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Abstract

We analyse the effect of learning by doing on firm performances when profit maximization follows a rule of thumb. Three regimes are compared: the technology sharing cartels, the oligopoly with spillovers, the proprietary regime. We show the dynamic implications on the industrial structure when firm production plan is revisited period by period.

Keywords: Oligopoly, Cartel, Industrial Structure, Learning, Dynamic Behaviour, Rule of Thumb.

JEL classification: L10, L13.

1 Introduction

A common assumption in game-theoretical models is that agents have perfect knowledge of the environment in which they act without any cognitive limitation: they know the consequences of their actions and of the actions of their competitor; consequently any consideration pertaining to whether and how agents will be able to arrive at some optimal equilibrium have been fully abstracted from. The success of this approach in oligopoly theory is due both to analytical tractability and conceptual reasons.

In a real oligopolistic context, players knowledge of the underlying game may be erroneous on several accounts; for instance, they may have only an estimate of the demand function in their market, or imperfect or lagged information about the production of rival firms. Such considerations lead to the question of what should be the reasonable features of the dynamic behaviour of the players and under which conditions the dynamic adjustment converges to a Nash equilibrium.

A central question in the literature on learning and adaptive process in dynamic games is whether the repeated interaction between players will eventually lead the system, in the long run, to the Nash equilibrium. In this growing literature there is an explicit description of the possible ways players attempt to learn the game, to recognize the behaviour of others, or to adapt over time (e.g., through reinforcement, imitation, belief updating).

We consider a very simple learning process that requires a very low cognitive effort of players and does require information about only previous rivals' actions and previous game payoff functions. We assume that the players behave as local maximizers: at each time period they adjust their quantities over time, proportional to their marginal profits. The players increase or decrease their strategy choice in response to profitability signals derived from marginal profits of the previous period. This kind of adjustment mechanism has been proposed by a few authors, in continuous-time formulations (see, for example, Arrow et al. (1958), Corchon and Mas-Colell (1996)), and in discrete-time framework (see, for example, Bischi and Naimzada (2000)).

We study the consequences of this learning process in the context of a quantity-setting duopoly with homogeneous goods where players can learn both by doing and from each other. It is widely recognized that production learning effects are not entirely firm-specific; indeed, they may spill over from one firm to another in many ways (Arrow(1962), Spence(1981)). Technological spillovers may be the result of explicit cooperation contracts between firms of a specific industrial sector. Many recent studies consider the consequences of sharing knowledge on market prices and quantities in a static context (recently among others Dasgupta and Stiglitz (1988), Katz and Ordover (1990), Baumol (1992)).

In a dynamic setting, Petit and Tolwinski (1992) consider a duopolistic framework with homogenous products where the spillover phenomena may take the form of full knowledge transfer; Tolwinski and Zaccour (1995) extend the framework by considering differentiated products and more general and realistic spillover scenarios.

Our model is presented in a discrete setting over an infinite time horizon as a two-person dynamic game with different assumption on spillover effects. The players' decision variables are quantities to produce, that are updated every period according to a local and correct estimate of the marginal profit obtained from the previous period, for example through market experiments.

In Section 2 is formulated the general framework of the model. In Section 3 we describe the adjustment process based on a rule of thumb mechanism and the two scenarios that can occur. In section 4 we present the simulations of the asymmetric case, while in Section 5 the symmetric case is discussed. Interpretation of the results concludes the paper.

2 The Model

The general framework of our model consists in two firms, producing a homogeneous good with constant returns to scale technology. The production process of each firm is influenced by a learning by doing process resulting in a reduction of the unit cost as the cumulative production of the firm increases. The unit production cost is further reduced if technological spillovers arise between firms. We are implicitly assuming complementarity of the firm innovation process: exchanging information is beneficial for each firm. The presence of technological spillovers and its intensity distinguish the three different regimes of our analysis. If the firm specific technological information is not spread in the economic system, the *proprietary regime* is defined. In this case strong information protection is assumed, preventing any outside information flow of the innovation process adopted by the firm. Instead if involuntary technological spillovers occur between firms, we are in the case of a *duopoly* in which the production process is influenced by a positive externality effect. On the contrary, voluntary and shared spillovers define our third regime, the *technological sharing cartel* setting (TSC).

In every regime, each firm is facing a dynamic maximisation problem in discrete time *à la Cournot.* We are assuming a bounded rationality framework. The firm is not maximising the present discounted value of the profit over an infinite horizon, but in each period the firm is adjusting its production according to its marginal profit of the previous period, assuming that the rival profit is constant. The aim of this paper is to analyse the consequences of this assumption on the industrial structure of the economic system.

To fix the notation, let $i = 1, 2$ the firm index and $q_{it} \geq 0$ the output produced by firm i at time t. Since we are assuming a dynamic Cournot game, q_{it} is also the control variable of our model. The aggregate quantity produced at time t is defined as: $Q_t = q_{1t} + q_{2t}$. We assumed a constant elasticity demand function of the type: $p(Q_t) = \frac{A}{Q_t^\beta}$ with $\beta = 1/B$ where B is the demand elasticity. The cumulative output equation is given by:

$$w_{it+1} = w_{it} + q_{it} \tag{1}$$

where w_{it} is the total output accumulated at time t, interpreted as a proxy of the firm level of experience in term of innovation process capability. The state variable w_{it} influences the cost condition of the firm, reducing the unit cost of production. With the presence of technological spillovers, the unit production cost is further reduced. The learning curve of each firm in the more general setting when there are involuntary technological spillovers is represented as follows:

$$c_i(w_i) = c_i^0(1 + w_i + \alpha w_j)^{-D_i} + c_i^{min} \tag{2}$$

where c_i^{min} is the asymptotic value of the marginal cost function, α indicates the intensity of the positive spillover externality between firm, D_i the rate of cost decreasing occurring in the production process. In this case, thus we are in the second regime previously mentioned: we are depicting a Cournot duopoly with involuntary transmission of technological experience between firms. The unit cost reduction is not only caused by the firm specific learning by doing process (the firm specific cumulative experience w_i) but also by involuntary changing of information (w_j). If, instead, the information is voluntary fully shared ($\alpha = 1$), the learning curve of the firm becomes:

$$c_i(w_i) = c_i^0(1+w)^{-D_i} + c_i^{min} \quad (3)$$

where $w = w_1 + w_2$ is the aggregate cumulative output. The law of motion of w is given by:

$$w_{t+1} = w_t + q_{1t} + q_{2t} \quad (4)$$

In this case, firms agree to form technological consortium in order to take advantages in fully interchange the firm specific technological experience. This is the case of the *technological sharing cartel* regime. On the contrary, if $\alpha = 0$, we are in the opposite case where the technological experience is fully protected by the firm: any exchange of information is not allowed. No spillovers arise in the system. This condition defines the *proprietary regime*, in which the cost reduction is due only to the own firm cumulative output. More specifically:

$$c_i(w_i) = c_i^0(1+w_i)^{-D_i} + c_i^{min} \quad (5)$$

The profit maximisation function differs in each regimes. In the *proprietary regime* (P) the profit function in time t is given by:

$$\pi_{it}^P = q_{it}[A(q_{1t}+q_{2t})^{-\beta} - c_i^0(1+w_i)^{-D_i} + c_i^{min}] \quad (6)$$

In the *duopoly with technological spillovers* (DRS) instead the profit function is as follow:

$$\pi_{it}^{DRS} = q_{it}[A(q_{1t}+q_{2t})^{-\beta} - c_i^0(1+w_i+\alpha w_j)^{-D_i} + c_i^{min}] \quad (7)$$

In the technological sharing cartels (TSC), the profit is defined as:

$$\pi_{it}^{TSC} = q_{it}[A(q_{1t}+q_{2t})^{-\beta} - c_i^0(1+w)^{-D_i} + c_i^{min}] \quad (8)$$

