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The Leninist Vanguard's Dilemma: Information and Coordination in Revolutions

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Abstract

We model the role of a Leninist vanguard as a sequential coordination game in which the vanguard and a follower must decide whether to revolt to secure an uncertain prize. The true value of the prize is unknown, but each agent privately observes a binary signal about its worth. The vanguard must act first and publicly, absorbing the strategic risk of punishment if isolated. The follower then decides whether to join, weighing the vanguard's costly action against her own private information. We characterize the generically unique Perfect Bayesian Equilibrium surviving Forward Induction. The parameter space structurally partitions into four regions based on the relative precision of the agents' signals and on their joint precision. We show that through the application of Forward Induction, these four regions collapse into three distinct equilibrium outcomes. When the vanguard is better informed about the prize, the equilibrium is first-best efficient. However, when the follower is better informed, coordination partially or entirely breaks down. If their joint precision is too low, we get to total inactivity. If joint precision is high enough, a separating equilibrium exists but remains strictly inefficient. Ultimately, improving the follower's information about the uncertain prize paradoxically widens the efficiency gap, while improving the vanguard's precision can restore the first-best outcome. These results show that the role of a Leninist vanguard is effective if and only if it is better informed than the followers.

Key Words: Vanguard, Revolution, Information, Coordination.

JEL Codes: C72, D74, D82.

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1 Introduction

1.1 The Role of Revolutionary Vanguards

Collective endeavours are faced with a simple but formidable obstacle: free riding. When the product of a collective activity, such as a regime or a policy change, cannot be withheld from noncontributors, individuals will attempt to avoid contributing to the activity: why incur costs when the benefit is provided to all, regardless of who participates? "Free-riders occur because individuals acting in a self-interested fashion see incentives to withhold their resources, hoping that the efforts of others will provide the good. By the collective nature of the good, the individual will receive the good even if they did not contribute to its supply."[1] Vanguards could provide a rational and parsimonious solution to this collective action problem: many theorists, and also many activists, stress the idea that leadership plays a significant role in overcoming both the initial and the continuing problems associated with the pursuit of collective action. The leadership of a collective endeavour may play a vital role in identifying, challenging, and testing the likelihood of a successful mass protest. This is usually unknown and its estimate is based upon broadly held expectations of the status quo's ability to maintain control.[2] In general, vanguards may attempt to alter the perceived value of a change in the status quo and as a result alter the citizens' propensities to join the protest, or they can alter the perceived estimate of the success threshold and as a result alter the perceived probabilities of success.[3]

However, this view is not generally shared, and the role of vanguards in regime change movements is hotly debated in political reflections of revolutionary intellectuals and activists. If indeed vanguards may help to overcome the collective action problem, on the other hand, they may deter some citizens from joining protest movements because of their extremism and may even endanger the effectiveness of protests by encouraging repression activities. To present these two contrasting points of view, we consider two extreme visions on the leaders' role, associated respectively with Leninism and Anarchism.

The role of revolutionary vanguards is at the core of Lenin's reflections and praxis: one of his most conspicuous contributions to twentieth-century politics is his conception of the Communist Party and its revolutionary vanguard as a creative history-making force and as the general staff of the world revolution. The key element of the Leninist notion of a vanguard is that a skilled vanguard must lead for a revolution to succeed. In What Is to be Done,[4] Lenin argued that there would be no spontaneous socialist revolution: it could be achieved only through the political leadership of revolutionary vanguards organized in a (communist) party. Fighting against the government forces, the vanguard might ally with the forces of spontaneity and might even yield to them temporarily.

[1] (Frolich and Oppenheimer, 1970, p. 105).
[2] See e.g. (Frolich and Oppenheimer, 1970; DeNardo, 1985; Elster, 1989).
[3] See (Kuran, 1989, 1991; Van Belle, 1996; Ginkel and Smith, 1999).
[4] (Lenin, 1902, CW 5).

Indeed revolution, according to Lenin, cannot be the work of conspirators alone, during a revolution initiative almost inevitably goes to the masses, at least for a while. This is exactly the meaning of the revolutionary situation according to Lenin: the masses have taken the initiative, or are about to take it, but it is the test of an able revolutionary vanguard to make use of these "elemental forces" and ride to power on the waves of spontaneity. Thus, Lenin, in seeking to bring about the revolution, relies on the spontaneous support of the masses, generated if necessary, by what Lenin calls "agitation", the activity of inciting the masses to action by playing on their instincts and passions.[5]

Lenin's vanguardism has been widely criticized, both at that time and since. For example, Rosa Luxemburg contended that Lenin's ideas "would restrict the revolutionary potential of the masses".[6] Similarly, Menshevik's theory was that the working class spontaneously develops towards revolutionary consciousness, so activists should rely on spontaneous militancy to reflect mass behaviour. Anarchists too were and are against any inequality in revolutionary groups: everyone should have an equal say in the group, and revolutions should not be led by a vanguard but by the citizens themselves. In pursuing regime changes, they advocated networked and horizontal forms of organization that reject the centralized and vertical hierarchies associated with revolutionary vanguards.[7] In Anarchism and Socialism, Lenin delineated what he thought were the revolutionary limitations of anarchism, identifying the circumstances in which anarchism can empower or instead weaken the working class, also establishing the organization, struggle, and leading role of the vanguard as a crucial distinction between Marxism and anarchism. Lenin emphasized that Marxism did not differ from anarchism in wanting to abolish the capitalist state. Their major disagreement concerned not the end goal, but the means of getting there, specifically how to promote a successful regime change from capitalism to socialism and then to communism. Indeed, under Lenin's leadership, the Third International codified the irreconcilable distinctions between Marxism and anarchism by emphasizing the organization, struggle, and leading role of the vanguard working class party.[8] In the last thirty years, following the collapse of the Soviet Union, the Leninist approach has been discredited in protest movements because of its authoritarian drifts, while anarchist spontaneity and leaderless movements gained momentum, especially in anti-globalization protest movements. Contemporary protest movements are often either explicitly or implicitly anarchist, and their identity is anchored around the opposition to hierarchical norms, promoting the adoption of horizontalism, decentralization, and mutual aid.

This quick review of these two opposite approaches towards revolts and regime changes shows that it is important to distinguish between the different roles that a revolutionary vanguard may play. In particular, the comparison between formal models of mass protests with revolutionary vanguards in different settings may help to understand the differences in these two different ways of

[5](Lane, 2021).
[6](Luxemburg, 1904, p. 102).
[7](Franks, 2013).
[8]See (Pateman, 2022).

approaching regime or policy changes.

However, whatever the role played by vanguards is, a basic question remains: when does a revolutionary vanguard succeed in mobilizing a broader movement? As seen, according to the Leninist point of view, successful revolutions require not only that conditions are ripe (the regime is weak, the prize of change is valuable) but that followers believe they are ripe enough to act alongside their leader. This belief, in turn, depends on the information each side possesses. A vanguard organization must publicly commit to a revolt before knowing whether followers will join. Followers observe the vanguard's costly action and update their beliefs, but also bring their own private intelligence to bear. When a follower is sufficiently well-informed to trust her own signal over the vanguard's costly demonstration, the vanguard's sacrifice may go unrewarded.

History is rich with illustrations of this tension. The Bolshevik seizure of power in October 1917 is perhaps the paradigmatic case: a compact, well-organized vanguard moved first, anticipating that a sufficient mass of soldiers, workers, and peasants would follow once the revolution's viability was demonstrated (Rabinowitch, 1976). Yet the same logic explains spectacular failures. The Spartacist uprising in Berlin in January 1919, led by Rosa Luxemburg and Karl Liebknecht, saw a small vanguard initiate a revolt against the provisional government in the belief that the broader German working class would join. That belief was wrong: followers, uncertain of the revolt's prospects and reading their own private signals about the regime's remaining strength, largely abstained, leaving the vanguard isolated and brutally suppressed (Broué, 2005). Conversely, the Cuban Revolution of 1959 stands as the paradigmatic success of the guerrilla vanguard: Fidel Castro's small armed *foco* in the Sierra Maestra successfully catalysed a nationwide mass movement against the Batista regime by demonstrating its underlying vulnerability (Guevara, 1961; Goodwin, 2001). Yet, attempting to replicate this exact model a decade later, Che Guevara's Bolivian campaign of 1966–1967 repeated the Spartacist pattern of failure with stark clarity: a small vanguard of guerrillas attempted to ignite a peasant revolution, but the Bolivian peasantry (whose private information about local conditions, loyalties, and the military's capabilities differed fundamentally from the vanguard's) declined to join (Anderson, 1997).

1.2 The Aim and the Results of the Paper

The common thread across previous cases is the information asymmetry between those who move first and those who wait: the vanguard's public sacrifice can only catalyse a broader movement if followers, weighing both what the vanguard's action reveals and their own private intelligence, find it rational to join. This paper studies this tension formally. We analyse a sequential coordination game between a vanguard and a follower, each observing a private binary signal about the value of a successful revolt. The vanguard acts first and publicly; the follower, having observed both the vanguard's action and her own signal, decides whether to join. Coordination succeeds only if both agents revolt; an isolated vanguard faces severe punishment regardless of the underlying state.

This setting means that we leave aside the collective action problem, to focus instead on the vanguard's leading role. The paper accordingly adopts the Leninist notion of the vanguard described above: a revolution cannot be the work of conspirators alone, but it is the vanguard that must lead if the masses' support is to be generated and turned into coordinated action.

Our analysis yields four main contributions.

First, we fully characterize the unique Perfect Bayesian Equilibrium surviving Forward Induction across the entire parameter space, establishing that no mixed or semi-separating equilibrium exists anywhere. The parameter space structurally partitions into four regions depending on the relative precision of the agents' signals and on their joint signals' precision. Using Forward Induction, these four regions collapse into three distinct equilibrium outcomes. A side result of this analysis is that the joint signals' precision is defined by the product of individual precisions, i.e. the signals' precision act as complements.

Second, we identify a dual layer of ex post inefficiency when the follower is better informed, operating through two *distinct* events. On one hand, coordination fails when conditions are ripe but the vanguard's private intelligence is discouraging: the planner, using the follower's superior signal, would coordinate both agents, whereas in equilibrium the vanguard's low signal leads her to abstain. On the other hand, the vanguard acts and is left isolated when she receives an encouraging signal but the follower does not: the vanguard absorbs a punishment that the planner avoids entirely by prescribing abstention.

Third, we show that when the follower's precision exceeds the vanguard's but their joint precision remains below a sharp threshold, the game ends up in inactivity and welfare drops to zero. The collapse occurs because the vanguard's payoff from acting is constant in her own mixing probability and everywhere strictly negative, so no interior indifference condition can be satisfied and no semi-separating equilibrium can arise.

Fourth, our comparative statics relative to the equilibrium welfare reveal a fundamental asymmetry: improving the vanguard's precision always narrows the efficiency gap and can restore first-best outcomes. Improving the follower's precision, however, creates a non-monotone relationship with welfare: it is destructive in a "valley of inactivity" and only partially recovers at sufficiently high joint precision, remaining strictly below the first-best throughout the region.

Crucially, these results hinge on distinguishing the conditional success of a revolution from the unconditional welfare of the movement. When the vanguard takes the initiative, she absorbs the strategic risk of moving first. If the follower requires too much independent certainty before joining, welfare losses accumulate on two fronts simultaneously: coordination fails when the follower's signal is unfavourable, and the vanguard suffers the punishment of isolation on that same event. A social planner who can direct both agents avoids both losses. It is this gap between decentralized and centralized action, not merely the probability of coordination, that drives our welfare results.

1.3 Related Literature

Our paper contributes to the formal political economy literature on sequential coordination, collective action, and information aggregation in revolutions. The notion that early movers or "vanguards" can catalyse broader rebellions by absorbing strategic risk and signalling vulnerability is central to models of informational cascades and protest dynamics (e.g., Kuran, 1991; Lohmann, 1993, 1994). A specific role for revolutionary entrepreneurs has been proposed by Bueno de Mesquita (2010), who aims to show how an uninformed vanguard through an unobservable costly effort may try to favour citizens' protests; however, the correct interpretation of this model is indeed the non-relevance of vanguards under these assumptions, as argued in Bosco et al. (2026). The analysis of protest movements with and without revolutionary vanguards is the aim of two companion papers, Shadmehr and Bernhardt (2011) and Shadmehr and Bernhardt (2019). Our paper most directly intersects with the latter, Shadmehr and Bernhardt (2019), on vanguards in revolutions. Shadmehr and Bernhardt model a sequential game where a vanguard initiates a costly revolt, and a follower with private information decides whether to join, elegantly highlighting how a first mover absorbs the strategic risk of isolated action to catalyse joint coordination. However, their analysis primarily explores the consequences of payoff heterogeneity: when a vanguard is "radicalized", deriving a strictly higher payoff from success than the follower, the vanguard acts more frequently but is followed less, as the follower discounts the informational value of the vanguard's action. Our paper abstracts away from payoff asymmetries entirely, isolating instead a fundamentally different friction: heterogeneous informational precision. By adopting a tractable binary signal structure, we establish four results that are distinct from those obtainable from payoff heterogeneity alone. We prove global equilibrium outcome uniqueness across the entire parameter space; we identify a dual-channel ex post inefficiency that is absent from payoff-heterogeneity models; we document a discontinuous expected welfare in an intermediate precision region; and we characterize a fundamental asymmetry in comparative statics with no counterpart in the radicalism literature.

A further related strand studies sequential social learning and herd behaviour. In the canonical models of Banerjee (1992) and Bikhchandani et al. (1992), agents act in turn and observe the actions of predecessors; under certain conditions, private information ceases to affect choices and a herd forms on a potentially incorrect action. Our setting shares the sequential structure but differs fundamentally in scope and focus. In herding models the chain of agents is long and the question is whether private information eventually gets aggregated; in our model there are exactly two agents and the question is whether the first mover's observable sacrifice is sufficient to trigger the second mover's participation. More importantly, the vanguard's act carries a direct coordination externality (it is a costly action with payoff consequences for both parties, not merely a signal about an exogenous state) that the canonical cascade literature abstracts away from entirely. Moreover, the specific result that better information for the follower can *destroy* coordination has no analog in the herding

literature, where more informative private signals tend to improve early-stage decision accuracy and reduce the probability of incorrect cascades.

