ρ_r	Interest rate persistence	0.80	—
s	Schurage factor	1.30	—
τ^f	Social security contribution	0.257	Busato and Chiarini (2004)
τ^h	Household income tax	0.335	Busato and Chiarini (2004)
$(1 - \theta)$	Share of underground workers	0.265	Schneider et al. (2010)
ζ	Share of high-skilled households	0.7250	Gali et al. (2004)
B	Share of high-skilled in production	0.62	Dolado et al. (2021)
ξ	Elasticity of substitution	-0.41	Pappa (2021)

Table 1: Structural Parameters: Quarterly Calibration

The table above reports the key parameters calibrated to match the main empirical regularities, while the remaining parameters take standard values commonly used in the literature. In particular, the discount factor is set to $\beta = 0.99$, implying a steady-state nominal interest rate of approximately 1.04 percent on an annualized basis. The debt-to-GDP ratio, D , is calibrated to 0.19, in line with empirical evidence on fiscal policy (Pappa et al. (2015)). The steady-state markup, e_p , is chosen to yield an average marginal cost of 0.833. Public expenditure, g , is set to 20 percent of GDP, consistent with the fiscal calibration of Pappa et al. (2017).

Regarding nominal and real rigidities, the parameters governing capital and price adjustment costs, KI and KP , are set to 2.8 and 56.6, respectively, following Smets and Wouters (2003a). The Calvo parameter for price stickiness, ω , is set to 0.75, consistent with Galí (2008).

The disutility weights on labor effort for high- and low-skilled agents, h_H and h_L , are calibrated ex post to match steady-state hours worked. Specifically, total hours worked by high-skilled households are normalized to one ($n_H = 1$), and the corresponding disutility parameter h_H is computed from the steady-state labor supply condition. An analogous approach is used for low-skilled workers, with total hours worked set to $n_L = 1$. Following Busato and Chiarini (2004) and Schneider et al. (2010), the share of low-skilled workers employed in the formal sector is fixed at 0.735. In Section 5, I show that deviating from this benchmark share in order to match specific empirical moments leads to indeterminacy issues. An optimal combination of Ricardian and hand-to-mouth consumers is thus required to ensure model determinacy.

Once the labor shares are fixed, the inverse Frisch elasticities for market and underground labor, γ and η , are easily computed. For high-skilled workers, I assume the same Frisch elasticity across formal and informal employment, differing only in their disutility weights. In particular, the lower disutility for high-skilled workers reflects their higher opportunity cost of leisure and productivity premium.

The probability of detection of underground activities, p , follows Busato and Chiarini (2004), while the Schurage factor, s , is fixed to 1.3 according to standard values in the literature. The share of Ricardian consumers, ζ , is set to 0.725 following Galí et al. (2004). Finally, steady-state tax rates are calibrated as in Busato and Chiarini (2004), and the share of high-skilled labor in the production function, $B = 0.62$, ensures an efficient steady state consistent with imperfect substitutability between factors (Dolado et al. (2021)).

5 Results

5.1 Informal Labor Supply shock

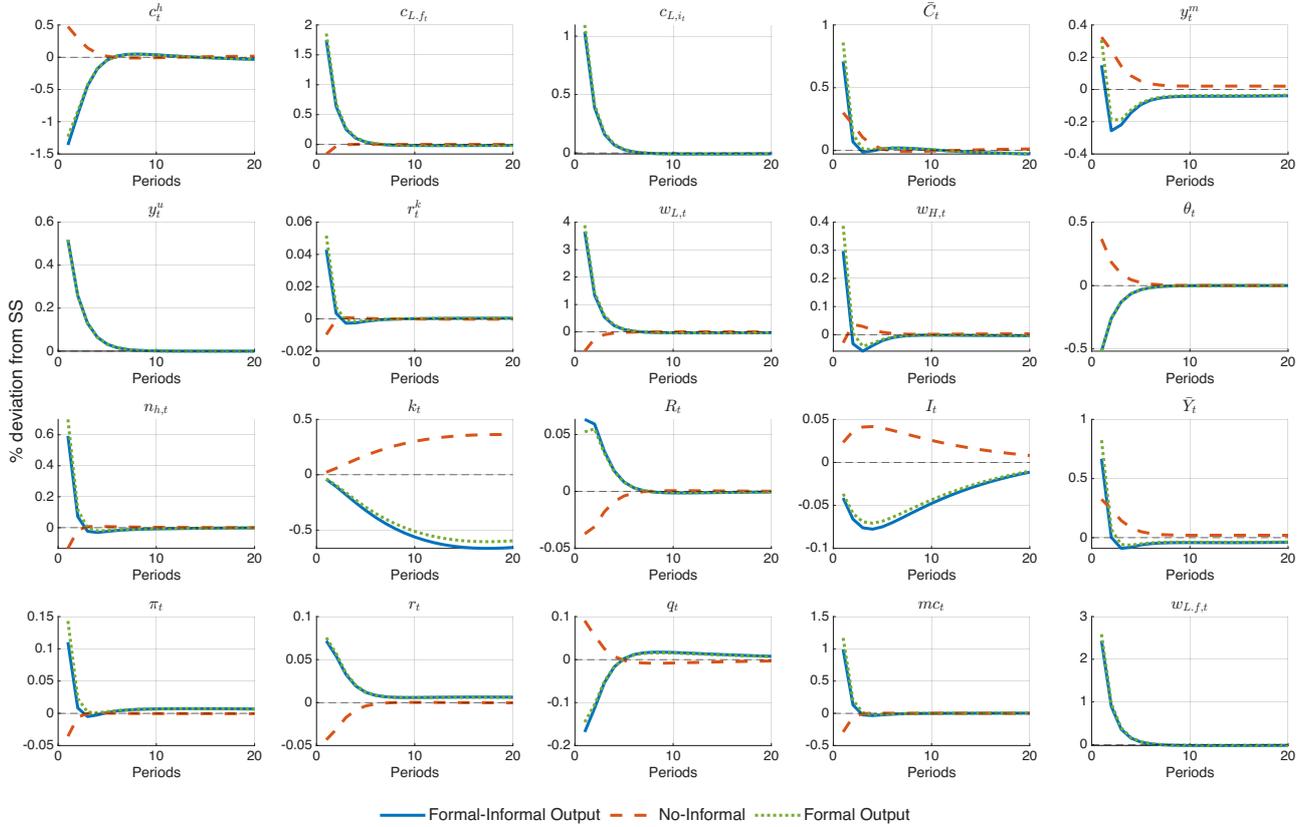


Figure 3: Impulse responses to an **Informal Labor supply shock** (ε_t^{in}).

Figure 3 reports the impulse response functions (IRFs) following a negative labor supply shock. As in the previous figures, the red line represents the response to a standard negative labor supply shock in an economy without informality. The dotted green and solid blue lines instead correspond to the responses to a negative *informal labor supply shock* under two alternative Taylor rule specifications. The dotted green line refers to the case in which the central bank reacts only to developments in the formal sector, while the blue line represents the case where monetary policy is set with reference to the aggregate output of the entire economy, including the informal component.

Naturally, the IRFs generated by these three model specifications convey different economic mechanisms and interpretations. Consistent with the core objective of this research, the analysis first focuses on the blue line—corresponding to the Taylor rule that takes into account the whole economy.

Following a negative informal labor supply shock, low-skilled individuals perceive a lower disutility from supplying labor in the informal sector. This shock operates through two distinct channels. First, it directly reduces the overall disutility of work for low-skilled workers. Second, it lowers the idiosyncratic risk or perceived cost associated with engaging in informal activities.¹¹

As a result, low-skilled, hand-to-mouth workers find it less costly, both in terms of effort and perceived risk, to work informally. This behavioral response manifests in a clear increase

¹¹See the Appendix for the mathematical derivation.

in hours worked in the informal sector, denoted by n_L^I . While this increase is pronounced in the informal sector, the adjustment in the formal sector differs markedly. In a standard framework without informality (red line), a negative labor supply shock would typically induce an increase in hours worked by low-skilled agents. However, the presence of an informal sector modifies this mechanism substantially.

The key difference lies in the wage response and in the absorption of the shock across sectors. As informal employment rises due to the reduced perceived risk of detection, the wage w_L does not decline, as classical labor market theory would predict, but instead increases. This outcome occurs because the higher informal labor supply is fully absorbed by firms that benefit from employing untaxed workers exempt from social security contributions. Consequently, informal output y^u expands proportionally (by approximately 0.6 in both cases). In this setting, the entire effect of the shock is absorbed within the informal sector, where firms take advantage of the lower effective labor cost.