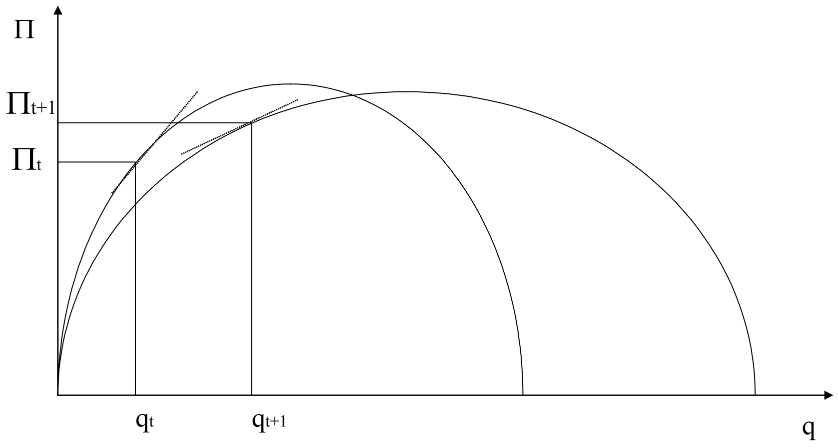

Figure 1: Firm decisional process when the marginal profit is positive.

3 The Dynamics

On the contrary with the literature of this field of research (Tolwinski and Zaccour (1995), Petit and Tolwinski (1996) (1998)), we are assuming a bounded rationality framework, where the two producers have no global knowledge of the market. So they are not able to reach a Nash equilibrium in one shot.They have to behave following a rule of thumb adjustment process based only on a local knowledge of the marginal profit of the previous period, $\partial\pi_{it}^{R}\backslash q_{it}^{R}$, obtained, for example, through market research. In each regime *R*= *P*, *DRS*, *TSC,* a firm decides to increase its production in time $t+1$ if it perceives a positive marginal profit in the previous period and to decrease its production if the marginal profit is negative:

$$q_{it+1}^{R} = q_{it}^{R} + v\frac{\partial\pi_{it}^{R}}{\partial q_{it}^{R}}.$$

Graphically, the dynamical behaviours analysed in this paper can be qualitatively interpreted by the following figures (Figure 1, Figure 2). In Figure 1 is represented the situation of the firm in which the profit area increases

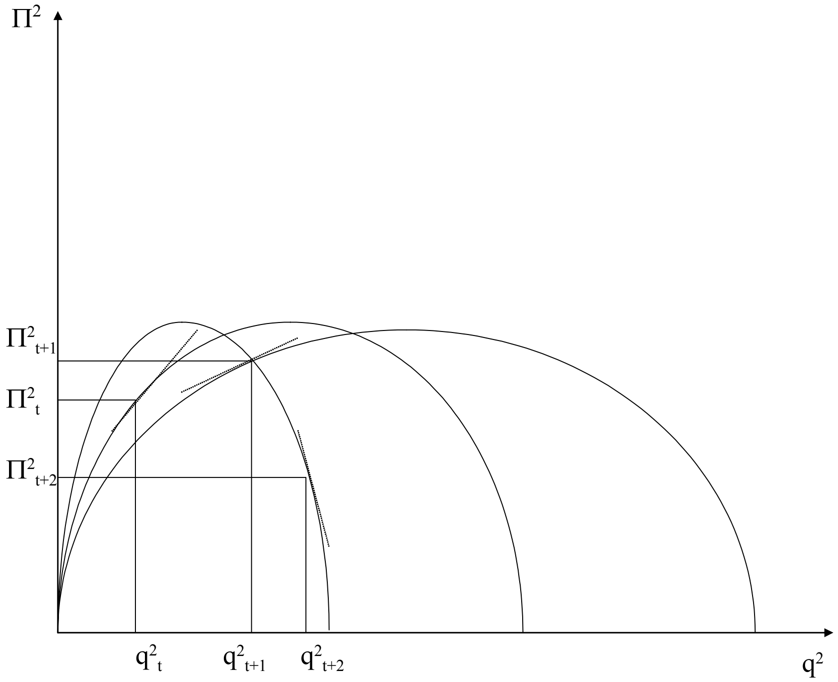

Figure 2: Firm 2 decisional process when the marginal profit is negative in $t+2$.

period by period due to the own learning by doing effect and or the positive spillover of the other firm. If in period t the firm produces q_t with positive marginal profit, in the following period it will expand its production to q_{t+1} according to the adaptive adjustment process previously described. At this production level, the marginal profit are still positive because of the expansion of its profit area resulting from learning by doing activities and eventual positive spillover effects. Such benchmark can be used to describe both the advantage firm behaviour period by period and the behaviour of the less advantaged firm when the positive effect of learning by doing and eventual spillovers overcome the negative effect on profits induced by the increasing market power of the rival.

In the Figure 2, instead, we show the case in which the firm is forced to exit the market since its market conditions are highly unfavourable. Indeed, the marginal profits in q_{t+2} are negative because the marginal cost reduction does not compensate the profit area reduction due to the market power of the other firm. In the following period the firm is forced to decrease its production. A vicious process will start: the production is reduced period by period till the exit of the firm from the market.

4 The Asymmetric Case

In this case we compare the time paths of price, individual quantity and profit of the firms under the different three regimes assuming asymmetric cost conditions. The asymmetry in cost conditions may arise from three different reasons. Firstly, we consider the effect of different initial unit costs on the variable paths ($c_1^0 \neq c_2^0$). Secondly, the effect of different asymptotic values of the unit cost are analysed ($c_1^{min} \neq c_2^{min}$). Thirdly, we focus on differences in term of the rate of cost decreasing ($D_1 \neq D_2$).

In all the simulations performed, we have tried to select numerical values of the initial conditions and of the parameters as more sensitive as possible. We have assigned the value of 1.5 for the elasticity of demand since it is supported by empirical studies (see Malerba (1992)). Considering that different magnitudes of the demand scaling parameter A and v do not qualitatively affect the variable paths, we have chosen $A = 10$ and $v = 0.8$. The initial individual quantities have been set equal to 1 and the initial cumulative experience null for both the firm. In each simulation we have performed 101 iterations.

In Figure 3, 4, 5 we present the time paths respectively for the individual quantity, price and individual profit, when the source of asymmetry is given only by a small difference in the initial costs. We have assumed an initial cost advantage for firm 1, fixing $c_1^0 = 1$ and $c_2^0 = 3$. The values of the other cost parameters are identical for each firm: $D_1 = D_2 = 0.3$[1] and $c_1^{min} = c_2^{min} = 0.1$. In this case, only the firm having a comparatively unfavorable position has incentives to create the technology sharing cartel. After few periods, it can improve its economic conditions since having access to the experience of the other firm can shortly diminish its cost disadvantage. Thus, it is not in the interest of the dominant firm to join the cartel. In brief time, its market

[1] This numerical value is supported by empirical evidence (see Malerba (1992)).