Finally, our framework naturally dialogues with the literature on global games of regime change (Morris and Shin, 1998, 2003; Edmond, 2013). While global games resolve equilibrium multiplicity in mass uprisings by assuming simultaneous action by a continuum of citizens observing noisy private signals, our approach explores a fundamentally different mechanism for coordination. By focusing on a strict sequential structure, we highlight how the visible, costly commitment of a first-mover (the vanguard) interacts with the private information of followers. This allows us to explicitly model the strategic risk of vanguard isolation and the resulting cliff-and-valley welfare dynamics, frictions that simultaneous-move global games abstract away from.

1.4 Structure

The paper is organized as follows: Section 2 presents the model and the interpretation of the exogenous variables. Section 3 derives the agents' equilibrium behaviour and characterizes the Perfect Bayesian Equilibria for the different values of the exogenous variables. Section 4 presents the welfare analysis, comparing equilibrium outcomes with the first-best social planner solution. Section 5 discusses the comparative statics as the precision of the signals changes, and Section 6 concludes.

2 The Model

Consider a sequential coordination game between two agents: a vanguard (indexed by $i = 1$) and a follower (indexed by $i = 2$). The game features an uncertain prize $V \in \{0, 1\}$ representing the net benefit of a successful revolution, and costly individual actions. Players share an uninformative prior $\Pr(V = 1) = \Pr(V = 0) = \frac{1}{2}$.

2.1 Signals, Actions, Conflict Technology and Payoffs

Each agent $i \in \{1, 2\}$ privately observes a binary signal $X_i \in \{H, L\}$ about the true value of V. Signals are conditionally independent given V and have accuracy $p_i > \frac{1}{2}$. Formally,

$$\Pr(X_i = H \mid V = 1) = \Pr(X_i = L \mid V = 0) = p_i \in \left(\tfrac{1}{2}, 1\right).$$

Each agent chooses an action $a_i \in \{0, 1\}$, interpreted as $a_i = 1$ (revolt) or $a_i = 0$ (do not revolt). Let $n = a_1 + a_2$ denote the number of agents who revolt. Payoffs are determined as follows:

- If both agents $i, j \in \{1, 2\}$ revolt ($n = 2$), the revolution succeeds, which we denote by $Z = 1$, and each receives $V - c$.

- If exactly one agent revolts ($n = 1$), the revolt fails, which we denote by $Z = 0$, and the acting agent receives $-c$ regardless of V. The non-acting agent receives 0.
- If neither agent revolts ($n = 0$), both receive 0.

Thus the conflict technology is given by the following conditional probability

$$\Pr(Z = 1|a_i, a_j) = \begin{cases} 1 & \text{if } a_i = 1, a_j = 1 \\ 0 & otherwise \end{cases}$$

while the payoffs are

$$u_i(a_i, a_j, V) = \begin{cases} V - c & \text{if } a_i = 1, a_j = 1 \\ -c & \text{if } a_i = 1, a_j = 0 \\ 0 & \text{if } a_i = 0, a_j \in \{0, 1\} \end{cases}$$

or as a payoff matrix

		2	
		$a_2 = 0$	$a_2 = 1$
1	$a_1 = 0$	$0, 0$	$0, -c$
	$a_1 = 1$	$-c, 0$	$V - c, V - c$

Social welfare is defined as the sum of realized payoffs:

$$W(n, V) = \begin{cases} 2(V - c) & \text{if } n = 2, \\ -c & \text{if } n = 1, \\ 0 & \text{if } n = 0. \end{cases} \tag{1}$$

To maintain symmetry in payoffs, the cost of acting is normalized to $c = \frac{1}{2}$. Then, when $V = 1$ a successful coordination yields total welfare $+1$, an isolated revolt yields $-\frac{1}{2}$, and mutual abstention yields 0.

2.2 Timing

The sequence of events unfolds as follows:

1. The value of the prize V and the private signals (X_1, X_2) are drawn according to the common prior and the signal structure.
2. Both agents simultaneously observe their own signal X_i.
3. The vanguard (Agent 1) chooses an action $a_1 \in \{0, 1\}$, which is publicly observed by the follower (Agent 2).
4. The follower, who has already observed their own signal X_2, updates their belief about V in light of the vanguard's observed action a_1, and then chooses $a_2 \in \{0, 1\}$.

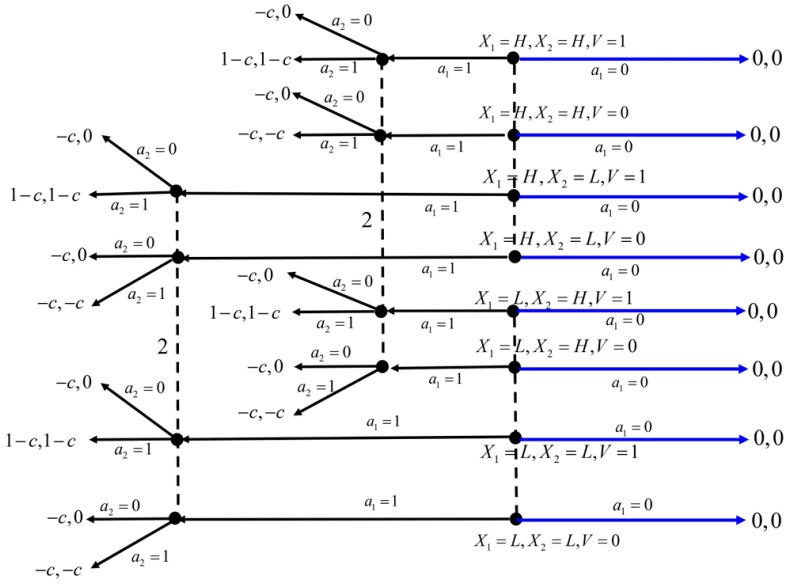

Figure 1: The essential tree of the game.

5. Payoffs are realized.

The following figure represents the basic tree of our game.

Because of the timing, the players' strategies are the following:

- Player 1, **the vanguard,** strategies are the following maps

$$s_1 : \{H, L\} \to \{0, 1\} \text{ so that } S_1 = \{(0,0), (1,0), (0,1), (1,1)\}$$

where the first element of the vector denotes player 1 action when $X_1 = H$ and the second element the action when $X_1 = L$;

- Player 2, **the citizens,** strategies are the following maps

$$\begin{aligned} s_2 &: \{H, L\} \times \{0, 1\} \to \{0, 1\} \text{ so that} \\ S_2 &= \{(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,1,0,0), (1,0,0,0)\, (0,0,1,1), (0,1,0,1), (1,0,0,1), \\ &\quad (0,1,1,0), (1,1,0,0), (1,0,1,0), (0,1,1,1), (1,0,1,1), (1,1,0,1), (1,1,1,0), (1,1,1,1)\} \end{aligned}$$

where the first element of the vector denotes player 2 action when $X_2 = H, a_1 = 1$, the second element the action when $X_2 = L, a_1 = 1$, the third element the action when $X_2 = H, a_1 = 0$, the fourth element the action when $X_2 = L, a_1 = 0$.

The game defined by our model form now on will be called the *Vanguard Game.*

3 Equilibrium Analysis

3.1 The Equilibrium Concept

Denote by $\Delta(\cdot)$ the set of probabilities on the set $\cdot$. The equilibrium concept we use to analyse the model is Perfect Bayesian Equilibrium (PBE). The definition of PBE is standard, however it is reported here for completeness.

Definition 1 (PBE of Vanguard Game) *A Perfect Bayesian Equilibrium for the vanguard game is a couple of behavioural strategies and beliefs (an assessment)*

$$(\pi_1^*, \pi_2^*), (\mu_1^*, \mu_2^*)$$

such that

1. $$\pi_1^* : \{H, L\} \to \Delta(\{0,1\}) \quad \textit{and} \quad \pi_2^* : \{H, L\} \times \{0,1\} \to \Delta(\{0,1\})$$

   are sequentially rational, i.e. maximize the continuation payoff given beliefs (μ_1^*, μ_2^*);

2. $$\mu_1^* : \{H, L\} \to [0,1] \quad \textit{and} \quad \mu_2^* : \{H, L\} \times \{0,1\} \to [0,1]$$

   are the posterior probabilities that $V = 1$ derived from the equilibrium strategies (π_1^*, π_2^*) using Bayes rule whenever possible.

3.2 Preliminary Results

Given the finite and sequential structure, we solve by backward induction, starting from the citizens (Player 2).

Lemma 1 (The Follower's Sequentially Rational Choices) *The citizens sequentially rational strategy $\pi_2^{\mathrm{SR}}(X_2, a_1)$ satisfies:*

1. $\forall \mu_2(V = 1 \mid a_1 = 0, X_2 = L) \in [0,1], \quad \pi_2^{\mathrm{SR}}(L, 0) = 0$;
2. $\forall \mu_2(V = 1 \mid a_1 = 0, X_2 = H) \in [0,1], \quad \pi_2^{\mathrm{SR}}(H, 0) = 0$;
3. $\pi_2^{\mathrm{SR}}(L, 1) = 1 \iff \mu_2(V = 1 \mid a_1 = 1, X_2 = L) \geq c$;
4. $\pi_2^{\mathrm{SR}}(H, 1) = 1 \iff \mu_2(V = 1 \mid a_1 = 1, X_2 = H) \geq c$.

Proof. If $a_1 = 0$, the follower knows that acting alone yields $-c < 0$, so abstention is strictly optimal regardless of X_2, establishing cases 1 and 2. For cases 3 and 4, when $a_1 = 1$ the follower's expected utility from joining is

$$\mathbb{E}u_2(a_2 = 1 \mid a_1 = 1, X_2) = \mu_2(V = 1 \mid a_1 = 1, X_2) - c,$$

while the payoff from abstaining is 0. Hence the citizens act if and only if $\mu_2(V = 1 \mid a_1 = 1, X_2) \geq c$. ■

Corollary 1 (Vanguard's Inaction and Beliefs Irrelevance) *When* $a_1 = 0$, *citizens' beliefs are irrelevant .*

Hence, we can omit the derivation of citizens' beliefs when $a_1 = 0$, while when $a_1 = 1$ they depend on the vanguard's strategies, as the following lemma shows.

Lemma 2 (Citizens' Beliefs) *In the vanguard game, citizens' beliefs depend on vanguard's strategies. In particular, when* $a_1 = 1$*:*

1.

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) =$$

$$= \frac{p_2\left[\pi_1^*(a_1 = 1|X_1 = H)\, p_1 + \pi_1^*(a_1 = 1|X_1 = L)(1 - p_1)\right]}{D(V = 1|X_2 = H, a_1 = 1)}$$

where

$$D(V = 1|X_2 = H, a_1 = 1) =$$

$$\begin{aligned} = \quad & p_2\left[\pi_1^*(a_1 = 1|X_1 = H)\, p_1 + \pi_1^*(a_1 = 1|X_1 = L)(1 - p_1)\right] + \\ & + (1 - p_2)\left[\pi_1^*(a_1 = 1|X_1 = H)(1 - p_1) + \pi_1^*(a_1 = 1|X_1 = L)\, p_1\right]. \end{aligned}$$

2.

$$\mu_2^*(V = 1|X_2 = L, a_1 = 1) =$$

$$= \frac{(1 - p_2)\left[\pi_1^*(a_1 = 1|X_1 = H)\, p_1 + \pi_1^*(a_1 = 1|X_1 = L)(1 - p_1)\right]}{D(V = 1|X_2 = L, a_1 = 1)}$$

where

$$D(V = 1|X_2 = L, a_1 = 1) =$$

$$\begin{aligned} = \quad & (1 - p_2)\left[\pi_1^*(a_1 = 1|X_1 = H)\, p_1 + \pi_1^*(a_1 = 1|X_1 = L)(1 - p_1)\right] + \\ & + p_2\left[\pi_1^*(a_1 = 1|X_1 = H)(1 - p_1) + \pi_1^*(a_1 = 1|X_1 = L)\, p_1\right]. \end{aligned}$$

Proof. Using Bayes rule and the game's structure:

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) =$$

$$= \frac{\Pr(V = 1)\Pr(X_2 = H, a_1 = 1 \mid V = 1)}{\Pr(V = 1)\Pr(X_2 = H, a_1 = 1 \mid V = 1) + \Pr(V = 0)\Pr(X_2 = H, a_1 = 1 \mid V = 0)} =$$

$$= \frac{\Pr(X_2 = H \mid V = 1)\Pr(a_1 = 1 \mid V = 1)}{\Pr(X_2 = H \mid V = 1)\Pr(a_1 = 1 \mid V = 1) + \Pr(X_2 = H \mid V = 0)\Pr(a_1 = 1 \mid V = 0)} =$$

since $X_1|V$ and $X_2|V$ are conditionally independent, then

$$\Pr(X_2 = H, a_1 = 1 \mid V = 1) = \Pr(X_2 = H \mid V = 1)\Pr(a_1 = 1 \mid V = 1) =$$

$$= p_2 \left[\underbrace{\pi_1^* (a_1 = 1|X_1 = H) \Pr (X_1 = H \mid V = 1) + \pi_1^* (a_1 = 1|X_1 = L) \Pr (X_1 = L \mid V = 1)}_{\Pr(a_1=1|V=1)} \right] =$$

$$= p_2 \left[\underbrace{\pi_1^* (a_1 = 1|X_1 = H) p_1 + \pi_1^* (a_1 = 1|X_1 = L) (1 - p_1)}_{\Pr(a_1=1|V=1)} \right].$$