Contrary to the standard supply–demand mechanism, the increase in labor supply does not depress wages; rather, the expansion in informal production sustains both higher wages and higher employment in the informal economy. This dynamic generates a partial equilibrium outcome characterized by greater informal activity and upward pressure on informal wages.

The rise in wages earned by low-skilled workers produces a wealth effect, particularly among those also employed in the formal sector. Feeling wealthier, they reduce their hours supplied to the formal market. While this behavior may appear counterintuitive, given that hand-to-mouth households are typically expected to increase labor supply to sustain consumption, it illustrates how the presence of informality can fundamentally alter macroeconomic transmission mechanisms.

As formal hours decline, total output produced through market (formal) technology contracts. This contrasts sharply with the benchmark model without informality, where the same shock would elicit the opposite response. Overall, these results underscore how accounting for the informal economy not only changes the quantitative dynamics but also the qualitative interpretation of standard labor market adjustments.

The contraction in formal output is associated with a reduction in labor demand for high-skilled workers, while, somewhat counterintuitively, it is accompanied by an increase in the demand for capital, r^k . These two mechanisms deserve a separate discussion.

Starting with the former, the decline in labor demand for high-skilled workers follows standard macroeconomic mechanisms. As formal output contracts, firms rationally adjust by reducing the utilization of their most expensive input: skilled labor. The reduction in production activity induces a downward pressure on equilibrium wages, which, in turn, generates a conventional substitution effect: as real wages decline, high-skilled individuals are incentivized to increase their labor supply in an attempt to compensate for the income loss, although the overall equilibrium adjustment still reflects a contraction in total labor demand.

More intriguing, however, is the observed increase in capital demand. From a conventional macroeconomic perspective, a contraction in output should lead to a reduction in the use of all production factors. Yet, in this framework, that relationship no longer holds. The reason lies in the reallocation of resources across sectors: firms respond to the shock by shifting production from the formal to the informal economy. To sustain and expand informal production, firms

require additional capital inputs, which remain legally recorded but are effectively employed in informal activities.

In other words, even though capital is officially allocated to the market (formal) sector, it can also be employed—de facto—to support informal production. This mechanism highlights how the informal economy acts as a hidden extension of the productive system: the same capital stock generates output both within and outside the official boundaries of the economy. Consequently, the increase in capital demand is only partially surprising. It reflects firms' strategic response to preserve profitability by expanding activity in the informal sector, where labor is cheaper and tax or regulatory constraints are weaker.

This result provides a clear macroeconomic insight: when informality is pervasive, shocks that would normally contract all factors of production may instead trigger a reallocation of capital toward unobserved sectors, thereby dampening the aggregate impact on total output while amplifying distortions between formal and informal markets.

Comparing this result with the benchmark case in which informality is absent reveals a fundamental asymmetry. At the micro level, the adjustment of key variables—such as hours worked and wages—moves in the opposite direction. Nevertheless, at the aggregate level, the dynamics of output growth, \bar{Y} , remain broadly similar across the two economies. This apparent paradox arises because the expansion in the informal sector compensates for the contraction in the formal one. Hence, correctly accounting for informal activities is crucial: models that neglect informality systematically overestimate the effective level of aggregate output and mismeasure the economy's true productive capacity.

The wage and labor-hour dynamics also translate into heterogeneous consumption patterns. In contrast to the benchmark model, the presence of an informal sector produces a markedly non-uniform response of aggregate consumption. Low-skilled households, benefiting from higher informal wages and reduced taxation, increase consumption, while high-skilled households—more exposed to formal labor market contraction—adjust downward. The result is a reallocation of purchasing power that alters the aggregate demand composition without significantly changing total output.

An important question naturally arises: how does an informal labor supply shock affect the price dynamics and, consequently, the response of Central Bank? The answer follows from the cost-channel propagation. The reallocation of production toward the informal economy raises the effective price of inputs. Firms, facing higher marginal costs, adjust by increasing their selling prices. Under price stickiness, this cost-push mechanism induces a temporary rise in inflation. However, the magnitude of the inflationary response is moderate compared to the case (dotted green line) in which the Taylor rule responds only to the formal sector. When the central bank reacts to aggregate variables—including informal activity—the overall response of inflation is more contained, reflecting a broader information set in policy formulation.

The central bank's policy response, nevertheless, remains largely homogeneous across regimes. In both settings, monetary authorities react to the inflationary pressure by raising the nominal interest rate to steer the economy back toward its steady state. Yet, this response is suboptimal. By responding uniformly to aggregate inflation, the central bank fails to internalize the structural heterogeneity introduced by the informal sector.

This mechanism reveals a structural inefficiency in conventional monetary policy design. A

Taylor-type rule calibrated on aggregate inflation and output gaps overlooks the reallocation effects between formal and informal production, leading to an “expansionary contraction.” Moreover, the altered composition of production flattens the Phillips curve, attenuating the sensitivity of inflation to real activity and reducing the effectiveness of interest rate adjustments.

From a broader perspective, this dynamic underscores that informality cannot be treated merely as a structural weakness. While it erodes the fiscal base, it simultaneously provides a buffer mechanism that stabilizes debt and mitigates the real costs of monetary tightening. Failing to account for this dual role may lead policymakers to systematically misjudge the true stance of the economy and to adopt suboptimal policy mixes.

These results highlight limitations of standard policy frameworks in economies with pervasive informality. Incorporating indicators of informal sector dynamics—such as the share of informal output or the shadow wage differential—into an augmented Taylor rule could enhance stabilization performance and support monetary–fiscal coherence when a significant share of economic activity is hidden.

5.2 Monetary Policy shock

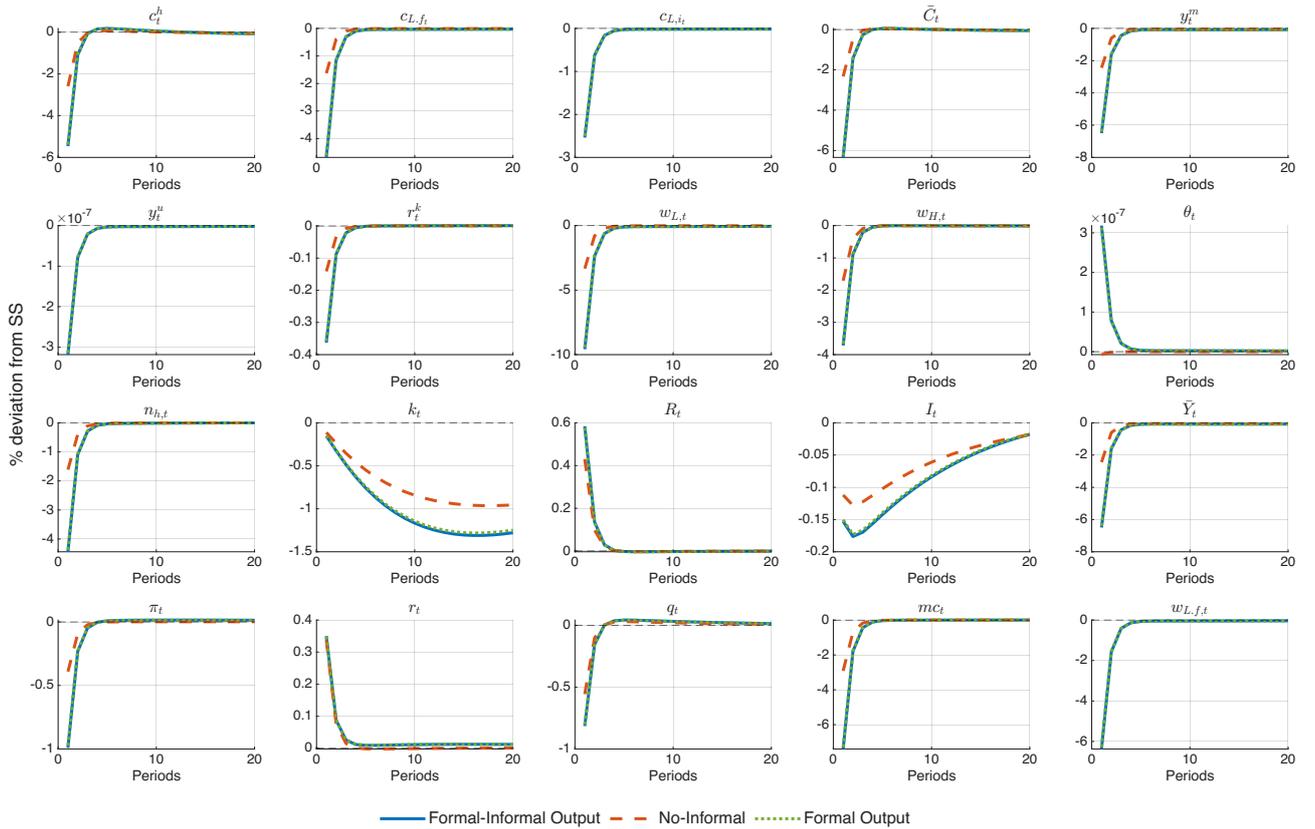


Figure 4: Impulse responses to a Monetary policy shock (ε_t^r).

