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Spillovers, Innovation Difficulty, and the Dynamics of Productivity

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Abstract

Is the aggregate productivity slowdown in the U.S. driven by a decline in successful innovation? This paper addresses this question using a medium-scale DSGE model with endogenous technology growth. The model distinguishes between two innovation channels: a spillover channel, which governs the efficiency with which aggregate R&D advances the technological frontier, and a difficulty channel, which governs the probability that sectoral R&D efforts successfully generate innovation. We estimate the model on U.S. macroeconomic and R&D data over the period 1984-2019, using macroeconomic observables and incorporating a patent-text-based measure of technological creativity that is informative about innovation probability. The results show that spillover shocks are the main drivers of short and medium run fluctuations in TFP growth, while R&D difficulty shocks mainly explain the probability of successful innovation. Once creativity data are included, the estimated difficulty shock becomes less volatile and more persistent, suggesting that innovation difficulty is a slow moving force shaping successful innovation. However, its quantitative contribution to TFP fluctuations remains substantially smaller than that of spillover shocks, although it matters in specific episodes.

Keywords: Innovation Difficulty, Endogenous growth, R&D investments, Bayesian estimation
JEL classification: E3, O3, O4, C11, C13.

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1 Introduction

Is the aggregate productivity slowdown in the U.S. driven by a decline in successful innovation? Short answer: no.

Over the last few decades, the U.S. economy has experienced an extraordinary wave of technological change. The diffusion of digital technologies, the expansion of platform-based business models, and, more recently, the rapid progress of artificial intelligence, machine learning, and data-intensive production have deeply reshaped economic activity. These developments have been supported by substantial investments in innovative sectors and by the growing concentration of entrepreneurial finance in technological hubs such as Silicon Valley (Davis et al., 2020). Yet, despite this visible acceleration in technological change and investment, aggregate productivity growth has weakened markedly since the early 2000s. The coexistence of rapid innovation and sluggish productivity has therefore become one of the central puzzles in modern macroeconomics (Goldin et al., 2024).

A large literature has proposed several explanations for this apparent paradox. Some contributions emphasize that the diffusion of digital technologies may be slower and more uneven than in previous technological waves, with gains concentrated among frontier firms and only gradually transmitted to the rest of the economy (Brynjolfsson and McAfee, 2014; Andrews et al., 2016). Others stress implementation lags associated with intangible capital and organizational restructuring (Brynjolfsson et al., 2019), the decline in business dynamism and resource reallocation (Decker et al., 2016), or the possibility that recent innovations, while privately valuable, have smaller aggregate effects than earlier general-purpose technologies (Gordon, 2016). A related and influential view is that ideas themselves are becoming harder to find, so that increasingly larger research efforts are required to sustain a given pace of technological progress (Bloom et al., 2020). While these explanations are often discussed together, they do not necessarily refer to the same economic mechanism.

This paper argues that a useful way to organize the debate is to distinguish between two different channels through which R&D affects productivity. The first is the *spillover channel*: how effectively aggregate R&D advances the technological frontier? The second is the *difficulty channel*: how likely it is that firm-level R&D efforts successfully generate innovation and frontier adoption? In our framework, these are not two names for the same decline in innovation efficiency. A spillover shock changes the extent to which aggregate research effort moves the frontier forward. A difficulty shock instead changes the probability that innovation attempts succeed for a given level of private R&D. This distinction is economically important because frontier progress and innovation need not move together. The frontier may continue to advance while fewer firms successfully incorporate frontier technologies, or, conversely, innovation attempts may remain relatively successful even when aggregate spillovers into frontier growth weaken.

In the growth literature, these two dimensions of the innovation process are typically modeled through different channels. Endogenous growth models in the tradition of Romer (1990) emphasize knowledge spillovers and the aggregate external effects of research activity, whereas semi-endogenous models such as Jones (1995) stress diminishing returns in idea production. Schumpeterian models, beginning with Aghion and Howitt (1992), instead place the probability of successful innovation and creative destruction at the center of growth dynamics. Quantitative DSGE models with endogenous innovation have shown that R&D and technology adoption can generate persistent medium-run fluctuations and long-lasting responses to macroeconomic shocks (Comin and Gertler, 2006;

Comin et al., 2009; Anzoategui et al., 2019). More recent medium-scale frameworks incorporate stochastic innovation processes and knowledge spillovers within an estimated general equilibrium environment (Cozzi et al., 2021). Our contribution is to bring these two innovation margins together and to identify them separately in the data.

To do so, we develop and estimate a medium-scale DSGE model with endogenous technological change using U.S. macroeconomic and R&D data over the period 1984-2019. The model builds on Cozzi et al. (2021) but allows for two separate exogenous innovation forces: a spillover shock affecting the efficiency with which aggregate R&D advances the frontier, and a difficulty shock affecting the probability of successful innovation. Aggregate productivity depends both on the pace at which the frontier expands and on the rate at which firms successfully innovate and incorporate frontier technologies into average productivity. In a baseline specification, the model is estimated using macroeconomic observables only. In an augmented specification, we further discipline the innovation block by incorporating a patent-based measure of technological creativity that is informative about the evolution of innovation probability. To the best of our knowledge, this is the first paper to use a patent-text-based measure of technological creativity to discipline the estimation of a DSGE model with endogenous innovation.

Our findings show that the two shocks generate different macroeconomic dynamics and play different quantitative roles. Spillover shocks are the main drivers of short and medium run fluctuations in TFP growth because they act directly on frontier advancement. Difficulty shocks, by contrast, primarily account for movements in innovation probability and affect aggregate productivity only indirectly, through slower creative destruction and weaker innovation. Once creativity data are incorporated, the estimated difficulty shock becomes less volatile but more persistent. This suggests that innovation difficulty is a slow moving force shaping successful innovation. However, its quantitative contribution to TFP fluctuations remains much smaller than that of spillover shocks, even though it matters in specific episodes.

The paper therefore provides a structural interpretation of the productivity slowdown that clarifies a distinction often left implicit in the existing debate. A decline in measured innovation performance may reflect either weaker spillovers from aggregate R&D to frontier growth or greater difficulty in translating sectoral research effort into successful innovation. In this sense, our results do not support the view that the productivity slowdown is primarily driven by a decline in successful innovation. Distinguishing these channels matters for both measurement and policy, because policies that raise R&D effort or improve innovation success need not strengthen the mechanism through which research advances the frontier.

The remainder of the paper is organized as follows. The next section reviews the related literature. The model is then presented, followed by the description of the estimation strategy and the discussion of the results. Section 6 concludes.

2 Related Literature

This paper speaks to two broad literatures. The first studies the productivity slowdown and the apparent disconnect between rapid technological change and weak aggregate productivity growth. The second develops growth models in which research activity affects productivity through knowledge spillovers, frontier expansion, and creative destruction. Our contribution lies at the intersection of these literatures. As discussed before, we argue that it is useful to distinguish between two conceptually different margins of the innovation process: the efficiency with which aggregate R&D advances the technological frontier, and the probability that sector-level innovation efforts successfully translate frontier knowledge into realized productivity gains.

2.1 Productivity slowdown, diffusion, and the productivity of research

A first strand of the literature emphasizes that the recent productivity slowdown need not imply an absence of technological progress. Rather, new technologies may diffuse slowly, unevenly, or with long implementation lags. In this view, digital technologies can generate substantial gains at the frontier without immediately raising aggregate productivity, either because adoption is concentrated among a limited set of firms or because complementary investments in organization, skills, and intangible capital take time to materialize (Brynjolfsson and McAfee, 2014; Brynjolfsson et al., 2019; Andrews et al., 2016). This perspective is particularly relevant for interpreting the gap between frontier innovation and average productivity, since the aggregate effects of new technologies depend not only on invention itself but also on the extent to which firms and sectors successfully adopt them.

A second and related strand argues that the productivity of research may itself have declined. Bloom et al. (2020) document that maintaining a given rate of technological progress requires increasing amounts of research effort across a wide range of sectors. Their evidence has revived the idea that the economy may be facing a gradual increase in innovation difficulty, in the sense that new ideas become harder to generate or implement as the stock of knowledge expands. A more pessimistic interpretation is offered by Gordon (2016), who argues that recent innovations may be less transformative than earlier general-purpose technologies and therefore have smaller aggregate productivity effects. Although these arguments are often grouped together, they point to different mechanisms: one concerns the effectiveness of research in pushing the frontier forward, while the other concerns the difficulty of translating research effort into economically relevant innovation.

Other contributions emphasize structural forces that may further weaken the macroeconomic effects of innovation. Lower business dynamism and weaker resource reallocation may reduce the speed with which new technologies displace less productive incumbents (Decker et al., 2016). More generally, the broader survey in Goldin et al. (2024) makes clear that the productivity slowdown is unlikely to admit a single explanation. Our paper does not attempt to arbitrate among all competing hypotheses. Instead, it isolates within a structural macroeconomic model two innovation-side channels that are often conflated in the debate: a spillover channel operating through frontier growth, and a difficulty channel operating through the success probability of innovation and creative destruction.

2.2 Innovation margins in growth theory

The growth literature has treated spillovers and innovation difficulty through different theoretical mechanisms. In endogenous growth models, research activity generates nonrival knowledge and aggregate spillovers that sustain long-run growth (Romer, 1986, 1990; Lucas, 1988). In these frameworks, the key margin is the extent to which current R&D expands the stock of usable knowledge. This makes them a natural benchmark for thinking about the spillover side of innovation, namely the mapping from aggregate research effort into frontier technological progress.

Semi-endogenous growth models modify this mechanism by introducing diminishing returns to idea production at the aggregate level (Jones, 1995). These models eliminate the scale effects of early endogenous growth formulations and imply that frontier growth depends on the interaction between research effort and the existing stock of knowledge. They are therefore particularly useful for thinking about the possibility that the social effectiveness of R&D may vary over time, either because of duplication, congestion, or weaker knowledge externalities.

A different perspective is offered by Schumpeterian models of growth, where innovation is inherently uncertain and occurs through creative destruction (Aghion and Howitt, 1992; Grossman and Helpman, 1991). In this class of models, firms invest in R&D in order to become the next technological leader, but success is probabilistic and generates turnover among incumbents. The central margin is therefore not only how much society invests in research, but also how likely research efforts are to succeed. This makes the Schumpeterian tradition especially relevant for the notion of innovation difficulty emphasized in this paper. Related work has also shown that measured productivity may understate true technological progress when creative destruction is imperfectly captured in the data (Aghion et al., 2019).

Our framework combines these insights. On the one hand, frontier growth is semi-endogenous and depends on knowledge spillovers from aggregate R&D. On the other hand, the adoption of frontier technology occurs through sector-specific innovation with a time-varying probability of success. This structure allows us to separate a shock that changes the efficiency of frontier advancement from a shock that changes the difficulty of successful innovation. In that sense, the model brings together the spillover logic of endogenous and semi-endogenous growth with the probabilistic innovation mechanism of Schumpeterian models.

2.3 Endogenous growth in DSGE models

A growing literature has embedded endogenous technological change into DSGE models in order to study jointly business-cycle dynamics and long-run productivity growth. Early contributions by Comin and Gertler (2006) and Comin et al. (2009) showed that R&D investment and gradual technology adoption can generate medium-run persistence and help explain the propagation of macroeconomic shocks. Subsequent work has developed medium-scale frameworks in which endogenous innovation interacts with standard nominal and real frictions, making it possible to estimate these mechanisms using aggregate data (Anzoategui et al., 2019).

This literature has also emphasized that the macroeconomic consequences of innovation depend on financial conditions, uncertainty, and policy. Credit frictions can depress R&D and generate persistent productivity losses (Moran and Queralto, 2018; Queralto, 2020; Guerrón-Quintana and Jinnai, 2019), while uncertainty can raise the effective hurdle rate for innovative investment and induce prolonged growth slowdowns (Bloom et al., 2007; Bianchi et al., 2019; Kozlowski et al., 2020). At the same time, monetary and fiscal policies may affect long-run productivity by altering innovation incentives and the allocation of resources toward research activity (Nuño, 2011; Jordà et al., 2020; Impullitti, 2010; Akcigit et al., 2022; Papaioannou, 2021). These contributions make clear that innovation is not merely a long-run residual, but a state variable through which macroeconomic disturbances can have persistent effects.