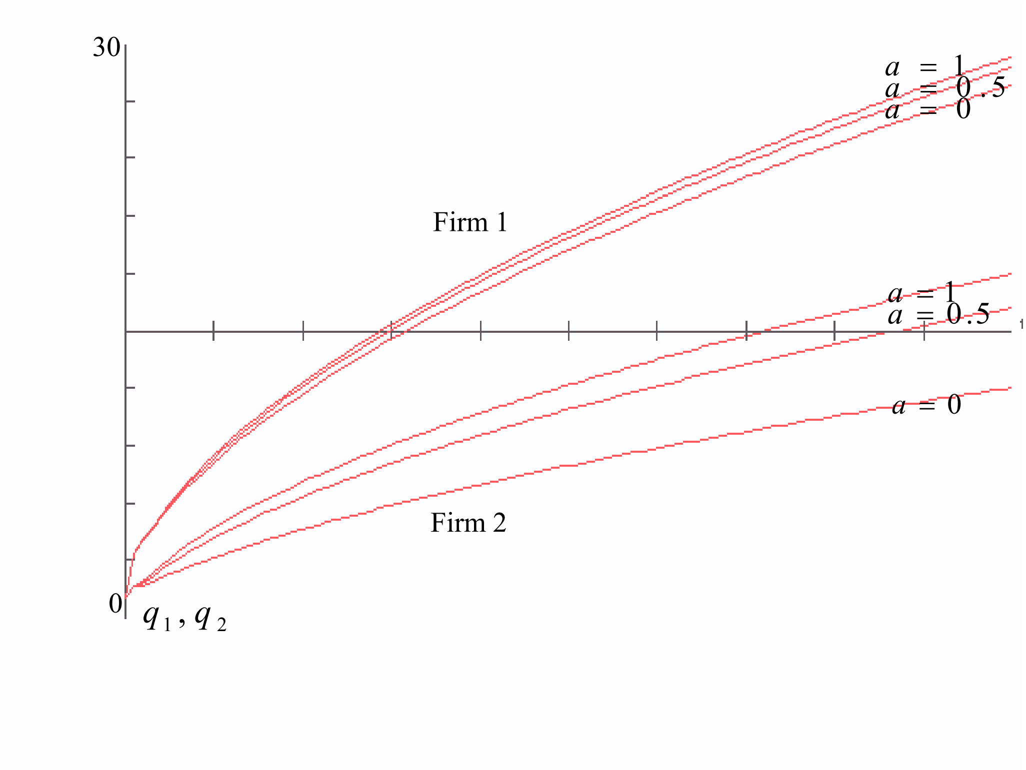

Figure 3: Time paths of each firm output under a small difference in initial costs ($c_1^0 = 1$ and $c_2^0 = 3$) and different spillover effects.

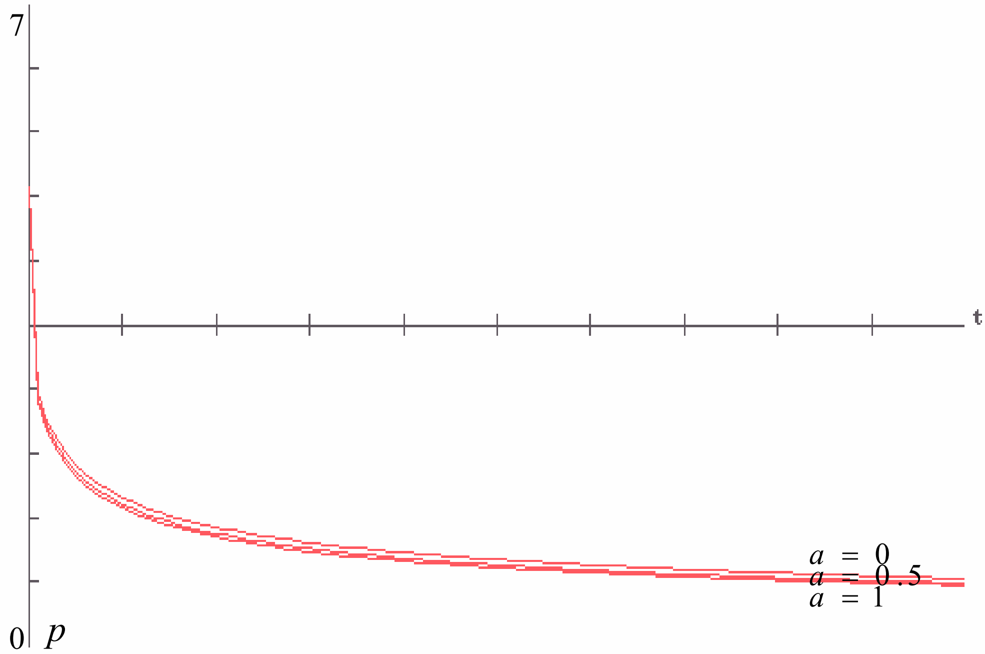

Figure 4: Time paths of prices under a small difference in initial costs ($c_1^0 = 1$ and $c_2^0 = 3$) and different spillover effects.

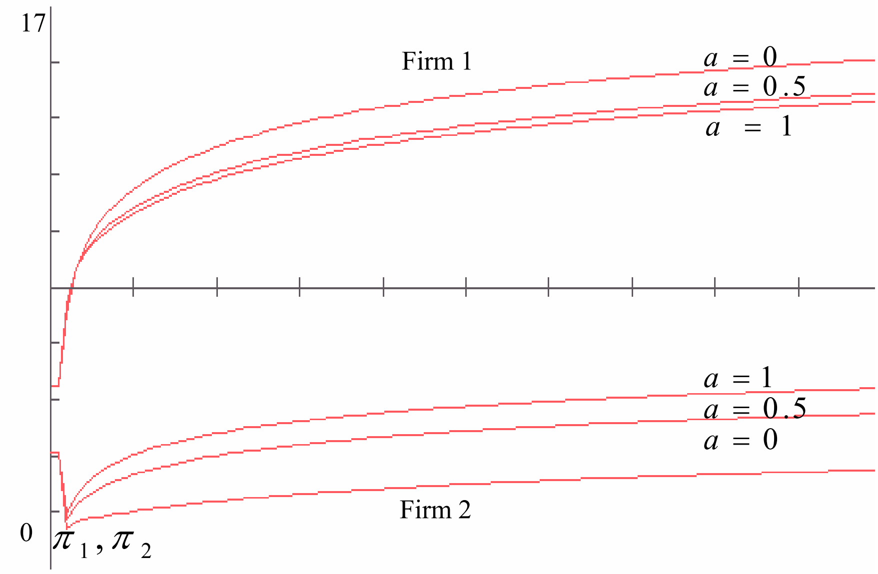

Figure 5: Time paths of each firm profits under a small difference in initial costs ($c_1^0 = 1$ and $c_2^0 = 3$) and different spillover effects.

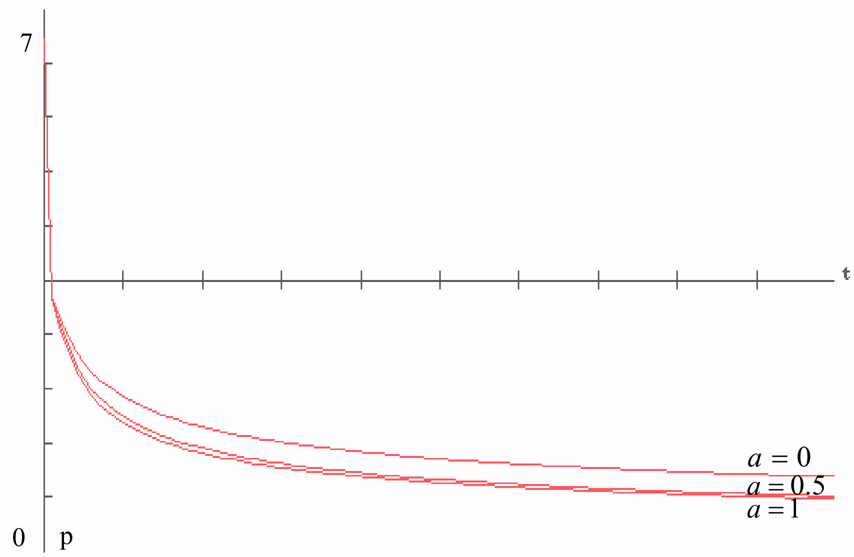

Figure 6: Time paths of prices under a bigger asymmetry in the initial costs ($c_1^0 = 0.5$ and $c_2^0 = 3.6$) and different spillover effects.

power position can be threatened by the rival . The dominant firm prefers to keep its knowledge privately: the cost reduction due to the accumulated quantity of the competitor does not compensate the profit reduction due to the increasing favourable position of the rival. In this case, monopolisation never occurs in the market.

