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Abstract

We analyse the effects on industry structure of non strategic learning by doing with spillovers in a differentiated oligopoly *à la* Bertrand. The dynamics is driven by a non linear learning curve. Conditions for shakeouts are analysed, focusing on the key factors affecting them. Policy interventions to limit shakeouts are suggested.

Keywords: Industry dynamics, Non linear learning curve, Spillover, Shakeouts.

JEL Classification: L11, L13, O31.

1 Introduction

The economic literature provides empirical evidence of how learning by doing and spillovers shape the industrial structure (Zimmerman (1982), Lieberman (1989), Foster and Rosenzweig (1995)). Consequently, the analysis of the effects of learning by doing and spillovers has emerged as an important research topic for consideration of industrial policies. In this paper, following Jin, Perota-Pena and Troege (2004), we present and characterize the dynamic behaviour of a simple model representing a differentiated oligopoly *à la* Bertrand whose dynamics is driven by non strategic learning by doing and spillovers. The model of Jin *et al.* has the merit of specifying with surprisingly simple equations the dynamic evolution of this industry. However, the learning process is very stylised. They assume a linear learning curve with spillovers. This assumption does not allow to consider the diminishing returns to scale which typically characterise the learning process and moreover does not identify the crucial role of the asymptotic value of the cost function in the dynamic outcome of the system. We depart from the paper by Jin, Perota-Pena and Troege (2004) in two ways. First, we propose a nonlinear dynamic cost function that enable us to find rigorously a positive stationary point as the asymptotic outcome of the market evolution. Second, the dynamic cost function has a richer structure in the sense that considers the rate of cost decreasing, the asymptotic value of marginal cost function and the initial cost value. This richer specification of the learning curve allows us to determine rigorously all the key factors affecting the industry evolution. Particularly, we find a short cut condition for the occurrence of shakeouts.

The structure of the paper is as follows. In section 2 we recall the Jin, Perota-Pena and Troege model. In section 3, we analyse the dynamic outcome of the system assuming a non linear learning curve with spillovers.

2 The Jin, Perota-Pena and Troege Model

For the convenience of the reader, we recall the demand function by Jin, Perota-Pena and Troege model (2004). The setting is characterised by n firms competing *à la* Bertrand in a differentiated market with n products $(x_1, ..., x_n)$ and a numeraire good (x_0). As usual, the price of the numeraire good is normalised to 1. Firm i demand function in each period t is obtained solving the following consumer maximisation problem:

$$\max_{x_{it}}\{x_{0t}+A\sum_{i=1}^{n}x_{it}-0.5\delta\sum_{i=1}^{n}x_{it}^{2}-r\delta\sum_{i=1}^{n}x_{it}\sum_{j\neq i}^{n}x_{jt}\}$$

$$s.t.\sum_{i=1}^{n}p_{it}x_{it}+x_{0t}=y,$$

where $A,\delta>0$, x_{it}, p_{it} denote respectively firm i's output and price in period t, y the representative consumer income, $0<r<1$ the degree of substitutability between goods. The extreme cases of perfect substitutability between goods ($r=1$) and complete independence ($r=0$) are excluded. Each firm thus in each period is facing the demand function:

$$x_{it}=\frac{A}{[1+(n-1)r]\delta}-\frac{p_{it}}{(1-r)\delta}+\frac{r}{(1-r)[1+(n-1)r]\delta}\sum_{j\neq i}^{n}p_{jt}.$$

Each oligopolist is playing a Bertrand game in each period, choosing p_{it} in order to maximise the current profit: $\Pi_{it}=x_{it}(p_{it}-c_{it})$ where c_{it} is firm i's marginal cost.

Solving the oligopolist first order conditions, the equilibrium price is obtained:

$$\begin{aligned}p_{it} &= \frac{(1-r)A}{2+(n-3)r}+\frac{[1+(n-2)r]c_{it}}{2+(2n-3)r}+\\ &+\frac{r[1+(n-2)r]}{[2+(n-3)r][2+(2n-3)r]}\sum_{j=1}^{n}c_{jt}.\end{aligned}$$

Substituting this equilibrium value in the demand function, the equilibrium output in each period is given by:

$$x_{it}=\phi A+\frac{\lambda-\phi}{n}\sum_{j=1}^{n}c_{jt}-\lambda c_{it}, \tag{1}$$

where $\phi=[1+(n-2)r]/\{[2+(n-3)r][1+(n-1)r]\delta\}$ and $\lambda=[1+(n-2)r]\{2+(n-2)r]/\{[2+(2n-3)r](1-r)\delta\}$.