Similarly

$$\Pr (X_2 = H \mid V = 0) \Pr (a_1 = 1 \mid V = 0) =$$

$$= (1 - p_2) \left[\underbrace{\pi_1^* (a_1 = 1|X_1 = H) \Pr (X_1 = H \mid V = 0) + \pi_1^* (a_1 = 1|X_1 = L) \Pr (X_1 = L \mid V = 0)}_{\Pr(a_1=1|V=1)} \right] =$$

$$= (1 - p_2) \left[\underbrace{\pi_1^* (a_1 = 1|X_1 = H) (1 - p_1) + \pi_1^* (a_1 = 1|X_1 = L) p_1}_{\Pr(a_1=1|V=1)} \right].$$

Hence

$$\mu_2^* (V = 1|X_2 = H, a_1 = 1) =$$

$$= \frac{p_2 [\pi_1^* (a_1 = 1|X_1 = H) p_1 + \pi_1^* (a_1 = 1|X_1 = L) (1 - p_1)]}{D (V = 1|X_2 = H, a_1 = 1)}$$

where

$$D (V = 1|X_2 = H, a_1 = 1) =$$

$$\begin{aligned} = \quad & p_2 [\pi_1^* (a_1 = 1|X_1 = H) p_1 + \pi_1^* (a_1 = 1|X_1 = L) (1 - p_1)] + \\ & + (1 - p_2) [\pi_1^* (a_1 = 1|X_1 = H) (1 - p_1) + \pi_1^* (a_1 = 1|X_1 = L) p_1]. \end{aligned}$$

Moreover,

$$\mu_2^* (V = 1|X_2 = L, a_1 = 1) =$$

$$= \frac{\Pr (V = 1) \Pr (X_2 = L, a_1 = 1 \mid V = 1)}{\Pr (V = 1) \Pr (X_2 = L, a_1 = 1 \mid V = 1) + \Pr (V = 0) \Pr (X_2 = L, a_1 = 1 \mid V = 0)} =$$

$$= \frac{\Pr (X_2 = L \mid V = 1) \Pr (a_1 = 1 \mid V = 1)}{\Pr (X_2 = L \mid V = 1) \Pr (a_1 = 1 \mid V = 1) + \Pr (X_2 = L \mid V = 0) \Pr (a_1 = 1 \mid V = 0)} =$$

since $X_1|V$ and $X_2|V$ are conditionally independent, then

$$\Pr (X_2 = L, a_1 = 1 \mid V = 1) = \Pr (X_2 = L \mid V = 1) \Pr (a_1 = 1 \mid V = 1) =$$

$$= (1 - p_2) \left[\underbrace{\pi_1^* (a_1 = 1|X_1 = H) \Pr (X_1 = H \mid V = 1) + \pi_1^* (a_1 = 1|X_1 = L) \Pr (X_1 = L \mid V = 1)}_{\Pr(a_1=1|V=1)} \right] =$$

$$= (1-p_2)\left[\underbrace{\pi_1^*(a_1=1|X_1=H)\,p_1+\pi_1^*(a_1=1|X_1=L)\,(1-p_1)}_{\Pr(a_1=1|V=1)}\right].$$

Similarly

$$\Pr(X_2=L\mid V=0)\Pr(a_1=1\mid V=0)=$$

$$=p_2\left[\underbrace{\pi_1^*(a_1=1|X_1=H)\Pr(X_1=H\mid V=0)+\pi_1^*(a_1=1|X_1=L)\Pr(X_1=L\mid V=0)}_{\Pr(a_1=1|V=1)}\right]=$$

$$=p_2\left[\underbrace{\pi_1^*(a_1=1|X_1=H)\,(1-p_1)+\pi_1^*(a_1=1|X_1=L)\,p_1}_{\Pr(a_1=1|V=1)}\right].$$

Hence

$$\mu_2^*(V=1|X_2=L,a_1=1)=$$

$$=\frac{(1-p_2)\left[\pi_1^*(a_1=1|X_1=H)\,p_1+\pi_1^*(a_1=1|X_1=L)\,(1-p_1)\right]}{D(V=1|X_2=L,a_1=1)}$$

where

$$D(V=1|X_2=L,a_1=1)=$$

$$\begin{aligned}=\quad&(1-p_2)\left[\pi_1^*(a_1=1|X_1=H)\,p_1+\pi_1^*(a_1=1|X_1=L)\,(1-p_1)\right]+\\&+p_2\left[\pi_1^*(a_1=1|X_1=H)\,(1-p_1)+\pi_1^*(a_1=1|X_1=L)\,p_1\right].\end{aligned}$$

■

On the other hand, vanguard's beliefs are immediately derived from prior and signal probabilities using Bayes rule.

Lemma 3 (Vanguard's Beliefs) *In the vanguard game, vanguard's beliefs are uniquely defined by Bayes rule:*

$$\mu_1^*(V=1|X_1=H)=p_1\quad\textit{and}\quad\mu_1^*(V=1|X_1=L)=1-p_1.$$

Proof. Using Bayes rule:

$$\begin{aligned}\mu_1^*(V=1|X_1=H)&=\frac{\Pr(V=1)\Pr(X_1=H\mid V=1)}{\Pr(V=1)\Pr(X_1=H\mid V=1)+\Pr(V=0)\Pr(X_1=H\mid V=0)}=\\&=\frac{\frac{1}{2}p_1}{\frac{1}{2}p_1+\frac{1}{2}(1-p_1)}=p_1\end{aligned}$$

$$\begin{aligned}\mu_1^*(V=1|X_1=L)&=\frac{\Pr(V=1)\Pr(X_1=L\mid V=1)}{\Pr(V=1)\Pr(X_1=L\mid V=1)+\Pr(V=0)\Pr(X_1=L\mid V=0)}=\\&=\frac{\frac{1}{2}(1-p_1)}{\frac{1}{2}(1-p_1)+\frac{1}{2}p_1}=1-p_1.\end{aligned}$$

■

Now, we can derive the game's PBE.

3.3 Separating Equilibria

Proposition 1 (Separating PBE) *The model has a separating PBE with if and only if $p_1 \geq p_2$, or $p_1 < p_2$ and $p_1 p_2 \geq c = \frac{1}{2}$. In particular, in each region the separating PBE are the following*

1. Region 1 *($p_1 \geq p_2$):*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = x) = 1 \quad \forall x \in \{H, L\} \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\begin{aligned} \mu_2^*(V = 1 | X_2 = H, a_1 = 1) &= \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} \\ \mu_2^*(V = 1 | X_2 = L, a_1 = 1) &= \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2} \end{aligned}$$

with the following equilibrium outcome

$$Z^* = \begin{cases} 1 & \textit{with probability } p_1 \\ 0 & \textit{with probability } 1 - p_1; \end{cases}$$

2. Region 2 *($p_1 < p_2$, $p_1 p_2 \geq \frac{1}{2}$):*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0, \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\begin{aligned} \mu_2^*(V = 1 | X_2 = H, a_1 = 1) &= \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} \\ \mu_2^*(V = 1 | X_2 = L, a_1 = 1) &= \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2} \end{aligned}$$

with the following equilibrium outcome

$$Z^* = \begin{cases} 1 & \textit{with probability } p_1 p_2 \\ 0 & \textit{with probability } 1 - p_1 p_2. \end{cases}$$

Proof. Suppose

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

so that

$$\begin{aligned} \mu_2^*(V = 1 | X_2 = H, a_1 = 1) &= \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} \;\; and \\ \mu_2^*(V = 1 | X_2 = L, a_1 = 1) &= \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2}; \end{aligned}$$

Then

$$Eu_2(a_1 = 1, a_2 = 1 | X_2 = H) = \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} - \frac{1}{2} \geq Eu_2(a_1 = 1, a_2 = 0 | X_2 = H) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2p_1 p_2 \geq p_1 p_2 + 1 - p_1 - p_2 + p_1 p_2 \Leftrightarrow p_1 + p_2 \geq 1$$

which is always satisfied. Hence, $\pi_2^{\mathrm{SR}}(H, 1) = 1$ always holds. Moreover,

$$Eu_2(a_1 = 1, a_2 = 1 | X_2 = L) = \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2} - \frac{1}{2} \geq Eu_2(a_1 = 1, a_2 = 0 | X_2 = H) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2p_1 - 2p_1 p_2 \geq p_1 - p_1 p_2 + p_2 - p_1 p_2 \Leftrightarrow p_1 \geq p_2.$$

Thus $\pi_2^{\mathrm{SR}}(L, 1) = 1$ if and only if $p_1 \geq p_2$. Then, consider the two cases

1. $p_1 \geq p_2$ so that $\pi_2^{\mathrm{SR}}(H, 1) = 1$ and $\pi_2^{\mathrm{SR}}(L, 1) = 1$. Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}} | X_1 = H\right) = p_1 - \frac{1}{2} \geq Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}} | X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}} | X_1 = L\right) = 0 \geq Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}} | X_1 = L\right) = (1 - p_1) - \frac{1}{2} \end{cases} \Leftrightarrow p_1 \geq \frac{1}{2}$$

which always satisfied. Thus in this case

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0.$$

2. $p_1 \leq p_2$ so that $\pi_2^{\mathrm{SR}}(H, 1) = 1$ and $\pi_2^{\mathrm{SR}}(L, 1) = 0$. Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}} | X_1 = H\right) = p_1 p_2 - \frac{1}{2} \geq Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}} | X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}} | X_1 = L\right) = 0 \geq Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}} | X_1 = L\right) = (1 - p_1) p_2 - \frac{1}{2} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1 p_2 \geq \frac{1}{2} \\ p_1 p_2 \geq p_2 - \frac{1}{2}. \end{cases}$$

Thus in this case

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow p_1 p_2 \geq \frac{1}{2}$$

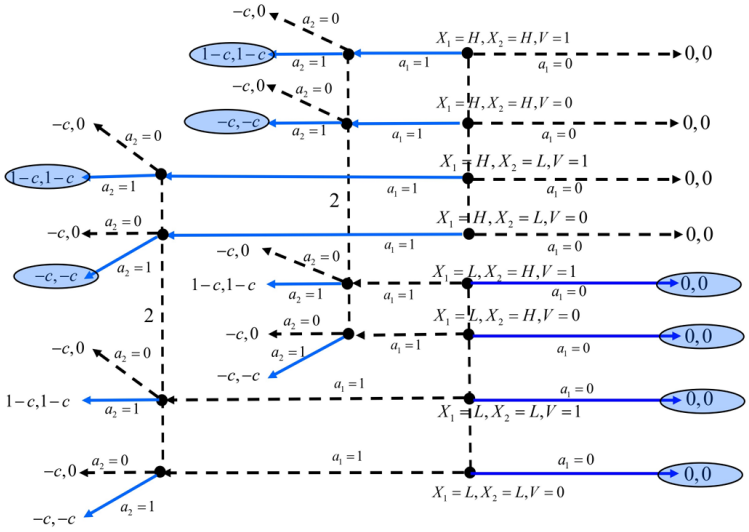

Figure 2: The path of the first separating PBE

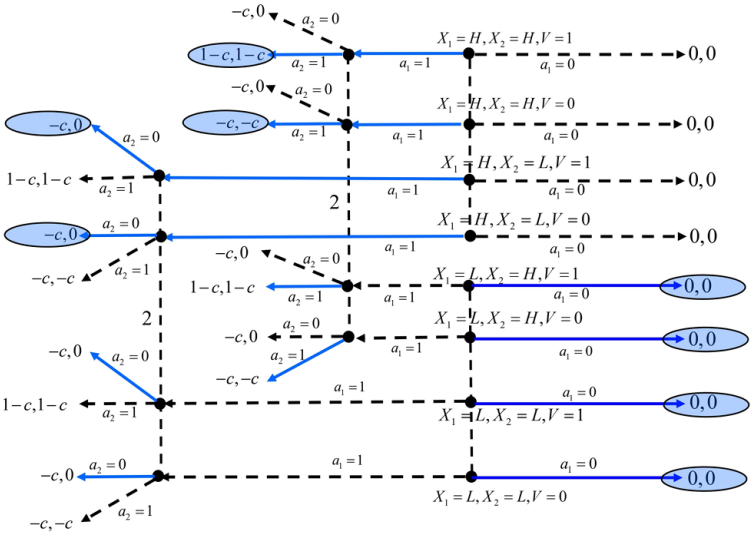

Figure 3: The path of the second separating PBE

■

The following figures represent the first and the second separating equilibrium in the game tree

The following proposition shows that there are no alternative separating PBE.

Proposition 2 (Non-Existence of Alternative Separating PBE) *The model has no separating PBE in which*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 1.$$

Proof. Suppose

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 1$$

so that

$$\begin{aligned} \mu_2^*(V = 1|X_2 = H, a_1 = 1) &= \frac{p_2(1-p_1)}{p_2(1-p_1)+(1-p_2)p_1} \;\; and \\ \mu_2^*(V = 1|X_2 = L, a_1 = 1) &= \frac{(1-p_2)(1-p_1)}{(1-p_2)(1-p_1)+p_2p_1}. \end{aligned}$$

Then

$$Eu_2(a_1 = 1, a_2 = 1|X_2 = H) = \frac{p_2(1-p_1)}{p_2(1-p_1)+(1-p_2)p_1} - \frac{1}{2} \geq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2p_2(1-p_1) \geq p_2(1-p_1)+(1-p_2)p_1 \Leftrightarrow p_2 - p_1p_2 \geq p_1 - p_1p_2 \Leftrightarrow p_2 \geq p_1.$$

Hence,

$$\pi_2^{\mathrm{SR}}(H, 1) = 1 \Leftrightarrow p_2 \geq p_1.$$

Moreover,

$$Eu_2(a_1 = 1, a_2 = 1|X_2 = L) = \frac{(1-p_2)(1-p_1)}{(1-p_2)(1-p_1)+p_2p_1} - \frac{1}{2} \geq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2(1-p_2)(1-p_1) \geq (1-p_2)(1-p_1)+p_2p_1 \Leftrightarrow (1-p_2)(1-p_1) \geq p_2p_1 \Leftrightarrow 1-p_1-p_2 \geq 0$$

which is impossible, thus $\pi_2^{\mathrm{SR}}(L, 1) = 0$. Then, consider the two cases

1. $p_1 \leq p_2$ so that $\pi_2^{\mathrm{SR}}(H, 1) = 1$ and $\pi_2^{\mathrm{SR}}(L, 1) = 0$. Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; and \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 1 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}}|X_1 = H\right) = 0 \geq Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}}|X_1 = H\right) = p_1p_2 - \frac{1}{2} \\ Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}}|X_1 = L\right) = (1-p_1)p_2 - \frac{1}{2} \geq Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}}|X_1 = L\right) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1p_2 \leq \frac{1}{2} \\ p_2 - p_1p_2 \geq \frac{1}{2} \end{cases}$$

which can't be jointly satisfied.