Different results arise if the difference in initial cost is bigger (in Figure 7 we show the profit behaviors when $c_1^0 = 0.5$ and $c_2^0 = 3.6$). The results of the literature in this field that voluntary or involuntary spillovers can prevent monopolisation in the market are still confirmed. Circulation of know-how lowers the price level and increases the quantity, improving social welfare (see Figure 6 and 7). In addition, both the firms have great incentives to enter into a knowledge agreement (see Figure 7). Individual profits are higher in

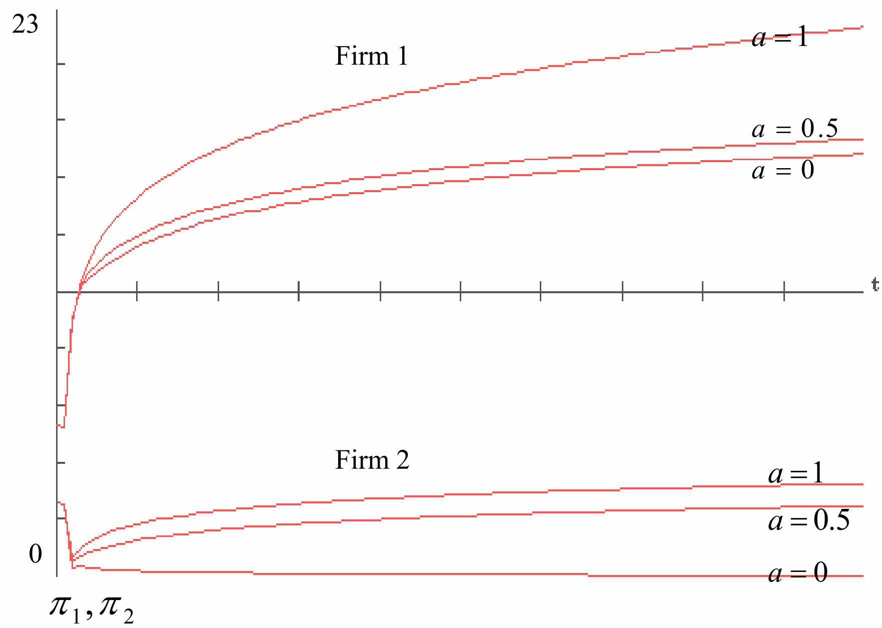

Figure 7: Time paths of individual profits under a bigger asymmetry in the initial costs ($c_1^0 = 0.5$ and $c_2^0 = 3.6$) and different spillover effects.

the technological sharing cartel ($\alpha_1 = \alpha_2 = 1$) than in the duopoly with technological spillover ($\alpha_1 = \alpha_2 = 0.5$) and in the case in which information is not spread in the market ($\alpha_1 = \alpha_2 = 0$) even for the dominant firm (see Figure 7). In this case the initial cost difference is so consistent that the dominant firm does not occur the risk of loosing the benefits of its market power position.

We are now analysing the effect on the variable trajectories assuming different asymptotic values of the unit cost parameters. We have set the initial cost for each firm equal to 3, while all the other numerical values are identical to the simulations previously presented. It is remarkable to notice that in this case only with a difference of the parameter close to

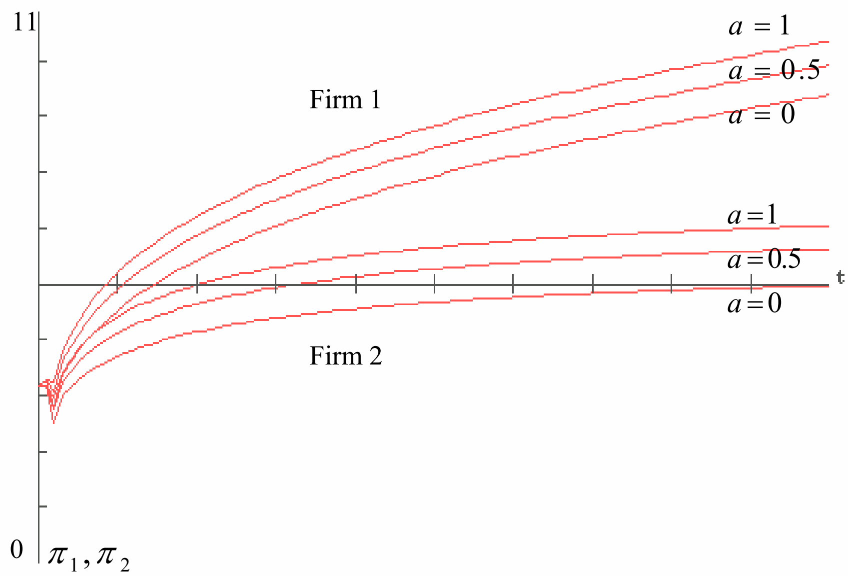

Figure 8: Time paths of each firm profits under a small difference in the asymptotic value of the marginal cost ($c_1^{min} = 0$ and $c_2^{min} = 0.1$) and different spillover effects.

the symmetric case ($c_1^{min} = 0$ and $c_2^{min} = 0.1$) the market benefits from technological transfers between firms (Figure 8) and monopolisation does not occur.

The less advantaged firm still will operate in the long run in the market even if there isn't knowledge dissemination. Only assuming a slightly bigger difference in the minimum cost ($c_1^{min} = 0$ $c_2^{min} = 0.9$), the less advantaged firm will leave the market in all the three regimes analysed (Figure 9). The cost gap difference can not be compensated by learning by doing cost reduction even in the case of voluntary sharing knowledge. The less advantaged firm periodically reduces the quantity produced since it always faces negative

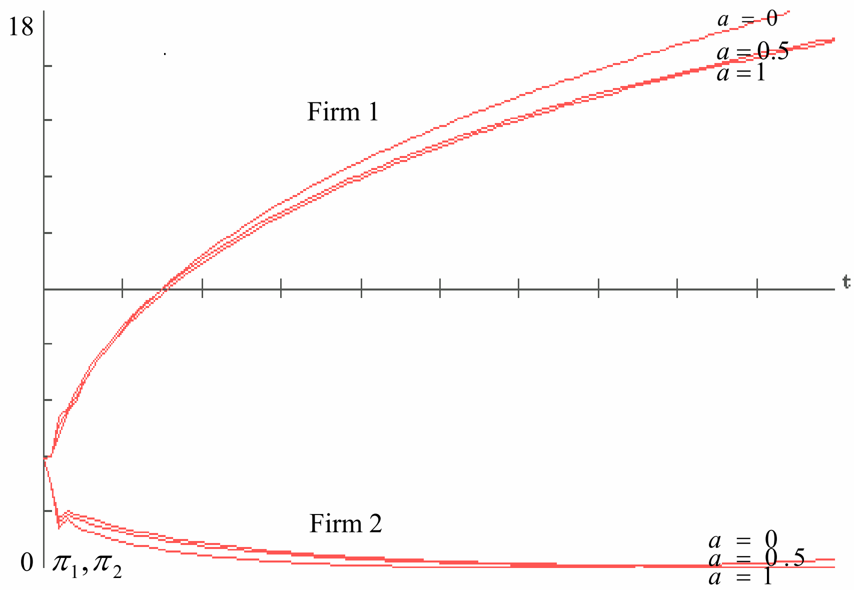

Figure 9: Time paths of each firm profits under a bigger asymmetry in the asymptotic value of the marginal cost ($c_1^{min} = 0$ and $c_2^{min} = 0.9$) and different spillover effects.

marginal profits. In this case the dominant firm has significant incentives to protect its know how because of the negligible benefits of the learning by doing activity of the competitor.