Our paper is closest to medium-scale DSGE models that combine endogenous growth with stochastic innovation processes, especially Cozzi et al. (2021). Their framework shows how knowledge spillovers, diminishing returns to R&D, and creative-destruction dynamics can be embedded in an estimated general equilibrium model. We build on that approach but depart from it in one crucial dimension. Rather than treating innovation frictions as a single reduced-form disturbance, we explicitly distinguish between a spillover shock, which changes the efficiency with which aggregate R&D advances the frontier, and a difficulty shock, which changes the probability that sectoral R&D successfully generates innovation. This distinction allows us to map different theoretical mechanisms into different macroeconomic dynamics and to assess their separate quantitative roles in explaining the U.S. productivity slowdown.

3 The Model

We develop a DSGE model based on the endogenous-growth structure of Cozzi et al. (2021). While the specification is simplified along some dimensions relative to their original framework, we retain the main real and nominal frictions typically employed in medium-scale New Keynesian models, namely, external habit formation in consumption, variable capital utilization, investment adjustment costs, and nominal price rigidity. This modelling choice allows us to preserve the standard transmission mechanisms used in the empirical DSGE literature, while focusing attention on the role of endogenous innovation and diffusion in shaping technology dynamics.

3.1 Households

There is a continuum of households, who choose how much to consume and how much to work maximizing their lifetime utility function, which is defined as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_t - bC_{t-1}\right) - \frac{\varepsilon_t^l}{1+\phi_l}\left(\frac{h_t}{g_h^t}\right)^{1+\phi_l} \right\} \tag{1}$$

where $0 < \beta < 1$ is the subjective discount factor and ϕ_l is the inverse of Frisch elasticity. $\varepsilon_t^l \sim AR(1)$ is labor supply shock. We assume that hours grow at a constant trend g_h.

Households allocate their resources between consumption C_t, government-issued bonds B_t and firm shares S_t. They receive income from labor services $W_t h_t$ and from holding government bonds and shares. Π_t are the dividends from a collective investment fund. This investment fund finances entrepreneurs' R&D investments in exchange for the ownership of the new incumbent firms from which it collects the profits. Moreover, the investment fund takes decisions also on physical capital investment. Bond returns are pre-determined whereas the stock market return is uncertain indicated by the timing $t+1$. Their budget constraint is:

$$P_t C_t + \frac{B_t}{\varepsilon_t^b} + \frac{P_t^S S_t}{\varepsilon_t^s} = R_{t-1}B_{t-1} + R_t^s P_{t-1}^S S_{t-1} + W_t h_t + P_t \Pi_t - T_t \tag{2}$$

where $\varepsilon_t^b \sim AR(1)$ and $\varepsilon_t^s \sim AR(1)$ are risk premium shocks, creating a wedge between the interest rate controlled by the central bank and the returns on assets held by the households. T_t are lump-sum taxes.

3.2 Investment decisions

Investment decisions are taken by an investment fund owned by the households. Capital evolves according to:

$$K_{t+1} = (1-\delta) K_t + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3}$$

where:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - g_Y\right)^2 \tag{4}$$

defines adjustment costs on investments (Christiano et al. (2005)). g_Y is the steady state of the trend in output.

Capital can be used more or less intensively, according to the degree of capital utilization u_t. Utilizing capital more intensively implies a cost, which is defined by function $a(u_t)$ (see Christiano et al. (2005)):

$$a(u_t) = \gamma_{u1}(u_t - 1) + \frac{\gamma_{u2}}{2}(u_t - 1)^2 \tag{5}$$

Optimal choices of capital, investment and capital utilization imply:

$$E_t \Xi_{t,t+1}\left[\frac{R_{t+1}^k}{P_{t+1}} u_{t+1} - a(u_{t+1})\right] - Q_t + E_t \Xi_{t,t+1} Q_{t+1}(1-\delta) = 0 \tag{6}$$

$$\begin{aligned} 1 = Q_t \varepsilon_t^i &\left\{1 - \gamma_I\left(\frac{I_t}{I_{t-1}} - g_Y\right)\frac{I_t}{I_{t-1}} - \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - g_Y\right)^2\right\} \\ &+ E_t \Xi_{t,t+1} Q_{t+1} \varepsilon_{t+1}^i \gamma_I \left(\frac{I_{t+1}}{I_t} - g_Y\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned} \tag{7}$$

$$\frac{1}{P_t} K_t R_t^k - [\gamma_{u1} + \gamma_{u2}(u_t - 1)] K_t = 0 \tag{8}$$

3.3 Firms

3.3.1 Final goods

The final good is produced by a representative perfectly competitive firm that aggregates a continuum of differentiated intermediate goods indexed by $i \in [0,1]$. The technology exhibits constant returns to scale and is given by:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\sigma_y - 1}{\sigma_y}} di\right)^{\frac{\sigma_y}{\sigma_y - 1}}, \tag{9}$$

where $\sigma_y > 1$ is the elasticity of substitution across goods. Profit maximization by the final goods producer implies the standard demand schedule for each intermediate variety:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma_y} Y_t. \tag{10}$$

3.3.2 Intermediate good firms

The intermediate good is produced by a monopolistically competitive firm. Each differentiated intermediate-good variety ($i \in [0,1]$) can be interpreted as a sector or product line. At any date t, variety i is produced by an incumbent intermediate-good firm that has market power over its own variety. Production combines labor services, h_t^i, and effective capital services, $u_t^i K_t^i$ as inputs. The technology is given by:

$$Y_t^i = A_{i,t}\left(u_t^i \frac{K_t^i}{A_t}\right)^{\alpha}\left(h_t^i\right)^{1-\alpha}$$

where α defines the capital share and $A_{i,t}$ represents the endogenous sector-specific productivity. As technologies become more advanced, production relies relatively more on capital services. $A_t = \int_0^1 A_{i,t} di$ is the average productivity across all sectors.

Total factor productivity is defined as:

$$TFP_t = \left(A_t\right)^{1-\alpha}. \tag{11}$$

Rearranging the production function highlights the role of technology diffusion across sectors. In particular, firms with higher relative productivity benefit more from the average technological level:

$$Y_t^i = \left(\frac{A_{i,t}}{A_t}\right)\left(u_t^i K_t^i\right)^{\alpha}\left(A_t h_t^i\right)^{1-\alpha} \tag{12}$$

Intermediate good firms choose prices, labor, and capital services to maximize dividends, taking final goods demand and technology as given. Dividends are given by:

$$D_{i,t} = \frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}h_{i,t} - R_t^k u_t^i K_t^i - \frac{\gamma_P}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - \exp\left(\bar{\pi}\right)\right)^2 Y_t\left(\frac{A_{i,t}}{A_t}\right) \tag{13}$$

3.3.3 Price setting

Price adjustment follows Rotemberg (1982) mechanism. Each firm chooses its optimal price to maximize the expected discounted value of future profits:

$$\max_{P_t^i} E_t \sum_{s=0}^{\infty} \Xi_{t,t+s} \quad \left\{\left(\tfrac{P_{i,t+s}}{P_{t+s}} - \tfrac{MC_{t+s}^i}{P_{t+s}}\right)Y_{i,t+s} - \tfrac{\gamma_P}{2}\left(\tfrac{P_{i,t+s}}{P_{i,t+s-1}} - \exp\left(\bar{\pi}\right)\right)^2 Y_{t+s}\left(\tfrac{A_{i,t+s}}{A_{t+s}}\right)\right\}$$

$$\text{s.t. } Y_{i,t+s} = \left(\tfrac{P_{i,t+s}}{P_{t+s}}\right)^{-\sigma_y} Y_{t+s}$$

where

$$E_t \Xi_{t,t+s} = \beta^s E_t\left[\varepsilon_t^s \frac{\Lambda_{t+s}}{\Lambda_t}\right]$$

is the stochastic *(stock market)* discount factor.

The optimal pricing condition can be written as:

$$\begin{aligned} 0 \;\; = \;\; & \left[1 - \sigma_y\left(1 - \frac{MC_t}{P_t}\right)\right] - \gamma_P\left(\pi_t - \exp\left(\bar{\pi}\right)\right)\pi_t \\ & + E_t \Xi_{t,t+1}\gamma_P\left(\pi_{t+1} - \exp\left(\bar{\pi}\right)\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} \end{aligned}$$

3.4 Endogenous Innovation

3.4.1 Innovations

In each period, productivity improvements occur through sector-specific innovations. Following Nuño (2011), productivity in sector i may jump to the technological frontier with probability $n_{i,t}$. This is due to R&D Investments of an entrepreneur in this sector. The frontier is defined as the highest productivity level currently available:

$$A_t^{max} = \max\{A_{i,t} | i \in [0,1]\}. \tag{14}$$

Productivity in each sector evolves according to:

$$A_{i,t+1} = \begin{cases} A_t^{max} \text{with probability } n_{i,t}, \\ A_{i,t} \text{with probability } 1 - n_{i,t}. \end{cases} \tag{15}$$

3.4.2 Entrepreneurs

Innovations are generated through R&D investments undertaken by entrepreneurs. In each sector and period, one entrepreneur is randomly selected as a potential entrant and invests resources $X_{i,t}^{RD}$ to attempt an innovation. These investments result in a probability $n_{i,t}$ of a successful innovation, leading to the discovery of a new intermediate good with next period's frontier productivity. Thus, this probability is endogenous and depends positively on R&D investments. The entrepreneur is distinct from the incumbent producer before innovation takes place. If the innovation is successful, the entrepreneur discovers a new intermediate good with next period's frontier productivity and becomes the new incumbent producer in that sector. Research becomes increasingly challenging as the technological frontier advances, capturing the idea that incremental improvements are harder to achieve when the stock of knowledge is already high. In addition, innovation efficiency may vary over time due to other factors, such as changes in research organization, institutional environments, or scientific opportunities. These influences are summarized by an exogenous difficulty shock, $\varepsilon_t^{n,RD}$. The probability of a successful innovation is given by:

$$n_{i,t} = \begin{cases} \left(\frac{X_{i,t}^{RD}}{\lambda^{RD} \varepsilon_t^{n,RD} g_h^t A_t^{max}}\right)^{\frac{1}{\eta+1}}, \text{ if } X_{i,t}^{RD} < \lambda^{RD} A_t^{max} \\ 1, \quad \text{if } X_{i,t}^{RD} \geq \lambda^{RD} A_t^{max} \end{cases} \tag{16}$$

where $\lambda^{RD} > 0$ governs R&D difficulty, and $\eta > 0$ captures decreasing returns to R&D. $\varepsilon_t^{n,RD}$ is a shock to innovation difficulty and evolves as an AR(1) process.

A successful innovator replaces the incumbent producer through Bertrand competition and collects monopoly profits in the next period. This creates the incentives to invest in R&D at time t.

Following Cozzi et al. (2021), R&D investment is subject to adjustment costs,:

$$adj_{i,t}^{RD}(X_{i,t}^{RD}) = \frac{\gamma^{RD}}{2Y_t} \left(X_{i,t}^{RD} - X_{t-1}^{RD} g_Y\right)^2, \tag{17}$$

Innovation outcomes are risky. To eliminate idiosyncratic risk, R&D projects are financed through a mutual fund owned by the households that pools investments across sectors. The entrepreneurial problem reduces to maximizing expected profits,

$$\max_{X^{RD}_{i,t}} n_{i,t} \left(\frac{P^{S,max}_{i,t}}{P_t} \right) - \left[X^{RD}_{i,t} + adj^{RD}_{i,t}(X^{RD}_{i,t}) \right] \varepsilon^{RD}_t$$

where $P^{S,max}_t$ is the nominal stock market value of the firm at the technological frontier. The value of becoming the incumbent is the same across sectors. Symmetry across sectors implies $X^{RD}_{i,t} = X^{RD}_t$ and $n_{i,t} = n_t$ in equilibrium. The optimal R&D condition is therefore:

$$\frac{n_t}{\eta + 1} \left[\frac{P^{S,max}_t}{P_t} \right] = X^{RD}_t \left(1 + \frac{\gamma^{RD}}{Y_t} \left(X^{RD}_t - X^{RD}_{t-1} g_Y \right) \right) \varepsilon^{RD}_t, \tag{18}$$

where ε^{RD}_t is a shock to R&D expenditures, which is modeled as a wedge in the private cost of R&D faced by entrepreneurs, rather than as a shock to the real resources absorbed by R&D activity. As such, it affects R&D incentives but does not directly enter the aggregate resource constraint.