2. $p_1 \leq p_2$ so that $\pi_2^{\text{SR}}(H,1) = 0$ and $\pi_2^{\text{SR}}(L,1) = 0$. Then, given the vanguard sequential best response are

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0.$$

Thus there exist no separating PBE such that

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 1.$$

■

3.4 Pooling Equilibria

The following results investigate whether in the vanguard's game there exist pooling PBE.

Proposition 3 (Pooling PBE) *The model has three outcome equivalent pooling PBE:*

1. *for any* $p_1 \in \left(\frac{1}{2}, 1\right)$ *and* $p_2 \in \left(\frac{1}{2}, 1\right)$:

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$
$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = x) = 0 \quad \forall x \in \{H, L\} \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) \leq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1 | X_2 = L, a_1 = 1) \leq \frac{1}{2};$$

2. *for any* $p_1 \in \left(\frac{1}{2}, 1\right)$ *and* $p_2 \in \left(\frac{1}{2}, 1\right)$:

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$
$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = H) = 0, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 1, \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) \leq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1 | X_2 = L, a_1 = 1) \geq \frac{1}{2};$$

3. *when* $p_1 p_2 \leq \frac{1}{2}$:

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$
$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0, \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) \geq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1 | X_2 = L, a_1 = 1) \leq \frac{1}{2}$$

All pooling PBE have the same equilibrium outcome

$$Z^* = 0 \quad \textit{with probability } 1.$$

Proof. Suppose

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

so that $X_2 \in \{H, L\}, a_1 = 1$ are out of the equilibrium path, thus beliefs are not determined by Bayes rule:

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \in [0,1] =: \mu_{2H}^* \;\; \textit{and} \;\; \mu_2^*(V = 1|X_2 = L, a_1 = 1) \in [0,1] =: \mu_{2L}^*.$$

Then

$$Eu_2(a_1 = 1, a_2 = 1|X_2 = H) = \mu_{2H}^* - \frac{1}{2} \leq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \Leftrightarrow \mu_{2H}^* \leq \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \pi_2^{\mathrm{SR}}(H,1) = \begin{cases} 0 & iff \;\; \mu_{2H}^* \leq \frac{1}{2} \\ 1 & iff \;\; \mu_{2H}^* \geq \frac{1}{2}. \end{cases}$$

Similarly

$$Eu_2(a_1 = 1, a_2 = 1|X_2 = L) = \mu_{2L}^* - \frac{1}{2} \leq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \Leftrightarrow \mu_{2L}^* \leq \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \pi_2^{\mathrm{SR}}(L,1) = \begin{cases} 0 & iff \;\; \mu_{2L}^* \leq \frac{1}{2} \\ 1 & iff \;\; \mu_{2L}^* \geq \frac{1}{2}. \end{cases}$$

Then, we should distinguish four possible cases.

1. Suppose

   $$\mu_{2H}^* \leq \frac{1}{2} \quad \& \quad \mu_{2L}^* \leq \frac{1}{2} \;\; \text{so that} \;\; \pi_2^{\mathrm{SR}}(H,1) = 0 \quad \text{and} \quad \pi_2^{\mathrm{SR}}(L,1) = 0.$$

   Then, given the vanguard continuation payoff,

   $$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

   $$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}}|X_1 = H\right) = -\frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}}|X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 1, \pi_2^{\mathrm{SR}}|X_1 = L\right) = -\frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\mathrm{SR}}|X_1 = L\right) = 0 \end{cases}$$

   which is always satisfied. Thus, for any $p_1 \in \left(\frac{1}{2}, 1\right)$ and $p_2 \in \left(\frac{1}{2}, 1\right)$, we have a pooling PBE

   $$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

   $$\mu_1^*(V = 1|X_1 = H) = p_1 \;\; \textit{and} \quad \mu_1^*(V = 1|X_1 = L) = 1 - p_1$$

   $$\pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = x) = 0 \quad \forall x \in \{H, L\} \;\; \textit{and} \quad \pi_2^*(a_2 = 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\}$$

   $$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \leq \frac{1}{2} \;\; \textit{and} \;\; \mu_2^*(V = 1|X_2 = L, a_1 = 1) \leq \frac{1}{2}.$$

2. Suppose

$$\mu_{2H}^* \leq \frac{1}{2} \quad \& \quad \mu_{2L}^* \geq \frac{1}{2} \text{ so that } \pi_2^{\text{SR}}(H,1) = 0 \quad \text{and} \quad \pi_2^{\text{SR}}(L,1) = 1.$$

Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = H\right) = (1-p_2)\, p_1 - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = L\right) = (1-p_2)\,(1-p_1) - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = L\right) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (1-p_2)\, p_1 \leq \frac{1}{2} \\ (1-p_2)\,(1-p_1) \leq \frac{1}{2} \end{cases}$$

which is always satisfied. Thus, for any $p_1 \in \left(\frac{1}{2}, 1\right)$ and $p_2 \in \left(\frac{1}{2}, 1\right)$, we have a pooling PBE

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*\left(V = 1 | X_1 = H\right) = p_1 \;\; \textit{and} \;\; \mu_1^*\left(V = 1 | X_1 = L\right) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = H) = 0, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 1, \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\mu_2^*\left(V = 1 | X_2 = H, a_1 = 1\right) \leq \frac{1}{2} \;\; \textit{and} \;\; \mu_2^*\left(V = 1 | X_2 = L, a_1 = 1\right) \geq \frac{1}{2}.$$

3. Suppose

$$\mu_{2H}^* \geq \frac{1}{2} \quad \& \quad \mu_{2L}^* \leq \frac{1}{2} \text{ so that } \pi_2^{\text{SR}}(H,1) = 1 \quad \text{and} \quad \pi_2^{\text{SR}}(L,1) = 0.$$

Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = H\right) = p_2 p_1 - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = L\right) = p_2\,(1-p_1) - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = L\right) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1 p_2 \leq \frac{1}{2} \\ (1-p_1)\, p_2 \leq \frac{1}{2} \end{cases} \Leftrightarrow p_1 p_2 \leq \frac{1}{2}.$$

Thus, when $p_1 p_2 \leq \frac{1}{2}$ we have a pooling PBE

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*\left(V = 1 | X_1 = H\right) = p_1 \;\; \textit{and} \;\; \mu_1^*\left(V = 1 | X_1 = L\right) = 1 - p_1$$

$$\begin{aligned} \pi_2^*(a_2 &= 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0, \\ \pi_2^*(a_2 &= 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\} \end{aligned}$$

$$\mu_2^*\left(V = 1 | X_2 = H, a_1 = 1\right) \geq \frac{1}{2} \;\; \textit{and} \;\; \mu_2^*\left(V = 1 | X_2 = L, a_1 = 1\right) \leq \frac{1}{2}.$$

4. Suppose

$$\mu_{2H}^* \geq \frac{1}{2} \quad \& \quad \mu_{2L}^* \geq \frac{1}{2} \text{ so that } \pi_2^{\text{SR}}(H,1) = 1 \quad \text{and} \quad \pi_2^{\text{SR}}(L,1) = 1.$$

Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \;\; \textit{and} \;\; \pi_1^*(a_1 = 1 \mid X_1 = L) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = H\right) = p_1 - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}} | X_1 = L\right) = (1 - p_1) - \frac{1}{2} \leq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}} | X_1 = L\right) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1 \leq \frac{1}{2} \\ (1 - p_1) \leq \frac{1}{2} \end{cases}$$

which can't be satisfied.

■

The following figure represents the three pooling PBE

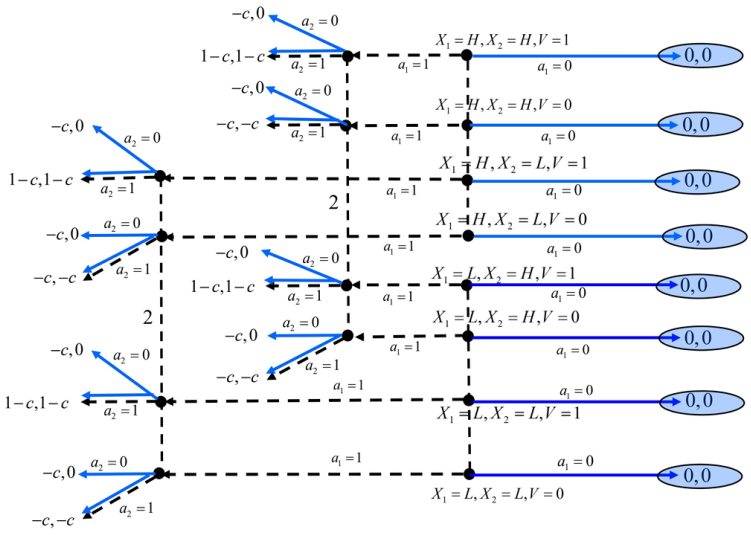

Figure 4: The three pooling PBE.

It is possible to prove that some of these pooling equilibria are not plausible. Throughout the paper, plausibility is assessed using Forward Induction, operationalized as an equilibrium-dominance restriction on off-path beliefs in the spirit of the Intuitive Criterion of Cho and Kreps (1987).

Definition 2 (Forward Induction) *Fix a candidate PBE* (π^*, μ^*) *and an off-path information set reached only by a deviation* a_1 *of Player 1. Type* X_1*'s deviation to* a_1 *is* equilibrium dominated *if Player 1's payoff from* a_1*, evaluated at type* X_1*'s* best-case *continuation payoff over all possible follower responses, is strictly lower than the type's equilibrium payoff under* (π^*, μ^*)*. An assessment survives Forward Induction only if, for every conjecture* $\pi_1^C(a_1 \mid X_1)$ *that assigns no equilibrium-dominated type a higher probability of playing* a_1 *than a non-dominated type, the belief* $\mu_2^*(\cdot \mid a_1)$ *derived from it via Bayes' rule does not make* a_1 *a profitable deviation for any non-dominated type.*

Proposition 4 (Out-of-equilibrium beliefs) *Consider the out-of-equilibrium beliefs that support the pooling PBE:*

1. *when* $p_1 \leq p_2$*, the beliefs*

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \leq \frac{1}{2}$$

are neither consistent nor structurally consistent,[9] *while the beliefs*

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \geq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1|X_2 = L, a_1 = 1) \leq \frac{1}{2}$$

are both consistent and structurally consistent;

2. *when* $p_1 > p_2$*, the beliefs*

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \leq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1|X_2 = L, a_1 = 1) \leq \frac{1}{2}$$

are not compatible with Forward Induction.

Proof. Let denote by π_1^C the citizens' conjecture on the vanguard's deviation form the equilibrium path. Then, because of Bayes rule

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) =$$

$$= \frac{p_2\left[\pi_1^C(a_1 = 1|X_1 = H)\, p_1 + \pi_1^C(a_1 = 1|X_1 = L)(1 - p_1)\right]}{D(V = 1|X_2 = H, a_1 = 1)}$$

Suppose

1. $p_1 \leq p_2$, then the beliefs

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) \leq \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow 2p_2\left[\pi_1^C(a_1 = 1|X_1 = H)\, p_1 + \pi_1^C(a_1 = 1|X_1 = L)(1 - p_1)\right] \leq$$

$$\leq p_2\left[\pi_1^C(a_1 = 1|X_1 = H)\, p_1 + \pi_1^C(a_1 = 1|X_1 = L)(1 - p_1)\right] +$$

[9] For the definition of consistency and structural consistency, see (Kreps and Ramey, 1987).

$$+(1-p_2)\left[\pi_1^C(a_1=1|X_1=H)(1-p_1)+\pi_1^C(a_1=1|X_1=L)p_1\right]\Leftrightarrow$$

$$\begin{aligned}\Leftrightarrow\quad & p_2\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]\leq\\ \leq\quad & (1-p_2)\left[\pi_1^C(a_1=1|X_1=H)(1-p_1)+\pi_1^C(a_1=1|X_1=L)p_1\right]\Leftrightarrow\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad & \pi_1^C(a_1=1|X_1=H)p_1p_2+\pi_1^C(a_1=1|X_1=L)(1-p_1)p_2\leq\\ \leq\quad & \pi_1^C(a_1=1|X_1=H)(1-p_1)(1-p_2)+\pi_1^C(a_1=1|X_1=L)p_1(1-p_2)\Leftrightarrow\end{aligned}$$

$$\Leftrightarrow \pi_1^C(a_1=1|X_1=H)\left[p_1p_2-(1-p_1)(1-p_2)\right]\leq\pi_1^C(a_1=1|X_1=L)\left[p_1(1-p_2)-(1-p_1)p_2\right]\Leftrightarrow$$

$$\Leftrightarrow \pi_1^C(a_1=1|X_1=H)\left[p_1p_2-1+p_1+p_2-p_1p_2\right]\leq\pi_1^C(a_1=1|X_1=L)\left[p_1-p_1p_2-p_2+p_1p_2\right]\Leftrightarrow$$

$$\Leftrightarrow \underbrace{\pi_1^C(a_1=1|X_1=H)(p_1+p_2-1)}_{>0}\leq\underbrace{\pi_1^C(a_1=1|X_1=L)(p_1-p_2)}_{>0\Leftrightarrow p_1>p_2}.$$