As the last source of asymmetry, we consider different learning rates. Figure 10 represents the long run behavior of the profits assigning a small difference in the rate of learning ($D_1 = 0.35$ $D_2 = 0.3$): each firm maximises its profit under TSC and the less advantaged firm survives even in the proprietary regime. As before, the results are quite different if the magnitude of cost asymmetry is bigger.

In Figure 11 we can observe the time paths of the individual profits when

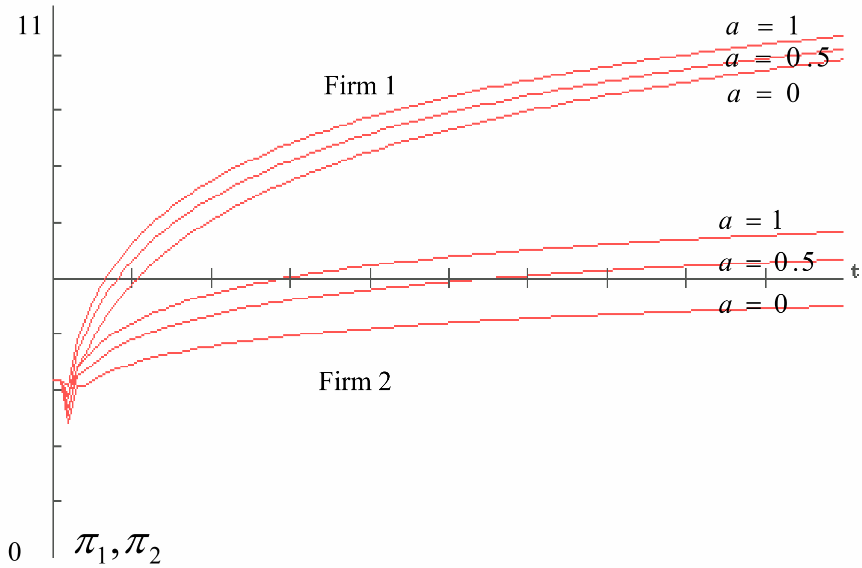

Figure 10: Time paths of each firm profits under a small difference in learning rates and different spillover effects.

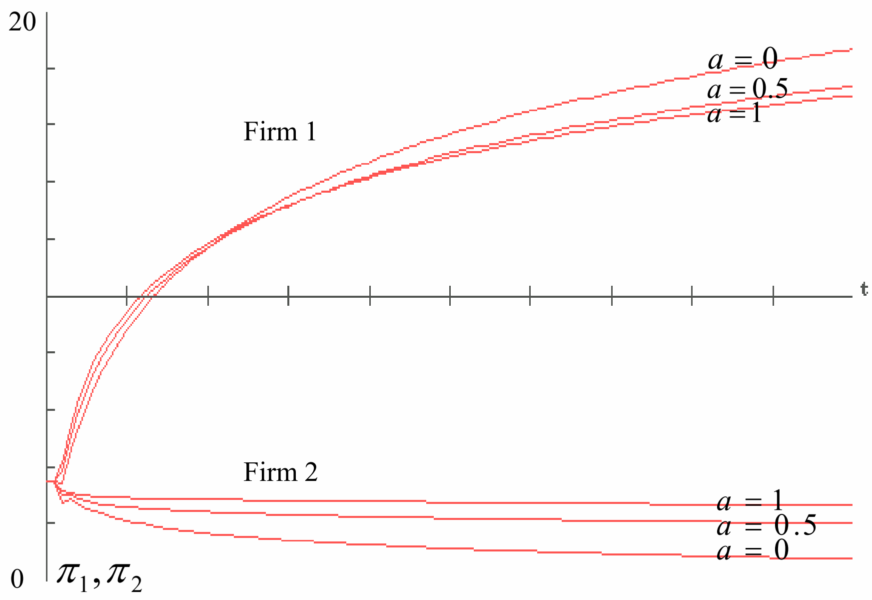

Figure 11: Time paths of each firm profits under a bigger asymmetry in learning rates ($D_1 = 0.25$ $D_2 = 0.5$) and different spillover effects.

$D_1 = 0.25$ $D_2 = 0.5$. In this case only the access to the technological experience of the dominant firm can prevent the market exit of the less advantaged firm. But the dominant firm has very little incentive to join the technology cartels since its profits are higher in the proprietary regime. In this case only an institutional intervention can encourage technological sharing agreements.

5 The Symmetric Case

After having discussed the asymmetric cases, in this section we present the variable trajectories assuming that the firms operate with identical market

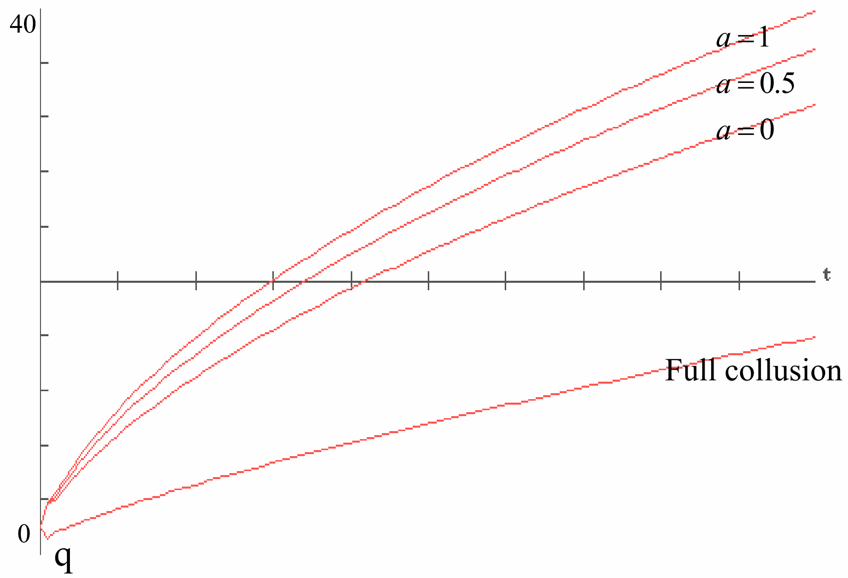

Figure 12: Output of the firm in different regimes and in the full collusion.

characteristics and conditions. In this case the results of the dynamic cartels analysed by Petit & Tolwinski (1996) maximising the actual value of the flow of profits are substantially confirmed even assuming a bounded rationality context. The performances of the three regimes are valuated also comparing them with the monopoly situation (the collusion case: see the appendix for a detailed description). We have selected the following values for the cost parameters: $c_1^0 = c_2^0 = 2$, $D_1 = D_2 = 0.3$, $c_1^{min} = c_2^{min} = 0.1$. The numerical values of the other parameters are identical to the previous section ($A = 10$, $v = 0.8$, $B = 1.5$). We have assumed identical initial conditions in each market structure analysed ($q_{10} = q_{20} = 1$ and $w_{10} = w_{20} = 0$). Under this conditions, the technology sharing cartels generates higher levels of quantities and lower prices than the other market configurations both in the short and in the long run (see Figure 12 and Figure 13).