On a balanced growth path, the innovation probability n_t also corresponds to the share of sectors that successfully innovate in each period, and therefore to the rate at which firms exit and enter the market. A higher n_t implies more frequent creative destruction, stronger innovative activity, and greater turnover among technologically leading industries.

3.4.3 Stock Market Value

Households hold shares in intermediate goods producers, whose aggregate market value is denoted by $P^S_t S_t$. For convenience, the total number of shares is normalized to one. Firms are heterogeneous, and firm turnover within intermediate-good sectors is endogenously driven by innovation. As a result of Schumpeterian creative destruction, a fraction n_{t-1} of firms in the portfolio held at time $t-1$ becomes obsolete at time t and is replaced by new firms with a higher market value, $P^{S,max}_t$. Accounting for firm entry and exit, the gross nominal return on the aggregate stock market portfolio reflects both dividend payments and value changes. The average portfolio value at time t, P^S_t, incorporates the newly created firms that replace the fraction n_{t-1} of displaced incumbents. Consequently, the aggregate value of these entrants, $n_{t-1} P^{S,max}_t$, must be netted out when computing portfolio returns. The gross nominal return is then:

$$R^s_t = \frac{D_t P_t + P^S_t - n_{t-1} P^{S,max}_t}{P^S_{t-1}} \tag{19}$$

where D_t represents average dividend payments.

3.4.4 Frontier Value

The growth rate of the technological frontier, $g_{A^{max}_t}$, is defined as:

$$g_{A^{max}_t} - 1 = \frac{A^{max}_t - A^{max}_{t-1}}{A^{max}_{t-1}}. \tag{20}$$

Entrepreneurs raise funds from households to finance R&D activities aimed at reaching the technological frontier. Upon success, they patent the sector-specific adaptation of the frontier technology and obtain monopoly rights over production. Firms operating at the frontier earn monopoly profits by supplying the highest quality intermediate good, which translates into dividend flows that exceed those of the average firm in proportion to their technological advantage.

The market value of a frontier firm reflects the expected discounted stream of future dividends as well as its continuation value. Innovations require one period to be implemented, so production and profit generation begin in the period following the R&D investment. The continuation value accounts for the possibility of further innovation by competitors: with probability n_{t+1}, the incumbent is replaced by a new innovator in the subsequent period, while with complementary probability the firm continues operating, albeit with a relative disadvantage compared to future frontier entrants.

$$\frac{P_t^{S,max}}{P_t} = \mathbb{E}_t \left[\Xi_{t,t+1} \left(D_{t+1} \frac{A_{t+1}^{max}}{A_{t+1}} + \frac{P_{t+1}^{S,max}}{P_{t+1} g_{A_{t+1}^{max}}} (1 - n_{t+1}) \right) \right], \tag{21}$$

where $\Xi_{t,t+1}$ is the stochastic discount factor.

3.4.5 Frontier Evolution

The growth of the technological frontier, $g_{A_t^{max}}$, is driven by knowledge spillovers arising from aggregate innovation activity, in line with the Schumpeterian growth literature ((Aghion and Howitt, 1998)).

Following Cozzi et al. (2021), we consider the following semi-endogenous technology:

$$A_t^{max} = A_{t-1}^{max} + \left(A_{t-1}^{max}\right)^{\phi} \left(\frac{X_{t-1}^{RD}}{Y_{t-1}} h_{t-1} \right)^{\lambda_A} \sigma_t^{RD}, \tag{22}$$

where λ_A is an externality parameter that captures forces such as research duplication, imitation, or knowledge leakage and $\phi < 1$ measures decreasing returns to the intertemporal knowledge spillover as in Jones (1995).

In addition, the contribution of R&D to frontier growth is subject to time-varying and stochastic spillovers, as emphasized by Nuño (2011). This feature captures fluctuations in the effectiveness of basic research, potentially driven by changes in scientific focus, institutional settings, or intellectual property regimes. The spillover shock σ_t^{RD} follows a stationary AR(1) process around its steady state:

$$\log \left(\frac{\sigma_t^{RD}}{\sigma^{RD}} \right) = \varepsilon_t^{\sigma} \sim AR\left(1\right)$$

The growth rate of the technological frontier is then:

$$g_{A_t^{max}} - 1 = \left(A_{t-1}^{max}\right)^{\phi-1} \left(\frac{X_{t-1}^{RD}}{Y_{t-1}} h_{t-1} \right)^{\lambda_A} \sigma_t^{RD}. \tag{23}$$

The condition for having a balanced growth path implies the following relationship among parameters:

$$log(g_{A^{max}}) = \lambda_A \frac{log(g_h)}{1 - \phi}. \tag{24}$$

3.4.6 Average Technology

The average productivity level evolves as successful innovators adopt the frontier technology while non-innovating firms retain their existing productivity. Then, aggregating across sectors:

$$A_t = \int_0^1 \left(n_{i,t-1} A_{t-1}^{max} + (1 - n_{i,t-1}) A_{i,t-1}\right) di, \tag{25}$$

Under symmetry across sectors, the law of motion for average productivity simplifies and highlights the role of creative destruction: technological progress depends on the gap between the frontier and the current average level, scaled by the fraction of sectors that successfully innovate:

$$A_t = n_{t-1}\left(A_{t-1}^{max} - A_{t-1}\right) + A_{t-1}. \tag{26}$$

On a balanced growth path, the frontier A_t^{max} grows at the same rate as the average technological level A_t.

3.5 Fiscal and Monetary authorities

The government budget constraint is:

$$P_t G_t + R_{t-1} B_{t-1} = B_t + T_t$$

We assume it is always balanced and omit it from the set of equations.

The central bank sets interest rates according to the following Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho^R} \varepsilon_t^r$$

3.6 Market clearing

Aggregate resource constraint:

$$Y_t = C_t + G_t + I_t + X_t^{RD} + \frac{\gamma_P}{2}\left(\pi_t - \exp\left(\bar{\pi}\right)\right)^2 Y_t + a\left(u_t\right) K_t + adj_t^{RD}(X_t^{RD})$$

4 Estimation methodology

We stationarize the model to guarantee that it has a balanced growth, adjusting the variables for their growth trends. The final equations are collected in the Appendix. Lower case letters stand for detrended variables, for example, $y_t = \frac{Y_t}{g_h^t A_t^{max}}$, We estimate the model with Bayesian techniques using U.S. quarterly data from 1984Q1 to 2019Q4. In the baseline specification we use eight macro time series, which are related to the endogenous variables through the following set of measurement equations:

$$
\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlXRD_t \\ dlWAG_t \\ lH_t \\ INTRATE_t \\ dlP_t \end{bmatrix} = \begin{bmatrix} log\left(g_{A_t^{max}}\right) \\ log\left(g_{A_t^{max}}\right) \\ log\left(g_{A_t^{max}}\right) \\ log\left(g_{A_t^{max}}\right) \\ log\left(g_{A_t^{max}}\right) \\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix} log(y_t) - log(y_{t-1}) \\ log(c_t) - log(c_{t-1}) \\ log(i_t) - log(i_{t-1}) \\ log(x_t) - log(x_{t-1}) \\ log(w_t) - log(w_{t-1}) \\ log(h_t) - log(h) \\ logR_t \\ log\pi_t \end{bmatrix} \tag{27}
$$

where dl denotes the percentage change measured as log difference and l denotes the log. The series considered are: growth rate in real GDP, consumption, investment, R&D investments, wages, log of hours worked (demeaned), the short-term interest rate, measured using Krippner's shadow rate[1] and inflation measured as the growth rate of the GDP deflator. GDP, consumption, investment and R&D investments are adjusted by a labor force index, so that they are in per capita terms. Our augmented specification also include a direct measure related to innovation probability. Rather than using firm entry and exit rates, which are often employed in the literature as proxies for creative destruction, we use the creativity measure proposed by Kalyani (2024). This index is constructed from micro-level patent data and is designed to capture economically meaningful innovative success more closely than turnover-based indicators, which are also influenced by cyclical conditions, regulation, and reallocation forces not directly related to technological progress. The creativity series is available at annual frequency, so we use annual percentage changes as observables and estimate the model in a mixed-frequency setting. To remain consistent with the stationarity of model-implied innovation probability (n_t), we also include an estimated constant in the measurement equation for the growth rate of n_t.[2][3]

The patent-based creativity measure is obtained by Kalyani (2024) and it is available at annual frequency. We use the growth rate of the original variable and adopt a mixed frequency approach. We also include a constant in the measurement equation.

We include ten fundamental shock processes in the estimation, several of which are the same as in Smets and Wouters (2007). In particular, we include two risk premium shocks (to bonds and to stocks returns), an investment (MEI) shock, a monetary policy shock, a government spending shock, a price markup shock, a labor supply shock and an R&D expenditure shock. In addition, we include a spillover shock and a difficulty shock. All shocks have an autoregressive component of order 1.

4.1 Calibration and Priors

We calibrate a number of parameters. In particular, the discount factor β is fixed at 0.996. The steady-state depreciation rate δ is 0.025, corresponding to a 10% depreciation rate per year. The elasticity of the demand for goods is set at 6, which implies a 20% net price markup in steady state. We set the government spending-to-GDP ratio at 20%, in line with its sample average. The capital share in the Cobb-Douglas bundle of capital and labor is set at 1/3. The steady state growth rate of technology, $g_{A^{max}}$ and the

[1] See Krippner (2013).

[2] We make robustness on this and estimate a version of the model were we do not include the constant. We discuss this below.

[3] Data sources are discussed in detail in Appendix C.

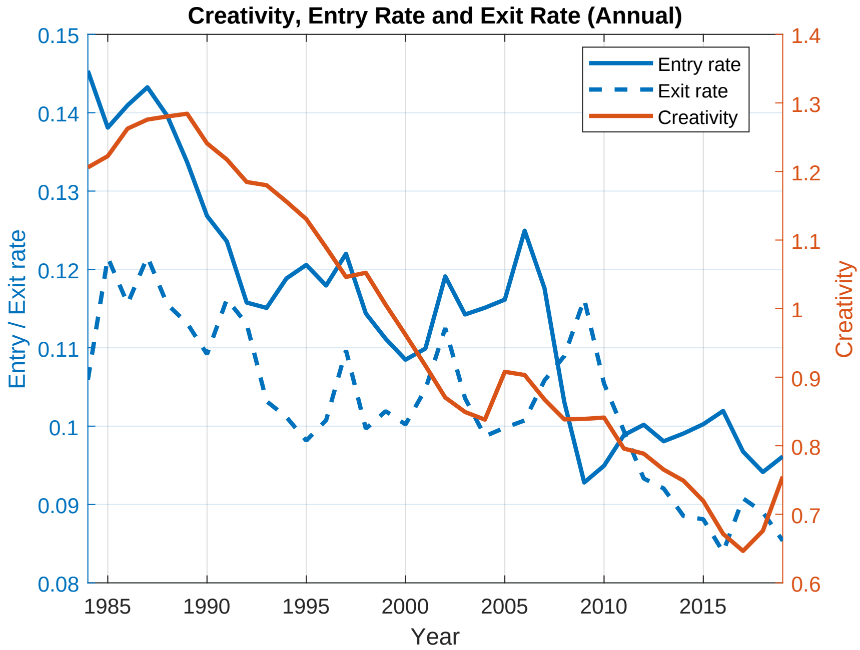

Figure 1: Entry rate, exit rate (LHS) and creativity measure (RHS).

constant trend g_h are set in line with the average of the data, at 1.6% and 1.2% annual respectively. Table 1 show the prior distributions of parameters. Notice that we estimate both the steady state of the probability of innovating (n_{ss}) and the degree of returns to the knowledge spillover (ϕ). We obtain λ_A and λ^{RD} from steady state relationships.