3 Non Linear Learning Curve With Spillovers

In (1), firm's current output is linked to the level of current costs. To describe the dynamic evolution of the system, another equation linking the level of current costs to the previous output is required. The learning curve provides this additional equation. In contrast with Jin *et al.*, we are not assuming a linear relation between current cost and own or total industry output in the previous period, but a more complex cost dynamic behaviour. The learning curve that we are using in this work is given by:

$$c_{it} = c_{it-1}(c_{io} + a_i x_{it-1} + b_i \sum_{j=1}^{n} x_{jt-1})^{-D_i} + c_i^{\min}, \tag{2}$$

where D_i is the rate of cost decreasing, $c_i^{\min}$ is the asymptotic value of the marginal cost function, c_{io} is the initial marginal cost value, a_i and b_i denote respectively the intensity of the learning by doing and of the spillover effect. We propose this functional dependence since it is able to describe the most general non linear behaviour, allowing for diminishing returns and asymptotic value of the cost function.

Industry dynamics is given by equations (2) and (1). Plugging (2) in (1), we get the following system of n difference equations:

$$c_{it} = c_{it-1}(c_{io} + \Psi_i + \Omega_i \sum_{j=1}^{n} c_{jt-1} - \lambda a_i c_{it-1})^{-D_i} + c_i^{\min} \qquad i = 1, 2, ..., n, \tag{3}$$

where $\Psi_i = \phi A(a_i + b_i n)$ and $\Omega_i = [a_i(\frac{\lambda - \phi}{n}) - b_i \phi]$. Let us define $\mathbf{c}_0 = [c_{10}, ..., c_{n0}]$ as the initial condition vector.

Proposition 1 *If each* $c_i^{\min}$ *is sufficiently small, the dynamic system (3) exhibits two equilibria given by:* $\mathbf{c}^* = [c_1^*, ..., c_n^*]$ *and* $\mathbf{c}^{**} = [c_1^{**}, ..., c_n^{**}]$ *such that* $c_i^* < c_i^0 < c_i^{**}$, $\forall i = 1, ..n$*; with* $c_i^* = 0$ *if* $c_i^{\min} = 0$ *and* $c_i^* > c_i^{\min}$ *if* $c_i^{\min} > 0$.

Proof. The LHS of each equation of (3) is the 45° line. The RHS is a continuous convex increasing function with $c_i^{\min}$ as vertical intercept and a vertical positive asymptote. If $c_i^{\min}$ is such small that the imagine of c_{io} is less then c_{io}, since $c_i^{\min} < c_0$, by the continuity of the function there must be two fixed points $(\mathbf{c}^*, \mathbf{c}^{**})$ and the initial condition vector must be always between the two fixed point vectors. If $c_i^{\min} = 0$, one of the zeros of the system coincides with the origin. ■

Proposition 2 *Under the conditions stated in Proposition 1, for each* c_{io} *the dynamic system (3) converges to the stationary state* $\mathbf{c}^*$.

Proof. Each equation defines a sequence of maps converging to an asymptotic map with the two equilibria $\mathbf{c}^*, \mathbf{c}^{**}$. Since (3) is a monotone decreasing bounded sequence, then it converges to the lower stationary state $\mathbf{c}^*$. ■

Proposition 3 *In* $\mathbf{c}^*$*, increasing levels of the parameters* a_i*,* b_i*,* D_i *and* A *decrease the value of the stationary state* $\mathbf{c}^*$*, while increasing levels of* $c_i^{\min}$ *decrease the value of* $\mathbf{c}^*$.

Proof. A graphical proof is given for this proposition. As previously said, $\mathbf{c}^*$ is given by the intersection of the 45° line and the map defined by the system (3). Increasing levels of the $c_i^{\min}$ increase the positive value of the vertical intercept. Increasing levels of a_i, b_i, D_i and A decrease the value of $\mathbf{c}^*$. ■

In the following figures (1-3), we present simulations of the system dynamics in the duopoly case, assuming different asymptotic values of firm marginal cost (firm 2 is always the advantaged firm). We set the demand parameters as $A = 10$, $\lambda = 0.5$ and $\phi = 0.3$. Changes in the values of these demand parameters do not affect qualitatively the results. Figure 1 represents the effect of different rates of cost decreasing (D_i) on the long run cost behaviour. We show the case of $D_i = 0.3$ and $D_i = 1$ with $c_1^{\min} = 1.5$ for the less advantaged firm and $c_2^{\min} = 0.5$ for the other firm. An increase in D_i lowers the cost stationary value for both firms, even if the level of the cost asymptotic value is never reached. Whichever is the value of D_i, the cost level of the stationary state is bigger than its asymptotic value.