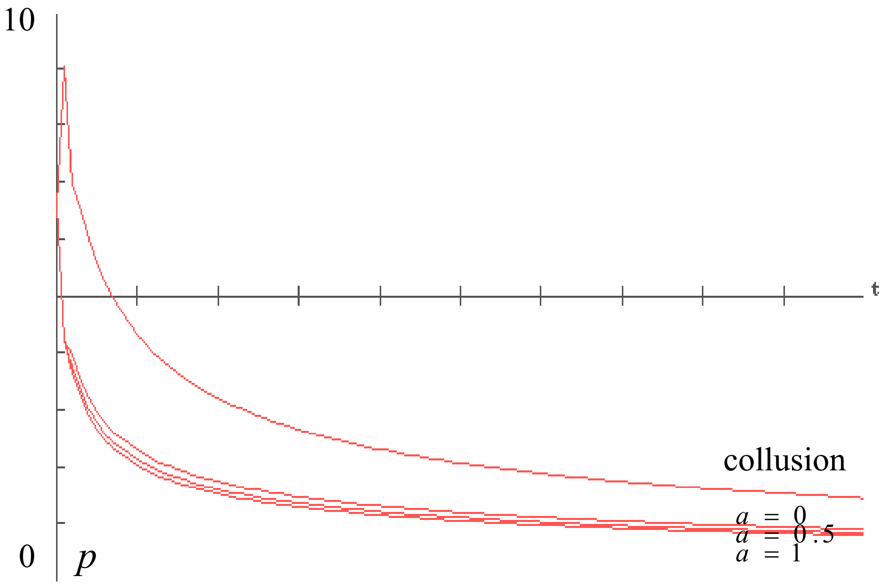

Figure 13: Prices under different regimes and the full collusion.

It doesn't seem that the free-rider effect in the transmission of knowledge analysed by Spence (1981) and Fudenberg & Tirole (1983, 1986) arise in this context. In fact both the firms have great incentives in investing in learning by doing activity from the beginning. The aggressiveness of the firm in term of output target doesn't change from the short to the long run period. A possible interpretation of this fact can be found in the particular decisional process adopted by the firms. In each period the firm decides its output target on the basis of the marginal profit of the previous period. Both the firm are periodically responding with the same intensity to the marginal profit obtained ($v_1 = v_2$). The parameter v can be interpreted as a proxy of the firm aggressiveness, that in our case is constant in every single interaction computed.

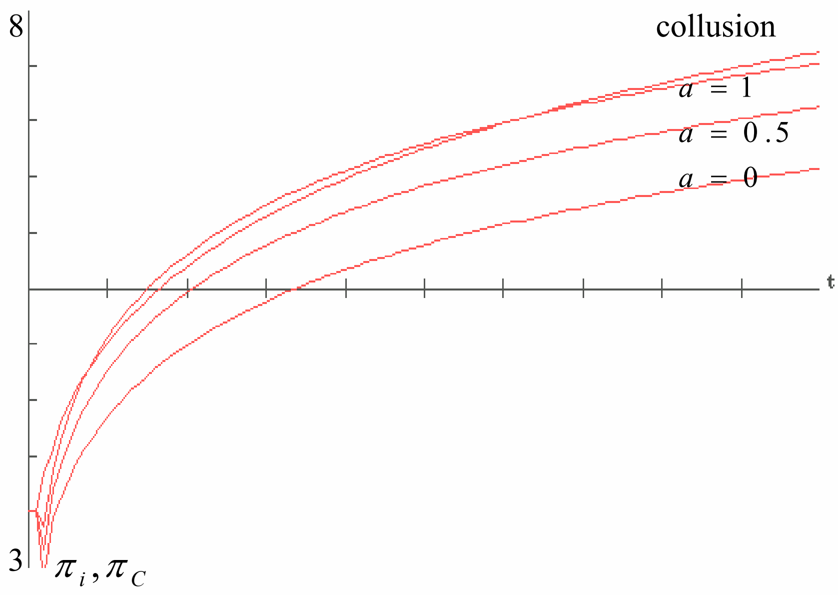

Figure 14: Profits of the firm in different regimes and in the full collusion case.

In brief, observing figure 12, 13 the standard results of the beneficial effects of voluntary knowledge transmission on social welfare are thus confirmed since they can guarantee higher quantity and lower price in comparison to the other market structure. But surprisingly, the profit trajectory of the technological sharing cartel is similar to the collusion case: only in the very short run the monopoly profits are higher than in the case in which there is an agreement in the knowledge transmission. They cross twice: after few interactions and in the long run. In the middle period the technological consortium can even lead to higher profits than the collusion case (see Figure 14).

6 Interpretation and Conclusion

This paper analyses the importance of technological spillover and TSC agreements in a dynamic setting, in which two firms adopt a rule of thumb decisional process. Output of each firm is revised period by period according to profitability signals of the previous period. The long run behaviour of the market variables is different according to the source of asymmetry in the cost function and the entity of this asymmetry.

From our simulations, the principal results of the asymmetric case can be summarised as follows:

- in the case of a mild asymmetry in the initial costs, the monopolisation never occurs despite the voluntary or involuntary degree of information transfers between the firms (Figure 3,4,5);

- we observe also that, if the gap in the initial costs is low, the dominant firm does not have any incentive to join the Technology Sharing Cartel. In this case, the less advantaged firm has more benefits form the access to the dominant firm know-how. The increase in profits of the dominant firm due to the technological agreement does not compensate the loss of profit due to the increasing market power of the other firm (Figure 5);

- if instead the asymmetry in the initial costs is larger, only the flow of knowledge from the dominant firm (both in the duopoly with involuntary spillover and in the sharing cartel) can prevent market monopolisation (Figure 7);

- both the firms have strong incentives to join the cartel if the initial cost gap is larger. The dominant firm acts in a so favourable position than it can be never threatened by the improving economic conditions of the rival (Figure 7);

- under a low difference in the asymptotic value of the marginal cost or in the rate of learning, even if monopolisation never occurs, it is in the interest of both the firms to make the Technological Sharing Cartel (Figure 8, Figure 10);

- if the asymmetry in the asymptotic value of the marginal cost is bigger, no kind of information flow can prevent market monopolisation (Figure 9);

- if the bigger asymmetry is due to rate the of learning, knowledge transmission is beneficial since can make the weaker firm survive. But, the dominant firm perceives bigger profits protecting its knowledge due to learning by doing activity (Figure 11).

  In the symmetric case, the following conclusions are remarkable:

- the profit level of the firm acting in a technology sharing cartel is very close to the profit level perceived when the firm colludes. But the individual quantity in the case of collusion is sensitively lower that in the case of the Technology Sharing Cartel. Consequently, the level of price in the Technological Sharing Cartel scenario is lower than in the collusion case.

In this paper we analyse the impact of learning spillovers on prices, profits and market structure when the decision mechanism of the duopolist is based on a rule of thumb. The next step should be to build a model in which the flow of information is the result of specific R&D decisions. In the real market, we observe the coexistence of firms participating to TSC and firms outside it; we think that models to describe this stylised fact can be useful.