5 Results

This section presents the quantitative results by focusing on the two innovation mechanisms that the model is designed to separate. We focus on the results around the transmission of the two innovation shocks. The baseline specification uses only macroeconomic observables, while the augmented specification adds the patent-text-based creativity measure to discipline the innovation-probability block.

Table 1 reports the posterior estimates for the baseline and augmented specifications. Most parameters are stable across the two models, indicating that the inclusion of the creativity measure does not substantially alter the broader macroeconomic structure. The main difference concerns the difficulty shock. Once creativity is included, the innovation-difficulty process becomes markedly more persistent and less volatile. This does not change the qualitative transmission mechanism of the difficulty shock, but it changes its interpretation: the augmented specification points to innovation difficulty as a more persistent force rather than a high-frequency disturbance.

Table 1: Prior and Posterior Parameter Estimates

Parameter	Prior			Posterior					
	Shape	Mean	Std. Dev.	Baseline			Augmented		
				Mean	90% HPD		Mean	90% HPD	
ρ_g	beta	0.5	0.2	0.948	0.917	0.980	0.948	0.917	0.979
ρ_b	beta	0.5	0.2	0.374	0.290	0.459	0.381	0.295	0.466
ρ_s	beta	0.5	0.2	0.939	0.926	0.953	0.943	0.931	0.956
ρ_i	beta	0.5	0.2	0.037	0.010	0.063	0.036	0.010	0.061
$\rho_{s,RD}$	beta	0.5	0.2	0.420	0.137	0.694	0.402	0.130	0.674
$\rho_{n,RD}$	beta	0.5	0.2	0.277	0.017	0.669	0.907	0.836	0.983
ρ_p	beta	0.5	0.2	0.944	0.914	0.975	0.948	0.920	0.977
ρ_l	beta	0.5	0.2	0.971	0.952	0.991	0.971	0.952	0.991
ρ_{RD}	beta	0.5	0.2	0.646	0.356	0.908	0.501	0.369	0.632
n_{ss}	beta	0.03	0.01	0.105	0.081	0.132	0.110	0.089	0.136
b	beta	0.5	0.2	0.639	0.580	0.699	0.639	0.580	0.699
ϕ_l	gamma	2.5	0.5	1.298	0.963	1.626	1.300	0.971	1.628
γ_I	gamma	60	40	139.2	100.1	176.8	151.1	108.4	192.1
γ_P	gamma	40	10	22.31	16.03	28.45	21.64	15.77	27.35
γ_{u2}	gamma	0.02	0.01	0.144	0.130	0.159	0.144	0.129	0.159
γ^{RD}	gamma	60	40	703.3	679.7	721.9	703.5	680.0	721.9
ρ^R	beta	0.7	0.2	0.352	0.227	0.477	0.343	0.224	0.465
ϕ_π	norm	2	0.4	4.319	4.086	4.545	4.328	4.102	4.545
ϕ_y	norm	0.125	0.05	0.058	0.026	0.090	0.059	0.027	0.090
ϕ	beta	0.7	0.2	0.778	0.538	1.000	0.777	0.537	1.000
const_n	norm	0	1.5	–	–	–	-1.828	-2.216	-1.438
Standard deviation of shocks									
η_g	gamma	1	0.4	0.026	0.024	0.029	0.026	0.024	0.029
η_b	gamma	1	0.4	0.025	0.025	0.026	0.025	0.025	0.026
η_s	gamma	1	0.4	0.027	0.025	0.029	0.027	0.025	0.029
η_i	gamma	1	0.4	0.555	0.496	0.612	0.561	0.504	0.619
η_r	gamma	1	0.4	0.013	0.013	0.013	0.013	0.013	0.013
$\eta_{s,RD}$	gamma	1	0.4	0.802	0.323	1.265	0.786	0.308	1.243
$\eta_{n,RD}$	gamma	1	0.4	1.385	0.858	1.865	0.244	0.178	0.308
η_p	gamma	2	0.8	0.054	0.046	0.062	0.054	0.046	0.061
η_l	gamma	1	0.4	0.021	0.018	0.024	0.021	0.018	0.024
η_{RD}	gamma	1	0.4	0.174	0.112	0.236	0.247	0.218	0.275

5.1 Impulse responses: frontier growth versus successful innovation

The impulse responses are particularly informative about the transmission mechanisms of the two shocks affecting the innovation side of the economy.[4] The spillover shock changes the efficiency with which aggregate R&D generates frontier growth, while the difficulty shock changes the probability that sectoral R&D successfully produces an innovation. The relevant distinction is therefore structural: the first shock acts directly on the frontier-growth margin, whereas the second acts directly on the success/probability margin.[5]

Both shocks affect aggregate productivity through the law of motion for average technology,

$$g_{A_t^{\max}} a_t = n_{t-1}(1 - a_{t-1}) + a_{t-1}. \tag{28}$$

This equation is the key bridge between frontier growth and successful innovation. A spillover shock affects average technology mainly by changing $g_{A_t^{\max}}$, while a difficulty shock affects average technology mainly by changing n_t, the fraction of sectors that successfully reach the frontier. Since TFP is defined as $TFP_t = A_t^{1-\alpha}$, both shocks ultimately affect measured productivity, but through different channels.

5.1.1 The spillover shock

An adverse spillover shock is a decline in the efficiency with which aggregate R&D produces frontier technological progress. In stationarized form, frontier growth is given by

$$g_{A_t^{\max}} - 1 = \left(\frac{x_{t-1}^{RD}}{y_{t-1}} \right)^{\lambda_A} z_{t-1} \sigma_t^{RD}. \tag{29}$$

The shock therefore hits the economy by changing the mapping from aggregate research effort into $g_{A_t^{\max}}$. Its direct effect is visible in the response of frontier growth. Figure 2 [6] shows that $g_{A_t^{\max}}$ falls sharply on impact and then gradually returns to steady state. This is the mechanical implication of equation (29): holding predetermined R&D effort fixed, a lower σ_t^{RD} immediately lowers frontier growth.

The fall in frontier growth accumulates into a persistent decline in the level of frontier technology. Since average technology depends on the frontier through equation (28), TFP also falls persistently. Output and investment decline as the lower technology path reduces expected future production possibilities and the return to capital accumulation. The effect is therefore broad and persistent: the shock lowers the frontier itself, and the rest of the economy adjusts to a lower path of technology.

The response of R&D is also part of the propagation mechanism. The spillover shock does not directly make entrepreneurial R&D less likely to succeed. Instead, it reduces the expected value of becoming a frontier firm. The value of a frontier firm satisfies

$$p_t^{S,\max} = E_t \left[\beta \varepsilon_t^s \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{d_{t+1}}{a_{t+1}} + \frac{p_{t+1}^{S,\max}}{g_{A_{t+1}^{\max}}} (1 - n_{t+1}) \right) \right]. \tag{30}$$

[4] The standard deviations of the two shocks are set on the estimated values. Differences in response magnitudes across the baseline and augmented specifications reflect both changes in the propagation mechanism and changes in the estimated volatility of the shocks.

[5] For variables reported in levels, the impulse responses are reconstructed by cumulating the corresponding growth-rate responses and are shown as deviations from the balanced-growth path. A negative response of a level variable therefore means that the variable lies below its balanced growth path, not that it decreases relative to the previous period in every quarter.

[6] Additional responses are shown in Figure A1 in the Appendix.

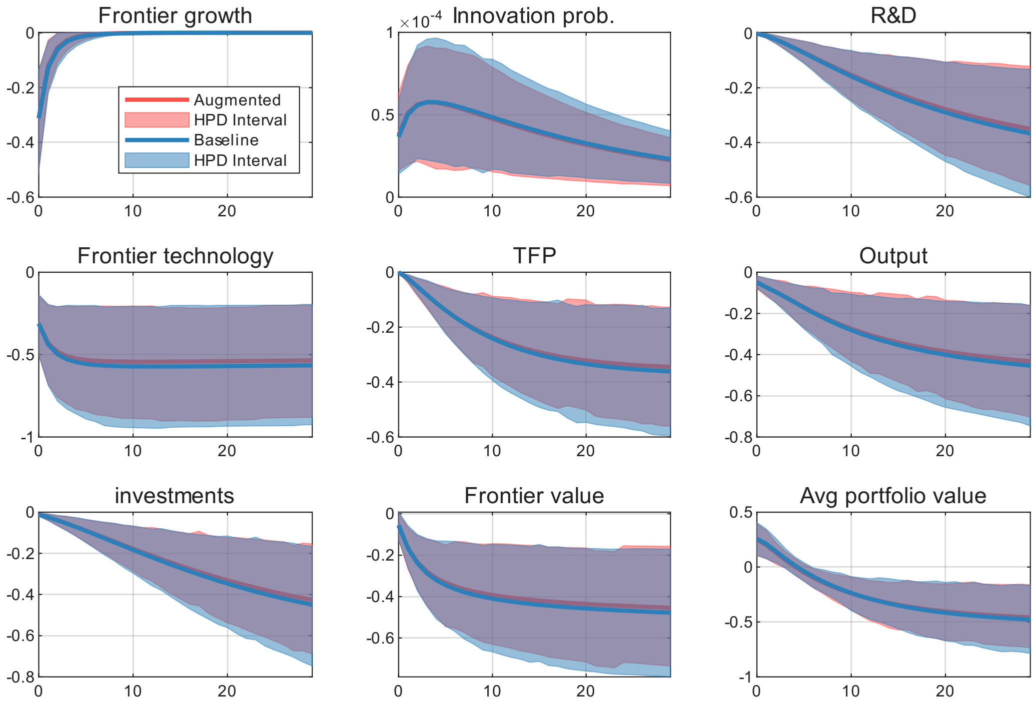

Figure 2: Impulse responses to an adverse spillover shock

Notes. The solid lines report posterior mean responses and the shaded areas denote 90% highest posterior density intervals.

After a spillover shock, weaker expected frontier growth lowers the value of future monopoly profits associated with frontier leadership. This reduces the payoff from innovative activity and leads to a decline in R&D. The R&D response reflects the fall in the expected return to innovation, not a direct deterioration in the probability of success.

This distinction is confirmed by the innovation probability response. Innovation probability moves very little and on a much smaller scale than the other technology variables. This is exactly what the model predicts. The spillover shock changes how much aggregate R&D advances the frontier; it does not directly change the probability that a given innovation attempt succeeds. Any movement in n_t arises only indirectly through the endogenous response of R&D. Frontier value falls because the shock reduces the productivity value of frontier leadership, reinforcing this interpretation. Since innovation probability reacts only weakly, changes in the probability of incumbent displacement play a minor role in this case.

5.1.2 The difficulty shock

The difficulty shock operates through a different channel. It enters the innovation-probability equation,

$$n_t = \left(\frac{x_t^{RD}}{\lambda^{RD} \varepsilon_t^{n,RD}} \right)^{\frac{1}{\eta+1}}. \tag{31}$$

Since $\varepsilon_t^{n,RD}$ appears in the denominator, a positive realization of this disturbance reduces the probability of successful innovation for a given level of R&D. In what follows, an adverse difficulty shock should therefore be understood as a shock that lowers n_t. Accordingly, Figure 3 [7] shows that innovation probability falls in both specifications. The decline is considerably larger on impact in the baseline model, whereas the augmented specification produces a smaller but more persistent response.