Figures 2 and 3 represent respectively the effect of different learning by doing and spillover parameters on the dynamic behaviour of the system. Increasing levels of both parameters lower the cost in each period. In our model, thus, spillovers are definitely beneficial to the system. Even with a more complex learning curve, we confirm the results of beneficial spillover effects obtained with non strategic learning and linear learning curves (see Jin et al. (2004)). Since also in our model learning is passive (it is driven by quantity decision of the previous period), the usual trade-off between incentives and cost reduction of strategic learning models does not occur

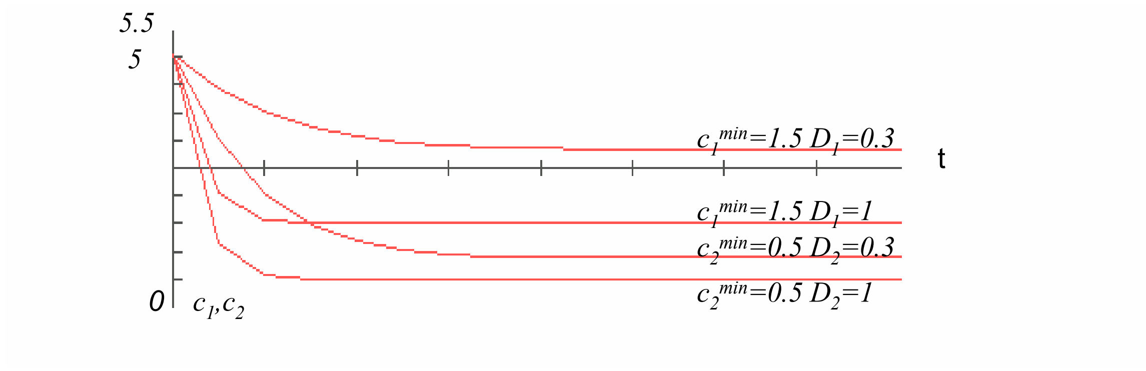

Figure 1: The effect of different rates of cost decreasing on the value of the stationary state $\mathbf{c}^*$ (firm 2 advantaged).

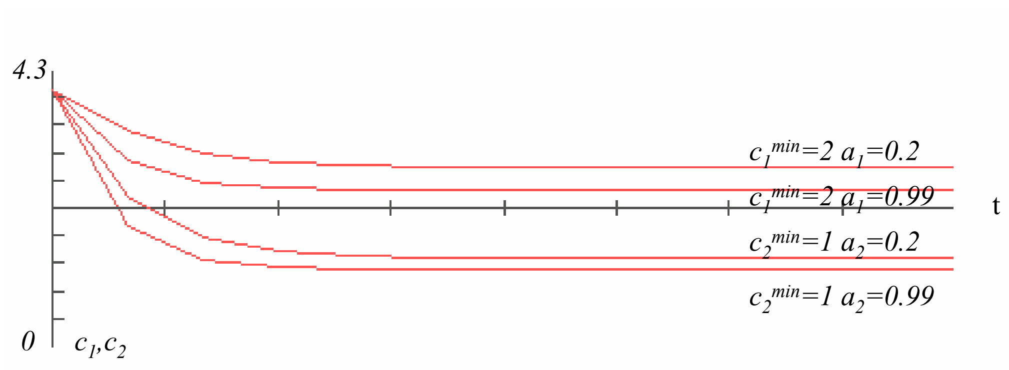

Figure 2: The effect of the different own learning by doing coefficients on the level of the stationary state $\mathbf{c}^*$ (firm 2 advantaged).

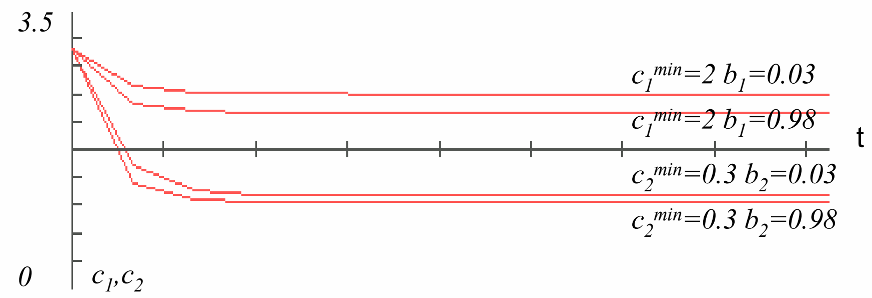

Figure 3: The effect of different spillover parameters on the stationary level of $\mathbf{c}^*$ (firm 2 advantaged).

(see, for example, Fudenberg and Tirole (1983) or Ghemawat and Spence (1985)). In order to improve welfare, government should encourage information exchange between firms. Specifically, it should promote the formation of technological consortia. A technological sharing agreement will improve the efficiency of the production process of both firms and will reduce costs. Also the learning parameter a_i has a straightforward effect on dynamics behaviour. Costs decrease as a_i increases. In contrast with the result of the linear cost case, the effect of the learning-by-doing parameter does not seem to have difficult interpretation in our case. Whichever is the entity of the spillover effect, the learning-by-doing coefficient will ever improve firm performance.