Figure 4 presents the Impulse Response Functions (IRFs) to a contractionary monetary policy shock, modeled as an exogenous increase in the nominal interest rate. The blue line depicts the dynamics of the full economy with informality when monetary policy responds to aggregate output, including both formal and informal sectors. The green dotted line represents the case in which the interest rate responds only to formal-sector output, while the red line refers to an economy without informality.

Focusing on the blue line (full economy with informality), the analysis examines how the economy responds to a nominal interest rate hike and how this response differs from that observed in an economy without an informal sector. The shock activates the standard transmission channels extensively documented in the DSGE literature—namely, the income, substitution, and wealth effects—collectively referred to as “distributional effects” following Colciago et al. (2019). The income effect reflects changes in disposable income induced by variations in asset returns and borrowing costs, the substitution effect captures intertemporal consumption decisions driven by expected returns, and the wealth effect accounts for the revaluation of financial assets such as bonds, equities, and real estate following monetary shocks (Galí, 2015; Christiano et al., 2005; Smets and Wouters, 2003b).

In the model, the contractionary shock primarily affects high-productivity (high-skilled) agents and firms. Ricardian high-skilled households reduce current consumption to reallocate wealth toward higher-yield financial assets, in line with substitution and wealth effects. At the same time, hours worked decline as higher asset returns increase perceived lifetime wealth, reflecting the income effect. While the qualitative dynamics are similar across model specifications, important quantitative differences emerge. In particular, the decline in consumption of high-skilled agents is more pronounced in economies with informality—under both Taylor rule regimes (blue and green lines)—than in the economy without informality (red line). As a result, hours worked contract more sharply in the presence of an informal sector. The wealth effect is also stronger with informality, reflecting greater reallocation margins and amplified balance-sheet adjustments induced by sectoral heterogeneity.

These differences are closely related to firms’ behavior and the allocation of capital across formal and informal sectors. Although the impact response of formal-sector output is similar across economies, the presence of informality leads to a more persistent and gradually shaped output contraction over time. Informality therefore modifies not only the magnitude but also the temporal profile of the adjustment.

Capital accumulation plays a central role in this mechanism. The increase in the nominal interest rate lowers inflation expectations and, through the Fisher equation, raises the real interest rate on nominal assets. However, the effective rental rate of capital r^k for firms decreases because the contraction in output reduces the demand for capital. Compared to the no-informality economy (red line), the decline in capital demand is more pronounced when informality is present.

This difference reflects a “capital-sharing” mechanism across sectors: an increase in the nominal interest rate induces firms to reduce investment. Unlike the standard literature, the informal sector does not act as a buffer; rather, it amplifies the effects of the nominal interest rate shock across sectors.

This highlights an important insight: depending on the nature of the shock, the informal economy can either smooth or amplify standard macroeconomic fluctuations.

In the case of a monetary policy shock, the reduction in firms’ investment affects not only formal output but also informal output. As borrowing becomes more costly, firms scale down their demand for inputs in both sectors, causing aggregate output to decline more than in a standard model without informality.

The contraction in formal-sector output reduces demand for production inputs, leading to

declines in the rental rate of capital r^k and in wages w^H and w^L . Notably, the decline in low-skill wages w^L is more pronounced in the presence of informality. While the informal sector cushions aggregate output and capital dynamics, it also increases labor supply elasticity, intensifying downward wage pressures for low-skilled workers.

Another important difference concerns the fraction of hours worked in the formal sector by low-skilled workers, θ_t . In an economy without informality, this share slightly declines following the shock. In contrast, once informality is introduced, θ_t increases.

The mechanism operates through the behavior of low-skilled workers, who are modeled as rule-of-thumb (hand-to-mouth) consumers. A decline in wages generates a negative wealth effect. Since these households cannot smooth consumption intertemporally, the wealth effect dominates the substitution effect, inducing them to supply more labor.

The increase in labor supply is concentrated in the formal sector because the decline in formal-sector wages is stronger than in the informal sector. This larger negative wealth effect induces low-skilled, hand-to-mouth households to supply more labor formally.

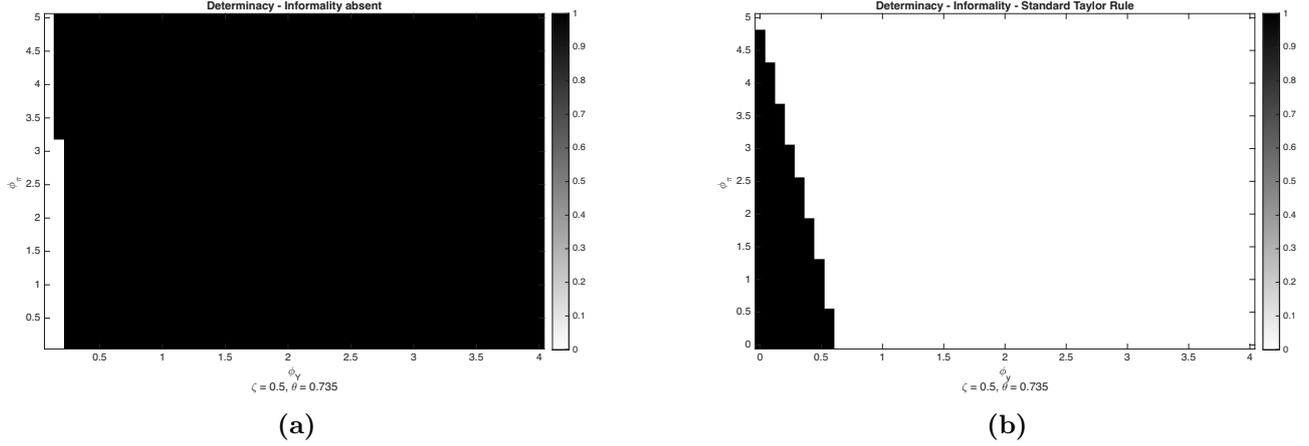
In the partial equilibrium version of the model, the wealth effect is therefore stronger in the formal market than in the informal one, explaining the rise in the formal labor share of low-skilled workers.

Lower input costs translate into a stronger reduction in marginal costs mc_t . Due to nominal rigidities, this cost adjustment feeds into inflation dynamics. Inflation declines more sharply and persistently in economies with informality than in the economy without an informal sector, regardless of whether the Taylor rule responds to aggregate or formal-sector output. This stronger disinflation reflects deeper and more prolonged adjustments in wages, capital demand, and sectoral reallocation when informal production is explicitly modeled.

Despite the larger and more persistent decline in inflation, the central bank's nominal interest rate response is less aggressive in economies with informality than in the no-informality benchmark. When informal activity is ignored, inflation dynamics appear more muted, yet monetary policy reacts more strongly, suggesting a misalignment between observed inflationary pressures and the underlying real adjustments. This mismatch indicates that neglecting informal production may lead the central bank to misinterpret the strength and persistence of monetary transmission.

Overall, the presence of an informal sector fundamentally alters the propagation of monetary policy shocks. Informality acts as both an amplification and a persistence mechanism, generating responses that are quantitatively stronger and longer-lasting than in standard DSGE models without informal activity. Crucially, these effects emerge independently of the specific formulation of the Taylor rule, underscoring the importance of explicitly accounting for informality when analyzing inflation dynamics and the transmission of monetary policy.

5.3 Determinacy



In Figures 5a and 5b I analyze the determinacy region in the absence and in the presence of informality, respectively. All parameters are fixed at their baseline values reported in Table 1, while the monetary policy coefficients ϕ_π and ϕ_y are allowed to vary.

Figure 5a reproduces the standard case discussed in Gali et al. (2004). Fixing the share of Ricardian consumers at $\zeta = 0.5$ and the fraction of hours worked by low-skilled agents in the formal sector at $\theta = 0.735$, determinacy requires the monetary authority to respond more aggressively to deviations of inflation from its target than to deviations of the output gap. In other words, the Taylor principle must hold: ϕ_π must be sufficiently large relative to ϕ_y .