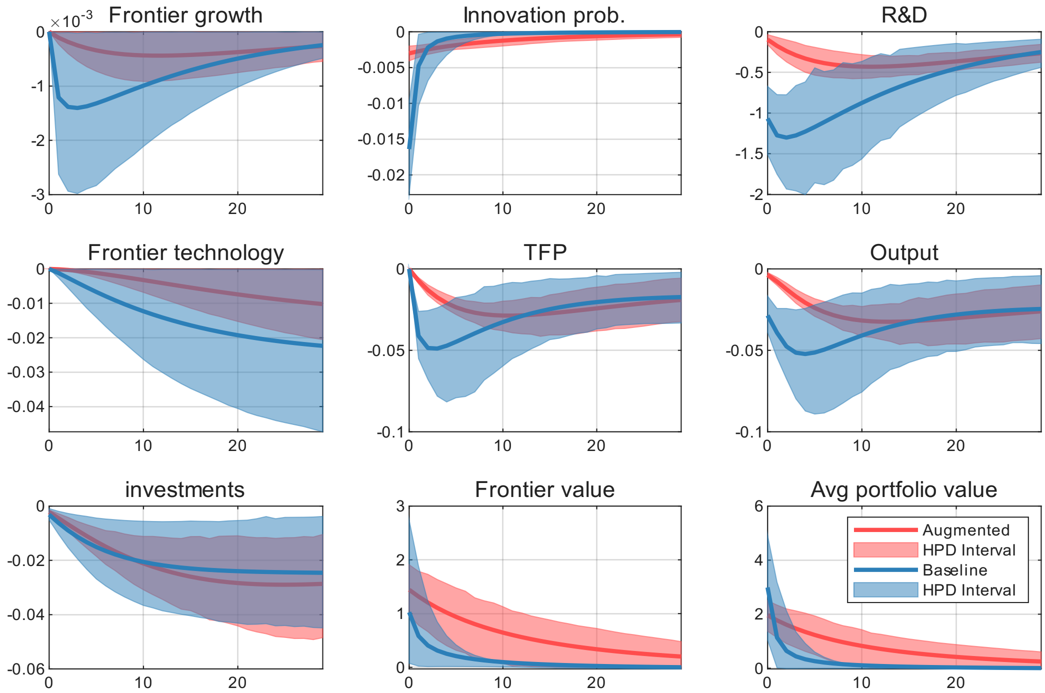

Figure 3: Impulse responses to an adverse difficulty shock

Notes. The solid lines report posterior mean responses and the shaded areas denote 90% highest posterior density intervals.

The effect on productivity is transmitted through equation (28). When n_t falls, fewer sectors adopt the frontier technology. Average productivity therefore catches up more slowly to the frontier, and TFP declines. Consistently, frontier growth reacts only mildly relative to the spillover shock. The response of frontier growth comes indirectly through the decline in R&D, which enters the frontier-growth equation with a lag.

R&D falls because the expected return to research declines when success becomes less likely. The optimality condition for R&D equates the expected marginal payoff from innovation to its marginal cost. A lower success probability weakens the payoff from undertaking R&D, leading entrepreneurs to reduce research expenditure. This in turn feeds back into frontier growth, but only indirectly. In the spillover case, frontier growth falls as a direct effect of the shock and R&D responds to a lower frontier value; in the difficulty case, the shock has a direct effect on the innovation probability which falls and R&D declines because successful innovation becomes harder.

The responses of frontier value and the average portfolio value highlight the Schumpeterian nature of the difficulty shock. After a difficulty shock, both variables rise on impact even though TFP and output fall. A lower innovation probability reduces the probability of creative destruction. In equation (30), a fall in n_{t+1} increases the survival term $1 - n_{t+1}$, raising the continuation value of incumbent firms. Thus, a difficulty shock is bad for aggregate productivity because it reduces successful innovation, but it can temporarily protect incumbents by lowering displacement risk. This valuation effect is

[7] Additional responses are shown in Figure A2 in the Appendix

especially persistent in the augmented specification, consistent with the higher estimated persistence of the difficulty shock.

Summing up, the spillover shock is a shock to the productivity of aggregate research in generating frontier growth. It produces a large and immediate fall in frontier growth, a persistent decline in frontier technology, and a broad contraction in TFP, output, investment, R&D, and firm portfolio values. Innovation probability is largely unaffected because the shock does not change the probability of success conditional on R&D. The difficulty shock is instead a shock to the success probability of innovation. It directly lowers n_t, reduces the number of sectors that reach the frontier, and slows the diffusion of frontier technology into average productivity. Its effect on frontier growth is secondary and operates through the endogenous fall in R&D. The distinctive feature of this shock is the creative destruction channel: lower innovation probability depresses aggregate productivity but can raise incumbent values by reducing replacement risk.

The baseline and augmented specifications deliver the same qualitative transmission mechanisms. For the spillover shock, the posterior mean responses are very similar and the corresponding HPD intervals overlap almost completely, confirming that the frontier-growth channel is robust to the inclusion of the creativity observable. Differences are more visible for the difficulty shock. In the augmented specification, the posterior mean responses of innovation probability and the main macroeconomic aggregates are generally smaller on impact but more persistent. The HPD intervals nevertheless overlap substantially for several variables. In addition, part of the smaller impact response reflects the considerably lower estimated volatility of the difficulty shock in the augmented model.

5.2 Quantitative importance: variance decompositions

The forecast error variance decompositions confirm that the two shocks matter for different objects. Table 2 reports the decompositions for the baseline and augmented specifications.

For TFP growth, spillover shocks are the dominant source of fluctuations. In the baseline model, they explain between about 58 and 87 percent of TFP-growth fluctuations, depending on the horizon and on whether TFP growth is measured quarter-on-quarter or year-on-year. In the augmented model, their role becomes even stronger: spillover shocks explain roughly 88-97 percent of TFP growth variance at the horizons considered. This is consistent with the model structure, since the spillover shock directly moves frontier growth, and frontier growth is a central component of measured technology dynamics.

For innovation probability, the ranking is reversed. Difficulty shocks explain almost all of the variance of innovation probability at short and medium horizons in both specifications. In the baseline model, the difficulty shock accounts for about 99 percent of the variance of n_t at 4 and 8 quarters, and still more than 92 percent at 40 quarters. In the augmented model, the corresponding shares are about 97 and 92 percent at four and eight quarters, with a lower but still dominant contribution at the longer horizon. This supports the interpretation of the difficulty shock as the disturbance governing the success/probability margin.

The FEVDs also clarify the role of R&D. R&D growth is mostly explained by other shocks, rather than by the spillover or difficulty shocks. This is important because the innovation shocks are not simply R&D expenditure shocks. They affect the efficiency and success of innovation, while the level of R&D spending itself also responds to other macroeconomic disturbances and to the separate R&D expenditure shock.

Table 2: FEVD decomposition: baseline and augmented specifications

		Spillover shock			Difficulty shock			Other shocks		
Specification	Variable	4	8	40	4	8	40	4	8	40
Baseline specification										
Baseline	Innovation prob	0.00	0.01	0.02	99.74	98.99	92.95	0.25	0.99	7.04
Baseline	TFP growth q/q	57.62	73.17	73.28	42.30	26.39	19.90	0.07	0.44	6.81
Baseline	TFP growth y/y	62.39	87.17	84.04	37.54	12.45	7.74	0.08	0.38	8.22
Baseline	R&D growth q/q	0.01	0.02	0.05	15.02	13.17	12.40	84.96	86.80	87.55
Baseline	R&D growth y/y	0.00	0.02	0.06	9.99	6.46	5.98	90.03	93.53	93.97
Augmented specification										
Augmented	Innovation prob	0.04	0.05	0.06	96.63	92.29	66.73	3.34	7.66	33.21
Augmented	TFP growth q/q	95.60	96.73	88.46	4.22	2.61	1.96	0.18	0.67	9.58
Augmented	TFP growth y/y	95.09	97.02	88.29	4.77	2.49	1.61	0.14	0.49	10.08
Augmented	R&D growth q/q	0.01	0.02	0.04	0.27	0.28	0.29	99.72	99.72	99.66
Augmented	R&D growth y/y	0.00	0.02	0.06	0.30	0.33	0.34	99.72	99.64	99.60

5.3 Historical decompositions

The historical decompositions show how the two mechanisms contribute to the estimated paths of innovation probability and TFP growth over the sample.

For innovation probability, we confirm the result obtained from the variance decomposition: fluctuations in n_t are mainly driven by the difficulty shock. This is visible in both specifications. In the baseline model, Figure 4 shows a relatively volatile innovation probability decomposition, with high frequency movements largely attributed to the difficulty shock. In the augmented model, Figure 5 shows a smoother path, because innovation probability matches our creativity observable. However, the underlying interpretation is unchanged: the difficulty shock remains the main source of movements in the probability of successful innovation.

For TFP growth, the decomposition points mainly to the spillover channel. In the baseline model, Figure 6 shows that spillover shocks account for a large share of the movements in TFP growth, although difficulty shocks also contribute in specific episodes. In the augmented model, Figure 7 makes the distinction sharper: spillover shocks continue to dominate TFP growth fluctuations, while the difficulty component becomes smoother.

This distinction is useful for interpreting the productivity slowdown. The model does not attribute weak productivity growth to a single innovation force. Weak TFP growth can arise because spillovers from aggregate R&D to the frontier are weak, or because successful innovation becomes more difficult and fewer sectors adopt frontier technologies. The historical decompositions suggest that spillover shocks are the dominant source of TFP-growth fluctuations, while difficulty shocks matter more for the evolution of innovation probability and for medium run episodes in which successful innovation deteriorates.

The estimated difficulty shock, shown in Figure 8, illustrates the contribution of the augmented specification. In the baseline model, the difficulty shock is more volatile. Once creativity is included, it becomes smoother and more persistent. This supports an interpretation of innovation difficulty as a slow-moving force affecting the success rate of innovation rather than as a standard high-frequency business-cycle disturbance. The post-2010 period is especially informative: the augmented specification points to a persistent increase in difficulty, consistent with the idea that successful innovation became progressively harder over this period. Still the contribution of this shock remains limited

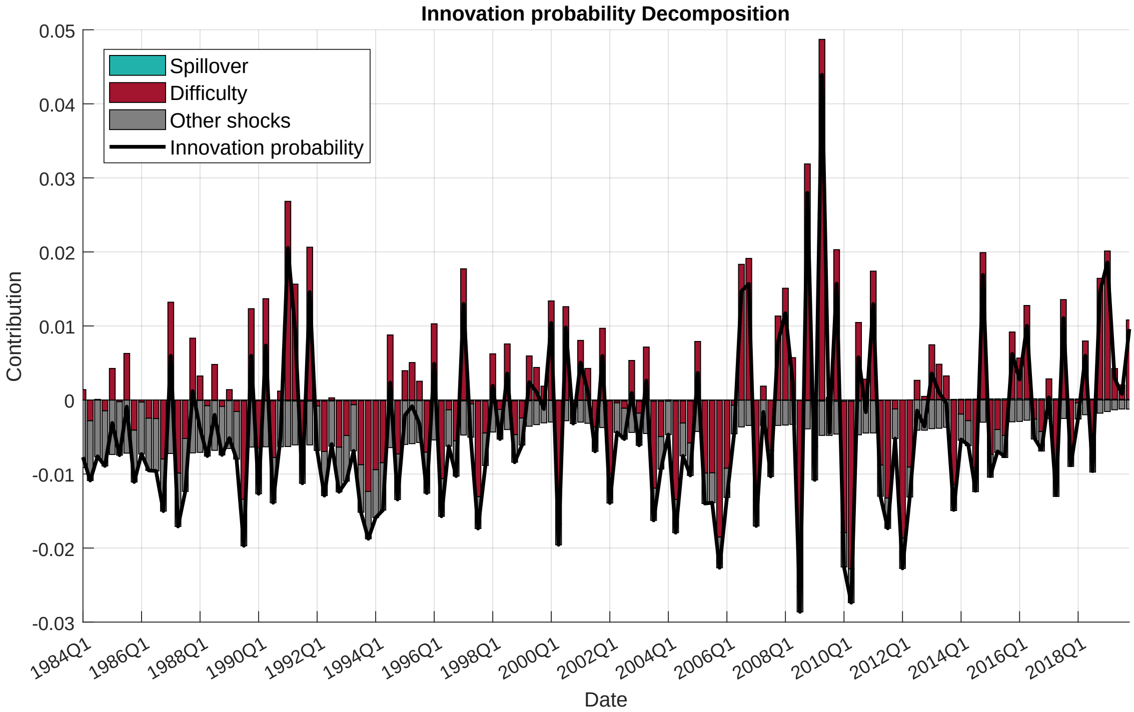

Figure 4: Historical decomposition of innovation probability under the baseline specification.

with respect to the spillover's.