Then, the inequality can't be satisfied if $p_1\leq p_2$ even when $\pi_1^C(a_1=1|X_1=H)\rightarrow 0^+$ and $\pi_1^C(a_1=1|X_1=L)\rightarrow 0^+$. On the other hand,

$$\mu_2^*(V=1|X_2=H,a_1=1)\geq\frac{1}{2}\Leftrightarrow$$

$$\begin{aligned}\Leftrightarrow\ & 2p_2\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]\geq\\ \geq\ & p_2\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]+\\ & +(1-p_2)\left[\pi_1^C(a_1=1|X_1=H)(1-p_1)+\pi_1^C(a_1=1|X_1=L)p_1\right]\Leftrightarrow\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad & p_2\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]\geq\\ \geq\quad & (1-p_2)\left[\pi_1^C(a_1=1|X_1=H)(1-p_1)+\pi_1^C(a_1=1|X_1=L)p_1\right]\Leftrightarrow\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad & \pi_1^C(a_1=1|X_1=H)p_1p_2+\pi_1^C(a_1=1|X_1=L)(1-p_1)p_2\geq\\ \geq\quad & \pi_1^C(a_1=1|X_1=H)(1-p_1)(1-p_2)+\pi_1^C(a_1=1|X_1=L)p_1(1-p_2)\Leftrightarrow\end{aligned}$$

$$\Leftrightarrow \pi_1^C(a_1=1|X_1=H)\left[p_1p_2-(1-p_1)(1-p_2)\right]\geq\pi_1^C(a_1=1|X_1=L)\left[p_1(1-p_2)-(1-p_1)p_2\right]\Leftrightarrow$$

$$\Leftrightarrow \pi_1^C(a_1=1|X_1=H)\left[p_1p_2-1+p_1+p_2-p_1p_2\right]\geq\pi_1^C(a_1=1|X_1=L)\left[p_1-p_1p_2-p_2+p_1p_2\right]\Leftrightarrow$$

$$\Leftrightarrow \underbrace{\pi_1^C(a_1=1|X_1=H)(p_1+p_2-1)}_{>0}\geq\underbrace{\pi_1^C(a_1=1|X_1=L)(p_1-p_2)}_{>0\Leftrightarrow p_1>p_2}.$$

Then, the inequality is always satisfied if $p_1\leq p_2$. Finally:

$$\mu_2^*(V=1|X_2=L,a_1=1)\leq\frac{1}{2}\Leftrightarrow$$

$$\begin{aligned}\Leftrightarrow\ & 2(1-p_2)\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]\leq\\ \leq\ & (1-p_2)\left[\pi_1^C(a_1=1|X_1=H)p_1+\pi_1^C(a_1=1|X_1=L)(1-p_1)\right]+\end{aligned}$$

$$+p_2\left[\pi_1^C\left(a_1=1|X_1=H\right)\left(1-p_1\right)+\pi_1^C\left(a_1=1|X_1=L\right)p_1\right]\Leftrightarrow$$

$$\Leftrightarrow\left(1-p_2\right)\left[\pi_1^C\left(a_1=1|X_1=H\right)p_1+\pi_1^C\left(a_1=1|X_1=L\right)\left(1-p_1\right)\right]\leq$$

$$\leq p_2\left[\pi_1^C\left(a_1=1|X_1=H\right)\left(1-p_1\right)+\pi_1^C\left(a_1=1|X_1=L\right)p_1\right]\Leftrightarrow$$

$$\Leftrightarrow\pi_1^C\left(a_1=1|X_1=H\right)\left[p_1\left(1-p_2\right)-\left(1-p_1\right)p_2\right]\leq\pi_1^C\left(a_1=1|X_1=L\right)\left[p_1p_2-\left(1-p_1\right)\left(1-p_2\right)\right]\Leftrightarrow$$

$$\Leftrightarrow\pi_1^C\left(a_1=1|X_1=H\right)\left[p_1-p_1p_2-p_2+p_1p_2\right]\leq\pi_1^C\left(a_1=1|X_1=L\right)\left[p_1p_2-1+p_1+p_2-p_1p_2\right]\Leftrightarrow$$

$$\Leftrightarrow\underbrace{\pi_1^C\left(a_1=1|X_1=H\right)\left(p_1-p_2\right)}_{>0\Leftrightarrow p_1>p_2}\leq\underbrace{\pi_1^C\left(a_1=1|X_1=L\right)\left(p_1+p_2-1\right)}_{>0}.$$

Then, the inequality is always satisfied if $p_1\leq p_2$.

2. $p_1>p_2$, then the beliefs

$$\mu_2^*\left(V=1|X_2=H,a_1=1\right)\leq\frac{1}{2}\Leftrightarrow$$

$$\Leftrightarrow\underbrace{\pi_1^C\left(a_1=1|X_1=H\right)\left(p_1+p_2-1\right)}_{>0}\leq\underbrace{\pi_1^C\left(a_1=1|X_1=L\right)\left(p_1-p_2\right)}_{>0\Leftrightarrow p_1>p_2}\Leftrightarrow$$

$$\Leftrightarrow\frac{\pi_1^C\left(a_1=1|X_1=H\right)}{\pi_1^C\left(a_1=1|X_1=L\right)}\leq\frac{p_1-p_2}{p_1+p_2-1}<1\Rightarrow$$

$$\Rightarrow\pi_1^C\left(a_1=1|X_1=H\right)<\pi_1^C\left(a_1=1|X_1=L\right).$$

However, the conjecture $\pi_1^C\left(a_1=1|X_1=H\right)<\pi_1^C\left(a_1=1|X_1=L\right)$ does not satisfy forward induction, since

(a) $\pi_1^C\left(a_1=1|X_1=L\right)$ at most produces an expected payoff $Eu_1\left(a_1=1,a_2=1|X_2=L\right)=\left(1-p_1\right)-\frac{1}{2}$ which is smaller than the expected equilibrium payoff;

(b) $\pi_1^C\left(a_1=1|X_1=H\right)$ may generate an expected payoff $Eu_1\left(a_1=1,a_2=1|X_2=H\right)=p_1-\frac{1}{2}$ greater than the expected equilibrium payoff, provided that the vanguard's is able to convince the citizens to change their out-of-equilibrium beliefs.

■

Proposition 5 (Non existence of alternative pooling PBE) *The model has no pooling PBE in which*

$$\pi_1^*(a_1=1\mid X_1=H)=1\ \ and\ \ \ \pi_1^*(a_1=1\mid X_1=L)=1.$$

Proof. Suppose

$$\pi_1^*(a_1=1\mid X_1=H)=1\ \ and\ \ \ \pi_1^*(a_1=1\mid X_1=L)=1$$

so that $X_2 \in \{H, L\}, a_1 = 0$ are out of the equilibrium path, however they do not matter because of Corollary 1. On the equilibrium path, instead

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) = p_2 \text{ and } \mu_2^*(V = 1|X_2 = L, a_1 = 1) = 1 - p_2.$$

Then

$$\begin{cases} Eu_2(a_1 = 1, a_2 = 1|X_2 = H) = p_2 - \frac{1}{2} \geq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \\ Eu_2(a_1 = 1, a_2 = 1|X_2 = L) = 1 - p_2 - \frac{1}{2} \leq Eu_2(a_1 = 1, a_2 = 0|X_2 = H) = 0 \end{cases}$$

which are always satisfied Hence, $\pi_2^{\text{SR}}(H, 1) = 1$ and $\pi_2^{\text{SR}}(L, 1) = 0$. Then, given the vanguard continuation payoff,

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \text{ and } \pi_1^*(a_1 = 1 \mid X_1 = L) = 1 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}}|X_1 = H\right) = p_1 p_2 - \frac{1}{2} \geq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}}|X_1 = H\right) = 0 \\ Eu_1^C\left(a_1 = 1, \pi_2^{\text{SR}}|X_1 = L\right) = (1 - p_1) p_2 - \frac{1}{2} \geq Eu_1^C\left(a_1 = 0, \pi_2^{\text{SR}}|X_1 = L\right) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} p_1 p_2 \geq \frac{1}{2} \\ p_2 - p_1 p_2 \geq \frac{1}{2}. \end{cases}$$

which can't be satisfied. ■

3.5 A General Overview of the Game's Pure Strategy Perfect Bayesian Equilibria

The previous subsections have fully characterized the set of pure strategy plausible PBE of the vanguard's game, as summed up in the following corollary.

Corollary 2 (Pure strategy PBE of the vanguard's game) *The vanguard's game has the following plausible PBE:*

1. *when $p_1 \geq p_2$, then there exists a unique separating PBE such that:*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \text{ and } \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1|X_1 = H) = p_1 \text{ and } \mu_1^*(V = 1|X_1 = L) = 1 - p_1$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = x) = 1 \quad \forall x \in \{H, L\} \text{ and } \pi_2^*(a_2 = 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\}$$

$$\mu_2^*(V = 1|X_2 = H, a_1 = 1) = \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} \text{ and}$$

$$\mu_2^*(V = 1|X_2 = L, a_1 = 1) = \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2}$$

with the following equilibrium outcome

$$Z^* = \begin{cases} 1 & \text{with probability } p_1 \\ 0 & \text{with probability } 1 - p_1; \end{cases}$$

2. *when* $p_1 < p_2$ *and* $p_1 p_2 \geq \frac{1}{2}$ *then there exists a unique separating PBE such that:*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 1 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0,$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) = \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)} \quad \textit{and}$$

$$\mu_2^*(V = 1 | X_2 = L, a_1 = 1) = \frac{p_1 (1 - p_2)}{p_1 (1 - p_2) + (1 - p_1) p_2}$$

with the following equilibrium outcome

$$Z^* = \begin{cases} 1 & \textit{with probability } p_1 p_2 \\ 0 & \textit{with probability } 1 - p_1 p_2; \end{cases}$$

3. *when* $p_1 \leq p_2$ *and* $p_1 p_2 \leq \frac{1}{2}$ *then there exists a unique pooling PBE such that*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0,$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) \geq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1 | X_2 = L, a_1 = 1) \leq \frac{1}{2}$$

with the following equilibrium outcome

$$Z^* = 0 \quad \textit{with probability } 1.$$

4. *when* $p_1 > p_2$, *and* $p_1 p_2 \leq \frac{1}{2}$ *then there exists a unique pooling PBE not consistent with Forward Induction such that*

$$\pi_1^*(a_1 = 1 \mid X_1 = H) = 0 \quad \textit{and} \quad \pi_1^*(a_1 = 1 \mid X_1 = L) = 0$$

$$\mu_1^*(V = 1 | X_1 = H) = p_1 \quad \textit{and} \quad \mu_1^*(V = 1 | X_1 = L) = 1 - p_1$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = H) = 1, \quad \pi_2^*(a_2 = 1 \mid a_1 = 1, X_2 = L) = 0,$$

$$\pi_2^*(a_2 = 1 \mid a_1 = 0, X_2 = x) = 0 \quad \forall x \in \{H, L\}$$

$$\mu_2^*(V = 1 | X_2 = H, a_1 = 1) \leq \frac{1}{2} \quad \textit{and} \quad \mu_2^*(V = 1 | X_2 = L, a_1 = 1) \leq \frac{1}{2}$$

with the following equilibrium outcome

$$Z^* = 0 \quad \textit{with probability } 1.$$

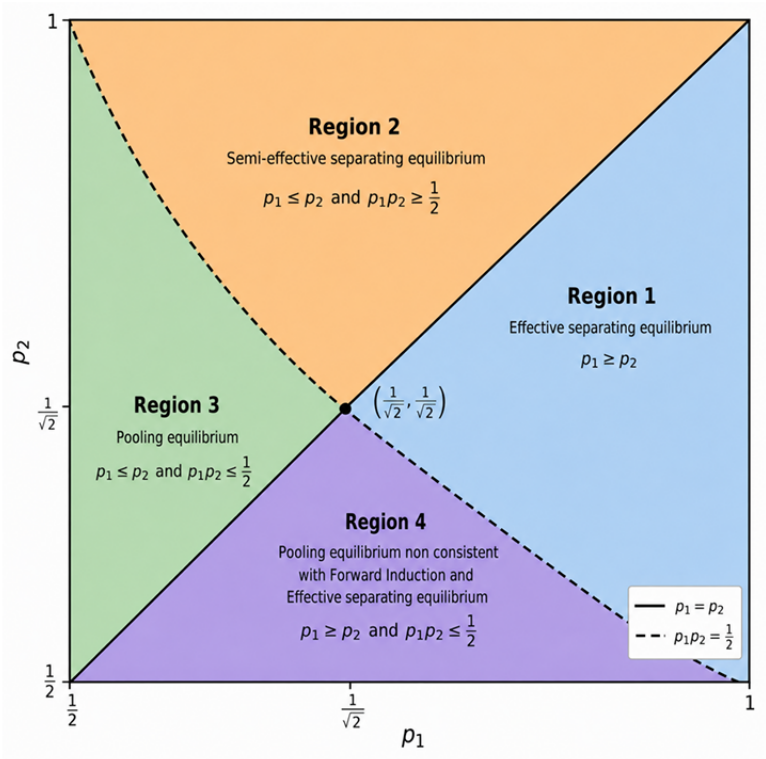

Figure 5: The equilibrium outcomes and the four parameters' regions.