A Appendix

A.1 The Symmetric Case

A.1.1 The Proprietary Regime

Demand function is given by:

$$p(Q_t) = \frac{A}{Q_t^{\beta}}$$

The firm 1 profit function is $(\pi_{1t}^{P}; x3)$:

$$\pi_{1t}^{P} = q_{1t}p_t - [c^0(1 + w_{1t})^{-D} + c^{min}]q_{1t}$$

$$\pi_{1t}^{P} = \frac{Aq_{1t}}{(q_{1t} + q_{2t})^{\beta}} - [c^0(1 + w_{1t})^{-D} + c^{min}]q_{1t}$$

Firm 2 profit function is$(\pi^P_{2t}; x4)$:

$$\pi^P_{2t} = q_{2t}p_t - [c^0(1+w_{2t})^{-D} + c^{min}]q_{1t}$$

$$\pi^P_{2t} = \frac{Aq_{2t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_{2t})^{-D} + c^{min}]q_{2t}$$

Marginal profit for firm 1 is:

$$\frac{\partial \pi^P_{1t}}{\partial q_{1t}} = \frac{A(q_{1t}+q_{2t})^{\beta} - Aq_{1t}\beta(q_{1t}+q_{2t})^{\beta-1}}{(q_{1t}+q_{2t})^{2\beta}} - [c^0(1+w_{1t})^{-D} + c^{min}]$$

$$\frac{\partial \pi^P_{1t}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(q_{1t}+q_{2t}) - \beta q_{1t}\right] - [c^0(1+w_{1t})^{-D} + c^{min}]$$

$$\frac{\partial \pi^P_{1t}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_{1t})^{-D} + c^{min}]$$

Symmetrically, marginal profit for firm 2 is:

$$\frac{\partial \pi^P_{2t}}{\partial q_{2t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_{1t})^{-D} + c^{min}]$$

Equation of output produced by firm 1 $(q_{1t}; x5$:)

$$q_{1t+1} = q_{1t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_{1t})^{-D} + c^{min}]\}$$

Equation of output produced by firm 2 $(q_{2t}; x6)$:

$$q_{2t+1} = q_{2t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{2t} + q_{1t}\right] - [c^0(1+w_{2t})^{-D} + c^{min}]\}$$

Equation of aggregate quantity $(Q_t; x1)$:

$$Q_{t+1} = Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] - [c^0(1+w_{1t})^{-D} + c^0(1+w_{2t})^{-D} + 2c^{min}]\}$$

Price equation $(p_t; x2)$:

$$p_{t+1} = \frac{A}{\{Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] - [c^0(1+w_{1t})^{-D} + c^0(1+w_{2t})^{-D} + 2c^{min}]\}\}^{\beta}}$$

The cumulative output equation for firm 1 is given by $(w_{1t}; x7)$:

$$w_{1t+1} = w_{1t} + q_{1t}$$

The cumulative output equation for firm 2 is given by $(w_{2t}; x8)$:

$$w_{2t+1} = w_{2t} + q_{2t}$$

A.1.2 Duopoly with Spillovers

Demand function is given by:

$$p(Q_t) = \frac{A}{Q_t^{\beta}}$$

The firm 1 profit function is $(\pi_{1t}^{DS}; x3)$:

$$\pi_{1t}^{DS} = q_{1t}p_t - [c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^{min}]q_{1t}$$

$$\pi_{1t}^{DS} = \frac{Aq_{1t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^{min}]q_{1t}$$

Firm 2 profit function is($\pi_{2t}^{DS}; x4$):

$$\pi_{2t}^{DS} = q_{2t}p_t - [c^0(1 + w_{2t} + \alpha w_{1t})^{-D} + c^{min}]q_{2t}$$

$$\pi_{2t}^{DS} = \frac{Aq_{2t}}{(q_{1t} + q_{2t})^{\beta}} - [c^0(1 + w_{2t} + \alpha w_{1t})^{-D} + c^{min}]q_{2t}$$

Marginal profit for firm 1 is:

$$\frac{\partial \pi_{1t}^{DS}}{\partial q_{1t}} = \frac{A(q_{1t} + q_{2t})^{\beta} - Aq_{1t}\beta(q_{1t} + q_{2t})^{\beta-1}}{(q_{1t} + q_{2t})^{2\beta}} - [c^0(1 + w_{1t} + \alpha w_{2t})^{-D} + c^{min}]$$

$$\frac{\partial \pi_{1t}^{DS}}{\partial q_{1t}} = A(q_{1t} + q_{2t})^{-(\beta+1)}\left[(q_{1t} + q_{2t}) - \beta q_{1t}\right] - [c^0(1 + w_{1t} + \alpha w_{2t})^{-D} + c^{min}]$$

$$\frac{\partial \pi_{1t}^{P}}{\partial q_{1t}} = A(q_{1t} + q_{2t})^{-(\beta+1)}\left[(1 - \beta)q_{1t} + q_{2t}\right] - [c^0(1 + w_{1t} + \alpha w_{2t})^{-D} + c^{min}]$$

Symmetrically, marginal profit for firm 2 is:

$$\frac{\partial \pi_{2t}^{DS}}{\partial q_{2t}} = A(q_{1t} + q_{2t})^{-(\beta+1)}\left[(1 - \beta)q_{1t} + q_{2t}\right] - [c^0(1 + w_{2t} + \alpha w_{1t})^{-D} + c^{min}]$$

Equation of output produced by firm 1 ($q_{1t}; x5$:)

$$q_{1t+1} = q_{1t} + v\{A(q_{1t} + q_{2t})^{-(\beta+1)}\left[(1 - \beta)q_{1t} + q_{2t}\right] - [c^0(1 + w_{1t} + \alpha w_{2t})^{-D} + c^{min}]\}$$

Equation of output produced by firm 2 ($q_{2t}; x6$):

$$q_{2t+1} = q_{2t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{2t}+q_{1t}\right] - [c^0(1+w_{2t}+\alpha w_{1t})^{-D} + c^{min}]\}$$

Equation of aggregate quantity ($Q_t; x1$):

$$\begin{aligned} Q_{t+1} &= Q_t + v\{A\left[(2-\beta)q_t^{-\beta}\right] + \\ &\quad -[c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^0(1+w_{2t}+\alpha w_{1t})^{-D} + 2c^{min}]\} \end{aligned}$$

Price equation ($p_t; x2$):

$$p_{t+1} = \frac{A}{\begin{array}{c}\{Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] + \\ -[c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^0(1+w_{2t}+\alpha w_{1t})^{-D} + 2c^{min}]\}\}^{\beta}\end{array}}$$

The cumulative output equation for firm 1 is given by ($w_{1t}; x7$):

$$w_{1t+1} = w_{1t} + q_{1t}$$

The cumulative output equation for firm 2 is given by ($w_{2t}; x8$):

$$w_{2t+1} = w_{2t} + q_{2t}$$

A.1.3 Technology Sharing Cartels

Demand function is given by:

$$p(Q_t) = \frac{A}{Q_t^{\beta}}$$

The firm 1 profit function is ($\pi_{1t}^{TSC}; x3$):

$$\pi_{1t}^{TSC} = q_{1t}p_t - [c^0(1+w_t)^{-D} + c^{min}]q_{1t}$$

$$\pi_{1t}^{TSC} = \frac{Aq_{1t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_t)^{-D} + c^{min}]q_{1t}$$

Firm 2 profit function is($\pi_{2t}^{TSC}; x4$):

$$\pi_{2t}^{TSC} = q_{2t}p_t - [c^0(1+w_t)^{-D} + c^{min}]q_{2t}$$

$$\pi_{2t}^{TSC} = \frac{Aq_{2t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_t)^{-D} + c^{min}]q_{2t}$$

Marginal profit for firm 1 is:

$$\frac{\partial \pi_{1t}^{TSC}}{\partial q_{1t}} = \frac{A(q_{1t}+q_{2t})^{\beta} - Aq_{1t}\beta(q_{1t}+q_{2t})^{\beta-1}}{(q_{1t}+q_{2t})^{2\beta}} - [c^0(1+w_t)^{-D} + c^{min}]$$

$$\frac{\partial \pi_{1t}^{TSC}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(q_{1t}+q_{2t}) - \beta q_{1t}\right] - [c^0(1+w_t)^{-D} + c^{min}]$$

$$\frac{\partial \pi_{1t}^{TSC}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_t)^{-D} + c^{min}]$$

Symmetrically, marginal profit for firm 2 is:

$$\frac{\partial \pi_{2t}^{TSC}}{\partial q_{2t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_t)^{-D} + c^{min}]$$