Finally, Figure 9 compares the estimated TFP-growth series under the two specifications. The series share a similar medium run profile, but the augmented series is smoother, reflecting the smoother creativity observable. Both model implied series are less volatile than standard empirical TFP measures, such as the utilization-adjusted series of Fernald (2014) and Penn World Tables TFP. This lower volatility follows from the structure of the model: technology is an endogenous state variable driven by R&D, frontier spillovers, and successful innovation. High frequency movements in output, labor productivity, or empirical TFP residuals are absorbed mainly by utilization, markups, and other non-technology shocks rather than appearing as short run fluctuations in model implied technology. The estimated TFP series should therefore be interpreted as a measure of persistent innovation dynamics rather than as a high-frequency productivity residual.

5.4 What the creativity observable adds

The augmented specification is best understood as a discipline on the innovation success channel. It does not overturn the spillover and difficulty transmission mechanisms. Instead, it changes the estimated properties of the difficulty shock. The spillover channel is robust: a spillover shock lowers frontier growth, frontier technology, TFP, output, investment, R&D, and frontier firm value in very similar ways across the two specifications.

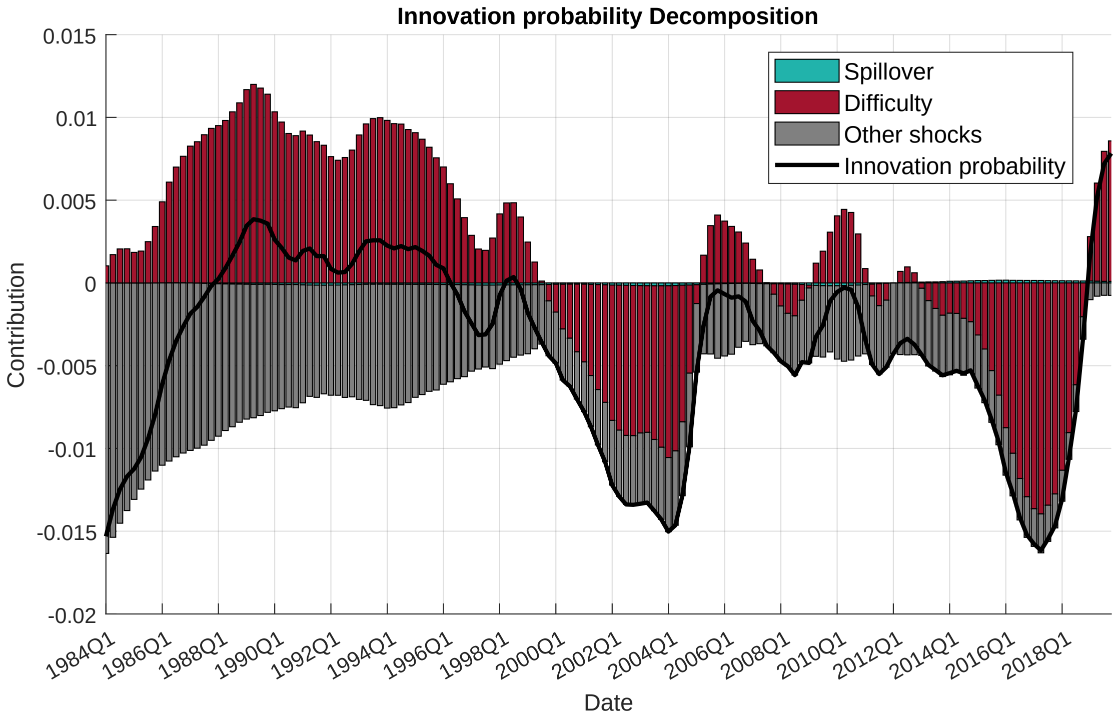

Figure 5: Historical decomposition of innovation probability under the augmented specification.

The difficulty channel is more affected by the additional observable: when creativity is included, the model assigns less volatility and more persistence to innovation difficulty, although at longer horizons the HPD intervals overlap substantially for several macroeconomic variables.

This result is important for the economic interpretation of innovation difficulty. Without the creativity observable, the difficulty shock can absorb relatively transitory fluctuations in innovation probability and TFP. With the creativity observable, the model is forced to match an additional measure that is informative about innovative success. The resulting difficulty shock is smoother and more persistent, suggesting that innovation difficulty is better interpreted as a slow moving force that affects the probability of successful innovation and the pace of creative destruction.

5.5 Robustness

We conclude with three robustness exercises. First, we assess whether the difficulty shock is important by re-estimating both the baseline and augmented specifications after excluding it. The comparison of log marginal data densities strongly favors the inclusion of the difficulty shock. In the augmented specification, the log marginal data density decreases from -1771.6 to -2125.8 when the shock is removed. The decline is smaller but still substantial in the baseline specification, where the log marginal data density falls from -1661.8 to -1692.66. The evidence in favor of the difficulty shock is therefore

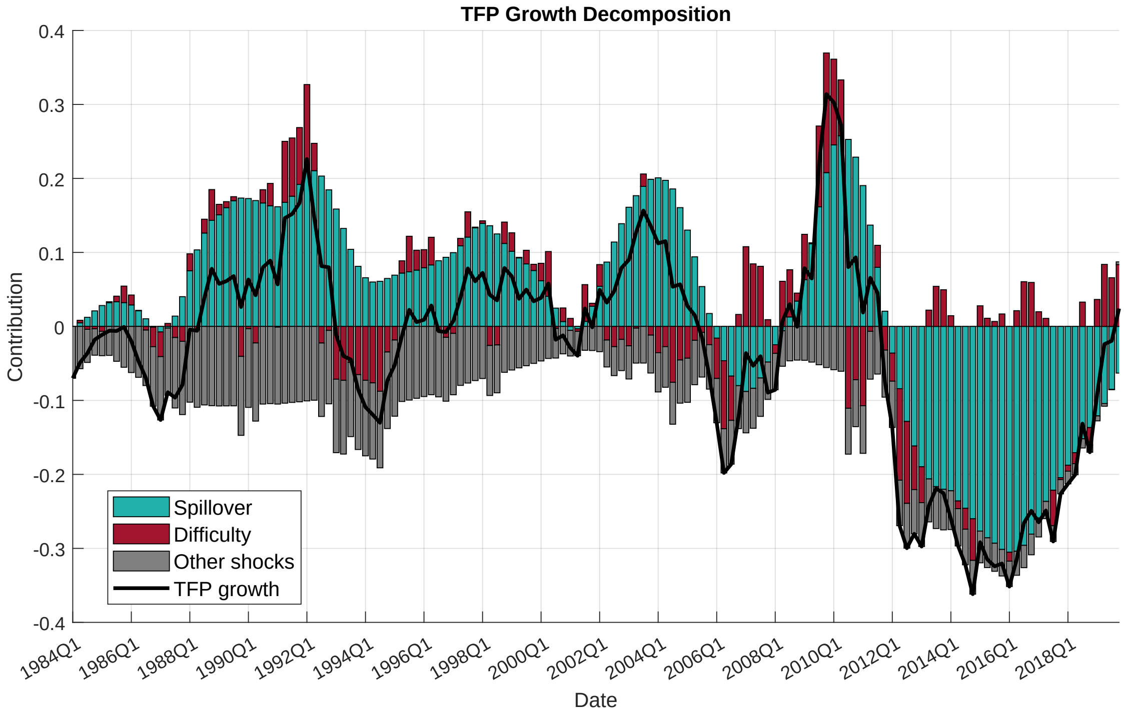

Figure 6: Historical decomposition of TFP year-on-year growth under the baseline specification.

especially strong when the creativity observable is included, consistent with its role in directly disciplining the innovation-success channel.

The variance decompositions also clarify how the restricted model reallocates the sources of productivity fluctuations. When the difficulty shock is excluded, the spillover shock explains almost all of the variance of TFP growth, absorbing the contribution attributed to the difficulty shock in the unrestricted specification. This should not be interpreted as evidence that the difficulty channel is irrelevant. Rather, once the model is prevented from distinguishing shocks to frontier advancement from shocks to innovation success, the spillover disturbance acts as a residual innovation-side shock and captures variation generated by both channels. Taken together, the marginal data density and variance decomposition results support the inclusion of a separate difficulty shock and indicate that distinguishing the two innovation mechanisms improves both model fit and the structural interpretation of TFP fluctuations.

Second, we estimate the augmented model without the constant in the measurement equation for creativity growth. In that case, the observed series retains a persistent negative mean over the sample, in line with the downward tendency visible in the annual data. As expected, this translates into a more persistent and rising difficulty shock. The main results, including posterior estimates, historical decompositions, and variance decompositions, are only mildly affected.

Third, we estimate two alternative specifications in which innovation probability is

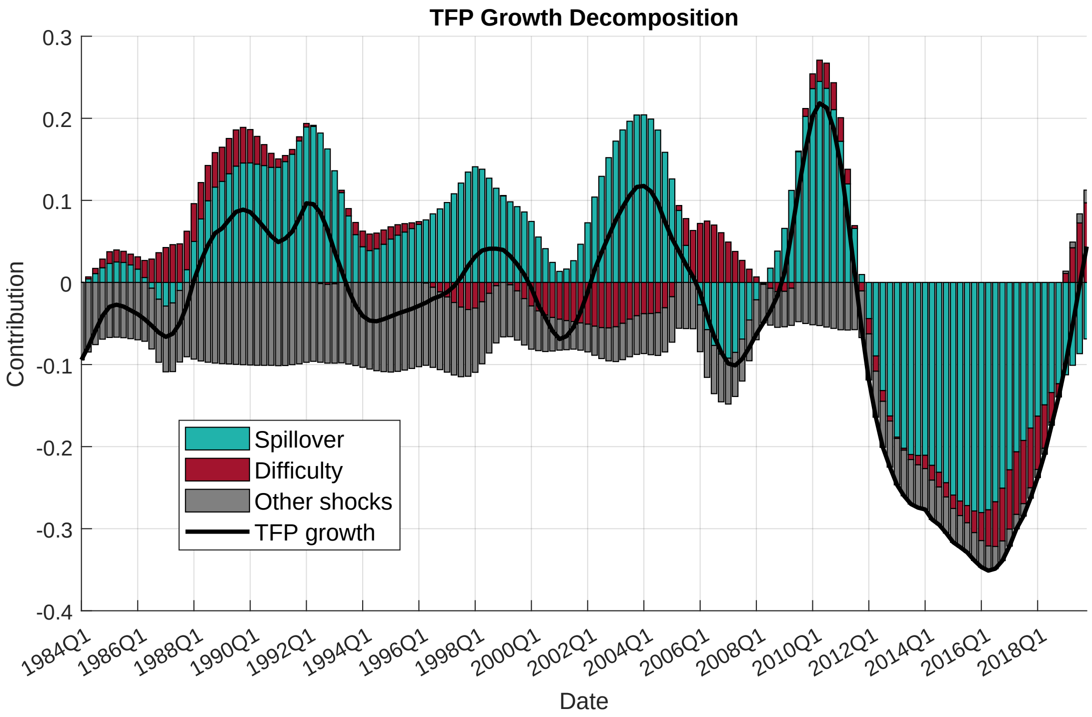

Figure 7: Historical decomposition of TFP year-on-year growth under the augmented specification.

measured using entry rates and exit rates rather than creativity. As in the creativity specification, the observables are introduced in annual changes because the available data are annual. The results remain close to those obtained with the creativity measure. These exercises therefore confirm both that the difficulty shock is empirically relevant and that the main results do not depend on one specific proxy for the innovation-success margin.

6 Conclusions

This paper asks whether the slowdown in innovation is the main force behind the slowdown in productivity. The short answer is no, at least not in the sense of a decline in the probability of successful innovation. We develop and estimate a DSGE model with endogenous R&D and frontier growth in which productivity depends on two distinct innovation mechanisms. The first is a spillover channel, which governs how effectively aggregate R&D advances the technological frontier. The second is a difficulty channel, which governs the probability that sectoral R&D efforts successfully generate innovation and creative destruction.