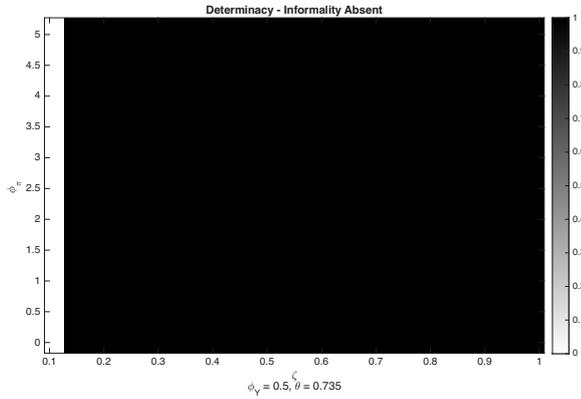
Figure 5b shows the corresponding determinacy region when informality is introduced. The overall qualitative pattern remains similar to the no-informality benchmark, but the geometry of the determinacy region changes. In particular, the region appears “inverted”: for any given response to inflation, determinacy is ensured only when the response to the output gap is sufficiently small.

More precisely, as ϕ_π increases, the admissible range of ϕ_y consistent with determinacy shrinks. For sufficiently high values of ϕ_y (above approximately 0.7), the equilibrium becomes indeterminate regardless of the value of ϕ_π . Thus, in the presence of informality, assigning excessive weight to output-gap stabilization may generate indeterminacy even when the response to inflation satisfies the Taylor principle.

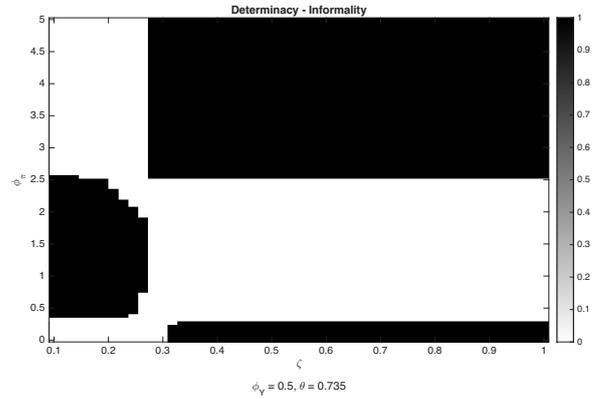
The inversion of the determinacy region can be traced back to a modification of the aggregate demand channel induced by informality and household heterogeneity. The presence of rule-of-thumb agents, combined with sectoral reallocation between formal and informal activities, alters the effective slope of the IS curve. As a result, the output gap becomes less tightly linked to inflation dynamics. A strong monetary response to output-gap fluctuations may therefore destabilize expectations, shrinking the determinacy region even when the Taylor principle is satisfied. This mechanism is closely related to the inverted determinacy results in Bilbiie (2008), although here it emerges through the interaction between informality and capital reallocation.

To further investigate the mechanism, I explore for which values of ζ and θ the determinacy region is restored. In Figures 6a and 6b, I analyze how the determinacy region varies with respect to ζ , keeping all other parameters fixed as in Table 1.

In the absence of informality (Figure 6a), the Taylor principle holds for values of ζ above approximately 0.15. In this case, determinacy is ensured independently of the weight assigned to



(a)

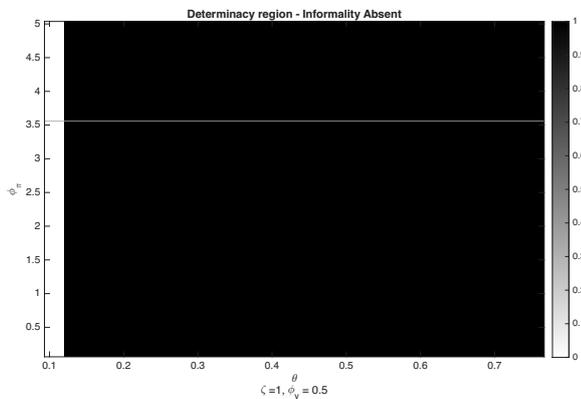


(b)

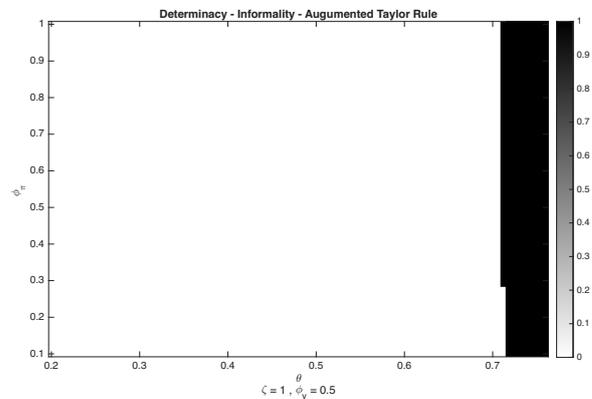
output-gap stabilization. The model therefore behaves in line with the standard New Keynesian framework.

Once informality is introduced (Figure 6b), the determinacy region changes dramatically. For intermediate values of ζ , approximately between 0.1 and 0.29, determinacy holds only for intermediate values of ϕ_π , roughly between 0.5 and 2.5. Outside this range, indeterminacy emerges.

In particular, determinacy may require either a sufficiently low response to inflation (consistent with an inverted aggregate-demand logic) or a sufficiently high response (consistent with the conventional Taylor principle). Fixing $\phi_y = 0.5$, inflation stabilization must generally be aggressive to ensure determinacy. However, for certain intermediate values of ζ , stability may also arise under relatively passive monetary policy, which departs from the standard Taylor principle prescription.



(a)



(b)

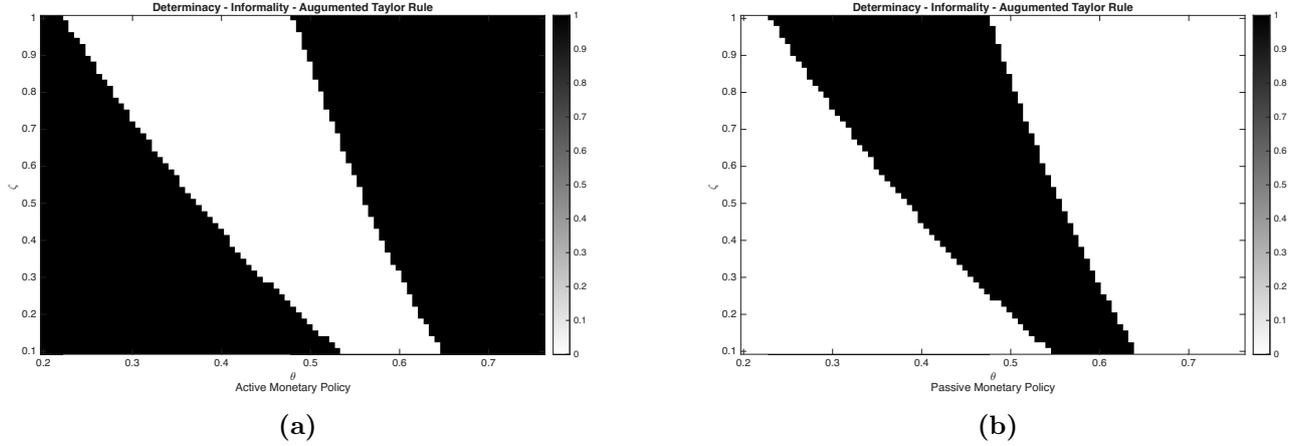
In Figures 7a and 7b, I analyze determinacy when the share of Ricardian households is set equal to one ($\zeta = 1$). In this case, all agents are forward-looking and the rule-of-thumb channel is shut down.

In the absence of informality (Figure 7a), the results are fully consistent with the standard New Keynesian framework and align with the intuition derived from Figure 5a. Since household heterogeneity in intertemporal behavior is absent, determinacy is governed by the conventional Taylor principle, and no additional distortions emerge.

In contrast, once informality is introduced (Figure 7b), important differences arise. As shown in Proposition 1, admissibility of the steady state requires θ to lie approximately within the interval $[0.2, 0.76]$, given the calibration in Table 1. Therefore, the determinacy analysis

focuses on this admissible range.

The results indicate that, even with $\zeta = 1$, determinacy requires θ to be sufficiently high, specifically above approximately 0.7. Hence, a sufficiently large share of hours worked in the formal sector is necessary to ensure macroeconomic stability when informality is present.