The four regions with the different PBE are represented in the following figure

Remark 1 (Reading the Four Regions) *Regions 1–4 above are not four disjoint tiles of the parameter square* $(\frac{1}{2}, 1)^2$*. Regions 1, 2 and 3 do partition it:* $p_1 \geq p_2$ *(Region 1),* $p_1 < p_2$ *with* $p_1 p_2 \geq \frac{1}{2}$ *(Region 2), and* $p_1 < p_2$ *with* $p_1 p_2 < \frac{1}{2}$ *(Region 3). Region 4, by contrast, is the subset* $\{p_1 > p_2,\ p_1 p_2 \leq \frac{1}{2}\}$ *of Region 1's own domain, and identifies exactly those parameter values at which a second, non-robust pooling PBE mathematically coexists alongside the separating PBE of Region 1. Corollary 3 below shows that Forward Induction eliminates this spurious pooling equilibrium throughout Region 4, so that Regions 1 and 4 always deliver the identical (separating, first-best efficient) outcome. For this reason, from Section 4 onward we follow the convention of writing "Region 1" for the full domain* $p_1 \geq p_2$*, i.e., for the union of Regions 1 and 4 as defined here.*

3.6 The Inactivity Region

The analysis so far has characterized equilibria when the separating PBE exists, i.e., when $p_1 \geq p_2$ or $p_1 < p_2$ with $p_1 p_2 \geq \frac{1}{2}$. We now turn to the remaining region $p_1 < p_2$ and $p_1 p_2 < \frac{1}{2}$, in which Proposition 1 guarantees no separating PBE exists. The natural conjecture is that this region admits a semi-separating equilibrium in which the vanguard randomizes when $X_1 = H$, partially coordinating with the follower. The main result of this subsection is that this conjecture is false: the mixed strategy equilibrium one would expect to replace the separating equilibrium does not exist. The reason, as we establish below, is that the vanguard's expected payoff from acting is constant and everywhere strictly negative for any mixing probability, so no interior indifference condition can be satisfied. As a consequence, the unique equilibrium surviving Forward Induction involves no action of any kind. A key analytical tool is the *likelihood ratio* of the vanguard's action. For any strategy profile $(\sigma_H, \sigma_L) \in [0,1]^2$ with $\sigma_H \geq \sigma_L$ (which must hold in any equilibrium, as we verify below), define:

$$\lambda(\sigma_H, \sigma_L) \equiv \frac{\Pr(a_1 = 1 \mid V = 1)}{\Pr(a_1 = 1 \mid V = 0)} = \frac{p_1 \sigma_H + (1 - p_1)\sigma_L}{(1 - p_1)\sigma_H + p_1 \sigma_L}.$$

Since $p_1 > \frac{1}{2}$ and $\sigma_H \geq \sigma_L$, we have $\lambda \geq 1$, with equality only when $\sigma_H = \sigma_L$. The follower's posterior upon observing $a_1 = 1$ can be written compactly as:

$$\mu_2(V = 1 \mid a_1 = 1, X_2 = H) = \frac{\lambda p_2}{\lambda p_2 + (1 - p_2)},$$
$$\mu_2(V = 1 \mid a_1 = 1, X_2 = L) = \frac{\lambda(1 - p_2)}{\lambda(1 - p_2) + p_2}.$$

Lemma 4 (Follower's Best Response under Any Vanguard Mixing) *Denote by* $\rho_H \equiv \pi_2^{\mathrm{SR}}(H, 1)$ *and* $\rho_L \equiv \pi_2^{\mathrm{SR}}(L, 1)$ *the follower's joining probabilities upon*

observing $a_1 = 1$ with signal $X_2 = H$ and $X_2 = L$ respectively. In any PBE with $\sigma_H \geq \sigma_L \geq 0$ and $\sigma_H > 0$:

1. *$\rho_H = 1$: the follower always joins upon observing $(a_1 = 1, X_2 = H)$.*
2. *$\rho_L = 0$: the follower never joins upon observing $(a_1 = 1, X_2 = L)$, whenever $p_1 < p_2$ and $\sigma_L = 0$.*

Proof. *Part 1.* Since $p_2 > \frac{1}{2}$, we have $(1-p_2)/p_2 < 1 \leq \lambda$, so $\lambda p_2 > 1 - p_2$ and $\mu_2(V = 1 \mid a_1 = 1, X_2 = H) > \frac{1}{2} = c$ for any $\lambda \geq 1$. Hence $\rho_H = 1$ is strictly optimal for the follower.

Part 2. When $\sigma_L = 0$, $\lambda = p_1/(1-p_1)$. The condition $\mu_2(V = 1 \mid a_1 = 1, X_2 = L) \geq c$ reduces to $p_1(1-p_2) \geq (1-p_1)p_2$, i.e., $p_1 \geq p_2$. Since $p_1 < p_2$ by assumption, the condition fails strictly, so $\rho_L = 0$. ■

Proposition 6 (Inactivity and Equilibrium Uniqueness) *Suppose $p_1 < p_2$ and $p_1 p_2 < \frac{1}{2}$. Then:*

1. *There exists no PBE, whether pure or mixed, in which the vanguard acts with positive probability on any type.*
2. *The pooling equilibrium with zero activity survives Forward Induction and is the unique equilibrium doing so.*

Proof. *Part 1: Non-existence of any other equilibrium.*

We consider all possible classes of strategy profiles (σ_H, σ_L).

Step 1: We may restrict to $\sigma_H \geq \sigma_L$.

Since $p_1 > \frac{1}{2}$, the expected payoff from acting is strictly increasing in $\Pr(V = 1 \mid X_1)$, so any type H who is willing to act strictly prefers acting over any type L who does so. More formally, for any follower response (ρ_H, ρ_L):

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = H) - \mathbb{E}u_1(a_1 = 1 \mid X_1 = L) = (2p_1 - 1)\big[p_2\rho_H + (1-p_2)\rho_L\big] > 0.$$

Hence $\sigma_L > \sigma_H$ cannot arise in equilibrium. We therefore restrict to $\sigma_H \geq \sigma_L$.

Step 2: No equilibrium with $\sigma_H > 0, \sigma_L = 0$.

By Lemma 4, $\rho_H = 1$ and $\rho_L = 0$ under any such profile. The vanguard's payoff from acting when $X_1 = H$ is then:

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = H) = \tfrac{1}{2}(2p_1p_2 - 1) < 0,$$

since $p_1 p_2 < \frac{1}{2}$. Crucially, this expression is independent of σ_H: the vanguard's incentive is constant in her own mixing probability (because the follower's posterior and best response are determined entirely by the likelihood ratio $\lambda = p_1/(1-p_1)$, which is independent of σ_H when $\sigma_L = 0$) and strictly negative throughout $\sigma_H \in (0, 1]$. There is therefore no interior mixing probability that makes type H indifferent, ruling out both the pure separating equilibrium ($\sigma_H = 1$) and any semi-separating equilibrium ($\sigma_H \in (0, 1)$) simultaneously.

Step 3: No equilibrium with $\sigma_L > 0$.

With $\sigma_L > 0$ and $\sigma_H \geq \sigma_L > 0$, we have $\rho_H = 1$ by Lemma 4. For type L to be willing to act we need:

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = L) = (1 - p_1)\big[p_2 + (1 - p_2)\rho_L\big] - \tfrac{1}{2} \geq 0.$$

Since $\rho_L \leq 1$, the left-hand side is at most $(1 - p_1) < \frac{1}{2}$, a contradiction.

Steps 1–3 together exhaust all cases with positive coordination.

Part 2: Survival of the pooling equilibrium under Forward Induction.

In the pooling equilibrium $\sigma_H = \sigma_L = 0$, the equilibrium payoff is 0 for both types. Type L's maximum possible payoff from deviating is $(1 - p_1) \cdot \frac{1}{2} + p_1 \cdot (-\frac{1}{2}) = \frac{1}{2} - p_1 < 0$, so the deviation is equilibrium-dominated for type L. The Forward Induction criterion therefore restricts off-path beliefs to type H. Given this restricted belief, $\rho_H = 1$ and $\rho_L = 0$ by Lemma 4, and type H's payoff from deviating is:

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = H; \text{ Forward Induction belief}) = \tfrac{1}{2}(2p_1p_2 - 1) < 0.$$

Since the deviation is unprofitable even under the most favourable belief Forward Induction can assign, the pooling equilibrium is not upset by the refinement. By Part 1, it is the unique surviving equilibrium. ■

Corollary 3 (Global Equilibrium Outcome Uniqueness) *For all $(p_1, p_2) \in (\frac{1}{2}, 1)^2$ except the knife-edges $\{p_1 = p_2\}$ and $\{p_1p_2 = \frac{1}{2},\ p_1 < p_2\}$, the unique PBE outcome surviving Forward Induction is:*

1. Regions 1 and 4 *($p_1 > p_2$): the separating equilibrium with $\sigma_H = 1$, $\sigma_L = 0$, $\rho_H = \rho_L = 1$. The pooling equilibrium that mathematically exists in Region 4 ($p_1p_2 \leq \frac{1}{2}$) is eliminated by Forward Induction.*

2. Region 2 *($p_1 < p_2$, $p_1p_2 > \frac{1}{2}$): the separating equilibrium with $\sigma_H = 1$, $\sigma_L = 0$, $\rho_H = 1$, $\rho_L = 0$.*

3. Region 3 *($p_1 < p_2$, $p_1p_2 < \frac{1}{2}$): the pooling equilibrium with $\sigma_H = \sigma_L = 0$.*

No mixed or semi-separating equilibrium exists in any region.

Proof. Regions 1 and 2 are covered by Proposition 1 and Proposition 4. Type L is always strictly better off abstaining (since $(1-p_1)\cdot\frac{1}{2} + p_1 \cdot (-\frac{1}{2}) = \frac{1}{2} - p_1 < 0$ in any case), so $\sigma_L = 0$ in any such equilibrium. Given this, the vanguard's expected payoff from acting is:

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = H) = \begin{cases} p_1 - \frac{1}{2} > 0 & \text{Region 1,} \\ \frac{1}{2}(2p_1p_2 - 1) > 0 & \text{Region 2.} \end{cases}$$

Both expressions are independent of σ_H, so the vanguard's payoff from acting is strictly positive for all $\sigma_H \in (0, 1]$. No interior mixing probability makes type H indifferent, ruling out any semi-separating equilibrium. Hence $\sigma_H = 1$,

$\sigma_L = 0$ is the unique best response. Region 3 is covered by Proposition 6. The knife-edge sets $\{p_1 = p_2\}$ and $\{p_1 p_2 = \frac{1}{2},\, p_1 < p_2\}$ have Lebesgue measure zero in the parameter space and admit a continuum of mixed equilibria; they do not affect the welfare analysis. ■

Remark 2 (Off-Path Beliefs in the Separating Equilibrium) *A standard requirement of PBE is that beliefs must be specified at every information set, including those reached off the equilibrium path. In the separating equilibrium* $(\sigma_H = 1, \sigma_L = 0)$*, however,* both *actions* $a_1 = 0$ *and* $a_1 = 1$ *are reached on the equilibrium path, the former by type* L*, the latter by type* H*. The follower therefore has no off-path information sets, and Bayes' rule pins down all beliefs uniquely:*

$$\begin{aligned}
&\mu_2(V = 1 \mid a_1 = 1, X_2) \text{ : computed in Proposition 1 above;} \\
&\mu_2(V = 1 \mid a_1 = 0, X_2) = \Pr(V = 1 \mid X_1 = L, X_2),
\end{aligned}$$

where the second line follows because observing $a_1 = 0$ *leads the follower to infer* $X_1 = L$ *with probability one. This is in sharp contrast to the pooling equilibrium, where* $a_1 = 1$ *is off-path and beliefs must be assigned by assumption. The separating equilibrium is therefore fully pinned down by Bayes' rule alone, with no free parameters in the belief system. Note also that the follower's strategy at* $a_1 = 0$ *is* $\rho = 0$ *regardless of* $\mu_2(V = 1 \mid a_1 = 0, X_2)$*: since acting alone yields* $-c < 0$ *with certainty, abstention is strictly dominant (Lemma 1, cases 1–2). The exact value of* $\mu_2(\cdot \mid a_1 = 0)$ *is therefore payoff-irrelevant.*

Remark 3 (The Boundary Case $p_1 = p_2$**)** *The knife-edge* $p_1 = p_2 = p$ *deserves explicit treatment. At this boundary, the follower's posterior upon observing* $a_1 = 1$ *and* $X_2 = L$ *is:*

$$\mu_2(V = 1 \mid a_1 = 1, X_2 = L)\big|_{p_1 = p_2 = p} = \frac{p(1-p)}{p(1-p) + (1-p)p} = \tfrac{1}{2} = c,$$

so the follower is exactly indifferent *between joining and abstaining upon observing* $(a_1 = 1, X_2 = L)$*. Any* $\rho_L \in [0, 1]$ *is sequentially rational, generating a continuum of equilibria indexed by* ρ_L*. Whether the vanguard is willing to act depends on* ρ_L*. A straightforward calculation gives:*

$$\mathbb{E}u_1(a_1 = 1 \mid X_1 = H;\ \rho_L) = p^2 - \tfrac{1}{2} + p(1-p)\rho_L \geq 0 \iff \rho_L \geq \rho_L^* \equiv \frac{\frac{1}{2} - p^2}{p(1-p)}.$$

Two sub-cases arise. If $p \geq \frac{1}{\sqrt{2}}$*, then* $p^2 \geq \frac{1}{2}$*, so* $\rho_L^* \leq 0$ *and the vanguard strictly prefers to act for any* $\rho_L \in [0, 1]$*: the entire continuum constitutes valid equilibria. If* $p < \frac{1}{\sqrt{2}}$*, then* $\rho_L^* \in (0, 1)$ *and only* $\rho_L \in [\rho_L^*, 1]$ *sustains the vanguard's participation. Forward Induction does not refine within this continuum, since the follower's indifference at* $X_2 = L$ *means any* $\rho_L \geq \rho_L^*$ *is consistent with sequential rationality.*

Remark 4 (The Boundary Case $p_1p_2 = \frac{1}{2}$, $p_1 < p_2$) *At this locus, $\frac{1}{2}(2p_1p_2 - 1) = 0$, so the vanguard with $X_1 = H$ is* exactly indifferent *between acting and abstaining when $\rho_H = 1$ and $\rho_L = 0$ (the follower's uniquely rational response at this parameter point). Any $\sigma_H \in [0, 1]$ is therefore a best response for type H, generating a continuum of equilibria indexed by σ_H; type L strictly prefers to abstain in all of them. Forward Induction does not refine within this continuum: type L's deviation is equilibrium-dominated for all σ_H, and type H's payoff from deviating is exactly zero, so the refinement applies but cannot select a unique value of σ_H. As with the $p_1 = p_2$ boundary, this knife-edge has Lebesgue measure zero in the parameter space and does not affect the analysis for generic parameter values.*

4 Welfare Analysis

This section compares equilibrium welfare to the full-information social planner (the first-best benchmark) under $c = \frac{1}{2}$. Recall the welfare function (1). We evaluate efficiency in two distinct ways: first, conditionally on the prize being valuable (ex post efficiency), and second, unconditionally over the entire distribution of uncertainty (ex ante efficiency).