The quantitative results show that these two channels play different roles. Spillover shocks are the main source of TFP-growth fluctuations because they affect frontier growth directly. Difficulty shocks, by contrast, mainly shape the probability of successful innovation. They influence productivity through slower adoption of frontier technologies and weaker creative destruction, but they are not the dominant driver of short run TFP dy-

Table 3: Posterior Parameter Estimates of Alternative Specifications

	Augmented			Entry rate			Exit rate		
Parameter	Mean	90% HPD		Mean	90% HPD		Mean	90% HPD	
ρ_g	0.948	0.917	0.980	0.948	0.917	0.980	0.948	0.917	0.980
ρ_b	0.382	0.297	0.468	0.377	0.292	0.463	0.378	0.292	0.462
ρ_s	0.944	0.932	0.956	0.944	0.932	0.956	0.943	0.930	0.956
ρ_i	0.036	0.010	0.062	0.036	0.010	0.063	0.036	0.010	0.062
$\rho_{s,RD}$	0.403	0.122	0.667	0.418	0.138	0.690	0.403	0.132	0.675
$\rho_{n,RD}$	0.989	0.980	0.998	0.818	0.700	0.944	0.642	0.365	0.895
ρ_p	0.953	0.926	0.981	0.947	0.918	0.976	0.947	0.917	0.976
ρ_l	0.971	0.951	0.991	0.971	0.953	0.991	0.971	0.952	0.991
ρ_{RD}	0.518	0.382	0.654	0.534	0.401	0.664	0.517	0.368	0.658
n_{ss}	0.112	0.092	0.137	0.111	0.090	0.136	0.107	0.084	0.133
b	0.639	0.579	0.698	0.640	0.581	0.699	0.640	0.582	0.700
ϕ_l	1.318	0.994	1.645	1.305	0.970	1.632	1.301	0.977	1.626
γ_I	153.7	110.4	194.6	153.8	111.5	196.2	151.2	107.9	193.0
γ_P	21.18	15.54	26.72	21.68	15.73	27.39	21.98	15.77	27.71
γ_{u2}	0.145	0.130	0.159	0.145	0.130	0.159	0.144	0.129	0.159
γ^{RD}	704.7	683.0	721.9	703.7	680.9	721.9	703.8	681.1	721.9
ρ^R	0.337	0.217	0.459	0.342	0.222	0.460	0.349	0.229	0.466
ϕ_π	4.331	4.105	4.545	4.331	4.109	4.545	4.324	4.096	4.545
ϕ_y	0.058	0.027	0.090	0.057	0.026	0.089	0.058	0.027	0.091
ϕ	0.776	0.526	1.000	0.772	0.529	1.000	0.778	0.537	1.000
constn	—	—	—	-1.239	-1.506	-0.965	-0.905	-1.132	-0.671
Standard deviation of shocks									
η_g	0.026	0.024	0.029	0.026	0.024	0.029	0.026	0.024	0.029
η_b	0.025	0.025	0.026	0.025	0.025	0.026	0.025	0.025	0.026
η_s	0.026	0.025	0.028	0.026	0.025	0.029	0.027	0.025	0.029
η_i	0.562	0.503	0.618	0.562	0.504	0.619	0.559	0.503	0.615
η_r	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013
$\eta_{s,RD}$	0.787	0.333	1.237	0.818	0.333	1.287	0.770	0.311	1.216
$\eta_{n,RD}$	0.231	0.181	0.281	0.341	0.239	0.437	0.561	0.322	0.830
η_p	0.053	0.046	0.060	0.054	0.046	0.061	0.054	0.046	0.061
η_l	0.021	0.018	0.024	0.021	0.018	0.024	0.021	0.018	0.024
η_{RD}	0.248	0.219	0.276	0.248	0.220	0.276	0.242	0.211	0.275

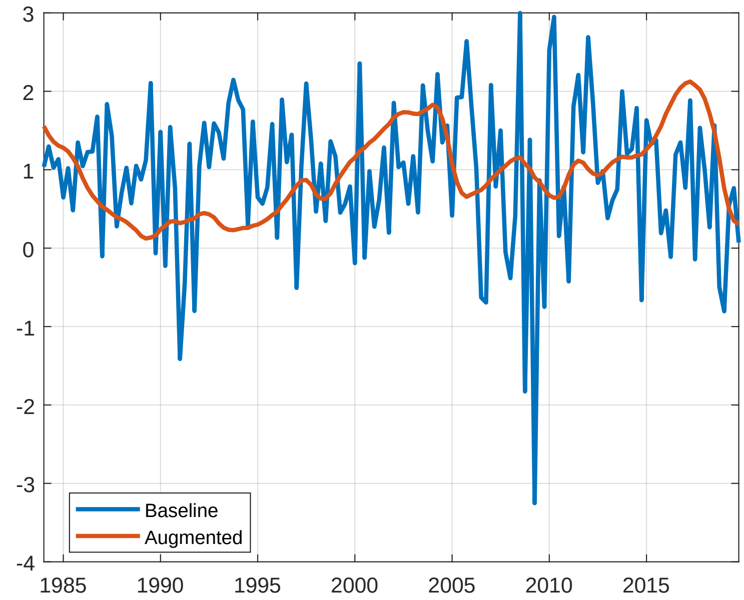

Figure 8: Estimated difficulty.

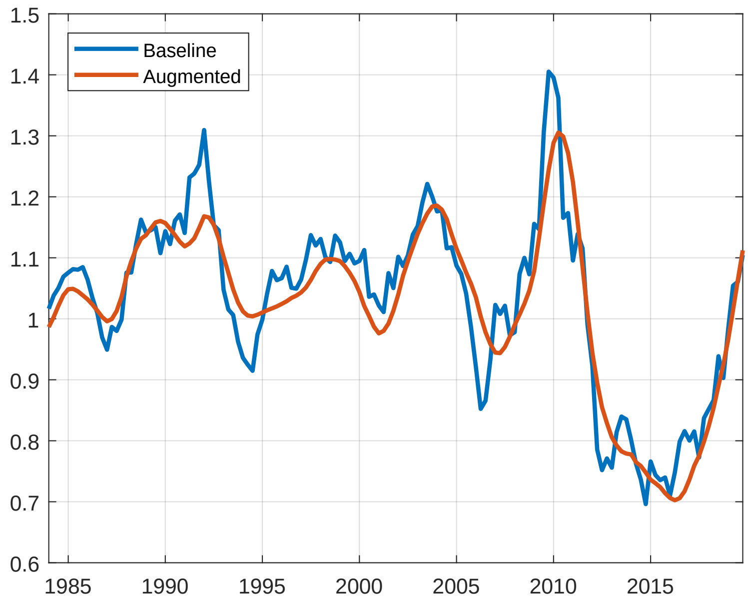

Figure 9: Estimated TFP growth rate (percent, year-on-year).

namics. In this sense, the model does not support the view that the productivity slowdown is primarily driven by a decline in successful innovation.

The historical decompositions reinforce this interpretation. Movements in innovation probability are accounted for mainly by the difficulty shock, while movements in TFP growth are driven predominantly by spillover shocks. Difficulty shocks do matter in some episodes, especially when successful innovation deteriorates persistently, but their contribution to TFP growth is secondary relative to the spillover channel. Weak productivity growth therefore reflects mainly a weakening of the mechanism through which aggregate R&D translates into frontier advancement, rather than a collapse in the success probability of innovation itself.

Adding the patent-text-based creativity measure sharpens this conclusion. The augmented specification does not overturn the distinction between the two channels. Instead, it disciplines the innovation success channel and changes the estimated properties of the difficulty shock. Once creativity is included, innovation difficulty becomes less volatile and more persistent. This suggests that difficulty is better interpreted as a slow moving force affecting successful innovation and creative destruction, rather than as a high-frequency driver of productivity fluctuations.

These findings have two implications. First, an innovation slowdown and a productivity slowdown should not be treated as the same phenomenon. A decline in successful innovation may coexist with weak productivity growth, but the latter need not be primarily caused by the former. Second, policies aimed only at increasing R&D effort or supporting innovation attempts may be insufficient if they do not also strengthen the spillover and diffusion mechanisms through which research advances the frontier and raises aggregate productivity. At the same time, policies that reduce barriers to successful innovation remain relevant, because innovation difficulty affects the pace of creative destruction and the medium-run transmission of frontier technologies to average productivity.

Overall, the paper shows that distinguishing between spillovers and innovation difficulty is crucial for interpreting the productivity slowdown. The main lesson is not that innovation is irrelevant for productivity. Rather, it is that the margin of innovation that matters most for measured TFP dynamics is the frontier growth spillover, while the innovation success channel operates as a distinct, persistent, but quantitatively secondary channel.

Declaration on the Use of Artificial Intelligence

The authors used artificial intelligence tools to support language editing, improve clarity, and refine the presentation of the manuscript. All theoretical arguments, model development, empirical analysis, results, and conclusions are the sole responsibility of the authors. The authors reviewed and edited all AI-assisted text and take full responsibility for the content of the paper.

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Appendix

A System of equations

Marginal utility of consumption:

$$\left(c_t - b\frac{c_{t-1}}{g_h g_{A_t^{max}}}\right)^{-1} = \lambda_t \tag{A1}$$

Labor supply:

$$w_t = \frac{\varepsilon_t^l \left(\tilde{h}_t\right)^{\phi_l}}{\lambda_t} \tag{A2}$$

Euler equation for bonds:

$$R_t = E_t \left[\pi_{t+1} \frac{\lambda_t g_h g_{A_{t+1}^{max}}}{\beta \varepsilon_t^b \lambda_{t+1}}\right] \tag{A3}$$

Euler equation for shares:

$$\beta \varepsilon_t^s E_t \frac{\lambda_{t+1} R_{t+1}^s}{g_h g_{A_{t+1}^{max}} \lambda_t \pi_{t+1}} = 1 \tag{A4}$$

Optimality for investments:

$$\begin{aligned} 1 \quad = \quad & Q_t \varepsilon_t^i \left\{1 - \gamma_I \left(g_h g_{A_t^{max}} \frac{i_t}{i_{t-1}} - g_Y\right) \frac{i_t}{i_{t-1}} - \frac{\gamma_I}{2} \left(\frac{g_h g_{A_t^{max}} i_t}{i_{t-1}} - g_Y\right)^2\right\} \\ & + \frac{\lambda_{t+1}}{g_h g_{A_{t+1}^{max}} \lambda_t} \varepsilon_t^s Q_{t+1} \varepsilon_{t+1}^i \beta \gamma_I \left(g_h g_{A_{t+1}^{max}} \frac{i_{t+1}}{i_t} - g_{A^{max}}\right) \left(g_h g_{A_{t+1}^{max}} \frac{i_{t+1}}{i_t}\right)^2 \end{aligned} \tag{A5}$$

Optimality for physical capital:

$$Q_t = E_t \frac{\lambda_{t+1}}{g_h g_{A_{t+1}^{max}} \lambda_t} \varepsilon_t^s \beta \left[r_{t+1}^k u_{t+1} - a\left(u_{t+1}\right) + Q_{t+1}\left(1 - \delta\right)\right] \tag{A6}$$

Optimality for capital utilization:

$$r_t^k = \gamma_{u1} + \gamma_{u2}\left(u_t - 1\right) \tag{A7}$$

Capital accumulation:

$$k_{t+1} = \left(1 - \delta\right) \frac{k_t}{g_h g_{A_t^{max}}} + \varepsilon_t^i \left[1 - \frac{\gamma_I}{2} \left(g_h g_{A_t^{max}} \frac{i_t}{i_{t-1}} - g_Y\right)^2\right] i_t \tag{A8}$$

Aggregate resource constraint:

$$\begin{aligned} y_t \quad = \quad & c_t + i_t + x_t^{RD} + g_t + a\left(u_t\right) \frac{k_t}{g_h g_{A_t^{max}}} + \frac{\gamma_P}{2} \left(\pi_t - \exp\left(\bar{\pi}\right)\right)^2 y_t \\ & + \frac{\gamma^{RD}}{2 y_t} \left(x_t^{RD} - \frac{x_{t-1}^{RD}}{g_h g_{A_t^{max}}} g_Y\right)^2 \end{aligned} \tag{A9}$$