In Figures 8a and 8b, I examine the cases of active and passive monetary policy, respectively, allowing both θ and ζ to vary simultaneously. The results confirm the three mechanisms discussed above.

Under active monetary policy (Figure 8a), determinacy is ensured only for specific combinations of θ and ζ . In general, higher values of ζ must be associated with sufficiently large values of θ to guarantee stability. For θ in the range $[0.2, 0.55]$, determinacy requires a decreasing relationship between ζ and θ , and the determinacy region appears “inverted.” By contrast, for θ in the range $[0.6, 0.76]$, the region becomes standard (or “normal”), and determinacy is restored under more conventional parameter configurations.

Figure 8b reports the combinations of θ and ζ for which determinacy holds under passive monetary policy. In this case, determinacy is achieved for θ values approximately between 0.2 and 0.65. Notably, when the informal sector is relatively large (i.e., low θ), stability requires a very high share of Ricardian households. In other words, as the informal economy becomes more persistent, determinacy can only be restored if the economy is predominantly composed of forward-looking agents.

Interestingly, parameter configurations that fail to ensure determinacy under active monetary policy (Figure 8a) may become stable under passive monetary policy (Figure 8b). This highlights the non-monotonic interaction between household heterogeneity, informality, and the monetary policy stance.

The analysis highlights that the effectiveness of monetary activism is structurally dependent on the degree of informality and household heterogeneity. In economies with a large informal sector and a significant share of non-Ricardian households, aggressive inflation stabilization may amplify sectoral reallocation and redistribution effects, weakening the link between output and inflation. As a result, the standard Taylor principle may no longer be sufficient to guarantee determinacy.

These findings suggest that monetary policy frameworks calibrated for advanced, predominantly formal economies may not be directly transferable to economies characterized by widespread informality. In such contexts, a more moderate or state-contingent policy stance may be required to ensure macroeconomic stability.

6 Conclusions

This paper studies the macroeconomic and monetary policy implications of introducing an informal sector into a New Keynesian framework with heterogeneous households. The analysis combines empirical evidence from a structural VAR for Colombia with a Two-Agent New Keynesian (TANK) model featuring formal and informal sectors, allowing for both Ricardian and hand-to-mouth households. The model explicitly captures sectoral reallocation, heterogeneous labor supply, and the differential response of wages and capital across formal and informal markets. By linking empirical regularities to the theoretical framework, the analysis highlights two central findings. First, informality fundamentally alters the transmission of both real and monetary shocks. Second, it reshapes the conditions for equilibrium determinacy, challenging the universal validity of the standard Taylor principle.

Empirically, the VAR evidence shows that shocks affecting informal labor supply generate persistent sectoral adjustments rather than a simple aggregate contraction. The model reproduces this pattern: an informal labor supply shock increases employment in the informal sector, sustaining aggregate output while redistributing production, wages, and capital across sectors. In this sense, informality acts as a buffer at the aggregate level, but at the cost of amplifying distortions between formal and informal markets. By contrast, under a monetary policy shock, informality amplifies and propagates the contractionary effects of interest rate hikes through stronger distributional and capital-reallocation channels. These results demonstrate that informality can either smooth or magnify fluctuations depending on the nature of the disturbance.

Beyond shock propagation, the paper uncovers a deeper structural implication: informality modifies the determinacy properties of the economy. When the informal sector is sufficiently large and household heterogeneity is significant, aggressive monetary policy does not necessarily guarantee equilibrium uniqueness. The interaction between sectoral reallocation and non-Ricardian behavior alters the effective slopes of the IS and Phillips curves, weakening the link between output and inflation. As a result, the standard Taylor principle may become insufficient, and in some regions of the parameter space, even destabilizing. Determinacy depends critically on the joint configuration of three structural elements: the share of Ricardian households, the size of the formal sector, and the stance of monetary policy. In economies with high informality and a large fraction of hand-to-mouth agents, active monetary policy may amplify volatility and generate self-fulfilling fluctuations. Conversely, more moderate or relatively passive policy rules may restore stability. These findings suggest that the appropriate degree of monetary activism is endogenous to the structural composition of the economy.

From a policy perspective, the results carry important implications for emerging and developing economies, where informal production represents a substantial share of total activity. Monetary frameworks calibrated for advanced, predominantly formal economies may not be directly transferable. Ignoring informality can lead central banks to misinterpret inflation dynamics, mismeasure output gaps, and adopt excessively aggressive stabilization strategies. Incorporating indicators of informal activity into policy rules—or, more broadly, designing state-contingent monetary frameworks—may improve stabilization outcomes.

More generally, the analysis underscores that informality is not merely a fiscal or regulatory issue but a macroeconomic force that shapes equilibrium dynamics and policy effectiveness.

Accounting explicitly for informal production is therefore essential for understanding monetary transmission, distributional effects, and the conditions under which equilibrium is stable and unique. Future research could extend this framework by incorporating endogenous entry into informality, fiscal–monetary interactions, or optimal policy design under commitment, further clarifying how structural dualism influences macroeconomic stabilization and long-run development.

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Appendix: Empirical SVAR Model

The empirical analysis is conducted using a Structural Vector Autoregression (SVAR) model of order p , specified as:

$$A_0 Y_t = B + \sum_{i=1}^p A_i Y_{t-i} + \varepsilon_t,$$

where:

- $Y_t \in \mathbb{R}^{N \times 1}$ is the vector of endogenous variables, with $N = 4$ in the empirical application;
- $A_0 \in \mathbb{R}^{N \times N}$ captures contemporaneous interactions among the endogenous variables and provides the structural mapping from reduced-form innovations to economically interpretable shocks;
- $A_i \in \mathbb{R}^{N \times N}$ for $i = 1, \dots, p$ are the coefficient matrices associated with the lagged endogenous variables;
- $B \in \mathbb{R}^{N \times 1}$ is a vector of intercepts;
- $\varepsilon_t \in \mathbb{R}^{N \times 1}$ is the vector of structural shocks, assumed to be serially uncorrelated with zero mean and diagonal covariance matrix.

Multiplying the system by A_0^{-1} yields the reduced-form representation:

$$Y_t = \alpha_0 + \sum_{i=1}^p C_i Y_{t-i} + u_t,$$

where:

- $\alpha_0 = A_0^{-1} B$ is the vector of reduced-form intercepts;
- $C_i = A_0^{-1} A_i$ are the reduced-form coefficient matrices;
- $u_t = A_0^{-1} \varepsilon_t$ is the vector of reduced-form innovations, with covariance matrix $\Sigma_u = \text{Cov}(u_t)$.

In the empirical application, the system includes $N = 4$ endogenous variables and is estimated over $T = 71$ time periods with a single lag ($p = 1$). The reduced-form covariance matrix is:

$$\Sigma_u = \begin{bmatrix} 1.0929 & 1.3456 & -0.13395 & 0.29232 \\ 1.3456 & 10.679 & -0.26923 & -0.036123 \\ -0.13395 & -0.26923 & 1.0681 & -0.19206 \\ 0.29232 & -0.036123 & -0.19206 & 1.016 \end{bmatrix}.$$

To identify the structural shocks, I perform a Cholesky decomposition of the reduced-form covariance matrix. Formally, I seek a lower-triangular matrix L such that:

$$\Sigma_u = LL',$$

where L is lower-triangular. In the empirical application, the Cholesky decomposition yields:

$$L = \begin{bmatrix} 1.0454 & 0 & 0 & 0 \\ 1.2871 & 3.0036 & 0 & 0 \\ -0.12814 & -0.034726 & 1.0249 & 0 \\ 0.27962 & -0.13185 & -0.1569 & 0.94646 \end{bmatrix}.$$

The Cholesky factor L corresponds to the inverse of the structural impact matrix:

$$A_0^{-1} = L \quad \implies \quad A_0 = L^{-1}.$$

Structural shocks are obtained from the reduced-form residuals as:

$$\varepsilon_t = A_0 u_t = L^{-1} u_t.$$

This decomposition imposes a recursive ordering of the variables, identifying the contemporaneous effects of each structural shock. The economic interpretation of the Cholesky decomposition matrix is as follows:

- Inflation: The contemporaneous impact is confined to itself (1.0454), indicating that initial inflation shocks are independent of the other variables in the system.
- GDP growth: Responds contemporaneously to both inflation (1.2871) and its own structural shock (3.0036), highlighting the sensitivity of economic activity to price-level movements.
- Aggregate inflation cycle: Reacts to shocks in both inflation (-0.128) and GDP growth (-0.035), in addition to its own structural shock (1.0249), reflecting the interaction between real and nominal dynamics.
- Nominal interest rate: Adjusts contemporaneously to all previous shocks as well as its own structural shock (0.94646), indicating that monetary policy reacts immediately to fluctuations in both inflation and output, in line with a responsive policy rule.