Equation of output produced by firm 1 ($q_{1t}; x5$:)

$$q_{1t+1} = q_{1t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c^0(1+w_t)^{-D} + c^{min}]\}$$

Equation of output produced by firm 2 ($q_{2t}; x6$):

$$q_{2t+1} = q_{2t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{2t}+q_{1t}\right] - [c^0(1+w_t)^{-D}+c^{min}]\}$$

Equation of aggregate quantity ($Q_t; x1$):

$$Q_{t+1} = Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] - [2c^0(1+w_t)^{-D}+2c^{min}]\}$$

Price equation ($p_t; x2$):

$$p_{t+1} = \frac{A}{\{Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] - [2c^0(1+w_t)^{-D}+2c^{min}]\}\}^{\beta}}$$

The cumulative output equation for firm 1 is given by ($w_t; x7$):

$$w_{t+1} = w_t + q_{1t} + q_{2t}$$

A.1.4 Full Collusion

Demand function is given by:

$$P(Q_t) = \frac{A}{Q_t^{\beta}}$$

Firm1 profit function is :

$$\pi_{1t} = \frac{Aq_{1t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_{1t}+\alpha w_{2t})^{-D}+c^{min}]q_{1t}$$

Firm 2 profit function is :

$$\pi_{2t} = \frac{Aq_{2t}}{(q_{1t}+q_{2t})^{\beta}} - [c^0(1+w_{2t}+\alpha w_{1t})^{-D}+c^{min}]q_{2t}$$

Total profit function ($\pi_t; x4$):

$$
\begin{aligned}
\pi_t &= \pi_{1t} + \pi_{2t} = \\
&= \frac{Aq_{1t}}{(q_{1t}+q_{2t})^\beta} - [c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^{min}]q_{1t} + \\
&\quad + \frac{Aq_{2t}}{(q_{1t}+q_{2t})^\beta} - [c^0(1+w_{2t}+\alpha w_{1t})^{-D} + c^{min}]q_{2t} = \\
&= \frac{AQ_t}{(Q_t)^\beta} - [c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^{min}]Q_t
\end{aligned}
$$

Marginal profit for firm 1 is:

$$\frac{\partial \pi_t^C}{\partial Q_t} = A(1-\beta)(Q_t)^{-\beta} - [c^0(1+w_{1t}+\alpha w_{2t})^{-D} + c^{min}]$$

Equation of output produced by firm 1 ($q_{1t}; x5$:)

$$q_{1t+1} = \{Q_t + v\{A(1-\beta)(Q_t)^{-\beta} - [c^0(1+w_t)^{-D} + c^{min}]\}/2$$

Equation of output produced by firm 2 ($q_{2t}; x6$):

$$q_{2t+1} = \{Q_t + v\{A(1-\beta)(Q_t)^{-\beta} - [c^0(1+w_t)^{-D} + c^{min}]\}/2$$

Equation of aggregate quantity ($Q_t; x1$):

$$Q_{t+1} = Q_t + v\{A(1-\beta)(Q_t)^{-\beta} - [c^0(1+w_t)^{-D} + c^{min}]\}$$

Price equation ($p_t; x2$):

$$p_{t+1} = \frac{A}{\{Q_t + v\{A(1-\beta)(Q_t)^{-\beta} - [c^0(1+w_t)^{-D} + c^{min}]\}^\beta}$$

The firm 1 profit function is ($\pi_{1t}^C; x3$):

$$\pi_{1t}^{TSC} = q_{1t}p_t - [c^0(1+w_t)^{-D} + c^{min}]q_{1t}$$

$$\pi_{1t}^{C} = \{Q_t p_t - [c^0(1+w_t)^{-D} + c^{min}]Q_t\}/2$$

Firm 2 profit function is($\pi_{2t}^{C}; x4$):

$$\pi_{2t}^{C} = \{Q_t p_t - [c^0(1+w_t)^{-D} + c^{min}]Q_t\}/2$$

The cumulative output equation for firm 1 is given by ($w_{1t}; x7$):

$$w_{1t+1} = w_{1t} + q_{1t}$$

The cumulative output equation for firm 2 is given by ($w_{2t}; x8$):

$$w_{2t+1} = w_{2t} + q_{2t}$$

A.2 The Asymmetric Case (but with $v_1 = v_2 = v$)

A.2.1 Duopoly with Spillovers

Demand function is given by:

$$p(Q_t) = \frac{A}{Q_t^{\beta}}$$

The firm 1 profit function is ($\pi_{1t}^{DS}; x3$):

$$\pi_{1t}^{DS} = q_{1t} p_t - [c_1^0(1 + w_{1t} + \alpha_1 w_{2t})^{-D_1} + c_1^{min}]q_{1t}$$

$$\pi_{1t}^{DS} = \frac{A q_{1t}}{(q_{1t} + q_{2t})^{\beta}} - [c_2^0(1 + w_{1t} + \alpha_2 w_{2t})^{-D_1} + c_2^{min}]q_{1t}$$

Firm 2 profit function is($\pi_{2t}^{DS}; x4$):

$$\pi_{2t}^{DS} = q_{2t} p_t - [c_2^0(1 + w_{2t} + \alpha_2 w_{1t})^{-D_2} + c_2^{min}]q_{2t}$$

$$\pi_{2t}^{DS} = \frac{Aq_{2t}}{(q_{1t}+q_{2t})^{\beta}} - [c_2^0(1+w_{2t}+\alpha_2 w_{1t})^{-D_2} + c_2^{min}]q_{2t}$$

Marginal profit for firm 1 is:

$$\frac{\partial \pi_{1t}^{DS}}{\partial q_{1t}} = \frac{A(q_{1t}+q_{2t})^{\beta} - Aq_{1t}\beta(q_{1t}+q_{2t})^{\beta-1}}{(q_{1t}+q_{2t})^{2\beta}} - [c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_1^{min}]$$

$$\frac{\partial \pi_{1t}^{DS}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(q_{1t}+q_{2t}) - \beta q_{1t}\right] - [c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_1^{min}]$$

$$\frac{\partial \pi_{1t}^{P}}{\partial q_{1t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_1^{min}]$$

Symmetrically, marginal profit for firm 2 is:

$$\frac{\partial \pi_{2t}^{DS}}{\partial q_{2t}} = A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c_2^0(1+w_{2t}+\alpha_2 w_{1t})^{-D_2} + c_2^{min}]$$

Equation of output produced by firm 1 ($q_{1t}; x5$:)

$$q_{1t+1} = q_{1t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{1t} + q_{2t}\right] - [c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_1^{min}]\}$$

Equation of output produced by firm 2 ($q_{2t}; x6$):

$$q_{2t+1} = q_{2t} + v\{A(q_{1t}+q_{2t})^{-(\beta+1)}\left[(1-\beta)q_{2t} + q_{1t}\right] - [c_2^0(1+w_{2t}+\alpha_2 w_{1t})^{-D_2} + c_2^{min}]\}$$

Equation of aggregate quantity ($Q_t; x1$):

$$Q_{t+1} = Q_t + v\{A\left[(2-\beta)q_t^{-\beta}\right] + \\ -[c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_2^0(1+w_{2t}+\alpha_2 w_{1t})^{-D_2} + c_1^{min} + c_2^{min}]\}$$

Price equation $(p_t; x2)$:

$$p_{t+1} = \frac{A}{\{Q_t + v\{A\left[(2-\beta)Q_t^{-\beta}\right] + -[c_1^0(1+w_{1t}+\alpha_1 w_{2t})^{-D_1} + c_2^0(1+w_{2t}+\alpha_2 w_{1t})^{-D_2} + c_1^{min} + c_2^{min}]\}\}^{\beta}}$$

The cumulative output equation of the cartel is given by $(w_{1t}; x7)$:

$$w_{t+1} = w_t + Q_t$$

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