Production function:

$$y_t + \Phi = \varepsilon_t^a \left(\frac{u_t k_t}{g_h g_{A_t^{max}}} \right)^\alpha \left(a_t \tilde{h}_t \right)^{1-\alpha} \tag{A10}$$

Capital-labor ratio:

$$\frac{u_t k_t}{g_h g_{A_t^{max}} \tilde{h}_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} \tag{A11}$$

Real marginal cost:

$$mc_t = \left(\varepsilon_t^a\right)^{-1} \frac{\alpha^{-\alpha}}{(1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \left(r_t^k\right)^\alpha a_t^{\alpha-1} \tag{A12}$$

Price setting equation:

$$0 = \left[1 - \sigma_y \left(1 - mc_t\right)\right] - \gamma_P \left(\pi_t - \exp\left(\bar{\pi}\right)\right) \pi_t + E_t \beta \varepsilon_t^s \frac{\lambda_{t+1}}{\lambda_t} \gamma_P \left(\pi_{t+1} - \exp\left(\bar{\pi}\right)\right) \pi_{t+1} \frac{y_{t+1}}{y_t} \tag{A13}$$

Frontier productivity dynamics:

$$g_{A_t^{max}} - 1 = \left(\frac{x_{t-1}^{RD}}{y_{t-1}} \right)^{\lambda_A} z_{t-1} \sigma_t^{RD} \tag{A14}$$

with

$$\frac{z_t}{z_{t-1}} = \left(g_{A_t^{max}}\right)^{\phi-1} \left(\frac{g_h \tilde{h}_t}{\tilde{h}_{t-1}} \right)^{\lambda_A}$$

Average productivity:

$$g_{A_t^{max}} a_t = n_{t-1} \left(1 - a_{t-1}\right) + a_{t-1} \tag{A15}$$

Optimality R&D investments:

$$\frac{n_t}{\eta+1} p_t^{S,max} = x_t^{RD} \left[1 + \frac{\gamma^{RD}}{y_t} \left(x_t^{RD} - \frac{x_{t-1}^{RD}}{g_h g_{A_t^{max}}} g_Y \right) \right] \varepsilon_t^{RD} \tag{A16}$$

Probability of innovation:

$$n_t = \left(\frac{x_t^{RD}}{\lambda^{RD} \varepsilon_t^{n,RD}} \right)^{\frac{1}{\eta+1}} \tag{A17}$$

Frontier stock market value:

$$p_t^{S,max} = \mathbb{E}_t \left[\beta \varepsilon_t^s \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{d_{t+1}}{a_{t+1}} + \frac{p_{t+1}^{S,max}}{g_{A_{t+1}^{max}}} \left(1 - n_{t+1}\right) \right) \right] \tag{A18}$$

Aggregate stock market portfolio:

$$\frac{p_{t-1}^S R_t^s}{g_h g_{A_t^{max}} \pi_t} = \left(d_t + p_t^S - n_{t-1} p_t^{S,max} \right) \tag{A19}$$

Intermediate firms' profits:

$$d_t = y_t - w_t h_t - r_t^k u_t \frac{k_t}{g_h g_{A_t^{max}}} - \frac{\gamma_P}{2} \left(\pi_t - \exp\left(\bar{\pi}\right)\right)^2 y_t \tag{A20}$$

Taylor rule (write it in terms of adjusted output):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y}\right)^{\phi_y}\right]^{1-\rho^R} \varepsilon_t^r \tag{A21}$$

B Definition of observable variables

The variables are defined following Smets and Wouters (2007). In particular:

- $gy\_obs = 100 * \Delta LN(RGDP/LNSindex)$
- $gc\_obs = 100 * \Delta LN((CONS/GDPDEF)/LNSindex)$
- $gi\_obs = 100 * \Delta LN(((INV - XRD)/GDPDEF)/LNSindex)$
- $gx\_obs = 100 * \Delta LN((XRD/GDPDEF)/LNSindex)$
- $gw\_obs = 100 * \Delta LN(WAGE/GDPDEF)$
- $h\_obs = 100 * (LN(HOURS * EMPL/100)/LNSindex) - mean$
- $pi\_obs = 100 * \Delta LN(GDPDEF)$
- $R\_obs = INT\_RATE/4$
- $n\_obs = 100 * \Delta LN(creativity)$

C Source of data

- RGDP: Real Gross Domestic Product (GDPC1), Billions Chained 2017 Dollars, Quarterly, Seasonally Adjusted Annual Rate.

  Source: U.S. Bureau of Economic Analysis via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDPC1.

- LNS: Population Level (CNP16OV), Thousands of Persons, Quarterly, Not Seasonally Adjusted.

- LNSindex: Index of LNS, 2017=1.

  Source: U.S. Bureau of Labor Statistics via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CNP16OV.

- GDPDEF: Gross Domestic Product: Implicit Price Deflator (GDPDEF), Quarterly, Index 2017=100, Seasonally Adjusted.

  Source: U.S. Bureau of Economic Analysis via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDPDEF.

- CONS: Personal Consumption Expenditures (PCEC), Billions Dollars, Quarterly, Seasonally Adjusted Annual Rate.

  Source: U.S. Bureau of Economic Analysis via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEC.

- INV: Fixed Private Investment (FPI), Billions Dollars, Quarterly, Seasonally Adjusted Annual Rate.

  Source: U.S. Bureau of Economic Analysis via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/FPI.

- XRD: Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (Y006RC1Q027SBEA), Billions Dollars, Quarterly, Seasonally Adjusted Annual Rate.

  Source: U.S. Bureau of Economic Analysis via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/Y006RC1Q027SBEA.

- WAGE: Nonfarm Business Sector: Hourly Compensation for All Workers (COMPNFB), Quarterly, Index 2017=100, Seasonally Adjusted .

  Source: U.S. Bureau of Labor Statistics via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/COMPNFB.

- HOURS: Nonfarm Business Sector: Average Weekly Hours for All Workers (PRS85006023), Index 2017=100, Quarterly, Seasonally Adjusted.

  Source: U.S. Bureau of Labor Statistics via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PRS85006023.

- EMPL: Employment Level (CE16OV), Thousands of Persons, Quarterly, Seasonally Adjusted.

  Source: U.S. Bureau of Labor Statistics via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CE16OV.

- INT_RATE:

  - 1984Q1-2008Q3, Federal Funds Effective Rate (FEDFUNDS), Percent, Quarterly. Source: Board of Governors of the Federal Reserve System, via FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/FEDFUNDS.
  - 2008Q4-2019Q4, Shadow rate by Krippner (2013).
    Source: https://www.ljkmfa.com/visitors/

- creativity: Patent creativity mean by Kalyani (2024), Annual.

  Source: https://www.aakashkalyani.com/

D Additional Impulse Responses

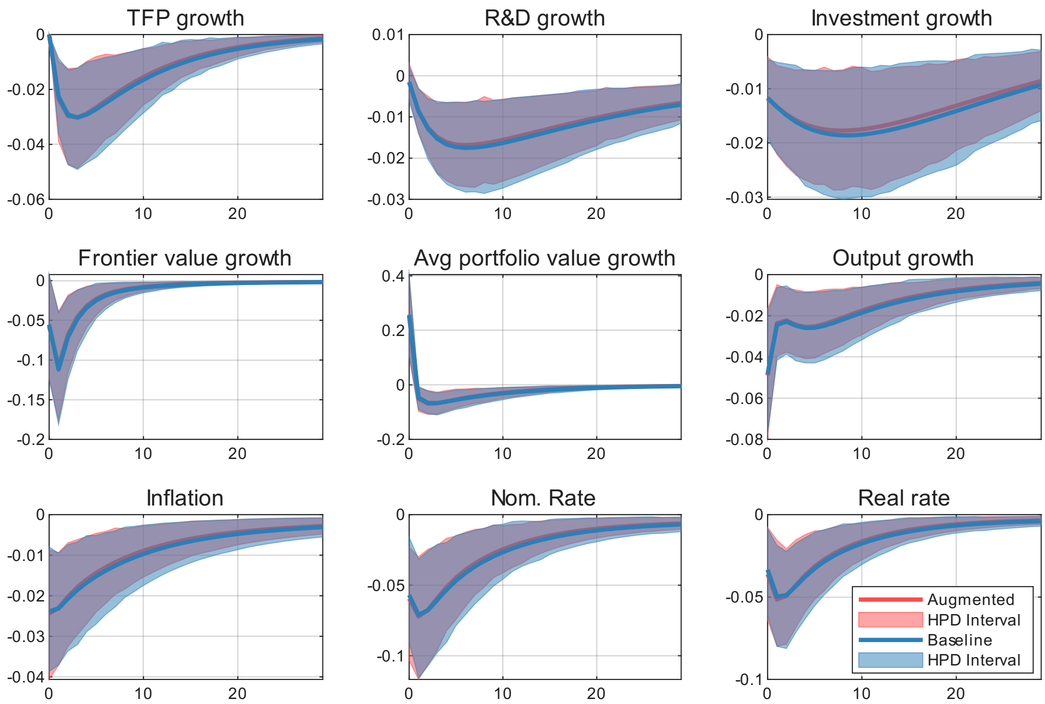

Figure A1: Impulse responses to an adverse spillover shock

Notes. The solid lines report posterior mean responses and the shaded areas denote 90% highest posterior density intervals.

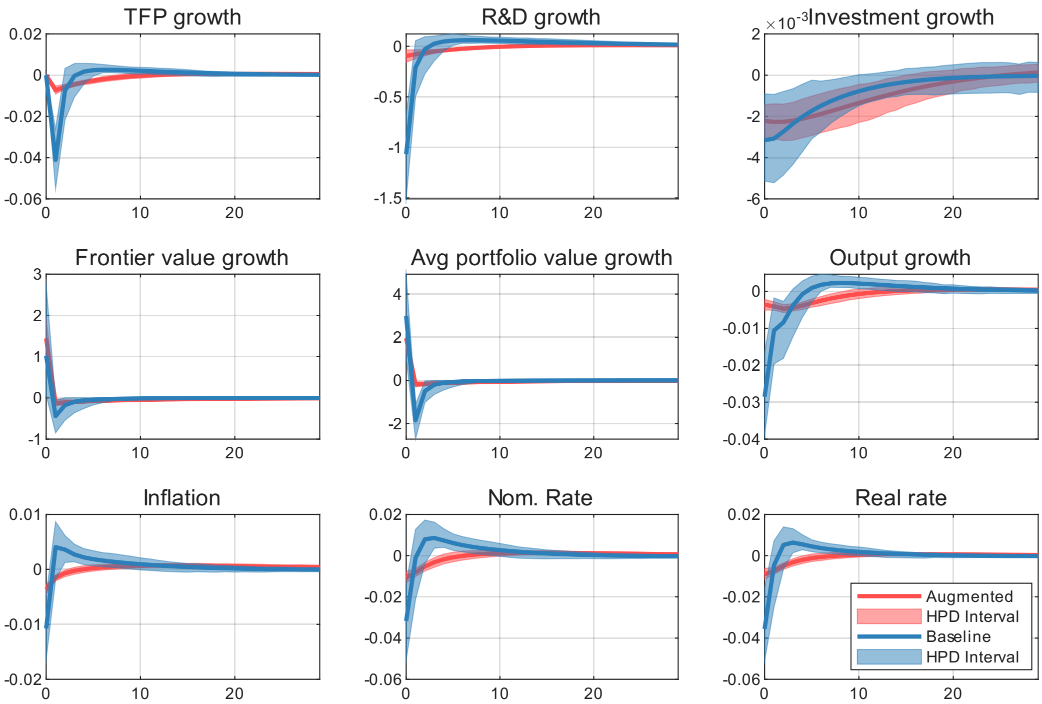

Figure A2: Impulse responses to an adverse difficulty shock

Notes. The solid lines report posterior mean responses and the shaded areas denote 90% highest posterior density intervals.