Appendix A.2

My model is composed by 21 endogenous equations

$$c_h, n_h, k, B, I, q, c_{L,f}, c_{L,i}, \theta_t, y^m, y^u, \pi, r^k, w_t^H, w_t^L, r, R, \bar{Y}, \bar{C}, \bar{c}_L, \bar{N}$$

and 3 exogenous process

$$M_t, Z_t, \iota_t$$

6.1 High Skilled Consumer

The maximization problem:

$$\max_{\{c_{H,t}, n_{H,t}, k_t, B_t, i_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{H,t}^{1-\sigma}}{1-\sigma} - h_h \frac{n_{H,t}^{1+\gamma}}{1+\gamma} \right) \quad (26)$$

$$\text{s.t.} \quad \begin{cases} p_t c_{H,t} + p_t i_t + p_t B_t = r_t^k p_t k_{t-1} + p_t r_{t-1} B_{t-1} + p_t (1 - \tau_t^H) w_{H,t} n_{H,t} + p_t \Gamma_t^F \\ k_t = (1 - \delta) k_{t-1} + \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \end{cases}$$

I obtain the following equations:

$$\lambda_{H,t} = c_{H,t}^{-\sigma} \quad (27)$$

$$w_{H,t} (1 - \tau_t^H) = h_h n_t^{H\gamma} \lambda_{H,t} \quad (28)$$

$$\lambda_{H,t} = \mathbb{E}_t \beta^t \left(\lambda_{H,t+1} \frac{r_t}{\pi_{t+1}} \right) \quad (29)$$

$$k_t = (1 - \delta) k_{t-1} + \left[1 - \frac{\kappa_I}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \quad (30)$$

$$\begin{aligned} 1 = q_t & \left[1 - \frac{K_I}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - K_I \left(\frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - 1 \right) \right) \right] \\ & + \mathbb{E}_t \beta K_I \left(\frac{\lambda_{H,t+1}}{\lambda_{H,t}} q_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 \left(\frac{i_{t+1}}{i_t} - 1 \right) \right) \end{aligned} \quad (31)$$

$$q_t = \mathbb{E}_t \beta \frac{\lambda_{H,t+1}}{\lambda_{H,t}} (r_{t+1}^k + (1 - \delta) q_{t+1}) \quad (32)$$

6.2 Low Skilled Consumers

The maximization problem

$$u(c_t, \theta_t) = \frac{(c_t)^{1-q} - 1}{1-q} - h \frac{(\theta_t)^{1+\gamma}}{1+\gamma} (1 - \theta_t) - f \frac{(1 - \theta_t)^{1-\eta}}{1-\eta} \quad (33)$$

$$c_{L,f,t} = w_{L,t}(1 - \tau_t^h)\theta_t \quad (34)$$

$$c_{L,i,t} = w_{L,t}(1 - \theta_t) \quad (35)$$

$$C_{L,t} = c_{L,f,t} + c_{L,i,t} \quad (36)$$

From 34, 35 and 36 I obtain

$$\bar{C}_{L,t} = w_{L,t}(1 + \theta_t \tau_t^h) \quad (37)$$

Then the maximization problem 33 become:

$$\max_{\{C_{L,t}, \theta_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{L,t})^{1-q} - 1}{1-q} - h \frac{(\theta_t)^{1+\gamma}}{1+\gamma} (1 - \theta_t) - f \frac{(1 - \theta_t)^{1-\eta}}{1-\eta} \right] \quad (38)$$

s.t

$$\bar{C}_{L,t} = w_{L,t}(1 + \theta_t \tau_t^h), \quad (39)$$

from which I obtain the following equations

$$\lambda_{L,t} = \bar{C}_{L,t}^{-q} \quad (40)$$

$$c_{L,t} = c_{L,f,t} + c_{L,i,t} \quad (41)$$

$$w_{L,t} \tau_t^h \lambda_{L,t} = h(\theta_t - 1)(\theta_t)^\gamma + f(1 - \theta_t)^{-\eta} + h \frac{(\theta_t)^{1+\gamma}}{1+\gamma} \quad (42)$$

In the case of informal labor supply shock I obtain:

$$w_{L,t} \tau_t^h \lambda_{L,t} = h(\iota \theta_t - 1)(\theta_t)^\gamma + \iota f(1 - \theta_t)^{-\eta} + h \frac{(\theta_t)^{1+\gamma}}{1+\gamma} \quad (43)$$

$$\log\left(\frac{\iota_t}{\iota}\right) = \rho_\iota \log\left(\frac{\iota_{t-1}}{\iota}\right) + \varepsilon_{\iota,t} \quad (44)$$

6.3 Intermediate Producers

$$Y_t^m(i) = M_t \left\{ k_{t-1}^\alpha \left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1-\alpha}{\xi}} \right\} \quad (45)$$

$$\bar{Y}_t(i) = Y_t^m(i) + Y_t^u(i) \quad (46)$$

$$Y_t^u(i) = Z t (1 - \theta_t) \quad (47)$$

$$\bar{Y}_t(i) = \bar{Y}_t \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon} \quad (48)$$

The problem is the following

$$\max_{k_{t-1}, n_{H,t}, \theta_t, p_t(i)} \Pi_t(i) : \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}^h}{\lambda_t^h} \left[\begin{array}{l} \frac{p_t(i)}{p_t} \bar{Y}_t(i) - r_t^k k_{t-1} - w_{H,t} (1 + \tau_t^f) n_{H,t} \\ - w_{L,t} (1 + \theta_t \tau_t^f) - \frac{K_P}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi} \right)^2 \bar{Y}_t \end{array} \right] \right\} \quad (49)$$

s.t (45 - 48)

I obtain the following equations:

$$r_t^k = MC_t(i) \left\{ (1 - \tau_t^f) \alpha M_t k_{t-1}^{\alpha-1} \left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1-\alpha}{\xi}} \right\} \quad (50)$$

$$w_{h,t} (1 + \tau_t^f) = MC_t(i) \left\{ (1 - \tau_t^f) \frac{1-\alpha}{\xi} M_t k_{t-1}^\alpha \left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} \xi B n_{h,t}^{\xi-1} \right\} \quad (51)$$

$$w_{L,t} \tau_t^f = MC_t(i) \left\{ (1 - \tau_t^f) \frac{1-\alpha}{\xi} M_t k_{t-1}^\alpha \left(B n_{h,t}^\xi + G \theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} \xi G \theta_t^{\xi-1} - (1 - p s \tau_t^f) Z t \right\} \quad (52)$$

where 50 is the capital demand, 51 is the high skilled labor demand and 52 is the low skilled labor demand.

Proof of Proposition 1. In steady state, all endogenous variables can be expressed as functions of the formal employment share θ . For any $\theta \in (0, 1)$, the capital stock, aggregate output, and investment are well-defined and strictly positive.

However, the sign of the informal wage

$$w_l = \frac{mc}{tf} \left[(1 - tf)(1 - \alpha) A k^\alpha G \theta^{\xi-1} (B + G \theta^\xi)^{\frac{1-\alpha-\xi}{\xi}} - (1 - p s t f) Z \right]$$

plays a pivotal role in determining the economic admissibility of the steady state.

Due to the nonlinearity induced by the CES labor aggregator with $\xi < 0$ and its interaction with fiscal distortions, the informal wage is a non-monotonic function of θ .

For the calibrated parameter values, w_l is strictly positive if and only if θ belongs to the two disjoint intervals

$$\theta \in [0.2, 0.76].$$

For values of θ outside these regions, the informal wage becomes non-positive, implying negative consumption or labor allocations, and therefore violating economic feasibility. \square

Remark 1. *The admissible set Θ is not connected. This property arises from the non-monotonic response of informal wages to changes in the formal employment share, driven by the interaction between labor aggregation, production complementarities, and fiscal distortions. Since an analytical characterization of Θ is not available in closed form, the admissible parameter region is identified numerically.*

Proof of Proposition n. 2. The firm maximizes net profit:

$$\begin{aligned} \Pi_t(i) = & (1 - \tau_t^f)M_t \left[k_{t-1}^\alpha (Bn_{h,t}^\xi + G\theta_t^\xi)^{\frac{1-\alpha}{\xi}} \right] \\ & + (1 - ps\tau_t^f)Z_t(1 - \theta_t) - r_t^k k_{t-1} - w_{H,t}(1 + \tau_t^f)n_{H,t} - w_{L,t}(1 + \theta_t\tau_t^f). \end{aligned} \quad (53)$$

The associated Lagrangian for the constraint $0 \leq \theta_t \leq 1$ is

$$\mathcal{L} = \Pi_t(i) - \Upsilon_2(-\theta_t) - \Upsilon_3(\theta_t - 1),$$

with Kuhn–Tucker multipliers $\Upsilon_2, \Upsilon_3 \geq 0$.