Let us recall the definition of social welfare as $W(a_1, a_2, V) \equiv u_1 + u_2$, the sum of individual payoffs. Let us define the social planner as a benevolent planner who observes *both* signals (X_1, X_2) and chooses (a_1, a_2) to maximize $\mathbb{E}[u_1 + u_2]$. The first-best benchmark is $W^{FB}_{\text{ante}} = \max_{(a_1,a_2)} \mathbb{E}[u_1 + u_2]$.

4.1 Ex Post Efficiency (Conditional Welfare)

We analyse welfare conditional on the favourable state $V = 1$. When $V = 1$ and $c = \frac{1}{2}$, total welfare equals $+1$ if both agents act, $-\frac{1}{2}$ if the vanguard acts alone, and 0 if neither acts. Unlike a simple coordination probability measure, conditional expected welfare must account for the deadweight loss of vanguard isolation even within the favourable state.

Proposition 7 (Conditional Efficiency Comparison) *Assume $c = \frac{1}{2}$ and $p_i \in (\frac{1}{2}, 1)$ for $i = 1, 2$. Let $W^{\text{eq}}_{\text{post}}$ be the equilibrium conditional expected welfare given $V = 1$, and $W^{\text{FB}}_{\text{post}}$ the planner's conditional welfare given $V = 1$. Then:*

1. *If $p_1 \geq p_2$, then $W^{\text{eq}}_{\text{post}} = W^{\text{FB}}_{\text{post}} = p_1$ (ex post efficient).*
2. *If $p_1 < p_2$ and $p_1p_2 \geq \frac{1}{2}$, then $W^{\text{eq}}_{\text{post}} = \frac{3}{2}p_1p_2 - \frac{1}{2}p_1 < W^{\text{FB}}_{\text{post}} = p_2$ (ex post inefficient).*
3. *If $p_1 < p_2$ and $p_1p_2 < \frac{1}{2}$, then $W^{\text{eq}}_{\text{post}} = 0 < W^{\text{FB}}_{\text{post}} = p_2$ (ex post zero welfare).*

Proof. *Planner's benchmark.* The full-information planner observes both signals and prescribes coordination whenever $\Pr(V = 1 \mid X_1, X_2) \geq \frac{1}{2}$, never prescribing unilateral action. Hence welfare under the planner is either $+1$ (both

act) or 0 (neither acts), and the planner never incurs the isolation penalty. If $p_1 \geq p_2$, the planner's decision rule simplifies to "act if and only if $X_1 = H$": since (i) $\Pr(V = 1 \mid H, H) > \frac{1}{2}$ always, (ii) $\Pr(V = 1 \mid H, L) \geq \frac{1}{2}$ if and only if $p_1 \geq p_2$, and (iii) $\Pr(V = 1 \mid L, \cdot) < \frac{1}{2}$ when $p_1 > p_2$, the planner coordinates both agents whenever $X_1 = H$ and has them abstain otherwise. (If $p_1 = p_2$ and signals conflict, the posterior is exactly $\frac{1}{2} = c$; any tie-breaking rule yields the same expected welfare, since both acting and not acting are payoff-equivalent at this posterior.) Given $V = 1$, this occurs with probability p_1, so $W^{\mathrm{FB}}_{\mathrm{post}} = p_1$. When $p_1 < p_2$, the more informative signal is X_2, and the planner acts whenever $X_2 = H$. Isolation is excluded by definition. Conditional on $V = 1$: $W^{\mathrm{FB}}_{\mathrm{post}} = \Pr(X_2 = H \mid V = 1) = p_2$.

Equilibrium outcomes. In the separating equilibrium, the vanguard acts when $X_1 = H$ and abstains when $X_1 = L$. The follower, upon observing the vanguard act, always joins when $X_2 = H$. The key question, resolved differently across cases, is whether the follower also joins when $X_2 = L$. Conditional on $V = 1$, the relevant event probabilities are:

$$\Pr(X_1 = H, X_2 = H \mid V = 1) = p_1 p_2,$$
$$\Pr(X_1 = H, X_2 = L \mid V = 1) = p_1(1 - p_2).$$

Whether the event $\{X_1 = H, X_2 = L\}$ results in coordination or vanguard isolation depends on the follower's strategy at $X_2 = L$, which differs across regions. We now address each case in turn.

Case 1: $p_1 \geq p_2$. Here $\pi_2^{\mathrm{SR}}(L, 1) = 1$, so the follower joins on both signals. Isolation never occurs, and both events lead to successful coordination. Hence:

$$W^{\mathrm{eq}}_{\mathrm{post}} = p_1 p_2 \cdot 1 \; + \; p_1(1 - p_2) \cdot 1 = p_1 = W^{\mathrm{FB}}_{\mathrm{post}},$$

confirming ex post efficiency.

Case 2: $p_1 < p_2$ and $p_1 p_2 \geq \frac{1}{2}$. Here $\pi_2^{\mathrm{SR}}(L, 1) = 0$, so the follower joins only when $X_2 = H$. The event $\{X_1 = H, X_2 = L\}$ therefore results in vanguard isolation, yielding welfare $-\frac{1}{2}$. Hence:

$$W^{\mathrm{eq}}_{\mathrm{post}} = p_1 p_2 \cdot 1 \; + \; p_1(1 - p_2) \cdot \left(-\tfrac{1}{2}\right) = \tfrac{3}{2} p_1 p_2 - \tfrac{1}{2} p_1.$$

Comparing to the planner's benchmark $W^{\mathrm{FB}}_{\mathrm{post}} = p_2$, the gap is:

$$\begin{aligned} W^{\mathrm{FB}}_{\mathrm{post}} - W^{\mathrm{eq}}_{\mathrm{post}} &= p_2 - \tfrac{3}{2} p_1 p_2 + \tfrac{1}{2} p_1 \\ &= p_2(1 - \tfrac{3}{2} p_1) + \tfrac{1}{2} p_1. \end{aligned}$$

When $p_1 \leq \frac{2}{3}$, we have $1 - \frac{3}{2} p_1 \geq 0$ and the inequality is trivial. When $p_1 > \frac{2}{3}$, the coefficient $(1 - \frac{3}{2} p_1)$ is strictly negative; but since $p_2 < 1$, we have $p_2(1 - \frac{3}{2} p_1) > 1 \cdot (1 - \frac{3}{2} p_1)$, and therefore $p_2(1 - \frac{3}{2} p_1) + \frac{1}{2} p_1 > (1 - \frac{3}{2} p_1) + \frac{1}{2} p_1 = 1 - p_1 > 0$. The gap is thus strictly positive throughout, confirming strict ex post inefficiency.

Case 3: $p_1 < p_2$ and $p_1 p_2 < \frac{1}{2}$. By Proposition 6, the unique equilibrium is the pooling equilibrium in which no agent ever acts, so $W^{\mathrm{eq}}_{\mathrm{post}} = 0 < p_2 = W^{\mathrm{FB}}_{\mathrm{post}}$, confirming the zero welfare outcome. ■

Remark 5 (Sources of Inefficiency in Decentralized Equilibria) *The source of ex post inefficiency in Case 2 is twofold, and the two channels operate on* distinct *events. (i)* Missed coordination*: on the event* $\{X_1 = L, X_2 = H, V = 1\}$*, the planner observes the follower's high signal and coordinates both agents (welfare* $+1$*), while in the decentralized equilibrium the vanguard observes* $X_1 = L$ *and abstains, closing the game with welfare* 0*. (ii)* Isolation deadweight*: on the event* $\{X_1 = H, X_2 = L, V = 1\}$*, the vanguard acts and is left isolated. Crucially, a social planner would not coordinate on a specific event where a vanguard acts in isolation; instead, the planner avoids the punishment* $-c$ *entirely by prescribing abstention (welfare* 0*). The equilibrium welfare gap is exactly* $(1 - p_1)p_2 + \frac{1}{2}p_1(1 - p_2)$*, comprising the full-coordination gain foregone on the first event and the isolation penalty on the second. Both margins are absent when* $p_1 \geq p_2$ *(the collapsed Region 1 and 4), since the follower then joins unconditionally.*

4.2 Ex Ante Efficiency (A Priori Welfare)

The ex ante analysis captures the full spectrum of outcomes: successful coordination, coordinated mistakes, and the deadweight loss of vanguard isolation.

Proposition 8 (A Priori Efficiency Comparison) *Assume* $c = \frac{1}{2}$ *and* $p_i \in (\frac{1}{2}, 1)$*. Let* $W^{\text{eq}}_{\text{ante}} = \mathbb{E}[W]$ *be the unconditional expected equilibrium welfare and* $W^{\text{FB}}_{\text{ante}}$ *the planner's unconditional welfare. Then:*

1. *If* $p_1 \geq p_2$*,* $W^{\text{eq}}_{\text{ante}} = W^{\text{FB}}_{\text{ante}} = p_1 - \frac{1}{2}$ *(ex ante efficient).*
2. *If* $p_1 < p_2$ *and* $p_1 p_2 \geq \frac{1}{2}$*,* $W^{\text{eq}}_{\text{ante}} = \frac{1}{2}\big(p_1 p_2 + \frac{1}{2}p_1 + \frac{1}{2}p_2 - 1\big) < W^{\text{FB}}_{\text{ante}} = p_2 - \frac{1}{2}$ *(ex ante inefficient).*
3. *If* $p_1 < p_2$ *and* $p_1 p_2 < \frac{1}{2}$*,* $W^{\text{eq}}_{\text{ante}} = 0 < W^{\text{FB}}_{\text{ante}} = p_2 - \frac{1}{2}$ *(ex ante zero welfare).*

Proof. *Planner's benchmark.* With a uniform prior, the planner's expected welfare is:

$$W^{\text{FB}}_{\text{ante}} = \tfrac{1}{2}\Pr(\text{act} \mid V = 1) \cdot 1 + \tfrac{1}{2}\Pr(\text{act} \mid V = 0) \cdot (-1).$$

When $p_1 \geq p_2$, the planner acts on $X_1 = H$:

$$W^{\text{FB}}_{\text{ante}} = \tfrac{1}{2}[p_1 \cdot 1 + (1 - p_1) \cdot (-1)] = p_1 - \tfrac{1}{2}.$$

When $p_1 < p_2$, the planner acts on $X_2 = H$:

$$W^{\text{FB}}_{\text{ante}} = \tfrac{1}{2}[p_2 \cdot 1 + (1 - p_2) \cdot (-1)] = p_2 - \tfrac{1}{2}.$$

Case 1: $p_1 \geq p_2$. The follower always joins the vanguard, so both act if and only if $X_1 = H$, perfectly replicating the planner. Hence $W^{\text{eq}}_{\text{ante}} = p_1 - \frac{1}{2} = W^{\text{FB}}_{\text{ante}}$, confirming ex ante efficiency.

Case 2: $p_1 < p_2$ *and* $p_1 p_2 \geq \frac{1}{2}$. The vanguard acts on $X_1 = H$ and the follower joins only on $X_2 = H$. We compute equilibrium expected welfare by summing over both states.

- *Given $V = 1$*: coordination ($X_1{=}H, X_2{=}H$) occurs with probability p_1p_2, yielding welfare $+1$; isolation ($X_1{=}H, X_2{=}L$) occurs with probability $p_1(1-p_2)$, yielding welfare $-\frac{1}{2}$:

$$\mathbb{E}[W \mid V = 1] = p_1p_2 - \tfrac{1}{2}p_1(1-p_2) = \tfrac{3}{2}p_1p_2 - \tfrac{1}{2}p_1.$$

- *Given $V = 0$*: a coordinated mistake ($X_1{=}H, X_2{=}H$) occurs with probability $(1-p_1)(1-p_2)$, yielding welfare -1; isolation ($X_1{=}H, X_2{=}L$) occurs with probability $(1-p_1)p_2$, yielding welfare $-\frac{1}{2}$:

$$\mathbb{E}[W \mid V = 0] = -(1-p_1)(1-p_2) - \tfrac{1}{2}(1-p_1)p_2 = -1 + p_1 + \tfrac{1}{2}p_2 - \tfrac{1}{2}p_1p_2.$$

Weighting by the uniform prior:

$$\begin{aligned} W^{\text{eq}}_{\text{ante}} &= \tfrac{1}{2}\left(\tfrac{3}{2}p_1p_2 - \tfrac{1}{2}p_1\right) + \tfrac{1}{2}\left(-1 + p_1 + \tfrac{1}{2}p_2 - \tfrac{1}{2}p_1p_2\right) \\ &= \tfrac{1}{2}\left(p_1p_2 + \tfrac{1}{2}p_1 + \tfrac{1}{2}p_2 - 1\right). \end{aligned}$$

To confirm strict inefficiency, we compare to $W^{\text{FB}}_{\text{ante}} = p_2 - \frac{1}{2} = \frac{1}{2}(2p_2 - 1)$. The gap is:

$$\begin{aligned} W^{\text{FB}}_{\text{ante}} - W^{\text{eq}}_{\text{ante}} &= \tfrac{1}{2}\left[(2p_2 - 1) - \left(p_1p_2 + \tfrac{1}{2}p_1 + \tfrac{1}{2}p_2 - 1\right)\right] \\ &= \tfrac{1}{2}\left[\tfrac{3}{2}p_2 - p_1p_2 - \tfrac{1}{2}p_1\right] \\ &= \tfrac{1}{4}\left[3p_2 - 2p_1p_2 - p_1\right]. \end{aligned}$$

This is strictly positive if and only if $3p_2 > p_1(2p_2 + 1)$. Since $p_1 < p_2$ and $p_2 < 1$, we have $p_1(2p_2 + 1) < p_2(2p_2 + 1) = 2p_2^2 + p_2 < 2p_2 + p_2 = 3p_2$. The inequality thus holds strictly throughout, confirming strict ex ante inefficiency.