Finally, we analyse the effect of learning by doing and spillover effects on industry shakeouts. The index $\overline{c} = \sum_j^n c_j^*/n$ measures the average stationary cost level of the industry. It synthesises the level of efficiency of the system. Let us define $\mu = \frac{\phi}{\lambda}$ and $\xi = \frac{(\lambda-\phi)}{\lambda}$.

Proposition 4 *If $c_i^* < \mu A + \xi\overline{c}$ shakeouts never occur in the system.*

Proof. Firm i's market share (s_i) is given by:

$$s_i = \frac{\phi A + (\frac{\lambda-\phi}{n})\sum_j^n c_j^* - \lambda c_i^*}{n\phi A - \phi\sum_j^n c_j^*}$$

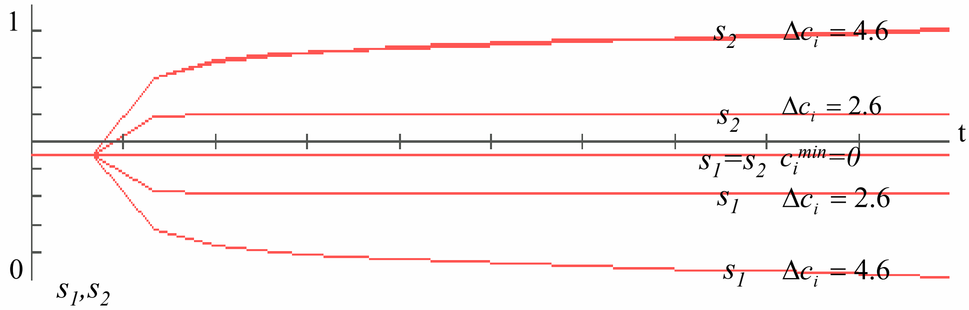

Figure 4: Industry dynamics under different cost gaps

Since $(n\phi A - \phi \sum_j^n c_j^*) > 0$, $s_i > 0$ if $c_i^* < \mu A + \xi \overline{c}$. ■

Two factors influence the occurrence of shakeouts in the system: market size and the general level of efficiency of the system. Higher the size of the market (higher value of A), lower the probability that the firm exits the market. Indeed, learning-by-doing activities and spillover transmissions are magnified and amplified with large markets. As far as the system efficiency is concerned, lower levels of $\overline{c}$ increase the probability of shakeouts. More efficient the system, lower the probability for the less advantaged firm to survive in the competition process. Information transmission should be encouraged by government intervention. Increasing values of the learning-by-doing and the spillover effect increase the efficiency of the single firm relatively to the others, reducing the possibility of concentration in the system.

Corollary 5 *A market converges to equal shares if* $c_i^{\min} = 0$, $i = 1, ..., n$ *whichever are the values of the other parameters in the system.*

Proof. The proof is straightforward. Since

$$s_i = \frac{\phi A + (\frac{\lambda - \phi}{n}) \sum_j^n c_j^* - \lambda c_i^*}{n\phi A - \phi \sum_j^n c_j^*}$$

if $c_i^{\min} = 0$ then $c_i^* = 0$ and so $s_i = 1/n$. ■

In figure 4, we show how the firm cost comparative condition affects the industry evolution of a duopoly in which the source of asymmetry is only due to the difference in the asymptotic cost value. We observe that if one firm is in a quite favorable cost condition with respect to the other (cost gap is sufficiently great) only the most efficient firm survives (firm 2). If, instead, this gap is relatively small, shakeout does not occur. In the case that the asymptotic cost conditions are equal, all firms survive in the market with equal shares. This result holds whichever is the magnitude of the spillover effect, the learning-by-doing parameters, the market size. Qualitatively, the same market evolution occurs when the entity of the cost gap is due to asymmetries in other parameters affecting the efficiency of firms.

4 Conclusion

We analysed the dynamic behaviour of a differentiated oligopoly *à la* Bertrand with a non linear learning curve and spillovers. The learning process and the spillovers are non strategic. The non linearity of the learning curve contributes to clarify the role of the cost asymptotic value on the industry structure. We show the conditions for the system to exhibit two fixed points. The industry approaches the lower stable equilibrium, whichever are the initial conditions. We study the key factors affecting the level of this stationary point. We show that learning by doing activities, transmission of information among firms, higher rate of cost decreasing improve firm performance. Moreover, we analyse the conditions for the occurrence of shakeouts in the system. These conditions are strictly connected to the market size and the comparative cost condition of the single firm in the market. Information exchange is beneficial to the system since it prevents market concentration by reducing the probability of shakeouts. We suggest that policy prescriptions such as technological sharing agreements should be encouraged and welcomed.

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