The first-order condition with respect to θ_t is

$$\frac{\partial \Pi_t(i)}{\partial \theta_t} + \Upsilon_2 - \Upsilon_3 = 0, \quad (54)$$

with complementary slackness

$$\Upsilon_2\theta_t = 0, \quad \Upsilon_3(\theta_t - 1) = 0.$$

Corner $\theta_t = 0$. Evaluate the marginal profit at $\theta_t = 0$:

$$\left. \frac{\partial \Pi_t(i)}{\partial \theta_t} \right|_{\theta_t=0} > 0.$$

- with $\xi < 0$: $0^{\xi-1} \rightarrow +\infty \Rightarrow f(0^+) \rightarrow +\infty > 0$
- with $\xi > 1$: $0^{\xi-1} \rightarrow 0 \Rightarrow f(0^+) = -(1 - ps\tau_t^f)Z_t < 0$

Hence, increasing θ_t raises profit, ruling out $\theta_t = 0$ as optimal.

Corner $\theta_t = 1$. Evaluate at $\theta_t = 1$:

$$\left. \frac{\partial \Pi_t(i)}{\partial \theta_t} \right|_{\theta_t=1} < 0.$$

Hence, reducing θ_t increases profit, ruling out $\theta_t = 1$.

Since both corner solutions are suboptimal, the optimum lies strictly in the interior:

$$0 < \theta_t^* < 1,$$

ensuring the firm hires both high-skilled and low-skilled workers. \square

I start from the first-order condition (FOC) with respect to labor inputs, which yields the following expression for the marginal cost:

$$MC_t(i) = \frac{w_{L,t}\tau_t^f}{(1 - \tau_t^f)(1 - \alpha)M_t k_{t-1}^\alpha \left(Bn_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} G\theta_t^{\xi-1} - (1 - ps\tau_t^f)Z_t} \quad (1)$$

To facilitate the derivation, I define the following auxiliary variables:

$$\Gamma_t \equiv \frac{k_{t-1}}{n_{h,t}} \quad \Rightarrow \quad k_{t-1} = \Gamma_t \cdot n_{h,t} \quad (55)$$

$$X_t \equiv \frac{n_{h,t}}{\theta_t} \quad \Rightarrow \quad \theta_t = \frac{n_{h,t}}{X_t} \quad (56)$$

I now simplify each component of the denominator accordingly.

$$k_{t-1}^\alpha = (\Gamma_t \cdot n_{h,t})^\alpha = \Gamma_t^\alpha n_{h,t}^\alpha \quad (57)$$

$$\theta_t^\xi = \left(\frac{n_{h,t}}{X_t} \right)^\xi = n_{h,t}^\xi X_t^{-\xi} \quad (58)$$

$$Bn_{h,t}^\xi + G\theta_t^\xi = n_{h,t}^\xi \left(B + GX_t^{-\xi} \right) \quad (59)$$

$$\left(Bn_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} = n_{h,t}^{1-\alpha-\xi} \left(B + GX_t^{-\xi} \right)^{\frac{1-\alpha}{\xi}-1} \quad (60)$$

$$\theta_t^{\xi-1} = \left(\frac{n_{h,t}}{X_t} \right)^{\xi-1} = n_{h,t}^{\xi-1} X_t^{-(\xi-1)} \quad (61)$$

$$D_t = (1 - \tau_t^f)(1 - \alpha)M_t \Gamma_t^\alpha n_{h,t}^\alpha \cdot n_{h,t}^{1-\alpha-\xi} \left(B + GX_t^{-\xi} \right)^{\frac{1-\alpha}{\xi}-1} \cdot G n_{h,t}^{\xi-1} X_t^{-(\xi-1)} - (1 - ps\tau_t^f)Z_t \quad (62)$$

$$\alpha + (1 - \alpha - \xi) + (\xi - 1) = 0 \quad \Rightarrow \quad n_{h,t}^0 = 1$$

$$D_t = (1 - \tau_t^f)(1 - \alpha)M_t \Gamma_t^\alpha \left(B + GX_t^{-\xi} \right)^{\frac{1-\alpha}{\xi}-1} GX_t^{-(\xi-1)} - (1 - ps\tau_t^f)Z_t \quad (63)$$

$$MC_t(i) = \frac{w_{L,t}\tau_t^f}{(1 - \tau_t^f)(1 - \alpha)M_t \left(\frac{k_{t-1}}{n_{h,t}}\right)^\alpha \left(B + G \left(\frac{n_{h,t}}{\theta_t}\right)^{-\xi}\right)^{\frac{1-\alpha}{\xi}-1} G \left(\frac{n_{h,t}}{\theta_t}\right)^{-(\xi-1)} - (1 - ps\tau_t^f)Z_t} \quad (64)$$

For compute the relative ratio, I start from the first-order conditions for capital and high-skill labor, which take the following form:

$$r_t^k = MC_t(i) (1 - \tau_t^f) \alpha M_t k_{t-1}^{\alpha-1} \left(Bn_{h,t}^\xi + G\theta_t^\xi\right)^{\frac{1-\alpha}{\xi}} \quad (65)$$

$$w_{h,t} (1 + \tau_t^f) = MC_t(i) (1 - \tau_t^f) (1 - \alpha) M_t k_{t-1}^\alpha \left(Bn_{h,t}^\xi + G\theta_t^\xi\right)^{\frac{1-\alpha}{\xi}-1} B n_{h,t}^{\xi-1} \quad (66)$$

To eliminate the marginal cost $MC_t(i)$, I divide equation (66) by equation (65), obtaining:

$$\frac{w_{h,t}(1 + \tau_t^f)}{r_t^k} = \frac{(1 - \alpha)k_{t-1}^\alpha (Bn_{h,t}^\xi + G\theta_t^\xi)^{\frac{1-\alpha}{\xi}-1} B n_{h,t}^{\xi-1}}{\alpha k_{t-1}^{\alpha-1} (Bn_{h,t}^\xi + G\theta_t^\xi)^{\frac{1-\alpha}{\xi}}} \quad (67)$$

$$= \frac{1 - \alpha}{\alpha} k_{t-1} B n_{h,t}^{\xi-1} \left(Bn_{h,t}^\xi + G\theta_t^\xi\right)^{-1} \quad (68)$$

where I used the simplifications:

$$k_{t-1}^\alpha / k_{t-1}^{\alpha-1} = k_{t-1}, \quad \left(\frac{1 - \alpha}{\xi} - 1\right) - \frac{1 - \alpha}{\xi} = -1$$

I isolate k_{t-1} from equation (68):

$$k_{t-1} = \frac{\alpha}{1 - \alpha} \frac{w_{h,t}(1 + \tau_t^f)}{r_t^k B} n_{h,t}^{1-\xi} \left(Bn_{h,t}^\xi + G\theta_t^\xi\right) \quad (69)$$

Dividing by $n_{h,t}$, I obtain the capital-to-labor ratio:

$$\frac{k_{t-1}}{n_{h,t}} = \frac{\alpha}{1 - \alpha} \frac{w_{h,t}(1 + \tau_t^f)}{r_t^k B} n_{h,t}^{-\xi} \left(Bn_{h,t}^\xi + G\theta_t^\xi\right) \quad (70)$$

I factor $n_{h,t}^\xi$ inside the parentheses:

$$Bn_{h,t}^\xi + G\theta_t^\xi = n_{h,t}^\xi \left[B + G(\theta_t/n_{h,t})^\xi\right]$$

Substituting into equation (70), I obtain:

$$\frac{k_{t-1}}{n_{h,t}} = \frac{\alpha}{1-\alpha} \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} n_{h,t}^{-\xi} \cdot n_{h,t}^\xi \left[B + G(\theta_t/n_{h,t})^\xi \right] \quad (71)$$

$$= \frac{\alpha}{1-\alpha} \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} \left[B + G(\theta_t/n_{h,t})^\xi \right] \quad (72)$$

This is the final clean expression for the capital-to-labor ratio:

$$\boxed{\frac{K_{t-1}}{n_{h,t}} = \frac{\alpha}{1-\alpha} \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} \left[B + G(\theta_t/n_{h,t})^\xi \right]} \quad (73)$$