Case 3: $p_1 < p_2$ and $p_1p_2 < \frac{1}{2}$. By Proposition 6, the unique equilibrium is the pooling equilibrium in which no agent ever acts, so $W^{\text{eq}}_{\text{ante}} = 0 < p_2 - \frac{1}{2} = W^{\text{FB}}_{\text{ante}}$, confirming zero welfare. ■

Remark 6 (The Welfare Threshold and Discontinuous Jump) *Propositions 7 and 8 reveal that $p_1p_2 = \frac{1}{2}$ is a sharp welfare threshold, but with a surprising direction. Since Region 2 ($p_1p_2 \geq \frac{1}{2}$) lies* above *the threshold and Region 3 lies below it, welfare is* increasing *as p_2 crosses $\frac{1}{2p_1}$ from below, a discrete upward jump from zero to a strictly positive level. Formally (for $p_1 < \frac{1}{\sqrt{2}}$, the range where Region 3 is non-empty):*

$$\lim_{p_2 \nearrow \frac{1}{2p_1}} W^{\text{eq}}_{\text{ante}} = 0, \qquad \lim_{p_2 \searrow \frac{1}{2p_1}} W^{\text{eq}}_{\text{ante}} = \tfrac{1}{2}\left(\tfrac{1}{2}p_1 + \tfrac{1}{4p_1} - \tfrac{1}{2}\right) > 0.$$

The positivity of the right-hand limit follows from the AM–GM inequality: $\frac{1}{2}p_1 + \frac{1}{4p_1} \geq 2\sqrt{\frac{p_1}{2} \cdot \frac{1}{4p_1}} = \frac{1}{\sqrt{2}} > \frac{1}{2}$. This non-monotone recovery, welfare collapsing to zero as p_2 crosses p_1, then recovering as p_2 reaches $\frac{1}{2p_1}$, is fully characterized in the comparative statics of Section 5. The non-monotonicity pattern thus follows a clear sequence as p_2 increases: Region 1 (constant welfare) → Region 3 (zero welfare, downward cliff) → Region 2 (positive and rising, upward jump).

5 Equilibrium Welfare and Comparative Statics

Corollary 3 partitions the parameter space $(\frac{1}{2}, 1)^2$ into four structural regions that using forward induction collapse into three distinct equilibrium outcomes. We now study how equilibrium welfare responds to changes in p_1 and p_2 across and within these regions. The results reveal a striking non-monotonicity: the relationship between the follower's information precision and social welfare follows a cliff-and-valley pattern, dropping discontinuously to zero at $p_2 = p_1$ (for $p_1 < \frac{1}{\sqrt{2}}$), remaining at zero throughout Region 3, then recovering discontinuously upward at the boundary $p_1 p_2 = \frac{1}{2}$ and increasing monotonically thereafter in Region 2.

5.1 Effect of Follower Precision (p_2)

Proposition 9 (Comparative Statics in p_2) *Fix $p_1 \in (\frac{1}{2}, 1)$ and consider $W^{\text{eq}}_{\text{ante}}$ as a function of $p_2 \in (\frac{1}{2}, 1)$.*

1. Within Region 1 *($p_2 < p_1$): $W^{\text{eq}}_{\text{ante}} = p_1 - \frac{1}{2}$ is constant in p_2. The follower's precision is irrelevant because the vanguard's signal alone governs coordination.*

2. Within Region 2 *($p_2 > \max\{p_1, \frac{1}{2p_1}\}$). Note that this max condition unifies the boundary for $p_1 \geq \frac{1}{\sqrt{2}}$ (where p_1 is the binding lower bound) and $p_1 < \frac{1}{\sqrt{2}}$ (where $\frac{1}{2p_1}$ is the binding lower bound): equilibrium welfare is strictly increasing in p_2,*

$$\frac{\partial W^{\text{eq}}_{\text{ante}}}{\partial p_2} = \tfrac{1}{2}\big(p_1 + \tfrac{1}{2}\big) > 0,$$

but strictly below the first-best rate ($= 1$). The efficiency gap $\frac{1}{4}(3p_2 - 2p_1 p_2 - p_1)$ is strictly increasing in p_2:

$$\frac{\partial (W^{\text{FB}}_{\text{ante}} - W^{\text{eq}}_{\text{ante}})}{\partial p_2} = \tfrac{1}{4}(3 - 2p_1) > 0.$$

3. Boundary transitions and non-monotonicity*:*

   (a) *At $p_2 = p_1$: welfare drops discontinuously. For any $p_1 \in (\frac{1}{2}, 1)$:*

$$\lim_{p_2 \nearrow p_1} W^{\text{eq}}_{\text{ante}} = p_1 - \tfrac{1}{2}, \qquad \lim_{p_2 \searrow p_1} W^{\text{eq}}_{\text{ante}} = \begin{cases} 0 & p_1 < \frac{1}{\sqrt{2}}, \\ \frac{1}{2}(p_1^2 + p_1 - 1) & p_1 \geq \frac{1}{\sqrt{2}}. \end{cases}$$

   In both cases the right-hand limit is strictly below $p_1 - \frac{1}{2}$.

   (a) *At $p_2 = \frac{1}{2p_1}$ (exists only when $p_1 < \frac{1}{\sqrt{2}}$): welfare jumps discontinuously* upward *as p_2 increases:*

$$\lim_{p_2 \nearrow \frac{1}{2p_1}} W^{\text{eq}}_{\text{ante}} = 0, \qquad \lim_{p_2 \searrow \frac{1}{2p_1}} W^{\text{eq}}_{\text{ante}} = \tfrac{1}{2}\left(\tfrac{1}{2}p_1 + \tfrac{1}{4p_1} - \tfrac{1}{2}\right) > 0.$$

For $p_1 < \frac{1}{\sqrt{2}}$, welfare is therefore non-monotone *in p_2:*

constant on Region 1, dropping to zero on Region 3, and recovering on Region 2.

Proof. Part 1 is immediate from Proposition 8, case 1.

Part 2: differentiating $W^{\text{eq}}_{\text{ante}} = \frac{1}{2}(p_1p_2+\frac{1}{2}p_1+\frac{1}{2}p_2-1)$ with respect to p_2 gives $\frac{1}{2}(p_1+\frac{1}{2}) < 1$. The gap formula and its derivative follow by direct computation from Proposition 8. Part 3(a): the left limit lies in Region 1 where $W^{\text{eq}} = p_1 - \frac{1}{2}$. For the right limit: when $p_1 \geq \frac{1}{\sqrt{2}}$, any $p_2 > p_1$ satisfies $p_1p_2 > p_1^2 \geq \frac{1}{2}$, placing us in Region 2; evaluating at $p_2 = p_1$ gives $\frac{1}{2}(p_1^2+p_1-1) < p_1 - \frac{1}{2}$ because $p_1^2 < p_1$ for $p_1 < 1$. When $p_1 < \frac{1}{\sqrt{2}}$, any p_2 just above p_1 satisfies $p_1p_2 < \frac{1}{2}$, placing us in Region 3 where $W^{\text{eq}} = 0$. Part 3(b): $\lim_{p_2 \nearrow \frac{1}{2p_1}} W^{\text{eq}} = 0$ since Region 3 holds below. The upper limit follows by evaluating the Region 2 formula at $p_1p_2 = \frac{1}{2}$; positivity follows from AM–GM: $\frac{1}{2}p_1 + \frac{1}{4p_1} \geq \frac{1}{\sqrt{2}} > \frac{1}{2}$. ■

Remark 7 (The Valley of Zero Welfare) *For $p_1 \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$, the non-monotonicity in Part 3 reveals a "valley of zero welfare" in Region 3. The follower's signal is precise enough to make her unwilling to trust the vanguard's action alone, but not precise enough for the joint signal to compensate the vanguard's isolation risk. Only when $p_1p_2 \geq \frac{1}{2}$ does the joint quality of information restore the vanguard's willingness to act. Better follower information is thus neither uniformly good nor uniformly bad: it is destructive in the range $(p_1, \frac{1}{2p_1})$ and beneficial beyond it.*

5.2 Effect of Vanguard Precision (p_1)

Proposition 10 (Comparative Statics in p_1) *Fix $p_2 \in (\frac{1}{2}, 1)$. Within Region 2 (i.e., $p_1 \in (\frac{1}{2p_2}, p_2)$):*

1. *Equilibrium welfare is strictly increasing in p_1:*
$$\frac{\partial W^{\text{eq}}_{\text{ante}}}{\partial p_1} = \tfrac{1}{2}\big(p_2 + \tfrac{1}{2}\big) > 0.$$
2. *The first-best $W^{\text{FB}}_{\text{ante}} = p_2 - \frac{1}{2}$ is independent of p_1: the planner always uses the more precise agent.*
3. *The efficiency gap is strictly decreasing in p_1:*
$$\frac{\partial (W^{\text{FB}}_{\text{ante}} - W^{\text{eq}}_{\text{ante}})}{\partial p_1} = -\tfrac{1}{4}(2p_2 + 1) < 0.$$
4. *As p_1 increases past p_2 (entering Region 1), full efficiency is restored and $W^{\text{eq}}_{\text{ante}} = W^{\text{FB}}_{\text{ante}} = p_1 - \frac{1}{2}$.*
5. *As p_1 decreases past $\frac{1}{2p_2}$ (entering Region 3), welfare drops discontinuously from $\frac{1}{2}(\frac{1}{2}p_2 + \frac{1}{4p_2} - \frac{1}{2}) > 0$ to 0.*

Proof. Parts 1–3 follow by differentiating $W^{\text{eq}}_{\text{ante}} = \frac{1}{2}(p_1p_2 + \frac{1}{2}p_1 + \frac{1}{2}p_2 - 1)$ with respect to p_1. Part 4: as $p_1 \nearrow p_2$, the follower's posterior at $X_2 = L$ reaches exactly $\frac{1}{2} = c$, so ρ_L switches from 0 to 1, eliminating all isolation events and restoring first-best efficiency. Part 5: below $p_1 = \frac{1}{2p_2}$, $p_1p_2 < \frac{1}{2}$ and Proposition 6 applies. The left-limit follows by evaluating the Region 2 formula at $p_1p_2 = \frac{1}{2}$; positivity is the same AM–GM argument as before. ■

Remark 8 (Fundamental Asymmetry Between Agents) *Propositions 9 and 10 together establish a fundamental asymmetry. Within Region 2, improving p_2 raises absolute welfare but* widens *the efficiency gap, while improving p_1 raises absolute welfare and* narrows *it. More strikingly, only p_1 can restore full efficiency (by returning the game to Region 1), whereas no level of p_2 can ever do so from within Region 2. This asymmetry is a direct consequence of the sequential structure: the vanguard's signal bears the public coordination burden, and strengthening it directly addresses the source of inefficiency. Strengthening the follower's signal, by contrast, only makes her more selective, raising her private threshold for joining while doing nothing to reduce the vanguard's isolation risk.*

6 Conclusion

This paper models the notion of Leninist vanguard using a sequential coordination game between a revolutionary vanguard and a follower, each endowed with a private binary signal about the value of a successful revolt. The vanguard acts first and publicly; the follower, observing both the vanguard's costly action and her own signal, decides whether to join. The action succeeds only if both agents act; an isolated vanguard faces punishment regardless of the underlying state.

Our analysis yields a coherent set of results that jointly characterize both the positive and normative properties of the game. On the positive side, using Forward Induction, we establish global equilibrium outcome uniqueness: the parameter space admits up to four cases of equilibrium (co)existence based on the agents' signal precisions, but the application of Forward Induction collapses these into three distinct equilibrium outcomes. The unique equilibrium surviving Forward Induction is a separating equilibrium in Region 1 (which nests Region 4) and in Region 2, and an inactivity pooling equilibrium in Region 3.

On the normative side, our welfare analysis identifies a hierarchy of inefficiency that deepens as the follower becomes better informed relative to the vanguard. When the vanguard is at least as well informed as the follower (Region 1), the equilibrium is first-best efficient: the follower joins unconditionally upon observing the vanguard act, regardless of her own signal, and the equilibrium perfectly replicates the planner's allocation. When the follower is better informed but the two signals are jointly precise enough (Region 2), inefficiency operates through two distinct channels on two separate events, even conditional on the prize being valuable. In the case where the vanguard's signal is low while the follower's is high, coordination fails: the planner, exploiting the follower's superior signal, would direct both agents to act, while in equilibrium the vanguard's low signal leads her to abstain and welfare is zero. In the separate case

where the vanguard's signal is high but the follower's is low, the vanguard acts and is left isolated by the follower, absorbing the punishment cost; the planner avoids this cost entirely by prescribing abstention whenever the follower's signal is low. In Region 3, the inefficiency is total: welfare collapses to zero as the vanguard rationally refuses to initiate a revolt she knows will go unsupported.

The comparative statics of Section 5 sharpen these welfare results into a precise and asymmetric picture. Improving the vanguard's precision always narrows the efficiency gap and restores full efficiency once her precision exceeds the follower's. Improving the follower's precision, by contrast, widens the gap within Region 2 and triggers a discrete welfare drop exactly at the point where the two precisions become equal. If the vanguard's own precision is still below one over the square root of two at that point, this drop is a collapse to zero, and coordination is only partially restored once the product of the two precisions reaches one half. If instead the vanguard's own precision is already at or above that threshold, crossing the point where the two precisions become equal produces a downward jump but not a collapse to zero: the product of the two precisions is then automatically above one half, so the game moves directly into Region 2 and Region 3 cannot occur. Welfare is therefore non-monotone in the follower's precision: constant in Region 1, zero in Region 3, and positive but inefficient in Region 2, with a "valley of zero welfare" (Region 3) separating the two. This non-monotonicity is the sharpest formal expression of the paper's central paradox: arming the follower with better information is not merely unhelpful; it is, in an intermediate range, strictly counterproductive for the movement. We believe that these results support the idea that a Leninist vanguard is effective if and only if it is better informed than the followers.

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