Then, to understand what the *CSC* i.e. $\frac{\theta}{n_h}$ is, I start from the first-order conditions for the wages of the two types of labor:

$$w_{h,t}(1+\tau_t^f) = MC_t(i)(1-\tau_t^f)(1-\alpha)M_t k_{t-1}^\alpha \left(B n_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} B n_{h,t}^{\xi-1} \quad (74)$$

$$w_{L,t}\tau_t^f = MC_t(i)(1-\tau_t^f)(1-\alpha)M_t k_{t-1}^\alpha \left(B n_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1} G\theta_t^{\xi-1} - (1-ps\tau_t^f)Z_t \quad (75)$$

I observe that both right-hand sides share the common factor

$$\Psi_t := MC_t(i)(1-\tau_t^f)(1-\alpha)M_t k_{t-1}^\alpha \left(B n_{h,t}^\xi + G\theta_t^\xi \right)^{\frac{1-\alpha}{\xi}-1}.$$

Rewriting, I have:

$$w_{h,t}(1+\tau_t^f) = \Psi_t B n_{h,t}^{\xi-1} \quad (76)$$

$$w_{L,t}\tau_t^f + (1-ps\tau_t^f)Z_t = \Psi_t G\theta_t^{\xi-1} \quad (77)$$

Dividing equation (77) by equation (76) to eliminate Ψ_t , I get

$$\frac{w_{L,t}\tau_t^f + (1-ps\tau_t^f)Z_t}{w_{h,t}(1+\tau_t^f)} = \frac{G\theta_t^{\xi-1}}{B n_{h,t}^{\xi-1}}.$$

From this, it follows that

$$\left(\frac{\theta_t}{n_{h,t}} \right)^{\xi-1} = \frac{B}{G} \cdot \frac{w_{L,t}\tau_t^f + (1-ps\tau_t^f)Z_t}{w_{h,t}(1+\tau_t^f)}.$$

Raising both sides to the power $\frac{1}{\xi-1}$, I obtain the ratio

$$\boxed{\frac{\theta_t}{n_{h,t}} = \left[\frac{B}{G} \cdot \frac{w_{L,t}\tau_t^f + (1-ps\tau_t^f)Z_t}{w_{h,t}(1+\tau_t^f)} \right]^{\frac{1}{\xi-1}}}. \quad (78)$$

In the case without penalty (i.e., when $Z_t = 0$), this expression simplifies to

$$\frac{\theta_t}{n_{h,t}} = \left[\frac{B}{G} \cdot \frac{w_{L,t}\tau_t^f}{w_{h,t}(1+\tau_t^f)} \right]^{\frac{1}{\xi-1}}. \quad (79)$$

I define the following auxiliary term for clarity:

$$\Phi_t := \left(\frac{B}{G} \cdot \frac{w_{L,t}\tau_t^f + (1 - ps\tau_t^f)Z_t}{w_{h,t}(1+\tau_t^f)} \right)^{\frac{\xi}{\xi-1}}. \quad (80)$$

Then the capital-to-labor ratio can be expressed as

$$\frac{k_{t-1}}{n_{h,t}} = \frac{\alpha}{1-\alpha} \cdot \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} [B + G\Phi_t]. \quad (81)$$

Also, the transformations for the labor ratio are

$$\left(\frac{n_{h,t}}{\theta_t} \right)^{-\xi} = \left(\frac{\theta_t}{n_{h,t}} \right)^{\xi} = \Phi_t, \quad \left(\frac{n_{h,t}}{\theta_t} \right)^{-(\xi-1)} = \left(\frac{\theta_t}{n_{h,t}} \right)^{\xi-1} = \Phi_t^{\frac{\xi-1}{\xi}}.$$

Substituting these expressions into the marginal cost definition gives

$$MC_t(i) = \frac{w_{L,t}\tau_t^f}{(1-\tau_t^f)(1-\alpha)M_t \left[\frac{\alpha}{1-\alpha} \frac{w_{h,t}(1+\tau_t^f)}{r_t^k B} (B + G\Phi_t) \right]^\alpha (B + G\Phi_t)^{\frac{1-\alpha}{\xi}-1} G\Phi_t^{\frac{\xi-1}{\xi}} - (1 - ps\tau_t^f)Z_t}. \quad (82)$$

Finally, if desired, one can expand Φ_t explicitly from (80):

$$\begin{aligned} & (1-\varepsilon) \left(\frac{p_t(i)}{p_t} \right)^{-\varepsilon} \frac{\bar{Y}_t}{p_t} \\ & + \varepsilon MC_t(i) \left(\frac{p_t(i)}{p_t} \right)^{-\varepsilon-1} \bar{Y}_t \\ & - k_p \left(\frac{p_t(i)}{p_{t-1}(i)} - \bar{\pi} \right) \frac{\bar{Y}_t}{p_{t-1}(i)} \\ & + \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[k_p \left(\frac{p_{t+1}(i)}{p_t(i)} - \bar{\Pi} \right) \frac{\bar{Y}_{t+1}}{p_t(i)} \frac{p_{t+1}(i)}{p_t(i)} \right] = 0 \end{aligned} \quad (83)$$

In a symmetric equilibrium, where $p_t(i) = p_t$:

$$\begin{aligned} (\varepsilon - 1) \frac{\bar{Y}_t}{p_t} &= \varepsilon MC_t(i) \frac{\bar{Y}_t}{p_t} - k_p \left(\frac{p_t}{p_{t-1}} - \bar{\pi} \right) \frac{\bar{Y}_t}{p_{t-1}} \\ &+ \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[k_p \left(\frac{p_{t+1}}{p_t} - \bar{\Pi} \right) \frac{\bar{Y}_{t+1}}{p_t} \frac{p_{t+1}}{p_t} \right] \end{aligned}$$

Multiply both sides by $\frac{p_t}{Y_t}$:

$$k_p \left(\frac{p_t}{p_{t-1}} - \bar{\pi} \right) \frac{p_t}{p_{t-1}} + (\varepsilon - 1) = \varepsilon MC_t(i) + \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[k_p \left(\frac{p_{t+1}}{p_t} - \bar{\Pi} \right) \frac{\bar{Y}_{t+1} p_{t+1}}{\bar{Y}_t p_t} \right]$$

$$\left(\frac{p_t}{p_{t-1}} - \bar{\pi} \right) \frac{p_t}{p_{t-1}} = \frac{1}{k_p} (1 - \varepsilon + \varepsilon MC_t(i)) + \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_{t+1}}{p_t} - \bar{\Pi} \right) \frac{\bar{Y}_{t+1} p_{t+1}}{\bar{Y}_t p_t}$$

Rewrite the constant term:

$$\frac{1}{k_p} (1 - \varepsilon + \varepsilon MC_t(i)) = \frac{\varepsilon}{k_p} \left(MC_t(i) - \frac{\varepsilon - 1}{\varepsilon} \right)$$

$$(\pi_t - \bar{\pi})\pi_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[(\pi_{t+1} - \bar{\pi}) \frac{\bar{Y}_{t+1}}{\bar{Y}_t} \pi_{t+1} \right] + \frac{\varepsilon}{k_p} \left(MC_t(i) - \frac{\varepsilon - 1}{\varepsilon} \right) \quad (84)$$

As discussed above, unlike the standard New Keynesian framework in which inflation dynamics are primarily driven by marginal costs and formal output, incorporating informality into the model—as done in the analysis above—alters this relationship. In particular, inflation becomes a function of total output, which now includes both the formal and informal sectors.

$$\log \left(\frac{M_t}{M} \right) = \rho_m \log \left(\frac{M_{t-1}}{M} \right) + \epsilon_m \quad (85)$$

$$\log \left(\frac{Z_t}{Z} \right) = \rho_z \log \left(\frac{Z_{t-1}}{Z} \right) + \epsilon_z \quad (86)$$

.

6.4 Monetary policy

$$\frac{r_t}{r} = \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{\bar{Y}_t}{\bar{Y}} \right)^{\phi_{\bar{Y}}} \right]^{1-\rho_r} \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \epsilon_r \quad (87)$$

6.5 Aggregation

$$\bar{C}_t = \zeta C_{H,t} + (1 - \zeta) C_{L,t} \quad (88)$$

$$\bar{N}_t = \zeta n_{h,t} + (1 - \zeta) (\theta_t n_{l,t} + (1 - \theta_t) n_{l,t}), \quad (89)$$

$$R = \frac{r}{\pi_{t+1}} \quad (90)$$

$$\bar{Y} = \bar{C} + I \quad (91)$$